On the Comparison of Different Methodologies for the Computation of Damped Nonlinear Normal Modes and Resonance Prediction of Systems with Non-conservative Nonlinearities

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Abstract

To improve the structural design of a future engineering component, various nonlinear characteristics have to be taken into consideration in the dynamic analysis. The inclusion of nonlinearities makes the dynamic analysis of structural components exceptionally complicated, especially in the case of non-conservative nonlinearities. Non-conservative nonlinear forces are a class of nonlinear forces that either dissipate or provide energy to the structure, i.e. introduce nonlinear damping in the system. Frictional contact, for instance, can be considered as a nonlinear damping term with non-smooth characteristics. The existence of nonlinear damping terms makes the autonomous system non-conservative, therefore classic nonlinear modal analysis methods for conservative systems cannot be applied. Furthermore, the use of the nonlinear modal analysis is also limited by the disadvantage that the relation between the nonlinear normal modes and resonant solutions in forced response cannot be directly accessed. Thus, a direct relationship between the NNM and the resonant solutions in forced response has great practical importance in that the response amplitude at resonance is the information of most interest in engineers.

To overcome these challenges, the concept of damped Nonlinear Normal Modes (dNNMs) is employed to analyse the complicated nonlinear dynamics of non-conservative systems. In this work, two different concepts, namely Complex Nonlinear Mode (CNM) [1] and Extended Periodic Motion Concept (EPMC) [2], used to compute dNNMs for dynamic systems with non-conservative nonlinearities have been compared. An alternative damping assumption in the EPMC is introduced using artificial hysteretic damping. The CNM and two variants of the EPMC are derived in the frequency domain. In order to predict the forced resonance response through dNNMs, two numerical methods, namely Extended Energy Balance Method (EEBM) [3] and Nonlinear Modal Synthesis (NMS), for predicting the resonances in the forced response using damped nonlinear normal modes are also assessed. A thorough comparison: (a) two concepts for dNNM computation including CNM and two variants of EPMC; (b) two numerical methods for resonant prediction including E-EBM and NMS, is achieved using two illustrative 1-DoF systems and a full-scale blisk with friction ring damper shown in Figure 1.

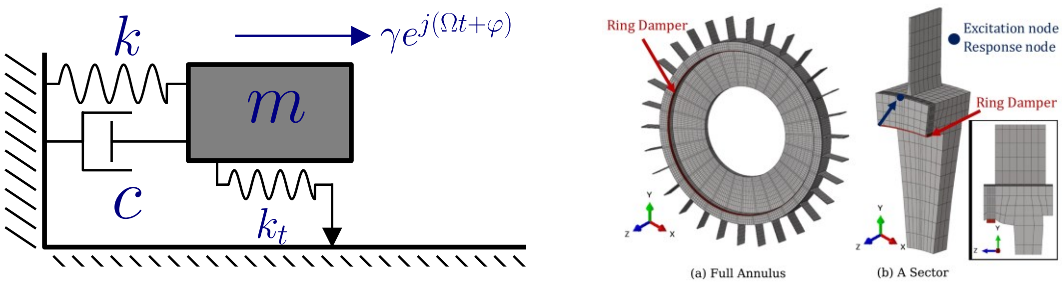

Figure 1 1 DOF System (left) and full-scale blisk with friction ring damper (right) [4]

With the detailed study carried out in [4] i.e. Figure 2, several useful statements can be concluded. Firstly, the accuracy of the CNM can be significantly affected by the presence of large nonlinear damping and higher-order harmonics. Then, both hysteretic damping and viscous damping can be used as artificial damping in EPMC. The two variants of EPMC are mathematically equivalent if only the 1st harmonic is considered in the computation of dNNM and perform equally well. Furthermore, if higher-order nonlinear damping is present with a large damping ratio, the accuracy of EPMC can be slightly reduced. Finally, for a full-scale structure with friction dampers, similar results are obtained from CNM and two variants of EPMC. The EPMC with artificial hysteretic damping can improve the computational efficiency by 3% than the original EPMC because of the simple formula of the analytical Jacobian matrices.

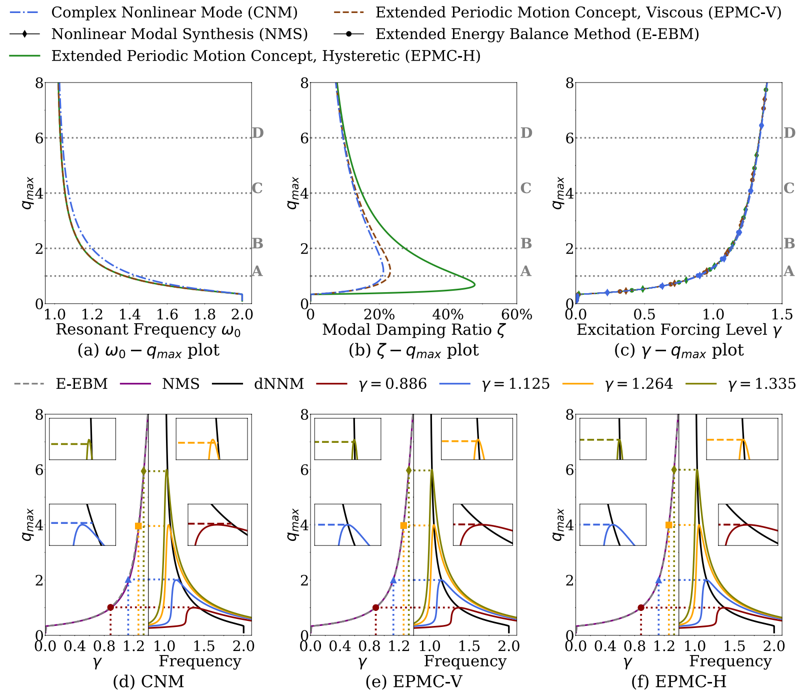

Figure 2 Testcase I: (a, b, c) Numerical results from the damped nonlinear normal mode within a range of modal amplitude; (d, e, f) Using nonlinear forced response to validate frequency -modal amplitude curves and force level-modal amplitude curves [4].

The resonant solutions are predicted using dNNMs through E-EBM and NMS. It is found that both methods have a similar principle that is, the solutions in the forced response share a great similarity with those in the dNNM. E-EBM and NMS are mathematically equivalent when only 1 harmonic is used. The study also shows that the existence of higher-order harmonic response will reduce the accuracy of NMS where E-EBM is more accurate since all the harmonics are reserved in the formulation. In addition, E-EBM is only able to predict the solutions at the resonance frequency while NMS synthetises of the full forced response possible.

Reference

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