IRREVERSIBLE COSMOLOGICAL MODELS

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ABSTRACT

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In this research a mathematical model of the universe is constructed based upon three standard postulates; (1) The Roberston Walker Metric, (2) The perfect fluid energy momentum tensor, and (3) General Relativity. In addition the thermodynamic interaction between the matter and radiation phases is included via the Thomson interaction. Oscillations of a closed model universe are studied and the irreversibility generated by the inclusion of the interaction is considered.

The set of differential equations that represents the model universe are solved numerically. Detailed study is made of the initial conditions and constants of motion to be assigned to the equations and their physical meaning. The effect of differing initial conditions on the numerical classification of the equations is touched upon.

It is found that for initial conditions based strictly on the physical universe that the irreversibility generated is less than the minimum error made in the solution of the equations. This error is probably in excess of 10^{-3}.

For initial conditions that are more numerically convenient to solve, but which are strongly guided by the physical condition of the universe the ireversibility induced by the interaction is calculable. For such conditions pleasing results appear as the irreversible effects build up over many cycles. These are, (1) later and later cycles appear more and more 'flat', this offers a non-inflationary solution to the flatness problem, (2) later and later cycles tend to be more and more dominated by radiation at the initial point. In addition it is also predicted by the model that as the cycles continue then these effects will also have two other results , (1) the universe will never enter a 'matter dominated era', (2) the radiation temperature will eventually forbid the evolution of life !

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CHAPTER ONE

Introduction

" In the beginning God created the heavens and the earth"

[Genesis Chapter 1 Verse 1 N.I.V.]

S1.1 Cosmology - The study of what ?

The Holy Bible begins with the above words describing the creation of our universe. It seems that as long as man has existed he has wondered about the universe, its creation and his place in it.

Almost all the myths of the ancient cultures have a creation fantasy, the Babylonians, Egyptians, Greeks, Indians and Chinese. Even the Hitch Hikers Guide to the Galaxy speaks of the universe being sneezed out of a great nostril ![1]

Cosmology proper begins when the scientific method of observation and measurement of phenomena is applied to the universe. Mathematical cosmology attempts to make predictions based on the observed physical laws of the universe.

As a science, cosmology can be seen to have begun with the classical studies of the Greeks from 580 B.C. to 140 A.D.. Pythagoras, Plato, Eudoxus, Aristotle, Aristarchus and Ptolemy did much to prepare the ground for their post dark ages successors.

Cosmology resurfaced after the dark ages with the Copernican revolution, when it was at last realised that the earth did not occupy a special place in the cosmos. With the work of this Polish cleric we can see the seed of the cosmological principle that was to serve science so well in the 20th century.

With the work of Kepler showing that the motion of the planets could be understood with geometry and that simple relationships connected various parameters of the planets' orbits the mathematical theory of our universe was born.

Newton made a significant contribution to cosmology by showing that the same physical process that led an apple to drop to the earth's surface could explain the motion of the heavens. In deriving Kepler's laws, Newton began the dynamical investigation of our universe. In fact, the great man was the first person to apply mechanics to the universe, but his study was flawed by the inherent problems associated with an infinite system . The problem of an infinite Newtonian gravitational potential would not disappear until the second decade of the twentieth century.

In 1916, the genius of Einstein left a lasting mark on cosmology with the General Theory of Relativity. With his interpretation of gravity as the manifestation of non-Euclidian geometry, allowing solutions to the field equations of the gravitional field, representing the universe that were finite and yet unbounded, the problems of an infinite universe vanished. (However, the problem of infinities was to return to plague cosmology in the question of singularities and the big bang)

With the cosmological principle embedded in the metric of Robertson and Walker, and Einstein's equations of general relativity, Friedman, in 1922, and Lemaitre were able to produce evolving 'big bang' theories of the universe, which, as a basic framework, still dominate cosmological thinking today.

When in 1929, Hubble discovered the expansion of the universe, the cosmological models of Friedman and Lemaitre were reinforced as being more than idle mathematical speculation. With the discovery of the cosmological microwave background by Penzias and Wilson in 1965 the big bang theory seems triumphant.

Thus today we have a standard model of the universe that seems to explain a vast number of its large scale features - the expansion of the universe, the microwave background, the element abundance etc.

One would not claim that the standard big bang theory was without fault. It does have problems that are not easy to explain. Apart from the problem of Galaxy formation, [2] that is how did the inhomogeneity that we observe in galaxies and their clusters originate and evolve in the homogeneous and isotropic Robertson-Walker Space-time. There are three main problems with the standard model:

i) The horizon problem [3] - the universe at present consists of approximatly 10^{80} causually disconnected regions of space-time, this very large number is arrived at by calculating the size of a causually connected region of space time just after classical General Relativity becomes applicable to the standard hot big bang model and comparing this volume to the present size of the universe . Why should the microwave background be isotropic over length scales far greater than the horizon length ?

ii) The flatness problem [4] - For our present universe to have evolved for us to observe it, the original value of the density parameter must have been tuned to within 10^{-55} of unity. Had the deviation been significantly larger the Universe would have reached the point of maximum extension with a radiation temperature far too high for intelligent life to form, or it would have expanded much too fast for galaxies to form. No process is known capable of producing such a fine tuning.

iii) The monopole problem [5] - Quantum field theory applied to the very early universe predicts that enough of these exotic species should have been produced to dominate the present mass density of the universe. This prediction is made by considering a unified quantum field theory applied to the very early universe, this predicts that at this time a very large number of magnetic monopoles should have been produced. The number of these thus produced seems to be insensitive to the particular unified field theory that is chosen, to eventually yield, via symmetry breaking, the three quantum fields that we observe at our much lower temperature today. An example of such a calculation in a particular unified field theory may be found in reference [6]. To date, no conclusive monopole has been produced. Where are they or where did they go ?

The inflationary universe theory [7] explains each of these problems by postulating a period of exponential expansion in the size of the early universe. This is caused by a phase transition in the field describing the matter content of the very early universe-the exact detail of the phase transition depends on the particular G.U.T. chosen, however, it always seems to be associated with spontaneous symmetry breaking. With this massive expansion, any factors such as curvature or monopoles would have been violently diluted. One single horizon length could have been blown up to larger than the present observable size of the universe.

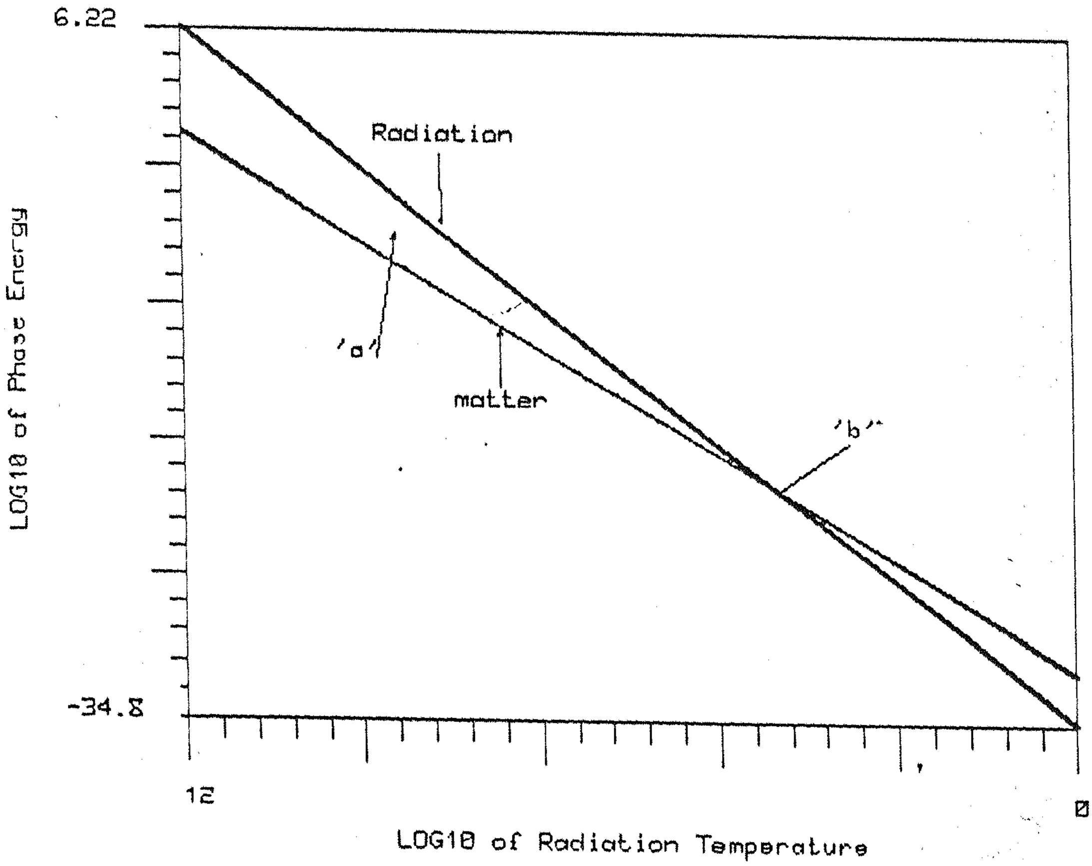
It will be seen in this work, if the interaction between the phases in the late universe are considered, over many cycles the universe becomes more flat as the cycles progress. The particular region of interest is shown as lying between the points marked 'a' and 'b' on the standard thermal history of the universe shown in Graph G1.1 and taken from reference [8], i.e. when the radiation temperature is below 10^9K but above 4,000K. This increase in flatness is due to the work done against the gravitational field, increasing the initial rate of expansion. Hence one of the problems of the standard big bang theory may be overcome in a non-inflationary way.

S1.2 The question of reversibility in the standard model

The standard cosmological model as described above considers the Universe on the large scale to consist of an isotropic blackbody radiation field and a matter field. After nucleosynthesis this matter field is dominated by Hydrogen, this is ionized until the recombination temperature of 4,000K is reached.

GRAPH G1.1

A standard thermal history of the universe taken from reference [8], showing the era, lying between the points 'a' and 'b', with which this work is concerned.



GRAPH G1.1

It is assumed that until after the recombination occurred, the interaction of the ionized gas with the photons was so strong that thermal equilibrium occurred and the temperatures of the two phases remained locked.

This assumption leads to the slightly puzzling state of affairs that, in a model in which the thermal interaction between the phases is very strong, the model is well represented by a zero interaction case. This is because an interaction is self defeating - it tends to equalise temperatures and then vanishes.

In such a strongly coupled model the generation of entropy, which is caused by the interaction, is zero. The universe is totally reversible (if it is closed), totally symmetric about its point of maximum extent and possesses no arrow of time.

This study considers the inclusion of the interaction between the phases explicitly in the set of differential equations representing the model universe. These equations are solved numerically, to study the validity of the reversibility statement and to assign limits to any irreversibility in a closed universe.

Although this irreversibility effect is expected to be small over a single cycle, in a connected polycycled model (if such can exist) the combined effects can build up to a sizeable amount.

It is possible that these irreversible effects could build up over many cycles and solve one of the problems of the standard big bang model, namely the flatness problem, in a non-inflationary way.

S1.3 This study

In this study we wish to consider how well justified the assumption of thermal equilibrium is in the post nucleosynthesis but pre-decoupling universe. To do this the standard cosmological model is adapted to include a thermal interaction between the matter and radiation.

This adaption means replacing the equation of energy conservation in the normal Friedmann-Robertson-Walker model with an equation of energy conservation for each phase. Mathematically this is easily accomplished, but results in equations that are much harder to solve.

The resulting set of equations will be solved numerically as the universe moves through a cycle. The irreversibility produced by the interaction acting over the cycle will be calculated. The value of this quantity will show how justifiable is the approximation of reversibility and equilibrium.

We proceed to present and discuss the 'ingredients' of the model and their applicability to the real universe. Many of these are parts of the standard model. Space does not allow a full critique of the assumptions in the standard model, for more information the interested reader is directed to the references, especially [9,10,11,12]

The standard model assumes both the large scale homogeneity and isotropy of the universe and Einstein's theory of General Relativity, with zero cosmological constant, as a description of gravity.

In this case the future is dependent on the spacial 3 curvature, k. If k is +1 the universe is closed and will eventually stop expanding and enter a recontracting phase. If however k is -1 (open) or 0 (flat) then the universe will expand for ever.

The curvature is dependent on the average energy density of the universe. If this exceeds a critical value ($\rho_c \simeq 10^{-9}$ J/M^3) then the universe is closed, if it is less then the universe is open. if the density should be equal to this value then the universe is spatially flat. The ratio of the actual density to this critical density is the density parameter, Ω.

Measurement of the luminous matter in the universe indicates $\Omega \simeq 0.2$ [13], however this conflicts with the observed motion of the galaxies [14]. Attempts to measure the actual geometry of space, using the galaxies as test particles, lead to values in the region of unity [15]. These results have lead to much speculation concerning the question of dark matter in the universe [16,17].

The standard model indicates that whatever the ultimate fate of the universe, it began at a finite time in the past. This occurred at a singularity with infinite density - the big bang. As the universe expanded from this stage it cooled. In fact in recent years big-bang type singularities have been reported in inhomogeneous cosmologies [18].

The microwave background is a relic of this early hot stage of the universe and allows us to calculate the thermal history of the universe to very near its birth. This allows the calculation of the production of complex elements during the era of nucleosynthesis. A definite prediction is that the abundance of Helium should be approximately 27% by weight [19]. This may be borne out by observation [20].

From its origin in the big bang, until the era of recombination of the ionized plasma, the standard model assumes that the universe was in exact thermal equilibrium. The matter and radiation phases had exactly the same temperature. After recombination each phase cooled independently.

The metric for the space-time of the universe is the Robertson-Walker metric [21]. This choice is dictated by the assumption that, on a sufficiently large scale, the universe is, in some sense, isotropic and homogeneous - the cosmological principle. This metric is independent of the choice of the theory of gravity, provided it is of a geometrical nature. There are essentially four separate reasons for making these symmetry assumptions.

These are simplicity assumptions. The field equations of any geometric theory of gravity give rise in general to a very complicated set of non-linear partial differential equations. The greater the symmetry chosen for the metric, the less complicated the set becomes. The Robertson-Walker metric is the simplest metric which allows universal expansion.

Philosophically, cosmology and astronomy have progressed as the Earth's position in the cosmos has been accepted as less and less special. The cosmological principle of the 3 dimensional spatial symmetry is thus attractive. Philosophy in science must always be restrained by hard facts - the perfect cosmological principle yields apparently false results [9].

The isotropic microwave background radiation accidentally discovered in 1965 [22] seems to indicate that, prior to the decoupling of the phases, the dominant phase was isotropic to a factor of 10^{-3}.

The question of the isotropy and homogeneity of the matter in galaxies is a far more open question [23]. However, it has been shown that above a smoothing length scale of order 200 MPc the universe can be considered as homogeneous and isotropic. [24]

Any significant deviation from homogeneity and isotropy in the present distribution is less relevant to our study of the effect of the pre-decoupling interaction among the phases, as it is believed that galaxies formed around the time of decoupling [10]. If, however, the inhomogeneity/anisotropy is indicative of serious asymmetry in the material content of the earlier structure of the universe it is far more serious. Such an occurence would invalidate the choice of the Robertson-Walker metric.

The energy momentum tensor taken for the space-time is that of a perfect fluid. Physically this implies that we are assuming that the universe is filled with a 'fluid' of galaxies. These galaxies are the test particles of the universe and, according to General Relativity, they move along the geodesics of the space-time. The assumption of this particular energy momentum tensor is justified for two reasons:

i) As with the choice of the metric for the space-time it is a simplicity assumption. The perfect fluid is the most physically realistic tensor for which there is any hope of solving the resultant field equations.

ii) The assumptions of spatial homogeneity and isotropy, when applied to the matter content of the universe, as represented by the energy momentum tensor, require that this object is form invariant under those transformations which leave the metric of the space-time form invariant. The only tensor which satisfies these spatial isometries is that of the perfect fluid [25].

The field equations for the gravitational field are Einstein's. General Relativity seems to be the best geometrical theory of gravity at present. The classical tests of General Relativity, although often not involving the actual field equations, appear to hold true [26]. Other theories have been tried but none seems as natural or explains physical facts with so few extra assumptions. The cosmological constant is set identically equal to zero. There seem to be many good reasons for supposing this to be true in the late universe, both from astrophysical observations and Quantum field theory [27].

The interaction chosen to couple the phases is the Thomson interaction of radiation with charged particles. How good is this approximation is tied to the question of the accuracy of the rest of the model. Suffice it to say that if the universe is well modelled by a charged plasma and a blackbody radiation field, then this interaction is a good approximation in the post nucleosynthesis universe [28].

The Thomson interaction is chosen in particular because with the temperature range of our model, from nucleosynthesis (10^9K) to decoupling (4,000K), it represents best the thermal interaction of an ionized plasma with a blackbody radiation field. Up to a temperature of 10^7K studies show that the Thomson interaction is an excellent approximation to the more general Klein Nishima interaction [29]. Above 10^7K to 10^9K it has been shown to be about 85% accurate. This interaction is adopted as it is the most accurate process which can be incorporated simply into the standard cosmological model.

It must also be noted in passing that the Thomson interaction is derived from the scattering of quantum particles on a flat static space-time background. The model, however, also assumes General Relativity, which describes gravity as being due to a non-static curved space-time. We can only assume that the interaction carries over into a more general space-time. There are problems with such hybrid theories which have been discussed elsewhere [30]. Further justification of such hybrid models has appeared in recent years as it has been proved that the Plank radiation law carries over exactly to an open universe, and that although a modification does indeed occur for a closed universe that this is small and within the current bounds of experimental error [31].

It has been shown that the overall effects of the interaction in easily calculable models is not strongly dependent upon the interaction chosen [32].

As the universe is chosen to be closed, it will eventually encounter an era of nucleobreak-up followed by a space-time singularity. As the model is built upon both classical General Relativity and thermodynamics, with a constant number of particles, the model is not valid in this region.

In order to study further cycles, a method of cycle truncation is employed such that a 'bounce' occurs and the universe emerges after a period of nucleosynthesis, in exactly the same state as it was prior to nucleobreak-up, but with the opposite sense of motion. This bounce, although crude, is the only method available for the study of a poly-cycled universe. It has been used by several notable authors to date [33].

This method of truncation is not as arbitrary as it might first appear. It is necessary to stop the model once the era of nucleobreak-up is encountered, as the model is based upon classical thermodynamics, with a constant number of particles. However, classical General Relativity remains valid very near to the Plank region [34]. Once the radiation temperature is above that required for element break-up, the appropriate interaction with which to couple the phases is the Klein Nishima interaction [35]. At these temperatures this is strong enough to maintain thermal equilibrium until the Plank region is reached.

In effect the 'bounce' employed in this work only assumes that the as yet unknown Quantum Gravitational forces, acting in the Plank domain, cause the universe to reappear afterwards with a new expansion, but exactly the same phase energies as before.

It has been argued [36] that consideration of the principles of equivalence and Birkhoff, applied to a reasonable theory of gravity would indicate that a contracting universe would always be governed by a Friedmann equation of the form of of equations 2.6 or 2.42, and hence may not bounce. If this is the case, which is still subject to conflicting publications [37], then, although the study of the entropy etc. generated in a single closed cycle by the irreversible interaction is still applicable, the polycycled predictions become a mere computational exercise. However, the following points are worthy of mention in defence of a bouncing model.

The limit of the Friedmann equation of motion, as the scale factor tends to zero, does not indicate a point of classical stability and, as the scale factor by definition cannot become negative then the question of what does the scale factor do begs to be asked, one may thus suggest that a re-expansion may occur.

It has been shown that in several models of super gravity the big bang and big crunch singularities do not occur and that the latter can even be converted into a bounce [38,39]. In addition, consideration of the growth of quantum uncertainty near to the initial singularity seems to indicate that there will be a finite probability of non-classical, non-singular states near to the classical singularity as the scale factor goes to zero [40]. In addition, it is possible that any universal rotation may avoid the singularity [41]..

It seems to the author inadvisable to make any definite predictions as to the behaviour of any true quantum theory of the gravitational field until such a time as such a theory is available, especially when predictions are being made so close to a space-time singularity where even the topology of space-time itself may be subject to quantum fluctuations. Indeed such an authority as Penrose has commented that, in his opinion, a quantum theory of gravity will in some respects be significantly different from classical General Relativity [42]. It is with a mind to these considerations that it is sugested tentativly that a future quantum theory of the universe may lead to a re-expansion of the universe.

As the model is taken to be closed, it will have a life of less than 10^{11} years, this means that the effects of proton decay may be safely ignored as the half life of this decay is known to be in excess of 10^{30} years [43].

Since the first appearance of the inflationary universe theory, research into that topic has appeared to dominate cosmological research to a large degree. However, in the past few years, three strands of cosmological research have appeared that consider the interaction of matter and radiation in a cosmological model.

The first of these [44], by considering the interaction of two ideal fluids around the time of nucleosynthesis, has shown that although the interaction does not affect the element abundancies, and is thus consistent with observed data, the interaction does allow for the universe to be closed by baryonic matter alone.

The second of these [45] the extra 'Thermal mass' that a particle acquires due to its interactions at a non-zero temperature is considered. By using first order perturbation theory it is found that for all species except the electron the effects are negligible. However, consideration of the electron's extra thermal mass prior to the period of nucleosynthesis indicates that a correction to the equation giving the temperature dependence of the scale factor should occur.

In the third of these [46] it has been shown that an interacting inhomogeneous cosmological model, consisting of an ideal fluid and dust, can in a sub-class of exact solutions evolve into a standard Robertson-Walker type cosmological model.

In view of the relative dearth of studies of the effects of thermal interaction on the structure of the universe, the present interest in the flatness problem, and the interesting conclusions of the papers cited above, the structure of the universe caused by thermal interaction during the later universe cried out for careful study. The need for just such a study was amplified by the arbitrary way that initial conditions and constants of motion were assigned to the earlier works on the effects of the thermal interaction in the post nucleosynthesis universe [29,30,32,33].

CHAPTER TWO

An Irreversible Cosmological Model

S2.1 The General Relativity of the model

We now proceed to use the physical assumptions stated and discussed in S1.3 to develop a three phase irreversible cosmological model.

These assumptions which are discussed in the previous chapter are,

i) The Robertson-Walker metric
ii) The perfect fluid energy momentum tensor
iii) General Relativity
iv) The Thomson Interaction

The assumptions of the large scale homogeneity and isotropy of the universe, ' The Cosmological Principle ' lead, on purely geometrical grounds, to a unique metric for the space-time - the Robertson-Walker metric [21]. The associated line element is,

$$ds^2=c^2dT^2-R^2(T)[dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)]/A^2(r) \quad (2.1)$$

where,

$A(r) = 1 + \frac{1}{4}kr^2$

$R(T)$ = Cosmological Scale Factor

k = Curvature Constant (Scaled to 0, ±1)

T = Cosmological time

r = Radial Coordinate

$\theta \& \phi$ = Normal Polar Angles

As discussed in S1.3 the assumptions of spatial symmetry force upon us the perfect fluid energy momentum tensor. This is [25]

$$T_{ab} = pg_{ab} + (\rho + p/c^2)U_a U_b \qquad (2.2)$$

where,

ρ = co-moving mass density

p = co-moving pressure

U_a = Fluid 4 velocity vector (in a co-moving frame U_a = [c,0])

g_{ab} = The metric of the space-time.

Einstein's field equations of General Relativity, without the cosmological constant, will be used as a description of gravity. Thus the field equations are, [47],

$$G_{ab} = (8\pi G/c^2)T_{ab} \qquad (2.3)$$

Calculation of the Einstein Tensor from (2.1) using an algebraic computing package and substitution of the results and (2.2) into (2.3) give two independent equations. Letting ' denote differentiation w.r.t. cosmological time T, these are [11]

$$2R''/R + R'^2/R^2 + kc^2/R^2 = -8\pi Gp/c^2 \qquad (2.4)$$

$$R'^2/R^2 + kc^2/R^2 = 8\pi G\rho/3 \qquad (2.5)$$

(2.5) immediately yields an equation of motion,

$$R'^2 = 8\pi G\rho R^2/3 - kc^2 \qquad (2.6)$$

multiplying (2.4) by R^2 gives,

$$\frac{1}{R'}\frac{d\,[RR'^2]}{dT} = \frac{-8\pi pR^2 - kc^2}{c^2} \qquad (2.7)$$

so using (2.5) in (2.7) gives the conservation equation,

$$\frac{d(8\pi G\rho R^3/3)}{dT} = \frac{-8\pi G\rho R^2 R'}{c^2} \qquad (2.8)$$

defining the co-moving element V by,

$$V = 4\pi R^3/3 \qquad (2.9)$$

and the energy U in a co-moving volume element V to be,

$$U = \rho V c^2 \qquad (2.10)$$

(2.8) & (2.10) give.

$$U' = 4\pi pR^2R' \qquad (2.11)$$

This is the most useful form of the conservation equation.

S2.2 The thermodynamics of the model

Decompose the pressure and energy of the model into that from a sum of three phases,

i) Radiation - subscript $_r$

ii) Protons - subscript $_p$

iii) Electrons - subscript $_e$

so,

$$U = U_r + U_e + U_p \qquad (2.12)$$

$$p = p_r + p_e + p_p \qquad (2.13)$$

Now introduce 3 dissipative interactions E_r, E_e & E_p where E_i is the rate at which the i'th phase loses energy by interaction .

with (2.12) & (2.13), (2.11) can be replaced by three equations,

$$U_r' + 4\pi p_r R' R^2 = E_r \qquad (2.14)$$

$$U_e' + 4\pi p_e R' R^2 = E_e \qquad (2.15)$$

$$U_p' + 4\pi p_p R' R^2 = E_p \qquad (2.16)$$

provided that the conservation requirment,

$$E_r + E_e + E_p = 0 \qquad (2.16a)$$

is obeyed.

With (2.12) the equation of motion (2.6) becomes.

$$R'^2 = 2G(U_r + U_e + U_p)/Rc^2 - kc^2 \qquad (2.17)$$

Choosing the electron - radiation interaction to be the Thomson scattering gives [28],

$$E_e = A(T_r)^4(T_r - T_e) \qquad (2.18)$$

where,

T_r = Radiation Temperature

T_e = Electron Temperature

$$A = 4\sigma akN/(m_{oe}c) \qquad (2.19)$$

with

σ = Thomson cross section of the electron
a = Blackbody radiation constant
k = Boltzman constant
N = Number of electrons
m_{oe} = Electron rest mass

Choosing the Proton - radiation interaction to be the Thomson scattering gives,

$$E_p = B(T_r)^4(T_r - T_p) \qquad (2.20)$$

where,

T_p = Proton Temperature

$$B = 4\sigma' akN'/(m_{op}c) \qquad (2.21)$$

with,

m_{op} = Proton rest mass
σ' = Thomson cross section of the Proton

N' = Number of protons

For a charge neutral universe we require,

$$N = N'$$

Thus from (2.19) & (2.21) we find, using the fact that cross sections are inversly proportional to the square of a particle's mass.

$$A/B = (m_{op}/m_{oe})^{3} \equiv (1/\alpha)^{3} \qquad (2.22)$$

Before we can use the above interactions to study the irreversibility, we need equations of state for the three phases. For the radiation we choose the blackbody formulae. These give [48]

$$p_r = U_r/3V \qquad (2.23)$$

$$T_r = (U_r/aV)^{1/4} \qquad (2.24)$$

For the electrons we use the ideal massive quantum gas and an interpolation formulae due to Honl [49]. This gives,

$$p_e = U_e\psi(U_e/U_{oe})/3V \equiv U_e\psi_e/3V \qquad (2.25)$$

where,

$$\psi(x)=1-3/(2x^2)+1/x^3-1/(2x^4) \approx 2(x-1)/x \qquad (2.26)$$

The approximation being true for x-1, small and representing the ideal massive classical gas. The electron temperature is given by,

$$T_e = p_e V/Nk \tag{2.26a}$$

For the protons, we take an ideal massive quantum gas. Thus similarly to the electron case,

$$p_p = U_p \psi(U_p/U_{op})/3V \equiv U_p \psi_p/3V \tag{2.27}$$

The proton temperature is given by,

$$T_p = p_p V/Nk \tag{2.28}$$

Using the equations of state ,the equation of energy transfer for each phase, together with the equation of motion and the conservation requirement, we can form the four fundamental equations of the model.

Using (2.9) (2.14) & (2.23) gives for the radiation.

$$U_r' = E_r - R'U_r/R \tag{2.29}$$

Using (2.9) (2.15) & (2.25) gives for electrons,

$$U_e' = E_e - R'U_e\psi_e/R \tag{2.30}$$

Using (2.9) (2.16) & (2.27) gives for the Protons

$$U_p' = E_p - R'U_p\psi_p/R \tag{2.31}$$

So equations (2.29) - (2.31) together with the equation of motion,

$$R'^2 = 2G(U_p + U_e + U_r)/(Rc^2) - kc^2 \qquad (2.32)$$

make up the fundamental differential equations of the model in S.I. units.

S2.3 The reduction of the equations defining the model

Prior to considering the question of initial conditions and constants of motion for this system it is most useful to introduce changes of variable to remove many of the constants.

Let us make the changes of variable,

$$R(T) = R_o r \qquad (2.33)$$

$$T = T_o t \qquad (2.34)$$

$$U_r = U_{oe} y \qquad (2.35)$$

$$U_e = U_{oe} x \qquad (2.36)$$

$$U_p = U_{oe} z \qquad (2.37)$$

where R_o & T_o are arbitrary positive constants to be chosen later for convenience. We will refer to the variable r , the scaled scale factor as the radius of the model universe.

The total electron rest energy is,

$$U_{oe} = Nm_{oe}c^2 \qquad (2.38)$$

These changes of variable introduce two arbitrary constants so that there are two degrees of freedom. We use these to eliminate as many constants as possible.

We now proceed to perform the above changes of variable on the fundamental equations of the model (2.29) - (2.32). We define short hand 'dot' $\dot{}$ to denote differentiation w.r.t. scaled cosmological time t.

i.e.,

$$\dot{p} \equiv \frac{dp}{dt}$$

With (2.23) - (2.37) the equation of motion (2.32) becomes,

$$\frac{\dot{r}^2}{2} + \frac{kc^2(T_o/R_o)^2}{c^2} = \frac{2GU_{oe}(x+y+z)T_o^2}{rR_o^3} \qquad (2.39)$$

Using one of our degrees of freedom by choosing,

$$\frac{GU_{oe}T_o^2}{R_o^3c^2} = 1 \qquad (2.40)$$

and defining the constant C by,

$$C = -T_o^2kc^2/2R_o^2 \qquad (2.41)$$

Is important to note that in the above the capital C, is the constant as defined by equation (2.41), and lower case c, is the speed of light. With the above definition of C, (2.39) then becomes the reduced equation of motion,

$$\dot{r}^2/2 - C = (x + y + z)/r \qquad (2.42)$$

With (2.23) - (2.27), (2.29) becomes

$$\dot{y} = (T_o/U_{oe})E_r - y\dot{r}/r \qquad (2.43)$$

Defining,

$$I_r = (T_o/U_{oe})E_r \qquad (2.44)$$

gives the reduced radiation equation,

$$\dot{y} = I_r - y\dot{r}/r \qquad (2.45)$$

With (2.23) - (2.27), (2.30) becomes

$$\dot{x} = (T_o/U_{oe})Ee - x\psi_e\dot{r}/r \qquad (2.46)$$

Defining,

$$I_e = (T_o/U_{oe})E_e \qquad (2.47)$$

gives the reduced electron equation,

$$\dot{x} = I_e - x\psi_e\dot{r}/r \qquad (2.48)$$

With (2.23) - (2.27), (2.30) becomes

$$\dot{z} = (T_o/U_{oe})Ep - z\psi_p\dot{r}/r \qquad (2.49)$$

Defining,

$$I_p = (T_o/U_{oe})E_p \tag{2.50}$$

gives the reduced proton equation,

$$\dot{z} = I_p - z\psi_p\dot{r}/r \tag{2.51}$$

In order to give the differential equations in a computable form we must find expressions for the I's as explicit functions of r,x,y,z. As a necessary prerequsite the phase temperatures must be found in reduced units.

Using (2.33) - (2.37) & (2.9) in (2.24) gives,

$$T_r^4 = (3U_{oe}/4\pi aR_o^3)(y/r^3) \tag{2.52}$$

Using (2.33) - (2.37) & (2.9) in (2.25) & (2.26a) gives

$$T_e = U_{oe}x\psi_e/3Nk \tag{2.53}$$

Defining,

$$H(p) = p\psi(p)$$

gives

$$T_e = U_{oe}H(x)/3Nk \tag{2.54}$$

Using (2.33) -(2.37) & (2.9) in (2.27) & (2.28) gives

$$T_p = U_{oe}z\psi(\alpha z)/3Nk \tag{2.55}$$

$$= U_{oe}H(\alpha z)/3Nk\alpha \tag{2.56}$$

Having, in equations (2.52) (2.54) & (2.56) explicit expressions for the phase temperatures we move on to calculating expressions for the interactions.

From (2.18) & (2.47) we obtain,

$$I_e = AT_oT_r^5/U_{oe} - T_oAT_r^4T_e/U_{oe} \qquad (2.57)$$

using (2.52) & (2.53) in (2.57) gives

$$I_e = \frac{T_oA(3U_{oe}/4\pi aR_o^3)^{5/4}(y/r^3)^{5/4}}{U_{oe}} - \frac{(3U_{oe}/4\pi aR_o^3)AT_oH(x)(y/r^3)}{3Nk} \qquad (2.58)$$

Defining,

$$P = (3U_{oe}/4\pi aR_o^3)^{5/4}AT_o/U_{oe} \qquad (2.59)$$

$$Q = AT_o3U_{oe}/(3Nk4\pi aR_o^3) \qquad (2.60)$$

gives in (2.58)

$$I_e = y/r^{15/4}(Py^{1/4} - Qr^{3/4}H(x)) \qquad (2.61)$$

Using our final degree of freedom given by the changes of variable and setting ,

$$P = 1 \qquad (2.62)$$

gives in (2.61)

$$I_e = y/r^{15/4}(y^{1/4} - Qr^{3/4}H(x)) \qquad (2.61)$$

Using (2.20) & (2.50) gives

$$I_p = BT_oT_r^5/U_{oe} - T_oBT_r^4T_p/U_{oe} \qquad (2.62)$$

Using (2.52) & (2.56) in (2.62) gives

$$I_p = \frac{T_oB(3U_{oe}/4\pi aR_o^3)^{5/4}(y/r^3)^{5/4}}{U_{oe}} - \frac{(3U_{oe}/4\pi aR_o^3)AT_oH(\alpha z)(y/r^3)}{3Nk\alpha} \qquad (2.63)$$

Defining,

$$V = (3U_{oe}/4\pi aR_o^3)^{5/4}BT_o/U_{oe} \qquad (2.64)$$

$$S = BT_o3U_{oe}/(3Nk4\pi aR_o^3) \qquad (2.65)$$

gives in (2.63)

$$I_p = y/r^{15/4}(Vy^{1/4} - Sr^{3/4}H(\alpha z)/\alpha) \qquad (2.66)$$

Comparing (2.64) & (2.59) gives,

$$P/V = A/B = 1/\alpha^3$$

so by (2.62)

$$V = \alpha^3 \simeq 1/1836^3 \qquad (2.67)$$

comparing (2.65) & (2.60) gives,

$$Q/S = A/B = 1/\alpha^3 \qquad (2.68)$$

so

$$S = \alpha^3Q \qquad (2.69)$$

(2.69) & (2.2.67) give in (2.66)

$$I_p = \alpha^3 y/r^{15/4}(y^{1/4} - Qr^{3/4}H(\alpha z)/\alpha) \qquad (2.70)$$

From (2.16a) (2.44) & (2.47) we get

$$I_r = -T_o(\ E_m + E_p\ )/U_{om} = -I_m - I_p \qquad (2.71)$$

so (2.71) (2.70) & (2.61) give

$$I_r = -y/r^{15/4}(y^{1/4}(1+\alpha^3) - Qr^{3/4}[H(x)+\alpha^2 H(\alpha z)]) \qquad (2.72)$$

Thus we arrive at the set of differential equations to be computed to give a three phase interacting cosmological model. They are:

The equation of motion of the model, (2.42)

$$\dot{r}^2/2 - C = (\ x + y + z\ )/r \qquad (2.73)$$

The three thermodynamic equations (2.45) (2.48) & (2.51)

$$\dot{y} = I_r - y\dot{r}/r \qquad (2.74)$$

$$\dot{x} = I_m - x\psi_m\dot{r}/r \qquad (2.75)$$

$$\dot{z} = I_p - z\psi_p\dot{r}/r \qquad (2.76)$$

Together with the internal relations (2.61) & (2.70)

$$H(p) \equiv p\psi(p) = p - 3p/2 + 1/p^2 - 1/(2p^3) \qquad (2.77)$$

$$I_e = y/r^{15/4}(y^4 - Qr^{3/4}H(x)) \qquad (2.78)$$

$$I_p = \alpha^3 y/r^{15/4}(y^4 - Qr^{3/4}H(\alpha z)/\alpha) \qquad (2.79)$$

$$I_r = -(I_e + I_p) \qquad (2.80)$$

The above system of differential equations together with three constants of motion ,

- Q Interaction constant
- C Curvature constant
- α Mass ratio

and a set of initial conditions,

at $t = t_o$

$$r = r(t=t_o) \equiv r_o$$

$$x = x(t=t_o) \equiv x_o$$

$$y = y(t=t_o) \equiv y_o$$

$$z = z(t=t_o) \equiv z_o$$

determine the time evolution of the model.

CHAPTER THREE

Initial Conditions and Constants

Before proceeding to solve numerically the set of initial value differential equations developed in chapter 2 to describe a three phase interacting cosmological model, it is necessary to assign initial conditions and constants of motion to the differential equations .

In this chapter this will be done in three different ways:

i) Assigning numbers in such a way that the differential equations are convenient for numerical solution.

ii) Assigning numbers based on sensible astrophysical data and information about the universe at present.

iii) A method intermediate to i) & ii) so that numbers are chosen to be numerically convenient but in a way guided by the real universe.

S3.1 Assigning initial conditions and constants to be numerically convenient:

In this section we follow closely the method and numerical values used in the previous work on interacting cosmological models [50].

We choose an initial point (suffix $_o$) at which the electrons are neither totally classical ($x_o \simeq 1.0$) nor totally relativistic ($x_o \gg 1.0$) but at an intermediate stage. We chöose,

$$x_o = 2 \tag{3.1}$$

In an attempt to maximize the effect of the interaction we choose the initial radiation energy to be equal to that of the electrons, so thus-, .

$$y_o = 2 \tag{3.2}$$

As the interaction of the protons with the radiation will be much less than that of the electrons (seen by the factor α^3 in equation (2.79)), and for the sake of simplicity, the protons are totally ignored in this treatment, we thus choose ,

$$z_o = z(t) = \dot{z}(t) = 0 \tag{3.3}$$

In terms of the thermodynamics of the model this is well justified, because the protons, which are far more massive than the electrons, are more weakly coupled to the radiation. This can be seen from the factor of α^3 in equation (2.70). In terms of the equation of motion this approximation is not well justified as the protons would add a minimum contribution of α^{-1} to the energy sum. This term is clearly not negligble. Henceforth when using this set of initial conditions we will refer to the electrons as the matter.

As inverse factors of r^3 and similar powers occur in the model r_o must be chosen of order 1 to give computationally convenient numbers so that any effects are neither washed out nor too violent. We thus choose, following earlier work [50]

$$r_o = 0.27 \qquad (3.4)$$

We impose the physical condition that initially the phases were in thermodynamic equilibrium i.e. the interaction vanished due to the equality of the phase temperatures, thus from (2.78) requiring

$$I_o = 0$$

gives,

$$y_o/r_o^3 = [QH(x_o)]^4 \qquad (3.5)$$

So (3.1) (3.2) (3.4) (2.77) & (3.5) give,

$$Q = 2.208 \qquad (3.6)$$

It is necessary to choose the curvature constant so that the model will not expand for too long and take up an excess of computer time, but will expand long enough for the irreversibility in the interaction to have a noticeable effect. In common with earlier work we choose,

$$C = -0.445 \qquad (3.7)$$

At variance with earlier works; and for the sake of simplicity we choose the initial conditions to occur at t = 0.

We thus have one set of initial conditions and constants for computation for the set of differential equations at the end of Chapter two. This set of data corresponding to computationally convenient numbers is

$x_o = 2$

$y_o = 2$

$r_o = 0.27$

at $t_o = 0$

with

$Q = 2.208$

$C = -0.454$

The results from these initial conditions may be found in Chapter four

S3.2 Assigning initial conditions and constants via astrophysical data

Before values for the constants and initial conditions can be calculated from astrophysical data, it is necessary to find expressions for the constants induced by our choice of relationships among the scaling constants T_o & R_o. The equations that we need from Chapter two are, (2.40)

$$\frac{GU_{oe}T_o^2}{R_o^3c^2} = 1 \qquad (3.8)$$

(2.59) & (2.62)

$$\frac{T_oA(3U_{oe}/4\pi aR_o^3)^{5/4}}{U_{oe}} = 1 \qquad (3.9)$$

(2.60)

$$Q = AT_o3U_{oe}/(3Nk4\pi aR_o^3) \qquad (3.10)$$

(2.41)

$$-C = T_o^2kc^2/(2R_o^2) \qquad (3.11)$$

Dividing (3.10) by (3.9) gives

$$Q = U_{oe}/3Nk(4\pi aR_o^3/3U_{oe})^{1/4} \qquad (3.12)$$

or

$$R_o^3 = 3U_{oe}/4\pi a(3NkQ/U_{oe}) \qquad (3.13)$$

Putting (3.13) into (3.8) gives

$$T_o^2 = (3NkQ/U_{oe})^4 3c^2/(4\pi aG) \qquad (3.14)$$

Using (3.13) & (3.14) into (3.12) gives

$$Q^3/A = U_{oe}^2(3c^2/4\pi aG)^{1/2}/(3Nk)^{1/3} \qquad (3.15)$$

Using (2.19) for A gives in (3.15)

$$Q^3 = (c/3)^3(12ac^2/\pi G)^{1/2}\sigma m_{oe}/k^2 \qquad (3.16)$$

Substitution of numerical constants of nature [51] into (3.16) gives

$$Q = 8.558 \times 10^5 \qquad (3.17)$$

Using (3.13) & (3.14) in (3.11) together with k=+1 for a closed universe, gives

$$-2C = c^4/G(3/4\pi a)^{1/3}(3NkQ)^{4/3}/U_{oe} \qquad (3.18)$$

Using (2.38) in (3.18) gives

$$C = \frac{-(3kQ)^{4/3}(3/4\pi a)^{1/3}}{2G} \cdot \frac{1}{m_{oe}{}^2N^{2/3}} \qquad (3.19)$$

Substitution of numerical values of constants of nature and (3.17) into (3.19) gives,

$$C = -7.164 \times 10^{52}/N^{2/3} \qquad (3.20)$$

Using an estimate of N consistent with a closed universe [47] gives,

$$N \simeq 4 \times 10^{80}$$

And hence

$$C = -0.132 \qquad (3.21)$$

Using the above, we are now able to proceed to assign initial conditions to our model using astrophysical data. We choose to assign our initial conditions at the end of the period of nucleosynthesis. This is chosen as the point for two reasons:

i) It is the earliest point in the universe's history when the number of particles N was constant. This is an implicit assumption in our model's development [52].

ii) The very elements which we view today were created at nucleosynthesis. It is the earliest event which is directly observable [53].

At the end of the period of nucleosynthesis it is believed that the following conditions applied :

a) Thermal Equilibrium held, $T_r=T_e=T_p$ [52]
b) $T_r = 10^9$ K [12]
c) $\rho_r/\rho_m \approx 10^5/2.35$ [8]

from a) b) & (2.54) we get

$$H(x_o) = 0.505$$

Using a package for numerical solution of polynomials the real root of the above was found to be,

$$x_o = 1.3 \qquad (3.22)$$

from a) b) and (2.56) we get.

$H(\alpha z_o) = 2.771 \times 10^{-4}$

Using the same package the real root of the above was found to be,

$$z_o = 1836.25 \quad \text{or} \quad \alpha z_o = 1.00014 \qquad (3.23)$$

from c)

$$y_o/(\ x_o + z_o\ ) = \ 10^5/2.35$$

(3.23) & (3.24) give

$$y_o = 7.77 \times 10^7 \qquad (3.24)$$

c) together with equation (3.5) give

$$r_o = 1.305 \times 10^{-5} \qquad (3.25)$$

We thus have a second set of initial conditions and constants of motion corresponding to our universe.

As the universe was ≃ 180 seconds old at the end of nucleosynthesis [54], we still regard these initial conditions to apply at t=0 as this timescale is dwarfed by the 10^{10} years for which the model will expand until it reaches 'now'. [55]

The set of initial conditions and constants of motion corresponding to our present universe is:

$x_o = 1.3$

$y_o = 7.77 \times 10^7$

$z_o = 1.8625 \times 10^3$

at $t = 0$

and

$Q = 8.558 \times 10^5$

$C = -0.132$

$\alpha = 1/1836$

The results from this set of initial conditions may be found in in Chapter five

S3.3 Assigning initial conditions and constants guided by the real universe

As we will see in Chapter five there are some intrinsic computational problems associated with the initial conditions and constants as developed in S3.2. For this reason, we develop a third set which, although not as accurately physically based as those in S3.2, will be devoid of the associated problems, without being as arbitrary as those developed in S3.1

The initial values of x used in S3.1 & S3.2 are similar so there is no problem with this parameter. To consider a model in which the matter is fairly relativistic, the value 2 is retained

As was seen in S3.2 the real universe originally had the radiation dominant over all the matter . We cannot have a radiation energy dominant over the proton rest energy (the proton rest energy is 1836 - this value being the proton to electron rest mass ratio) without recovering the problems associated with the conditions of S3.2 (see Chapter five). we can, however, have a radiation energy which is originally dominant over the electron energy. We choose,

$$y_o = 30$$

As in S3.1 we ignore the interaction of the radiation with the protons, as much weaker than that of the electrons. We do, however, include a static cold contribution of the protons to the equation of motion. This means that the only contribution made by the protons to the equation of motion is via their constant rest mass energy . From (3.23) it can be seen that this is a good approximation. We thus set

$$z_o = 1/\alpha \approx 1836$$

As in S3.1 we wish to keep r_o of order one but for the sake of variety and to show that results are not finely dependent on initial conditions, as was found to be the case for a distribution of initial conditions based around this value, we choose,

$$r_o = 0.25$$

Similarly we alter the curvature constant, although not drastically. We set,

$C = -0.20$

Imposing the requirement of initial thermal equilibrium again via (3.5) gives,

$Q = 4.605$

We thus have a third set of initial conditions and constants based upon physical facts, but which still give computationally convenient numbers. The set is

$x_o = 2$

$y_o = 30$

$z_o = 1/\alpha$

at $t = 0$

and

$Q = 4.605$
$C = -0.20$
$\alpha = 1/1836$

The results of this set of initial conditions are in Chapter six.

CHAPTER FOUR

Results from Numerically Convenient Initial Conditions

S4.1 Results

The initial value differential equations given at the end of Chapter two were integrated using the set of initial conditions presented in S3.1. With these parameters this was possible using simple numerical routines [56]. In this Chapter, some of the results are presented, illustrated and discussed.

As so many physically meaningful functions can be calculated from the basic solution set [t; r(t), x(t), y(t)] the results we present must be seen as a subset of all possible results.

The first problem was to confirm numerically the results of previous studies of irreversibility in the universe using the Thomson interaction [29] and others [30]. This was done in two independent ways using both a program constructed from first principles and the numerical library routines cited above [56]. The previous results were found to be accurate by both methods which were themselves consistent. In view of the far greater computer efficiency of the routines these were used, in various forms, for all following calculations.

Earlier works had concentrated heavily on the question of entropy and the arrow of time in a closed universe. These studies have shown numerically that entropy was non-decreasing in both the expanding and contracting phases of the motion. However, the internal consistency of the entropy calculations was not clear, and worried at least one of the authors [57].

To attempt to settle this question of the entropy generation, the entropy was calculated in two different ways,

i) Via adding the individual phase entropies at each point

ii) Via numerically integrating the rate of entropy generation, treating the entropy as a fourth dependent variable in the system of differential equations.

The results were found to be the same, up to the arbitrary additive constant representing the initial entropy. This result was pleasing to one of the earlier authors as it allayed his fears concerning the internal consistancy of the entropy calculations [57]. In essence, these calculations confirmed that the sum of the phase entropies gave the same total entropy as did integrating the rate of entropy production given by the interaction, once a similar 'zero' of entropy was chosen.

Next, the question of studying more cycles than the three originally considered was addressed. Using the cycle truncation method (stopping the contracting phase and starting the expanding phase of the next cycle when the radius reaches its original value), as described in S1.3, indicates that a cycle is determined by the initial phase energies. The initial energies of matter and radiation x_o, y_o will be considered as functions of the cycle number n, thus,

$$x_o = x_o(n)$$

and,

$$y_o = y_o(n)$$

Ten cycles of the model were calculated and the results tabulated in Table T4.1. These are illustrated in graphs G4.1 and G4.2. The two initial energies show a slow, approximately linear increase with cycle number.

It is important when studying all of the graphical results in this Chapter to note that none of the scaling factors between the scaled variables given, and actual astrophysical observables is given. The reason for this is as the current set of initial conditions is not derived from astrophysically based data the results are for a universe with different physical constants, and number of particles than our own, so that presentation of these scaling factors would be at best unhelpful and at worst misleading.

In order to see whether any limiting behaviour exists in these trends, a much larger number of cycles was studied using more computer time. The results of this study of 1600 cycles are presented in Table T4.2 and illustrated in the Graphs G4.3-G4.6.

The results from these studies of convenient initial conditions for an irreversible, oscillating

Table T4.1

The variation of the initial scaled matter and radiation energy of an interacting, oscillating cosmological model, subject to numerically convenient initial conditions, with cycle number, N, for the first ten cycles.

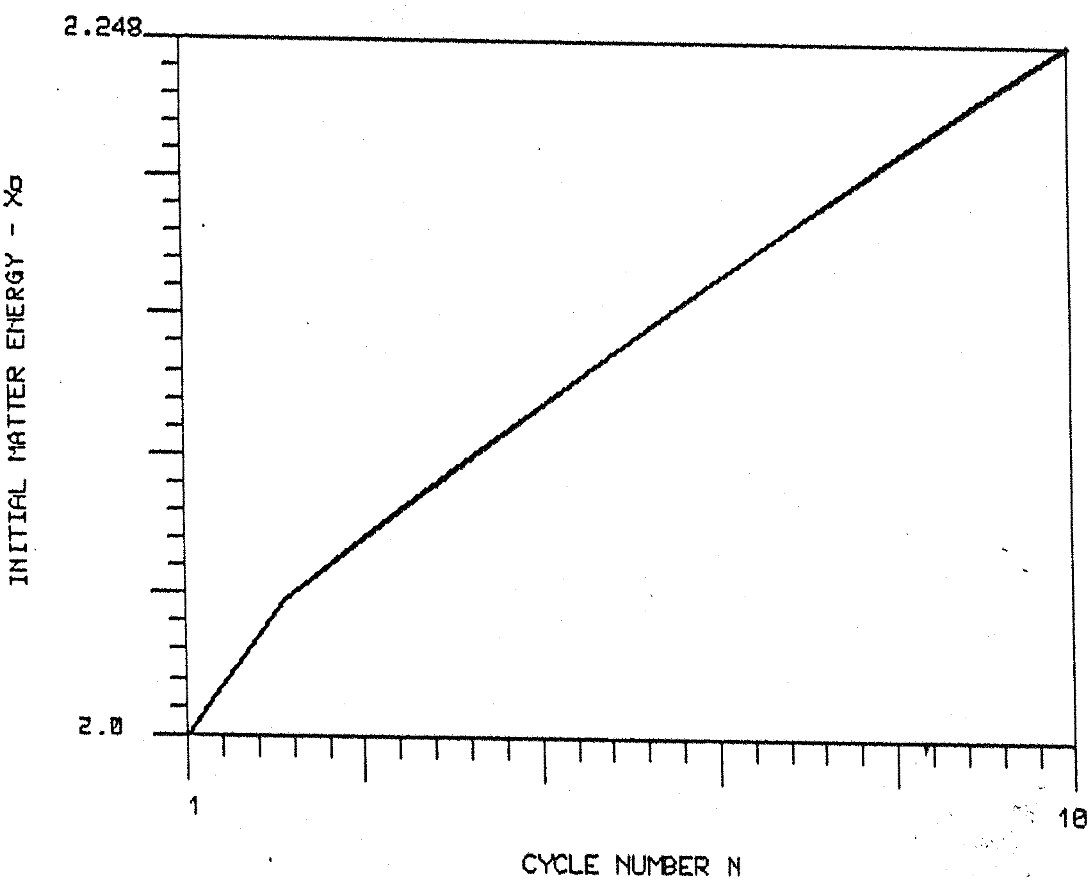
Cycle Number N	Initial Matter Energy x_o	Initial Radiation Energy y_o
1	2.0000	2.0000
2	2.0475	2.1862
3	2.0747	2.3997
4	2.1012	2.6207
5	2.1272	2.8489
6	2.1525	3.0841
7	2.1773	3.3259
8	2.2015	3.5739
9	2.2252	3.8285
10	2.2483	4.0888

Table T4.1

GRAPH G4.1

The variation of the initial scaled matter energy x_o, of an interacting, oscillating cosmological model, subject to numerically convenient initial conditions, with cycle number, N, for the first ten cycles.

Data : Table T4.1 Results column 1

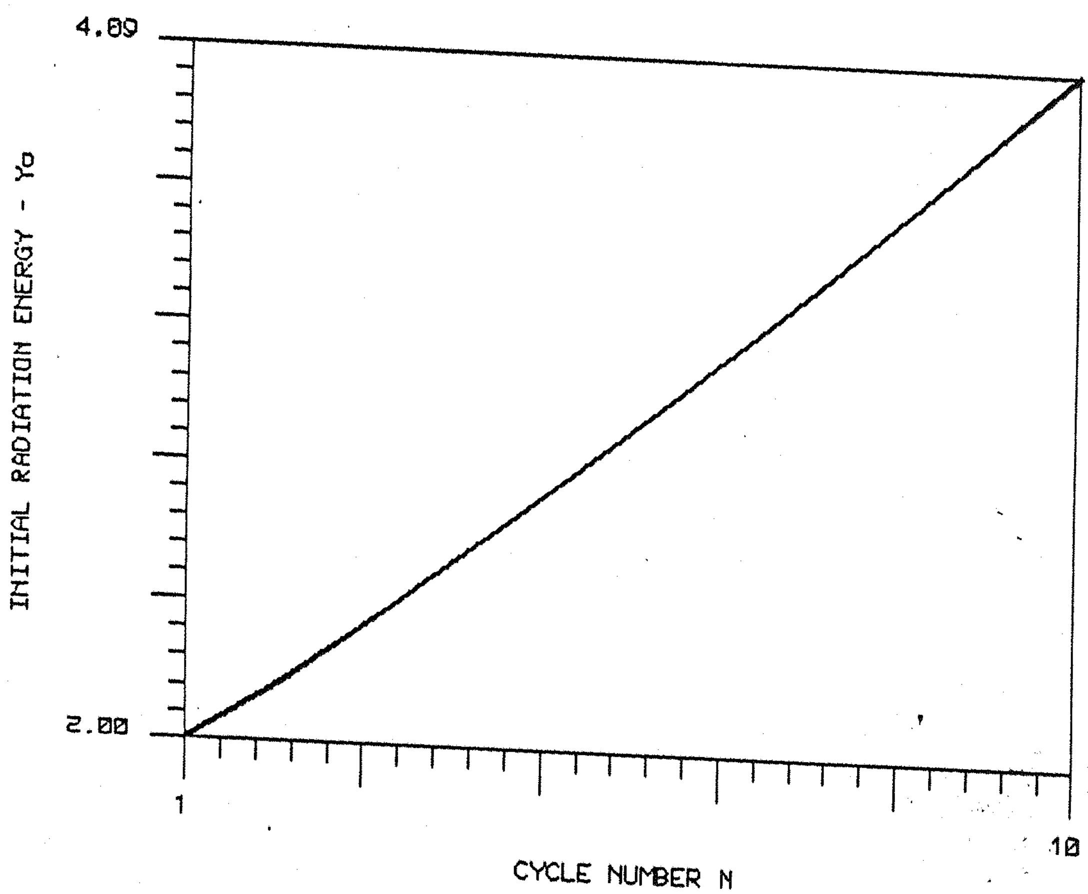


GRAPH G4.1

GRAPH G4.2

The variation of the initial scaled radiation energy y_0, of an interacting, oscillating cosmological model, subject to numerically convenient initial conditions, with cycle number, N, for the first ten cycles.

Data : Table T4.1 , Results column 2



GRAPH G4.2

universe, taken over many cycles, show certain interesting trends. If we can expect these trends, if not the actual numerical results themselves, to be reproduced in the real universe, then we may make a comparison with the astrophysics and cosmography that is observed in the cosmos.

Comparing Graphs G4.3 and G4.4 shows conclusively that the initial energies of the matter and radiation, although both increasing with cycle number, increase at different rates. G4.3 shows that although x_o increases monotonically with n it is concave downwards. Graph G4.4 shows that y_o increases monotonically with n but that it is convex upwards. We may thus conclude from a study of G4.3 and G4.4 that in such model universes, the initial radiation energy grows much faster with cycle number than does the matter energy.

If, in our identification of the model universe with the real one, we may tie the initial point to the end of the era of nucleosynthesis (or to any point in the radiation-dominated phase of the universe), then we may explain the fact that at the end of this period the ratio of the radiation to matter energy was high [8]. Our study predicts that just such an initial state could have evolved, via a closed irreversible, oscillating universe from a state many cycles before which had a phase energy ratio much closer to unity.

Consideration of Graph G4.5 shows that the initial value of the dimensionless density parameter $\Omega_o(n)$, goes to unity from above with increasing cycle number

Table T4.2

The variation of cosmological parameters in an interacting, oscillating cosmological model, subject to numerically convenient initial conditions, with cycle number, N, for the first 1500 cycles.

Cycle Number	Initial Matter Energy	Initial Radiation Energy	(Initial Density Parameter $-1)\times10^{-3}$	Minimum Value of $[y/(x+y)]$
1	2.0000	2.0000	30.7000	.1306
50	2.9379	18.6904	5.5358	.4930
100	3.5181	47.3786	2.5454	.6409
150	3.9766	83.1327	1.3687	.7188
200	4.3627	127.9753	.9005	.7674
250	4.7148	180.4211	.6436	.8007
300	5.0245	239.4500	.4728	.8251
350	5.3013	304.3393	.3847	.8400
400	5.5656	375.5964	.3125	.8583
450	5.8046	451.7177	.2603	.8702
500	6.0340	533.3758	.2208	.8800
550	6.2514	620.0273	.1902	.8883
600	6.4550	711.0517	.1660	.8954
650	6.6542	807.1486	.1463	.9015
700	6.8386	906.4832	.1304	.9069
750	7.0128	1009.9630	.1171	.9117
800	7.1815	1117.2258	.1059	.9159
850	7.3537	1229.9285	.0962	.9197
900	7.5058	1343.7373	.0881	.9231

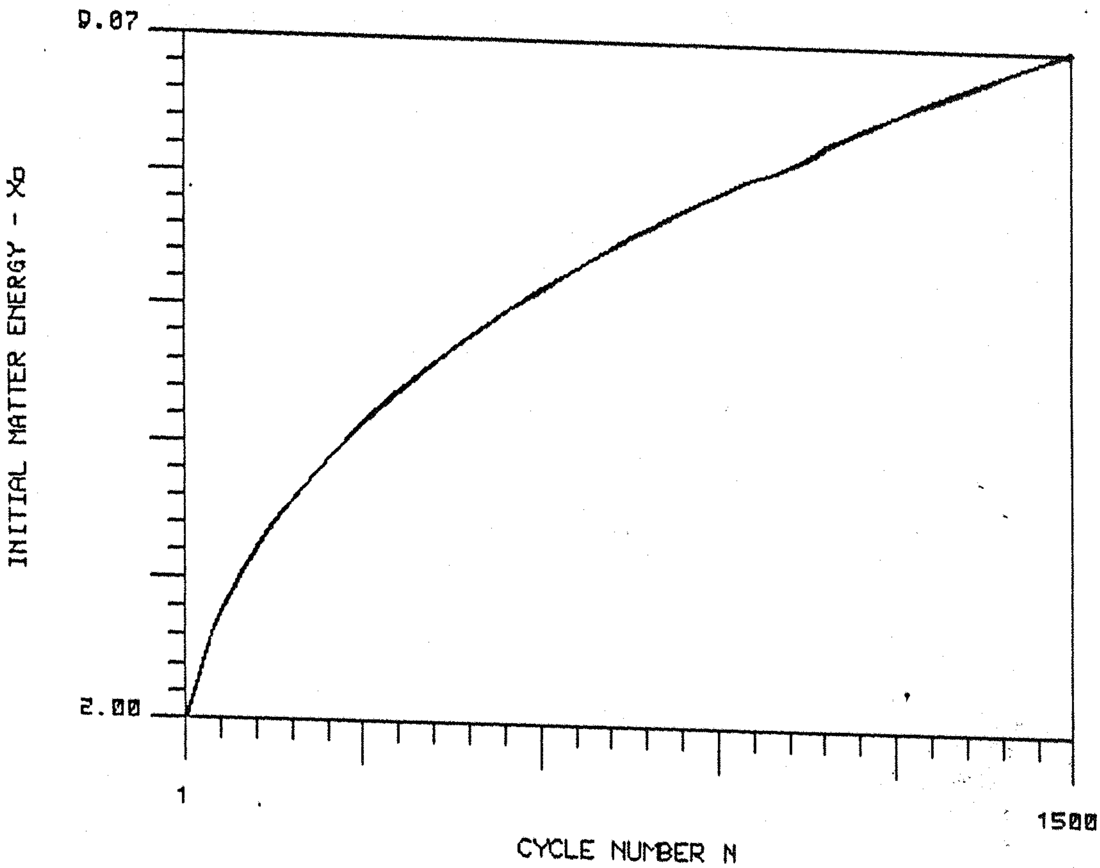
950	7.6606	1462.4870	.0810	.9262
1000	7.7537	1583.6789	.0748	.9290
1050	7.8060	1709.6040	.0693	.9315
1100	8.0912	1837.3794	.0645	.9339
1150	8.2305	1969.6422	.0602	.9361
1200	8.3593	2103.1144	.0564	.9381
1250	8.4895	2240.7969	.0529	.9400
1300	8.6083	2379.0045	.0499	.9418
1350	8.7283	2521.1127	.0471	.9434
1400	8.8497	2667.1578	.0445	.9449
1450	8.9723	2817.1806	.0421	.9464
1500	9.0740	2963.3705	.0401	.9477

Table T4.2

GRAPH G4.3

The variation of the initial scaled matter energy x_o, of an interacting, oscillating cosmological model, subject to numerically convenient initial conditions, with cycle number, N, for the first 1500 cycles.

Data : Table T4.2 Results column 1

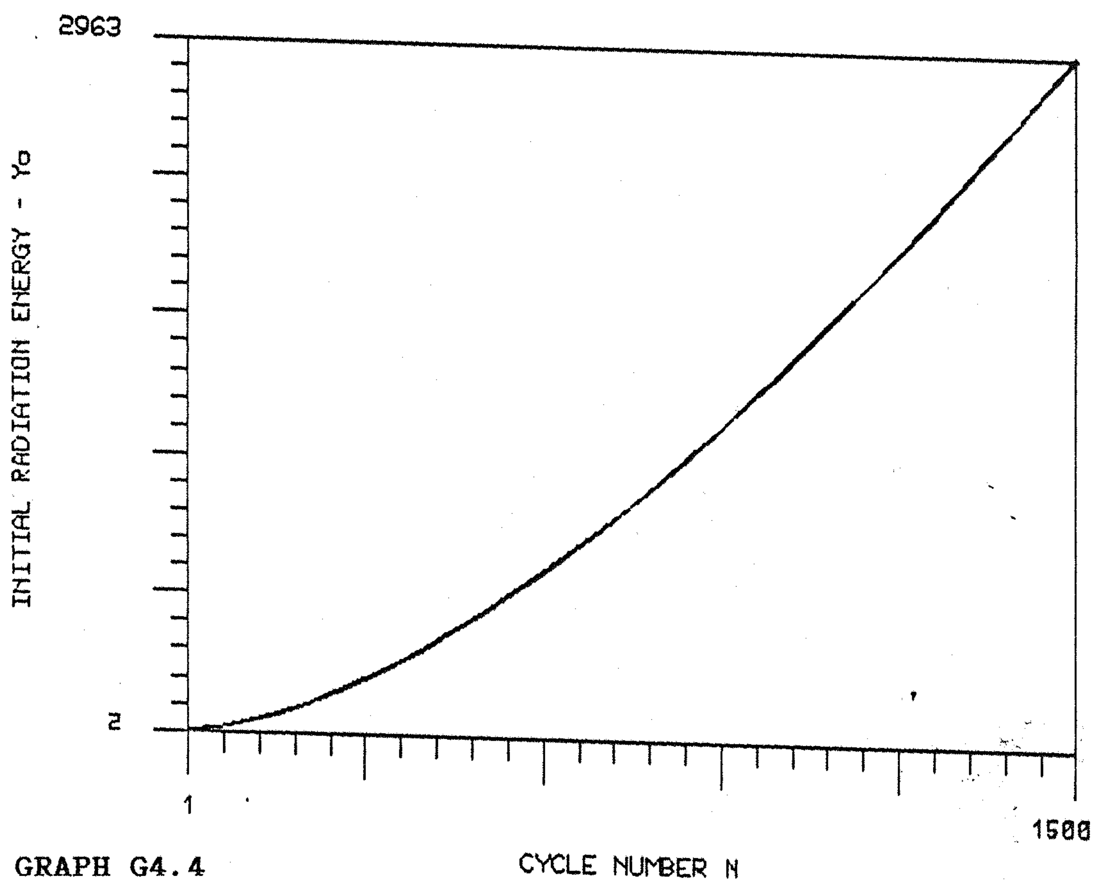


GRAPH G4.3

GRAPH G4.4

The variation of the initial scaled radiation energy y_o, of an interacting, oscillating cosmological model, subject to numerically convenient initial conditions, with cycle number, N, for the first 1500 cycles.

Data : Table T4.2 Results column 2

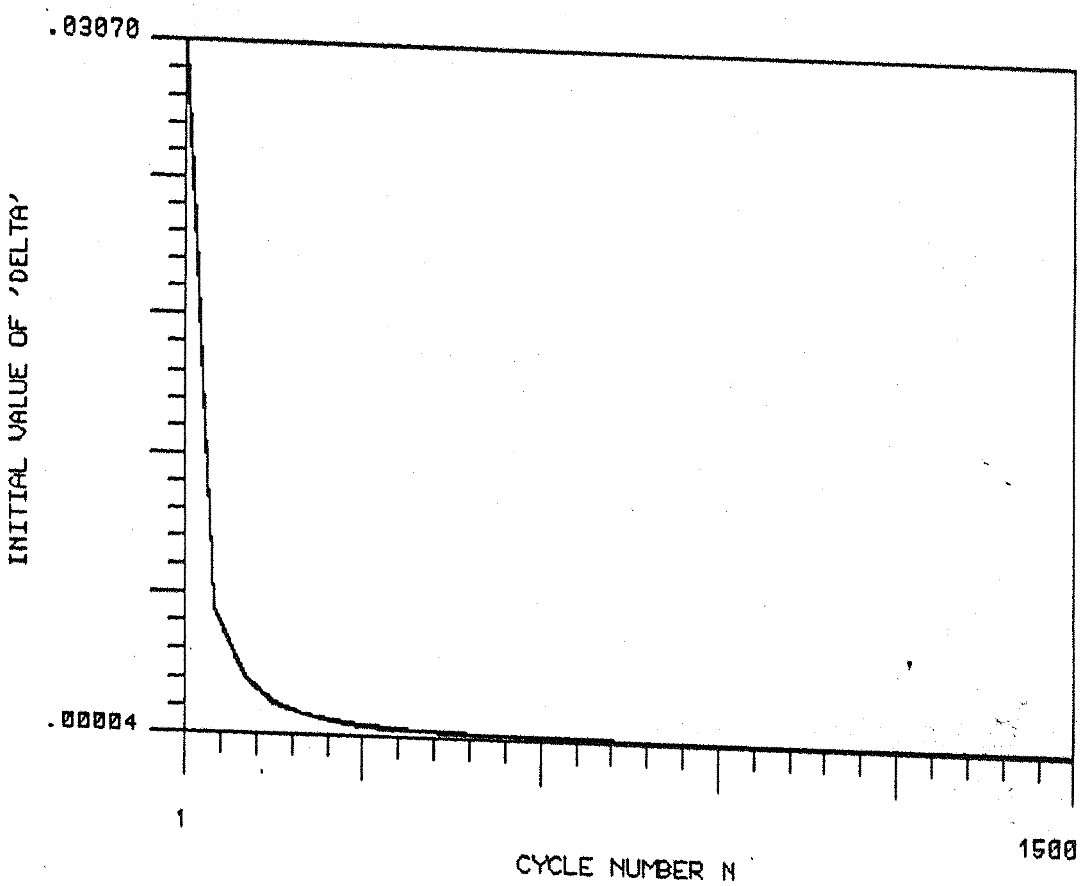


GRAPH G4.4

GRAPH G4.5

The variation of the difference between the dimensionless density parameter and unity, defined as DELTA, of an interacting, oscillating cosmological model, subject to numerically convenient initial conditions, with cycle number, N, for the first 1500 cycles.

Data : Table T4.2 Results column 3



GRAPH G4.5

This observation can lead into a natural non-inflationary solution of the flatness problem (see S1.2 & [4]). Summarized, the problem to consider is, why the initial value of Ω was so very close to unity if it was not equal to it

This work indicates that a possible answer to this question is that, in an irreversible , closed oscillating universe, the density parameter goes to unity with increasing cycle number. Ω was close to unity at the start of our present cycle because the universe had previously been through many cycles.

Having shown that the many cycled, closed, irreversible, cosmological model with numerically convenient initial conditions can provide two desirable results, if our universe is preceded by many previous cycles, it is necessary to consider one unfortunate prediction of the model.

Study of graph G4.6 shows that, at the point of maximum extension ($\dot{r} = 0$), in the first cycle, the energy content of the model, although not tending to be dominated by radiation, is by no means as strongly dominated by matter as is the universe now [52]. This is an unfortunate prediction in view of two facts,

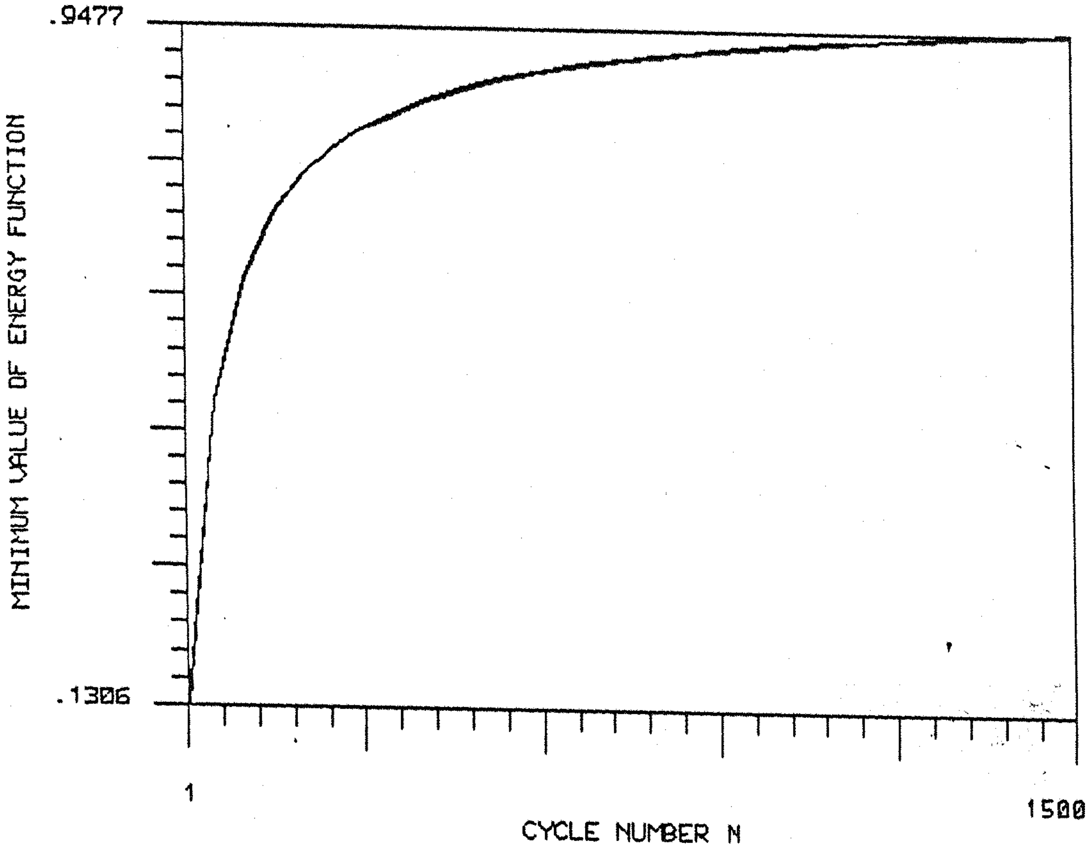
i) At the point of maximum extension in a closed universe, the value of the radiation energy has reached its lowest value.

ii) At our present epoch of the universe (about 2/3 of the way to the point of maximum extension, if it is closed), the matter energy is believed to dominate the radiation [52,4].

GRAPH G4.6

The variation of the minimum value of the energy function, defined as the ratio of radiation energy to total energy, of an interacting, oscillating cosmological model, subject to numerically convenient initial conditions, with cycle number, N, for the first 1500 cycles.

Data : Table T4.2 Results column 4



GRAPH G4.6

Thus our many cycled model indicates the non-physical fact that, throughout the cycles, a strongly matter-dominated stage never occurs. As we wish our present cycle to be a 'later' one to explain the flatness of the universe in the present cycle, we see that the model predicts a universe that is eventually radiation dominated.

It is believed that this non-physical prediction is due to the total neglect of the protons in the model. The inclusion of even a cold, static proton contribution would have the following advantages.

i) The model would expand much further due to the addition of a term $\simeq 1/\alpha \simeq 1836$ in the energy part of the equation of motion. This modification would allow the radiation to cool much more.

ii) The matter energy, when considered in the energy ratio, would have a term $\simeq 1836$ added to it.

The results of improving the model by the inclusion of a protonic contribution may be found in full in Chapter six

S4.2 Accuracy and stability of results

In the preceding section much use has been made of the results of the numerical solution of a set of differential equations. It has been possible to draw some interesting conclusions from the results. However, before the results are accepted we must satisfy ourselves that they are stable and of sufficient accuracy.

When we speak of stability in this context, we wish to know if a small alteration in the input data to the numerical solution routine (initial conditions and constants of motion) gives only a small change in the results of the computation. If the results do not show this behaviour, i.e. we see a decrease in a phase's energy over a cycle if r_o =.25, then it may be suspected that the results are unstable. This is often indicative of a significant inaccuracy in the results.

In order to investigate this question the program used to calculate the results presented in this Chapter was run with a distribution of initial conditions and constants centred on the values used for S4.1. In all cases only a small change in results appeared. The salient features, trends and results of the model were maintained. It may thus be concluded that the results of the calculations are numerically stable.

We next address the important question of the accuracy of the numerical results presented above.

In the routines used for the solution of these equations the input error parameter is called the tolerance. This is the maximum error allowed in the numerical solution at any one time step. However, as our results are based on calculations over a whole cycle (and many cycles) we need to know the total error over the whole calculation. This quantity is known as the Global error of the calculation. It represents the overall result of the tolerance errors at each time step and is, of course, dependent on the tolerance chosen as input data.

For the simple routines used to calculate the results quoted in this Chapter a special sub-routine exists that will compute an upper bound on the global error over a whole calculation.

A program incorporating this subroutine was run, using different tolerances and the initial conditions and constants of motion in S3.1. The results are presented in Table T4.3. It may be seen that as a tolerance of 10^{-10} was used for the calculation of the above results, we are justified in quoting our results to four decimal places. The typical error in a result of S4.1 was $\leq 10^{-7}$.

Thus we conclude that the results presented in S4.1 are both numerically stable and of a consistent accuracy. We therefore have strong faith in these results being a good approximation to the solution of the set of differential equations developed in Chapter two, when subject to the initial conditions and constants of motion of S3.1

Table T4.3

The variation of the maximum global error in the solution of an interacting, oscillating cosmological model, subject to numerically convenient initial conditions, with the maximum local error (tolerance).

Tolerance	Component with Max. error	Max. error	Max. error / Tolerance
1 Expanding Phase			
10^{-4}	r	1.6×10^{-5}	.16
10^{-5}	r	4.4×10^{-6}	.44
10^{-6}	r	8.4×10^{-7}	.84
10^{-7}	r	1.1×10^{-7}	1.10
10^{-8}	r	1.6×10^{-8}	1.60
10^{-9}	r	1.7×10^{-9}	1.70
10^{-10}	r	1.8×10^{-10}	1.80
2 Contracting Phase			
10^{-4}	y	1.7×10^{-2}	170.0
10^{-5}	y	3.6×10^{-3}	360.0
10^{-6}	y	5.4×10^{-4}	540.0
10^{-7}	y	6.5×10^{-5}	650.0
10^{-8}	y	5.8×10^{-6}	580.0
10^{-9}	y	4.5×10^{-7}	450.0
10^{-10}	y	3.5×10^{-8}	350.0

Table T4.3

CHAPTER FIVE

Results From Initial Conditions Based On Astrophysical Data

S5.1 Problems associated with these initial conditions

Here we present the rather disappointing results which may be gleaned from the set of differential equations developed in Chapter two, when they are subjected to the initial conditions and constants of motion as assigned in S3.2.

The resulting system is totally inappropriate for solution via the simple numerical library routines used in Chapters four and six.

The imposition of these initial conditions and constants results in the system becoming 'stiff'. A system which is stiff contains rapidly decaying, transient terms. The Jacobian of such a system has some eigen values which are large and negative compared to others [58].

In our case the term causing the problem is the factor in the interaction,

$$y/r^{15/4}$$

With the various sets of initial conditions this term has a value:

Initial conditions of S3.1 271.2788
Initial conditions of S3.2 1.6102×10^{26}
Initial conditions of S3.3 5430.5801

The large numerical differences between the value of this term in the different sets of initial conditions is obvious. Physically we may understand this problem in the following manner. Although the interaction has only a small effect because of the near equality of the phase temperatures, it does represent a very large flow of energy between each phase. However this flow is very well balanced.

When numerical solution is attempted, a small time step size is chosen, over which the equations are assumed to be linear to a good approximation. Unless this step size is impractically small the energy flow from the hotter to the cooler phase is massive, resulting in a large rise in the second phase's temperature. In the next time step a large temperature difference is thus in existence, so the interaction is much larger. This process continues and instability in the numerical solution sets in.

The system, although physically well behaved, is numerically unstable if simple routines as used for the convenient initial conditions are used in the solution. In order to solve such a system a special routine, suited to the problem, is needed. This type of routine was discovered by Gears [59].

S5.2 Essential transformation of the basic equations

The equations given at the end of Chapter two,

$$\dot{r}^2/2 - C = (x + y + z)/r \qquad (5.0)$$

$$\dot{y} = I_r - y\dot{r}/r \qquad (5.1)$$

$$\dot{x} = I_e - H(x)\dot{r}/r \qquad (5.2)$$

$$\dot{z} = I_p - \alpha H(z)\dot{r}/r \qquad (5.3)$$

$$H(p) = p\psi(p) = p-1.5/p+1/p^2-.5/p^3 \qquad (5.4)$$

$$I_e = y/r^{15/4}(y^4 - Qr^{3/4}H(x)) \qquad (5.5)$$

$$I_p = \alpha^3 y/r^{15/4}(y^4 - Qr^{3/4}H(\alpha z)/\alpha) \qquad (5.6)$$

$$I_r = - (I_e + I_p) \qquad (5.7)$$

together with the initial conditions and constants of motion developed in S3.2,

$x_o = 1.3$
$y_o = 7.77 \times 10^7$
$z_o = 1.83625 \times 10^3$

$Q = 8.652 \times 10^5$
$C = -.132$
$\alpha = 1/1836$

are in their present form too stiff to be computed by even the most sophisticated Gear's routine available [59].

In order to tackle this problem changes of variable are made . These will be guided by the known solutions of the above set of differential equations when the interaction is neglected. We now proceed to develop these zero interaction solutions. For simplicity's sake the protons are considered to be cold and static ($z = 1/\alpha$, $\dot{z} = 0.0$). With the interaction set equal to zero the two important equations are,

$$\frac{dy}{dr} = \frac{-y\dot{r}}{r} \qquad (5.8)$$

$$\frac{dx}{dr} = \frac{-H(x)\dot{r}}{r} \qquad (5.9)$$

(5.8) immediately integrates to,

$$y(r) = b/r \qquad (5.10)$$

where b is a constant

however it is not possible to perform the integral required to solve (5.9) exactly. Guided by numerical solution of the integral [60] and the classical limit we use the approximation to H(x).

$$H(x) = 2(x-1) \qquad (5.11)$$

We are thus restricting the matter to be represented as an ideal classical gas rather than a quantum one.

With this approximation to H (5.9) can be integrated easily to,

$$x(r) = 1 + a/r^2 \qquad (5.12)$$

a, another constant.

We will now include the interaction in the equations and consider the constants a & b to be arbitrary functions of r which we expect to be slowly varying if the zero interaction solutions are a good approximation to the real case.

i.e.

a becomes a(r)

b becomes b(r)

then from (5.11) & (5.12) ,

$$a(r) = (x-1)r^2 \qquad (5.13)$$

$$b(r) = yr \qquad (5.14)$$

differentiating (5.13) & (5.14) w.r.t. time gives,

$$\dot{a} = \dot{x}r^2 + 2(x-1)\dot{r}r \qquad (5.15)$$

$$\dot{b} = \dot{y}r + \dot{r}y \qquad (5.16)$$

using (5.1) & (5.2) for x & y in (5.15) & (5.16) and defining,

$$I \equiv I_{\infty} \tag{5.17}$$

gives,

$$a = [\ I - 2(x-1)\dot{r}/r\ ]r^2 + 2(x-1)r\dot{r} \ = \ Ir^2 \tag{5.18}$$

$$b = \ [-I - y\dot{r}/r\ ]r + y\dot{r} \ = -Ir \tag{5.19}$$

thus to solve this set we need an expression for I as a function of a, b, r.

Using (5.17) & (5.5) gives,

$$I = (y/r^3)^{5/4} - QyH(x)/r^3 \tag{5.20}$$

Using (5.12) - (5.14) in (5.20) gives,

$$I = (b/r^4)^{5/4} - 2Qba/r^6 \tag{5.21}$$

then,

$$I = b^{5/4}/r^5 - 2Qba/r^6 \tag{5.22}$$

This expression involving the difference between two very large, but nearly equal quantities, is very hard to evaluate numerically, the variables are therefore changed again:

$$b \equiv b_o(\ 1 + p\ ) \tag{5.23}$$

$$a \equiv a_o(1+q) \tag{5.24}$$

$$r \equiv r_o(1+s) \tag{5.25}$$

then automatically,

$$p_o = q_o = s_o = 0,$$

thus (5.23) - (5.25) (5.18) (5.19) & (5.0) yields,

$$\dot{p} = \dot{b}/b_o = -Ir_o(1+s)/b_o \tag{5.26}$$

$$\dot{q} = \dot{a}/a_o = Ir_o^2(1+s)^2/a_o \tag{5.27}$$

$$\dot{s}^2=(\dot{r}/r_o)^2\left[C+\frac{z_o+1}{r_o(1+s)}+\frac{a_o(1+q)}{r_o^3(1+s)^3}+\frac{b_o(1+q)}{r_o^2(1+s)^2}\right]r_o^{-2} \tag{5.28}$$

Thus we need an expression for I in terms of p,q,s. This will now be calculated.

(5.23) - (5.25) into (5.22) gives,

$$I = \frac{bo^{5/4}(1+p)^{5/4}}{r_o^5(1+s)^5} - \frac{2Qb_oa_o(1+p)(1+q)}{r_o^6(1+s)^6}$$

so rearranging ,

$$I = \frac{b_o(1+p)}{r_o^5(1+s)^5}\left[(1+p)^{1/4}b_o^{1/4}-2Qa_o(1+q)/\{r_o(1+s)\}\right] \tag{5.29}$$

the initial requirment of thermal equilibrium imposed (56)

$$(y_o/r_o^3)^{1/4} = QH(x_o) \tag{5.30}$$

gives, using (5.23) - (5.25),

$$b_o^{1/4} = 2Qa_o/r_o \qquad (5.31)$$

with (5.31) (5.29) reads,

$$I = \frac{b_o^{5/4}(1+p)[\ (1+p)^{1/4} - (1+q)/(1+q)\ ]}{r_o^5(1+s)^5} \qquad (5.32)$$

defining the function f by,

$$f(p,q,s) = (1+p)^{1/4} - (1+q)/(1+s) \qquad (5.33)$$

gives a set of differential equations transformed so that they are in the form best suited to numerical solution,

$$\dot{p} = -\alpha'(1+p)f/(1+s)^4 \qquad (5.34)$$

$$\dot{q} = \beta(1+p)f/(1+s)^3 \qquad (5.35)$$

$$\dot{s}^2 = r_o^{-2}\{\ C+\gamma/(1+s)+(1+p)\delta/(1+s)^2 + (1+q)\epsilon/(1+s)^3\ \} \qquad (5.36)$$

where the new constants are given by,

$$\alpha' = b_o^{1/4}/r_o^4 \qquad (5.37)$$

$$\beta = b_o^{5/4}/r_o^3 \qquad (5.38)$$

$$\gamma = (z_o + 1)/r_o \qquad (5.39)$$

$$\delta = b_o/r_o^2 \qquad (5.40)$$

$$\epsilon = a_o/r_o^3 \qquad (5.41)$$

If s, rather than t is made the independent variable, the system is soluble via a Gear's method designed for stiff systems of equations.

The results of numerical solution of the above equations represent an interacting cosmological model as developed in Chapter five. The only simplifications are that:

i) The protons are taken as static and cold

ii) The electrons are represented by an ideal massive classical gas rather than a quantum one.

The results are presented in the following section.

S5.3 The results, their stability and accuracy

In this section we will present the results available from numerical solution of the simplified, rescaled and transformed set of differential equations developed in the preceding section.

When considering the irreversibility produced by the interaction in the real universe, we do best to consider the fractional change, over a cycle or cycles, in the initial energies of each phase.

i.e. the critical results to consider are,

$$\Delta x_o/x_o \quad \text{and} \quad \Delta y_o/y_o$$

Unfortunatly it is not possible to give actual numerical values to these quantities and upper limits can only be assigned tentatively to them. The explanation for this is contained in the accuracy and stability of the results that have been obtained. These questions will now be considered.

Before the result of the numerical solution of a differential equation is accepted it must be shown that it is stable and of sufficient accuracy. [These words are discussed and their meanings referenced in S4.3].

When the numerical stability of the results is considered via perturbations in the initial conditions, it is found that, for a sensible physical spread of initial conditions and constants, both matter and radiation irreversibilities stay small. However both their sign and magnitude vary considerably for small changes in the input data.

This sort of behaviour is highly indicative of numerical instability in the results. We are thus led to suspect that the results may well be simply the manifestation of the global error of the calculation. [Global error is also discussed and referenced in S4.3]. It is very unfortunate that no routine exists in the library to calculate the global error for a Gear's routine. Thus it is necessary to use less direct and less satisfactory methods to compare the results with some measure of the global error.

In order to ascertain whether our results are merely systemisation of the total error in the calculation, we consider how the results change for a range of tolerances. [This represents the maximum local error, again see S4.3].

Table T5.1

Matter and radiation irreversibilities for various maximum local error (tolerance) values in the solution of an interacting, oscillating cosmological model subject to physical initial conditions

Tolerance	$(\Delta x_o/x_o)$/Tol.	$(\Delta y_o/y_o)$/Tol.
1.0×10^{-8}	875.3	1789.0
5.0×10^{-8}	162.8	329.4
1.0×10^{-7}	94.2	189.5
5.0×10^{-7}	558.4	2174.0
1.0×10^{-6}	7.6	15.3
5.0×10^{-6}	2.2	4.3
1.0×10^{-5}	.8	1.3
5.0×10^{-5}	.1	-.1
1.0×10^{-4}	2.4	13.4
5.0×10^{-4}	.2	.5
1.0×10^{-3}	-.1	-.4

Table T5.1

The results for a range of possible tolerances using the physical initial conditions are in Table T5.1.

Consideration of this data leads us to conclude that,

i) The result is quite strongly tolerance-dependent, even for the smaller, more accurate values. Even the sign of the result is dependent on the tolerance.

ii) The result is never more than 2,000 times the tolerance chosen.

The first observation is a strong indication that the results we are seeing are simply the global error of the solution routine. This we would expect to be tolerance (local error) dependent.

The second observation confirms our conclusion drawn from the first. We have seen in S4.3 and will see in S6.2 that, for much more numerically convenient numbers, which can be solved using far simpler routines, the global error can be 600 to 130,000 times the the smallest tolerance that can be used with the stiff routines (10^{-8}). The physical initial conditions seem more similar to the intermediate set than to the numerically convenient set. We would expect the higher figure to be nearest to the mark for these initial conditions. However Table T5.1 shows that the phase irreversibilities are never more than 2200 times the tolerance (local error). We would thus assert strongly that the results are the global error of the routine.

From the preceding we can only conclude that,

i) The irreversibility in a model universe when subject to physically based initial conditions, is very probably less than the global error associated with a Gear's routine with the smallest local error (10^{-8}).

ii) This global error is most likely to be in excess of 1.0×10^{-3}.

CHAPTER SIX

Results from Intermediate Initial Conditions

In this Chapter we present the results of numerical solution of the set of differential equations developed in Chapter two when subject to the initial conditions and constants assigned in S3.3. These results, in general, will be seen to contain the pleasing features of the results in Chapter four without the non-physical problems encountered in Chapter 5.

S6.1 Results

The differential equations, when subjected to the initial conditions and constants of S3.3, are still soluble by the simple numerical routines used in Chapter four, rather than by the more complicated, stiff, routines necessary for the conditions assigned in S3.2.

These routines were used on a powerful main-frame computer to calculate 1,500 cycles of an oscillating universe. The values of important cosmological results were calculated at two important and well defined points in the model's history.

i) The initial point, from where the model is started for the first cycle, and after the cycle truncation has occurred for subsequent cycles.

ii) The point of maximum extension, when the rate of expansion is zero

($\dot{r}$ = 0.0). Here the model has just finished expanding and is about to start contracting.

At the initial point the energy of each phase, and the density parameter were studied as functions of the cycle number, N. The results of this are in table T6.1 and are illustrated in graphs G6.1, G6.2, and G6.3.

It is important when studying all of the graphical results in this chapter to note that none of the scaling factors between the scaled variables used here, and actual astrophysical observables is given. The reason for this is, as the current set of initial conditions is not derived from astrophysically based data the results are for a universe with different physical constants, and number of particles than our own, so that presentation of these scaling factors would be at best unhelpful and at worst misleading

Graphs G6.1, of the initial electron energy, and G6.2, of the initial radiation energy, are both monotonically increasing functions of N, but are of different concavity. G6.1 is concave downwards whereas G6.2 is convex upwards. This shows that although both the initial electron and radiation energies increase with cycle number, the radiation energy increases faster Eventually the radiation energy will totally dominate the electron energy at the initial point.

Thus, if we may identify our initial point with some early point in the real universe we may explain why the early universe was radiation dominated. Our model predicts that this was the case because irreversible effects in an oscillating universe .

Table T6.1

The variation of cosmological parameters in an interacting, oscillating cosmological model, subject to intermediate initial conditions, with cycle number, N, for the first 1500 cycles.

Cycle Number N	Initial Matter Energy x_o	Initial Radiation Energy y_o	Initial Density Parameter $(\Omega_o-1)/10^{-4}$
1	2.000	30.000	5.352
50	2.132	45.066	5.309
100	2.252	62.663	5.259
150	2.350	81.947	5.206
200	2.445	103.251	5.148
250	2.530	126.058	5.089
300	2.611	150.471	5.0262
350	2.688	176.329	4.965
400	2.763	203.990	4.894
450	2.829	232.318	4.827
500	2.896	262.237	4.758
550	2.9573	293.074	4.688
600	3.019	325.394	4.619
650	3.075	358.423	4.550
700	3.132	392.829	4.479
750	3.183	427.737	4.4103
800	3.235	463.916	4.341
850	3.287	501.370	4.271

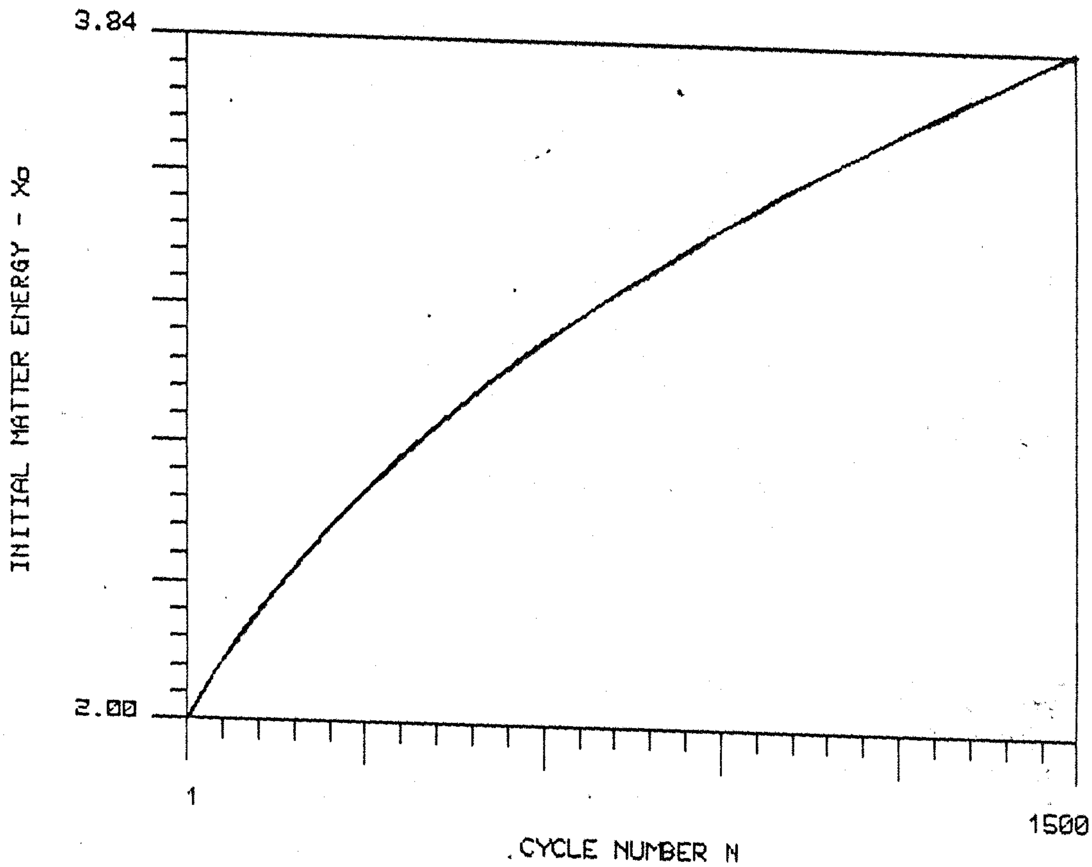
900	3.338	540.098	4.200
950	3.385	579.014	4.132
1000	3.432	619.101	4.065
1050	3.479	660.363	3.998
1100	3.521	701.549	3.933
1150	3.563	743.805	3.869
1200	3.605	787.132	3.805
1250	3.647	831.534	3.742
1300	3.689	877.015	3.680
1350	3.727	922.045	3.619
1400	3.764	968.044	3.560
1450	3.807	1016.663	3.499
1500	3.840	1063.216	3.446

Table T6.1

GRAPH G6.1

The variation of the initial scaled matter energy x_0, of an interacting, oscillating cosmological model, subject to intermediate initial conditions, with cycle number, N, for the first 1500 cycles.

Data : Table T6.1 Results column 1

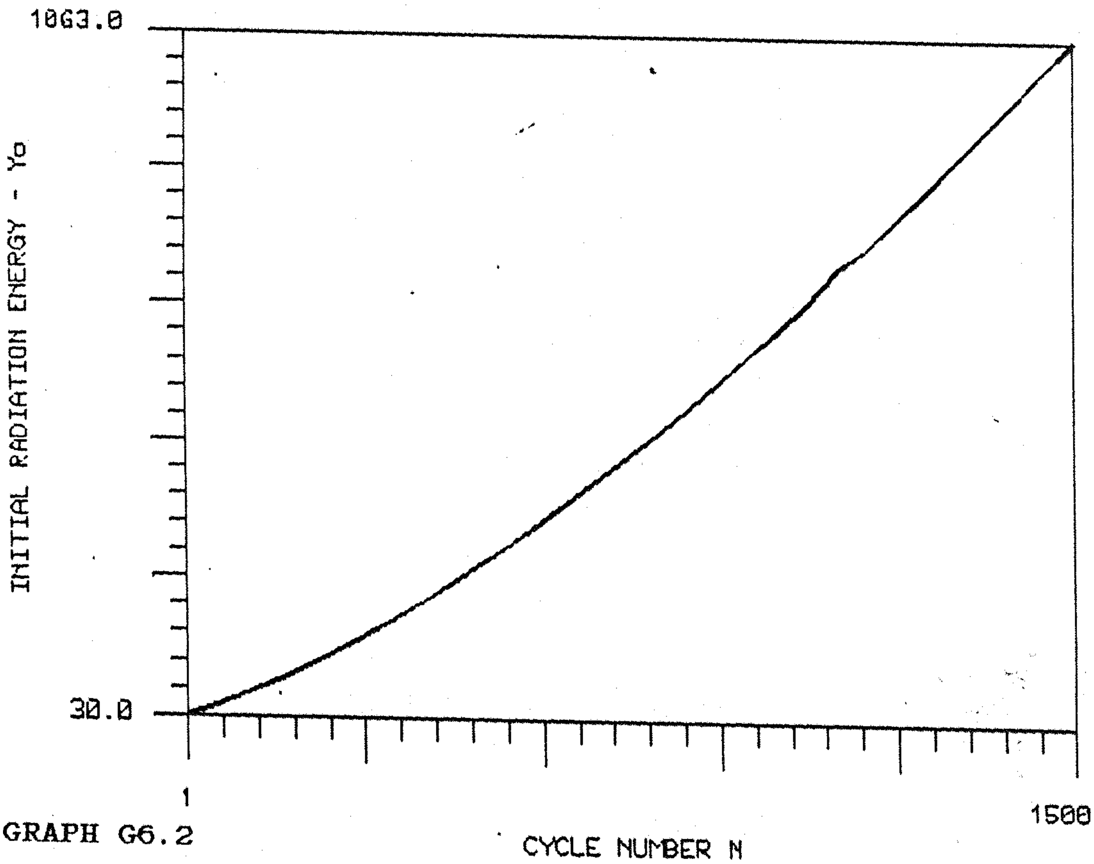


GRAPH G6.1

GRAPH G6.2

The variation of the initial scaled radiation energy y_o, of an interacting, oscillating cosmological model, subject to intermediate initial conditions, with cycle number, N, for the first 1500 cycles.

Data : Table T6.1 Results column 2

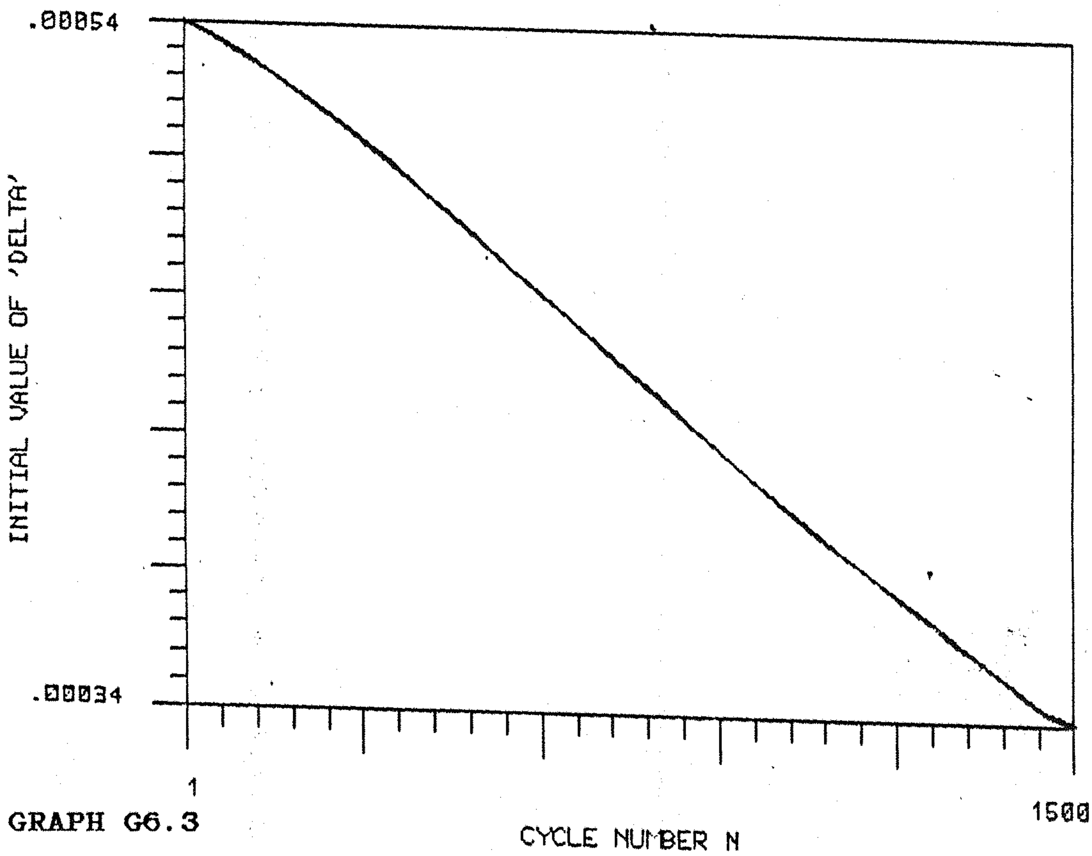


GRAPH G6.2

GRAPH G6.3

The variation of the difference between the dimensionless density parameter and unity, defined as DELTA, of an interacting, oscillating cosmological model, subject to intermediate initial conditions, with cycle number, N, for the first 1500 cycles.

Data : Table T6.1 Results column 3



GRAPH G6.3

increase the radiation energy faster than they do the matter energy. We would then claim that our present universe, with its initially high phase energy ratio, may well have developed from a previous state with a much more equal phase energy ratio.

Graph G6.3 of the initial density perameter shows that Ω_o decreases with the cycle number N in an approximately linear fashion. We thus arrive at a non-inflationary solution to the flatness problem [4] The universe is so nearly flat, our model indicates, because it has been through many irreversible cycles, and non-equilibrium effects during these cycles bring the initial value of the density parameter towards unity from above.

Thus, the model predicts that our present cycle has been preceded by many earlier ones that were less flat. In addition, they contained less energy at their start. Their initial phase energy ratio was much nearer to unity than was our cycle's. Our model is thus able to explain two observed facts of the early universe in our present cycle; the initial flatness, and the initial radiation dominance, by postulating many previous, irreversible cycles.

At the point of maximum extension we consider the energy fraction - that fraction of the total energy that is contained in the radiation phase. We also consider the radiation temperature. These results, as functions of cycle number, N are in Table T6.2 and are illustrated in Graphs G6.4 & G6.5.

Graph G6.4 shows that, although the energy fraction increases with cycle number, it is small compared with one for all the cycles studied. This is a significant

Table T6.2

The variation of critical cosmological parameters at the point of maximum extension, in an interacting, oscillating cosmological model, subject to intermediate initial conditions, with cycle number, N, for the first 1500 cycles.

Cycle Number	Radiation Energy Ratio	Radiation temperture Model Units
N	$y/(x+y+z)/10^{-7}$	$3NkQT_r/10^{-4}$
1	4.380	1.795
50	6.584	1.988
100	9.157	2.159
150	12.024	2.311
200	15.154	2.448
250	18.530	2.575
300	22.134	2.692
350	25.953	2.801
400	29.974	2.904
450	34.189	3.000
500	38.587	3.093
550	43.162	3.183
600	47.905	3.265
650	52.813	3.345
700	57.876	3.423
750	63.093	3.497
800	68.456	3.570

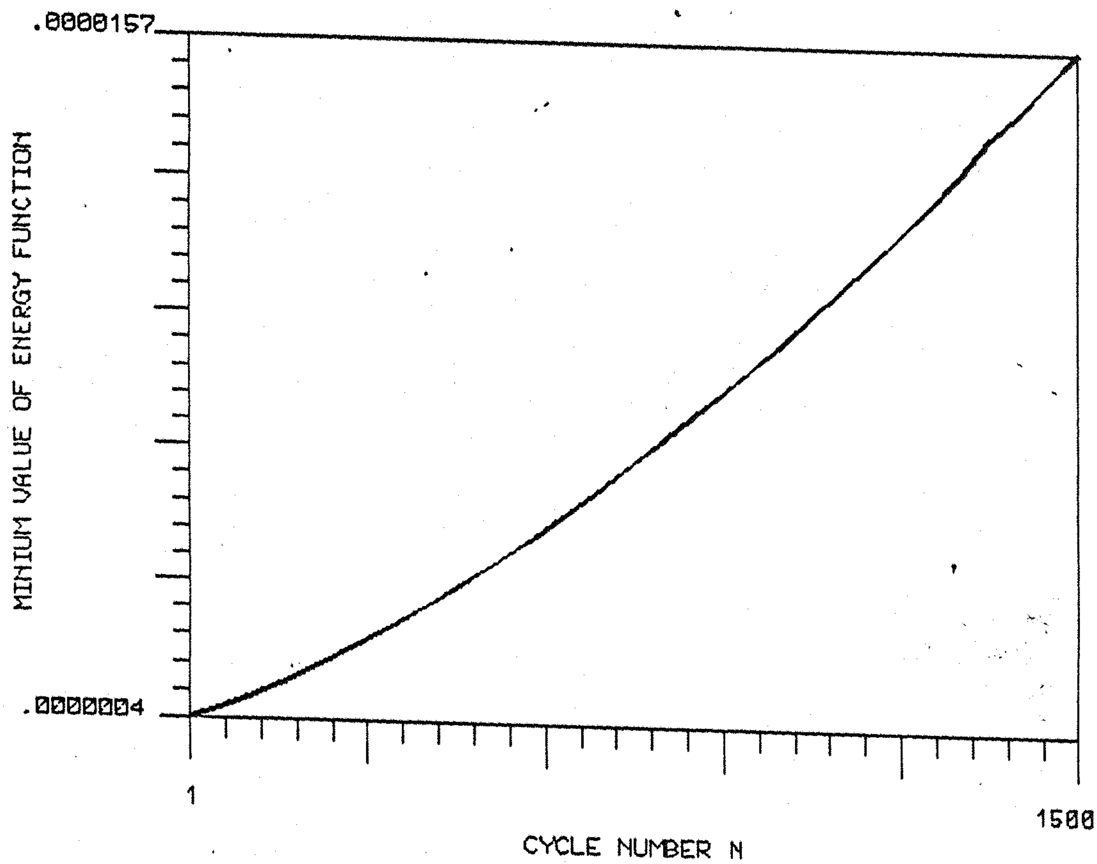
850	73.960	3.639
900	79.610	3.722
950	85.392	3.772
1000	91.310	3.836
1050	97.349	3.898
1100	103.519	3.958
1150	109.812	4.017
1200	116.223	4.075
1250	122.752	4.131
1300	129.403	4.186
1350	137.509	4.239
1400	143.036	4.292
1450	150.002	4.331
1500	156.678	4.390

Table T6.2

GRAPH G6.4

The variation of the minimum value of the energy function, defined as the ratio of radiation energy to total energy, of an interacting, oscillating cosmological model, subject to intermediate initial conditions, with cycle number, N, for the first 1500 cycles.

Data : Table T6.2 Results column 1

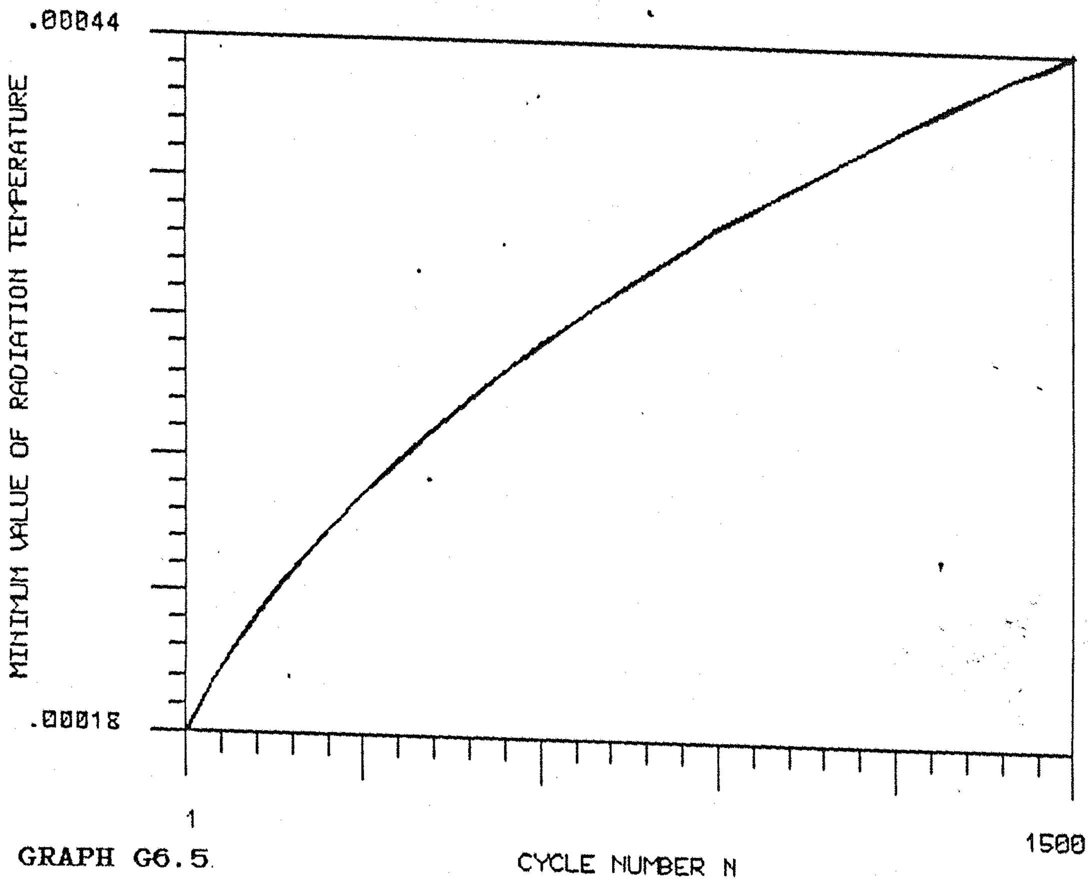


GRAPH G6.4

GRAPH G6.5

The variation of the minimum value of the scaled radiation temperature of an interacting, oscillating cosmological model, subject to intermediate initial conditions, with cycle number, N, for the first 1500 cycles.

Data : Table T6.2 Results column 2



GRAPH G6.5

improvement when compared with the same variable considered in Chapter four. This is caused by the inclusion of the protons, in two distinct ways,

i) A proton contribution increases the total energy without increasing the radiation energy. This automatically increases the denominator of the energy fraction.

ii) A proton contribution to the equation of motion causes the model to expand much further. As the radiation energy is approximatly inversely proportional to the radius of the model, this causes a decrease in the numerator of the energy fraction.

We thus recover an observed fact of the universe - that at present the energy content is dominated by the matter phase. This indicates that this will continue to be true until the point of maximum extension. Our model shows this feature too, and indicates that this will be the case for many cycles, but not for ever. This dominance will become weaker and weaker and eventually be overturned. Eventually the universe will become radiation dominated throughout the whole cycle.

Graph G6.5 shows that the radiation temperature at the point of maximum extension is monotonically increasing and is concave downwards. The model thus predicts that as the cycles progress the minimum radiation temperature achieved in each cycle will grow larger and larger. Thus, eventually, life as we know it will become impossible, even when the universe is at its coolest. Galaxies will be unable to form. This prediction is at odds with the final anthropic principle [61].

The model under consideration thus shows one fact of the observed universe, a matter dominated phase. In addition it predicts that in cycles to come, this phase will eventually cease to exist and that galaxies and life as we know it will no longer be possible.

S6.2 Accuracy and stability of the results

The results presented above were found to be numerically stable to perturbations in the initial conditions in exactly the same way as was described in some detail in S4.2. This indicates strongly that we may believe these to be actual results rather than a manifestation of the accumulation of numerical errors.

As the initial conditions used allowed the use of simple non-stiff numerical routines, a global error calculation was possible. This was performed for the set of initial conditions and constants under consideration. The results of the global error calcualtion for a range of tolerances are in Table T6.3. This table shows that the global error was a much larger multiple of the tolerance (local error) than was the case in S4.2. Even so with the tolerance used for the calculation (10^{-10}) the maximum global error was $\simeq 10^{-5}$. We are thus justified in quoting our results to four places of decimals.

We thus conclude that the results presented in this Chapter are numerically stable, and of sufficent accuracy for us to draw our conclusions safely. Thus we assert strongly that they represent accurately the solution of the set of differential equations .

Table T6.3

The variation of the maximum global error in the solution of an interacting, oscillating cosmological model, subject to intermediate initial conditions, with the maximum local error (tolerance).

Tolerance	Component with Max. error	Max. error	Max. error / Tolerance
1 Expanding Phase			
10^{-4}	r	-.94	-9400.0
10^{-5}	r	-2.8×10^{-2}	-2800.0
10^{-6}	r	-3.1×10^{-3}	-3100.0
10^{-7}	r	-1.8×10^{-4}	-1800.0
10^{-8}	r	1.5×10^{-5}	1500.0
10^{-9}	r	2.1×10^{-6}	2100.0
10^{-10}	r	2.0×10^{-7}	2000.0
2 Contracting phase			
10^{-4}	r	23.5	235000
10^{-5}	r	4.4	440000
10^{-6}	r	.3	300000
10^{-7}	r	2.0×10^{-2}	200000
10^{-8}	r	1.3×10^{-3}	130000
10^{-9}	r	-9.8×10^{-5}	98000
10^{-10}	r	-7.6×10^{-6}	76000

Table T6.3

developed in Chapter two, when subject to the initial conditions and constants developed in S3.3

S6.3 Summary, conclusions and discussion of results

A cosmological model has been developed, based generally on the hot big bang theory, but which includes the dissipative interaction between the matter and radiation phases in the post nucleosynthesis and pre-decoupling era. This model, although simple, is less limited , and a more accurate description of the real universe than the standard big bang theory, in that it contains all of the attractive features of that model, and the thermal interaction. This model is represented by a set of initial value differential equations, which define the time development of the model from a set of initial conditions and subject to a set of constants of motion.

Unlike earlier works in a similar vein [29,30,32,33], detailed discussion has been given regarding the assignment of numerical values to the initial conditions and constants of motion. It has been found that when subject to initial conditions and constants of motion derived from physically observed astrophysical data the set of differential equations becomes exceptionally difficult to solve numerically. Solution shows that any irreversibility produced by the interaction in a closed cycle of the universe and, hence, any cumulative effects caused by the irreversibility in a polycycled model, is almost certainly less than the global error associated with the numerical solution of the set of equations. Put simply , the irreversible effects induced by the interaction are lost in the 'noise' of the solution routine. However the solution of the system of differential equations under less severe initial conditions shows trends that one would strongly expect to see reproduced in a model subject to such physically based initial conditions. If a significant improvement in the techniques for the numerical solution of such stiff equations should occur the re-anaylsis of a closed polycycled universe containing an interaction between the matter and radiation should be one of the first priorities.

However the system of equations is solvable when subject to a set on initial conditions, which are assigned in a way guided by astrophysical trends. Under such conditions, in a polycycled, 'bouncing' universe, if the irreversible effects introduced by the interaction are allowed to build up over many cycles interesting trends appear in the various cosmological parameters. The main points of interest are;

1 The energy of both matter and radiation at the initial point increase with cycle number.

2 The radiation energy increases at a much greater rate, and with a different concavity, than does the matter.

3 The initial value of the dimensionless density parameter is seen to decrease to unity from above as the cycles proceed.

4 The minimum values of the radiation energy and temperature, which occur at the point of maximum extension, are seen to increase with the cycle number.

If the above trends may be expected to be reproduced in the real universe, then by postulating that our present universe is one of a series of closed, oscillating universes in which a thermal interaction between the phases produces irreversible effects, then two observed features of our present universe may be explained, and one prediction may be made for further cycles.

Point 2) may explain why our universe had such a high ratio of radiation to matter energy early on, by postulating that in earlier cycles the phase energy ratio was much closer to unity but has been increased to its high value in the present cycle by the interaction acting over many cycles.

Point 3) provides a non-inflationary solution to the flatness problem by suggesting that our present universe has a density parameter so close to unity because the effect of the interaction is to lower this parameter to unity as the cycles progress.

Point 4) predicts that in future cycles the minimum radiation temperature will eventually become so high that the formation of galaxies and the existence of any life as we know it will become impossible.

Several authors [62], have studied entropy in a series of closed universes from the point of view of the entropy generated by stella and galactic sources in each cycle. By assuming that this was approximately the same in each cycle they show that a maximum of about 100 previous cycles is possible. As a useful solution of the interacting cosmological model subject to physical initial conditions was not possible it is impossible to say whether the entropy introduced by the interaction would have been dominant, insignificant, or comparable to the entropy generated by star light. For exactly the same reason it is impossible for this work to add any refinement to that figure. In any case the two entropies must be added, and if one assumes that the entropy produced by star light was constant per cycle one would set the upper limit at less than 100 cycles.

However the present work gives strong indication that the entropy due to starlight produced was less in previous cycles, this work indicates that previous cycles had shorter lives than subsequent cycles, hence the time for starlight to produce entropy was less. In even earlier cycles, of short enough duration, it was not possible for galaxies or stars to form, hence entropy production due to the phase interaction was the dominant factor. An accurate estimate of the number of previous cycles could only be made if solution of the differential equations when subject to physical initial conditions and constants of motion were possible.

Other authors [63], by considering the effects of hydrogen burning on the entropy of the universe, predict that the next cycle of the present universe would have a maximum extent twice that of the present cycle. If this prediction should be correct and applicable to several cycles before the present one then it would almost certainly dominate any irreversible effects introduced in those cycles by the phase interaction. However, in a universe that lasts too short a time for star birth to occur, the phase interaction would again become the dominant entropy producer. A reduction in size as large as halving each cycle, caused by stellar sources, would rapidly lead to a universe in which no star formation could occur.

In view of the above considerations, it appears likely that the irreversible effects produced by the phase interaction studied in this work may well have been responsible for the thermal and geometric structure of our present universe. It is also possible that they may spell the end of life in some future cycle.

APPENDIX A

LIST OF SYMBOLS AND MEANINGS OF SUBSCRIPTS

LIST OF SYMBOLS

Symbol	Meaning
k	Curvature constant
ρ_c	Critical density for closure
Ω	Dimensionless density parameter
ds	Increment of Space-Time
c	Velocity of light
$R(T)$	Scale factor
T	Cosmological time
r, θ, ϕ	Cosmological co-ordinates
$A(r)$	Curvature factor
T_{ab}	Energy-Momentum tensor
P	Pressure
g_{ab}	Metric tensor
ρ	Energy density
U_a	Fluid 4 velocity
G_{ab}	Einstein Tensor
G	Newtonian Gravitational Constant
V	Co-moving volume element
U	Energy in a co-moving volume element
E_i	Dissipative interaction for the i'th phase
U_i	Energy of the i'th phase in a co-moving volume element
T_i	Temperature of the i'th phase
ρ_i	Energy density of the i'th phase in a co-moving volume element

A	Interaction constant
σ	Thomson cross section for the electron
k	Boltzman constant
N	Number of electrons
m_{oe}	Rest mass of electron
B	Interaction constant
σ'	Thomson cross section of proton
N'	Number of protons
m_{op}	Rest mass of proton
α	Ratio of electron to proton rest mass
a	Blackbody constant
$\psi(x)$	Honl function of x
H(x)	Honl function of x times x $= x\Psi(x)$
R_o	Change of variable for scale factor
T_o	Change of variable for time
U_{oe}	Change of variable for energy
r	Scaled scale factor ('radius' of universe)
y	Scaled radiation energy
x	Scaled electron energy
z	Scaled proton energy
t	Scaled time
C	Scaled curvature constant
I_i	Scaled interaction for the i'th phase
P, Q, V, S	Scaled interaction constants
DELTA	Anomalous part of density parameter
E fun	That fraction of the total energy that is contained in the radiation
a, b	Constants of integration
p, q, s	Changes of variable for a, b, r
f	Function representing interaction in terms of p, q, s
α', β'	Collections of constants
γ, δ, ϵ	Collections of constants
Δx_o	Change in initial matter energy
Δy_o	Change in initial radiation energy

MEANINGS OF SUBSCRIPTS

Subscript	Meaning
r	Radiation
e	Electron
p	Proton
m	Matter = electrons + protons
o	Initial

APPENDIX B

SUMMARY OF CRITICAL EQUATIONS AND INITIAL CONDITIONS

SUMMARY OF CRITICAL EQUATIONS

The model universe developed is defined by the set of initial value first order differential equations

$$\dot{r}^2/2 - C = (x + y + z)/r$$

$$\dot{y} = I_r - y\dot{r}/r$$

$$\dot{x} = I_e - x\psi_e\dot{r}/r$$

$$\dot{z} = I_p - z\psi_p\dot{r}/r$$

where,

$$H(P) = P\psi = P - 3P/2 + 1/P^2 - 1/(2P_3)$$

$$I_e = y/r^{15/4}(y^{1/4} - Qr^{3/4}H(x))$$

$$I_p = \alpha^3 y/r^{15/4}(y^{1/4} - Qr^{3/4}H(\alpha z)/\alpha)$$

$$I_r = -(I_e + I_p)$$

SUMMARY OF INITIAL CONDITIONS AND CONSTANTS

The state of the model universe at any time t is specified by the solution to the above set of equations subject to a set of initial conditions,

at $t = t_0$, $r = r_0$, $x = x_0$, $y = y_0$, $z = z_0$

given the values of 3 constants of motion Q, C, α

In this work 3 sets of initial conditions and constants of motion are used, these are given below again as a reference.

INITIAL CONDITIONS I

Numerically convenient Initial conditions

$t_0 = 0$

$x_0 = 2$

$y_0 = 2$

$z_0 = 0 \quad (= z(t))$

$r_0 = .27$

and

$Q = 2.208$

$C = -.445$

$\alpha = 1/1836$

INITIAL CONDITIONS II

Physical Initial conditions

$t_o = 0$

$x_o = 1.3$

$y_o = 7.77 \times 10^7$

$z_o = 1836.25$

$r_o = 1.305 \times 10^{-5}$

and

$Q = 8.558 \times 10^5$

$C = -.132$

$\alpha = 1/1836$

INITIAL CONDITIONS III

Intermediate Initial conditions

$t_o = 0$

$x_o = 2$

$y_o = 30$

$z_o = 1836 \quad (= z(t))$

$r_o = .25$

and

$Q = 4.605$

$C = -.2$

$\alpha = 1/1836$

APPENDIX C

NUMERICAL SOLUTION OF INITIAL VALUE DIFFERENTIAL EQUATIONS

In this work much use has been made of the results of the solution of a set of initial value differential equations. In this Appendix we present a summary of the ideas and methods relevant to this branch of numerical anaylsis. Although this can by no means be a full treatise on this wide subject, it is hoped that it will provide more insight into the techniques than it was possible to include in the necessarily terse summaries in Chapters 4,5 and 6.

Consider the solution of the system of equations,

$$x(t)' = g(t,x)$$

where,

$$x' = \frac{dx}{dt}, \quad \text{and at } t=t_0,\ x=x_0$$

this system can also be extended to a system with more than one dependent variable, however, for the present discussion, it will prove sufficient to consider a system with only one dependent variable.

The problem is to solve for x(t), for times later than t_0. The first method used would be a simple difference method, where one would use a short time dt, over which the time variation of x would be linear to a good approximation, one could then write,

$$x(t_o+dt) = x_o + dt\,g(t_o, x_o)$$

By using the x values thus generated as new initial conditions and repeatedly applying the above equation the solution $x(t)$ can be built up at times $t=t_o+ndt$, where n is an integer. If desired dt can be changed as the solution proceeds if the time derivative $g(t,x)$ becomes more or less severe. It was using just such methods as above that the results stated in Chapter 4 were calculated from first principles.

The above method can be made more accurate if the basic approximation equation is refined to [64],

$$x_n = x_{n-1} + h(x_{n+1}'+x_n')/2$$

where from now on we use the short hand

$$h = dt$$
$$x_n = x(t_o+nh)$$
$$x' = g(t,x)$$

The above formulation, although sufficient for simple problems, is only accurate to first order in h. To enable certain types of equations to be solved, without prohibitively small h values being required, other methods have been derived. The most obvious method is to notice that the above equations are in essence, Taylor expansions to first order in h. To obtain approximations accurate to a greater order in h, say to order h^n, one simply expands the Taylor series to this desired order. However this method has the disadvantage that more input information is needed than just the initial point (t_o, x_o)

In view of these problems with directly expanding the Taylor series to greater order in h, other methods, namely Runge-Kutta techniques have been developed [65]. The idea of such routines was to produce a series expansion for x_n accurate to a desired order in h, without it being necessary to calculate derivatives of higher than first order. A typical equation that does this to fourth order in h is [66],

$$x_{n+1} = x_n + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

where,

$$k_1 = hg(t_n, x_n)$$
$$k_2 = hg(t_n+h/2,\ x_n+k_1/2)$$
$$k_3 = hg(t_n+h/2,\ x_n+k_2/2)$$
$$k_4 = hg(t_n+h,\ x_n+k_3)$$

It was just such a method as this that was used for the step by step solution of the set of equations representing the cosmological model to produce the majority of results as presented in Chapters 4 and 6.

In the numerical solution of any system of equations the degree of accuracy achieved is of vital importance. The error introduced by the numerical solution of a system of equations is two fold. At each step in the solution an error of order h^{n+1} is introduced if the solution is made to order n. This error causes the input data to the next step of the solution itself to be inaccurate by this much before any effect of the truncation of the series in h occurs at this step. The error at any step is termed local error in the solution. Because the local error at each step has a 'knock on' effect to the next step, the error in the final solution, termed global error may be very much more than the sum of the local errors at each solution step.

One of the great advantages of the R-K method is that it is posible to arrive at an upper bound on the global error introduced in the final solution due to the effects of cumulative local errors. It has been shown that [67],

$$|x_f - x_a| < \frac{6MN|x-x_o|^5|N5-1|}{|N-1|}$$

where,

x_f = The final solution from the R-K method

x_a = The accurate true solution to the equation

and,

$M < |g(t,x)|$

$N < 1/|x-x_o|$

in addition various other formulae are available to give, M,N in terms of various partial derivatives of g. It was such techniques for giving global error bounds that were used to calculate global errors in Chapters 4 and 6.

The numerical methods presented above only work well for certain systems of equations, however, when a system of equations becomes mathematically stiff, then the above methods will need a very small value of h to obtain sufficient accuracy.

There are several definitions for determining whether a system is stiff [68],

i) A system of equations is said to be stiff if it contains a rapidly decaying exponential term.

ii) A system of equations is stiff over a region [a,b], if there is a component of the solution which in that region varies largely compared to 1/(b-a).

iii) A system of equations which may be written as a matrix equation,

$$X' = AX$$

where, X' and X are column vectors and A is a martrix of coefficients, is stiff if the eigen values of A are large compared to each other.

By considering infinitesimal perturbations away from the initial point (I=0) of the cosmological model, subject to physical initial conditions, and identifying X as a column vector of r,x,y we find that the eigen values are indeed large compared to each other; the ratio of the largest to the smallest finite eigen value is approximatley 1.4×10^4. Thus the system of equations representing the model when subject to physical initial conditions is mathematically stiff.

The method of Gears [69] to solve such stiff equations is a more complicated and involved method than the R-K methods discussed above, as such it is beyond the scope of the present work, however a brief summary will be given to illustrate the main points of that method applicable to this study.

The essence of the solution of stiff differential equations by the Gears method [70] is that a predictor-corrector equation for the value of the dependent variable, x at the n'th time step is used. The predictor equation gives the dependent variable as a linear combination of itself and its derivatives at k earlier time steps. The coefficients in the expansion are determined in a similiar way to those involved in the R-K method. The corrector equation, which is also a linear combination of the same terms as the predictor equation, plus a term in the derivative at the current time step, allows iterative improvements to be made to the value of the dependent variable at the current time step. The algorithm for anaylsis is thus as follows; use the predictor equation to generate a first approximation to the value of the dependent variable, then use the corrector equation as a form of regression formula to obtain sucessively more accurate approximations to this value.

The Gears method thus provides a method of finding the solution of stiff systems of differential equations. However because of the greater complexities involved compared to the more simple R-K methods no expression has yet been found for the overall global error associated with the solution of a system of differential equations in this way. It is this unfortunate gap in the arsenal of numerical anaylsis techniques that resulted in many of the problems discussed in chapter 5, and lead ultimately to it being only possible to place tentative limits on the irreversibility generated during a cycle of a closed universe.

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