## ANALYSIS OF LOCAL MPPS ON THE P-V CURVE OF A PARTIALLY SHADED PV STRING

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ABSTRACT: In this paper, a comprehensive analysis on the local maxima developed in a partially shaded PV string, experiencing multiple irradiance levels, is presented. The number and the trends of the local and global maximum power points (MPPs) are studied, initially considering two irradiance levels and subsequently extending to the more complicated three-irradiance levels scenario. The shading patterns at which each MPP appears are investigated, and the conditions under which they constitute the global MPP (GMPP) are identified, while the effect of the operating temperature and the intensity of irradiance is studied. Keywords: Global maximum power point (GMPP), maximum power point (MPP), multiple irradiance levels, partial shading, photovoltaic (PV) string, power peaks.

## 1 INTRODUCTION

When operating under partial shading conditions, the electrical response of a PV generator is significantly affected, giving rise to multiple maximum power points (MPPs). Previously published work on the performance of partially shaded PV systems deals with the appearance and characteristic trends of multiple MPPs [1]-[5]. In [1], two local MPPs are identified when a single PV module experiences two irradiance levels, while in [2] the general multi-irradiance case of a PV string is considered, leading to mathematical formulae for the evaluation of all potential local MPPs. Similarly, the partial shading phenomenon on PV arrays is studied in [6]-[10], while empirical expressions are proposed in [11] in order to quantify the voltage and current of the local MPPs appearing on the I-V and P-V curves of PV arrays.

Although these papers deal with the characterization and quantification of the potential MPPs, they do not shed light on the conditions for their appearance, since under some shading patterns certain MPPs appear while others do not. This is partially addressed graphically in [3]-[4] for the case of two irradiance levels, illustrating the relation of the number of MPPs to the extent and intensity of the shade. However, these papers consider only the number of MPPs and do not identify their type, nor do they study the global maximum (GMPP) trends.





In this paper, a detailed analysis is carried out on the number of MPPs developed in a partially shaded PV string, the conditions for their appearance and the identification of the GMPP. The analysis extends beyond the two irradiance levels, while various operating irradiances and temperatures are considered.

The modeling of PV string adopted in this study is described in Section 2, while its operating principles under partial shading are discussed in Section 3. In Sections 4 and 5 the number of developed MPPs and the global maximum trends of a PV string illuminated by two and three irradiance levels, respectively, are classified for various shading patterns. The main conclusions are summarized in Section 6.

### 2 MODELING OF THE PV STRING

The PV string comprises several series connected PV modules, while a PV module consists of series connected PV cells. A group of series connected cells, with a bypass diode connected in parallel at its terminals, is denoted a cell string. The layout of a typical PV string is illustrated in Fig. 1.



Figure 1: Layout of a PV string comprising 12 PV modules and its building components.



**Figure 3:** (a) *I-V* and (b) *P-V* curves of a PV string illuminated at 3 irradiance levels ( $G_1$ =1 p.u.,  $G_2$ =0.6 p.u.,  $G_3$ =0.3 p.u. / $T_c$ =25°C).

In this study, in order to simulate the electrical response of the PV string, the single-diode PV cell electrical equivalent of Fig. 2 is adopted, properly enhanced for negative voltage operation [12]-[15]. Its operation is described by the following eq. (1), employing the Lambert *W* function to reduce computational complexity [2].

$$V_{cell} = \begin{cases} R_{sh} \left( I_{ph} + I_s \right) - \left( R_s + R_{sh} \right) \cdot I - \\ a \cdot W \left\{ \frac{R_{sh} I_s}{a} e^{\frac{R_{sh} \left( I_{ph} + I_s - I \right)}{a}} \right\}, & I \le I_{ph} \end{cases}$$
(1)  
$$V_{br} - I \cdot R_s - Z_{R\min} , & I > I_{ph} \end{cases}$$

where  $I_{ph}$ ,  $I_s$ , a,  $R_s$  and  $R_{sh}$  are the five parameters of the model, determined according to [16], b,  $V_{br}$ , mare coefficients related to negative voltage operation [12]-[14],  $W{x}$  is the Lambert W function and  $z_{Rmin}$ the minimum real root of the following quartic function, as described in [2]:

$$\frac{1}{R_{sh}} \cdot z^4 + \left(I_{ph} - I - \frac{V_{br}}{R_{sh}}\right) \cdot z^3 + bV_{br}^3 \cdot z - bV_{br}^4 = 0$$
(2)

Eq. (3) gives the cell string voltage  $V_{cs}$  in an explicit manner, assuming a number of  $N_s$  cells connected in series.

$$V_{cs} = \begin{cases} \sum_{i=1}^{N_{s}} V_{cell-i}(I) & , I \le I_{SC,cs} \\ -a_{bp} \cdot \ln\left(\frac{I - I_{SC,cs}}{I_{sbp}} + 1\right) & , I > I_{SC,cs} \end{cases}$$
(3)

Assuming that each PV module is composed by  $N_{cs}$  cell strings and each PV string comprises  $N_m$  modules, eq. (4) is used to calculate PV string voltage:

$$V_{str} = \sum_{i=1}^{N_{m} \cdot N_{cs}} V_{cs-i}(I)$$
(4)

## 3 PV STRING OPERATION UNDER PARTIAL SHADING CONDITIONS

Under partial shading conditions, a PV string may present no more than two local maxima, assuming two irradiance levels, denoted as  $MPP_1$  and  $MPP_2[1]$ -[2]. Extending to *n* irradiance levels, a PV string develops up to *n* local MPPs, denoted as

 $MPP_1, MPP_2, ..., MPP_n$  [2].



**Figure 4:** *I-V* curve of a PV string illuminated at two irradiance levels. The red and yellow arrows show how the characteristic is modified when the extent  $(N_2/(N_1+N_2))$  or the intensity  $(G_2/G_1)$  of the shadow changes.

In Fig. 3, an indicative scenario of three irradiance levels  $G_1$ =1 p.u.,  $G_2$ =0.6 p.u. and  $G_3$ =0.3 p.u. is illustrated, forming three groups of  $N_1$ =14,  $N_2$ =12 and  $N_3$ =10 cell strings illuminated at the respective irradiance levels. Three MPPs appear in this case, which are identified and classified as different MPP types according to [2]. At each  $MPP_j$ , the cell string groups 1 to *j* (assuming  $G_i > G_j$ , *i*<*j*) operate at the reduced current imposed by the irradiance level  $G_j$ , while the rest of the groups are shorted by their bypass diodes.

In particular, at  $MPP_1$  only the group 1 ( $N_1$ =14) contributes to power generation, while groups 2 ( $N_2$ =12) and 3 ( $N_3$ =10) are bypassed. At  $MPP_2$ , cell string groups 1 and 2 operate at the current dictated by the irradiance level  $G_2$  and group 3 is shorted, while at  $MPP_3$  all cell string groups are forced to operate at the reduced current corresponding to irradiance level  $G_3$ , since no bypass diode conducts.

4 PARTIAL SHADING – 2 IRRADIANCE LEVELS

## 4.1 Number of MPPs

A PV string illuminated at two different irradiance levels may present one or two



**Figure 5:** (a) Number and (b) classification of the MPPs versus the extent  $(N_2/(N_1+N_2))$  and intensity  $(G_2/G_1)$  of the shadow, for a PV string illuminated at two irradiance levels.



**Figure 6:** Number of MPPs for (a) varying irradiance on the unshaded cell strings  $G_1$  ( $T_c=25^{\circ}$  C) and (b) varying operating temperature  $T_c$  ( $G_1=1.0$  p.u.).

MPPs, according to the shading pattern and intensity. In Fig. 4, the electrical response of the PV string of Fig. 1 is depicted in blue, for an indicative shading scenario ( $G_1$ =1 p.u.,  $G_2$ =0.5 p.u.,  $N_1$ =22,  $N_2$ =14), developing two local MPPs ( $MPP_1$  and  $MPP_2$ ). This shading pattern is subsequently varied to cover all possible cases, changing its extent ( $N_2/(N_1+N_2)$ ) from fully shaded to unshaded and its relative intensity ( $G_2/G_1$ ) from 0.01 to 1 ( $G_1$ =1 p.u. and operating temperature  $T_c$ =25°). Variation of the shade extent shifts  $MPP_1$  horizontally (red dashed *I*-*V* curves of Fig. 4), while the shade intensity shifts  $MPP_2$  vertically (yellow dashed *I*-*V* curves of Fig. 4), as indicated by arrows in Fig. 4.

This trend is displayed in Fig. 5. In Fig. 5(a), the number of MPPs developing on the I-V characteristic is depicted on the plane of the extent and intensity of shade. A single MPP will arise only at the boundary area, where the string experiences a shade either too deep or too extended.

The characterization of the MPPs as  $MPP_1$  or  $MPP_2$ , defined above, is shown in Fig. 5(b). Three areas are presented: the two-MPP area, where both  $MPP_1$  and  $MPP_2$  appear, is colored in yellow; the single MPP area is split into sub-regions where the

MPP is either an  $MPP_1$  (light blue) or  $MPP_2$  (green). Hatching indicates whether the  $MPP_1$  or the  $MPP_2$  is the global MPP.

A detailed observation of Fig. 5 leads to the conclusion that when  $G_2$  is higher than 95% of  $G_1$  or the number of shaded cell strings ( $N_2$ ) exceeds the 95% of the total number ( $N_1+N_2$ ), only  $MPP_2$  is developed on the *P*-*V* curve of the string. On the other hand, the single MPP is  $MPP_1$  only when the number of shaded cell strings ( $N_2$ ) is less than 2% of the string or at unrealistically heavy shade levels. Consequently, a single MPP appears at boundary shading scenarios, where the shaded area is very high (>95% - MPP\_2) or too small (<2% - MPP\_1), or the depth of shade is negligible (>95% - MPP\_2). In the two-MPP region, the global maximum GMPP may be  $MPP_1$  or  $MPP_2$ , depending on the shading pattern, as shown in Fig. 5(b).

#### 4.2 Various operating irradiance and temperature

This analysis is extended in Fig. 6 for cases that the irradiance on the unshaded part  $G_I$  and the operating temperature  $T_c$  deviate from STC. As shown in Fig. 6(a), the absolute value of  $G_I$  does not



**Figure 7:** (a) Number and (b) classification of the MPPs versus the extent  $(N_2/(N_1+N_2))$  and intensity  $((G_2-G_3)/(G_1-G_3))$  of the shadow, for the PV string of Fig. 1, illuminated at three irradiance levels and assuming  $G_3/G_1=0.3$  and  $N_3=6$ .



**Figure 8:** *I*-*V* curve of a PV string illuminated at three irradiance levels, assuming  $N_3$  and  $G_3$  are constant. Red and yellow arrows show how the characteristic is affected when the extent  $(N_2/(N_1+N_2))$  or the relative intensity  $((G_2-G_3)/(G_1-G_3))$  of the shadow change.

have any notable effect, because the MPPs depend only on the relative shade intensity  $G_2/G_1$ . In Fig.6(b) the two-MPP region shrinks slightly at high cell temperatures, yet this effect is again not significant. As similar observations hold also for the relative importance of MPP<sub>1</sub> and MPP<sub>2</sub> in forming the global MPP (GMPP), it can be concluded that in the general case of a partially shaded PV string, MPPs are determined by the relative shade intensity and extent, rather than by the absolute irradiance values or the temperature.

# 5 PARTIAL SHADING – 3 IRRADIANCE LEVELS

### 5.1 Constant N<sub>3</sub> and G<sub>3</sub>

In order to extend the analysis to the general multi-irradiance case, the indicative scenario of three irradiance levels  $G_1$ =1.0 p.u.,  $G_2$ =0.7 p.u.,  $G_3$ =0.3 p.u., with  $N_1$ =18,  $N_2$ =12 and  $N_3$ =6, is examined in Fig. 8. The horizontal red double arrow

implies the relative variation of the lightly shaded cell strings  $N_2$  with respect to the sum  $N_I+N_2$ . The number  $N_3$  of heavily shaded cell strings is considered constant. The vertical yellow double arrow indicates variation of  $G_2$  in the range from  $G_3$ to  $G_1$ , assuming again that  $G_1$  and  $G_3$  are constant.

In this case, the relevant parameters are the relative shade intensity  $(G_2-G_3)/(G_1-G_3)$  on the lightly shaded part of the string, comprising  $N_2$  cell strings, and the relative extent  $N_2/(N_1+N_2)$  of the lightly shaded part of the string. The parameters  $G_3$ ,  $G_1$  and  $N_3$  are initially considered constant. In Fig. 7(a), the number of MPPs is illustrated assuming  $N_3$ =6 cell strings,  $G_1$ =1.0 and  $G_3$ =0.3.

Apparently, a number of up to three MPPs may arise depending to the shading pattern. Typically, 3 MPPs are presented, whereas one or two MPPs appear only at the boundary areas of Fig. 7(a). To further understand which MPP is presented and which one is suppressed in each case, the categorization in the MPP1, MPP2 and MPP3 types is shown in different color in Fig. 7(b). In the central yellow colored region all three MPPs appear, while the green and light blue areas correspond to two local maxima  $(MPP_2+MPP_3 \text{ and } MPP_1+MPP_3)$ respectively). The single MPP case appears only when the shading intensities are almost equal  $(G_2$  $\approx G_3$ ) and the unshaded part of the string is very limited  $(N_2/(N_1+N_2)\geq 0.9)$ . In this case, the single MPP is of the  $MPP_3$  type. It is evident that  $MPP_3$  is present in all cases because of the minimum heavy shade extent of  $N_3=6$  cell strings considered in all examined patterns. Despite the small extent of the latter, it is notable that the  $MPP_1$  and  $MPP_2$  are never presented as the single MPP of the string, nor does the combination  $MPP_1 + MPP_2$  ever appear as a two-MPP case. The hatching patterns in Fig. 7(b) indicate which MPP type  $(MPP_1, MPP_2 \text{ or } MPP_3)$  is the global one in each case,  $MPP_2$  being the most common case.



**Figure 9:** (a) Number of MPPs and (b) global maximum for a PV string illuminated at three irradiance levels, for a varying number  $N_3$  of heavily shaded cell strings ( $G_3/G_1=0.3$ ).

5.2 Impact of N<sub>3</sub> variation

In the scenarios studied above, the heavy shade extent has been considered constant ( $N_3$ =6 cell strings) for illustration purposes. In the following,  $N_3$  is varied in the range 6 to 24 out of 36 total cell strings. The effect on the I-V characteristic is shown with the green arrow in Fig. 10.

The number of MPPs is shown in Fig. 9(a), where the central three-MPP region seems to shrink with the increment of  $N_3$ , leaving more space to the two-MPP area and specifically to the  $MPP_2+MPP_3$  region. This is reasonable, since the unshaded and lightly shaded areas are reduced in these cases, leading to weakening of  $MPP_1$  and  $MPP_2$ , unlike  $MPP_3$  which is strengthened. This is clear in Fig. 9(b), in which the global maximum origination is depicted. As  $N_3$  increases, the  $MPP_3$  is more often the GMPP, until it becomes always the global maximum for  $N_3=24$  cell strings and beyond.



**Figure 10:** *I-V* curve of a PV string illuminated at three irradiance levels (assuming that  $G_3$  is constant). Red, yellow and green arrows show how the characteristic is modified when the extent  $(N_2/(N_1+N_2))$  and the intensity  $((G_2-G_3)/(G_1-G_3))$  of the light shadow or the number of heavily shaded cell strings  $N_3$  change.

### 5.3 Impact of G<sub>3</sub> variation

The impact of the deep shadow irradiance level  $G_3$  on the *I*-V curve of the PV string is shown in Fig.

11 (brown arrow), considering now its extent to be constant ( $N_3$ =6 cell strings).

In Fig. 12(a), the number of MPPs is examined for four different values of  $G_3$ . The central three-MPP region appears to shrink with the increment of  $G_3$ , as in Fig. 9(a), giving space to the  $MPP_2+MPP_3$ region. At the same time,  $MPP_3$  is the GMPP more often, as depicted in Fig. 12(b), while for  $G_3=0.8$  it becomes the global MPP in all cases.



**Figure 11:** *I-V* curve of a PV string illuminated at three irradiance levels, assuming a constant  $N_3$ . Red and yellow arrows show how the characteristic is modified when the relative extent  $(N_2/(N_1+N_2))$  and intensity  $((G_2-G_3)/(G_1-G_3))$  of the light shadow changes. The brown arrow presents the effect of irradiance  $G_3$  variations.

### 6 CONCLUSION

In this paper, the number and type of the local MPPs, as well as the global maximum origination, are analyzed for a PV string illuminated at multiple irradiance levels.

For two irradiance levels, it is concluded that two MPPs ( $MPP_1$  and  $MPP_2$ ) usually appear and they share equally the probability to act as the GMPP,  $MPP_2$  being the global maximum under light shadow or limited shade extent conditions. Furthermore, it isobserved that the absolute



**Figure 12:** (a) Number of MPPs and (b) global maximum of a PV string illuminated at three irradiance levels, for varying deep shadow irradiance level  $G_3(N_{3=}6)$ .

irradiance levels  $(G_1, G_2)$  and the operating temperature do not affect the correlation of the MPPs.

For three irradiance levels on the string, it is observed that in most cases all three MPPs ( $MPP_1$ ,  $MPP_2$  and  $MPP_3$ ) appear, but the global MPP in each case is determined by the extent and intensity of the light and heavy shadows. The gradual increment of the heavy shadow extent ( $N_3$ ) or its corresponding irradiance ( $G_3$ ) leads to the rightmost MPP ( $MPP_3$ ) becoming more frequently the global maximum.

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