Global Synchronization via Non-Reciprocal Coupling: A Route to Photonic Time Crystals

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***Abstract* – We report on a new mechanism for synchronization of noise-driven, linear oscillators based upon non-reciprocal coupling. Under such conditions – which may be achieved, by design, in a photonic metamaterial ensemble of nano-opto-mechanical oscillators – spontaneous synchronization can emerge through a first-order phase transition as a function of coupling strength, providing a route to the realization keenly-sought continuous photonic time crystals.**

I. INTRODUCTION

Entrainment - the synchronization of rhythms - is a ubiquitous phenomenon observed in various natural and artificial systems, from biological clocks to lasers. Self-entrainment is a process whereby individual units synchronize with each other without external influence. The Kuramoto model is a commonly used mathematical framework describing such synchronization in coupled oscillator systems, via nonlinearity. In contrast, we report on entrainment achieved in a noise-driven system of linear coupled oscillators through their non-reciprocal interaction. Our results highlight the significance of non-reciprocal coupling in breaking time translation symmetry – a defining characteristic of 'time crystals': a new state of matter with properties that are periodic in time, rather than in space, as is the case in conventional ('space') crystals. Time crystals offer wide-ranging applications potential in all-optical modulation, frequency conversion, and timing, as well as fundamental studies of dynamic classical many-body states in the strongly correlated regime.

II. NON-RECIPROCALLY COUPLED OSCILLATORS

The dynamics of a system of linear, noise-driven coupled oscillators are described by the Langevin equation: $\ddot{x}_i + \gamma\dot{x}_i + \omega_{0i}^2 x_i + \sum\xi_{ij}(x_i - x_j) = \sqrt{2k_B T\gamma/m_i}\eta_i(t)$, where ω_{oi} are the natural angular frequencies of individual oscillators with effective mass m_i, ξ_{ij} is the coupling coefficient describing their interaction, k_B is the Boltzmann constant, T is temperature, dissipation factor $\gamma = \omega_{oi}/Q$ where Q is the quality factor, and $\eta_i(t)$ is a normalized white noise term.

We consider a pair of oscillators with natural frequencies $f_{01} = 1.99MHz$ and $f_{02} = 2.01MHz$, effective mass $m_1 = m_2 = 1pg$ and $Q = 1000$ as illustrated in the inset to Fig. 1(a). Coupling coefficients are expressed as $\xi_{ij} = n\omega_{0i}^2$, where n is the real number. The power spectral density (PSD) of the positional difference $(x_1 - x_2)$ reveals the dynamics of the system under different conditions of coupling strength and type (Fig. 1(b), (c), (d)).

III. RESULTS AND ANALYSIS

In the absence of coupling (ξ_{12}, ξ_{21} = 0), each oscillator oscillates independently (asynchronous [AS] state), resulting in two distinct peaks in the PSD plot at their natural frequencies, f_{01} and f_{02} (Fig. 1(b)).

When coupling is reciprocal and attractive (ξ_{ij}, $\xi_{ji} > 0$) [or repulsive (ξ_{ij}, $\xi_{ji} < 0$)], both peaks blue [red] shift with increasing coupling strength; the lower [higher] frequency peak tends towards the mean frequency (f_m) while the other continues to higher [lower] frequencies with stronger coupling.

Non-reciprocal coupling (attractive in one direction and repulsive in the other) facilitates synchronization, manifested in the merger of the two peaks to a single higher amplitude peak at f_m (Fig. 1(c)), *only* when coupling

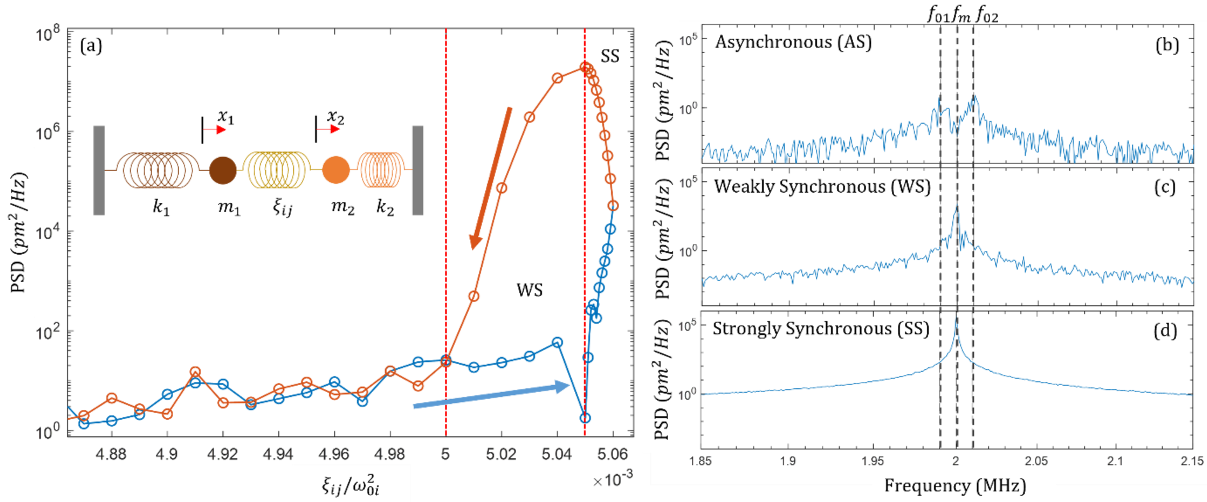

Fig. 1(a) Inset: Schematic representation of the system of two dissimilar coupled oscillators. Main panel: Synchronization as a 1st-order phase transition: Peak amplitude of oscillator relative displacement power spectral density [PSD] against increasing [blue] and decreasing [orange] coupling strength. Power spectral density of oscillator relative displacement under conditions of (b) zero coupling ($\xi_{12}/\omega_{01}^2 = -\xi_{21}/\omega_{02}^2 = 0$), asynchronous oscillation; (c) Weakly synchronized oscillation ($\xi_{12}/\omega_{01}^2 = -\xi_{21}/\omega_{02}^2 = 0.0050$ in Fig. 1a); (d) Strongly Synchronized oscillation ($\xi_{12}/\omega_{01}^2 = -\xi_{21}/\omega_{02}^2 = 0.00506$).

is attractive from the lower to higher frequency oscillator (i.e. in the present case when $\xi_{12} > 0$) and repulsive from higher to lower ($\xi_{21} < 0$). In the opposite case, the two peaks move apart rather than together.

With increasing coupling strength, above a certain critical value (blue data points in Fig. 1(a)), an abrupt transition from a weakly-synchronized (WS) to a strongly synchronized (SS) state is observed (Fig. 1(d)). In the strongly synchronized regime, the amplitudes of the oscillators grow exponentially, and noise is suppressed. When the coupling strength is decreased (orange data points in Fig.1(a)), we observe hysteretic loss of synchronization in reversion of the system to its lower coupling strength form, revealing the first order nature of the synchronization phase transition.

IV. Conclusion

In summary, we have shown that spontaneous synchronization of the motion of linear, noise-driven oscillators can be achieved via non-reciprocal coupling. The above model can be generalized to a system of N coupled oscillators, including near-neighbour and all-to-all coupling, to describe and inform understanding of dynamics in classical opto-mechanical metamaterial ensembles manifesting all the key features of a continuous time crystal [1], wherein coupling is mediated by optical forces.

References

[1] T. Liu, J. Y. Ou, K. F. MacDonald, N. I. Zheludev , "Photonic metamaterial analogue of a continuous time crystal" *Nat. Phys.* (in press) arXiv:2209.00324v3.