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Optimal remanufacturing strategies for original equipment manufacturers: partner and mode selection

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This study examines a supply chain with three players which are a contract manufacturer (CM), an independent remanufacturer (IR) and an original equipment manufacturer (OEM). The OEM outsources the production process of new products to the CM and enters the remanufacturing market by cooperating with the remanufacturer which can either be the CM or IR through outsourcing or authorization remanufacturing modes. We investigate the OEM's remanufacturing partner choice when the remanufacturing mode is given, and the remanufacturing mode choice when the remanufacturer is selected. We consider profitability and environmental performance as the OEM's decision-making criteria. When the wholesale price of the new product is exogenously determined, we find that cooperating with the remanufacturer that has a lower variable cost is the dominant strategy given a specific remanufacturing mode. When the CM strategically determines the wholesale price of the new product, the OEM should never select the CM as the remanufacturing partner, and the IR is always a preferable remanufacturing partner for the OEM. In addition, both outsourcing and authorization modes are optimal when the CM is the remanufacturer. However, when the IR is the remanufacturer, the OEM should always choose the outsourcing mode.

Keywords: Production outsourcing; remanufacturing partner; channel choice; cost structure

1. Introduction

The environmental and economic values of remanufacturing have attracted or forced many supply chain members to enter the remanufacturing market (Agrawal et al., 2015, 2019; Alev et al., 2020; Shi et al., 2020; Souza, 2013). It is reasonable to anticipate that such trend will continue in the future. For instance, remanufacturing operations and sales have been well established and supported by Dell, a manufacturer that is known for its direct selling model (Shi et al., 2020). In the automotive parts industry, almost all original equipment manufacturers (OEMs) in Europe have established their own remanufacturing business or have a supportive attitude toward the development of remanufacturing (Chen & Chen, 2019). However, OEMs' decision to remanufacture is inhibited for fear that selling new products will be cannibalized by remanufactured products (Shi et al., 2020; Timoumi et al., 2021; Xu et al., 2021). For example, Alpha Equipment destroyed the

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majority of its collected used products even though its annual return volume was over $800 M (Atasu et al., 2010).

The remanufacturing business not only attracts OEMs themselves to participate in remanufacturing but also contract manufacturers (CMs) and independent remanufacturers (IRs). Foxconn, for instance, a CM of Apple, has started to sell refurbished iPhones on its website since 2015 (Zhou, Meng, Yuen, & Sheu, 2021). In the Chinese market, Aihuishou, an IR in the electric and electronic industry, has established its own remanufacturing business and becomes a remanufacturing partner for many famous brands, such as Huawei, Xiaomi, Vivo and so on (Aihuishou, 2011). The fact that almost all supply chain members can manage remanufacturing operations complicates OEMs' remanufacturing-related decisions. A natural question for OEMs, especially for those who directly sell new products to the market, is should they cooperate with a CM, or select an IR as their remanufacturing partner?

The remanufacturing modes which are available for OEMs are also quite different and can be classified into two types, i.e. outsourcing and authorization, based on the marketing channel of remanufactured products (Zou et al., 2016). In the outsourcing mode, the OEM directly participates in remanufacturing operations by managing the marketing of remanufactured products itself. In the authorization mode, the remanufacturing market is indirectly affected by the OEM's authorization cooperation decision with the remanufacturer, and the marketing of remanufactured products is independently managed by the remanufacturer. Industry evidence shows that both modes have been adopted by OEMs and some may even adopt the two modes at the same time for different markets. For example, the authorization remanufacturing mode has been adopted by IBM, Cisco and Hewlett Packard (Oraiopoulos et al., 2012), while Caterpillar adopts an outsourcing remanufacturing mode for its automobile parts service (Zhang, Chen, & Mi, 2020). Apple adopts different remanufacturing modes in the China and European markets. The remanufacturing business in the Chinese market has been authorized to Foxconn which also acts as a CM. Phobio acts as the IR in the European market and is only responsible for remanufacturing used products for Apple but not for remarketing these remanufactured products (Zhou & Yuen, 2021). Considering the above choices that are available for OEMs, several questions arise, which are: what remanufacturing mode should be used when coping with a different remanufacturing partner, and which party (i.e. the CM or the IR) should the OEM cooperate with when a remanufacturing mode is given.

The efficiency of remanufacturing operations highly depends on the collection process of used products as remanufacturing brings used products to 'like-new' or 'as-new' conditions by recovering values from these used products (Atasu et al., 2008). This makes the process of remanufacturing quite different from the process of manufacturing new products. However, the reliance on collected used products complicates remanufacturing operations by affecting remanufacturing costs. This is because customers can return their used products during the whole product life cycle (Guide et al., 2003), and different customers have different use rates. As a result, collected used products vary in quality, and remanufacturing cost varies when processing these used products (Wang et al., 2017).

Collection uncertainty, especially quality uncertainty, plays a vital role in shaping a firm's remanufacturing decision as the firm's remanufacturing cost structure changes according to the quality of used products (Atasu & Souza, 2013). However, the impact of quality of collected used products on OEM's remanufacturing mode and partner decisions has not been examined by the current literature.

This research aims to examine an OEM's remanufacturing-related decisions by considering the quality of used products, i.e. how should an OEM choose its remanufacturing partner when the remanufacturing mode is given and what remanufacturing mode should the OEM use with a specific remanufacturing partner. This study examines a supply chain with three players, i.e. an OEM, a CM and an IR. The OEM outsources the production process of new products to the CM and then directly manages the marketing of new products. For the remanufacturing business, it can either be conducted by the CM or by the IR through two remanufacturing modes, i.e. outsourcing and authorization. In each remanufacturing mode, the remanufacturing process of used products is always conducted by the remanufacturer which can either be the CM or the IR, but the marketing of remanufactured products can either be managed by the OEM or by the remanufacturer. The OEM manages the marketing of remanufactured products itself in the outsourcing mode, while remanufactured products are sold by the remanufacturer in the authorization mode. Due to the quality of used products, the remanufacturer faces uncertainty in remanufacturing costs due to the various conditions of the collected used products, and this will affect the remanufacturer's decision in accepting OEM's remanufacturing cooperation. More specifically, the following research questions are addressed in this research.

(1) What are the equilibrium outcomes when the OEM cooperates with different remanufacturers (i.e. CM or IR) by using different remanufacturing modes (i.e. outsourcing and authorization)?
(2) How do the remanufacturing partner and mode selection decisions affect supply chain members' profit and environmental impact? Do the economic goal in line with the environmental goal?
(3) How the OEM's decisions on remanufacturing partner and mode are affected by the CM's pricing behavior on new products?

The OEM's remanufacturing operation decisions regarding remanufacturing mode and partner choice have been widely examined in the literature. For instance, Zou et al. (2016), Zhang, Chen, and Mi (2020) and Feng et al. (2020) studied an OEM's remanufacturing mode decision under different contexts in the IR remanufacturing scenario. Zhou and Yuen (2021) studied an OEM's remanufacturing mode and partner decisions, and Zhou et al. (2023) considered an OEM's strategic remanufacturing cooperation decision with its CM by considering the quality of used products. However, to the best of our knowledge, there is the first study that simultaneously examines the impact of the quality of used products on the OEM's remanufacturing partner and remanufacturing strategy decisions. Compared with the existing literature, this research has the following noteworthy features. First, this research incorporates collection uncertainty into the remanufacturing cost to examine the impact of the collected used products' quality on OEM's remanufacturing mode and partner decisions. As such, the remanufacturer needs to determine the quality level of used products needed for remanufacturing before conducting detailed remanufacturing work. This consideration can help the OEM and the remanufacturer make more accurate remanufacturing decisions. In terms of the OEM's remanufacturing partner and mode decisions, Zhou and Yuen (2021) also investigated similar questions. However, the main difference is that they did not consider collection uncertainty, which is an important parameter that would affect the OEM's remanufacturing-related decisions as remanufacturing processes are highly dependent on the collection of used products. Furthermore, the consideration of collection uncertainty by considering the quality of used products enables us to examine the environmental performance of different remanufacturing-related decisions, which is a dimension that has not been studied by Zhou and Yuen (2021). Second, given the fact that CMs manufacture new products for OEMs who do not have manufacturing capability, these CMs can naturally act as remanufacturing partners again as the process of remanufacturing is similar to that of manufacturing. Therefore, the CM competes with the OEM's new products by selling remanufactured products if it engages in remanufacturing. Similar relationships have been examined by Zhou, Meng, and Yuen (2021) and Zhou, Meng, Yuen, and Sheu (2021), but they only focused on a remanufacturing partner (i.e. CM) with a remanufacturing mode (i.e. authorization). Third, in addition to examining the OEM's remanufacturing mode cooperation decisions with the CM by considering the impact of collection uncertainty, the study also considers the traditional case that remanufacturing activities are managed by the IR whose primary business is remanufacturing. Henceforth, deciding which one to cooperate with in remanufacturing and which mode to use become decision problems for the OEM, which have not been considered by Zhou et al. (2023). Finally, considering the OME's remanufacturing partner decision, we find that, depending on the CM's strategic decisions on the wholesale price of new products, the IR is always the preferable remanufacturing partner when the wholesale price is strategically determined, which enriches the conclusions in Zhou et al. (2023).

The rest of this paper is organized as follows. A literature review is provided in Section 2. Model assumptions and model setup are shown in Section 3. The optimal responses of the supply chain members, i.e. the OEM, the CM and the IR, when different remanufacturing modes are implemented are examined in Section 4. Section 5 compares the models presented in Section 4. Section 6 provides a further discussion on how remanufacturing costs affect the supply chain members' decisions. The conclusions of this research are summarized in Section 7. All proofs are presented in the Appendix.

2. Literature review

This research is built on four streams of literature: (a) remanufacturing authorization, (b) remanufacturing outsourcing, (c) collection uncertainty and (d) channel decision. Each of which will be reviewed below.

It has been widely acknowledged that customers' demand for new products will be severely cannibalized by the existence of remanufactured products (Atasu et al., 2008, 2010; Ma et al., 2017; Ovchinnikov, 2011; Wu & Zhou, 2016; Xu et al., 2021). Conventionally, for OEMs who do not participate remanufacturing themselves, authorizing remanufacturers to conduct all the remanufacturing activities is a good way for OEMs to participate in the remanufacturing business without affecting their sale of new products. For OEMs, a critical trade-off decision for remanufacturing authorization is whether the profit loss of new products affected by the selling of remanufactured products can be compensated by the indirect benefit from the authorization fee (Oraiopoulos et al., 2012). Based on the unique industry practice for information technology equipment where relicensing is needed for refurbishing, Oraiopoulos et al. (2012) identified when the OEM should eliminate the remanufacturing market or try to embrace it. Liu et al. (2018) compared three remanufacturing models, i.e. the OEM does not participate in remanufacturing operations, the OEM remanufactures itself, and the OEM authorizes an IR to conduct remanufacturing on behalf of itself, to investigate when remanufacturing authorization is an optimal remanufacturing model for the OEM. Ma et al. (2018) used a game-theoretical model to investigate when the OEM should authorize its remanufacturing business to an IR where competition and cooperation exist between the two parties. Competition may exist between unauthorized and authorized remanufacturing parties if the OEM chooses to authorize one of the IRs as its remanufacturing partner. Zhou et al. (2020) examined how the OEM's remanufacturing authorization strategy decision was affected by market competition between unauthorized and authorized remanufactured products. Zhou, Meng, and

Yuen (2021) incorporated the OEM's and the remanufacturer's bargaining power into remanufacturing authorization cooperation to examine the determination process of the authorization fee and identified the conditions under which a Pareto improvement region will exist. A dealer who sells new products for the OEM may also become as an authorized remanufacturer, i.e. dealer authorization. Jin et al. (2022) compared dealer authorization with traditional remanufacturing authorization to examine the impact of cooperation and competition between an OEM and its dealer on the OEM's remanufacturing authorization decisions and investigated when dealer authorization becomes optimal for the OEM. OEMs who do not have the manufacturing capability and have outsourced their production process of new products to a CM may naturally cooperate with their CMs again in remanufacturing. Zhou, Meng, Yuen, and Sheu (2021) developed a Nash bargaining game to examine the OEM's remanufacturing authorization cooperation decision with a CM when it acts as a partner in the upstream and a competitor in the downstream. This research also examines trade-offs for OEMs in remanufacturing authorization decision process. However, this study extends current research from two perspectives. First, based on current industry practice where an IR may also become an authorized remanufacturer for OEMs who do not have manufacturing capability, this research compares the scenario where the CM is the remanufacturer with the scenario where the IR is the remanufacturer to select an optimal remanufacturing cooperator for the OEM. Second, the research not only considers remanufacturing authorization as the sole remanufacturing mode but remanufacturing outsourcing as well to examine which remanufacturing mode can give the OEM a higher profit.

Remanufacturing outsourcing is another way for OEMs to engage in remanufacturing market but in a more direct way, many supply chain members can conduct remanufacturing outsourcing activities for OEMs, and the outsourcing process is quite similar to the production outsourcing process of new products. First, retailers may enter the remanufacturing market though their main business is to sell new products for OEMs. Wang et al. (2017) examined the reverse channel design decision for a retailer and investigated when the retailer should conduct these reverse channel-related activities in-house or outsource this activity to other supply chain members. Timoumi et al. (2021) compared two remanufacturing models, i.e. model M where the manufacturer conducts remanufacturing itself, and Model R where the manufacturer outsources remanufacturing to its retailer, to answer the questions of whether and when model R is an optimal choice for the manufacturer. Second, for OEMs who do not have manufacturing capability, remanufacturing can also be outsourced to the OEM's original CM. Zhou and Yuen (2021) studied the remanufacturing mode decision for an OEM when its CM becomes a possible remanufacturing partner. In the scenario where remanufacturing is conducted by IRs, OEM's preference over remanufacturing outsourcing and authorization has also been examined. Zou et al. (2016) compared remanufacturing outsourcing with remanufacturing authorization to examine OEM's and IR's preferences over the two remanufacturing modes and found that the outsourcing mode is always beneficial for the OEM. Feng et al. (2020) incorporated environmentally friendly behaviors of customers and firms into models to investigate OEM's and IR's choices between the two remanufacturing modes, i.e. outsourcing and authorization. They found that the choice of the OEM and the IR is highly dependent on green customers' preference for new and remanufactured products. Considering the competitiveness of the supply chain, Zhang, Chen, and Mi (2020) studied the impact of chain competition on OEMs' and IRs' remanufacturing mode decisions. In a scenario where new products are directly sold to the market by the OEM, this research also examines the OEM's remanufacturing partner decision under two remanufacturing modes in a supply chain where all supply chain members have a connection with the OEM may become a possible remanufacturing partner. Moreover, this research considers the impact of uncertainty in the collection on remanufacturing cost and OEM's remanufacturing mode and partner decisions.

The fact that customers can return their used products during the whole life cycle of the product complicates the reverse supply chain and creates uncertainty for the used products collection (Guide et al., 2003; Zhou et al., 2022). A direct impact of collection uncertainty is that it changes a firm's sorting policy for used products. Galbreth and Blackburn (2006) incorporated used product condition variability into a total cost model for a firm's optimal sorting policy for used products when its goal is to minimize the total cost and when firms facing both deterministic and uncertain demand. Another impact of collection uncertainty is that it affects a firm's remanufacturing cost structure and then subsequently affects the firm's remanufacturing related decision. Atasu and Souza (2013) considered three general product recovery forms where each form of product recovery leads to different remanufacturing costs to examine how the firm's quality decision is affected by product recovery. Atasu et al. (2013) examined a manufacturer's design of a reverse channel in a scenario where diseconomies of scale may arise during collection, and how the manufacturer's decision is affected by collection cost structure. Chuang et al. (2014) investigated a manufacturer's choice of the reverse channel by considering two kinds of collection cost structures driven by collection uncertainty. This research contributes to this stream of literature by also incorporating collection uncertainty into a firm's remanufacturing decision and then examining the impact of collection uncertainty on the firm's remanufacturing mode and partner decisions. Driven by collection uncertainty, the remanufacturer will first sort used products based on their quality levels as different quality levels will cause different production costs, and then make the appropriate remanufacturing decision to maximize its profit.

Finally, this research also relates to the literature that examines channel decisions in a supply chain. Traditional research in supply chain management studies the channel decision for the collection of used products. For example, Savaskan et al. (2004) examined three classic collection models, i.e. models M, R and 3P where manufacturer, retailer and third-party respectively manage the collection of used products, to examine which model can help the OEM perform better with considering the coexistence of new and remanufactured products. They found that model R where the retailer manages the collection of used products can help the OEM gain a higher profit as the retailer is closer to customers. Savaskan and Van Wassenhove (2006) further examined manufacturers' reverse collection channel design when competition exists among retailers. In the context of dual channel setting, He et al. (2018) studied how the firm should design its extended warranty service when customers have different channel preferences. Xiao et al. (2021) extended customer returns to e-tailing and examined how should the manufacturer manage collection responsibility when e-tailers can take the collection responsibility and may have higher channel efficiency. The research that studies channel decisions for remanufactured products is growing. Yan et al. (2015) considered an OEM's market channel decision for remanufactured products in a traditional three-tier supply chain where new products are sold to the market through a retailer, and the marketing of remanufactured products can either be conducted by the OEM itself through e-channel or be outsourced to a third-party. Gan et al. (2017) investigated pricing decisions for new and remanufactured products when the two kinds of products are deliberately offered in separate channels. He et al. (2019) examined how an OEM's channel structure and pricing decisions are affected by government subsidy when new products are sold to the market through a retailer while the marketing of remanufactured products may either be conducted by the OEM itself or by a third-party. Zhang, Chen, Xiong et al. (2020) studied whether the manufacturer should authorize the marketing of remanufactured products to an authorized remanufacturer when it already cooperated with the remanufacturer in the collection of used products. This research also examines an OEM's marketing channel decision for remanufactured products when the collection of used products is managed by the remanufacturer. Furthermore, this research extends current studies by considering the channel decision of remanufactured products for a special kind of OEMs that do not have the manufacturing capability and have outsourced their production process.

3. Model assumptions and notations

This research considers a supply chain with three players which are a CM, an IR and an OEM. The three supply chain players compete in the market and interact as follows: the OEM cooperates with the CM in the contract manufacturing of new products and may either select the CM or the IR as the remanufacturing partner through outsourcing or authorization mode. In the outsourcing mode, the remanufacturer manages the contract remanufacturing of used products but not the marketing of these remanufactured used products. The marketing of new and remanufactured products is managed by the OEM, and new and remanufactured products coexist in the OEM's sale channel. In the authorization mode, all activity related to remanufacturing is authorized to the remanufacturer by the OEM. As such, the remanufacturer is not only responsible for remanufacturing but also for the marketing of remanufactured products. Hence, new products exist in the OEM's sale channel, while the marketing of remanufactured products is managed by the remanufacturer. This kind of supply chain structure can be found in practice. For example, Apple is referred to as the OEM, Foxconn is referred to as the CM and the IR may refer to Aihuishou. In the following, assumptions made in this study are discussed.

ASSUMPTION 1 *Customers are heterogeneous regarding their willingness to pay (WTP) either for new products or for remanufactured products.*

This assumption is widely used in the operations literature, such as Oraiopoulos et al. (2012). Customers' valuation for new products is$v$, and the valuation is uniformly distributed in the interval$[0, 1]$. Uniform distribution aims to ensure that the results are analytically tractable. Without loss of generality, the market size is normalized to 1 and each customer buys at most one unit of product.

ASSUMPTION 2 *Customers' WTP for new products is higher than that for remanufactured products.*

To model this, a customer with a WTP v for a new product has a WTP αv for a remanufactured product. For customers, new and remanufactured products provide exactly the same value when $\alpha = 1$, and remanufactured products provide no value when $\alpha = 0$. In fact, customers' WTP for remanufactured produts is still lower than that for new products even if the quality level of remanufactured products is the same as that of new products (Agrawal et al., 2015). Hence, let $\alpha \in (0, 1)$ denote customers' valuation discount for remanufactured products. Let p_n and p_r repectively represent the sales prices of new and remanufactured products. The utility a customer can gain from purchasing a new product is $U_n = v - p_n$, and the utility a customer can gain from purchasing a remanufactured products is $U_r = \alpha v - p_r$. Customers will buy the product that can give them the highest utility, and the outside option is normalized to 0. Therefore, customers will purchase a new product when $U_n \geq U_r$ and $U_n \geq 0$ and choose a remanufactured product when $U_r \geq U_n$ and $U_r \geq 0$. By comparing the two utility functions, customers' demand for new products is denoted as $q_n = 1 - \frac{p_n - p_r}{1 - \alpha}$ and customers' demand for remanufactured products is denoted as $q_r = \frac{p_n - p_r}{1 - \alpha} - \frac{p_r}{\alpha}$, where q_n and q_r respectively denote the demand function for new and remanufactured products.

Assumption 3 *The OEM cooperates with the CM through a wholesale price contract for production outsourcing.*

Let w_n denote the wholesale price of new products, and it is an exogenous parameter. The CM's production cost of new products is denoted as c_n, and $w_n \geq c_n$. The wholesale price contract has been commonly considered in the operations management literature to examine an OEM's production outsourcing decision (Bolandifar et al., 2016; Niu et al., 2015; Wang et al., 2013, 2015; Xu et al., 2018). Depending on the OEM's and the CM's bargaining powers, the value of w_n is negotiable between the two parties. For instance, Wang et al. (2013) used a generalized Nash bargaining game to examine the impact of the negotiation of the wholesale price on the OEM's production outsourcing cooperation decisions with a CM. However, this study assumes that w_n is an exogenously given parameter. This is to focus on the OEM's remanufacturing partner decision in a given remanufacturing mode, and the OEM's remanufacturing mode decision with a given remanufacturing partner. Hence, the determination process of new products' production outsourcing is simplified to focus on the OEM's decision relates to remanufacturing business. Second, price competition is severe among CMs given the prevalence of production outsourcing. As a result, price alliances or associations may exist for CMs, and this leads to an industry standard price for production outsourcing. Similar assumptions can also be found in the research done by Wang et al. (2013). Finally, the robustness of the main analysis will be discussed in Section 6 by endogenizing the CM's wholesale price decision.

Assumption 4 *In the outsourcing mode, the OEM and the remanufacturer cooperate through a wholesale price contract, while a licensing agreement contract is used in the authorization mode.*

In the outsourcing mode, the wholesale price of remanufactured products is denoted as w_r, and the OEM charges the remanufacturer a unit of authorization fee p_s for selling remanufactured products in the authorization mode. To focus on the OEM's remanufacturing mode and partner decisions, this researh simplifies the determination process of w_r and p_s by assuming that the wholesale price of remanufactured products is determined by the remanufacturer, while the authorization fee is decided by the OEM. For the wholesale price of remanufactured products, similar assumption can be found at the research done by Timoumi et al. (2021). The assumption on the determination of the authorization fee p_s can also be found in the research done by Liu et al. (2018), Zhou et al. (2020) and Zhou, Meng, Yuen, and Sheu (2021).

Assumption 5 *The number of used products that can be collected is subject to market conditions.*

The amount of collected used products is denoted as S, where $S \in (0, 1)$. This assumption seperates the market of used products with the market of new products though new products are the core source of used products. This is because the remanufacturing process of used products is usually decoupled from the collection process (Wang et al., 2017). Besides, the market size of used products is generally much smaller than that of new products (Chen & Chen, 2019). This reduces the volume uncertainty of collecting process and, the supply of cores for remanufacturing is abundant. However, the number of products that can be remanufactured is always smaller than the number of products that can be collected from the market. As such, $q_r \leq S$.

ASSUMPTION 6 *The remanufacturing decision of the remanufacturer depends on the quality of collected used products.*

Let random variable θ represent the quality of used products. For tractability of the decisions and being consistent with Wang et al. (2017), this research also assumes that the quality of used products follows a uniform distribution in the interval [0, 1]. Not all collected items will be remanufactured, and the remanufacturer decides the quality threshold of used products, which is denoted as θ_i, such that used products whose quality level is higher than θ_i will be remanufactured. Therefore, the total amount of remanufactured products is$S\int_{\theta=\theta_i}^{1} f(\theta)d\theta = S(1 - F(\theta_i))$. The number of products that can be remanufactured is always smaller than the number of products that can be collected from the market. Hence, $q_r \leq S\int_{\theta=\theta_i}^{1} f(\theta)d\theta = S(1 - F(\theta_i))$.

ASSUMPTION 7 *The remanufacturer's variable cost of remanufacturing depends on the quality of collected used products, and a high quality of collected used products leads to a low variable cost.*

Let c_i represent remanufacturer i's unit variable cost of remanufacturing, where $i \in \{m, r\}$ indexes the remanufacturer CM and IR. The unit variable cost of remanufacturing appears in an aggregate form, which means that all costs, such as collection fee, transportation cost, sorting fee and so on, happened in the collection and remanufacturing processes are incorporated into the variable cost (Örsdemir et al., 2014). More remanufacturing processes will be involved if the quality of used products is low and this will lead to a high remanufacturing cost. Hence, depending on the quality of used products, the actual variable cost of remanufacturing is denoted as $C_i(\theta) = c_i(1 - \theta)$. This form of remanufacturing cost is also considered by Wang et al. (2017) to examine the impact of quality uncertainty on the firm's remanufacturing-related decision, which incorporates the characteristics that the actual variable cost of remanufacturing decreases with the increase of the quality of used products. In other words, when the quality of the used products to be remanufactured is high, the cost of remanufacturing should be low. The remanufacturer i's total expected variable cost can be formulated as. $S\int_{\theta=\theta_i}^{1} C_i(\theta_i)f(\theta)d\theta = S\int_{\theta=\theta_i}^{1} c_i(1 - \theta_i)f(\theta)d\theta$.

ASSUMPTION 8 *In addition to the variable cost of remanufacturing, the remanufacturer (i.e. CM or IR) needs to incur fixed cost to engage in remanufacturing.*

The remanufacturer needs to incur a one-time investment cost to engage in remanufacturing so that it has the expertise or infrastructure to remanufacture. Given the fact that the CM also manufactures new products, the CM naturally has more cost advantage in remanufacturing than that of the IR as the remanufacturing process in most scenarios is to retore the original function of new products. Besides, the infrastructure that used to manufacture new products can also be reused in the remanufacturing process though additional capacity investment may be needed. Therefore, the fixed cost of the CM incurred for remanufacturing is normalized to 0, and the fixed cost that the IR incurred for remanufacturing is denoted as $\beta(1 - c_r)$ with $\beta > 0$. This cost function form incorporates the connection between variable cost and fixed cost, that is, a higher fixed cost leads to a lower variable cost.

ASSUMPTION 9 *The environmental impact of remanufacturing is measured by the percentage of collected used products that is remanufactured.*

One of the positive externalities of remanufacturing is that used products can be prevented from landfilling through remanufacturing (Timoumi et al., 2021). As such, the environmental impact of remanufacturing can be measured by the percentage of collected products that are remanufactured. In Assumption 6, the remanufacturer will only remanufacture the used product if its quality level is higher than θ_i. Hence, θ_i can also be used as an indicator of environmental impact of remanufacturing, and remanufacturing is more beneficial to the environment with a lower value of θ_i. Given a constant market condition, a lower value of θ_i means that more collected used products will be remanufactured. As such, less used products will be ended with landfilling. Similar measurement on environmental impact of remanufacturing can also be found in the research done by Wang et al. (2017), Timoumi et al. (2021) and so on. To reflect the relationship between the profit and environmental impact, 'congruence region' and 'conflict region' are defined. In the congruence region, the profit and environmental performance are in line with each other, and strategy selection dominates in profit and environmental impact. While in the conflict region, the profit and environmental performance conflict with each other. Therefore, the decision maker needs to determine the relative importance between environment and profit (Koo et al., 2014).

Based on these descriptions, the OEM can cooperate with two remanufacturers through two remanufacturing modes. Hence, there are four models available which are denoted as $X \in \{OM, OR, AM, AR\}$, where O and A respectively denote the outsourcing and authorization modes, and M and R respectively represent remanufacturer CM and IR. For example, model OM denotes the CM being the remanufacturer in the outsourcing mode. The four remanufacturing models are depicted in Figure 1.

Let $\prod_i^j$ denote member i's profit in model j, where $i \in \{O, M, R\}$ represents OEM, CM and IR, and $j \in \{OM, OR, AM, AR\}$ denotes model OM, model OR, model AM and model AR. The OEM's, the CM's and the IR's optimal response in these model structures are first examined in Section 4, and comparison of these models is presented in Section 5.

4. Model analysis

This section examines the optimal response of the three supply chain players (i.e. OEM, CM and IR) in the four remanufacturing models: *OM, OR, AM* and *AR*. The OEM's, the CM's and the IR's optimal decisions in the outsourcing and authorization modes are respectively discussed in Sections 4.1 and 4.2.

4.1. *Outsourcing modes – models OM and OR*

Similar to the production outsourcing of new products, the OEM outsources the remanufacturing activity of remanufactured products to the remanufacturer in the outsourcing mode and conducts the marketing of remanufactured products itself. As such, new and remanufactured products both exist in the OEM's sale channel, and the OEM benefits from selling the two kinds of products. As the CM and the IR can both be the remanufacturer, the profit generated from contract remanufacturing of used products may either belong to the CM or the IR. In addition, the CM also benefits from contract manufacturing

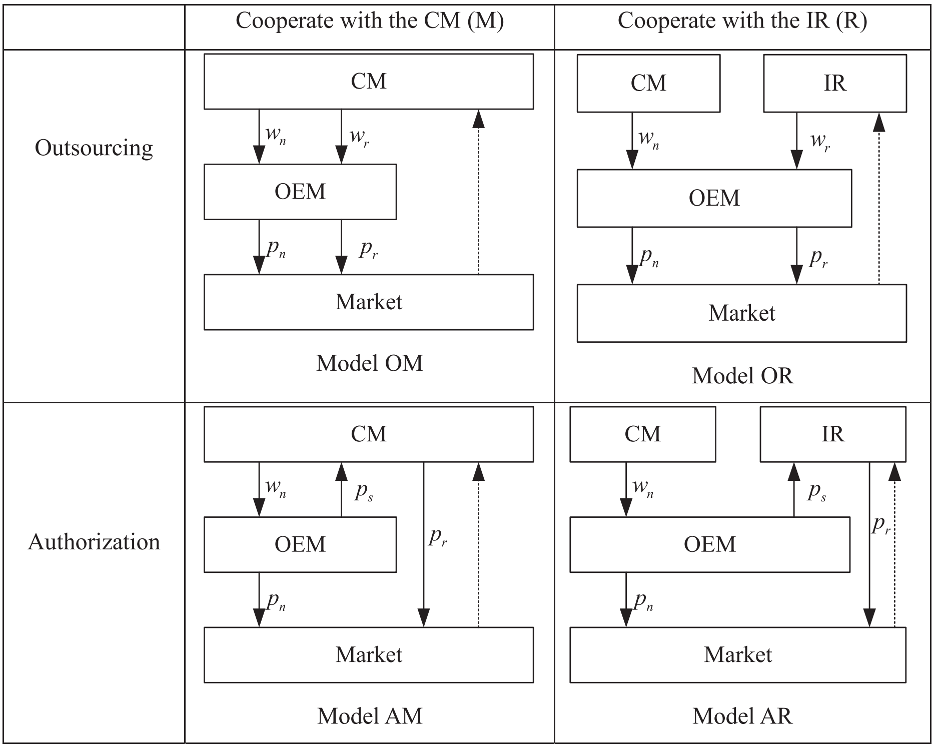

Figure 1. Model structures and decisions.

of new products. However, the IR will exit the market if it does not engage in remanufacturing. Therefore, in model OM where the CM is the remanufacturer, the OEM's and the CM's profit functions are formulated as follows:

$$\max \prod_{O}^{OM} (p_n, p_r) = (p_n - w_n)q_n + (p_r - w_r)q_r$$

$$\max \prod_{M}^{OM} (w_r, \theta_i) = (w_n - c_n)q_n + w_r q_r - \int_{\theta_i}^{1} c_m S(1 - \theta_i) f(\theta) d\theta$$

In model OM, the IR is not the OEM's selected remanufacturer. It may or may not enter the remanufacturing market, and its optimization problems are out of the discussion scope of this research. In model OR where the IR acts as the remanufacturer, the OEM's and the IR's profit functions are formulated as follows:

$$\max \prod_{O}^{OR} (p_n, p_r) = (p_n - w_n)q_n + (p_r - w_r)q_r$$

$$\max \prod_{R}^{OR} (w_r, \theta_i) = w_r q_r - \int_{\theta_i}^{1} c_r S(1 - \theta_i) f(\theta) d\theta - \beta(1 - c_r)$$

In model OR, the CM is not the selected remanufacturer. But it will manufacture new products, and its profit will still be affected by OEM's decisions on the new products though it does not make decisions itself. The CM's profit function in model OR is $\prod_{M}^{OR} = (w_n - c_n)q_n$. In the two models, customers' demand for remanufactured products is subject to the constraint $q_r \leq S\int_{\theta}^{1} f(\theta)d\theta = S(1 - F(\theta))$, which means that the number of products that can be remanufactured is always smaller than the number of products that can be collected from the market. This constraint comply with the current research in the operations management literature, such as Wang et al. (2017), and will finally affect the OEM's, the CM's and the IR's optimal decisions. There is a three-stage game between the OEM and the remanufacturer. The remanufacturer decides the quality threshold (θ) such that any collected products with a quality level higher than the threshold will be remanufactured in the first stage of the game. In the second stage, the remanufacturer decides the wholesale price of remanufactured products (w_r) charged to the OEM. Finally, the OEM determines the sales price of new and remanufactured products (p_n, p_r).

The concavity of the optimization problems can be proved by Hessian matrices, and we can solve the problem by backward induction. Table A1 (see Appendix 1 for details) summarizes the optimal solution for the outsourcing mode, and $c^{OM} = c_n\alpha - 4S\alpha(1 - \alpha)$ and $c^{OR} = w_n\alpha - 4S\alpha(1 - \alpha)$. The values of c^{OM} and c^{OR} should be strictly limited to (0, 1) to make the discussion feasible and valid, which means the parameters w_n, c_n, α and S shall satisfy certain conditions. In the following, we assume that $4S(1 - \alpha) < c_n < w_n < 1$ (i.e. $c^{OM}, c^{OR} \in (0, 1)$) always holds to ensure the validity of the discussion.

Table A1 implies how the CM's and IR's variable cost of remanufacturing (c_m and c_r) derives the optimal solution: (a) when the CM's and the IR's variable cost of remanufacturing is low (i.e. $c_m \in (0, c^{OM})$and $c_r \in (0, c^{OR})$), all collected items will be remanufactured ($\theta = 0$); (b) when the CM's and the IR's variable costs for remanufacturing are high (i.e. $c_m \in (c^{OM}, 1)$ and $c_r \in (c^{OR}, 1)$), the remanufacturer will become more selective on which cores to remanufacture, and not all collected items will be remanufactured ($0 < \theta < 1$). In addition, the wholesale price for remanufactured products and the sales price for new and remanufactured products are invariant to the variable cost of remanufacturing when c_m and c_r are at a low level. This is because the remanufacturer will remanufacture all the collected used products if it can produce at a low cost. However, when the variable cost of remanufacturing increases, the remanufacturer will pass a proportion of the cost increase to consumers and the OEM. The remanufacturer will increase the wholesale price of remanufactured products, and consequently will induce the OEM to increase the sales price of remanufactured products. In addition, the sales price for new products remains unchanged in the four cases (as shown in Table A1). The underlying intuition is that the OEM increases the sales price of remanufactured products with the increase of the wholesale price. As such, the OEM can maintain the competitiveness of new products by maintaining the sales price. The new product will become more attractive when the remanufactured product becomes more costly for the consumers.

The managerial insight of these results is that the remanufacturer should remanufacture all collected items available for remanufacturing when it has a relatively low remanufacturing cost. However, when the remanufacturing cost increases, the remanufacturer should set the quality threshold at a proper level so that it can control the total remanufacturing cost. The increase of the remanufacturing cost motivates the remanufacturer to increase the wholesale price of remanufactured products, and the OEM will adjust the sales of remanufactured products accordingly with the increase of the wholesale price.

Next, we investigate the optimal solutions' comparative statics with respect to consumers' preference for the remanufactured product (α), the amount of used products available for remanufacturing (S) and wholesale price of new product (w_n). All the results are summarized in Table A2 (see Appendix 1 for details).

Table A2 shows how the three parameters affect the optimal solutions in Table A1. The optimal solutions are divided into two cases according to the variable cost of remanufacturing. The remanufacturer will set the quality threshold at the lowest level and remanufacture all the collected used products when it can remanufacture at a low cost. As such, the OEM's and the remanufacturer's most optimal solutions are not affected by the three parameters. However, the remanufacturer will change its remanufacturing strategy and only remanufacture those qualified used products when the remanufacturing cost increases. The OEM's and remanufacturer's optimal decisions will be affected by the three parameters with the change of the remanufacturing strategy. Intuitively, consumers' high preference for remanufactured products and a lower availability of used products will motivate the remanufacturer to set the quality threshold θ at a low level so that it can remanufacture more items to satisfy consumers' demands. A higher wholesale price stimulates the remanufacturing activity when the IR is the remanufacturer. However, when the CM is the remanufacturer, its remanufacturing level is not affected by the wholesale price as the impact of the wholesale price on remanufacturing activity is already reflected through the production cost. The CM will decrease the quality threshold to remanufacture more used items when it has to produce new products at a high cost.

In addition, the sales price for new products is not affected by consumers' preference for the remanufactured product (α) or the amount of used products available for remanufacturing (S). However, the wholesale price and sales price for remanufactured products will change accordingly in response to different market conditions. As discussed earlier, the OEM can maintain its competitive position by not changing the sales price of new products. The OEM will increase the sales price for remanufactured products or increase the quantity available for sale when consumers have a high preference for remanufactured products. In addition, a higher wholesale price for the new product will also induce the OEM to increase p_r. This is because the OEM's new product's margin decreases as w_n increases. Therefore, the OEM needs to increase the sales price for remanufactured products to compensate the profit loss of new products.

4.2. *Authorization mode – models AM and AR*

In the authorization mode, the OEM only manages the marketing of new products and authorizes the remanufacturer to conduct the remanufacturing and marketing of remanufactured products on behalf of itself. As the cost of authorization, the remanufacturer shares its remanufacturing profit with the OEM by paying the OEM authorization fee. In this mode, new products are managed by the OEM, while the remanufacturer manages the marketing of remanufactured products. Hence, the OEM benefits from selling new products and the charge of authorization fee from the remanufacturer. The CM always benefits from new products' contract manufacturing, and it can also make profit from remanufacturing if it is the remanufacturer. However, the profit generated from remanufacturing business belongs to the IR if the OEM chooses to select the IR as the remanufacturing partner. Therefore, in model AM where the CM is the remanufacturer, the OEM's and the CM's profit functions are formulated as follows:

$$\max \prod_{O}^{AM}(p_n, p_s) = (p_n - w_n)q_n + p_s q_r$$

$$\max \prod_{M}^{AM}(p_r, \theta_i) = (w_n - c_n)q_n + (p_r - p_s)q_r - \int_{\theta_i}^{1} c_m S(1-\theta_i)f(\theta)d\theta$$

Similarly, in model AM, the IR is not the selected remanufacturer and its optimization problems in this model are out of the discussion scope of this research. In model AR where the OEM licenses the IR to manage the remanufacturing activity, the OEM's and the IR's profit functions are formulated as follows:

$$\max \prod_{O}^{AR}(p_n, p_s) = (p_n - w_n)q_n + p_s q_r$$

$$\max \prod_{R}^{AR}(p_r, \theta_i) = (p_r - p_s)q_r - \int_{\theta_i}^{1} c_r S(1-\theta_i)f(\theta)d\theta - \beta(1-c_r)$$

In model AR, the CM's profit function is $\prod_{M}^{AR} = (w_n - c_n)q_n$ and it will be affected by the OEM's decisions on new products though the CM does not make any decisions in the process. Similar to the outsourcing mode, customers' demand for remanufactured products in the two authorization models is also subject to a constraint $q_r \leq S\int_{\theta}^{1} f(\theta)d\theta = S(1 - F(\theta))$, showing that the number of products that can be remanufactured is always smaller than the number of products that can be collected from the market. There is a three-stage game between the OEM and the remanufacturer. The remanufacturer determines the quality threshold (θ) which identifies what quality level of used products is qualified for remanufacturing in the first stage of the game. In the second stage, the OEM decides the unit authorization fee (p_s) charged to the remanufacturer. Finally, the OEM and the remanufacturer simultaneously and respectively decide their sales prices for new and remanufactured products (p_n, p_r). We assume that the remanufacturer determines the quality threshold before the OEM determines the authorization fee. This is because the authorization fee is a revenue sharing mechanism between the remanufacturer and the OEM. The existence of such authorization fee would affect the remanufacturer's production cost, which in turn affects the decision of the quality threshold (θ).

The concavity of the optimization models can be proved by their Hessian matrices being negative definite. We can solve the problems by backward induction. Table A3 (see Appendix 1 for details) presents the equilibrium solutions, and $c^{AM} = c_n - (4S + w_n)(1-\alpha)$ and $c^{AR} = w_n\alpha - 4S(1-\alpha)$. Similarly, the values of c^{AM} and c^{AR} should strictly restrict to (0, 1) to make the discussion feasible. Hence, we assume that $(4S + w_n)(1-\alpha) < c_n < 1$ and $\frac{4S(1-\alpha)}{\alpha} < w_n < 1$. Note that, the setting of the discussion region is to make the analysis valid and tractable to focus on the main research questions. Other values that are outside of the constraints may be partially valid but are out of the discussion scope of this study and hence, we omit this part.

Most insights from Table A3 are similar to those from Table A1 when outsourcing mode is adopted by the OEM. First, the remanufacturer will remanufacture all the collected used products when it can remanufacture at a low variable cost, and as the variable cost increases, only a part of the collected used products will be remanufactured by the remanufacturer. Second, the authorization fee and sales price for new and remanufactured products are invariant to the variable cost when the variable cost for remanufacturing is low. In addition, the OEM will increase the authorization fee and sales price for new products when c_m and c_r are high.

However, in the previous section where outsourcing mode is adopted, we showed that the OEM will always maintain the sales price for new products whether the variable cost is high or low. By contrast, in this section, we find that the remanufacturer will always maintain the sales price for remanufactured products, whereas the OEM will adjust the authorization fee and the sales price for new products according to the variable cost for remanufacturing. This is because the marketing of remanufactured products is managed by the remanufacturer in the authorization mode. As such, a higher authorization fee does not necessarily lead to a higher sales price for remanufactured products as new and remanufactured products exist in different marketing channels. The OEM increases the sales price of new products correspondingly when the authorization fee increases and anticipates that the remanufacturer would increase sales price for remanufactured products. However, the optimal strategy for the remanufacturer is to maintain the sales price for remanufactured products since the sales price of new products is already increased. The net effect is to decrease customers' demand for new products but increase customers' demand for remanufactured products. In addition, we also notice that the remanufacturer needs to adjust its collecting quality threshold in the authorization mode (i.e. $\theta_1^{OM} < \theta_2^{AM}$, $\theta_2^{OM} < \theta_2^{AM}$, $\theta_1^{OR} < \theta_2^{AR}$ and $\theta_2^{OR} < \theta_2^{AR}$). A high collecting quality threshold does not only reduce the remanufacturer's remanufacturing cost but also leads to a smaller remanufacturing quantity. Hence, the remanufacturer can maintain the sales price of remanufactured products to maintain its market share.

Table A4 (see Appendix 1 for details) summarizes the comparative statics for the optimal solutions in the authorization mode.

Most insights from Table A4 are similar to those from Table A2. However, there are some exceptions. First, the quality threshold increases with the wholesale price of new products when the CM is the remanufacturer, while decreases with w_n when the IR is the remanufacturer. The latter result is consistent with the previous section where outsourcing mode is adopted. However, the CM will remanufacture less used products when the wholesale price of new products is high as it can earn enough profit from contract manufacturing. As a result, remanufacturing quantity is intentionally reduced by the CM. This implies that cooperating with the CM may not be environmentally friendly as the CM remanufacturing model does not necessarily lead to a high level of remanufacturing. Second, the sales price of new products will be affected by the three parameters as the remanufacturer maintains the sales price of remanufactured products in this scenario. This result is completely different from the result in the previous section where the OEM will always maintain the sales price of new products to maintain its competitiveness. In addition, the sales price of new products increases with the wholesale price and the availability of used products for remanufacturing but decreases with consumers' WTP for the remanufactured product.

5. Comparisons of different models

This section investigates three decisions: (1) the OEM's remanufacturing partner decision when the remanufactured product's distribution channel is given, (2) the remanufacturing mode decision when the OEM's remanufacturing partner selection strategy is given and (3) how partner and distribution choices affect the environmental performance. In Sections 5.1 and 5.2, we study the OEM's optimal remanufacturing partner choice under the outsourcing and authorization modes. In Section 5.3, we investigate the optimal distribution channel choice for remanufactured products when the OEM's remanufacturing partner selection strategy is given.

5.1. *Remanufacturing partner selection in the outsourcing mode*

In this section, we investigate the impacts of remanufacturing partner choice on the OEM's profit and the environment effect when new and remanufactured products are both managed by the OEM. We use subscript $k \in (1, 2)$ in the optimal solutions to denote low and high variable cost of remanufacturing, respectively. For instance, $\prod_{O1}^{OM}$ denotes the OEM's profit in model *OM* when the variable cost of remanufacturing is within the lower boundary (e.g. $c_m \in (0, c^{OM})$), and $\prod_{O2}^{OM}$ when the variable cost of remanufacturing is within the high boundary (e.g. $c_m \in (c^{OM}, 1)$). The following proposition characterizes the OEM's partner choice between the CM and the IR.

PROPOSITION 1 *In the outsourcing mode, depending on the cost of remanufacturing, cooperating with the CM and the IR can both become the dominant strategy that yields a higher profit and a better environmental impact.*

LEMMA 1 *Cooperating with the CM or the IR in the outsourcing mode has the following outcomes with respect to the OEM's profit and environmental impact*:

(a) When $c_m \in (0, c^{OM})$ and $c_r \in (0, c^{OR})$, cooperating with the CM and the IR yields an equal profit for the OEM and an equal (and 'best' possible) environmental impact;
(b) When $c_m \in (0, c^{OM})$ and $c_r \in (c^{OR}, 1)$, cooperating with the CM gives the OEM a higher profit and better environmental impact;
(c) When $c_m \in (c^{OM}, 1)$ and $c_r \in (0, c^{OR})$, cooperating with the IR gives the OEM a higher profit and better environmental impact;
(d) When $c_m \in (c^{OM}, 1)$ and $c_r \in (c^{OR}, 1)$, if $c_r > \dfrac{w_n(c_m + 4S\alpha(1-\alpha)) - 4Sc_n\alpha(1-\alpha)}{c_n}$, cooperating with the CM gives the OEM a higher profit and better environment outcome; otherwise, cooperating with the IR leads to the aforementioned outcome.

Based on Proposition 1 and Lemma 1, Figure 2 compares the two remanufacturing partner choices regarding the OEM's profit and environment impact, for the different combinations of c_m and c_r. Other parameters include $\alpha = 0.8$, $S = 0.2$, $w_n = 0.3$, $c_n = 0.2$. For consistency purposes, this set of parameters is applied to all the numerical results in this section. The regions bearing tags with the format $\prod$ and θ refer to the party's profit and remanufacturing impact in each model, respectively.

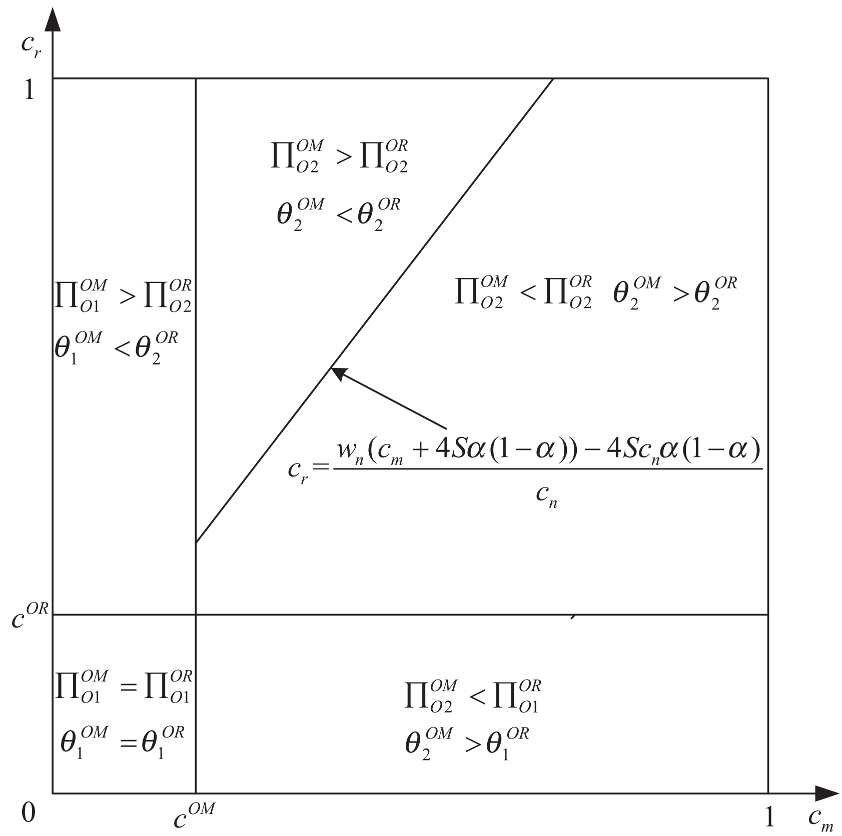

Figure 2. OEM's profit and environment impact in the outsourcing mode.

Proposition 1, Lemma 1 and Figure 2 yield how the OEM should select its remanufacturing partner when it determines to manage the marketing of remanufactured products itself. Intuitively, both cooperating with the CM and the IR can become the dominant strategy that achieves a higher profit and a better environment outcome. However, the condition that each remanufacturing partner becomes the optimal remanufacturing partner of the OEM may different. First, if both the CM and the IR produce the remanufactured product at low costs, the OEM can cooperate with either of them. If the CM remanufactures at a relatively low cost (i.e.c_m is small) and the IR remanufactures at a relatively high cost (i.e. c_r is large), then cooperating with the CM not only helps the OEM achieve a higher profit but also is friendly to the environment. The reverse favors cooperating with the IR. This indicates that, when the OEM can flexibly determine its remanufacturing partner, the OEM should cooperate with the one who has a lower variable cost. The OEM's remanufacturing partner decision will always make the OEM's economic goal in line with the environmental impact though the IR has to incur an additional fixed cost for remanufacturing. However, when both the CM and the IR remanufacture at relatively high costs, the optimal partner choice depends on specific conditions. If $c_r > \frac{w_n(c_m + 4S\alpha(1-\alpha)) - 4Sc_n\alpha(1-\alpha)}{c_n}$, then the OEM should select the CM as the remanufacturing partner; otherwise cooperating with the IR yields better outcomes.

Second, the congruence regions in the figure show that the profitability and environmental impact are not in conflict. The optimal remanufacturing partner choice for the OEM also dominates the other in terms of the environmental performance.

These insights are different with Zhou and Yuen (2021), which they found that, when the wholesale price is exogenously given, cooperation with the IR can always give the OEM a higher profit. However, we find that, if both the IR and the CM can remanufacture used products at a low cost, then either cooperating with the CM or the IR can help the OEM achieve the same profit, implying that remanufacturing partner decision becomes unimportant for the OEM. At the same time, cooperating with both the CM and the IR can achieve the best possible environment outcome, which is a conclusion that has not been found in Zhou and Yuen (2021). Furthermore, when the remanufacturing cost of the two parties increases, cooperating with the CM can still become a profitable choice, which serves as an explanation of the practice that OEM cooperates with the CM as considered in Zhou et al. (2023).

5.2. *Remanufacturing partner selection in the authorization mode*

This section examines the impacts of remanufacturing partner choice on the OEM's profit and the environmental impact when new and remanufactured products are managed by different parties. The OEM's partner choice between the CM and the IR are characterized in Proposition 2.

PROPOSITION 2 *In the authorization mode, depending on the cost of remanufacturing, cooperating with the CM and the IR can both become the dominant strategy that yields a higher profit and a better environmental impact.*

LEMMA 2 *Cooperating with the CM and IR in the authorization mode have the following outcomes with respect to the OEM's profit and environmental impact*:

(a) When $c_m \in (0, c^{AM})$ and $c_r \in (0, c^{AR})$, cooperating with the CM and the IR yields an equal profit for the OEM and an equal (and 'best' possible) environmental impact;
(b) When $c_m \in (0, c^{AM})$ and $c_r \in (c^{AR}, 1)$, cooperating with the CM gives the OEM a higher profit and better environmental impact;
(c) When $c_m \in (c^{AM}, 1)$ and $c_r \in (0, c^{AR})$, cooperating with the IR gives the OEM a higher profit and better environmental impact;
(d) When $c_m \in (c^{AM}, 1)$ and $c_r \in (c^{AR}, 1)$, if $\min(c_{a1}, c_{a2}) < c_r < \max(c_{a1}, c_{a2})$, cooperating with the IR gives the OEM a higher profit; if otherwise, cooperating with the CM leads to a higher profit. However, cooperating with the IR yields a better environmental impact if $c_r < c_{a1}$, where $c_{a1} = \dfrac{-4c_nS(1-\alpha) + w_n(4S(1-\alpha) + c_m\alpha)}{c_n - w_n(1-\alpha)}$ and $c_{a2} = \dfrac{4c_nS(1-\alpha) + w_n(c_m\alpha - 4S(1-\alpha)(1-2\alpha))}{w_n(1-\alpha) - c_n}$.

Most insights from Proposition 2, Lemma 2 and Figure 3 are similar to those from Proposition 1 and Figure 2 for the outsourcing mode. That is, whether the marketing of remanufactured products is managed by the OEM itself or not, either the CM or the IR

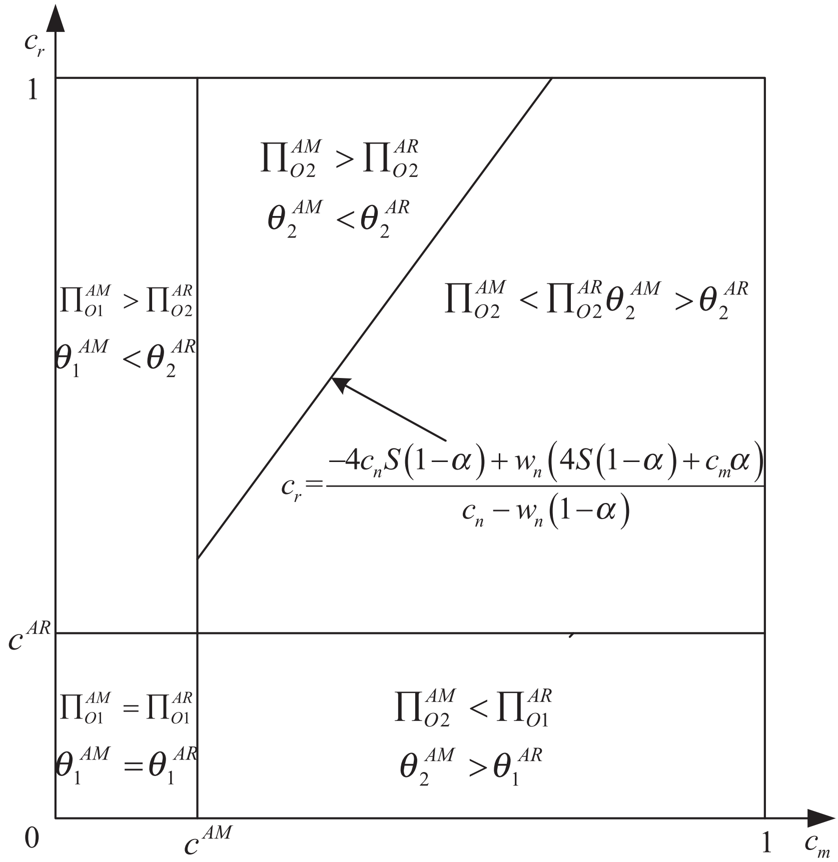

Figure 3. OEM's profit and environment impact in the authorization mode.

may become the preferrable partner choice. Figure 3 reflects the relative magnitudes of c_m and c_r affecting the OEM's optimal remanufacturing partner selection and the environmental performance. Consistent with the outsourcing mode, both the CM and the IR are the optimal remanufacturing partner for the OEM if they remanufacture at relatively low costs. The OEM should cooperate with the remanufacturer that has the lower variable cost of remanufacturing when the CM's and IR's variable costs of remanufacturing are at a high level. In addition, congruence regions in the figure shows that the profitability and environmental impact are in line with each other. These insights, however, are different with Zhou and Yuen (2021). They found that remanufacturing partner decision is not important for the OEM when authorization mode is used. However, we show that, consistent with the results in the outsourcing mode, depending on the remanufacturing cost, the OEM's decision on remanufacturing partner not only affects its own profitability but also the overall environmental performance of different modes.

5.3. *Optimal distribution channel strategy for remanufactured products*

In the previous two subsections, we investigated the OEM's remanufacturing partner decision when the remanufacturing mode is pre-determined. That is, in each remanufacturing mode, who, i.e. the CM or the IR, the OEM should choose to cooperate with. In this subsection, we investigate how the OEM should manage the marketing of remanufactured products with a given remanufacturing partner. We first derive the optimal distribution channel strategy when the CM is the remanufacturing partner in Proposition 3. In Proposition 4, we derive the optimal distribution channel strategy when the IR is the remanufacturer.

PROPOSITION 3 *When the CM is the remanufacturing partner, the OEM's decision over the two manufacturing modes depends on the amount of available used products. However, for the CM, it is always optimal to cooperate with the OEM by using outsourcing mode.*

LEMMA 3 *Cooperating with the CM in the two distribution channels (outsourcing and authorization) has the following outcomes with respect to the OEM's and CM's profits and the environmental impact*:

(a) When $c_m \in (0, c^{AM})$, the OEM's and CM's profits and the environmental impact are related as follows: $\prod_{O1}^{OM} < \prod_{O1}^{AM}$, $\prod_{M1}^{OM} > \prod_{M1}^{AM}$ and $\theta_1^{OM} = \theta_1^{AM}$.

(b) When $c_m \in (c^{AM}, c^{OM})$, then:

(i) for the OEM, if $S > S_{m1}$ and $w_n > \frac{c_n}{1-\alpha}$, for $c_m \in (c^{AM}, c_{m1})$, then $\prod_{O1}^{OM} < \prod_{O2}^{AM}$; for $c_m \in (c_{m1}, c^{OM})$, then $\prod_{O1}^{OM} > \prod_{O2}^{AM}$. If $S > S_{m1}$ and $w_n < \frac{c_n}{1-\alpha}$, for $c_m \in (c^{AM}, c^{OM})$, then $\prod_{O1}^{OM} > \prod_{O2}^{AM}$, where $c_{m1} = \frac{\sqrt{(c_n - (1-\alpha)w_n)^2\alpha - 4S(1-\alpha)\alpha}}{\alpha}$ and $S_{m1} = \frac{c_n(1 - \sqrt[3]{\alpha^2}) - w_n(1-\alpha)}{4(1-\alpha)^2\sqrt{\alpha}}$.

(ii) for the CM, $\prod_{M1}^{OM} > \prod_{M2}^{AM}$ always holds.

(iii) for the environmental impact, $\theta_1^{OM} < \theta_2^{AM}$ always holds.

(c) When $c_m \in (c^{OM}, 1)$, then:

(i) for the OEM, if $S < S_{m2}$, for $c_m \in (c^{OM}, 1)$, then $\prod_{O2}^{OM} < \prod_{O2}^{AM}$. If $S_{m2} < S < S_{m3}$, for $c_m \in (c^{OM}, c_{m5})$, then $\prod_{O2}^{OM} > \prod_{O2}^{AM}$; for $c_m \in (c_{m5}, 1)$, then $\prod_{O2}^{OM} < \prod_{O2}^{AM}$. If $S > S_{m3}$, for $c_m \in (c^{OM}, 1)$, then $\prod_{O2}^{OM} > \prod_{O2}^{AM}$, where $c_{m5} = S(w_n - c_n)\alpha(1-\alpha)((w_n + c_n)\alpha - w_n + c_n) + 4Sc_n\alpha(1-\alpha)$ $\frac{\sqrt{(c_n-(1-\alpha)w_n)^2\alpha}}{c_n^2(1+\alpha+\alpha^2)+w_n^2(1-\alpha)-2c_nw_n}$, $S_{m2} = \frac{c_n(c_n^2(1+\alpha+\alpha^2) - 2c_nw_n + w_n^2(1-\alpha))}{(1-\alpha)}$ $\left(3w_n^2(1-\alpha) + c_n^2\left(3 + 4\sqrt{\alpha} + 3\alpha + 4\alpha^2\right) - 2c_nw_n\left(3 + 2(1-\alpha)\sqrt{\alpha}\right)\right)$ and $S_{m3} = \frac{c_n^2(1+\alpha+\alpha^2) - 2c_nw_n + w_n^2(1-\alpha)}{(-1+\alpha)\alpha}$ $\left(-2c_nw_n\left(1 - 2(1-\alpha)\sqrt{\alpha}\right) + w_n^2(1-\alpha) + c_n^2\left(1 - 4\sqrt{\alpha} + \alpha\right)\right)$.

(ii) for the CM, $\prod_{M2}^{OM} > \prod_{M2}^{AM}$ always holds.

(iii) for the environmental impact, $\theta_2^{OM} < \theta_2^{AM}$ always holds.

Figure 4 visualizes Proposition 3. Each region is tagged with each party's profit and remanufacturing level, and the shaded areas in the figure show the congruence regions.

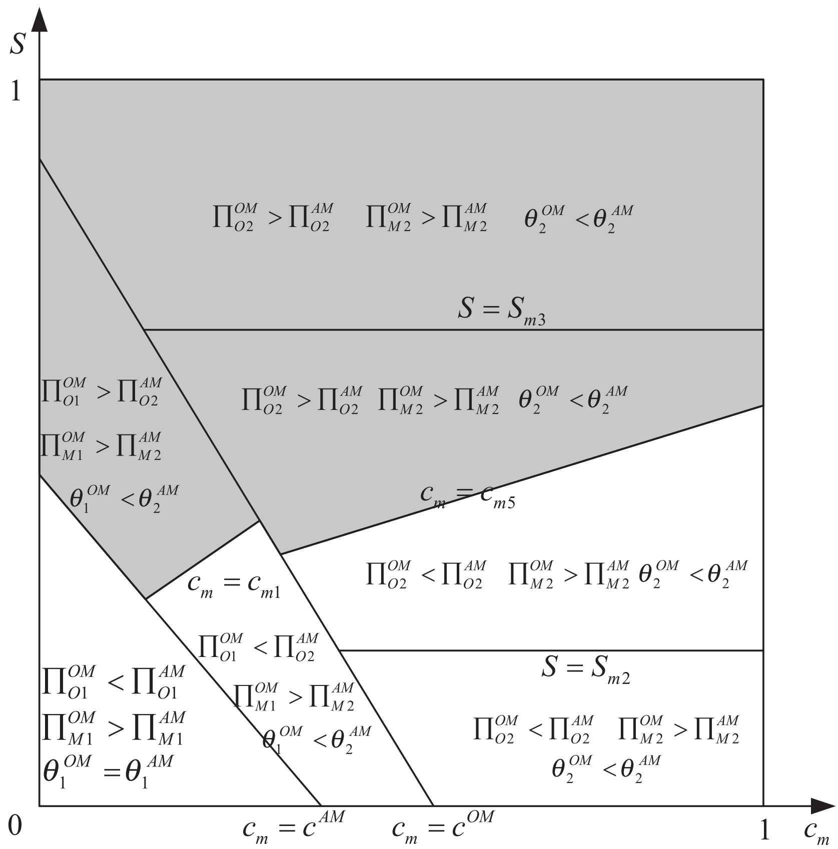

Figure 4. OEM's profit, CM's profit and environment impact in the two distribution channels when the CM is the remanufacturer.

Proposition 3, Lemma 3 and Figure 4 show the OEM's and CM's preferences for the distribution channel of the remanufactured product and how their preference affects the environment performance. It is surprising that the OEM and the CM have different preferences over the two remanufacturing modes. Specially, for the OEM, when the amount of collected products is at a low level (i.e. S is at a low level), it should authorize the CM to manage the marketing of remanufactured products. However, the OEM should use the outsourcing mode to cooperate with the CM when S increases regardless of the remanufacturing cost. In contrast to the OEM whose decision over the two manufacturing modes depends on the amount of available used products, outsourcing mode is always the optimal choice for the CM. Few products can be remanufactured when the amount of collected products is at a low level and hence, indirect selling of the remanfuactured products can give the OEM higher profits. However, the OEM should manage the marketing of remanufactured products itself by cooperating with the CM through the outsourcing mode when the CM can collect more used products from customers. The authorization and outsourcing modes can result in the same and best 'possible' environment impact when the remanufacturing cost and collecting rate are both at a low level (i.e. c_m and S are at a low level). However, when the remanufacturing cost and collecting rate increase, the outsourcing mode is always more eco-friendly.

These results are consistent with Zhou et al. (2023), but different from Zhou and Yuen (2021). They found that, when the wholesale price is exogenously given, authorization mode is always a dominant strategy for the OEM. However, we find that the OEM should use outsourcing mode not authorization mode when a large number of used products can be collected from the market.

In Figure 4, the shaded areas are the congruence regions in which the choice of the outsourcing mode dominates that of the authorization mode in terms of the OEM's and the CM's profitability and environmental impact. The remanufactured product should be sold by the OEM directly in shaded areas when the amount of the collected products (S) is at a high level and regardless of the variable cost of remanufacturing (c_m). Besides, the outsourcing mode is a win-win strategy for the OEM and the CM, and the cooperation can provide both firms a higher profit. However, the firms' profitability and environment impact are not always in congruent with each other. The blank areas are the conflict regions in which the choice of the outsourcing mode dominates the choice of the authorization mode in the CM's profitability and environment impact. However, outsourcing to the CM cannot give the OEM a higher profit in those regions. The existence of conflict regions arises mainly because of the profit conflict between the OEM and the CM. When the amount of collected products is at a low level, authorization mode can help the OEM achieve a higher profit, while this mode is not beneficial for the CM. As such, the CM is not the optimal remanufacturing partner for the OEM and remanufacturing cooperation may not be reached. In addition, the supply chain cannot be coordinated in terms of the environment impact outcome.

Next, the distribution channel choice for the remanufactured product when the IR is the remanufacturer is investigated.

PROPOSITION 4 *When the IR is the remanufacturing partner, depending on the wholesale price, the OEM and the CM may strategically choose between authorization mode and outsourcing mode. However, for the IR, it is always optimal to cooperate with the OEM by using outsourcing mode.*

LEMMA 4 *Cooperating with the IR in the two distribution channels (outsourcing and authorization) has the following outcomes with respect to the OEM's, CM's and IR's profit, and the environmental impact*:

(a) When $c_r \in (0, c^{AR})$, the OEM's, CM's and IR's profits, and the environmental impact are related as follows: $\prod_{O1}^{OR} < \prod_{O1}^{AR}$, $\prod_{M1}^{OR} > \prod_{M1}^{AR}$, $\prod_{R1}^{OR} > \prod_{R1}^{AR}$ and $\theta_1^{OR} = \theta_1^{AR}$.

(b) When $c_r \in (c^{AR}, c^{OR})$, then:

(i) for the OEM, if $S < S_{r1}$, for $c_r \in (c^{AR}, c^{OR})$, then $\prod_{O1}^{OR} < \prod_{O2}^{AR}$. If $S > S_{r1}$, for $c_r \in (c^{AR}, c_{r1})$, then $\prod_{O1}^{OR} < \prod_{O2}^{AR}$; for $c_r \in (c_{r1}, c^{OR})$, then $\prod_{O1}^{OR} > \prod_{O2}^{AR}$, where $c_{r1} = w_n\sqrt{\alpha} - 4S(1-\alpha)$ and $S_{r1} = \dfrac{w_n\sqrt{\alpha}}{4(1+\sqrt{\alpha}-\alpha-\sqrt[3]{\alpha^2})}$.

(ii) for the CM, when $w_n > 4S(1-\alpha)$, then $\prod_{M1}^{OR} > \prod_{M2}^{AR}$; when $w_n < 4S(1-\alpha)$, then $\prod_{M1}^{OR} < \prod_{M2}^{AR}$.

(iii) for the IR, $\prod_{R1}^{OR} > \prod_{R2}^{AR}$ always holds.

(iv) for the environmental impact, $\theta_1^{OR} < \theta_2^{AR}$ always holds.

(c) When $c_r \in (c^{OR}, 1)$, then:

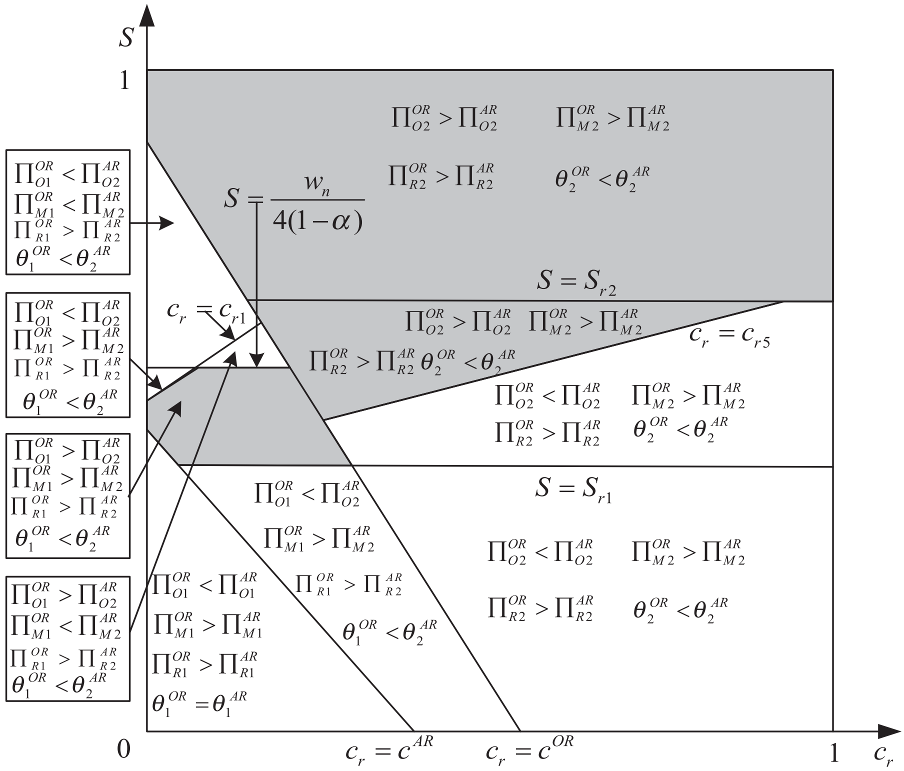

Figure 5. OEM's profit, CM's profit, IR's profit and environment impact in the two distribution channels when the IR is the remanufacturer.

(i) for the OEM, if $S < S_{r1}$, for $c_r \in (c^{OR}, 1)$, then $\prod_{O2}^{OR} < \prod_{O2}^{AR}$. If $S_{r1} < S < S_{r2}$, for $c_r \in (c^{OR}, c_{r5})$, then $\prod_{O2}^{OR} > \prod_{O2}^{AR}$; for $c_r \in (c_{r5}, 1)$, then $\prod_{O2}^{OR} < \prod_{O2}^{AR}$. If $S > S_{r2}$, for $c_r \in (c^{OR}, 1)$, then $\prod_{O2}^{OR} > \prod_{O2}^{AR}$, where $c_{r5} = 4S(1-\alpha)\sqrt{\alpha}$ and $S_{r2} = \dfrac{1}{4(1-\alpha)\sqrt{\alpha}}$.

(ii) (ii) for the CM, $\prod_{M2}^{OR} > \prod_{M2}^{AR}$ always holds.

(iii) for the IR, $\prod_{R2}^{OR} > \prod_{R2}^{AR}$ always holds.

(iv) for the environmental impact $\theta_2^{OR} < \theta_2^{AR}$ always holds.

The findings of Lemma 4 are numerically illustrated in Figure 5. Each region is tagged with each party's profit and the IR's remanufacturing level.

Proposition 4, Lemma 4 and Figure 5 illustrate the OEM's, the CM's and the IR's preference for the distribution channel of the remanufactured product and how their preference affects the environmental impact. The OEM and the CM will choose between the authorization and outsourcing modes depending on the wholesale price, consumers' valuation for the remanufactured product, the collecting rate and variable cost of remanufacturing. However, compared with the authorization mode, outsourcing mode can always give the remanufacturer, i.e. the IR, a higher profit, which is consistent with the results obtained in Zhou and Yuen (2021). That is, being the OEM's contract remanufacturer always achieves a higher profit for the IR. When the amount of collected products (S) is at a low level, the OEM should authorize the IR to produce and sell remanufactured products. However, the profit conflict exists among the three parties. Authorizing the IR to sell the remanufactured product is not optimal for the IR or CM though both distribution channels yield an equal environment impact. When the collecting rate is at a high level, the OEM should sell the remanufactured product itself. However, profit congruence and conflict still exist among the three parties, and there is no optimal distribution channel outcome for remanufactured products when the profit conflict exists. Note that when the IR is the remanufacturer, outsourcing mode is always beneficial to the environment than the authorization mode. The outsourcing mode dominates the authorization mode in terms of both profitability and environmental impact (see the shaded areas shown in Figure 5) when certain conditions are satisfied. As such, outsourcing mode is a win-win-win strategy for the OEM, the CM and the IR. However, the optimal distribution channel choice for the remanufactured product does not exist in the blank areas.

6. Discussions

This section considers several discussions to verify the robustness of our results. The OEM's performance in the four remanufacturing models is first examined in Section 6.1. In Section 6.2, we then extend the main model to the case where the wholesale price of new products is endogenously determined by the CM.

6.1. *The OEM'S performance in four remanufacturing models*

This section examines the OEM's performance in the above analyzed four remanufacturing models. For comparison purpose, we assume $c_m = c_r = c \in (0, 1)$ in the following analysis. This general form of the remanufacturing cost function is consistent with the current research in the operations management literature, such as, Savaskan et al. (2004), Yan et al. (2015) and Wu and Zhou (2016), and allows the tractability of the results.

PROPOSITION 5 *Depending on the cost of remanufacturing and the used products that are available for remanufacturing, the OEM should determine its remanufacturing partner and remanufacturing mode selectively. More specifically, when the cost of remanufacturing is at a low level, the OEM can either cooperate with the CM or the IR using authorization mode. However, cooperating with the IR either using authorization or outsourcing mode would become a preferable choice for the OEM in most scenarios as the cost of remanufacturing increases.*

LEMMA 5 *The OEM's profit in the four models has the following ordering*:

(a) When $c \in (0, c^{AM})$, then compared to the outsourcing mode, the OEM can achieve an equal or higher profit by cooperating with the CM and the IR in the authorization mode.

(b) (b) When $c \in (c^{AM}, 1)$, the OEM's profit in the four models satisfies the following three conditions:

(i) If $S < S_{r1}$, for any $c \in (c^{AM}, 1)$, then the OEM can perform better by cooperating with the IR in the authorization mode.

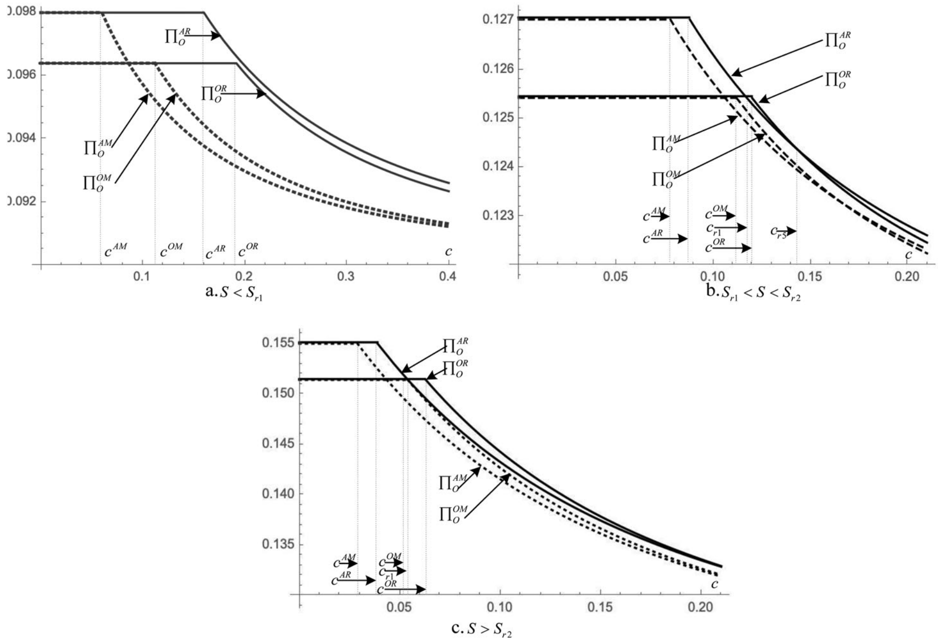

Figure 6. OEM's profit in the four models.

(ii) (ii) If $S_{r1} < S < S_{r2}$, when $c \in (c^{AM}, c_{r1})$, then the OEM can perform better by cooperating with the IR in the authorization mode; when $c \in (c_{r1}, c_{r5})$, then the OEM can perform better by cooperating with the IR in the outsourcing mode; when $c \in (c_{r5}, 1)$, then the OEM can perform better profit by cooperating with the IR in the authorization.

(iii) If $S > S_{r2}$, when $c \in (c^{AM}, c_{r1})$, then the OEM can perform better by cooperating with the IR in the authorization mode; when $c \in (c_{r1}, 1)$, then the OEM can perform better by cooperating with the IR in the outsourcing mode.

Figure 6 illustrates the findings of Proposition 5 and Lemma 5. Proposition 5, Lemma5 and Figure 6 show how the OEM should determine the remanufacturing partner and distribution channel for the remanufactured product. Authorizing the CM or the IR to remanufacture yields an equal profit for the OEM when the remanufacturing cost is at a low level ($c \in (0, c^{AM})$). However, when the variable cost of remanufacturing increases, the OEM should never select the CM as the remanufacturing partner. Cooperating with the IR either through outsourcing or authorization can give the OEM a higher profit in most cases, and the OEM's choice is further influenced by the amount of collected products (S). The OEM's remanufacturing mode decision (authorization or outsourcing) depends on the collecting rate and remanufacturing cost. When the colleting rate is at a low level (i.e. $S < S_{r1}$) and the cost of remanufacturing is at a high level (i.e. $c \in (c^{AM}, 1)$), the OEM should always authorize the IR to conduct the remanufacturing activity. As the collecting rate increases (i.e. $S_{r1} < S < S_{r2}$), the OEM should use the authorization mode when the remanufacturing cost is at a low or high level (i.e. $c \in (c^{AM}, c_{r1})$ and $c \in (c_{r5}, 1)$). The outsourcing mode is optimal for the OEM when the variable cost of remanufacturing is at a medium level (i.e. $c \in (c_{r1}, c_{r5})$). While the OEM should first authorize the IR to remanufacture (i.e. $c \in (c^{AM}, c_{r1})$) and then outsource the remanufacturing activity to the IR (i.e. $c \in (c_{r1}, 1)$) when the collecting rate is at a high level (i.e. $S > S_{r2}$). Consistent with Propositions 3 and 4, outsourcing mode is a more profitable choice for the OEM as the collecting rate increases. In addition, from Figure 6, we also notice that the OEM can earn more profit when more used products can be collected (i. e. S increases). This is because more remanufactured products can be remarketed to the market when the availability of used products increases.

In the following, the environmental performance of the four models is analyzed in Proposition 6.

PROPOSITION 6 *For the environment impact, when the variable cost of remanufacturing is at a low level, cooperating with the CM and the IR in two distribution channels result in the same environment impact. However, the IR performing remanufacturing is better for the environment when the variable cost of remanufacturing increases.*

Proposition 6 shows that the four remanufacturing models can achieve an equal and 'best' possible environmental performance when the cost of remanufacturing is at a low level. When the remanufacturing is costly for the remanufacturer, the IR conducting the remanufacturing is better for the environment. This is because, as an independent remanufacturer, the IR does not profit from contract manufacturing of new products. It will set a lower quality threshold than the CM to remanufacture more products for a better profit. This can be beneficial to the environment. For the CM, it has a contract for new products, which discourages it from remanufacturing more used products. Therefore, the quality threshold will not be lowered, and less used items will be qualified for remanufacturing.

6.2. *Endogenized wholesale price of the new product*

In the main analysis, the wholesale price of new products is considered as an exogenous parameter. In this section, this assumption is relaxed to the case where the CM can strategically decide the wholesale price of new products. This new assumption does not change the original game sequence except that the CM needs to determine the wholesale price in the first stage. The assumption that the CM determines the wholesale price before the remanufacturer determines the cut-off point for used products is slightly different from the assumption in Zhou et al. (2023) with the aim to examine the impact of the CM's leadership status in deciding the wholesale price on the OEM's remanufacturing-related decisions. That is, put the supply chain in an extreme condition, what the OEM should make its remanufacturing decisions when the CM can proactively and strategically adjust its wholesale price. Furthermore, since the CM is not necessarily the remanufacturer, but it would always be the manufacturer of new products and hence, it is reasonable that it determines the wholesale price at the first stage of the game. We also solve the problem by using backward induction. Based on the results in Section 4, we only need to consider the optimization problem in the first stage. For better readability, the optimal results of the four models are summarized in the Appendix. Here, we focus on the OEM's performance in the four models when the wholesale price of new products is endogenized. In the following, we add '*' in the superscript to differentiate the case with Section 4. Consistent with Oraiopoulos et al. (2012) and Liu et al. (2018), a numerical study is used to examine the OEM's profit in the four models. Proposition 7 shows the comparative results of the OEM's profit in the four models, and Figure 7 illustrates the observation with different parameter combinations.

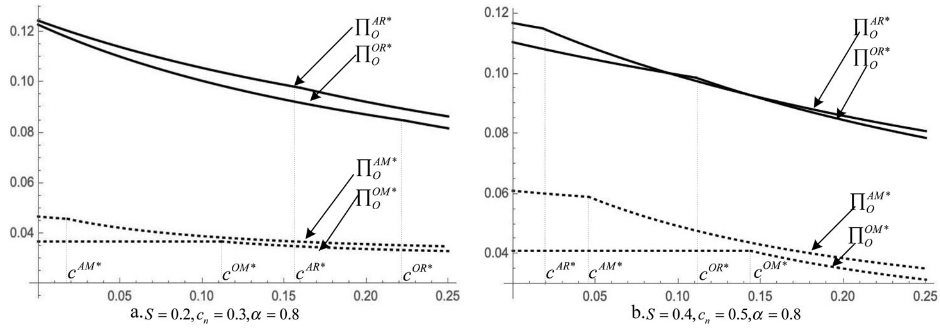

Figure 7. OEM's profit in the four models when the wholesale price is endogenized.

Proposition 7 *When the wholesale price of new products is endogenously determined by the CM, the OEM can perform better by cooperating with the IR either through outsourcing or authorization mode, while the CM is never an optimal remanufacturing partner for the OEM.*

Figure 7 illustrates the results of Proposition 7 by using two sets of parameters, i.e. $S = 0.2$, $c_n = 0.3$, $\alpha = 0.8$ and $S = 0.4$, $c_n = 0.5$, $\alpha = 0.8$. Figure 7 shows that, when S and c_n are both at low levels, the OEM should select the IR as the remanufacturing partner through authorization mode. The outsourcing mode becomes optimal under certain circumstances as the manufacturing cost and collecting rate increase. When the wholesale price of new products is an exogenous parameter, cooperating with the CM through authorization mode can become an optimal choice because the remanufacturing cost is at a low level. By contrast, we find that the OEM should never cooperate with the CM again in remanufacturing either through authorization or outsourcing mode. This implies that if the supply chain goes into an extreme condition, i.e. the CM can adjust the wholesale price at the very beginning, then cooperating with the CM again in remanufacturing is never an optimal choice for the OEM. In fact, cooperation and a win-win solution can be reached between the OEM and the CM in a wide range of parameter regions if the supply chain does not go into an extreme condition, i.e. the wholesale price is not determined at the first stage of the game (Zhou et al., 2023) (interested readers can refer to the cited literature for more discussions). Besides, from Figure 7, we can also see that the profit difference of the OEM between cooperating with the IR and the CM increases when the CM strategically decides the wholesale price. The CM can always set the wholesale price to maximize its own profit, and this can consequently decrease the OEM's profit. As such, remanufacturing cooperation is less likely between the OEM and the CM.

The corresponding environmental impact of the four models when the CM determines the wholesale price is illustrated in Figure 8. Note that in the figure, we only show the four models' environmental impact when the variable cost of remanufacturing is at a relatively high level (i.e. $c_m, c_r \in (\min(c^{AM*}, c^{OM*}, c^{AR*}, c^{OR*}), 1)$). Models *AM, OM, AR* and *OR* can achieve an equal and 'best' possible environmental impact when the remanufacturing cost is at a low level (i.e. $c_m, c_r \in (0, \min(c^{AM*}, c^{OM*}, c^{AR*}, c^{OR*}))$). From the figure, we can see that the outsourcing mode is always more eco-friendly than the authorization mode (i.e. $\theta^{AM*} > \theta^{OM*}$ and $\theta^{AR*} > \theta^{OR*}$). In the outsourcing mode, remanufactured products are directly sold by the OEM and hence, the remanufacturer always has the incentive to set the quality threshold at a low level to collect as many used products as possible.

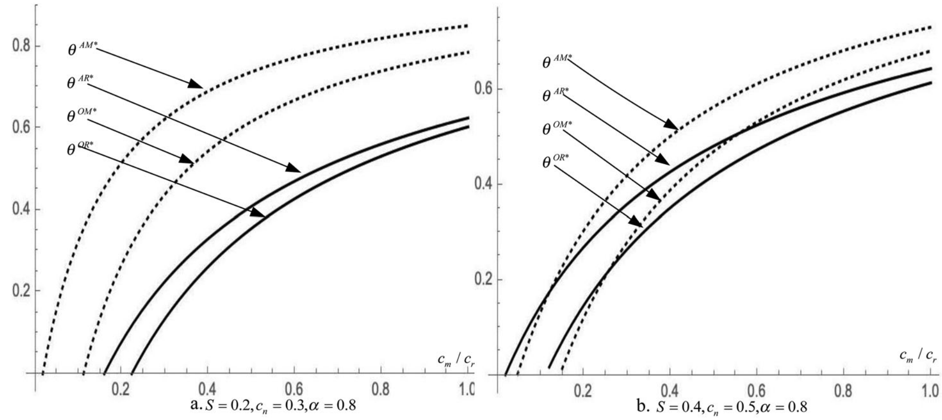

Figure 8. The environment impact in the four models when the wholesale price is endogenized.

However, in the authorization mode, the remanufacturer sells remanufactured products itself, and it cannot set the quality threshold too low. There are two reasons contributing to this observation. On the one hand, the remanufacturer can directly earn profits from the remanufactured product in the authorization mode, and a high level of θ can help the remanufacturer earn enough profit from the remanufacturing business. On the other hand, a low level of θ means a high rate of remanufacturing and a high level of remanufacturing cost. However, consumers' acceptance for the remanufactured product is uncertain, which prevents the remanufacturer from decreasing its quality threshold.

Figure 8 also implies that cooperating with the IR through outsourcing mode is more eco-friendly when S and c_n are both at a low level. However, a better environmental impact can first be achieved by cooperating with the CM and then by cooperating with the IR in the outsourcing mode as the collecting rate and production cost increase. As shown in Proposition 7 and Figure 7, cooperating with the CM can never become the optimal choice for the OEM when the CM determines the wholesale price strategically. However, we show that cooperating with the CM through outsourcing mode is more eco-friendly when the remanufacturing cost is at low level. As such, the model that gives the OEM a higher profit may not necessarily be environmentally friendly, and conflict arises between the profit and environmental impact. Therefore, in these scenarios, the OEM needs to determine the relative importance of profits and environmental impact.

6.3. *Managerial implications*

The findings of our analyses offer some managerial implications for OEMs in the selection of a remanufacturing partner and distribution channel for remanufactured products. Some OEMs' choices in the remanufacturing market are in line with our results. For instance, in the China market, Apple has authorized its original CM, Foxconn, to implement all the remanufacturing activity. While in the European market, Apple has contracted the remanufacturing to an IR, Phobio. In general, new product manufacturing can be globally located. However, remanufacturing requires proximity to the local market. The remanufacturer would incur a high cost of remanufacturing if used items collected in the European market were reshipped back to Foxconn for remanufacturing, meaning that cooperating with the original CM is less economical due to the transportation cost. For the distribution channel, whether remanufactured products should be sold by the OEM itself or the IR depends on the IR's remanufacturing cost. If the IR can remanufacture at a medium cost, the OEM should manage the marketing of remanufactured products itself. In the China market, Foxconn has a cost advantage compared to other IRs and became Apple's remanufacturing partner. However, we should acknowledge that cooperating with the CM may not be environmentally friendly because the CM may be conservative in collecting used products. Therefore, OEMs need to determine the relative importance of profits and the environmental impact when those two goals conflict.

7. Conclusion

This research considered a remanufacturing supply chain consisting of an OEM, a CM and an IR. The OEM manages the marketing but not the manufacturing of new products, and the production of new products is outsourced to the CM. Depending on the marketing channel of remanufactured products, the OEM participates in the remanufacturing market with two modes, i.e. outsourcing and authorization. The remanufacturing process can either be conducted by the CM or the IR. As such, either the CM or the IR can become the potential remanufacturing partner of the OEM. We derived the supply chain optimal strategy in terms of remanufacturing partner selection, the choice of distribution channel of the remanufactured product, pricing decisions and the quality threshold of used products for remanufacturing.

We first investigated the OEM's remanufacturing partner selection decision when the distribution channel of remanufactured products is given. We found that cooperating with the CM and the IR achieves an equal profit for the OEM when the two remanufacturers remanufacture at a low variable cost. This result is consistent with Zhou and Yuen (2021). They also found that the remanufacturing partner decision of the OEM has no impact on its profit when the wholesale price of new products is an exogenous parameter. However, as the remanufacturing cost increases, the OEM should cooperate with the party that has a lower variable cost indicating that collection uncertainty in the quality of collected used products will play a role and subsequently affects the OEM's decision in remanufacturing partner. This result enriches current research and can help the OEM make more accurate decisions in selecting remanufacturing partner. In addition, the OEM's partner selection can achieve the best profit and environmental outcomes.

We then investigated the distribution channel decision for remanufactured products when the OEM's remanufacturing partner selection is given. We found that the authorization mode can provide the OEM with a higher profit when the amount of collected products is at a low level. However, the remanufacturer (i.e. CM and IR) does not benefit from the authorization mode. As such, a win-win or win-win-win strategy does not exist among the OEM, the CM and the IR. As the collecting rate increases, the outsourcing mode can help the OEM perform better. The OEM selling the remanufactured product directly to the market can also give the remanufacturer a higher profit, and a win-win or win-win-win outcome may exist. These results are completely different from Zou et al. (2016). By Focusing on IR remanufacturing with two remanufacturing modes, Zou et al. (2016) found that the outsourcing mode can always give the OEM a higher profit, while the IR's preference over the two remanufacturing modes depends on customers' valuation for remanufactured products.

In addition, we also investigated the OEM's profit in the four models when the wholesale price is exogenously determined and when the wholesale price is endogenously determined by the CM. In the former case, we found that when the remanufacturing cost is at a low level, cooperating with the CM and the IR in authorization mode provides the OEM with an equal or higher profit compared to the outsourcing mode. While as the remanufacturing cost increases, the OEM's choice among the four models is further affected by the collecting rate, and the OEM should select the IR as the remanufacturing partner either through authorization or outsourcing mode. In the latter case, we found that cooperating with the CM can never give the OEM a higher profit, and the IR is always a preferable remanufacturing partner choice for the OEM. Note that, in the two cases, the OEM's profitability may conflict with the environment impact and then, the OEM needs to decide the relative importance of profits and environmental impact.

By incorporating collection uncertainty into the OEM's remanufacturing mode and remanufacturing partner decisions, some of the results confirm the findings of Zhou et al. (2023), and Zhou and Yuen (2021), but the rest are surprising. For instance, in contrast with Zhou and Yuen (2021), we found that the OEM's decision on the remanufacturing partner plays a role that not only affects its own profitability but also the environmental performance when the marketing of remanufactured products is managed by the OEM itself. Therefore, it is important to take the collection uncertainty or the quality of used products into consideration when making remanufacturing-related decisions. However, it should be noted that applying the detailed results of this study should consider the specific conditions of the market. For instance, for some kinds of products, such as automobile parts, the used products market is quite mature and with less uncertainty (Chen & Chen, 2019) and hence, the applications of the results are different.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

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Appendices

Appendix 1: tables

Table A1. Optimal Solutions for the Direct Selling Modes.

	Cooperate with the CM (Model OM) Range for c_m		Cooperate with the IR (Model OR) Range for c_r	
	$c_m \in (0, c_n\alpha - 4S\alpha(1-\alpha)]$	$c_m \in (c_n\alpha - 4S\alpha(1-\alpha), 1)$	$c_r \in (0, w_n\alpha - 4S\alpha(1-\alpha)]$	$c_r \in (w_n\alpha - 4S\alpha(1-\alpha), 1)$
θ	0	$1 - \frac{c_n\alpha}{c_m + 4S\alpha(1-\alpha)}$	0	$1 - \frac{w_n\alpha}{c_r + 4S\alpha(1-\alpha)}$
w_r	$\alpha(w_n - 2S(1-\alpha))$	$\alpha\left(w_n - \frac{2Sc_n\alpha(1-\alpha)}{c_m + 4S\alpha(1-\alpha)}\right)$	$\alpha(w_n - 2S(1-\alpha))$	$\frac{w_n\alpha(c_r + 2S\alpha(1-\alpha))}{c_r + 4S\alpha(1-\alpha)}$
p_n	$\frac{1+w_n}{2}$			
p_r	$\frac{\alpha(1+w_n-2S(1-\alpha))}{2}$	$\frac{\alpha(c_m(1+w_n) + 2S(2(1+w_n) - c_n)\alpha(1-\alpha))}{2(c_m + 4S\alpha(1-\alpha))}$	$\frac{\alpha(1+w_n-2S(1-\alpha))}{2}$	$\frac{\alpha}{4}\left(2 + w_n + \frac{w_nc_r}{c_r + 4S\alpha(1-\alpha)}\right)$
$\prod_O$	$\frac{1}{4}(1-w_n)^2 + S^2\alpha(1-\alpha)$	$\frac{1}{4}(1-w_n)^2 + \frac{S^2c_n^2\alpha^3(1-\alpha)}{(c_m + 4S\alpha(1-\alpha))^2}$	$\frac{1}{4}(1-w_n)^2 + S^2\alpha(1-\alpha)$	$\frac{1}{4}(1-w_n)^2 + \frac{S^2w_n^2\alpha^3(1-\alpha)}{(c_r + 4S\alpha(1-\alpha))^2}$
$\prod_M$	$\frac{1}{2}\begin{pmatrix} w_n - w_n^2 - c_mS \\ -4S^2\alpha(1-\alpha) \\ +c_n(w_n + 2S\alpha - 1) \end{pmatrix}$	$\frac{\begin{pmatrix} c_m(w_n - c_n)(1-w_n)+ \\ S\alpha(4w_n(1-w_n)(1-\alpha) + c_n^2\alpha \\ -4c_n(1 - w_n - \alpha + w_n\alpha)) \end{pmatrix}}{2(c_m + 4S\alpha(1-\alpha))}$	$\frac{1}{2}(w_n - c_n)(1 - w_n - 2S\alpha)$	$\frac{\begin{pmatrix} c_r(w_n - c_n)(1-w_n)+ \\ 2S(w_n - c_n)(2 - w_n(2-\alpha) - 2\alpha)\alpha \end{pmatrix}}{2(c_r + 4S\alpha(1-\alpha))}$
$\prod_R$	N/A	N/A	$S(w_n - 2S(1-\alpha))\alpha - c_r\frac{S}{2}\beta(1-c_r)$	$\frac{Sw_n^2\alpha^2}{2(c_r + 4S\alpha(1-\alpha))} - \beta(1-c_r)$

Table A2. Comparative statics for the outsourcing modes.

Parameter	Key Decisions																
	Model OM								Model OR								
	θ	p_n	p_r	w_r	q_n	q_r	$\prod_O$	$\prod_M$	θ	p_n	p_r	w_r	q_n	q_r	$\prod_O$	$\prod_M$	$\prod_R$
α	$NC/\downarrow$	NC	$\uparrow$	$\uparrow$	$\downarrow$	$NC/\uparrow$	$M/\uparrow$	$\uparrow$	$NC/\downarrow$	NC	$\uparrow$	$\uparrow$	$\downarrow$	$NC/\uparrow$	$M/\uparrow$	$\downarrow$	$\uparrow$
S	$NC/\uparrow$	NC	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$NC/\uparrow$	NC	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
w_n	NC	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	NC	$\downarrow$	M	$NC/\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$NC/\uparrow$	$\downarrow$	M	$\uparrow$

Table A3. Optimal solutions for the indirect selling modes.

	Cooperate with the CM (Model AM) Range for c_m		Cooperate with the IR (Model AR) Range for c_r	
	$c_m \in (0, c_n - (4S + w_n)(1-\alpha)]$	$c_m \in (c_n - (4S + w_n)(1-\alpha), 1)$	$c_r \in (0, w_n\alpha - 4S(1-\alpha)]$	$c_r \in (w_n\alpha - 4S(1-\alpha), 1)$
θ	0	$1 - \dfrac{c_n - w_n(1-\alpha)}{c_m + 4S(1-\alpha)}$	0	$1 - \dfrac{w_n\alpha}{c_r + 4S(1-\alpha)}$
p_s	$2S + \frac{1}{2}(1 - 4S - w_n)\alpha$	$\dfrac{c_m(1-w_n)\alpha + 4Sc_n(1-\alpha) + 4S(1-\alpha)(\alpha - w_n)}{2(c_m + 4S(1-\alpha))}$	$2S + \frac{1}{2}(1 - 4S - w_n)\alpha$	$\dfrac{c_r(1-w_n)\alpha + 4S(1-\alpha)\alpha}{2(c_r + 4S(1-\alpha))}$
p_n	$\dfrac{1 + w_n + 2S(1-\alpha)}{2}$	$\dfrac{c_m(1+w_n) + 2S(1-\alpha)(2 + c_n + w_n + w_n\alpha)}{2(c_m + 4S(1-\alpha))}$	$\dfrac{1 + w_n + 2S(1-\alpha)}{2}$	$\dfrac{c_r(1+w_n) + 2S(1-\alpha)(2 + 2w_n + w_n\alpha)}{2(c_r + 4S(1-\alpha))}$
p_r	$\dfrac{\alpha(1+w_n)}{2}$			
$\prod_O$	$\frac{1}{4}(1-w_n)^2 + S^2(1-\alpha)$	$\frac{1}{4}(1-w_n)^2 + \dfrac{S^2(c_n - w_n(1-\alpha))^2(1-\alpha)}{(c_m + 4S(1-\alpha))^2}$	$\frac{1}{4}(1-w_n)^2 + S^2(1-\alpha)$	$\frac{1}{4}(1-w_n)^2 + \dfrac{S^2 w_n^2 \alpha^2 (1-\alpha)}{(c_r + 4S(1-\alpha))^2}$
$\prod_M$	$\frac{1}{2}\begin{pmatrix} w_n - w_n^2 - c_m S \\ -4S^2(1-\alpha) - 2Sw_n(1-\alpha) \\ +c_n(w_n + 2S - 1) \end{pmatrix}$	$\dfrac{\begin{pmatrix} c_m(w_n - c_n)(1-w_n) + S(w_n(4 - w_n(3+\alpha))(1-\alpha) \\ +c_n^2 - 2c_n(2-w_n)(1-\alpha)) \end{pmatrix}}{2(c_m + 4S(1-\alpha))}$	$\frac{1}{2}(w_n - c_n)(1 - w_n - 2S)$	$\dfrac{\begin{pmatrix} (c_r + 4S)(w_n - c_n)(1-w_n) \\ -2S(w_n - c_n)(2-w_n)\alpha \end{pmatrix}}{2(c_r + 4S(1-\alpha))}$
$\prod_R$	N/A	N/A	$S(w_n + 2S)\alpha - \frac{1}{2}S(c_r + 4S) - \beta(1-c_r)$	$\dfrac{Sw_n^2\alpha^2}{2(c_r + 4S(1-\alpha))} - \beta(1-c_r)$

Table A4. Comparative statics for the authorization modes.

sParameter	Key Decisions																
	Model AM								Model AR								
	θ	p_n	p_r	p_s	q_n	q_r	$\prod_O$	$\prod_M$	θ	p_n	p_r	p_s	q_n	q_r	$\prod_O$	$\prod_M$	$\prod_R$
α	$NC/\downarrow$	$\downarrow/M$	$\uparrow$	$M/\uparrow$	$NC/\downarrow$	$NC/\uparrow$	$\downarrow/\uparrow$	$\uparrow$	$NC/\downarrow$	$\downarrow$	$\uparrow$	$M/\uparrow$	$NC/\downarrow$	$NC/\uparrow$	$\downarrow/\uparrow$	$NC/\downarrow$	$\uparrow$
S	$NC/\uparrow$	$\uparrow$	NC	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$NC/\uparrow$	$\uparrow$	NC	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
w_n	$NC/\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$NC/\downarrow$	$\downarrow$	M	$NC/\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$NC/\uparrow$	$\downarrow$	M	$\uparrow$

The directional relationships are abbreviated as follows, and apply over all $c_m \in (0, 1)$ and $c_r \in (0, 1)$. NC, $\uparrow$, $\downarrow$, and M indicate no change, increase, decrease and mixed change (nonmonoticity), respectively. For instance, $NC/\uparrow$ indicates NC when the remanufacturing cost within the lower boudary and $\uparrow$ when the remanufacturing cost within the higher boundary, whereas single $\uparrow$ indicates increase within the whole cost range.

Appendix 2: proofs

Proof of model OM *when the wholesale price is exogenous given and when the wholesale price is endogenously determined by the CM*

We first show the proof of model OM when the wholesale price is exogenously determined and then the proof of model OM when the CM can determine the wholesale price strategically. When the wholesale price is exogenously determined, the sequence of the game is as follows: First, the CM determines the quality threshold (θ) such that all collected products with at least this level of quality will be remanufactured; and then the CM determines the wholesale price of remanufactured product (w_r); the OEM finally decides the sales price of new and remanufactured product (p_n, p_r). The problem can be solved by backward induction. We first solve the OEM's optimal problem with respect to p_n and p_r, and then solve the CM's problem with respect to the wholesale price of remanufactured product (w_r), finally substituting these results back to obtain the CM's optimal quality threshold (θ).

The decision problem faced by the OEM is given by $\max \prod_O^{OM}(p_n, p_r) = (p_n - w_n)q_n + (p_r - w_r)q_r$. Because the Hessian matrix is negative definite, $\prod_O^{OM}$ is strictly concave with respect to p_n and p_r. The first-order condition of $\prod_O^{OM}(p_n, p_r)$ with respect to p_n and p_r leads to $p_n = \frac{1 + w_n}{2}$ and $p_r = \frac{\alpha + w_r}{2}$.

The profit-maximization problem for the CM is $\max \prod_M^{OM}(w_r, \theta) = (w_n - c_n)q_n + w_r q_r - \int_\theta^1 c_m S(1-\theta)f(\theta)d\theta$, and subjects to $q_r \leq S\int_\theta^1 f(\theta)d\theta = S(1 - F(\theta))$. The constraint in optimality must be blind, i.e. $q_r = S\int_\theta^1 f(\theta)d\theta = S(1 - F(\theta))$, which yields $p_r = \alpha p_n - \alpha(1-\alpha)S(1-\theta)$. Substituting this, p_n and p_r into the CM's optimal function. Because of the second-order sufficient condition, i.e. $\frac{\partial^2 \prod_M^{OM}}{\partial w_r^2} = -\frac{1}{\alpha(1-\alpha)} < 0$ is negative, the profit function $\prod_M^{OM}$is concave inw_r. Letting $\frac{\partial \prod_M^{OM}}{\partial w_r} = 0$, we have $w_r = \alpha(w_n - 2S(1-\alpha)(1-\theta))$.

Substituting p_n, p_r and w_r into the CM's profit function allows the reformulation of the CM's profit as $\prod_M^{OM} = \frac{1}{2}((1 - w_n)w_n + c_n(-1 + w_n + 2S\alpha(1-\theta)) - S(c_m + 4S(1-\alpha)\alpha)(1-\theta)^2)$. The CM's profit function has these first and second derivatives with respect to θ:

$$\frac{\partial \prod_M^{OM}}{\partial \theta} = \frac{1}{2}(-2c_n S\alpha + 2S(c_m + 4S(1-\alpha)\alpha)(1-\theta))$$

$$\frac{\partial^2 \prod_M^{OM}}{\partial \theta^2} = -S(c_m + 4S(1-\alpha)\alpha)$$

Because the second-order sufficient condition, i.e. $\frac{\partial^2 \prod_M^{OM}}{\partial \theta^2} = -S(c_m + 4S(1-\alpha)\alpha) < 0$, $\prod_M^{OM}$ is strictly concave in θ. While the first-order condition $\frac{\partial \prod_M^{OM}}{\partial \theta}$ can either positive or negative. When $c_m < c_n\alpha - 4S\alpha(1-\alpha)$, then $\frac{\partial \prod_M^{OM}}{\partial \theta}$ has to be negative. So the lowest possible θ will be optimal, i.e. $\theta = 0$. When $c_m > c_n\alpha - 4S\alpha(1-\alpha)$, then the zero of the first-order condition will be a unique global maximum. Letting $\frac{\partial \prod_M^{OM}}{\partial \theta} = 0$, we have $\theta = 1 - \frac{c_n\alpha}{c_m + 4S\alpha(1-\alpha)}$.

Then substituting the optimal θ back, the optimal results in model *OM* when the wholesale price is exogenously determined follow directly.

The proof of the models *OR*, *AM* and *AR* when wholesale price is exogenously given is similar to that in the mode *OM*. Therefore, we omit the proof process. The detailed results are summarized in Tables A1 and A2.

In the following, we go to the proof of the case when the wholesale price is strategically determined by the CM. The game sequence in this case is the same as when the wholesale price is exogenously given, except the CM needs to determine w_n in the first stage. The problem can be solved by backward induction, and the solving process is the same as that of case when the wholesale price is exogenously given. Hence, we substitute $p_n = \frac{1+w_n}{2}$, $p_r = \alpha p_n - \alpha(1-\alpha)S(1-\theta)$, $w_r = \alpha(w_n - 2S(1-\alpha)(1-\theta))$ and $\theta = 1 - \frac{c_n\alpha}{c_m + 4S\alpha(1-\alpha)}$ back into the CM's profit function. The CM's profit function has these first and second derivatives with respect to w_n:

$$\frac{\partial \prod_M^{OM*}}{\partial w_n} = \frac{1}{2}(1 + c_n - 2w_n)$$

$$\frac{\partial^2 \prod_M^{OM}}{\partial w_n^2} = -1$$

Because the second-order sufficient condition, i.e. $\frac{\partial^2 \prod_M^{OM*}}{\partial w_n^2} = -1 < 0$, $\prod_M^{OM*}$ is strictly concave in w_n. Then, the optimal wholesale price can be obtained by solving $\frac{\partial \prod_M^{OM*}}{\partial w_n} = 0$, we have $w_n^{OM*} = \frac{1+c_n}{2}$.

Then substituting the optimal w_n^{OM*} back, the optimal results in model $OM*$ when the wholesale price is endogenized follow directly.

The proof of models *OR**, *AM* *and *AR** is similar to that of the proof in the model *OM**, so we omit the proof process.

Proof of Lemma 1

To prove Proposition1, we compare the OEM's profit and the remanufacturer's quality threshold under the four scenarios derived in Proposition 1. Note that the quality threshold (θ) is our metric of environmental impact, with lower values implying a greater amount of remanufacturing and a better environmental impact.

Case 1: $c_m \in (0, c_n\alpha - 4S\alpha(1-\alpha))$ and $c_r \in (0, w_n\alpha - 4S\alpha(1-\alpha))$, we have $\prod_{O1}^{OM} - \prod_{O1}^{OR} = 0$ and $\theta_1^{OM} = \theta_1^{OR} = 0$. Hence, both cooperating with the CM and IR yield an equal profit for the OEM and an equal (and 'best' possible) environment outcome.

Case 2: $c_m \in (0, c_n\alpha - 4S\alpha(1-\alpha))$ and $c_r \in (w_n\alpha - 4S\alpha(1-\alpha), 1)$, we have.

$$\prod_{O1}^{OM} - \prod_{O2}^{OR} = \frac{S^2\alpha(1-\alpha)(c_r + w_n\alpha + 4S(1-\alpha)\alpha)(c_r - w_n\alpha + 4S(1-\alpha)\alpha)}{(c_r + 4S\alpha(1-\alpha))^2} > 0,$$

$$\theta_1^{OM} = 0 < \theta_2^{OR} = 1 - \frac{w_n\alpha}{c_r + 4S\alpha(1-\alpha)}$$

Hence, $\prod_{O1}^{OM} > \prod_{O2}^{OR}$ and $\theta_1^{OM} < \theta_2^{OR}$.

Case 3: $c_m \in (c_n\alpha - 4S\alpha(1-\alpha), 1)$ and $c_r \in (0, w_n\alpha - 4S\alpha(1-\alpha))$, we have.

$$\prod_{O2}^{OM} - \prod_{O1}^{OR} = -\frac{S^2\alpha(1-\alpha)(c_m + c_n\alpha + 4S(1-\alpha)\alpha)(c_m - c_n\alpha + 4S(1-\alpha)\alpha)}{(c_m + 4S\alpha(1-\alpha))^2} < 0$$

$$\theta_2^{OM} = 1 - \frac{c_n\alpha}{c_m + 4S\alpha(1-\alpha)} > \theta_1^{OR} = 0$$

Hence, $\prod_{O2}^{OM} < \prod_{O1}^{OR}$ and $\theta_2^{OM} > \theta_1^{OR}$.

Case 4: $c_m \in (c_n\alpha - 4S\alpha(1-\alpha), 1)$ and $c_r \in (w_n\alpha - 4S\alpha(1-\alpha), 1)$, we have.

$$\prod_{O2}^{OM} - \prod_{O2}^{OR} = \frac{S^2\alpha^3(1-\alpha)(c_n^2(c_r+4S\alpha(1-\alpha))^2 - w_n^2(c_m+4S\alpha(1-\alpha))^2)}{(c_m+4S\alpha(1-\alpha))^2(c_r+4S\alpha(1-\alpha))^2}$$

$$= \frac{S^2\alpha^3(1-\alpha)(c_n(c_r+4S\alpha(1-\alpha))+w_n(c_m+4S\alpha(1-\alpha)))(c_n(c_r+4S\alpha(1-\alpha))-w_n(c_m+4S\alpha(1-\alpha)))}{(c_m+4S\alpha(1-\alpha))^2(c_r+4S\alpha(1-\alpha))^2}$$

Then, the sign of $\prod_{O2}^{OM} - \prod_{O2}^{OR}$ depends on that of $c_n(c_r+4S\alpha(1-\alpha)) - w_n(c_m+4S\alpha(1-\alpha))$. To solve the roots of $c_n(c_r+4S\alpha(1-\alpha)) - w_n(c_m+4S\alpha(1-\alpha)) = 0$, we have $c_r = \frac{w_n(c_m+4S\alpha(1-\alpha)) - 4Sc_n\alpha(1-\alpha)}{c_n}$. Thus, if $c_r > \frac{w_n(c_m+4S\alpha(1-\alpha)) - 4Sc_n\alpha(1-\alpha)}{c_n}$, then $\prod_{O2}^{OM} > \prod_{O2}^{OR}$; if $c_r < \frac{w_n(c_m+4S\alpha(1-\alpha)) - 4Sc_n\alpha(1-\alpha)}{c_n}$, then $\prod_{O2}^{OM} < \prod_{O2}^{OR}$.

$$\theta_2^{OM} - \theta_2^{OR} = \frac{w_n\alpha}{c_r+4S\alpha(1-\alpha)} - \frac{c_n\alpha}{c_m+4S\alpha(1-\alpha)} = -\frac{(c_n(c_r+4S\alpha(1-\alpha)) - w_n(c_m+4S\alpha(1-\alpha)))}{(c_m+4S\alpha(1-\alpha))(c_r+4S\alpha(1-\alpha))}$$

To solve the roots of $\theta_2^{OM} - \theta_2^{OR} = 0$, we have $c_r = \frac{w_n(c_m+4S\alpha(1-\alpha)) - 4Sc_n\alpha(1-\alpha)}{c_n}$. Hence, if $c_r > \frac{w_n(c_m+4S\alpha(1-\alpha)) - 4Sc_n\alpha(1-\alpha)}{c_n}$, then $\theta_2^{OM} < \theta_2^{OR}$; if $c_r < \frac{w_n(c_m+4S\alpha(1-\alpha)) - 4Sc_n\alpha(1-\alpha)}{c_n}$, then $\theta_2^{OM} > \theta_2^{OR}$.

Straightforward algebra then produces the findings of the Lemma 1.

Proof of Lemma 2

To prove Lemma 2, we compare the OEM's profit and the remanufacturer's quality threshold when the OEM cooperates with the CM or the IR in the indirect selling mode.

Case 1: $c_m \in (0, c_n - (4S + w_n)(1-\alpha))$ and $c_r \in (0, w_n\alpha - 4S(1-\alpha))$, we have $\prod_{O1}^{AM} - \prod_{O1}^{AR} = 0$ and $\theta_1^{AM} = \theta_1^{AR} = 0$. Hence, cooperating with the CM and IR yields an equal profit for the OEM and an equal (and 'best' possible) environment outcome.

Case 2: $c_m \in (0, c_n - (4S + w_n)(1-\alpha))$ and $c_r \in (w_n\alpha - 4S(1-\alpha), 1)$, we have.

$$\prod_{O1}^{AM} - \prod_{O2}^{AR} = \frac{S^2(1-\alpha)(c_r + w_n\alpha + 4S(1-\alpha))(c_r - w_n\alpha + 4S(1-\alpha)\alpha)}{(c_r + 4S(1-\alpha))^2} > 0$$

$$\theta_1^{AM} = 0 < \theta_2^{AR} = 1 - \frac{w_n\alpha}{c_r + 4S(1-\alpha)}$$

Hence, $\prod_{O1}^{AM} > \prod_{O2}^{AR}$ and $\theta_1^{AM} < \theta_2^{AR}$.

Case 3: $c_m \in (c_n - (4S + w_n)(1-\alpha), 1)$ and $c_r \in (0, w_n\alpha - 4S(1-\alpha))$, we have.

$$\prod_{O2}^{AM} - \prod_{O1}^{AR} = -\frac{S^2(1-\alpha)(c_m - c_n + (4S + w_n)(1-\alpha))(c_m + c_n + (4S - w_n)(1-\alpha))}{(c_m + 4S(1-\alpha))^2}$$

Then, the sign of $\prod_{O2}^{AM} - \prod_{O1}^{AR}$ depends on that of $c_m + c_n + (4S - w_n)(1-\alpha)$. To solve the roots of $c_m + c_n + (4S - w_n)(1-\alpha) = 0$, we have $c_m = (w_n - 4S)(1-\alpha) - c_n$. As $(w_n - 4S)(1-\alpha) - c_n < c_n - (4S + w_n)(1-\alpha)$ always exists, then $\prod_{O2}^{AM} < \prod_{O1}^{AR}$ always holds.

$\theta_2^{AM} = 1 - \frac{c_n - w_n(1-\alpha)}{c_m + 4S(1-\alpha)} > \theta_1^{AR} = 0$, thus $\theta_2^{AM} > \theta_1^{AR}$.

Case 4: $c_m \in (c_n - (4S + w_n)(1-\alpha), 1)$ and $c_r \in (w_n\alpha - 4S(1-\alpha), 1)$, we have.

$\prod_{O2}^{AM} - \prod_{O2}^{AR} = S^2(1-\alpha)(w_n(c_m\alpha + (c_r + 4S)(1-\alpha)) - c_n(c_r + 4S(1-\alpha)))\big(w_n((c_r$ $\frac{+4S)(1-\alpha)-c_m\alpha-8S\alpha(1-\alpha))-c_n(c_r+4S(1-\alpha)))}{(c_m+4S(1-\alpha))^2(c_r+4S(1-\alpha))^2}$ To solve the roots of $\prod_{O2}^{AM} - \prod_{O2}^{AR} = 0$, we have $c_{a1} = \dfrac{-4c_nS(1-\alpha) + w_n(4S(1-\alpha) + c_m\alpha)}{c_n - w_n(1-\alpha)}$ and $c_{a2} = \dfrac{4c_nS(1-\alpha) + w_n(c_m\alpha - 4S(1-\alpha)(1-2\alpha))}{w_n(1-\alpha) - c_n}$. Thus, if $\min(c_{a1}, c_{a2}) < c_r < \max(c_{a1}, c_{a2})$, then $\prod_{O2}^{AM} < \prod_{O2}^{AR}$; otherwise $\prod_{O2}^{AM} > \prod_{O2}^{AR}$.

$$\theta_2^{AM} - \theta_2^{AR} = \frac{(c_r + 4S)(w_n - c_n) + (4Sc_n + (c_m - c_r - 4S)w_n)\alpha}{(c_r + 4S(1-\alpha))(c_m + 4S(1-\alpha))}$$

To solve the roots of $\theta_2^{AM} - \theta_2^{AR} = 0$, we have $c_r = \dfrac{-4c_nS(1-\alpha) + w_n(4S(1-\alpha) + c_m\alpha)}{c_n - w_n(1-\alpha)}$. Thus, if $c_r < c_{a1}$, then $\theta_2^{AM} > \theta_2^{AR}$; if $c_r > c_{a1}$, then $\theta_2^{AM} < \theta_2^{AR}$.

Straightforward algebra then produces the findings of the Lemma 2.

Proof of Lemma 3

To prove Lemma 3, we compare the OEM's profit, the CM's profit and the CM's remanufacturing level when the OEM cooperates with the CM in the two selling modes. We first compare the boundaries in the two selling modes. The boundary in the model OM is $c_m^{OM} = c_n\alpha - 4S\alpha(1-\alpha)$, whereas the boundary in the model AM is $c_m^{AM} = c_n - (4S + w_n)(1-\alpha)$.

We have $c_m^{OM} - c_m^{AM} = (1-\alpha)(w_n + 4S(1-\alpha) - c_n) > 0$. Then $c_m^{OM} > c_m^{AM}$, we divide the analysis into three cases.

Case 1: $c_m \in (0, c_n - (4S + w_n)(1-\alpha))$, we have $\prod_{O1}^{OM} - \prod_{O1}^{AM} = -S^2(1-\alpha) < 0$, $\prod_{M1}^{OM} - \prod_{M1}^{AM} = S(1-\alpha)(w_n + 2S(1-\alpha) - c_n) > 0$, $\theta_1^{OM} = \theta_1^{AM} = 0$. Hence, $\prod_{O1}^{OM} < \prod_{O1}^{AM}$, $\prod_{M1}^{OM} > \prod_{M1}^{AM}$, and $\theta_1^{OM} = \theta_1^{AM}$.

Case 2: $c_m \in (c_n - (4S + w_n)(1-\alpha), c_n\alpha - 4S\alpha(1-\alpha))$, we have.

$$\prod_{O1}^{OM} - \prod_{O2}^{AM} = \frac{S^2(1-\alpha)(\alpha(c_m + 4S(1-\alpha))^2 - (c_n - (1-\alpha)w_n)^2)}{(c_m + 4S(1-\alpha))^2}$$

To solve the roots of $\prod_{O1}^{OM} - \prod_{O2}^{AM} = 0$, we have $c_m = \dfrac{\pm\sqrt{(c_n - (1-\alpha)w_n)^2\alpha} - 4S(1-\alpha)\alpha}{\alpha}$ $\left(c_m = \dfrac{-\sqrt{(c_n - (1-\alpha)w_n)^2\alpha} - 4S(1-\alpha)\alpha}{\alpha} < 0\right)$ and hence, let $c_{m1} = \dfrac{\sqrt{(c_n - (1-\alpha)w_n)^2\alpha} - 4S(1-\alpha)\alpha}{\alpha}$. We compare the root c_{m1} with the two boundaries (i.e. c^{AM} and c^{OM}). We have: (a) if $S > \dfrac{c_n(1 - \sqrt[3]{\alpha^2}) - w_n(1-\alpha)}{4(1-\alpha)^2\sqrt{\alpha}} = S_{m1}$, then $c_{m1} < c^{OM}$; otherwise $c_{m1} > c^{OM}$. (b) if $w_n > \dfrac{c_n}{1-\alpha}$, then $c_{m1} > c^{AM}$; otherwise $c_{m1} < c^{AM}$. Note that, $S > S_{m1}$ equals to $w_n > 4\left(\dfrac{c_n(1-\alpha^{2/3})}{4(1-\alpha)^2\sqrt{\alpha}} - S\right)(1-\alpha)\sqrt{\alpha}$, and we can prove that $4\left(\dfrac{c_n(1-\alpha^{2/3})}{4(1-\alpha)^2\sqrt{\alpha}} - S\right)(1-\alpha)\sqrt{\alpha} < \dfrac{c_n}{1-\alpha}$. The OEM's profit in models OM and AM satisfies the following conditions: (a) If $S > S_{m1}$ and $w_n > \dfrac{c_n}{1-\alpha}$ (i.e. $w_n > \dfrac{c_n}{1-\alpha}$), for $c_m \in (c^{AM}, c_{m1})$, then $\prod_{O1}^{OM} < \prod_{O2}^{AM}$; for $c_m \in (c_{m1}, c^{OM})$, then $\prod_{O1}^{OM} > \prod_{O2}^{AM}$. (b) If $S > S_{m1}$ and $w_n < \dfrac{c_n}{1-\alpha}$ (i.e.

$4\left(\frac{c_n(1-\alpha^{2/3})}{4(1-\alpha)^2\sqrt{\alpha}}-S\right)(1-\alpha)\sqrt{\alpha}<w_n<\frac{c_n}{1-\alpha}$), for $c_m\in(c^{AM},c^{OM})$, then $\prod_{O1}^{OM}>\prod_{O2}^{AM}$.
$\prod_{M1}^{OM}-\prod_{M2}^{AM}=-S(c_m^2+c_n^2-2c_n(1-\alpha)(w_n\ \frac{+4S\alpha)+(1-\alpha)^2(w_n^2+16S^2\alpha)-2c_m(c_n\alpha-2S(1-\alpha^2)))}{2(c_m+4S(1-\alpha))}$

To solve the roots of $\prod_{M1}^{OM}-\prod_{M2}^{AM}=0$, we have $c_m=c_n\alpha-2S(1-\alpha^2)\pm\sqrt{(w_n+2S(1-\alpha)-c_n)(1-\alpha)((2S(1-\alpha)-w_n)(1-\alpha)+c_n(1+\alpha))}$. Let $c_{m2}=c_n\alpha-2S(1-\alpha^2)-\sqrt{(w_n+2S(1-\alpha)-c_n)(1-\alpha)((2S(1-\alpha)-w_n)(1-\alpha)+c_n(1+\alpha))}$ and $c_{m3}=c_n\alpha-2S(1-\alpha^2)+\sqrt{(w_n+2S(1-\alpha)-c_n)(1-\alpha)((2S(1-\alpha)-w_n)(1-\alpha)+c_n(1+\alpha))}$. We compare the roots of c_{m2} and c_{m3} with the two boundaries (i.e. c^{AM} and c^{OM}). We can prove that $c_{m2}<c^{OM}$ and $c_{m3}>c^{AM}$ always exist and hence, $\prod_{M1}^{OM}>\prod_{M2}^{AM}$ always holds. $\theta_1^{OM}=0<\theta_2^{AM}=1-\frac{c_n-w_n(1-\alpha)}{c_m+4S(1-\alpha)}$. Hence, $\theta_1^{OM}<\theta_2^{AM}$.

Case 3: $c_m\in(c_n\alpha-4S\alpha(1-\alpha),1)$, we have.

$$\prod_{O2}^{OM}-\prod_{O2}^{AM}=\frac{S^2c_n^2\alpha^3(1-\alpha)}{(c_m+4S\alpha(1-\alpha))^2}-\frac{S^2(c_n-w_n(1-\alpha))^2(1-\alpha)}{(c_m+4S(1-\alpha))^2}$$

To solve the roots of $\prod_{O2}^{OM}-\prod_{O2}^{AM}=0$, we have $c_m=\frac{S(w_n-c_n)\alpha(1-\alpha)((w_n+c_n)\alpha-w_n+c_n)\pm 4Sc_n\alpha(1-\alpha)\sqrt{(c_n-(1-\alpha)w_n)^2\alpha}}{c_n^2(1+\alpha+\alpha^2)+w_n^2(1-\alpha)-2c_nw_n}$. Let $c_{m4}=\frac{S(w_n-c_n)\alpha(1-\alpha)((w_n+c_n)\alpha-w_n+c_n)-4Sc_n\alpha(1-\alpha)\sqrt{(c_n-(1-\alpha)w_n)^2\alpha}}{c_n^2(1+\alpha+\alpha^2)+w_n^2(1-\alpha)-2c_nw_n}$ and $c_{m5}=\frac{S(w_n-c_n)\alpha(1-\alpha)((w_n+c_n)\alpha-w_n+c_n)+4Sc_n\alpha(1-\alpha)\sqrt{(c_n-(1-\alpha)w_n)^2\alpha}}{c_n^2(1+\alpha+\alpha^2)+w_n^2(1-\alpha)-2c_nw_n}$ ($c_{m4}<c_{m5}$ always exists as $S>0$). We compare the two roots c_{m4} and c_{m5} with the two boundaries (i.e. c^{OM} and 1). We first compare c_{m4} with c^{OM}, and find that $c_{m4}<c^{OM}$ always holds. We then compare c_{m5} with c^{OM} and 1, and have the following conditions: (a) if $S<\frac{c_n(c_n^2(1+\alpha+\alpha^2)-2c_nw_n+w_n^2(1-\alpha))}{(1-\alpha)\left(3w_n^2(1-\alpha)+c_n^2\left(3+4\sqrt{\alpha}+3\alpha+4\alpha^2\right)-2c_nw_n\left(3+2(1-\alpha)\sqrt{\alpha}\right)\right)}=S_{m2}$, then $c_{m5}<c^{OM}$; (b) if $S_{m2}<S<\frac{c_n^2(1+\alpha+\alpha^2)-2c_nw_n+w_n^2(1-\alpha)}{(-1+\alpha)\alpha\left(-2c_nw_n\left(1-2(1-\alpha)\sqrt{\alpha}\right)+w_n^2(1-\alpha)+c_n^2\left(1-4\sqrt{\alpha}+\alpha\right)\right)}=S_{m3}$, then $c^{OM}<c_{m5}<1$; (c) if $S>S_{m3}$, then $c_{m5}>1$. The OEM's profit in models OM and AM satisfies the following conditions: (a) if $S<S_{m2}$, for $c_m\in(c^{OM},1)$, then $\prod_{O2}^{OM}<\prod_{O2}^{AM}$. (b) if $S_{m2}<S<S_{m3}$, for $c_m\in(c^{OM},c_{m5})$, then $\prod_{O2}^{OM}>\prod_{O2}^{AM}$; for $c_m\in(c_{m5},1)$, then $\prod_{O2}^{OM}<\prod_{O2}^{AM}$. (c) if $S>S_{m3}$, for $c_m\in(c^{OM},1)$, then $\prod_{O2}^{OM}>\prod_{O2}^{AM}$.

$$\prod_{M2}^{OM}-\prod_{M2}^{AM}=\frac{S(1-\alpha)(4S(1-\alpha)\alpha(w_n^2\alpha-(w_n-c_n)^2)+c_m(w_n-c_n)((w_n+c_n)\alpha-w_n+c_n))}{2(c_m+4S\alpha(1-\alpha))(c_m+4S(1-\alpha))}$$

To solve the roots of $\prod_{M2}^{OM}-\prod_{M2}^{AM}=0$, we have $c_m=\frac{4S\alpha(1-\alpha)((w_n-c_n)^2-w_n^2\alpha)}{(w_n-c_n)((w_n+c_n)\alpha-w_n+c_n)}=c_{m6}$. We can prove that $c_{m6}<c^{OM}$ always holds. Thus, for $c_m\in(c^{OM},1)$, then $\prod_{M2}^{OM}>\prod_{M2}^{AM}$ always holds.

$\theta_2^{OM}-\theta_2^{AM}=-\frac{(1-\alpha)(c_m(w_n-c_n)+4Sw_n\alpha(1-\alpha))}{(c_m+4S\alpha(1-\alpha))(c_m+4S(1-\alpha))}<0$. Hence, $\theta_2^{OM}<\theta_2^{AM}$.

Straightforward algebra then produces the findings of the Lemma 3.

Proof of Lemma 4

To prove Lemma 4, we compare the OEM's profit, the CM's profit, the IR's profit and the IR's remanufacturing level when the OEM cooperates with the IR in the two selling modes. We first compare the boundaries in the two selling modes. The boundary in the model OR is $c_r^{OR} = w_n\alpha - 4S\alpha(1-\alpha)$, whereas the boundary in the model AR is $c_r^{AR} = w_n\alpha - 4S(1-\alpha)$. We have $c_r^{OR} - c_r^{AR} = 4S(1-\alpha)^2 > 0$. Then $c_r^{OR} > c_r^{AR}$, we divide the analysis into three cases.

Case 1: $c_r \in (0, w_n\alpha - 4S(1-\alpha))$, we have $\prod_{O1}^{OR} - \prod_{O1}^{AR} = -S^2(1-\alpha)^2 < 0$, $\prod_{M1}^{OR} - \prod_{M1}^{AR} = S(w_n - c_n)(1-\alpha) > 0$, $\prod_{R1}^{OR} - \prod_{R1}^{AR} = 2S^2(1-\alpha)^2 > 0$, and $\theta_1^{OR} = \theta_1^{AR} = 0$. Hence, $\prod_{O1}^{OR} < \prod_{O1}^{AR}$, $\prod_{M1}^{OR} > \prod_{M1}^{AR}$, $\prod_{R1}^{OR} > \prod_{R1}^{AR}$ and $\theta_1^{OR} = \theta_1^{AR}$.

Case 2: $c_r \in (w_n\alpha - 4S(1-\alpha), w_n\alpha - 4S\alpha(1-\alpha))$, we have.

$$\prod_{O1}^{OR} - \prod_{O2}^{AR} = \frac{S^2\alpha(1-\alpha)((c_r + 4S(1-\alpha))^2 - w_n^2\alpha)}{(c_r + 4S(1-\alpha))^2}$$

To solve the roots of $\prod_{O1}^{OR} - \prod_{O2}^{AR} = 0$, we have $c_r = \pm w_n\sqrt{\alpha} - 4S(1-\alpha)$. As , we let $c_{r1} = w_n\sqrt{\alpha} - 4S(1-\alpha) > w_n\alpha - 4S(1-\alpha)$. We then compare the difference between c_{r1} and c^{OR}. We have if $S < \frac{w_n\sqrt{\alpha}}{4(1+\sqrt{\alpha} - \alpha - \sqrt[3]{\alpha^2})}$, then $c_{r1} > c^{OR}$; otherwise, $c_{r1} < c^{OR}$. The OEM's profit in models AR and OR satisfies the following conditions: (a) if $S < \frac{w_n\sqrt{\alpha}}{4(1+\sqrt{\alpha} - \alpha - \sqrt[3]{\alpha^2})}$, for $c_r \in (c^{AR}, c^{OR})$, then $\prod_{O1}^{OR} < \prod_{O2}^{AR}$ holds; (b) if $S > \frac{w_n\sqrt{\alpha}}{4(1+\sqrt{\alpha} - \alpha - \sqrt[3]{\alpha^2})}$, for $c_r \in (c^{AR}, c_{r1})$, then $\prod_{O1}^{OR} < \prod_{O2}^{AR}$ holds; for $c_r \in (c_{r1}, c^{OR})$, then $\prod_{O1}^{OR} > \prod_{O2}^{AR}$ holds.

$$\prod_{M1}^{OR} - \prod_{M2}^{AR} = \frac{S(w_n - c_n)(w_n - c_r - 4S(1-\alpha))\alpha}{(c_r + 4S(1-\alpha))}$$

To solve the roots of $\prod_{M1}^{OR} - \prod_{M2}^{AR} = 0$, we have $c_r = c_{r2} = w_n - 4S(1-\alpha)$. We have, if $w_n > 4S(1-\alpha)$, then $c_{r2} > w_n\alpha - 4S\alpha(1-\alpha)$is held, and $\prod_{M1}^{OR} > \prod_{M2}^{AR}$ always hold; if $w_n < 4S(1-\alpha)$, then $\prod_{M1}^{OR} < \prod_{M2}^{AR}$.

$$\prod_{R1}^{OR} - \prod_{R2}^{AR} = \frac{S}{2}\left(w_n\alpha\left(2 - \frac{w_n\alpha}{c_r + 4S(1-\alpha)}\right) - 4S(1-\alpha)\alpha - c_r\right)$$

To solve the roots of $\prod_{R1}^{OR} - \prod_{R2}^{AR} = 0$, we have $c_r = w_n\alpha - 2S(1-\alpha^2) \pm 2(1-\alpha)\sqrt{S(S(1-\alpha)^2 + w_n\alpha)}$. Let $c_{r3} = w_n\alpha - 2S(1-\alpha^2) - 2(1-\alpha)\sqrt{S(S(1-\alpha)^2 + w_n\alpha)}$, $c_{r4} = w_n\alpha - 2S(1-\alpha^2) + 2(1-\alpha)\sqrt{S(S(1-\alpha)^2 + w_n\alpha)}$. We compare the roots c_{r3} and c_{r4} with the boundary conditions, we have $c_{r3} < w_n\alpha - 4S(1-\alpha)$ and $c_{r4} > w_n\alpha - 4S\alpha(1-\alpha)$. Thus, $\prod_{R1}^{OR} > \prod_{R2}^{AR}$ always holds.

$\theta_1^{OR} = 0 < \theta_2^{AR} = 1 - \frac{w_n\alpha}{c_r + 4S(1-\alpha)}$. Thus, $\theta_1^{OR} < \theta_2^{AR}$.

Case 3: $c_r \in (w_n\alpha - 4S\alpha(1-\alpha), 1)$, we have.

$$\prod_{O2}^{OR} - \prod_{O2}^{AR} = \frac{S^2\alpha^2(1-\alpha)^2 w_n^2(16S^2(1-\alpha)^2\alpha - c_r^2)}{(c_r + 4S(1-\alpha))^2(c_r + 4S\alpha(1-\alpha))^2}$$

To solve the roots of $\prod_{O2}^{OR} - \prod_{O2}^{AR} = 0$, we have $c_r = \pm 4S(1-\alpha)\sqrt{\alpha}$. As $c_r = -4S(1-\alpha)\sqrt{\alpha} < 0$, let $c_{r5} = 4S(1-\alpha)\sqrt{\alpha}$. We compare the root c_{r5} with the two boundaries (i.e.c^{OR} and 1). We have: (a) if $S < \frac{w_n\sqrt{\alpha}}{4(1+\sqrt{\alpha}-\alpha-\sqrt[3]{\alpha^2})}$, then $c_{r5} < c^{OR}$; otherwise $c_{r5} > c^{OR}$. (b) if $S > \frac{1}{4(1-\alpha)\sqrt{\alpha}}$, then $c_{r5} > 1$; otherwise $c_{r5} < 1$. The OEM's profit in models AR and OR satisfies the following conditions: (a) if $S < \frac{w_n\sqrt{\alpha}}{4(1+\sqrt{\alpha}-\alpha-\sqrt[3]{\alpha^2})}$, then $\prod_{O2}^{OR} < \prod_{O2}^{AR}$. (b) if $\frac{w_n\sqrt{\alpha}}{4(1+\sqrt{\alpha}-\alpha-\sqrt[3]{\alpha^2})} < S < \frac{1}{4(1-\alpha)\sqrt{\alpha}}$, for $c_r \in (c^{OR}, c_{r5})$, then $\prod_{O2}^{OR} > \prod_{O2}^{AR}$; for $c_r \in (c_{r5}, 1)$, then $\prod_{O2}^{OR} < \prod_{O2}^{AR}$. (c) if $S > \frac{1}{4(1-\alpha)\sqrt{\alpha}}$, for $c_r \in (c^{OR}, 1)$, then $\prod_{O2}^{OR} > \prod_{O2}^{AR}$.
$\prod_{M2}^{OR} - \prod_{M2}^{AR} = \frac{Sc_r(w_n - c_n)w_n(1-\alpha)\alpha}{(c_r + 4S(1-\alpha))(c_r + 4S\alpha(1-\alpha))} > 0$. Thus, $\prod_{M2}^{OR} > \prod_{M2}^{AR}$.

$\prod_{R2}^{OR} - \prod_{R2}^{AR} = \frac{2S^2 w_n^2 (1-\alpha)^2\alpha^2}{(c_r + 4S(1-\alpha))(c_r + 4S\alpha(1-\alpha))} > 0$. Thus, $\prod_{R2}^{OR} > \prod_{R2}^{AR}$.

$\theta_2^{OR} - \theta_2^{AR} = -\frac{4Sw_n(1-\alpha)^2\alpha}{(c_r + 4S\alpha(1-\alpha))(c_r + 4S(1-\alpha))} < 0$. Thus, $\theta_2^{OR} < \theta_2^{AR}$.

Straightforward algebra then produces the findings of the Lemma 4.

Proof of Lemma 5 and Proposition 6

To prove Lemma 5 and Proposition 6, we compare model *OM* with model *OR*, and model *AM* with model *AR*, respectively with the assumption that $c_m = c_r = c$. There are three cases respectively when the remanufactured products are directly sold by the OEM or through the remanufacturer, and we analyze them separately.

(1) Direct selling of the remanufactured products

Case 1: $c \in (0, c^{OM})$, we have $\prod_{O1}^{OM} - \prod_{O1}^{OR} = 0$, $\theta_1^{OM} = \theta_1^{OR} = 0$. Thus,$\prod_{O1}^{OM} = \prod_{O1}^{OR}$, $\theta_1^{OM} = \theta_1^{OR}$.

Case 2: $c \in (c^{OM}, c^{OR})$, we have.

$$\prod_{O2}^{OM} - \prod_{O1}^{OR} = -\frac{S^2\alpha(1-\alpha)(c + c_n\alpha + 4S(1-\alpha)\alpha)(c - c_n\alpha + 4S(1-\alpha)\alpha)}{(c + 4S\alpha(1-\alpha))^2} < 0,$$

$\theta_2^{OM} > \theta_1^{OR} = 0$.

Thus, $\prod_{O2}^{OM} < \prod_{O1}^{OR}$,$\theta_2^{OM} > \theta_1^{OR}$.

Case 3: $c \in (c^{OR}, 1)$, we have.

$$\prod_{O2}^{OM} - \prod_{O2}^{OR} = -\frac{S^2\alpha^3(1-\alpha)(w_n + c_n)(w_n - c_n)}{(c + 4S\alpha(1-\alpha))^2} < 0,$$

$\theta_2^{OM} - \theta_2^{OR} = \frac{(w_n - c_n)\alpha}{c + 4S(1-\alpha)\alpha} > 0$.

Thus, $\prod_{O2}^{OM} < \prod_{O2}^{OR}$ and $\theta_2^{OM} > \theta_2^{OR}$.

(1) Indirect selling of the remanufactured products

Case 1: $c \in (0, c^{AM})$, we have $\prod_{O1}^{AM} - \prod_{O2}^{AR} = 0$ and $\theta_1^{AM} = \theta_1^{AR} = 0$. Thus, $\prod_{O1}^{AM} = \prod_{O2}^{AR}$ and $\theta_1^{AM} = \theta_1^{AR}$.

Case 2: $c \in (c^{AM}, c^{AR})$, we have.

$$\prod_{O2}^{AM} - \prod_{O1}^{AR} = -\frac{S^2(1-\alpha)(c-c_n+(4S+w_n)(1-\alpha))(c+c_n+(4S-w_n)(1-\alpha))}{(c+4S(1-\alpha))^2}$$

To solve the roots of $\prod_{O2}^{AM} - \prod_{O1}^{AR} = 0$, we have $c = (w_n - 4S)(1-\alpha) - c_n$. As $c^{AM} > (w_n - 4S)(1-\alpha) - c_n$, then $\prod_{O2}^{AM} < \prod_{O1}^{AR}$ always holds.

$\theta_2^{AM} > \theta_1^{AR} = 0$, thus $\theta_2^{AM} > \theta_1^{AR}$.

Case 3: $c \in (c^{AR}, 1)$, we have.

$$\prod_{O2}^{AM} - \prod_{O2}^{AR} = -\frac{S^2(1-\alpha)(c_n - w_n(1-2\alpha))(w_n - c_n)}{(c+4S(1-\alpha))^2} < 0,$$

$$\theta_2^{AM} - \theta_2^{AR} = \frac{(w_n - c_n)}{c + 4S(1-\alpha)} > 0.$$

Thus, $\prod_{O2}^{AM} < \prod_{O2}^{AR}$ and $\theta_2^{AM} > \theta_2^{AR}$.

From the comparison of these cases, we find that cooperating with the CM and IR can give the OEM equal profit only when the variable cost of remanufacturing is at a low level. However, when the variable cost of remanufacturing increases, the OEM should always name the IR as its remanufacturing partner if the IR can remanufacture at the same cost as the CM. And whether the remanufactured products are directly sold by the OEM or by the remanufacturer does not affect the OEM's decision to cooperate with the IR. The distribution channel choice of the remanufactured products when the CM is the remanufacturer and when the IR is the remanufacturer are listed in the proofs of Lemmas 3 and 4. Therefore, combine these results together, it is easy to prove Lemma 5 and Proposition 6.

This completes the proof.