



Production, Manufacturing, Transportation and Logistics

Gain-sharing in urban consolidation centers

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ARTICLE INFO

Article history:

Received 26 February 2018

Accepted 22 May 2019

Available online 28 May 2019

Keywords:

Game theory

Last mile logistics

Urban consolidation centers

Gain-sharing

Core

ABSTRACT

Urban consolidation centers provide the logistical infrastructure for cooperation among less-than-truckload carriers with contiguous destinations. The rising number of initiatives to establish and operate urban consolidation centers and their low success rates signal the need for better mechanisms to manage cooperation in this context. We introduce and study cooperative situations comprising a set of carriers with time sensitive deliveries who can consolidate their cargo to obtain savings. We introduce the class of Dispatch Consolidation (DC) games and search for ways to fairly allocate the obtained savings among the participating carriers. When delivery capacities are not restrictive, i.e. when waiting costs trigger truck dispatches, we show that stable allocations in the core always exist and can, in their entirety, be found by solving a compact linear program. With restrictive capacities, however, the core of a DC game may become empty. We introduce the notion of component-wise core for DC games to preserve stability first and foremost among the carriers whose deliveries are dispatched together in the chosen optimal solutions. The novelty of our approach is to link the stability requirements of an allocation rule with the structure of selected solutions for the underlying optimization problems. We characterize the component-wise cores of DC games, prove their non-emptiness, and suggest proportionally calculated allocations therein. Finally, we discuss a refinement of component-wise core allocations that minimizes envy among the carriers who are dispatched separately.

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1. Introduction

The outlook of logistics and transportation industry presents one of the major paradoxes of our times. The ever increasing need for better, cheaper, and more responsive products and services drives the industry towards growth and efficiency on both local and global scales. On the other hand, modern life has never been so grappled with problems of pollution, congestion, and a myriad of environmental issues that are negatively impacted by the logistics and transportation sector. Road transportation alone is responsible for more than 20% of total carbon emissions in [European Commission \(2017\)](#). At the same time, more than 20% for all truck movements in Europe is completely empty ([Eurostat, 2018](#))—and the remainder is hardly ever full. Despite the fruitfulness of research on optimizing routes, schedules, and networks for individual organizations involved in transport and logistics, the next level of efficiency obtained by increasing the

economies of scale can only be made possible via collaboration among individual operators.

Although collaboration can positively affect almost all aspects of the transportation and logistics sector ([Newing, 2008](#)), an increasingly promising context for collaborative logistics is consolidation centers. A consolidation center is a logistical facility that is used to combine loads of various carriers and to construct delivery plans that are either more economical (e.g. via better utilized trucks) or have higher service levels (e.g. via faster turnouts) ([Morana, Gonzalez-Feliu, & Semet, 2014](#)). A recent study commissioned by Transport For London finds that the use of Construction Consolidation Centers can reduce freight traffic to construction sites by over 70% ([Mayor of London, 2016](#)). By 2005, there has been over 60 documented cases of consolidation centers in Europe ([Browne, Sweet, Woodburn, & Allen, 2005](#))—though with various levels of success. [Allen, Browne, Woodburn, and Leonardi \(2012\)](#) report 114 documented implementation cases of consolidation centers in 17 countries by 2012. There are several pieces of evidence showing that consolidation centers are heavily supported by governments and urban authorities to remedy increasing logistical side-effects

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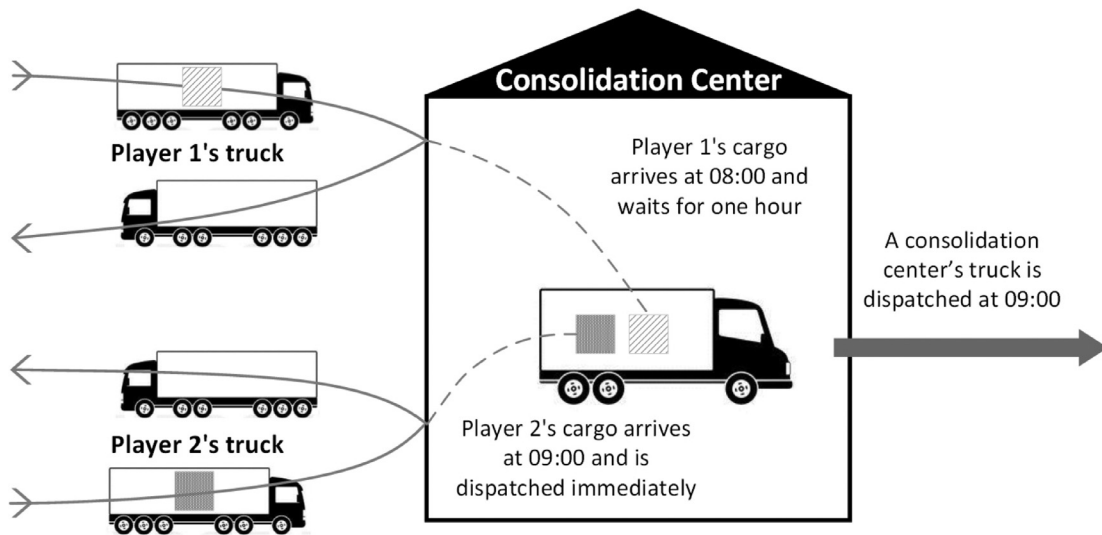


Fig. 1. Activities at a consolidation center.

of congestion and pollution (Duin, Quak, & Muñuzuri, 2010; Hoyer, Slikker, & Van Woensel, 2012; Paddeu, 2017).

Given the potential benefits of consolidation centers and the level of support that they receive, one would expect to see their successful and sustained operations all around the world. But this is not the case. Quak and Tavasszy (2011) report that among more than 100 initiatives in urban logistics collaborations, more than half of them fail during implementation. One of the main barriers to success of consolidation centers is the deficiency of the mechanisms they use to share the obtained savings among, or cover the incurred costs from, the participants. In fact, carriers may hesitate to collaborate as long as they do not have a clear understanding of the mechanisms employed and whether or not they receive a fair share out of collaborative operations. Nordtømme, Bjerkan, and Sund (2015) report, as one of the main success barriers for Oslo's consolidation center, that "there was no plan for how costs would be financed and who would benefit from saved costs". The lack of consensus on fair cost/gain sharing schemes as a barrier to collaboration in wider logistics context is empirically confirmed by Cruijssen, Cools, and Dullaert (2007). As a result, consolidation centers are often destined to disappear when governmental subsidies are lost (Verlinde, Macharis, & Witlox, 2012).

In this paper, we construct a model to study collaboration among carriers via urban consolidation centers. The carriers have deliveries that are destined for the same area (e.g. a city center or a commercial/construction site). Instead of driving to their destinations, the carriers' trucks can arrive at the consolidation center, unload their cargo, and move on to carry out their other fulfillments. Their cargo would sit in the consolidation center in order to be bundled into full-truck loads (see Fig. 1). The amounts of savings that the carriers obtain are dependent on their dispatch times, i.e. the deliveries are time-sensitive. This is in line with previous studies emphasizing that the main costs of deliveries in the consolidation centers are time-related costs (Janjevic & Ndiaye, 2017). To materialize the full benefits of load consolidation, collaborating parties delegate decision making authorities regarding bundling and dispatching of their cargo to a consolidation center operator.¹ Having the delivery information of all carriers, the consolidation center operator in our model first determines the set of carriers

whose deliveries are accepted to be handled by the consolidation center. Incorporating such selection option allows for the exclusion of deliveries that cannot be profitably consolidated—for example, when a carrier's delivery size is already close to a full-truckload. The consolidation center operator then decides the dispatch times of accepted deliveries. Finally, savings allocated to each carrier are determined. As we show, the problem of allocating the savings obtained by collaboration in the consolidation center is equivalent to determining the players' shares of dispatched trucks' costs. Knowing the rules of the game, players decide whether or not to collaborate with others in consolidating their loads, and if so who they are willing to collaborate with.

Finding appropriate gain/cost-sharing methods is the main focus of this paper. This problem is extensively studied within the framework of cooperative game theory (see for example Peleg & Sudhölter, 2007). One of the most important gain-sharing rules in cooperative game theory literature is the core (Shapley, 1955).² Based on the notion of stability, allocations in the core of a cooperative game distribute the total savings obtained by cooperation in a way that sub-groups of players, also called coalitions, cannot object to their combined allocated savings being less than what they could achieve on their own. The latter requirement for allocations have several other desirable properties as well (see for example Peleg, 1992). One of the practically appealing properties of core allocations is their efficiency with regard to the set of players that positively contribute to realization of total savings. That is, if a coalition of players together generates the entire savings, then every allocation in the core distributes the total savings exclusively among those players. Thus, the players whose exclusion does not reduce the savings are allocated with no additional savings (the so-called dummy property Peleg & Sudhölter, 2007). The example below illustrates this point.

Example 1. Three carriers, each with half a truck-load cargo, must deliver to a city center. An urban consolidation center provides opportunities for combining cargo and dispatching full truck-loads. It would not make economical sense for a carrier to come to the consolidation center and be dispatched individually in a half-full truck. As the result, only two carriers would be selected to arrive at the consolidation center and one has to transport his cargo

¹ The execution of collaboration via a decision making entity is common and can be seen, for example, in cooperative logistics (Özener & Ergun, 2008) and cooperative procurement (Hezarkhani & Sošić, 2018).

² It is worth mentioning that although the literature often associates the definition of the core to Gillies (1959), as shown by Zhao (2018), it was Shapley (1955) who first defined the core in its current form.

directly. Assume that carriers 1 and 2 as well as carriers 2 and 3 can combine their cargo together and obtain 60 units of savings. Carriers 1 and 3, arriving too far from each other to the consolidation center, cannot gain any savings from combining their cargo. Suppose eventually carriers 1 and 2 are selected to consolidate their cargo, and carrier 3, not being accepted to arrive at the consolidation center, transports his delivery directly. The (unique) stable allocation in the core gives 60 units of savings completely to carrier 2 and leaves other two players with zero allocated saving.

The stable allocation in example above does not allocate any savings to the carrier who is not selected to arrive at the consolidation center. If the latter condition is violated, carriers 1 and 2—whose cargoes are eventually dispatched together via the consolidation center—can object to the idea of giving away a positive part of their jointly generated savings to carrier 3.

Despite the desirable features of core allocations, it is not always possible to find allocations in the core. The following example exhibits this.

Example 2. In Example 1, assume that carriers 1 and 3 can also gain 60 units of saving by combining their cargo into a full truckload. The consolidation center operator has to choose between the three possible pairs of carriers to arrive at the facility. Suppose, again, that eventually carriers 1 and 2 are selected to consolidate their cargo. No matter how the 60 units of savings are distributed among the three carriers in this case, two of them will receive less than what they can potentially gain together. Thus, the core is empty.

How should the savings be allocated among the carriers in the above example? One might argue, as heard in our discussions with industry practitioners, that in the above situation the entire savings must (similar to the outcome of core allocations in Example 1) be completely distributed among carriers 1 and 2 simply because, as part of the selected decision for the cooperative situation, they are responsible for realization of total savings. Although the carriers seem to be symmetric before a particular solution is chosen for the system (every pair has the same potential in creating the savings), once a decision to include 1 and 2 and exclude 3 is made, the carriers may no longer be treated in a similar way. But there seems to be no solution in cooperative game theory literature that can accomplish the latter requirement.

In situations where the core of a game can be empty, cooperative game theory literature suggests alternative allocation rules that adopt a weaker notion of stability in order to suggest ways to share the gains. The least core (Maschler, Peleg, & Shapley, 1979), for instance, yields allocations wherein the maximum objection (instability) over all coalitions are minimized. The nucleolus (Schmeidler, 1969), as another instance, further refines the least-core by singling out a unique allocation wherein the objections of all coalitions are lexicographically minimal. Although both of these allocation rules incorporate the notion of stability to some extent and resolve the problem of existence, they do not necessarily distribute the savings among the players who are eventually responsible for creating the total savings. In Example 2, both the least core and the nucleolus suggest equal allocations for the three players, that is, 20 units of savings to each carrier. Subsequently, although cooperation among carriers 1 and 2 eventually brings about all the savings, carrier 3 also receives an equal share. The reason for this is that the available allocation rules in cooperative game theory are not necessarily linked with the final course of action in a situation and base their recommendations for allocated savings on the players' *potential*, not what they eventually do in a cooperative situation. In other words, with the classical allocation rules there is a disconnection between the allocation of gains and the chosen optimal solutions.

An important and well-studied allocation rule in cooperative game theory literature is the Shapley value (Shapley, 1953). The Shapley value distributes the savings by averaging what each player can contribute to the savings obtained in all coalitions (in all different permutations of arrivals). Unlike allocations in the core, the Shapley value may assign savings to players that might be inconsequentially removed from the situation. In Example 1 above, the Shapley value allocates 10 units of savings to both carriers 1 and 3, and 40 units to player 2. Thus, even though player 3 is not selected to arrive at the consolidation center, he receives as much profit as player 1. In Example 2, the Shapley value also prescribes the equal allocations: each carrier is allocated with 20. Again, this is because the Shapley value considers the potential contributions of the players and disregard the selected decision for a cooperative situation.

In light of the discussions above, there seems to be a gap in the literature regarding the existence of allocation rules that explicitly consider the selected decisions in cooperative situations. In this paper, when encountered with situations whose associated games have empty cores, we propose an alternative approach to incorporate the notion of stability in an *ex post* sense while taking into account the selected optimal decisions. In this way, we prioritize the stability of different coalitions based on the selected solution for the underlying optimization problem. To the best of our knowledge, this approach has never been considered for operations research games.

1.1. Contributions of this paper

In this paper, we introduce and study Dispatch Consolidation (DC) games as a new class of cooperative games associated with logistics and operations research situations. The main results of the paper are fourfold:

1. For DC games with non-restrictive capacities, i.e., when truck capacities do not impose any restriction on dispatching decisions and waiting costs trigger truck dispatches, we prove non-emptiness of their cores. We provide a complete characterization of the core by means of a linear program which has up to $n(n+1)/2$ constraints (n being the number of players in the system). This is significant because the generic linear program for obtaining core has up to $2^n - 1$ constraints.
2. We show that the capacity restricted version of the game can easily have an empty core. In this case, we introduce the component-wise core as a weaker notion of stability for DC games which imposes the no-objection requirement only for subsets of players in the same dispatched truck in the selected solution for the underlying optimization problem.
3. We prove the non-emptiness of component-wise cores of DC games in general. We provide a complete characterization of the component-wise cores of DC games—drawing upon a key property of component-wise stable allocations in DC games, which requires later arriving players to compensate the delay they cause for the earlier players in the same dispatch. To calculate an allocation in the component-wise core, we introduce an algorithm which uses a sequential procedure to distribute the cost of a dispatched truck among the carriers involved based on adjusted proportions of the benefits they receive.
4. We introduce the notion of envy-freeness for the allocations, pertaining to any player's willingness to swap places with some other player, and show that although core allocations are always envy-free, in DC games with restrictive capacities envy-free allocations cannot always be found. We formulate a linear program, in a similar vein as the least

core (Maschler et al., 1979), that finds allocations within the component-wise cores of DC games which minimize the maximum envy of players that are dispatched in separate trucks.

The rest of this paper is organized as follows. In Section 2, we briefly overview the relevant literature. In Section 3, we formally introduce the model for dispatch consolidation situations, discuss the underlying optimization problems, and formulate their associated cooperative games, i.e. DC games. The gain sharing problem in this context is presented in Section 4. Games with non-restrictive capacities are analyzed in Section 5 where we provide the main result regarding the non-emptiness of their cores. In Section 6, the notion of component-wise core is introduced to address stability in games with restrictive capacities. We provide a complete characterization of the component-wise cores of DC games in Section 7 and present a proportional allocation rule therein subsequently in Section 8. The notion of envy-freeness is introduced in Section 9 along with a procedure to obtain allocations in the component-wise core with minimum envy. Finally, Section 10 concludes the paper. All proofs are given in the Supplementary Materials.

2. Literature review

In the centralized setting, optimization problems at consolidation centers have been subject to several studies with various levels of details. In a deterministic system, Daganzo (1988) studies consolidation strategies of shipments from several departures with a common single destination. When arrivals at consolidation centers are stochastic, Çetinkaya and Bookbinder (2003) find optimal dispatch policies at a consolidation center while incorporating the time sensitive nature of cargo. Çetinkaya (2005) provides an overview of literature in this area. More recently, van Heeswijk, Mes, and Schutten (2017) propose an approximate dynamic program to plan dispatches at an urban consolidation center with random arrivals, heterogeneous fleet, multiple destinations, and options for spot market transport. Savelsbergh and Van Woensel (2016) review the literature on routing problems in the context of city logistics.

Given the nature of consolidation centers as a point of aggregating the cargo for different parties, the decentralized view of the problem is an indispensable part of the analysis. Zhou, Hui, and Liang (2011) study collaboration through a consolidation center between two players with a common destination and examine how different alliance settings could affect the performance of the system. Using a mechanism design approach, Zhang, Uhan, Dessouky, and Toriello (2016) study the application of Moulin cost-sharing mechanism (Moulin & Shenker, 2001) in a consolidation center used by carriers with small-sized and time-insensitive cargo. In an auction setting, Handoko, Nguyen, and Lau (2014) address the winner determination problem for the consolidation center's operator to decide how to accept bids from the carriers who are willing to use the service. van Heeswijk, Larsen, and Larsen (2019) present an agent-based simulation framework to evaluate the impact of urban logistics choices on the individual stakeholders in a case study of an urban consolidation center in the city of Copenhagen. Along the same lines, van Heeswijk, Mes, and Schutten (2016) look into urban logistics problems exploring coordination and collaboration among five types of autonomous agents (receivers, shippers, carriers, urban consolidation center and administrator) all with diverging interests and stakes. Nguyen, Dessouky, and Toriello (2014) propose a heuristic for consolidation of perishable agricultural products with stochastic demands and suggest a simple proportional rule to share the costs among participating suppliers. However, there is little research on the requirements for fairness

among carriers which motivates the applications of cooperative game theory in the context of consolidation centers.

In order to deal with the gain/cost allocation problems in logistics and transportation context, many authors have proposed the adoption of well-known allocation rules of cooperative game theory. The most investigated solution so far is the core. Özener and Ergun (2008) study a class of cooperative truckload delivery situations and show that the cores of their associated games are always non-empty and dual solutions provide allocations in their core. Hezarkhani, Slikker, and Van Woensel (2014) further delineate the possibilities and impossibilities for a complete characterization of the core of these games via dual solutions. Hezarkhani (2016) discusses a cooperative logistics game with punctual delivery times and proves the non-emptiness of the core. Skorin-Kapov (1998) examines several cooperative games associated with hub network games and highlights special cases where the core is non-empty. Still, in several key decentralized logistics problems—e.g. traveling salesman, vehicle routing, facility location, etc.—it is known that the cores of the associated games can be empty. In cooperative vehicle routing situations, where the cores could be empty, Göthe-Lundgren, Jörnsten, and Värbrand (1996) and Engevall, Göthe-Lundgren, and Värbrand (2004) elaborate on the implementation of the nucleolus as the allocation rule of choice. Hezarkhani, Slikker, and Van Woensel (2016) propose a cost-sharing rule for collaborative routing of full-truckloads to incorporate the competitive position of players while addressing the stability of the solution. Krajewska, Kopfer, Laporte, Ropke, and Zaccour (2007) discuss the implementation of the Shapley value as the solution in cooperative organizations of logistics providers. Computational complexities of finding Shapley values in large games have given rise to novel approximation techniques and heuristics (Bremer & Sonnenschein, 2013; Castro, Gómez, & Tejada, 2009). Several reviews of cost sharing problems in operations management and logistics are available in the literature (see for example Curiel, 2008 and Deng & Fang, 2008 among others). For a recent review on applications of cooperative game theory in gain/cost-sharing problems specific to collaborative logistics and transportation see Guajardo and Rönnqvist (2016).

3. Dispatch consolidation (DC) situations and games

A non-empty set of carriers, hereafter players, $N = \{1, \dots, n\}$ have deliveries destined for the same geographical area.³ The size (volume) of player i 's delivery is $c_i > 0$. Player i arrives at the consolidation center at time $r_i \geq 0$ and, without loss of generality, unload his cargo at the same time. The truck carrying a player's cargo leaves the consolidation center immediately after unloading.⁴ We use the terms player and delivery interchangeably. We call r_i the arrival time of delivery i . For ease of exposition we assume that deliveries have non-identical arrival times and that N is arranged by increasing order of arrival times, i.e., $r_1 < r_2 < \dots < r_n$. All results hold if the latter condition is relaxed.

Without using the consolidation center, each player could individually fulfill his delivery. The status quo cost of delivery for each player is known. The cost of inbound transportation to consolidation center for all players is also known. Thus, the potential benefit that a player can obtain by fulfilling his delivery via the consolidation center is subsequently assumed to be known. For delivery

³ The assumption of common geographical destination holds in several real-life case studies, e.g. the consolidation center to service Regent Street in central London (ARUP, 2019).

⁴ This is in line with real-life case studies where arrival times are pre-booked in advance and suppliers deliver their cargo to the consolidation center and leave immediately. For example, see the case study of Wilson James's London Construction Consolidation Centre (LCCC) (James, 2019).

$i \in N$, we denote that latter value with K_i and refer to it by player i 's potential.⁵

A player would achieve his potential if upon arrival at the consolidation center, his cargo is immediately dispatched and he pays no additional costs. Note that actual savings achieved by a player as the result of using the consolidation center depends on his waiting time at the consolidation center as well as his payment toward the cost of dispatch from the consolidation center to his destination. In our model we consider the latter costs separately. We assume $K_i > 0$ for all $i \in N$. That is, all players have the potential to achieve savings from using the consolidation center. This does not mean that all player will necessarily benefit from using the consolidation center—a player with positive potential may be worse off if he is dispatched without being bundled with other deliveries or if he (more specifically, his cargo) waits too long at the consolidation center.

As deliveries arrive over time, to be able to consolidate deliveries in the consolidation center some deliveries must wait for the arrival of the others. But deliveries are time-sensitive so waiting is costly.⁶ We let $p_i \geq 0$ be the waiting penalty rate for player i , that is the cost that he incurs when his cargo sits in the consolidation center for a unit of time. Thus, the benefit obtained by player i if dispatched from the consolidation center at time $d_i \geq r_i$ is $K_i - p_i(d_i - r_i)$.⁷

The consolidation center operates a homogeneous fleet with sufficient number of trucks. Each truck has the capacity $C > 0$. The cost of dispatching a truck from the consolidation center to the common destination is $W \geq 0$. We assume, without loss of generality, that the preparation time at the consolidation center for dispatching arrived deliveries is normalized to zero. Accordingly, a Dispatch Consolidation (DC) situation can be defined by the tuple $\Gamma = (N, \mathbf{c}, \mathbf{r}, \mathbf{K}, \mathbf{p}, C, W)$ with bold notation representing n -element vectors. In the remainder of this paper, unless mentioned explicitly, we assume that the situation is arbitrary but fixed.

3.1. Dispatch decisions

The consolidation center is responsible for making decisions, on behalf of the participating players, regarding the dispatching of their deliveries. Acting as a coordinator in the system, the objective of the consolidation center is to maximize the overall profit of all players. In doing so, however, the consolidation center can choose not to accept some deliveries for handling at the facility. Allowing a consolidation center to reject players enables the exclusion of non-profitable deliveries. Thus, having the information about the deliveries of all players, the consolidation center decides (a) the set of deliveries which are accepted to be handled by the consolidation center, and (b) dispatch times of accepted deliveries.⁸

⁵ There are several case studies that publish potential savings due to use of consolidation centers. For example, UK's Waste & Resources Action Programme (WRAP) provides comprehensive data on potential savings in the case study of London Construction Consolidation Center (WRAP, Waste & Resources Action Programme, 2019). The potential savings in this case comprise reduction in overall trips as well as avoiding congestion charges due to traveling into central London.

⁶ Waiting times and associated costs are measured and controlled in several case studies. Janjevic and Ndiaye (2017) report numerical values of average dwelling times in some real-life consolidation centers.

⁷ The waiting penalties can also reflect dispatch deadlines: let d_i such that $K_i - p_i(d_i - r_i) = 0$. Then d_i can be interpreted as the deadline of delivery i to be dispatched from the consolidation center as missing this deadline makes direct deliveries more profitable.

⁸ The decision on dispatching times essentially determines the duration that each accepted player needs to wait at the consolidation center. An alternative modeling approach, yet mathematically equivalent, is to consider ready times of players at their origins, explicitly considering the time distance of each player from the consolidation center, and to require the consolidation center to advise players on their postponed departure times so that all players in a dispatch arrive together at the consolidation center.

Suppose, for example, that the consolidation center accepts the deliveries $T \subseteq N$ and consolidates them within a single truck dispatched at time $d_T \geq 0$, hereafter a *dispatch*. To ensure feasibility, two conditions must be met: (1) the dispatch time must be no earlier than the arrival times of all included deliveries, i.e., $d_T \geq r_i$ for all $i \in T$, and (2) the total size of the consolidated deliveries in a dispatch must be less than or equal to the truck capacity, i.e. $\sum_{i \in T} c_i \leq C$. If these two conditions are maintained, the saving obtained by the dispatch is $\sum_{i \in T} [K_i - p_i(d_T - r_i)] - W$. The saving associated with a dispatch is thus the sum of potentials of the included deliveries, minus their waiting costs as well as the cost of a dispatched truck.

The consolidation center's decision structure comprises a collection of dispatches, representing consolidated subsets of players, and their associated dispatch times. Let \mathcal{N} be the set of all non-empty subsets of N and define $\mathcal{N}^f = \{T | T \in \mathcal{N}, \sum_{i \in T} c_i \leq C\}$ to be the set of capacity-feasible subsets of players. The combined cargo size of players in each subset $T \in \mathcal{N}^f$ is less than or equal to the truck capacity. The objective of the consolidation center operator is to maximize the sum of savings of all players. The observation below limits the choices for the optimal dispatch times of a dispatch.

Remark 1. The optimal time for the dispatch of players in $T \in \mathcal{N}$, coincides with the arrival time of the last player in T .

It is straightforward to verify the above remark. Reducing the dispatch time also reduces the waiting times and this can never decrease the associated savings. Thus, at optimality the dispatch time can be reduced until the feasibility condition $d_T \geq r_i$ for all $i \in T$, or equivalently for the last arriving player, becomes binding. Subsequently, in the rest of the paper we limit our attention to the optimal dispatch times as prescribed in Remark 1. In conjunction with the assumption of non-identical arrival times, Remark 1 implies that the optimal number of dispatched trucks at any point of time is at most one.

Given $T \in \mathcal{N}$, we denote the first and last arriving delivery in T with $b(T)$ and $e(T)$, respectively. Since the players are ordered by their arrival times, $b(T)$ and $e(T)$ also represent respectively the smallest and largest elements in T . Define the saving function u for a group of players $T \in \mathcal{N}$ as

$$u_T = \sum_{i \in T} [K_i - p_i(r_{e(T)} - r_i)] - W. \tag{1}$$

In this manner, u_T determines the total benefit obtained by deliveries in T when dispatched at $r_{e(T)}$, which is the earliest feasible dispatch time for them, minus the cost of the truck.

To find the best choice of selected players for handling at the consolidation center and, simultaneously, obtaining the best dispatching schemes, we can construct the optimization problem as a set packing formulation:

$$v(N) = \max \sum_{T \in \mathcal{N}^f} z_T u_T \tag{2}$$

$$s.t. \quad \sum_{T \in \mathcal{N}^f: T \ni i} z_T \leq 1 \quad \forall i \in N \tag{3}$$

$$z_T \in \{0, 1\} \quad \forall T \in \mathcal{N}^f \tag{4}$$

The optimization problem above chooses the best combination of dispatches to maximize total savings. Let $z^N = (z_T^N)_{T \in \mathcal{N}^f}$ be an optimal solution to the problem above. We denote the *optimal dispatching scheme* for N associated with z^N with

$$Z^N = \{T | T \in \mathcal{N}^f, z_T^N = 1\}, \tag{5}$$

and call every $T \in Z^N$ a *component* of Z^N . With this notation in place, the maximum saving of all players can be obtained via the sum of

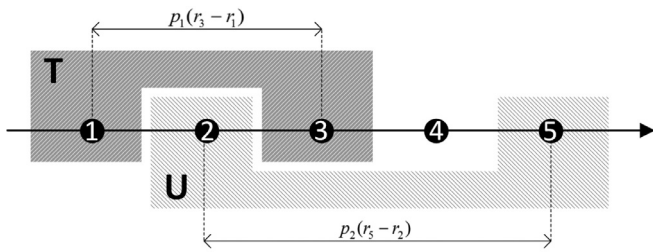


Fig. 2. Optimal dispatching scheme in Example 3.

savings over all components in an optimal dispatching scheme:

$$v(N) = \sum_{T \in Z^N} u_T. \tag{6}$$

It is straightforward to see that for every $T \in Z^N$ we have $v(T) = u_T \geq 0$.

Example 3. Assume that the DC situation is as follows. There are 5 players, $N = \{1, 2, 3, 4, 5\}$. The size of all deliveries are equal to one unit. Suppose $r_i = i$ for all $i \in N$, $K_i = 10$ for all $i \in N \setminus \{4\}$ and $K_4 = 5$. The waiting penalty rates are $p_1 = p_3 = 2$, $p_2 = 0.5$ and $p_4 = p_5 = 5$. The cost and capacity of trucks are $W = 15$ and $C = 2$. In the optimal dispatching scheme for the grand coalition in this example—illustrated in Fig. 2—players 1 and 3 are dispatched together at r_3 , while players 2 and 5 are dispatched together at r_5 . Player 4 in this example is not accepted for dispatching through the consolidation center.

For simplicity, hereafter we use the notation $\lambda_{i,j} = K_i - p_i(r_j - r_i)$. The value $\lambda_{i,j}$ is the benefit obtained by delivery i when dispatched at the arrival time of delivery j . Note that $\lambda_{i,i}$ coincides with the potential of delivery i , i.e. K_i . We provide some observations in connection with optimal dispatching schemes.

Lemma 1. Let Z^N be an optimal dispatching scheme for N . Suppose $T \in Z^N$. For all $i \in T$ the followings hold:

- (a) $\lambda_{i,e(T)} \geq 0$,
- (b) $(r_{e(T)} - r_i) \sum_{k \in T: k \leq i} p_k \leq W$.
- (c) $(r_{e(T)} - r_i) \sum_{k \in T: k \leq i} p_k \leq \sum_{k \in T: k > i} \lambda_{k,e(T)}$

The first part in the above lemma states that the benefit of each player included in a component of an optimal dispatching scheme must be non-negative. This is an intuitive feature because otherwise the exclusion of delivery i from the dispatch increases total savings. To interpret part (b), note that the term $(r_{e(T)} - r_i)p_k$ is the improvement in player k 's waiting cost if he is dispatched at r_i instead of his current dispatch time which is $r_{e(T)}$. In other words, this term is the saving in waiting cost that k can get if he is dispatched at r_i . Consequently, the inequality in part (b) means that if T is an optimal component, then the sum of savings in waiting costs that players before i , including i himself, obtain by organizing an alternative dispatch at r_i should be small enough so that it would not outweigh the cost of an additional dispatched truck. An optimal dispatching scheme must satisfy this condition otherwise dispatching those deliveries independently (or rejecting them in case their associated savings are negative) improves the total savings. For example, suppose an optimal component includes two deliveries—thus dispatching these two deliveries together are at least as profitable as two individual dispatches. The cost difference between the latter dispatch scheme and the optimal is the difference between the cost of an additional truck and the waiting cost of the first delivery. In this example the condition in part (b) states that the waiting cost of the first delivery must be less than W . Finally, the condition in part (c), requires that the sum of benefits obtained by including the deliveries after i in an optimal

dispatching scheme, $\sum_{k \in T: k > i} \lambda_{k,e(T)}$, must always outweigh the sum of extra waiting costs of earlier deliveries. In the two-delivery component example mentioned earlier, this observation requires the benefit of the second delivery to be at least as large as the waiting cost of the first in order to justify the wait.

3.2. DC games

A cooperative game is a pair (N, v) comprising a player set N , and a characteristic function v that assigns a real value to every subset, i.e. coalition, of N with $v(\emptyset) = 0$. A dispatch consolidation (DC) game is a cooperative game defined in association with a DC situation. For every $S \subseteq N$, let \mathcal{S} be the set of all non-empty subsets of S and $\mathcal{S}^f = \mathcal{S} \cap \mathcal{N}^f$ be the restriction of capacity-feasible subsets among players in S . The DC game associated with the given situation Γ is obtained by defining $v(S)$ for every $S \subseteq N$ via⁹

$$v(S) = \max \sum_{T \in \mathcal{S}^f} z_T u_T \tag{7}$$

$$\text{s.t.} \quad \sum_{T \in \mathcal{S}^f: T \ni i} z_T \leq 1 \quad \forall i \in S \tag{8}$$

$$z_T \in \{0, 1\} \quad \forall T \in \mathcal{S}^f \tag{9}$$

Accordingly, we can define an optimal dispatching scheme for S with Z^S . Natural analogs for Eq. (6) and Lemma 1 also hold for every Z^S , i.e., optimal dispatching schemes for coalitions.

A cooperative game is called superadditive if for every pair of disjoint subsets $S, T \in \mathcal{N}$, $S \cap T = \emptyset$, we have $v(S) + v(T) \leq v(S \cup T)$. It is straightforward to verify that DC games are indeed superadditive. The superadditivity of DC games implies that the sum of savings obtained by (optimally) consolidating players' deliveries in sub-coalitions never exceeds the savings obtained from consolidating deliveries in the grand coalition. Thus economies of scale are present in consolidation centers.

4. Gain sharing problem

An important problem in every cooperative game is the division of the grand coalition's savings among the players. Let $\mathbf{a} = (a_i)_{i \in N}$ be an allocation where $a_i \in \mathbb{R}$ is the allocated saving to player $i \in N$. We introduce some desirable properties that an allocation may satisfy.

The first property requires that savings of the grand coalition be fully distributed among the players.

Property 1. An allocation \mathbf{a} is efficient for (N, v) if $\sum_{i \in N} a_i = v(N)$.

In order for the players to participate in the game, their allocated savings must be at least as much as they can obtain individually. Otherwise they would be better off not participating. The next property formalizes this requirement.

Property 2. An allocation \mathbf{a} is individually rational for (N, v) if $a_i \geq v(\{i\})$ for all $i \in N$.

As an extension to the individual rationality property, it is desirable to distribute savings in such a way that all groups of players receive at least as much as they would if collaborate only among themselves. The notion of stability is accordingly defined.

Property 3. An allocation \mathbf{a} is stable for (N, v) if for every $S \subseteq N$ we have $\sum_{i \in S} a_i \geq v(S)$.

⁹ Note that a DC game is a saving game. One can define a dual cost game by incorporating the players status quo costs and introducing additional variables for the selection problem. This, however, renders the formulation of the problem more cumbersome.

With a stable allocation, no coalition of players can object that they would have been better off outside the grand coalition and on their own. Stability implies individual rationality, that is, every stable allocation is also individually rational. The core of a game incorporates the properties defined above, that is, the core of a game is the set of all efficient and stable allocations.

Definition 1. An allocation \mathbf{a} is in the core of (N, ν) whenever $\sum_{i \in N} a_i = \nu(N)$, and $\sum_{i \in S} a_i \geq \nu(S)$ for all $S \subset N$.

Given the desirable features of core allocations, one would be interested in examining their existence and subsequently finding them.

We showed earlier that the characteristic function of every DC game is defined via a set packing formulation. As a result, DC games are special instances of the class of set packing games (Deng, Ibaraki, & Nagamochi, 1999). The following result regarding the non-emptiness of the cores of set packing games in general holds for DC games as well. Consider the dual program associated with the integer relaxation of the program (2)–(4):

$$v^D(N) = \min \sum_{i \in N} a_i \tag{10}$$

$$s.t. \quad \sum_{i \in T} a_i \geq u_T \quad \forall T \in \mathcal{N}^f \tag{11}$$

$$a_i \geq 0 \quad \forall i \in N \tag{12}$$

Theorem 1. The core of the DC game (N, ν) is non-empty if and only if $v^D(N) = \nu(N)$, that is, the integer relaxation of the program does not affect optimality. In this case the core coincides with the set of solutions to (10)–(12).

The proof of Theorem 1, expressed in terms of set packing games, can be found in Deng et al. (1999). The core of a set packing game can in general be empty, and (as seen in Example 2 above) so does the core of a DC game. But whenever the integrality constraint of the optimization problem in a given situation turns out to be superfluous, the core of the associated game would also be non-empty.

5. DC games with non-restrictive capacities

Consider the special case where truck capacities would not impose any restriction on optimality. This can happen when either the capacity is ample and/or the waiting penalties are substantial enough that dispatches are executed before trucks exceed their capacity limits. Note that non-restrictive capacities do not mean that all deliveries are dispatched in one truck. For example, suppose that a series of deliveries all with a third of truck-load size arrive every half an hour to the consolidation center but, in order to make the use of consolidation center economically feasible, each delivery cannot wait more than forty five minutes in the facility. In this case, a truck would be dispatched every hour with only two-third of its capacity filled.

Definition 2. The DC game (N, ν) has non-restrictive capacities if replacing \mathcal{N}^f with \mathcal{N} in formulation of $\nu(N)$ in (7)–(9) would not affect its optimal value. The game has totally non-restrictive capacities if for every $S \subseteq N$ replacing \mathcal{S}^f with \mathcal{S} in formulation of $\nu(S)$ would not affect its optimal value.

With non-restrictive capacities, the value of optimal dispatching schemes would not change if one relaxes the capacity-feasible requirement for the grand coalition. If the same holds for all coalitions as well, the game would be said to have totally non-restrictive capacities. DC games with non-restrictive capacities exhibit special structures that we exploit in this section to prove that their cores are non-empty.

For games with non-restrictive capacities, we first provide an alternative formulation of the corresponding optimization problem. Given $S \subseteq N$, we call $T \subseteq S$, a connected subset of S if for every $k \in S$ such that $b(T) < k < e(T)$, it holds that $k \in T$. Recall that $b(T)$ and $e(T)$ are the first and last arriving deliveries in T respectively. Denote the set of connected subsets of S with $\bar{\mathcal{S}}$. Let $\lambda_{i,j}^+ = (K_i - p_i(r_j - r_i))^+$, and for every $T \in \mathcal{N}$ define $\bar{u}_T = \sum_{i \in T} \lambda_{i,e(T)}^+ - W$. For every set of deliveries T , $\bar{u}(T)$ gives the sum of benefits of those deliveries in T whose dispatch generates non-negative benefits. Consider the modified characteristic function $\bar{\nu}$, defined for $S \subseteq N$ as

$$\bar{\nu}(S) = \max \sum_{T \in \bar{\mathcal{S}}} z_T \bar{u}_T \tag{13}$$

$$s.t. \quad \sum_{T \in \bar{\mathcal{S}}: T \ni i} z_T \leq 1 \quad \forall i \in S \tag{14}$$

$$z_T \in \{0, 1\} \quad \forall T \in \bar{\mathcal{S}} \tag{15}$$

Compared to the original formulation in (7)–(9), the above formulation replaces u_T with \bar{u}_T and \mathcal{S}^f with $\bar{\mathcal{S}}$ so the packing is done over the set of connected subsets. Our first observation in this section states conditions under which such transformation is inconsequential in terms of the optimal value.

Lemma 2. If the DC game (N, ν) has non-restrictive capacities, then we have $\bar{\nu}(N) = \nu(N)$ and $\bar{\nu}(S) \geq \nu(S)$ for every $S \subset N$. If the game has totally non-restrictive capacities then $\bar{\nu} = \nu$.

We are now ready to provide the main result of this section regarding the non-emptiness of the core of every DC game with non-restrictive capacities.

Theorem 2. The core of every DC game with non-restrictive capacities is non-empty.

The proof of Theorem 2 draws upon the results of Barany, Edmonds, and Wolsey (1986) regarding zero duality gap of set packing problems on trees via their sub-trees and incorporates the necessary and sufficient condition for the core non-emptiness of a set packing game as described in Theorem 1. Therefore, one can always find allocations in the core of a DC game with non-restrictive capacities. In order to do so, one can solve the dual program in (10)–(12). The latter program can have up to $2^n - 1$ constraints. Unfortunately, one cannot use the dual problem associated with (13)–(15) for N —which contains only $n(n+1)/2$ constraints—to obtain allocations in the core. This is shown in the next example.

Example 4. Assume that the situation involves three players $N = \{1, 2, 3\}$. Let $c_i = 1$, $r_i = i$, $K_i = 10$, $p_i = 1$ for all $i \in N$, and furthermore $W = 4$. We have $\nu(\{i\}) = 6$ for $i \in N$, $\nu(\{1, 2\}) = \nu(\{2, 3\}) = 15$, $\nu(\{1, 3\}) = 14$, and $\nu(N) = 23$. The dual solution associated with relaxation of (13)–(15) for N —which is equivalent to (10)–(12) with \mathcal{N}^f replaced with $\bar{\mathcal{N}}$ —requires that $a_i \geq \nu(\{i\}) = 6$ for all $i \in N$, $a_1 + a_2 \geq \nu(\{1, 2\}) = 15$, $a_2 + a_3 \geq \nu(\{2, 3\}) = 15$, and $a_1 + a_2 + a_3 = \nu(N) = 23$. The allocation $\mathbf{a} = (6, 11, 6)$ satisfies the above requirements, but it is not in the core since $a_1 + a_3 = 12 < \nu(\{1, 3\}) = 14$.

As our next result indicates, in case of non-restrictive capacities, there exists an alternative program, drawing upon at most $n(n+1)/2$ inequalities, that obtains allocations in the core.

Theorem 3. Suppose that the DC game (N, ν) has non-restrictive capacities. Let \mathbf{a} be an efficient allocation such that for a collection of

¹⁰ We use the notation $(\cdot)^+$ instead of $\max\{\cdot, 0\}$.

non-negative pairwise weights $(w_{i,j})_{i,j \in N: i < j}$ it satisfies:

$$a_i + w_{i,j} \geq \lambda_{i,j}^+ \quad \forall i, j \in N : i < j \tag{16}$$

$$a_j - \sum_{i < j} w_{i,j} \geq \lambda_{j,j} - W \quad \forall j \in N \tag{17}$$

Then such an \mathbf{a} exists and any such \mathbf{a} is an allocation in the core. Furthermore, if the game has totally non-restrictive capacities, all allocations in the core can be obtained in this way.

The conditions in (16) and (17) balance the players' allocations through a set of non-negative pairwise weights. Once these conditions, along with the efficiency requirement, are satisfied the outcome is an allocation in the core. In case of totally non-restrictive capacities, if an allocation is in the core of the game, it always satisfies this set of conditions. Therefore, Theorem 3 gives a complete characterization of the core for DC games with totally non-restrictive capacities.

Example 5. In Example 4, the conditions in (16) and (17) can be written as: $a_1 + w_{1,2} \geq 9, a_1 + w_{1,3} \geq 8, a_2 + w_{2,3} \geq 9, a_1 \geq 6, a_2 - w_{1,2} \geq 6, a_3 - w_{1,3} - w_{2,3} \geq 6$. The efficiency also requires that $a_1 + a_2 + a_3 = 23$. Feasible solutions for these constraints, i.e. allocations in the core, include $a = (7, 9, 7)$, with corresponding pairwise weights $w_{1,2} = 2, w_{1,3} = 1, w_{2,3} = 0$, and $a = (8, 9, 6)$, with corresponding pairwise weights $w_{1,2} = 1, w_{1,3} = 0, w_{2,3} = 0$, among others. Observe that one cannot find a set of weights which, together with the non-core allocation $a = (6, 11, 6)$ discussed in Example 4, comprise a feasible solution to these set of constraints.

6. DC games with restrictive capacities and component-wise core

In the previous section we showed that in every DC game with non-restrictive capacities one can always find allocations in the core. When the delivery capacities impose restrictions, however, the core can be empty—as shown in the simple example below.

Example 6. Consider the situation in Example 4 with additional requirement that $C = 2$. We get $v(\{i\}) = 6$ for $i \in N, v(\{1, 2\}) = v(\{2, 3\}) = 15, v(\{1, 3\}) = 14$, and $v(N) = 21$. One can verify that the core of this game is empty.

Recall that the stability property imposes the no-objection requirement for all coalitions of players. As seen in Example 6, this can be too demanding and, accordingly, the core of a DC game with restrictive capacities can be empty. In this case, cooperative game theory literature suggests allocation rules such as least-core and nucleolus that minimize maximum objections over the set of all coalitions. These allocation rules treat all coalitions in the same way—i.e. all having the same chance of being formed. However, in some situations, e.g. DC games, one can argue that some coalitions are more likely to form and some objections are more likely to be raised. In DC games per se, players who are eventually dispatched together in a single truck are directly involved in working with each other, so if there are chances that players form coalitions and orchestrate their objections, players in the same dispatch are first to spot such opportunities. The novelty of our approach is to restrict the stability requirements of an allocation rule with the structure of the selected solution for the underlying optimization problem. Next, we introduce the concept of component-wise stability. In the following definitions we assume that the optimal dispatching scheme for the situation, Z^N , is arbitrary but fixed.

Property 4. An allocation \mathbf{a} for the DC game (N, v) is component-wise stable with regard to Z^N if for all $T \in Z^N$ and all $S \subseteq T$ we have $\sum_{i \in S} a_i \geq v(S)$.

Component-wise stability requires the no-objection condition only for coalitions of players within the components of an optimal dispatching scheme. In this manner, it disregards the objections that can be raised by the collections of players from different components. Note that it is necessary for this property to be given with an optimal dispatching scheme as there can be multiple choices in one situation. Subsequently, we define the component-wise core of a DC game with regard to an optimal dispatching scheme as the set of all efficient and component-wise stable allocations.

Definition 3. An allocation \mathbf{a} is in the component-wise core of the DC game (N, v) with regard to Z^N whenever $\sum_{i \in N} a_i = v(N), \sum_{i \in S} a_i \geq v(S)$ for all $S \subseteq T$ and all $T \in Z^N$, and $a_i = 0$ for every $i \notin T$ and all $T \in Z^N$.

Let $Z^N = \{S | S \subseteq T, T \in Z^N\}$ be the set of subsets of components of Z^N . In comparison with the core, the component-wise core relaxes the no-objection requirement for coalitions in $\mathcal{N}^f \setminus Z^N$. Clearly, the component-wise core is a weakening of the core. That is, every allocation in the core is also in the component-wise core. The reverse, however, is not necessarily true. That is, an allocation in the component-wise core may not lie in the core.

Our main result in this section shows that despite the possibility of encountering an empty core in a DC game with restrictive capacities, once an optimal dispatching scheme is fixed, the associated component-wise core is always non-empty.

Theorem 4. The component-wise core of every DC game with regard to every corresponding optimal dispatching scheme is non-empty.

In light of Theorem 4, the notion of component-wise stability in DC games with restrictive capacities offers an alternative way to partially incorporate stability with regard to a selected optimal dispatch scheme while resolving the issue of existence. Therefore, with an allocation in the component-wise core in this case, carriers whose deliveries are dispatched within the same truck can never object to their allocations when considering possible cooperation among themselves.

7. A Characterization of component-wise cores of DC games

In DC games, the component-wise cores can be characterized by a collection of properties defined for DC situations. The first property requires that the savings generated by players within each component of the selected optimal dispatching scheme be completely distributed among themselves and any player who is not part of a dispatch gets zero.

Property 5. An allocation \mathbf{a} for the DC game (N, v) satisfies Component-wise Efficiency (CE) property with regard to Z^N if we have $\sum_{i \in T} a_i = u_T$ for every $T \in Z^N$, and $a_i = 0$ for every $i \in N$ such that $i \notin T$ for all $T \in Z^N$.

So far we have considered the problem of allocating gains obtained via cooperation among the players. At this point we offer a complementary interpretation of allocations in terms of players' cost shares. Let Z^N be an optimal dispatch scheme, $T \in Z^N$ a component, and $i \in T$ a player in T . Given a_i as the allocated saving to a player i define

$$y_i = \lambda_{i,e(T)} - a_i. \tag{18}$$

The value y_i is the difference between the benefit that player i obtains when dispatched along with the other players in T , and his allocated saving. In this case, y_i can be regarded as i 's share of dispatch cost. The properties introduced in this section have intuitive interpretations in terms of a players' shares of dispatch costs. For example, the CE property above can be expressed alternatively in terms of the corresponding cost shares.

Property 6. An allocation \mathbf{a} for the DC game (N, v) satisfies CE property with regard to Z^N if for every $T \in Z^N$ we have $\sum_{i \in T} y_i = W$.

Whenever an allocation satisfies the CE property, the cost of each dispatched truck is distributed completely among players in a component of the optimal dispatching scheme.

The next property formalizes the requirement for the players' shares of dispatch costs to be non-negative.

Property 7. An allocation \mathbf{a} for the DC game (N, v) satisfies Non-negative Contribution to dispatch cost (NC) property with regard to Z^N if $y_i \geq 0$ for all $i \in T, T \in Z^N$.

The NC property caps the amount of savings that each player can obtain as the result of collaboration given the choice of optimal dispatching scheme. This implies that if an allocation satisfies the NC property then no player is subsidized for his waiting cost in the selected optimal dispatching scheme.

Next, we introduce a property with regard to the sum of allocated costs to the players in an optimal dispatch.

Property 8. An allocation \mathbf{a} for the DC game (N, v) satisfies Compensation for Delay-caused (CD) property with regard to Z^N , if for every $T \in Z^N, |T| \geq 2$, and every $i \in T \setminus \{e(T)\}$ it holds that $\sum_{k \in T: k > i} y_k \geq (r_{e(T)} - r_i) \sum_{k \in T: k \leq i} p_k$.

The above property states that the sum of the shares of dispatch costs of late arriving players in a component of the selected optimal dispatch is always large enough to compensate for the waiting cost of the early arriving players in that component. Thus, this property requires that later players collectively compensate for the delay that they cause for earlier players.

Our final property introduces a weaker version of individual rationality for the players in components of the optimal dispatching scheme.

Property 9. An allocation \mathbf{a} for the DC game (N, v) satisfies Component-wise Individual Rationality (CIR) property with regard to Z^N if for every $T \in Z^N$, and $i \in T$ it holds that $y_i \leq \lambda_{i,e(T)}$.

The CIR property requires non-negative allocations of savings to players who are dispatched in the selected optimal dispatching scheme for the grand coalition.

Before presenting the main result of this section, we provide an observation regarding the savings of sub-coalitions of players within components of optimal dispatching schemes.

Lemma 3. Let $T \in Z^N$. For every $S \subseteq T$ we have $v(S) = (\max_{i \in S} u_{\{k \in S: k \leq i\}})^+$.

Our main result in this section characterizes the component-wise core in terms of four properties defined earlier.

Theorem 5. With regard to Z^N , an allocation is in the component-wise core of the DC game (N, v) if and only if it satisfies the CE, NC, CD, and CIR properties.

The above theorem reveals the necessary and sufficient conditions for an allocation to be in the component-wise core with regard to an optimal dispatching scheme. With regard to Z^N , every allocation in the component-wise core of the DC game (N, v) satisfies the CE, NC, CD, and CIR property. Also, any allocation that satisfies these four properties simultaneously is an allocation in the component-wise core. In light of this characterization, one can obtain allocations in the component-wise core of a DC game with regard to Z^N by finding the feasible solutions for the following set

of constraints:¹¹

$$\sum_{i \in T} y_i = W \quad \forall T \in Z^N \tag{19}$$

$$\sum_{i \in T: k \leq i} y_i \geq (r_{e(T)} - r_{k-1}) \sum_{i \in T: i < k} p_i \quad \forall T \in Z^N, k \in T \tag{20}$$

$$y_i \leq \lambda_{i,e(T)} \quad \forall T \in Z^N, i \in T \tag{21}$$

$$y_i \geq 0 \quad \forall i \in T \in Z^N \tag{22}$$

Constraint (19), corresponding to CE property, requires that the sum of allocated costs to all players i within a component T of Z^N exactly covers the cost of a dispatched truck. Constraint (20), corresponding to CD, maintains that within a component T of Z^N , the total allocated costs to players that arrive after a player $k \in T$ including k himself, i.e. $i \in T$ such that $i \geq k$, be at least as large as the extra waiting costs that preceding players in T , i.e. $i \leq k - 1$, incur when being dispatched at $e(T)$ instead of r_{k-1} (which would be the earliest time that they can together dispatch a truck). Constraint (21), corresponding to CIR property, requires the allocated cost to each player to be at most as large as the benefit that the player gains in his designated component. Finally, constraint (22), corresponding to NC, sets the lower bound of zero for all allocated costs. Theorem 4 implies that there always exists a feasible solution to the above program. Once a feasible solution is found, it can be turned into a gain-sharing allocation using (18).

8. A proportional allocation rule in the component-wise core

The intuitive interpretation—and often simplicity—of proportional allocation rules make them appealing in real-life applications. In DC games, the benefits obtained by the players in an optimal dispatching scheme appear to be a logical yardstick for allocating the savings proportionally among them. Accordingly, one can choose to distribute the savings obtained in each component of an optimal dispatching scheme among the players in the component based on their individual benefits.

In the most simplistic approach to incorporate proportionality, the savings of a component can be distributed among the players involved directly in proportion to their individual benefits. Recall that, given an optimal dispatching scheme Z^N , the benefit of a player i dispatched in component $T \in Z^N$ is $\lambda_{i,e(T)}$. Subsequently, given $T \in Z^N$, for every $i \in T$ define the naive proportional allocation as $a_i^p = u_T \lambda_{i,e(T)} / \sum_{j \in T} \lambda_{j,e(T)}$. Also, for any carrier $i \in N$ that is not part of any component of Z^N let $a_i^p = 0$. The allocation \mathbf{a}^p obtained in this way satisfies some of the properties we introduced so far. Clearly, \mathbf{a}^p satisfies CE since the sum of the allocated savings to the players in a component is exactly the savings obtained in that component. It also satisfies the CIR property because by Lemma 1 we have $u_T \geq 0$ and also $\lambda_{i,e(T)} \geq 0$ for every $i \in T$. However, the allocation \mathbf{a}^p does not necessarily obtain allocations within the component-wise core. The following example illustrates this.

Example 7. Assume that the situation involves three players, $N = \{1, 2, 3\}$. Also, suppose $c_i = 1$ and $K_i = 50$ for $i \in N$, $r_1 = 0, r_2 = 10$, and $r_3 = 20$. In addition, $p_1 = 1, p_2 = 2, p_3 = 1, C = 3$ and $W = 50$. The optimal dispatching scheme is $Z^N = \{1, 2, 3\}$ so all players will be dispatched together at $r_3 = 20$ which results in total savings of $v(N) = 60$. The naive proportional allocation rule in this case obtains $\mathbf{a}^p = (16.36, 16.36, 27.27)$ thus players 1 and 2 together obtain 32.72. However, if these players do not wait for player 3 and

¹¹ Given $k \in T \subseteq N$, for simplicity of notation hereafter we use $k - 1$ to refer to the player in T which arrives immediately before k .

dispatch a truck at $r_2 = 10$, their saving would be $v(\{1, 2\}) = 40$ which implies that \mathbf{a}^p is not within the component-wise core.

It also follows from the characterization of allocations in the component-wise core in Theorem 5 that the naive proportional allocation rule defined above may fail to satisfy some of the characterizing properties.

In the remainder of this section we introduce an allocation rule that draws upon proportionality to obtain allocations in the component-wise core. Given an optimal dispatching scheme Z^N and a component $T \in Z^N$, for every $l \in T$ define R^l recursively as:

$$R^l = \begin{cases} (r_{e(T)} - r_{l-1}) \sum_{k \in T: k < l} p_k & l = e(T) \\ \left((r_{e(T)} - r_{l-1}) \sum_{k \in T: k < l} p_k - \sum_{k \in T: k > l} R^k \right)^+ & l \notin \{b(T), e(T)\} \\ W - \sum_{k \in T: k > l} R^k & l = b(T) \end{cases} \quad (23)$$

In components containing at least two players, $R^{e(T)}$ is the total waiting cost that the last player causes all other players. For all other players in the component, except the first one, R^l is the outstanding waiting cost caused by the players that come after l , including l himself, in T once the corresponding values for subsequent players are all subtracted. The value $R^{b(T)}$ is the difference between the cost of dispatching a truck and the sum of all other players' allocated waiting costs. We present a technical lemma with regard to the sum of outstanding caused waiting costs of players in a component.

Lemma 4. Let Z^N be an optimal dispatching scheme and $T \in Z^N$. For $l \in T, l \neq b(T)$ we have:

$$\sum_{k \in T, k \geq l} R^k = \max_{q \in T: l \leq q \leq e(T)} \left\{ (r_{e(T)} - r_{q-1}) \sum_{k \in T: k < q} p_k \right\}.$$

Simply put, the above observation states that the sum of outstanding caused waiting costs for the players in T that come after $l \in T$, including l himself, is the largest amount of delay caused by any group of last m players in that dispatch with $1 \leq m \leq e(T) - l + 1$. In conjunction with Lemma 1 part (b), the last observation implies that $R^{b(T)} \geq 0$.

Given a dispatch $T \in Z^N$, our proportional rule works recursively and in stages. It starts from $l = e(T)$, goes backwards, and divides the values of R^l among l and the players in T that comes after l . Let $i, l \in T, l \leq i$, and define:

$$y_i^l = \frac{a_i^l}{\sum_{k \in T: k \geq l} a_k^l} R^l, \quad (24)$$

where

$$a_i^l = \begin{cases} \lambda_{i, e(T)} & \text{if } l = i \\ \lambda_{i, e(T)} - \sum_{h \in T: l < h \leq i} y_i^h & \text{if } l < i \end{cases} \quad (25)$$

At stage l , the cost share of player $i \geq l$, i.e. y_i^l , is a portion of R^l that is calculated based on the players adjusted benefits at each stage after accounting for the shares allocated to them so far. Let

$$\dot{y}_i = \sum_{l \in T: l \leq i} y_i^l \quad \forall i \in T \quad (26)$$

and finally

$$\dot{a}_i^p = \lambda_{i, e(T)} - \dot{y}_i \quad \forall i \in T. \quad (27)$$

For any player $i \in N$ that is not part of any component of Z^N let $\dot{a}_i^p = 0$. The allocation rule $\dot{\mathbf{a}}^p$ thus is obtained from a recursive procedure for calculating allocations in components of an optimal dispatching scheme for a DC game in such a way that the delay

Table 1
Situation in Example 8.

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-------|----|----|----|----|-----|-----|-----|-----|-----|-----|
| r_i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| p_i | 5 | 5 | 2 | 2 | 10 | 50 | 10 | 50 | 10 | 10 |
| K_i | 50 | 50 | 20 | 20 | 100 | 100 | 100 | 100 | 100 | 100 |

caused by later deliveries are divided among those deliveries in proportion to their benefits that are adjusted to reflect their already allocated costs. Note that a player can only be responsible for waiting costs of those who arrive before him in a dispatch. In the last step, the procedure divides the remainder of costs in adjusted proportions of benefits as well.

We are now ready to prove the main result of this section regarding the component-wise stability of $\dot{\mathbf{a}}^p$.

Theorem 6. The allocation $\dot{\mathbf{a}}^p$ is in the component-wise core with regard to Z^N .

The following example exhibits the results of applying this allocation rule.

Example 8. The main data for the situation in this example is given in Table 1. In addition, assume $W = 120$ and $C = 4$. The optimal dispatching scheme in this example is $Z^N = \{\{1, 2, 5, 6\}, \{3, 4, 7, 8\}, \{9, 10\}\}$ which is illustrated in Fig. 3. We have $u_{\{1,2,5,6\}} = 125$, $u_{\{3,4,7,8\}} = 92$, $u_{\{9,10\}} = 70$, so $v(N) = 287$. The calculations for this example are given in Supplementary Material and the summary of the results are presented in Table 2.

Given an optimal dispatching scheme, although the allocation $\dot{\mathbf{a}}^p$ is always in the component-wise core, it may not necessarily be in the core. In the above example the aggregated allocation for coalition of players 5, 7, and 8, is 130 but these players together can generate 140.

9. Envy-free allocations

The notion of component-wise stability focuses on eliminating the objections that can be raised by collections of players within the same component of an optimal dispatching scheme. As we argued, once an optimal dispatching scheme for the system has been fixed, the orchestration of objections by coalitions of players from different dispatches may render to be more difficult. Nevertheless, objections by individual players when comparing themselves with other players in different dispatches may still be easily raised. Such individual objections can be of the following form: "I would rather be dispatched in place of another player in a different dispatch and receive that players' allocation". The latter type of objection corresponds to the notion of *envy*. In this section we introduce the envy-free property to formalize the aforementioned category of objections in DC games.

Property 10. An allocation \mathbf{a} for the DC game (N, v) satisfies the Envy-Free (EF) property with regard to Z^N , if for every non-identical $T, U \in Z^N$, every $i \in T$ such that $r_i \leq r_{e(U)}$, and every $j \in U$ such that $(U \setminus \{j\}) \cup \{i\} \in \mathcal{N}^f$, it holds that $y_i \leq y_j + p_i(r_{e(U)} - r_{e(T)})$.

Table 2
Proportional allocation $\dot{\mathbf{a}}^p$ in Example 8.

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---------------|-------|-------|------|------|-------|-------|-------|-------|----|----|
| \dot{a}_i^p | 6 | 6 | 8 | 8 | 6 | 6 | 8 | 8 | 10 | 10 |
| \dot{y}_i | 9.76 | 11.71 | 5.31 | 6.37 | 41.58 | 56.96 | 48.23 | 60.09 | 55 | 65 |
| \dot{a}_i^p | 15.24 | 18.29 | 4.69 | 5.63 | 48.42 | 43.04 | 41.77 | 39.91 | 35 | 35 |

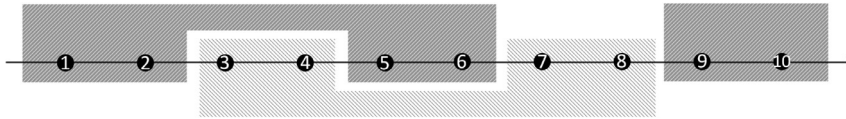


Fig. 3. The optimal dispatching scheme in Example 8.

The envy free property requires that the cost-share (payment) of each dispatched player be less than that of another player in an alternative dispatch which is feasible for the former plus the difference in waiting costs. In this context, the envy-free property is satisfied if no player would prefer to be dispatched in place of another and pays the latter player's cost share. Our result below shows that allocations in the core are always envy-free.

Theorem 7. *Every allocation in the core of every DC game satisfies the EF property with regard to every corresponding optimal dispatching scheme.*

Thus, whenever an allocation in the core is considered, envious objections of the above sort can never be raised. When an allocation not in the core is considered, however, players may be able to object to their allocated savings when comparing themselves to what other players in alternative dispatches attain. It should be noted that if the core is empty, as it can be in DC games with restrictive capacities, there may not exist allocations in the component-wise core with regard to any optimal dispatching scheme that satisfy the EF property. In fact, EF property cannot always be enforced in conjunction with the CE property, as shown in the next impossibility result.

Lemma 5. *In a DC game with restrictive capacities, there may exist no allocation that satisfies simultaneously the CE and EF properties with regard to an optimal dispatching scheme.*

The following counterexample proves the above statement.

Example 9. Consider the following situation: $N = \{1, 2, 3\}$, $c_i = 1$, $r_i = i$, $K_i = 10$, and $p_i = 1$ for all $i \in N$. Furthermore, let $W = 5$ and $C = 2$. We have $v(\{i\}) = 5$ for $i \in N$, $v(\{1, 2\}) = v(\{2, 3\}) = 14$, $v(\{1, 3\}) = 13$, and $v(N) = 19$. Also consider the optimal dispatching scheme $Z^N = \{\{1\}, \{2, 3\}\}$. By CE property for component $\{1\}$ we must have $y_1 = 5$. By EF we also must have $y_1 \leq y_2 + 2$ and $y_1 \leq y_3 + 2$. Together, these conditions require that $y_2 + y_3 \geq 6$ which violates the CE for component $\{2, 3\}$, i.e. $y_2 + y_3 = 5$.

In situations when envy cannot be completely eliminated, we propose a procedure to obtain allocations in the component-wise core with regard to an optimal dispatching scheme that reduces envy as much as possible. The program below obtains cost shares y corresponding to allocations in the component-wise core of the DC game (N, v) with regard to Z^N that minimize the maximum envy among the players dispatched in separate components:

$$\min \quad \epsilon \tag{28}$$

$$\text{s.t. } y_i \leq y_j + p_i(r_{e(U)} - r_{e(T)}) + \epsilon \quad \forall T, U \in Z^N, i \in T, j \in U : r_i \leq e(U), U \setminus \{j\} \cup \{i\} \in \mathcal{N}^f \tag{29}$$

$$\sum_{i \in T} y_i = W \quad \forall T \in Z^N \tag{30}$$

$$\sum_{i \in T: k \leq i} y_i \geq (r_{e(T)} - r_{k-1}) \sum_{i \in T: i < k} p_i \quad \forall T \in Z^N, k \in T \tag{31}$$

$$y_i \leq \lambda_{i, e(T)} \quad \forall T \in Z^N, i \in T \tag{32}$$

$$y_i \geq 0 \quad \forall i \in T \in Z^N \tag{33}$$

In the program above, decision variables are allocated costs to the players $(y_i)_{i \in N}$ as well as an auxiliary variable ϵ related to the players' envy. Constraints (30)–(33) in this program are identical to (19)–(22) and correspond to the CE, CD, CIR, and NC properties which together obtain allocations in the component-wise core of the DC game (N, v) with regard to Z^N . Taking into account the definition of EF Property, if a given $(y_i)_{i \in N}$ satisfies the family of constraints in (29) with an arbitrary $\epsilon \leq 0$, then $(y_i)_{i \in N}$ would satisfy envy-freeness as well. However, if EF property cannot be satisfied, the formulation of (29) allows allocated costs to violate envy-freeness by a maximum degree of $\epsilon > 0$. The auxiliary parameter ϵ in this formulation is thus the maximum envy that players experience with regard to $(y_i)_{i \in N}$. The objective function in (28) then distributes costs among players to obtain allocations in the component-wise core which minimize the maximum envy that players experience. Our approach in formulating this program resembles that of the least core (Maschler et al., 1979). When envy-free allocations within the component-wise core do exist, the procedure above obtains such allocations. But if envy-freeness cannot be enforced in conjunction with component-wise stability, the program above yields component-wise allocations that are least prone to envious objections. As an example, one can verify that the allocation \bar{a}^P in Example 8 is indeed envy-free.

10. Final remarks and conclusions

In this paper, we proposed a stylized model for collaboration in urban consolidation centers. The carriers have time sensitive cargo and the consolidation center operator solves a selection and dispatch problem to maximize the total savings that can be obtained by the players in the system. We addressed the requirements for having fair allocations of gains/costs among the players in order to attain stability, or reducing instability, in the cooperative system.

With non-restrictive capacities, we characterized the cores of associated cooperative games and proof their non-emptiness. However, when the capacity constraints hamper the existence of core allocations, we proposed an alternative way to incorporate stability into an allocation rule. The rationale behind the definition of component-wise core proposed in the paper is that once a decision for the system is made and implemented, not all players have the same chance of forming coalitions and object to their combined allocations. In DC situations in particular, we argue that the players who are dispatched together within a single truck in the final solution are directly in contact with each other and hence can organize their objections more easily while players in separate dispatches are less capable to do so. In this manner we explicitly linked the procedure for obtaining appropriate allocations with the selected optimal solution for the system. This is a distinctive feature which is not considered in existing solutions in the literature, as shown in our last example.

Example 10. Consider the following situation: $N = \{1, 2, 3\}$, $c_i = 1$, $r_i = i$, $K_i = 10$ for all $i \in N$. Let $p_1 = p_2 = 1$ and $p_3 = 2$. Furthermore, let $W = 8$ and $C = 2$. We have $v(\{i\}) = 2$ for $i \in N$, $v(\{1, 2\}) = 11$, $v(\{2, 3\}) = v(\{1, 3\}) = 10$, and $v(N) = 13$. The unique optimal dispatching scheme is $Z^N = \{\{1, 2\}, \{3\}\}$. Table 3 shows the results of applying different gain-sharing solutions in this example. The core

Table 3
Comparing different solutions in Example 10.

| Allocation rule | α_1 | α_2 | α_3 |
|------------------|----------------|----------------|----------------|
| Shapley value | $4\frac{1}{3}$ | $4\frac{1}{3}$ | 4 |
| Nucleolus | $4\frac{1}{3}$ | $4\frac{1}{3}$ | $3\frac{2}{3}$ |
| $\hat{\alpha}^p$ | 5.5 | 5.5 | 2 |

in this example is empty. The Shapley value and nucleolus give part of savings obtained by players 1 and 2 to player 3. With our proportional allocation rule, however, the savings obtained from the dispatch of players 1 and 2 are completely distributed among those two players.

To the best of our knowledge, the proposed allocation rule in this paper is the first to determine players' allocations with regard to the selected course of action for the situation and not just their potential in the cooperative situation. As discussed earlier, in situations whose associate games have empty-cores, other allocation rules such as the least-core, nucleolus, and the Shapley value fail to distribute the savings solely among the players who are responsible for creating the entire savings in the system. This means such allocation rules distribute a part of savings gained in the consolidation center to carriers who are not using the facility. The latter can be a serious flaw in some real-life situations. To strengthen the overall stability of our allocations, we incorporated individual objections to the allocated gains through the notion of envy-freeness. Our results imply that in DC games, the existence of envious objections to an allocation renders it unstable and thus outside the core. We provided a linear program to minimize the maximum envy caused by the allocations in the component-wise core. Our approach can be extended to other situations whose optimal solutions partition the players into distinct groups.

There are several possible extensions to our model to account for additional practical requirements in real world. Examples include, but are not limited to, incorporating time windows for deliveries, the need for heterogeneous fleet to cater for various delivery conditions such as temperature, and multiplicity of delivery destinations. Another challenging direction for future research is the online version of this problem. Note that in calculating our allocations, we have worked backwards from the last dispatched truck to the first and sequentially compensated the earlier deliveries for their waiting costs. In the online version of this problem where the information about future arrivals are not available at the time of accepting a delivery, the latter cannot be done any longer. Therefore, the dynamic gain/cost sharing problem in this context would require a different approach which remains an intriguing open problem. Finally, testing our results in conjunction with real data can bring insights about implementation challenges and shed light on ways to make the approach taken in the paper more practical.

Acknowledgments

The author is grateful to the associate editor and two anonymous referees whose suggestions improved the paper.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2019.05.028.

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