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Two-loop Sudakov Form Factor in ABJM

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Abstract

We compute the two-loop Sudakov form factor in three-dimensional $\mathcal{N}=6$ superconformal Chern-Simons theory, using generalised unitarity. As an intermediate step, we derive the non-planar part of the one-loop four-point amplitude in terms of box integrals. Our result for the Sudakov form factor is given by a single non-planar tensor integral with uniform degree of transcendentality, and is in agreement with the known infrared divergences of two-loop amplitudes in ABJM theory. We also discuss a number of interesting properties satisfied by related three-dimensional integral functions.

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1 Introduction

In this paper we continue our investigation of amplitudes and form factors in three-dimensional $\mathcal{N}=6$ Chern-Simons matter theory, also known as ABJM [1]. This theory is closely related to the maximally supersymmetric theories constructed in [2, 3], and provides an interesting example of holographic duality in three dimensions.

One interesting fact about ABJM theory is the presence of very similar, hidden structures as in $\mathcal{N}=4$ super Yang-Mills (SYM). In particular, dual conformal symmetry [4], integrability, [5–7], duality with Wilson loops at four points [8–10], uniform transcendentality of the two-loop four-point [11, 12] and six-point amplitude [13], colour-kinematics duality [14] have made their appearance in both theories. Furthermore, several powerful methods to compute four-dimensional amplitudes could be adapted and generalised to three dimensions – BCFW recursion relations [15], generalised unitarity [4, 16–18], and Grassmannian formulations [19] of amplitudes.

With the broad aim of further exploring these similarities between ABJM and $\mathcal{N}=4$ SYM, we initiate here a study of form factors in ABJM. In the present paper we focus on Sudakov form factors of half-BPS operators, i.e. the overlap of a state created by an operator built from two scalars and a two-particle on-shell state. In $\mathcal{N}=4$ SYM, Sudakov form factors were first studied (at one and two loops) by van Neerven in [20]. More recently, these quantities were revisited and various generalisations and extensions were achieved: form factors with more than two external on-shell particles were considered in [21–24] at one loop, and BPS operators with more than two scalars were studied in [22]. In [25] the two-loop, three-point from factor was calculated using generalised unitarity and, alternatively, from symbology [26], and a remarkable link to Higgs plus one-jet amplitudes in QCD [27] was unearthed, providing a new example of the principle of maximal transcendentality, first observed in [28] for anomalous dimensions of composite operators. The calculation of the Sudakov form factor was pushed to three loops in [29], where it was also found that the result could be expressed in terms of a set of integral functions that individually are uniformly transcendental, and more recently its four-loop integrand was constructed in [30] using colour-kinematics duality [31]. In parallel there have been studies of form factors at strong coupling using AdS/CFT duality and integrability, initially in [32] and more recently in [33] where insertions of multiple operators were also considered.

As alluded to above, these studies revealed that much of the technology and mathematical structures known from amplitudes also apply to form factors, e.g. recursion relations and unitarity, integrability and Y-systems, maximal transcendentality, symbology, colour-kinematics duality. However, there are also marked differences, in particular the appearance of non-planar integral topologies and the absence of dual conformal/Yangian symmetry.

In this paper we find that the result for the two-loop Sudakov form factor in ABJM has uniform degree of transcendentality and captures correctly the infrared divergences of two-loop amplitudes. The slightly unusual observation is that it is expressed in terms of single non-planar integral function with a very peculiar numerator. This is different from the story in $\mathcal{N}=4$ SYM, where the two-loop form factor [20] is expressed in terms of a planar and a non-planar integral function which both have trivial numerators and are separately transcendental. The role of this special numerator is two-fold: it removes unphysical infrared divergences linked to internal three-particle vertices, and at the same time makes the result transcendental. We have investigated these issues for several planar two-loop topologies with very similar findings, i.e. choosing the numerators in such a way to remove unphysical infrared divergences makes these integral transcendental. These planar topologies do not appear in the ABJM form factor but are ingredients of the form factors in ABJ which is also transcendental.

There are some parallels with $\mathcal{N} = 4$ SYM, namely there exist certain superficially dual-conformal integrands which upon integration lead to divergent results even with external massive kinematics [34]. There, this phenomenon was observed only at higher loops and it was shown that such integrals, which could potentially contribute to amplitudes, actually have vanishing coefficient. Similar observations were made in the study of two-loop amplitudes in ABJM [13], where important constraints on integral functions were inferred from the condition of vanishing triple cuts involving three-particle (and five-particle) amplitudes. It is the vanishing triple cuts, involving three-particle amplitudes, that we identify to guarantee the absence of unphysical infrared divergences and of terms which would spoil the uniform transcendentality of the result. Furthermore, these triple cuts give powerful constraints and cross-checks on the (numerators of) integral functions. This observation also points to more general questions in the study of amplitudes and form factors in three and four dimensions: what is a good basis of integral functions? What are the physical and practical criteria for the choice? What are the properties of this basis? Some of the desirable features would be that the expansion coefficients are ϵ-independent (in the case of ABJM this requires tensor integrals instead of scalar integrals) and that the integral functions individually are transcendental[1], but it would clearly be interesting to understand better the underlying physical and mathematical criteria for any potential choice. The ABJ(M) form factors seem to provide an interesting playground to study such issues.

The rest of the paper is organised as follows. After reviewing some general properties of ABJM amplitudes in Section 2, we present in Section 3 the calculation of the non-planar part of the one-loop four-point amplitude in this theory. The complete – planar plus non-planar – integrand of the four-point amplitude is a key ingredient in the construction of the Sudakov form factor, which is discussed in Section 4. There, we derive this quantity firstly at one and then at two loops, using two- and three-particle cuts at the level of the integrand. As mentioned earlier, the result is expressed in terms of a single non-planar integral topology with a special numerator, whose properties we discuss in detail. In particular, we consider certain three-vertex cuts, which put strong constraints on the form of these numerators. We also compare our result to the known infrared divergences of ABJM amplitude at two loops, finding complete agreement. Finally, in Section 5 we introduce three planar integral topologies which contribute to the ABJ form factor. We discuss their properties and present their maximally transcendental result. Three appendices contain details on half-BPS operators in ABJM, on the one-loop box function in terms of which the four-point amplitude is expressed, and on the reduction to master integrals of the integral topologies discussed in this paper.

[1] See [35] for a recent proposal in four dimensions.

2 Scattering amplitudes in ABJM theory

In the following we briefly review some key facts of ABJM theory, and in particular of its tree amplitudes, which appear in the construction of loop amplitudes and form factors using unitarity [36–39].[2]

2.1 Superamplitudes

Three-dimensional $\mathcal{N} = 6$ Chern-Simons matter theory [1] (or, in short, ABJM) is a quiver theory with gauge group $U_k(N) \times U_{-k}(N)$, where k and $-k$ are the Chern-Simons levels of the gauge fields A_μ and $\hat{A}_\mu$, respectively. The matter fields comprise four complex scalars ϕ^A and four fermions ψ_A^α, where $A = 1, \dots, 4$ is a $SU(4)$ R-symmetry index and $\alpha = 1, 2$ is a spin index. The fields $(\phi^A)^i_{\bar{j}}$ and $(\psi_A^\alpha)^i_{\bar{j}}$ transform in the bifundamental representation $(N, \bar{N})$ of the gauge group, while $(\bar{\phi}_A)^{\bar{i}}_j$ and $(\bar{\psi}_\alpha^A)^{\bar{i}}_j$ transform in the $(\bar{N}, N)$, with $i, \bar{j} = 1, \dots, N$. An interesting variant of ABJM is the so-called ABJ theory, i.e. $\mathcal{N} = 6$ Chern-Simons theory with gauge group $U_k(N) \times U_{-k}(N')$. In this case $i = 1, \dots, N$, $\bar{j} = 1, \dots, N'$. Note that in the ABJ(M) theory the gauge fields are non-dynamical because of the topological nature of the Chern-Simons action, and hence they cannot appear as external states.

The momenta of the particles can be written efficiently in the three-dimensional spinor helicity formalism as

$$p_{\alpha\beta} := \lambda_\alpha \lambda_\beta \,, \tag{2.1}$$

where λ_α are commuting spinors. The states of the ABJM theory can be packaged into two Nair superfields [40, 41],

$$\Phi(\lambda, \eta) = \phi^4(\lambda) + \eta^A \psi_A(\lambda) + \frac{1}{2}\epsilon_{ABC}\eta^A\eta^B\phi^C(\lambda) + \frac{1}{3!}\epsilon_{ABC}\eta^A\eta^B\eta^C\psi_4(\lambda) \,, \tag{2.2}$$

$$\bar{\Phi}(\lambda, \eta) = \bar{\psi}^4(\lambda) + \eta^A \bar{\phi}_A(\lambda) + \frac{1}{2}\epsilon_{ABC}\eta^A\eta^B\bar{\psi}^C(\lambda) + \frac{1}{3!}\epsilon_{ABC}\eta^A\eta^B\eta^C\bar{\phi}_4(\lambda) \,, \tag{2.3}$$

where η^A, $A = 1, 2, 3$ are Grassmann coordinates parameterising an $\mathcal{N} = 3$ superspace. The superfields Φ and $\bar{\Phi}$ carry colour indices $\Phi^i_{\bar{j}}$ and $\bar{\Phi}^{\bar{i}}_j$. Note that Φ is bosonic while $\bar{\Phi}$ is fermionic. This description breaks the $SU(4)$ R-symmetry of the theory down to a manifest $U(3)$.

Colour-ordered partial amplitudes were introduced in [42], and we denote them as $\mathcal{A}(\bar{\Phi}_1, \Phi_2, \dots, \Phi_n)$. An important feature of ABJ(M) is that any amplitude with an odd number of particles vanishes, as a simple consequence of gauge invariance. Invariance under translations and supersymmetry transformations ensures that amplitudes are proportional to $\delta^{(3)}(P)\delta^{(6)}(Q)$, where Q^A_α and $P_{\alpha\beta}$ are the total momentum and supermomentum of n particles, respectively:

$$P_{\alpha\beta} := \sum_{i=1}^{n} \lambda_{i,\alpha}\lambda_{i,\beta} \,, \qquad Q^A_\alpha := \sum_{i=1}^{n} \lambda_{i,\alpha}\eta_i^A \,. \tag{2.4}$$

[2] In this paper we follow the conventions of Section 2 and Appendix A of [17] for the ABJM superamplitudes and the three-dimensional spinor helicity formalism, respectively.

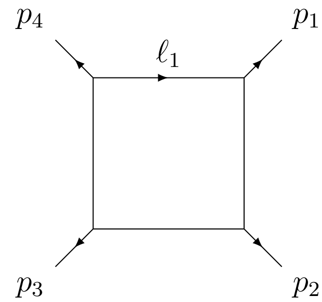

Figure 1: *The one-loop box function in* (2.7).

The first non-vanishing amplitude of the theory occurs at four points, and is the basic building block of higher-point amplitudes. At tree level it is given by the following compact expression [43],

$$\mathcal{A}_4^{(0)}(\bar{1},2,\bar{3},4) \;=\; i\,\frac{\delta^{(6)}(Q)\delta^{(3)}(P)}{\langle 1\,2\rangle\,\langle 2\,3\rangle}\;. \tag{2.5}$$

As usual, component amplitudes can be obtained by extracting the coefficient of the appropriate monomial in the η_i variables. For instance, in order to pick the component amplitude $A\big(\bar{\phi}_A(p_1),\phi^4(p_2),\bar{\phi}_4(p_3),\phi^A(p_4)\big)$ we need to expand the fermionic delta function $\delta^{(6)}(Q)$ and keep the term $(\eta_1)^1(\eta_2)^0(\eta_3)^3(\eta_4)^2\langle 1\,3\rangle\langle 3\,4\rangle^2$.

The one-loop colour-ordered four-point superamplitude[3] was constructed in [11], and is equal to[4]

$$\mathcal{A}^{(1)}(\bar{1},2,\bar{3},4) \;=\; i\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4)\,N\,I(1,2,3,4)\;, \tag{2.6}$$

where $s_{ij}=(p_i+p_j)^2$. The one-loop integral $I(1,2,3,4)$ is defined by

$$I(1,2,3,4)\;:=\;\int\frac{d^D\ell}{i\pi^{D/2}}\,\frac{s_{12}\,\mathrm{Tr}(\ell\,p_1\,p_4)\;+\;\ell^2\,\mathrm{Tr}(p_1\,p_2\,p_4)}{\ell^2(\ell-p_1)^2(\ell-p_1-p_2)^2(\ell+p_4)^2}\,, \tag{2.7}$$

with $D=3-2\epsilon$. Note that $\mathrm{Tr}(abc)=2\epsilon(a,b,c):=2\epsilon_{\mu\nu\rho}a^\mu b^\nu c^\rho$.

Explicit evaluation of the right-hand side of (2.6) shows that $\mathcal{A}^{(1)}(\bar{1},2,\bar{3},4)$ is of $\mathcal{O}(\epsilon)$, and hence vanishes in three dimensions [11]. This is consistent with the fact that all one-loop amplitudes in ABJM can be expanded in terms of one-loop triangle functions [16], as expected from dual conformal invariance. The vanishing of the four-point amplitude then follows since one-mass (and two-mass) triangles vanish in three dimensions. Very interestingly, the box function with the particular numerator in (2.7) is also dual conformal invariant, as was demonstrated in [11] using a five-dimensional embedding formalism. Furthermore, the expression for $\mathcal{A}^{(1)}(\bar{1},2,\bar{3},4)$ given in (2.6) is correct to all orders in the dimensional regularisation parameter ϵ. In the following, the integrand of (2.7) will be a crucial ingredient in applying unitarity at the integrand level.

[3] Here, and in what follows, we use the normalisation $1/(i\pi^{D/2})$ per loop. In [11], the normalisation is $1/(2\pi)^D$.

[4] We suppress the Chern-Simons level k, which will be reinstated at the end of our two-loop calculation.

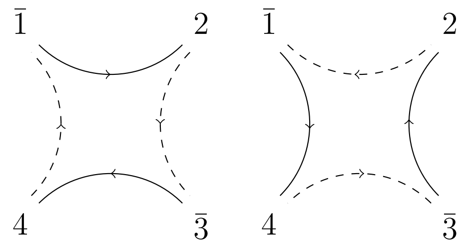

Figure 2: *The two possible colour orderings* $[1,2,3,4]$ *and* $[1,4,3,2]$ *appearing in the four-point tree-level amplitude* (2.10).

2.2 Colour ordering at tree level

As is well known from experience in $\mathcal{N}=4$ SYM, starting from two loops, cuts of form factors receive contributions from non-planar amplitudes which are as leading as those arising from the planar amplitudes (see for example [44, 25]). For our present purposes, we will need the complete (planar and non-planar) one-loop amplitude in ABJM at four points. We recall that complete tree amplitudes, denoted here by $\tilde{\mathcal{A}}$, are given by [42, 7]

$$\tilde{\mathcal{A}}(\bar{1},2,\ldots,n) \;=\; \sum_{\mathcal{P}_n} \mathrm{sgn}(\sigma)\mathcal{A}^{(0)}\big(\sigma(\bar{1}),\sigma(2),\sigma(\bar{3}),\ldots,\sigma(n)\big)\,\big[\sigma(\bar{1}),\sigma(2),\sigma(\bar{3}),\ldots,\sigma(n)\big]\,, \tag{2.8}$$

where $\mathcal{P}_n := (S_{n/2}\times S_{n/2})/C_{n/2}$ are permutations of n sites that only mix even (bosonic) and odd (fermionic) particles among themselves, modulo cyclic permutations by two sites, and the function $\mathrm{sgn}(\sigma)$ is equal to -1 if σ involves an odd permutation of the odd (fermionic) sites, and $+1$ otherwise. $\mathcal{A}^{(0)}(\bar{1},2,\bar{3}\ldots,n)$ are colour-ordered tree amplitudes, and we have also defined

$$\big[\bar{1},2,\bar{3},\ldots,n\big] \;:=\; \delta^{\bar{i}_1}_{\bar{i}_2}\delta^{i_2}_{i_3}\delta^{\bar{i}_3}_{\bar{i}_4}\cdots\delta^{i_n}_{i_1}\,. \tag{2.9}$$

In the following we will just write $\big[1,2,\cdots,n\big]$ without specifying if a particle is barred (i.e. fermionic) or un-barred (bosonic), with the understanding that the first entry in the bracket always represents a fermionic field.

As an example, we consider the complete four-point amplitude at tree level. It includes the two colour structures $[1,2,3,4]$ and $[1,4,3,2]$ (see Figure 2) and is given by the following expression:

$$\tilde{\mathcal{A}}^{(0)}(\bar{1},2,\bar{3},4) \;=\; \mathcal{A}^{(0)}(\bar{1},2,\bar{3},4)\Big(\big[1,2,3,4\big]\;-\;\big[1,4,3,2\big]\Big)\,. \tag{2.10}$$

We have also used that

$$\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4) \;=\; \mathcal{A}^{(0)}(\bar{3},2,\bar{1},4)\,, \tag{2.11}$$

a fact that follows from (2.5).

2.3 Symmetry properties of amplitudes

It is useful to recall the following general relations for colour-ordered amplitudes [16]:

$$\mathcal{A}^{(l)}(\bar{1},2,\bar{3}\cdots,n) \;=\; (-)^{\frac{n}{2}-1}\,\mathcal{A}^{(l)}(\bar{3},4,\cdots,\bar{1},2)\,, \tag{2.12}$$

and

$$\mathcal{A}^{(l)}(\bar{1},2,\bar{3}\cdots,n) \;=\; (-)^{\frac{n(n-2)}{8}+l}\,\mathcal{A}^{(l)}(\bar{1},n,\overline{n-1},n-2,\cdots,\bar{3},2)\ . \tag{2.13}$$

We note that complete amplitudes should behave under exchange of any two particles as the spin-statistics theorem requires. In particular we expect

$$\tilde{\mathcal{A}}^{(l)}(\bar{1},2,\bar{3},4) \;=\; -\,\tilde{\mathcal{A}}^{(l)}(\bar{3},2,\bar{1},4)\ , \tag{2.14}$$

at any loop order. This requires this explains the presence of the crucial minus sign between the two possible colour structures in (2.10).

3 The complete one-loop amplitude

3.1 Results

In this section we present our result for the complete four-point amplitude at one loop in ABJM. As mentioned earlier, this amplitude will be needed in order to construct the two-particle cuts of the two-loop form factor. The one-loop four-point amplitude is given by the sum of a planar and non-planar contribution:

$$\tilde{\mathcal{A}}^{(1)}(\bar{1},2,\bar{3},4) \;=\; \mathcal{A}^{(1)}_{\mathrm{P}}(\bar{1},2,\bar{3},4) \,+\, \mathcal{A}^{(1)}_{\mathrm{NP}}(\bar{1},2,\bar{3},4)\ , \tag{3.1}$$

where

$$\mathcal{A}^{(1)}_{\mathrm{P}}(\bar{1},2,\bar{3},4) = i\,N\,\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4)\,I(1,2,3,4)\Big([1,2,3,4]+[1,4,3,2]\Big)\,, \tag{3.2}$$

and

$$\begin{aligned}\mathcal{A}^{(1)}_{\mathrm{NP}}(\bar{1},2,\bar{3},4) = -\,2\,i\,\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4)\Big[\Big(I(1,2,3,4)-I(4,2,3,1)\Big)[1,2][3,4]\\ -\Big(I(2,3,4,1)-I(1,3,4,2)\Big)[1,4][3,2]\Big]\ .\end{aligned} \tag{3.3}$$

Note that the double-trace structure $[1,2]$ is

$$[1,2] \;=\; \delta^{\bar{i}_1}_{\bar{i}_2}\delta^{i_2}_{i_1}\ . \tag{3.4}$$

The complete one-loop amplitude can also be written in the following way,

$$\begin{aligned}\frac{\tilde{\mathcal{A}}^{(1)}(\bar{1},2,\bar{3},4)}{\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4)} = i\Big\{I(1,2,3,4)\Big[N\Big([1,2,3,4]+[1,4,3,2]\Big)-2[1,2][3,4]-2[1,4][3,2]\Big]\\ +\,2\Big[I(4,2,3,1)[1,2][3,4]-I(1,3,4,2)[1,4][3,2]\Big]\Big\}.\end{aligned} \tag{3.5}$$

3.2 Symmetry properties of the one-loop amplitude

Before discussing the derivation of (3.1), it is instructive to prove that $\mathcal{A}^{(1)}_{\rm P}$ and $\mathcal{A}^{(1)}_{\rm NP}$ are antisymmetric under the swap $\bar{1} \leftrightarrow \bar{3}$ (see (2.14)). In order to show this one needs to use (2.11) and the following relations satisfied by the one-loop box (2.7):

$$I(a,b,c,d) \ = \ -\,I(b,c,d,a) \ , \qquad I(a,b,c,d) = -I(c,b,a,d) \ . \tag{3.6}$$

These relations state that by cyclically shifting the labels of the external legs of the box function (2.7) by one unit one picks a minus sign; and similarly if one swaps two non-adjacent legs. Both relations are straightforward to prove using the definition (2.7) of the box function. One then finds,

$$\begin{aligned} I(3,2,1,4) - I(4,2,1,3) &= I(2,3,4,1) - I(1,3,4,2) \ , \\ I(2,1,4,3) - I(3,1,4,2) &= I(1,2,3,4) - I(4,2,3,1) \ . \end{aligned} \tag{3.7}$$

Using (3.7) we get

$$\begin{aligned} \mathcal{A}^{(1)}_{\rm P}(\bar{1},2,\bar{3},4) &= -\mathcal{A}^{(1)}_{\rm P}(\bar{3},2,\bar{1},4) \ , \\ \mathcal{A}^{(1)}_{\rm NP}(\bar{1},2,\bar{3},4) &= -\mathcal{A}^{(1)}_{\rm NP}(\bar{3},2,\bar{1},4) \ . \end{aligned} \tag{3.8}$$

Notice the presence of a minus sign between the two non-planar colour structure $[1,2][3,4]$ and $[1,4][3,2]$ appearing in the non-planar one-loop amplitude (3.3).

3.3 Derivation of the complete one-loop amplitude from cuts

We now briefly outline the strategy for the derivation of the complete one-loop amplitude (3.1), which is very similar to that in $\mathcal{N}=4$ SYM, see for example [45]. We consider the two-particle cuts of the complete one-loop amplitude, which are obtained by merging two tree-level amplitudes summed over all possible colour structures and internal particle species. We will see that each cut can be re-expressed in terms of cuts of sums of box functions (2.7). The sum over internal species is (partially) performed via an integration over the Grassmann variables η_{ℓ_1} and η_{ℓ_2} associated to the cut momenta. If one of the particles crossing is bosonic and the other is fermionic we also have to add to this the same expression with $\ell_1 \leftrightarrow \ell_2$ – this is necessary only for the s- and t-cuts. For instance, the s-cut integrand of the one-loop amplitude is[5]

$$\tilde{\mathcal{A}}^{(1)}(\bar{1},2,\bar{3},4)|_{s-{\rm cut}} \ = \ \frac{1}{2}\int d^3\eta_{\ell_1} d^3\eta_{\ell_2} \ \tilde{\mathcal{A}}^{(0)}(\bar{1},2,-\bar{\ell}_2,-\ell_1) \times \tilde{\mathcal{A}}^{(0)}(\bar{3},4,\bar{\ell}_1,\ell_2) + \ell_1 \leftrightarrow \ell_2 \, . \tag{3.9}$$

[5] For convenience we include here a factor of $\frac{1}{2}$ in the definition of the (symmetrised) cuts. In practice it means that we take the average of the two contributions in the s- and t-cuts, and multiply the u-cut with a symmetry factor as two identical (super)particles cross the cut.

The one-loop amplitude has cuts in the s-, t- and u-channels, for which we find the following integrands:

$$\begin{aligned}
\tilde{\mathcal{A}}^{(1)}(\bar{1},2,\bar{3},4)|_{s-\text{cut}} &= \frac{i}{2}\,\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4)\ c_s\,\mathcal{S}_{12}I(1,2,3,4)|_{s-\text{cut}}\ ,\\
\tilde{\mathcal{A}}^{(1)}(\bar{1},2,\bar{3},4)|_{t-\text{cut}} &= \frac{i}{2}\,\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4)\ c_t\,\mathcal{S}_{23}I(1,2,3,4)|_{t-\text{cut}}\ ,\\
\tilde{\mathcal{A}}^{(1)}(\bar{1},2,\bar{3},4)|_{u-\text{cut}} &= \frac{i}{2}\,\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4)\ c_u\,\mathcal{S}_{13}I(3,1,2,4)|_{u-\text{cut}}\ ,
\end{aligned} \tag{3.10}$$

where the colour factors c_s, c_t, c_u are

$$\begin{aligned}
c_s &= N[1,2,3,4] + N[1,4,3,2] - 2[1,2][3,4]\,,\\
c_t &= N[1,2,3,4] + N[1,4,3,2] - 2[1,4][3,2]\,,\\
c_u &= 2[1,2][3,4] - 2[1,4][3,2]\,,
\end{aligned} \tag{3.11}$$

and we recall that by $\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4)$ we denote the colour-ordered four-point amplitude. Furthermore, we indicate by $\mathcal{S}_{ab}I(a,b,c,d)|_{s_{ab}-\text{cut}}$, the s_{ab}-cut of the one-loop box function $I(a,b,c,d)$ in (2.7), symmetrised in the cut loop momenta ℓ_1 and ℓ_2, which are defined such that $\ell_1+\ell_2 = p_a+p_b$,

$$\begin{aligned}
\mathcal{S}_{12}I(1,2,3,4)|_{s-\text{cut}} &= \frac{s\,\mathrm{Tr}(\ell_1 p_1 p_4)}{(\ell_1-p_1)^2(\ell_1+p_4)^2} + \ell_1 \leftrightarrow \ell_2\,,\\
\mathcal{S}_{23}I(1,2,3,4)|_{t-\text{cut}} &= \frac{(-t)\mathrm{Tr}(\ell_1 p_1 p_2)}{(\ell_1-p_1)^2(\ell_1+p_2)^2} + \ell_1 \leftrightarrow \ell_2\,,\\
\mathcal{S}_{13}I(3,1,2,4)|_{u-\text{cut}} &= \frac{u\,\mathrm{Tr}(\ell_2 p_3 p_4)}{(\ell_2-p_3)^2(\ell_2+p_4)^2} + \ell_1 \leftrightarrow \ell_2\,.
\end{aligned} \tag{3.12}$$

We should stress here that despite the simplified notation the cut momenta ℓ_1 and ℓ_2 are different for the three distinct channels under considerations. For instance, $\ell_1+\ell_2 = p_1+p_2$ for the s-cut, while $\ell_1+\ell_2 = p_2+p_3$ in the t-cut and $\ell_1+\ell_2 = p_1+p_3$ in the u-cut. Recall that the symmetrisation in the cut momenta in the s- and t-channel coefficients originates from summing over all possible particle species that can propagate on the cut legs, while in the u cut there is a single configuration allowed, and the result turns out to be automatically symmetric in ℓ_1 and ℓ_2.

Next we merge the cuts into box functions. For the planar structures $[1,2,3,4]$ and $[1,4,3,2]$ this is immediate as the only function consistent with the s- and t-cuts in (3.10) and vanishing u-cut is $I(1,2,3,4)$. Hence, the corresponding planar amplitude is

$$i\,\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4)\ N\big([1,2,3,4]+[1,4,3,2]\big)\,I(1,2,3,4)\ , \tag{3.13}$$

thus arriving at the expression (3.2) for the planar part of the full one-loop amplitude.[6] For the non-planar terms $[1,2][3,4]$ and $[1,4][3,2]$ we need to use the results of Appendix B.2 and in particular (B.13), which we reproduce here,

$$\mathcal{S}_{ab}I(a,b,c,d)|_{s_{ab}-\text{cut}} \ = \ \mathcal{S}_{ab}I(a,b,d,c)|_{s_{ab}-\text{cut}}\ . \tag{3.14}$$

[6]Note that at the level of the integral we can simply replace $\mathcal{S}_{12}I(1,2,3,4)$ by $2\,I(1,2,3,4)$.

Firstly, we note that an immediate consequence of this result is that

$$\mathcal{S}_{23} I(2,3,4,1)|_{t-\text{cut}} \ - \ \mathcal{S}_{23} I(2,3,1,4)|_{t-\text{cut}} \ = \ 0 \, , \tag{3.15}$$

in other words the combination $I(2,3,4,1)-I(2,3,1,4)$, symmetrised in the loop momenta ℓ_1 and ℓ_2, with $\ell_1+\ell_2 = p_2+p_3$, has a vanishing t-channel cut as expected for the coefficient of the $[1,2][3,4]$ colour structure (see (3.11)). For the same combination we find, using $I(2,3,4,1) = -I(1,2,3,4)$, the symmetrised s-cut

$$- \, \mathcal{S}_{12} I(1,2,3,4)|_{s-\text{cut}} \, , \tag{3.16}$$

and similarly, for the symmetrised u-cut we obtain

$$\mathcal{S}_{13} I(3,1,4,2)|_{u-\text{cut}} = \mathcal{S}_{13} I(3,1,2,4)|_{u-\text{cut}} \, , \tag{3.17}$$

where we have used $I(2,3,1,4) = -I(3,1,4,2)$ and (B.13), which allows us to swap the last two legs on the symmetrised u-cut. Comparing with (3.10) and (3.11) we can uniquely fix the coefficient of the non-planar structure $[1,2][3,4]$:

$$2\, i\, \mathcal{A}^{(0)}(\bar{1},2,\bar{3},4) \ [1,2][3,4] \ \Big[I(2,3,4,1) - I(2,3,1,4) \Big] \, , \tag{3.18}$$

or, using the first relation of (3.6),

$$- \, 2\, i\, \mathcal{A}^{(0)}(\bar{1},2,\bar{3},4) \ [1,2][3,4] \ \Big[I(1,2,3,4) - I(4,2,3,1) \Big] \, . \tag{3.19}$$

One can proceed similarly for the coefficient of the other non-planar structure $[1,4][3,2]$, arriving at the result quoted earlier in (3.3). Note that in that result we use the freedom to rename loop momenta in order to eliminate the various symmetrisations introduced by the operation $\mathcal{S}_{ab}$ above.

4 The Sudakov form factor at one and two loops

We now move on to the form factors of gauge-invariant, single-trace scalar operators

$$\mathcal{O} = \text{Tr} \left(\phi^{A_1} \bar{\phi}_{B_1} \phi^{A_2} \bar{\phi}_{B_2} \ldots \phi^{A_L} \bar{\phi}_{B_L} \right) \chi^{B_1 \ldots B_L}_{A_1 \ldots A_L} \, , \tag{4.1}$$

where A and B are indices of the $\mathbf{4}$ and $\bar{\mathbf{4}}$ representation of the R-symmetry group $SU(4)$. The operators (4.1) are half BPS if χ is a symmetric traceless tensor in all the A_i and B_i indices separately (see for example [5, 6]). For $L = 2$, the relevant operator is[7]

$$\mathcal{O}^A{}_B = \text{Tr} \left(\phi^A \bar{\phi}_B - \frac{\delta^A{}_B}{4} \phi^K \bar{\phi}_K \right) \, . \tag{4.2}$$

In the rest of the paper we will focus on the Sudakov form factor

$$\langle (\bar{\phi}_A)^{\bar{i}_1}_{i_1}(p_1) \, (\phi^4)^{i_2}_{\bar{i}_2}(p_2) | \text{Tr}(\bar{\phi}_A \phi^4)(0) | 0 \rangle \ := \ [1,2] \ F(q^2) \ , \tag{4.3}$$

where $q := p_1 + p_2$ and $A \neq 4$, and we recall that $[1,2] := \delta^{\bar{i}_1}_{\bar{i}_2} \delta^{i_2}_{i_1}$. At tree level,

$$F^{(0)}(q^2) \ = \ 1 \ . \tag{4.4}$$

We will now derive this quantity at one and two loops.

[7] More details on half-BPS operators, as well as conventions are discussed in Appendix A.

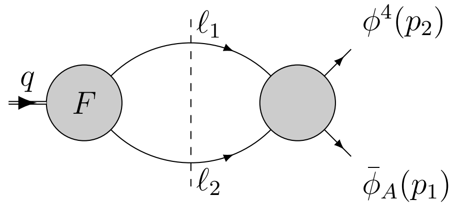

Figure 3: *The q^2 cut of the Sudakov form factor. Note that the amplitude on the right-hand side of the cut is summed over all possible colour orderings.*

4.1 One-loop form factor in ABJM

At one loop it is possible to determine the integrand of the form factor from a single unitarity cut in the q^2 channel. As shown in Figure 3, on one side of the cut there is the Sudakov form factor and on the other side the complete four-point amplitude, both at tree level. The colour-ordered tree amplitude is given in (2.5). Let us work out the colour factor first. It is given by

$$\delta^{\bar i_{\ell_2}}_{\bar i_{\ell_1}} \delta^{i_{\ell_1}}_{i_{\ell_2}} (\delta^{\bar i_1}_{\bar i_2} \delta^{i_2}_{i_{\ell_1}} \delta^{\bar i_{\ell_1}}_{\bar i_{\ell_2}} \delta^{i_{\ell_2}}_{i_1} - \delta^{\bar i_1}_{\bar i_{\ell_2}} \delta^{i_2}_{i_1} \delta^{\bar i_{\ell_1}}_{\bar i_2} \delta^{i_{\ell_2}}_{i_{\ell_1}}) \;=\; (N' - N) \delta^{\bar i_1}_{\bar i_2} \delta^{i_2}_{i_1} \,. \tag{4.5}$$

Obviously, the one-loop form factor vanishes identically in ABJM theory, because in this case $N' = N$.

We now consider the kinematic part. Since the operator is built solely out of scalars, only the four-point scalar amplitude can appear in the cut. To match the particles of the tree amplitude in Figure 3, we pick the $(\eta_1)^1(\eta_{\ell_1})^3(\eta_{\ell_2})^2(\eta_2)^0$ component from the $\delta^6(Q)$ to write the q^2 cut of the one-loop form factor as:

$$\frac{\delta^{(6)}(Q)\big|_{(\eta_1)^1(\eta_{\ell_1})^3(\eta_{\ell_2})^2(\eta_2)^0}}{\langle 1\,2\rangle\langle 2\,\ell_1\rangle} = \frac{\langle \ell_1\,\ell_2\rangle^2\langle 1\,\ell_1\rangle}{\langle 1\,2\rangle\langle 2\,\ell_1\rangle} = \frac{\langle 1\,2\rangle\langle 1\,\ell_1\rangle}{\langle 2\,\ell_1\rangle} = -\frac{\mathrm{Tr}(\ell_1 p_1 p_2)}{2(\ell_1\cdot p_2)}\,, \tag{4.6}$$

which can be immediately lifted to a full integral as it is the only possible cut of the form factor. Thus we get,

$$F^{(1)}(q^2) = (N' - N)\int \frac{d^D\ell_1}{i\pi^{D/2}} \frac{\mathrm{Tr}(\ell_1 p_1 p_2)}{\ell_1^2\,(\ell_1 - p_2)^2(\ell_1 - p_1 - p_2)^2}\,. \tag{4.7}$$

The integral in (4.7) is a linear triangle and is of $\mathcal{O}(\epsilon)$. Hence, we conclude that the one-loop Sudakov form factor in ABJ theory vanishes in strictly three dimensions. Moreover, the three-dimensional integrand vanishes in ABJM theory but is non-vanishing for $N \neq N'$ and can (and does) participate in unitarity cuts at two loops in ABJ theory. Note, that the vanishing of the one-loop form factors in ABJ(M) is consistent with the infrared finiteness of one-loop amplitudes in ABJ(M).

4.2 Two-loop form factor in ABJM

Next, we come to the computation of the two-loop Sudakov form factor. In order to construct an ansatz for its integrand we will make use of two-particle cuts, and fix potential remaining ambiguities with various three-particle cuts described in detail in Sections 4.2.2 and 4.2.3.

Three-particle cuts are very useful because they receive contributions from planar as well as non-planar integral functions at the same time, and thus are particularly constraining. A special feature of ABJM theory is that all amplitudes with an odd number of external particles vanish and, as a consequence, all cuts involving such amplitudes are identically zero [13]. In our case this observation will be important for triple cuts, where three- and five-particle amplitudes would appear.

A particular type of such cuts, first considered in [13] in the context of loop amplitudes in ABJM, involves three adjacent cut loop momenta meeting at a three-point vertex. The vanishing of these cuts imposes strong constraints on the form of the loop integrands. We will discuss and exploit this later in this section, where we will also make the intriguing observation that integral functions with numerators satisfying such constraints are transcendental and free of certain unwanted infrared divergences.

4.2.1 Two-particle cuts

We begin by considering the cut shown in Figure 4, which contains a tree-level Sudakov form factor merged with the integrand of the complete one-loop, four-point amplitude. The internal particle assignment is fixed and is determined by the particular operator we consider. The integrand of this cut is schematically given by

$$F^{(0)}(\bar{\ell}_2, \ell_1)[\ell_2, \ell_1]\ \tilde{\mathcal{A}}^{(1)}\left(\bar{\phi}_A(p_1), \phi^4(p_2), \bar{\phi}_4(-\ell_1), \phi^A(-\ell_2)\right)\ , \tag{4.8}$$

where we picked the relevant component amplitude of the complete one-loop amplitude $\tilde{\mathcal{A}}^{(1)}$, given in (3.1), and we recall that the colour factor $[a, b]$ is defined in (3.4).

We begin by working out the colour structures that will appear in the result. Firstly we consider the planar amplitude (3.2) and combine it with the part of the non-planar amplitude (3.3) containing $I(1, 2, -\ell_1, -\ell_2)$. Intriguingly, by contracting this with the tree-level form factor (given in (4.3) and (4.4)) we obtain a vanishing result:

$$\Big(N\big([1, 2, \ell_1, \ell_2] + [1, \ell_2, \ell_1, 2]\big) - 2[1, 2][\ell_1, \ell_2]\Big)[\ell_2, \ell_1]\ =\ 0\ . \tag{4.9}$$

We now consider the remaining contributions arising from the non-planar one-loop amplitude (3.3). There are two possible colour contractions to consider,

$$c^{(1)}_{\rm NP} := 2\,[1, 2][\ell_1, \ell_2][\ell_2, \ell_1]\ =\ 2\,N^2[1, 2]\ , \tag{4.10}$$

and

$$c^{(2)}_{\rm NP} := 2\,[\ell_1, 2][1, \ell_2][\ell_2, \ell_1]\ =\ 2\,[1, 2]\ . \tag{4.11}$$

Note that (4.11) is subleading in the large N limit, and can be discarded in the large-N limit. Moreover, the corresponding coefficient actually vanishes which implies that the two-loop form factor does not have non-planar corrections.

We now need to determine the coefficient of $c^{(1)}_{\rm NP}$. On the two-particle cut $\ell_1^2 = \ell_2^2 = 0$ its integrand is given by the appropriate component tree-level amplitude (4.6) times a

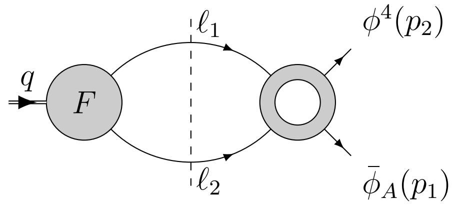

Figure 4: *Tree-level form factor glued to the complete one-loop amplitude.*

particular box integral (3.3):

$$\mathcal{C}_1^{(\mathrm{NP})}|_{s-\mathrm{cut}} := \frac{1}{2}\frac{\langle 12\rangle\langle 1\ell_1\rangle}{\langle 2\ell_1\rangle} I(-\ell_2, 2, -\ell_1, 1) + \ell_1 \leftrightarrow \ell_2 \ . \tag{4.12}$$

Recall that we have to symmetrise in order to include all particle species in the sum over intermediate on-shell states. Since $I(-\ell_2, 2, -\ell_1, 1)$ is antisymmetric under $\ell_1 \leftrightarrow \ell_2$ the complete cut-integrand can be written as[8]

$$\begin{aligned} \mathcal{C}_1^{(\mathrm{NP})}|_{s-\mathrm{cut}} &:= \frac{1}{2}\left(\frac{\langle 12\rangle\langle 1\ell_1\rangle}{\langle 2\ell_1\rangle} - \frac{\langle 12\rangle\langle 1\ell_2\rangle}{\langle 2\ell_2\rangle}\right) I(-\ell_2, 2, -\ell_1, 1) \\ &= -\frac{1}{2}\int \frac{d^D\ell_3}{i\pi^{D/2}} \frac{q^2\left[\mathrm{Tr}\,(p_1 p_2 \ell_1 \ell_3) - q^2 \ell_3^2\right]}{\ell_3^2\,(\ell_1-\ell_3)^2(p_1-\ell_3)^2(\ell_3-\ell_1+p_2)^2} \ . \end{aligned} \tag{4.13}$$

Summarising, the two-particle cut indicates that the two-loop form factor is expressed in terms of a single crossed triangle with a particular numerator, represented in Figure 5,

$$\mathbf{XT}(q^2) = \int \frac{d^D\ell_1 d^D\ell_3}{(i\pi^{D/2})^2} \frac{q^2\left[\mathrm{Tr}\,(p_1 p_2 \ell_1 \ell_3) - q^2 \ell_3^2\right]}{\ell_1^2\,\ell_2^2\,\ell_3^2\,(\ell_1-\ell_3)^2(p_1-\ell_3)^2(\ell_3-\ell_1+p_2)^2}\,, \tag{4.14}$$

so that

$$\mathcal{C}_1^{(\mathrm{NP})} = -\frac{1}{2}\mathbf{XT}(q^2)\,. \tag{4.15}$$

For future convenience we will define

$$\mathbf{xt} := \frac{q^2\left[\mathrm{Tr}\,(p_1 p_2 \ell_1 \ell_3) - q^2 \ell_3^2\right]}{\ell_1^2\,\ell_2^2\,\ell_3^2\,(\ell_1-\ell_3)^2(p_1-\ell_3)^2(\ell_3-\ell_1+p_2)^2}\,. \tag{4.16}$$

The result of the evaluation of $\mathbf{XT}(q^2)$ is quoted in (4.25) and shows that this quantity has maximal degree of transcendentality. Before evaluating $\mathbf{XT}(q^2)$, we use triple cuts in order to confirm the correctness of the ansatz obtained from two-particle cuts.

[8]Similarly as done earlier for the complete one-loop amplitude, we include a factor of 1/2 in the symmetrisation.

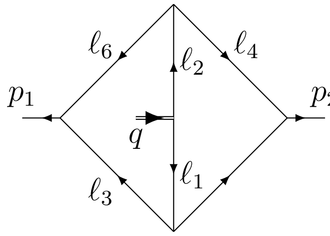

Figure 5: *The crossed triangle integral arising from gluing a tree form factor with the complete one-loop four-point amplitude. The arrow in the middle denotes the location where the momentum* $q = p_1 + p_2$ *is injected. We call these integrals "crossed triangles" because they have the topology of the master integral* (C.4). *Note however that the latter integral is non-transcendental, while the particular numerator in* (4.14) *makes this integral transcendental.*

4.2.2 Three-vertex cuts

To confirm the uplift of the two-particle cut to the integral (4.14), we will study additional cuts. We begin by considering three-point vertex cuts involving three adjacent legs meeting at a three-point vertex. These cuts were first examined in [13], where it was observed that they must vanish since there are no three-particle amplitudes in ABJM theory. Calling k_1, k_2 and k_3 the momenta meeting at the vertex, we have

$$k_1 + k_2 + k_3 = 0\,, \qquad k_1^2 = k_2^2 = k_3^2 = 0\,. \tag{4.17}$$

The conditions (4.17) imply that all spinors associated to these momenta are proportional, thus

$$\langle k_1\, k_2\rangle = \langle k_2\, k_3\rangle = \langle k_3\, k_1\rangle = 0\,. \tag{4.18}$$

As an example consider the three-point vertex cut of $\mathbf{XT}(q^2)$ with momenta ℓ_2, ℓ_4 and $\ell_6 := \ell_2 - \ell_4$ (see Figure 5 for the labelling of the momenta). Importantly, the form factor is expected to vanish as the three momenta belonging to a three-point vertex become null. By rewriting the numerator of (4.14) using only cut momenta, it is immediately seen that it vanishes, since

$$\begin{aligned} \mathrm{Tr}\big[p_1 p_2 (p_1 - \ell_2)(p_1 - \ell_6)\big] - q^2 (p_1 - \ell_6)^2 \;&=\; -\mathrm{Tr}\big[p_1 p_2 (p_1 - \ell_2)\ell_6\big] - q^2 (p_1 - \ell_6)^2 \\ =\; -\mathrm{Tr}(p_1 p_2 p_1 \ell_6) + 4(p_1 \cdot p_2)(p_1 \cdot \ell_6) \;&=\; 0\;, \end{aligned} \tag{4.19}$$

where we have used $\langle \ell_2 \ell_6\rangle = 0$ to set $\mathrm{Tr}(p_1 p_2 \ell_2 \ell_6) = 0$. It is easy to see that all other three-vertex cuts of the integral (4.14) vanish in a similar fashion because of the particular form of its numerator.

Important consequences of these specific properties of the numerator of the integral function (4.14) are that the result is transcendental as we will show below and is free of unphysical infrared divergences related to internal three-point vertices. These divergences appear generically in three-dimensional two-loop integrals with internal three-vertices even if the external kinematics is massive (unlike in four dimensions) and it appears that master integrals with appropriate numerators to cancel these peculiar infrared divergences are a preferred basis for amplitudes and form factors in ABJM. Related discussions in the context of ABJM amplitudes have appeared in [13,46]. Note that for form factors we do not have dual conformal symmetry, which gives further constraints on the structure of the numerators of integral functions appearing in amplitudes.

4.2.3 Three-particle cuts

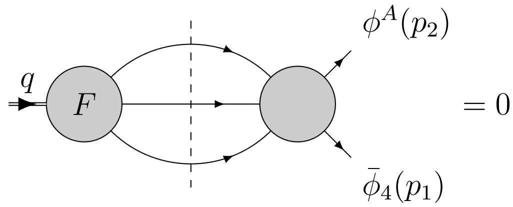

Figure 6: *The (vanishing) three-particle cut of the two-loop Sudakov form factor.*

The remaining cut we will study is a triple cut of the type illustrated in Figure 6. These cuts may potentially detect additional integral functions which have no two-particle cuts at all, and are thus very important. Moreover, such cuts are sensitive to both planar and non-planar topologies. In this triple cut, a tree-level amplitude is connected to a tree-level form factor by three cut propagators. Due to the vanishing of amplitudes with an odd number of external legs in the ABJM theory, the triple cut in question vanishes. We will now check that the triple cut of the two-loop crossed triangle **XT** of (4.14), which we have detected using two-particle cuts, is indeed equal to zero.

To this end, we note that there are two possible ways to perform a triple-cut on **XT**, shown in Figures 7a and 7b. The cut loop momenta are called ℓ_2, ℓ_5 and ℓ_3 and satisfy

$$\ell_2 + \ell_5 + \ell_3 = p_1 + p_2\,, \qquad \ell_2^2 = \ell_5^2 = \ell_3^2 = 0\,. \tag{4.20}$$

We observe that these two cuts cannot be converted into one another by a simple relabelling of the cut momenta because of the non-trivial numerators. The A-cut depicted in Figure 7a of the non-planar integrand is:

$$\mathbf{XT}\big|_{\text{3-p cut A}} = -q^2 \frac{\langle 1\,2\rangle}{\langle \ell_3\,\ell_5\rangle\langle \ell_5\,2\rangle\langle 1\,\ell_3\rangle}\ . \tag{4.21}$$

After a similar calculation, the B-cut of this integral, depicted in Figure 7b, turns out to be identical to the A-cut:

$$\mathbf{XT}\big|_{\text{3-p cut B}} = \mathbf{XT}\big|_{\text{3-p cut A}} = -q^2 \frac{\langle 1\,2\rangle}{\langle \ell_3\,\ell_5\rangle\langle \ell_5\,2\rangle\langle 1\,\ell_3\rangle}\ . \tag{4.22}$$

A quick way to establish the vanishing of the triple cuts consists in symmetrising in the particle momenta p_1 and p_2, which is allowed since the Sudakov form factor is a function of q^2. This symmetrisation gives

$$-\frac{q^2\langle 1\,2\rangle}{\langle \ell_3\,\ell_5\rangle}\left[\frac{1}{\langle \ell_5\,2\rangle\langle 1\,\ell_3\rangle} - \frac{1}{\langle \ell_5\,1\rangle\langle 2\,\ell_3\rangle}\right] \;=\; -\frac{q^4}{\langle 1|\ell_5|2\rangle\,\langle 1|\ell_3|2\rangle}\ . \tag{4.23}$$

(a) *The A-cut.* (b) *The B-cut.*

Figure 7: *The two triple cuts of the crossed triangle, with $\ell_2 + \ell_3 + \ell_5 = q$. In the second figure we have relabelled the loop momenta in order to merge the two contributions.*

This expression is symmetric in ℓ_5 and ℓ_3. In evaluating the triple cut one has to introduce a jabobian proportional to $\epsilon(\ell_2, \ell_3, \ell_5)$ [13] which effectively makes this triple cut vanish upon integration. This implies that the complete answer for the two-loop form factor in ABJM is proportional $\mathbf{XT}(q^2)$ and no additional integral functions have to be introduced.

4.2.4 Results and comparison to the two-loop amplitudes

Combining the information from the unitarity cuts discussed above, we conclude that the two-loop Sudakov form factor in ABJM is given by a single non-planar integral

$$F_{\mathrm{ABJM}}(q^2) \;=\; -2\left(\frac{N}{k}\right)^2\left(-\frac{1}{2}\right)\mathbf{XT}(q^2)\,, \tag{4.24}$$

where $\mathbf{XT}(q^2)$ is defined in (4.14) and we have reintroduced the dependence on the Chern-Simons level k. The integral $\mathbf{XT}(q^2)$ can be computed by reduction to master integrals using integration by parts identities. The details of the reductions are provided in Appendix C. The expansion of the result in the dimensional regularisation parameter ϵ can then be found using the expressions for the the master integrals (C.1)–(C.4). Plugging these masters into the reduction (C.5), we arrive at

$$\mathbf{XT}(q^2) = \left(\frac{-q^2 e^{\gamma_E}}{\mu^2}\right)^{-2\epsilon}\left[\frac{\pi}{\epsilon^2} + \frac{2\pi\log 2}{\epsilon} - 4\pi\log^2 2 - \frac{2\pi^3}{3} + \mathcal{O}(\epsilon)\right], \tag{4.25}$$

where γ_E is the Euler-Mascheroni constant. One comment is in order here. We have derived (4.25) in a normalisation where the the loop integration measure is written as $d^D l/(i\pi^{D/2})$. This should be converted to the standard one $d^D l/(2\pi)^D$. At two loops, this implies that (4.25) has to be multiplied by a factor of $-1/(4\pi)^D$. The result in the standard normalisation is then

$$\mathcal{F}_{\mathrm{ABJM}}(q^2) \;=\; -\frac{1}{(4\pi)^3}\left(\frac{N}{k}\right)^2\left(\frac{-q^2 e^{\gamma_E}}{4\pi\mu^2}\right)^{-2\epsilon}\left[\frac{\pi}{\epsilon^2} + \frac{2\pi\log 2}{\epsilon} - 4\pi\log^2 2 - \frac{2\pi^3}{3} + \mathcal{O}(\epsilon)\right]. \tag{4.26}$$

We note that $\mathcal{F}(q^2)$ can be expressed more compactly by introducing a new scale

$$\mu'^2 := 8\pi\, e^{-\gamma_E}\mu^2\;, \tag{4.27}$$

in terms of which we get

$$\mathcal{F}_{\rm ABJM}(q^2) \;=\; \frac{1}{64\pi^2}\left(\frac{N}{k}\right)^2\left(\frac{-q^2}{\mu'^2}\right)^{-2\epsilon}\left[-\frac{1}{\epsilon^2}\,+\,6\log^2 2\,+\,\frac{2\pi^2}{3}\,+\,\mathcal{O}(\epsilon)\right], \tag{4.28}$$

which is our final result.

We now discuss two consistency checks that confirm the correctness of (4.28). Firstly, we recall that the Sudakov form factor captures the infrared divergences of scattering amplitudes. We now check that (4.28) matches the infrared poles of the four-point amplitude evaluated in [11, 12]. Here we quote its expression as given in [12]:

$$\mathcal{A}_4^{(2)} \;=\; -\frac{1}{16\pi^2}\mathcal{A}_4^{(0)}\left[\frac{(-s/\mu'^2)^{-2\epsilon}}{4\epsilon^2}+\frac{(-t/\mu'^2)^{-2\epsilon}}{4\epsilon^2}-\frac{1}{2}\log^2\left(\frac{-s}{-t}\right)-4\zeta_2-3\log^2 2\right], \tag{4.29}$$

where μ' is related to μ in the same way as in (4.27). Hence, the Sudakov form factor (4.28) is in perfect agreement with the form of the infrared divergences of (4.29). Secondly, we have also checked that the expansion of our result in terms of master integrals (i.e. the expansion of the two-loop non-planar triangle **XT** defined in (4.14)) is identical to that obtained from the Feynman diagram based result of [47]. This implies that the cut-based calculation of this paper and the Feynman diagram calculation of [47] agree to all orders in ϵ – even though we have been using cuts in strictly three dimensions.

5 Maximally transcendental integrals in 3d

As discussed in section 4.2.2, the integrand **xt** that appears in the Sudakov form factor in ABJM has a particular numerator such that all the cuts which isolate a three-point vertex vanish. We have observed in this example that this property ensures that the integral **XT** has a uniform (and maximal) degree of transcendentality – failure to obey the triple-cut condition, for instance by altering the form of the numerator, would result in the appearance of new terms with lower degree of transcendentality. In this section we present further integrals that vanish in these three-particle cuts and have maximal degree of transcendentality. These integrals are expected to appear in the form factor of ABJ theory where cancellations between colour factors such as that in (4.9), do not occur.

We begin by considering the following planar integral function:

$$\begin{aligned}\mathbf{LT}(q^2) \;&=\int\!\frac{d^D\ell_1 d^D\ell_3}{(i\pi^{D/2})^2}\frac{-q^2\,[\,\mathrm{Tr}(p_1\,\ell_3\,p_2\,\ell_1)-(\ell_1-p_1)^2(\ell_3-p_2)^2]}{\ell_1^2\,(p_1+p_2-\ell_1)^2\,\ell_3^2\,(p_1+p_2-\ell_3)^2(\ell_1-\ell_3)^2(\ell_3-p_2)^2} \\ &=\left(\frac{-q^2e^{\gamma_E}}{\mu^2}\right)^{-2\epsilon}\left[-\frac{\pi}{4\epsilon^2}-\frac{\pi\log 2}{\epsilon}+2\pi\log^2 2-\frac{5\pi^3}{8}+\mathcal{O}(\epsilon)\right],\end{aligned} \tag{5.1}$$

which is shown in Figure 8a. It is easy to see that the three vertex cut $\{\ell_1,\ell_3,\ell_5\}$ vanishes, since on this cut the numerator can be placed in the form

$$\langle\ell_1\,1\rangle\langle\ell_3\,2\rangle\langle 1\,2\rangle\langle\ell_3\,\ell_1\rangle\,, \tag{5.2}$$

after using a Schouten identity. (5.2) vanishes because $\langle\ell_3\,\ell_1\rangle=0$ on this cut.

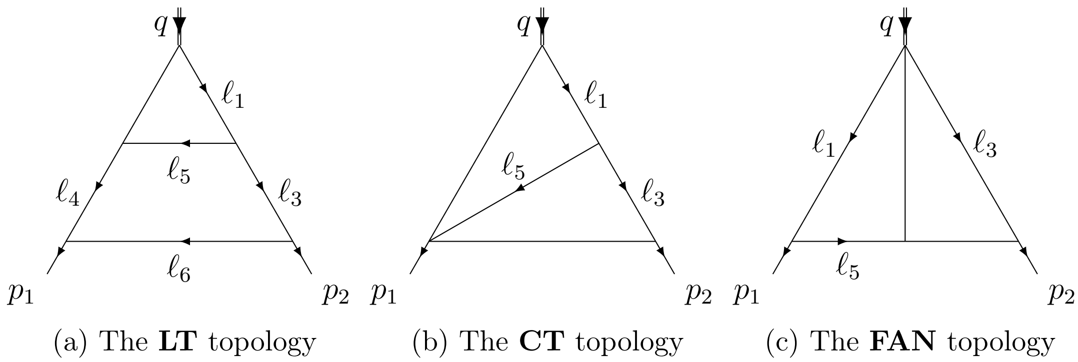

Figure 8: *The three maximally transcendental integrals considered in* (5.1), (5.6) *and* (5.7).

A further property of (5.1) emerges when we consider particular triple cuts involving two adjacent massless legs, which in three dimensions are associated with soft gluon exchange [13]. With reference to Figure 8a, we cut the three momenta ℓ_3, ℓ_6 and ℓ_4. The cut conditions $\ell_3^2 = \ell_6^2 = \ell_4^2 = 0$ together with the masslessness of p_1 and p_2 can only be satisfied if ℓ_6 becomes soft, that is

$$\ell_6 \to 0\,, \quad \ell_4 \to p_1\,, \quad \ell_3 \to p_2\,. \tag{5.3}$$

In this limit, the second term of (5.1) vanishes since $\ell_3 - p_2 = \ell_6 \to 0$. The first term becomes

$$-q^2 \frac{\mathrm{Tr}(p_1\,\ell_3\,p_2\,\ell_1)}{8\epsilon(\ell_3, p_1, p_2)} \to -q^2 \frac{\langle 2|\ell_1|1\rangle}{4\langle 12\rangle}\,, \tag{5.4}$$

where $8\epsilon(\ell_3, p_1, p_2)$ is the Jacobian.[9] After restoring the remaining propagators we are left with

$$\frac{2\epsilon(\ell_1, p_1, p_2)}{\ell_1^2(\ell_1 - p_2)^2(q - \ell_1)^2}\,, \tag{5.5}$$

which reproduces the one-loop integrand of the one-loop form factor, given earlier in (4.7).

Other examples of integrals with different topologies that satisfy the three-particle cut condition are depicted in Figures 8b and 8c. The definitions of the integrals as well as

[9]This Jacobian arises from re-writing the δ-functions of the cut momenta, $\ell_3^2 = \ell_4^2 = \ell_6^2 = 0$, in terms of p_1, p_2 and ℓ_6.

their values are listed below:

$$\mathbf{CT}(q^2) = \int \frac{d^D\ell_1 d^D\ell_3}{(i\pi^{D/2})^2} \frac{\mathrm{Tr}(p_1,p_2,\ell_3,\ell_1)}{\ell_1^2\,(p_1+p_2-\ell_1)^2\,\ell_3^2\,(\ell_1-\ell_3)^2\,(\ell_3-p_2)^2}$$
$$= \left(\frac{-q^2 e^{\gamma_E}}{\mu^2}\right)^{-2\epsilon}\left[-\frac{\pi}{4\epsilon^2}+\frac{7\pi^3}{24}+\mathcal{O}(\epsilon)\right], \tag{5.6}$$

$$\mathbf{FAN}(q^2) = \int \frac{d^D\ell_1 d^D\ell_3}{(i\pi^{D/2})^2} \frac{\mathrm{Tr}(p_1,p_2,\ell_3,\ell_1)}{\ell_1^2\,\ell_3^2\,(p_1+p_2-\ell_1-\ell_3)^2\,(\ell_1-p_1)^2\,(\ell_3-p_2)^2}$$
$$= \left(\frac{-q^2 e^{\gamma_E}}{\mu^2}\right)^{-2\epsilon}\left[-\frac{\pi}{4\epsilon^2}+\frac{7\pi^3}{24}+\mathcal{O}(\epsilon)\right]. \tag{5.7}$$

Note that the ϵ expansion of (5.6) and (5.7) agree up to $\mathcal{O}(1)$. It is simple to show that these integrals satisfy the properties discussed earlier, for example by setting $\{\ell_1, \ell_3, \ell_5\}$ on shell in **CT** and $\{\ell_1, p_1, \ell_5\}$ in **FAN** and similarly for all other possible three-vertex cuts.

The reductions of the integrals considered in this section in terms of scalar master integrals through IBP identities can be found in Appendix C.

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A Half-BPS operators

In this appendix we briefly recall how half-BPS operators are introduced in ABJM theory. Consider the variation of operators of the form $\mathrm{Tr}\left(\phi^I\bar{\phi}_J\right)$ with $I \neq J$. Setting for example $I = 1$ and $J = 4$, this expands to

$$\delta\mathrm{Tr}\left(\phi^1\bar{\phi}_4\right) = \mathrm{Tr}\left(\delta\phi^1\bar{\phi}_4 + \phi^1\delta\bar{\phi}_4\right). \tag{A.1}$$

Following [48], we use the transformations:

$$\delta\phi^I = i\,\omega^{IJ}\psi_J\,, \tag{A.2}$$
$$\delta\bar{\phi}_I = i\,\bar{\psi}^J\omega_{IJ}\,. \tag{A.3}$$

The ω_{IJ}'s are given in terms of the $(2+1)$-dimensional Majorana spinors, ϵ_i $(i = 1, \ldots, 6)$ which are the supersymmetry generators:

$$\begin{aligned} \omega_{IJ} &= \epsilon_i (\Gamma^i)_{IJ} \,, && \text{(A.4)} \\ \omega^{IJ} &= \epsilon_i \left((\Gamma^i)^* \right)^{IJ} \,, && \text{(A.5)} \end{aligned}$$

that are antisymmetric in I, J. The 4×4 matrices Γ^i are given by:

$$\Gamma^1 = \sigma_2 \otimes 1_2 \,, \qquad \Gamma^4 = -\sigma_1 \otimes \sigma_2 \,, \tag{A.6}$$

$$\Gamma^2 = -i\sigma_2 \otimes \sigma_3 \,, \qquad \Gamma^5 = \sigma_3 \otimes \sigma_2 \,, \tag{A.7}$$

$$\Gamma^3 = i\sigma_2 \otimes \sigma_1 \,, \qquad \Gamma^6 = -i 1_2 \otimes \sigma_2 \,, \tag{A.8}$$

and satisfy the following relations,

$$\left\{ \Gamma^i, \Gamma^{j\dagger} \right\} = 2\delta_{ij} \,, \qquad \left(\Gamma^i\right)_{IJ} = -\left(\Gamma^i\right)_{IJ} \,, \tag{A.9}$$

$$\frac{1}{2}\epsilon^{IJKL}\Gamma^i_{KL} = -\left(\Gamma^{j\dagger}\right)^{IJ} = \left((\Gamma^i)^*\right)^{IJ} \,, \tag{A.10}$$

leading to

$$\left(\omega^{IJ}\right)_\alpha = \left((\omega_{IJ})^*\right)_\alpha \,, \qquad \omega^{IJ} = \frac{1}{2}\epsilon^{IJKL}\omega_{KL} \,. \tag{A.11}$$

Explicitly, ω_{IJ} is given by the following matrix:

$$\omega_{IJ} = \begin{pmatrix} 0 & -i\epsilon_5 - \epsilon_6 & -i\epsilon_1 - \epsilon_2 & \epsilon_3 + i\epsilon_4 \\ i\epsilon_5 + \epsilon_6 & 0 & \epsilon_3 - i\epsilon_4 & -i\epsilon_1 + \epsilon_2 \\ i\epsilon_1 + \epsilon_2 & -\epsilon_3 + i\epsilon_4 & 0 & i\epsilon_5 - \epsilon_6 \\ -\epsilon_3 - i\epsilon_4 & i\epsilon_1 - \epsilon_2 & -i\epsilon_5 + \epsilon_6 & 0 \end{pmatrix} . \tag{A.12}$$

The term $\phi^1 \delta\bar{\phi}_4$ yields

$$\phi^1 \delta\bar{\phi}_4 = \phi^1 \left[-\bar{\psi}^1(\epsilon_3 + i\epsilon_4) + i\bar{\psi}^2(\epsilon_1 + i\epsilon_2) - i\bar{\psi}^3(\epsilon_5 + i\epsilon_6) + 0 \right] . \tag{A.13}$$

Therefore, requiring $\phi^1 \delta\bar{\phi}_4 = 0$ the conditions are:

$$\begin{aligned} \epsilon_1 + i\epsilon_2 &= 0 \,, \\ \epsilon_3 + i\epsilon_4 &= 0 \,, \\ \epsilon_5 + i\epsilon_6 &= 0 \,, \end{aligned} \tag{A.14}$$

which relate half of the generators with the other half by constraining the components $\omega_{4J} = 0$.

Note that because of the relations (A.11) which set components of the form ω_{4L} to zero, the entries ω^{IJ} with $I, J \in (1,2,3)$ automatically vanish implying that $\delta\phi^I = 0 \iff I \in (1,2,3)$. This procedure may be iterated to show that generally the operators $\mathrm{Tr}\left(\bar{\phi}_I \phi^J\right)$ for $I \neq J$ are indeed half-BPS. In the present work the operators under consideration are of the type

$$\mathcal{O} = \mathrm{Tr}\,(\phi^A \bar{\phi}_4) \,, \tag{A.15}$$

where $A \neq 4$.

B Properties of the box integral

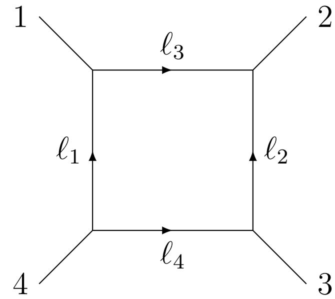


Figure 9: *Four-point one-loop box.*

The box integral function (2.7) was constructed and used in [11], and has several interesting properties that have been exploited in the present work. This section presents and proves (some of) these properties.

B.1 Rotation by 90°

The first property we wish to discuss is what could be called a $\pi/2$ rotation symmetry. Focusing on the numerator of the box integrand,

$$N \;=\; s\,\mathrm{Tr}(\ell_1 p_1 p_4) + \ell_1^2\,\mathrm{Tr}(p_1 p_2 p_4), \tag{B.1}$$

we can eliminate ℓ_1 in favour of ℓ_3 and arrange to have only the external legs p_2, p_3, p_1 appear in the numerator. Using momentum conservation, we can re-write N as

$$N = (-t-u)\,\mathrm{Tr}\left((\ell_3+1)p_1(-p_1-p_2-p_3)\right) + (\ell_3+p_1)\,\mathrm{Tr}\left(p_1 p_2(-p_1-p_2-p_3)\right) \tag{B.2}$$

$$= -\left[t\,\mathrm{Tr}(\ell_3 p_2 p_1) + \ell_3^2\,\mathrm{Tr}(p_2 p_3 p_4)\right] + \mathcal{R}\,,$$

where

$$\mathcal{R} = s\,\mathrm{Tr}(\ell_3 p_3 p_1) - u\,\mathrm{Tr}(\ell_3 p_2 p_1) - 2(\ell_3\cdot p_1)\mathrm{Tr}(p_2 p_3 p_1)\,. \tag{B.3}$$

In three dimensions the loop momentum ℓ_3 can be expressed as a function of the external momenta p_1, p_2, p_3 as

$$\ell_3 = \alpha p_1 + \beta p_2 + \gamma p_3\,, \tag{B.4}$$

where α, β, γ are arbitrary coefficients. When this identity is used in the expression for $\mathcal{R}$, we find that $\mathcal{R}$ vanishes identically zero in three dimensions. Hence

$$s\,\mathrm{Tr}(\ell_1 p_1 p_4) + \ell_1^2\,\mathrm{Tr}(p_1 p_2 p_4) \;=\; -t\left(\mathrm{Tr}(\ell_3 p_2 p_1) + \ell_3^2\,\mathrm{Tr}(p_2 p_3 p_4)\right). \tag{B.5}$$

It is also interesting to write down explicitly the s- and t-cut of the one-loop box. Starting from the expression of the box integral

$$I(1,2,3,4) \;:=\; \int \frac{d^D\ell}{i\pi^{D/2}}\,\frac{N}{\ell^2(\ell-p_1)^2(\ell-p_1-p_2)^2(\ell+p_4)^2}\,, \tag{B.6}$$

with N given in (B.1), we first consider the s-cut of this function. This gives

$$I(1,2,3,4)|_{s\text{-cut}} \;=\; \frac{s\,\mathrm{Tr}(\ell_1 p_1 p_4)}{\ell_3^2\,\ell_4^2}\,, \tag{B.7}$$

which upon using $\ell_3 = \ell_1 - p_1$ and $\ell_4 = -(\ell_1 + p_4)$ becomes

$$I(1,2,3,4)|_{s\text{-cut}} \;=\; \frac{s\langle 41\rangle}{\langle 4\ell_1\rangle\langle \ell_1 1\rangle}\,. \tag{B.8}$$

Similarly the t-channel expression of the full integrand is

$$I(1,2,3,4) = \frac{t\,\mathrm{Tr}(\ell_3 p_2 p_1) + \ell_3^2\,\mathrm{Tr}(p_2 p_3 p_1)}{\ell_1^2\,\ell_2^2\,\ell_3^2\,\ell_4^2}\,. \tag{B.9}$$

The t-cut of $I(1,2,3,4)$ is immediately written using the three-dimensional identity (B.5),

$$\begin{aligned} I(1,2,3,4)|_{t\text{-cut}} &= -\frac{t\,\mathrm{Tr}(\ell_3 p_2 p_1)}{\ell_1^2\,\ell_2^2} \\ &= \frac{t\langle 12\rangle}{\langle 1\ell_3\rangle\langle \ell_3 2\rangle}\,. \end{aligned} \tag{B.10}$$

Finally, if we re-introduce the tree-level amplitude prefactor $\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4) = 1/(\langle 12\rangle\langle 23\rangle)$, we can write down the two cuts of the one-loop amplitude,

$$\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4) \times I(1,2,3,4)|_{s\text{-cut}} = -\frac{\langle 34\rangle}{\langle 4\ell_1\rangle\langle \ell_1 1\rangle}\,, \tag{B.11}$$

$$\mathcal{A}^{(0)}(\bar{1},2,\bar{3},4) \times I(1,2,3,4)|_{t\text{-cut}} = \frac{\langle 23\rangle}{\langle 1\ell_3\rangle\langle \ell_3 2\rangle}\,. \tag{B.12}$$

B.2 An identity for the s-channel cuts of $I(1,2,3,4)$ and $I(1,2,4,3)$

Here we discuss an intriguing property of the three-dimensional cuts of $I(1,2,3,4)$. We consider the s-channel cut of this function and symmetrise it in the cut loop momenta ℓ_1 and ℓ_2, where $\ell_1 + \ell_2 = p_1 + p_2$. The result we wish to show is that the symmmetrised three-dimensional cuts of $I(1,2,3,4)$ and $I(1,2,4,3)$ are in fact identical:

$$I(1,2,3,4)|_{s\text{-cut}} \;+\; \ell_1 \leftrightarrow \ell_2 \;=\; I(1,2,4,3)|_{s\text{-cut}} \;+\; \ell_1 \leftrightarrow \ell_2\,. \tag{B.13}$$

In order to do so, we use (B.8) to write

$$\begin{aligned} I(1,2,3,4)|_{s\text{-cut}} \;+\; \ell_1 \leftrightarrow \ell_2 &= s\langle 41\rangle \left(\frac{1}{\langle 4|\ell_1|1\rangle} + \frac{1}{\langle 4|\ell_2|1\rangle} \right) \\ &= s\langle 41\rangle \left(\frac{\langle 4|\ell_1+\ell_2|1\rangle}{\langle 4|\ell_1|1\rangle\langle 4|\ell_2|1\rangle} \right) \\ &= \frac{s\,\langle 41\rangle\langle 4|2|1\rangle}{\langle 4|\ell_1|1\rangle\langle 4|\ell_2|1\rangle}\,, \end{aligned} \tag{B.14}$$

where in the last step momentum conservation was used. Again using (B.8) this time for the s-cut of $I(1,2,4,3)$ one can write,

$$I(1,2,4,3)|_{s\text{-cut}} = \frac{s\langle 31\rangle}{\langle 3\ell_1\rangle\langle \ell_1 1\rangle}\,, \tag{B.15}$$

and hence

$$\begin{aligned} I(1,2,4,3)|_{s\text{-cut}} \ +\ \ell_1 \leftrightarrow \ell_2 &= s\langle 31\rangle \left(\frac{\langle 3|\ell_1+\ell_2|1\rangle}{\langle 3|\ell_1|1\rangle\langle 3|\ell_2|1\rangle}\right) \\ &= \frac{\langle 31\rangle\langle 3|2|1\rangle}{\langle 3|\ell_1|1\rangle\langle 3|\ell_2|1\rangle}\,. \end{aligned} \tag{B.16}$$

Next we compare (B.14) to (B.16):

$$\begin{aligned} \frac{I(1,2,3,4)|_{s\text{-cut}}}{I(1,2,4,3)|_{s\text{-cut}}} &= \frac{\langle 41\rangle\langle 4|2|1\rangle}{\langle 31\rangle\langle 3|2|1\rangle}\frac{\langle 3|\ell_1|1\rangle\langle 3|\ell_2|1\rangle}{\langle 4|\ell_1|1\rangle\langle 4|\ell_2|1\rangle} \\ &= \frac{\langle 1|4|2\rangle}{\langle 1|3|2\rangle}\frac{\langle \ell_1|3|\ell_2\rangle}{\langle \ell_1|4|\ell_2\rangle} \\ &= 1\,, \end{aligned} \tag{B.17}$$

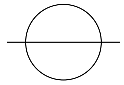
thus proving (B.13).

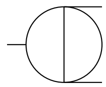
C Details on the evaluation of integrals

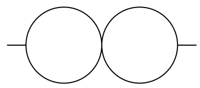
The integral in our result (4.24) can be reduced to a set of four scalar, single-scale, master integrals using integration by parts identities and the `FIRE` package [49] for `Mathematica.`In this appendix we present the details of this reduction as well as the values of these master integrals.

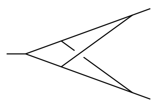
C.1 Two-loop master integrals in three dimensions

The master integrals that appear at two loops, in particular in our result appear in the reduction of our result (4.24), are given in $D = 3 - 2\epsilon$ dimensions by the following expressions:

$$\mathbf{SUNSET}(q^2) = $$  $$ = -\left(\frac{-q^2}{\mu^2}\right)^{-2\epsilon} \frac{\Gamma\left(\frac{1}{2}-\epsilon\right)^3 \Gamma(2\epsilon)}{\Gamma\left(\frac{3}{2}-3\epsilon\right)} \tag{C.1}$$

$$\mathbf{TRI}(q^2) = $$  $$ = -(-q^2)^{-1}\left(\frac{-q^2}{\mu^2}\right)^{-2\epsilon} \frac{\Gamma\left(\frac{1}{2}-\epsilon\right)^2 \Gamma(-2\epsilon)\Gamma\left(\frac{3}{2}+\epsilon\right)\Gamma(2+2\epsilon)}{\epsilon(1+2\epsilon)^2\Gamma\left(\frac{1}{2}-3\epsilon\right)} \tag{C.2}$$

$$\mathbf{GLASS}(q^2) = $$  $$ = (-q^2)^{-1}\left(\frac{-q^2}{\mu^2}\right)^{-2\epsilon} \frac{\Gamma\left(\frac{1}{2}-\epsilon\right)^4 \Gamma\left(\frac{1}{2}+\epsilon\right)^2}{\Gamma(1-2\epsilon)^2} \tag{C.3}$$

$$\mathbf{TrianX}(q^2) = $$  $$ = (-q^2)^{-3}\left(\frac{-q^2}{\mu^2}\right)^{-2\epsilon} e^{-2\gamma_E\epsilon}\left[\frac{4\pi}{\epsilon^2} + \frac{\pi(3+8\log 2)}{\epsilon} \right. \tag{C.4}$$

$$- \frac{2\pi}{3}\big(81 + 4\pi^2 + 6\log 2\,(4\log 2 - 9)\big) + \frac{\pi}{6}\Big(-\pi^2(7+40\log 2)$$

$$\left. + 8\big(69 + 6\log 2 + 2\log^2 2(8\log 2 - 27) - 113\zeta_3\big)\Big)\epsilon + \mathcal{O}(\epsilon)\right],$$

where we use the conventions of [50] for the integration measure. The first three of these integrals are planar and their expressions in all orders in ϵ can be easily obtained by first computing their Mellin-Barnes representations most conveniently using the `AMBRE` package [51] and then performing the Mellin-Barnes integrations using the MB tools, in particular `MB.m` [50] and `barnesroutines.m` by David Kosower. The expansion around $\epsilon = 0$ of the **TRI** and **GLASS** topologies has uniform degree of transcendentality, while this is not the case for the **SUNSET** and **TrianX** topologies.

C.2 Reduction to master integrals

Here we present the reductions of the integral (4.14) that appears in our result (4.24) in terms of the master integrals (C.1)–(C.4) of the previous section:

$$\mathbf{XT}(q^2) = \frac{7(D-3)(3D-10)(3D-8)}{2(D-4)^2(2D-7)}\mathbf{SUNSET}(q^2) \tag{C.5}$$
$$+ (-q^2)\frac{5(D-3)(3D-10)}{2(D-4)(2D-7)}\mathbf{TRI}(q^2) + (-q^2)^3\frac{D-4}{4(2D-7)}\mathbf{TrianX}(q^2)\,.$$

Note that the GLASS topology does not appear in $\mathbf{XT}(q^2)$. Two other integrals we have considered are:

$$\mathbf{LT}(q^2) \;=\; \frac{8-3D}{D-3}\mathbf{SUNSET}(q^2) + q^2\left(\mathbf{GLASS}(q^2) - \mathbf{TRI}(q^2)\right), \tag{C.6}$$

$$\mathbf{CT}(q^2) \;=\; \mathbf{FAN}(q^2) = \left(\frac{1}{4\epsilon} - \frac{3}{2}\right)\mathbf{SUNSET}(q^2)\,. \tag{C.7}$$

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