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Sphaleron freeze-in baryogenesis with gravitational waves from the QCD transition

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A large primordial lepton asymmetry is capable of explaining the baryon asymmetry of the Universe (BAU) through suppression of the electroweak sphaleron rates ("sphaleron freeze-in") which can lead to a first-order cosmic QCD transition with an observable gravitational wave (GW) signal. With next-to-leading order dimensional reduction and the exact 1-loop fluctuation determinant, we accurately compute the lepton asymmetry needed to realize this paradigm, finding it to be an order of magnitude smaller than previous estimates. Further, we apply an improved QCD equation of state capable of describing the phase transition line together with the critical endpoint leading to better agreement with lattice and functional QCD results. Based on this, we identify the range of lepton flavor asymmetries inducing a first-order cosmic QCD transition. We then extract the parameters relevant to the prediction of GW signal from a first-order cosmic QCD transition. Our result showcases the possibility of probing the sphaleron freeze-in paradigm as an explanation of BAU by future gravitational wave experiments like μAres.

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Introduction— We know little about the Universe at temperatures before Big Bang Nucleosynthesis (BBN). Based on our knowledge of particle physics, we however expect that the first second of the Universe was filled with many exciting events, like the generation of a baryon asymmetry and a transition from a quark-gluon plasma to a hadronic phase. The new era of gravitational wave (GW) cosmology provides a window towards these early stages because, if the QCD transition was strongly first-order, this would leave a relic stochastic GW background visible today. The recent findings of several pulsar timing array collaborations [1–9] have lead to an enormous increase of interest to this end. On the other hand, the origin of the observed baryon asymmetry of the Universe (BAU) is one of the biggest open puzzles in particle physics and cosmology. According to the standard leptogenesis picture, an asymmetry is created in the leptonic sector and converted to the baryonic sector by sphaleron processes [10]. This would imply the lepton asymmetry to be of the same order of magnitude as the baryon asymmetry (i.e. tiny) - for a review see [11–13]. It was however shown that in the case of *large primordial lepton asymmetries* the restoration of the electroweak symmetry gets prohibited, which can lead to the right amount of baryon asymmetry due to a suppressed sphaleron rate [14–19]. We will refer to this mechanism of exponential suppression for the rate for sphalerons due to non-restoration of the electroweak symmetry (such that the conversion of lepton asymmetry into baryon asymmetry happens extremely slowly) as "sphaleron freeze-in". At the same time, large lepton asymmetries have an impact on the QCD epoch [20–25] and it was shown very recently [26] that they can render the cosmic QCD transition to be first-order. In this Letter, we present the novel possibility of testing the sphaleron freeze-in paradigm in the presence of large lepton asymmetries via the GW imprint from a first-order cosmic QCD transition. This is based on a careful study of the impact of large lepton asymmetries from temperatures above the electroweak scale down to the QCD epoch, improving upon existing literature in several ways: To study symmetry non-restoration, we apply dimensional reduction techniques based on [27, 28] to obtain the thermal potential at finite density. We find that the lepton asymmetries required to produce the observed baryon asymmetry are about an order of magnitude smaller than previous estimates based on a perturbative effective potential [14, 19]. Concerning the QCD epoch, the new equation of state (EoS) [29] is applied which improves over [26] by a better agreement with lattice QCD and an improved description of the thermodynamic variables around the critical endpoint (CEP). Building upon these, we show for the first time that experiments like μAres [30] can potentially probe the sphaleron freeze-in paradigm.

Constraints on lepton flavor asymmetries— We will denote the lepton flavor asymmetries as follows,

$$Y_{L_\alpha} = \frac{n_{L_\alpha}}{s} = \frac{n_\alpha + n_{\nu_\alpha}}{s} \qquad (\alpha = e, \mu, \tau), \qquad (1)$$

where n_i are particle minus anti-particle number densities, s is the entropy density of the Universe, and the total lepton asymmetry is given by $Y_L = \sum_\alpha Y_{L_\alpha}$. Observational bounds on the lepton flavor asymmetries are many orders of magnitude weaker than on their baryonic counterpart. They are based on analyses of data from the anisotropy spectrum of the Cosmic Microwave Background (CMB) and measurements of primordial element abundances as predicted by Big Bang Nucleosynthesis (BBN). As it was shown in [31, 32] that neutrino oscillations tend to equilibrate lepton flavor asymmetries, it is usually assumed that during BBN and CMB the individual lepton flavor asymmetries Y_{L_α} are roughly one-third of the (conserved) total lepton asymmetry Y_L. Under this assumption, [33] found $|Y_L| \leq 1.2 \times 10^{-2}$ which is comparable to the constraints from BBN [34][35]. Note however, that the level of equilibration depends on the details of neutrino oscillations [36–41] and the bounds of [33] only hold approximately. Finally, Ref. [42] recently presented constraints on the values of *primordial* lepton flavor asymmetries (for $T > 10^6$ GeV) using the hypermagnetic fields inducing overproduction of the baryon asymmetry. However, such a constraint is not applicable in our case because it requires the electroweak symmetry to be restored at high temperatures [43].

Sphaleron freeze-in and baryogenesis— It is well known that the presence of a large background charge in the Universe can lead to non-restoration of symmetry at high temperatures. The various existing studies in the literature [14–19] employ the perturbative effective potential at finite temperature and chemical potential. However, at finite temperature, the long-distance behavior becomes strongly coupled [44] and the hierarchy between some terms at different loop order breaks down (see [45, 46]). It becomes then necessary to re-organize perturbation theory in powers of the weak gauge coupling constant, g. Then one has control of defining the theory to an appropriate level of accuracy. This is automatically achieved using the framework of dimensional reduction [27, 28]. At finite temperature, the imaginary time domain becomes a compact dimension of size $1/T$, resulting in a Kaluza Klein tower of heavy Matsubara modes. Integrating out such modes results in an effective three-dimensional theory where the hard modes screen the long-distance behavior and the theory becomes straightforward to organize in powers of g. The benefits of this approach can be quite drastic—GW predictions at one loop tend to differ from the predictions from a dimensionally reduced theory of $\mathcal{O}(g^4)$ by multiple orders of magnitude [45, 47]. Similarly, in our case, following Ref. [48] we will define our theory again to $\mathcal{O}(g^4)$ and find a substantially different prediction of the needed lepton asymmetry compared to the existent results in the literature e.g. of [14, 19]. A detailed account of the full potential with all the technicalities will be presented in a longer and separate communication [49]. However, at $\mathcal{O}(\mu^4/T^4, g^2\mu^2/T^2, g^3)$ an approximate form for the thermal potential is given by

$$\begin{aligned} V_{\rm eff}(\phi) \;\simeq\; & \frac{1}{2}\Big(-\frac{m_H^2}{2} + \frac{g^2T^2}{16m_W^2}(m_H^2 + 2m_W^2 + m_Z^2 + 2m_t^2) \\ & -\frac{16}{121}\mu^2\Big)\phi^2 - \frac{g^3T}{32\pi m_W^3}(2m_W^3 + m_Z^3)\phi^3 \\ & +\frac{1}{4}\left(\lambda + \frac{9}{1331}\frac{\mu^2}{T^2}\right)\phi^4 \,. \end{aligned} \qquad (2)$$

Note that the correction to the mass term, $-\mu^2\phi^2$, (which induces a negative correction to the quadratic) is responsible for the non-restoration of electroweak symmetry at high temperatures. The approach of dimensional reduction allows for numerical minimization of the effective 3-dimensional potential at a given finite temperature and chemical potential, which can be combined with an exact computation of the small-fluctuation determinant from e.g. [50] to estimate the sphaleron freeze-in rate and the final baryon asymmetry in a reliable manner.

Using this approach, we find the allowed initial lepton flavor asymmetries ($Y^{\rm ini}_{L_{\mu(\tau)}}$) that can lead to the correct baryon asymmetry as shown in Fig. 1. Note that we choose $Y^{\rm ini}_{L_e}$ to be vanishing without any loss of generality [51]. Interestingly, using our approach the prediction for primordial lepton asymmetry required for successful baryogenesis is reduced by an order of magnitude as compared to previous results in the literature, e.g. Refs. [14, 19]. The colored contours show the allowed region capable of reproducing the correct observed baryon asymmetry. The primordial asymmetry $Y^{\rm ini}_{L_{\mu(\tau)}}$ can get diluted before BBN in the presence of late-time entropy production. We define the entropy dilution factor as $\Delta \equiv Y_L^{\rm ini}/Y_L$. The differently colored contours correspond to different factors of entropy dilution required to produce the observed baryon asymmetry. We find that given the constraints from BBN and CMB on primordial lepton asymmetry (subject to flavor asymmetry equilibration due to neutrino oscillations), a minimum entropy dilution of $\Delta^{\rm NE}_{\rm min} \gtrsim 5$ (no-equilibration limit) and $\Delta^{\rm FE}_{\rm min} \gtrsim 18$ (full-equilibration limit) is required for reproducing the correct baryon symmetry. The entropy dilution can naturally occur via late-time decaying states in many common and well-motivated new physics scenarios e.g. long-lived moduli [52], supersymmetric condensate [53], gravitinos [54], inflatons [55], curvatons [56], dilatons [57], Q-balls [58], etc. We now proceed to show that in the presence of entropy dilution as discussed above (after the QCD transition and before BBN), the primordial lepton asymmetry can lead to a predictive first-order QCD phase transition with an observable GW signal.

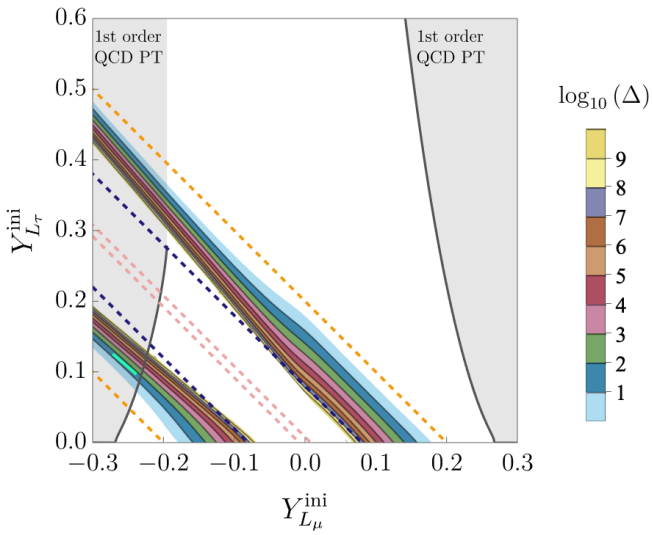

FIG. 1. The viable parameter space for successful baryogenesis via sphaleron freeze-in in the plane of $Y_{L_\mu}^{\rm ini}$ and $Y_{L_\tau}^{\rm ini}$. The color coding of the bands corresponds to different values of entropy dilution (as shown in the legend) required to produce the observed baryon abundance today. The area within the pink, blue, and orange pair of dashed lines represents the CMB constraint in the limit of fully equilibrated lepton asymmetries [33], corresponding to the cases $\Delta^{\rm FE} = 1$ (no entropy dilution), 10, and 25, respectively. The grey-shaded regions correspond to the parameter space that can induce a cosmic first-order QCD transition. The cyan line on the colored band corresponds to the benchmark line which we use to study the viability of probing GW signal from the first-order cosmic QCD phase transition (PT). Note that such a line leads to successful baryogenesis for an entropy dilution $\Delta \sim (18)$.

First-order QCD transition— After the electroweak phase transition ($T \sim 100$ GeV) and before the onset of neutrino oscillations ($T \sim 10$ MeV) the three lepton flavor numbers, the baryon number, and the electric charge of the Universe are conserved. In the absence of any entropy-producing process, the lepton asymmetries in Eq. (1) as well as the analogously defined baryon asymmetry Y_B and the charge asymmetry Y_Q are conserved quantities. Each of these conserved charges is assigned a charge chemical potential. Assuming chemical equilibrium, the charge chemical potentials $\mu_B, \mu_Q, \mu_{L_\alpha}$ can be expressed through the chemical potentials of the individual particle species, e.g. $\mu_B = \mu_u + 2\mu_d$ (where (u, d) stands for the (u, d)-quark). Given the values of $Y_B^{\rm ini}, Y_{L_\alpha}^{\rm ini}$ and $Y_Q^{\rm ini}$, the so called *cosmic trajectory* is the solution for $(\mu_B, \mu_{L_\alpha}, \mu_Q)$ at a given temperature T. As pointed out in [20–22], large lepton flavor asymmetries shift the cosmic trajectory towards large quark chemical potentials (and hence a large baryon chemical potential). Ref. [26] applied results from functional QCD to express the thermodynamic quantities of QCD matter and found that sufficiently large $Y_{L_\alpha}^{\rm ini}$ can indeed induce $\mu_{u,d}$ beyond the critical value at the $T_{\rm CEP}$, which implies a first-order QCD transition. The results of [26] were based on solving the EoS using a gap equation from the Dyson-Schwinger equation for the propagator in the Rainbow-ladder (RL) truncation [59]. At the time of the publication of Ref. [26], this was the only *QCD based* method delivering a complete computation of the phase diagram and the related thermodynamic quantities. However, as already pointed out in [26], the truncation scheme can be improved and in particular the location of the CEP from the RL truncation was not realistic. Therefore, by including the running of the quark-gluon vertex, the improved truncation gives the phase transition line (under the assumption of negligible surface tension) which agrees with lattice QCD results at low chemical potentials and makes predictions for the CEP location [60, 61]. The EoS of QCD can further be constructed from these results of functional QCD methods [29], giving the phase transition temperature at zero chemical potential at $T = 157$ MeV, the curvature of the transition line $\kappa = 0.016$ and a critical endpoint at $(T_{\rm CEP},\ \mu_{\rm CEP}) = (118,\ 200)$ MeV. The EoS has been incorporated in hydrodynamic simulations, allowing one to obtain particle yields and particle ratios that are consistent with commonly used EoSs at high collision energies [29]. We incorporate the improved thermodynamic quantities of [29] in the calculation of the cosmic trajectory. Compared to [26], the new method shows a significantly better agreement at low temperatures to the hadron resonance gas limit of [20–22]. In this work, we are particularly interested in the values of $Y_{L_\mu}^{\rm ini}$ and $Y_{L_\tau}^{\rm ini}$ that induce a first-order cosmic QCD transition. As described in [26], imposing the condition

$$|\mu_{u,d}| > \mu_{\rm CEP} \qquad \text{at T=}T_{\rm CEP} \tag{3}$$

additionally to the 5 conservation laws for the charge asymmetries allows us to find the values of $Y_{L_\mu}^{\rm ini}$ (for given $Y_{L_\tau}^{\rm ini}$ and $Y_{L_e}^{\rm ini}$ values) inducing a first-order transition [62]. In this way, we perform a systematic scan and show the resulting first-order region of $(Y_{L_\mu}^{\rm ini}, Y_{L_\tau}^{\rm ini})$ as the grey shaded region in Fig. 1. It turns out that we always have $|\mu_u| \sim |\mu_d|$ such that we expect a simultaneous first-order transition for the u and d quark. We find that there exists a region around $Y_{L_\mu}^{\rm ini} = -Y_{L_\tau}^{\rm ini}$ (c.f. Fig. 1) which is capable of inducing a first-order cosmic QCD transition while being consistent with the CMB and BBN bounds without any need for entropy dilution (assuming the stringent limiting case of full-equilibration). However, to obtain successful baryogenesis while inducing a first-order QCD transition, some amount of minimal entropy dilution between the epochs of QCD transition and BBN is needed as discussed in the previous section. A set of such benchmark points of interest is highlighted by the cyan curve in Fig. 1, which we use to further compute the possibility of observing a GW signal. Of particular importance for the calculation of the GW spectrum induced by a first-order QCD transition is the critical temperature T_c. Due to the

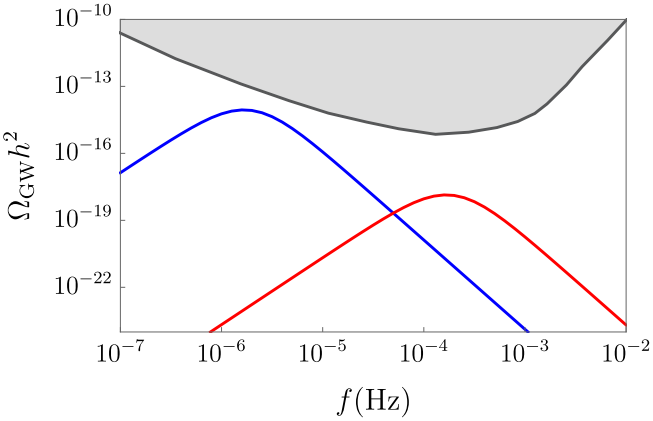

FIG. 2. Two gravitational wave spectra with the thermal parameters $(\alpha, v_w, T_c) = (0.1, 1, 0.109\,\text{GeV})$, and the inverse timescales $\beta/H_c = 10^2$ (blue) and $\beta/H_c = 10^4$ (orange). The sensitivity of μAres is only weakly sensitive to the timescale of the transition for a reasonably typical range.

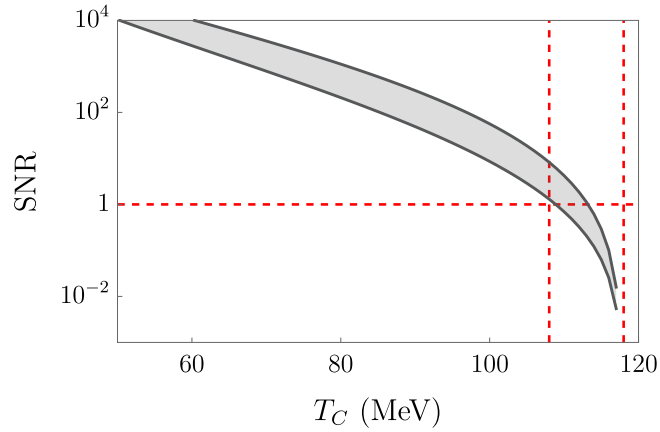

FIG. 3. Signal-to-noise ratio as a function of the critical temperature, taking the trace anomaly for two quarks for a phase transition that lasts between 10^{-2} and 10^{-4} of the Hubble time. The gridlines denote what we expect to be the realistic range of critical temperatures for the benchmark scenarios marked in cyan in Fig. 1.

so-called focusing effect of QCD [63] (the bending of trajectories towards the CEP in its vicinity) the density of trajectories with different $(Y_{L_\mu}^{\rm ini}, Y_{L_\tau}^{\rm ini})$-values in the region around the CEP is expected to be very high. Resolving different trajectories in the CEP region would therefore require extremely high numerical precision and in practice this means that we cannot determine the exact value of T_c for given $(Y_{L_\mu}^{\rm ini}, Y_{L_\tau}^{\rm ini})$-values. Nevertheless, a conservative estimate for the critical temperatures for the benchmark scenarios in Fig. 1 is provided by the gap in the trajectories, where no solutions can be found. This gives the interval $T_c \in [109, 118]$ MeV. Note that in principle, going deeper into the region of a first-order QCD transition in Fig. 1 can induce lower values of T_c. We here, however, restrict our analysis to the values of the benchmark line as chosen in Fig. 1 because the uncertainty of the determination of T_c increases the deeper we are in the first-order regime.

Gravitational wave imprint— Recently, it was proposed to launch three interferometers to the Lagrange points of the Earth's orbit around the sun as a new window into GWs at the μHz level. Such a proposal, known as "μAres" [30], seems perfectly designed to probe the nature of the QCD transition due to its sensitivity to microhertz gravitational waves. In principle if there is a very strong cosmological QCD phase transition, methods sensitive to nanohertz sources such as astrometry or pulsar timing arrays could also detect a signal [64, 65]. Either way, if such a signal manifests, there are very few known scenarios that can mimic the signal from a first-order QCD transition. The two exceptions, a supercooled electroweak phase transition [66] and a dark sector transition at the QCD scale [65], can likely be distinguished from a QCD transition by other means (see refs. [67, 68] for details), making a GW background from a QCD phase transition a near smoking gun for the sphaleron freeze-in paradigm. The GW signal from a strong first-order transition is generally dominated by a contribution arising from sound waves [69]. We calculate the spectrum using the sound shell model [70, 71] in an expanding background [72, 73]. In the sound shell model, the GW spectrum grows with the difference in the trace anomaly between the two phases which can be extracted from the EoS of QCD as [29]:

$$\theta = \Delta p - T\frac{1}{4}\Delta s - \mu\frac{1}{4}\Delta n \geq \left(-T\frac{1}{4}\Delta s - \mu\frac{1}{4}\Delta n\right)\Big|_{T_c}. \tag{4}$$

Here T_c is the critical temperature at which the pressure difference between the phases vanishes, $(\Delta p, \Delta s, \Delta n)$ are the pressure, entropy and density differences between the phases respectively. We follow the above argument and set the FOPT take place at $\Delta p = 0$ (implying percolation temperature $\sim T_c$), and the entropy and density differences $(\Delta s, \Delta n)$ can be directly read off from the EoS at the two sides of the PT line. For the range of values for θ that we consider, the signal today has an approximate scaling of $\Omega_{\rm GW}h^2 \sim 10^2\theta^3 v_w/\beta^2$ (for details and exact expressions see [72]). Note that the GW signal is monotonic with θ, so saturating the above bound by assuming the transition is at the critical temperature results in a conservative lower bound on the GW peak amplitude. We also assume that the walls are relativistic, noting that the efficiency of the sound-shell source for hybrid and deflagrations is higher unless the bubble wall velocity is very small ($v_w \lesssim 0.3$ [74]). The one remaining parameter is the mean bubble separation, usually denoted as R_*, or equivalently, the inverse time scale of the transition, usually denoted as β. This is unfortunately difficult to estimate without knowledge of the surface tension

of a bubble of hadronic phase in a medium of a quark-gluon plasma. However, due to the suggested sensitivity profile of μAres, the signal-to-noise ratio (SNR) of the expected GW background is not very sensitive to the timescale of the transition. A realistic range for a typical first-order phase transition is between a hundredth and a ten-thousandth of the Hubble time [69], and we plot the SNR for this range as a function of temperature in Fig. 3. The reason for the insensitivity to the transition time can be understood from Fig. 2. A reduction of the transition time as a fraction of the Hubble time reduces the peak GW amplitude, but also increases the peak frequency, pushing the spectrum closer to the peak sensitivity of μAres.

Conclusions— Using the framework of dimensional reduction at finite temperature and chemical potential we found that the required primordial lepton asymmetry for a successful baryogenesis via sphaleron freeze-in is reduced by an order of magnitude as compared to the previous results from perturbative effective potential. We improved the calculation of the lepton flavor asymmetries that induce a first-order QCD transition in [26] by taking into account QCD thermodynamic quantities with an improved equation of state [49]. We find that a first-order QCD phase transition is expected for a substantial part of the baryogenesis parameter space. For a strong first-order QCD transition, the GW spectrum can be detected at μAres. This can provide a smoking gun evidence for the sphaleron freeze-in baryogenesis paradigm. A better understanding of the surface tension for a bubble in the hadronic phase would play a key role in improving the predictions further.

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[1] **NANOGrav** Collaboration, G. Agazie et al., "The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background," Astrophys. J. Lett. **951** (2023) no. 1, L8, `arXiv:2306.16213 [astro-ph.HE]`.

[2] **NANOGrav** Collaboration, G. Agazie et al., "The NANOGrav 15 yr Data Set: Observations and Timing of 68 Millisecond Pulsars," Astrophys. J. Lett. **951** (2023) no. 1, L9, `arXiv:2306.16217 [astro-ph.HE]`.

[3] **EPTA** Collaboration, J. Antoniadis et al., "The second data release from the European Pulsar Timing Array III. Search for gravitational wave signals," `arXiv:2306.16214 [astro-ph.HE]`.

[4] **EPTA** Collaboration, J. Antoniadis et al., "The second data release from the European Pulsar Timing Array I. The dataset and timing analysis," `arXiv:2306.16224 [astro-ph.HE]`.

[5] **EPTA** Collaboration, J. Antoniadis et al., "The second data release from the European Pulsar Timing Array: V. Implications for massive black holes, dark matter and the early Universe," `arXiv:2306.16227 [astro-ph.CO]`.

[6] D. J. Reardon et al., "Search for an Isotropic Gravitational-wave Background with the Parkes Pulsar Timing Array," Astrophys. J. Lett. **951** (2023) no. 1, L6, `arXiv:2306.16215 [astro-ph.HE]`.

[7] A. Zic et al., "The Parkes Pulsar Timing Array Third Data Release," `arXiv:2306.16230 [astro-ph.HE]`.

[8] D. J. Reardon et al., "The Gravitational-wave Background Null Hypothesis: Characterizing Noise in Millisecond Pulsar Arrival Times with the Parkes Pulsar Timing Array," Astrophys. J. Lett. **951** (2023) no. 1, L7, `arXiv:2306.16229 [astro-ph.HE]`.

[9] H. Xu et al., "Searching for the Nano-Hertz Stochastic Gravitational Wave Background with the Chinese Pulsar Timing Array Data Release I," Res. Astron. Astrophys. **23** (2023) no. 7, 075024, `arXiv:2306.16216 [astro-ph.HE]`.

[10] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, "On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe," Phys. Lett. B **155** (1985) 36.

[11] S. Davidson, E. Nardi, and Y. Nir, "Leptogenesis," Phys. Rept. **466** (2008) 105–177, `arXiv:0802.2962 [hep-ph]`.

[12] E. J. Chun et al., "Probing Leptogenesis," Int. J. Mod. Phys. A **33** (2018) no. 05n06, 1842005, `arXiv:1711.02865 [hep-ph]`.

[13] G. Elor et al., "New Ideas in Baryogenesis: A Snowmass White Paper," in Snowmass 2021. 3, 2022. `arXiv:2203.05010 [hep-ph]`.

[14] B. Bajc, A. Riotto, and G. Senjanovic, "Large lepton number of the universe and the fate of topological defects," Phys. Rev. Lett. **81** (1998) 1355–1358, `arXiv:hep-ph/9710415`.

[15] J. McDonald, "Symmetry nonrestoration via order 10**-10 B and L asymmetries," Phys. Lett. B **463** (1999) 225–229, arXiv:hep-ph/9907358.

[16] B. Bajc and G. Senjanovic, "Large lepton number and high temperature symmetry breaking in MSSM," Phys. Lett. B **472** (2000) 373–381, arXiv:hep-ph/9907552.

[17] J. McDonald, "Naturally large cosmological neutrino asymmetries in the MSSM," Phys. Rev. Lett. **84** (2000) 4798–4801, arXiv:hep-ph/9908300.

[18] J. March-Russell, H. Murayama, and A. Riotto, "The Small observed baryon asymmetry from a large lepton asymmetry," JHEP **11** (1999) 015, arXiv:hep-ph/9908396.

[19] G. Barenboim and W.-I. Park, "A full picture of large lepton number asymmetries of the Universe," JCAP **04** (2017) 048, arXiv:1703.08258 [hep-ph].

[20] D. J. Schwarz and M. Stuke, "Lepton asymmetry and the cosmic QCD transition," JCAP **11** (2009) 025, arXiv:0906.3434 [hep-ph]. [Erratum: JCAP 10, E01 (2010)].

[21] M. M. Wygas, I. M. Oldengott, D. Bödeker, and D. J. Schwarz, "Cosmic QCD Epoch at Nonvanishing Lepton Asymmetry," Phys. Rev. Lett. **121** (2018) no. 20, 201302, arXiv:1807.10815 [hep-ph].

[22] M. M. Middeldorf-Wygas, I. M. Oldengott, D. Bödeker, and D. J. Schwarz, "Cosmic QCD transition for large lepton flavor asymmetries," Phys. Rev. D **105** (2022) no. 12, 123533, arXiv:2009.00036 [hep-ph].

[23] D. Bodeker, F. Kühnel, I. M. Oldengott, and D. J. Schwarz, "Lepton Flavour Asymmetries and the Mass Spectrum of Primordial Black Holes," arXiv:2011.07283 [astro-ph.CO].

[24] V. Vovchenko, B. B. Brandt, F. Cuteri, G. Endrődi, F. Hajkarim, and J. Schaffner-Bielich, "Pion Condensation in the Early Universe at Nonvanishing Lepton Flavor Asymmetry and Its Gravitational Wave Signatures," Phys. Rev. Lett. **126** (2021) no. 1, 012701, arXiv:2009.02309 [hep-ph].

[25] F. Hajkarim, J. Schaffner-Bielich, S. Wystub, and M. M. Wygas, "Effects of the QCD Equation of State and Lepton Asymmetry on Primordial Gravitational Waves," Phys. Rev. D **99** (2019) no. 10, 103527, arXiv:1904.01046 [hep-ph].

[26] F. Gao and I. M. Oldengott, "Cosmology Meets Functional QCD: First-Order Cosmic QCD Transition Induced by Large Lepton Asymmetries," Phys. Rev. Lett. **128** (2022) no. 13, 131301, arXiv:2106.11991 [hep-ph].

[27] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, "Generic rules for high temperature dimensional reduction and their application to the standard model," Nucl. Phys. B **458** (1996) 90–136, arXiv:hep-ph/9508379.

[28] E. Braaten and A. Nieto, "Effective field theory approach to high temperature thermodynamics," Phys. Rev. D **51** (1995) 6990–7006, arXiv:hep-ph/9501375.

[29] Y. Lu, F. Gao, B.-C. Fu, H.-C. Song, and Y.-X. Liu, "Constructing Equation of State of QCD in a functional QCD based scheme," to appear soon (2023) .

[30] A. Sesana et al., "Unveiling the gravitational universe at μ-Hz frequencies," Exper. Astron. **51** (2021) no. 3, 1333–1383, arXiv:1908.11391 [astro-ph.IM].

[31] A. D. Dolgov, S. H. Hansen, S. Pastor, S. T. Petcov, G. G. Raffelt, and D. V. Semikoz, "Cosmological bounds on neutrino degeneracy improved by flavor oscillations," Nucl. Phys. B **632** (2002) 363–382, arXiv:hep-ph/0201287.

[32] Y. Y. Y. Wong, "Analytical treatment of neutrino asymmetry equilibration from flavor oscillations in the early universe," Phys. Rev. D **66** (2002) 025015, arXiv:hep-ph/0203180.

[33] I. M. Oldengott and D. J. Schwarz, "Improved constraints on lepton asymmetry from the cosmic microwave background," EPL **119** (2017) no. 2, 29001, arXiv:1706.01705 [astro-ph.CO].

[34] C. Pitrou, A. Coc, J.-P. Uzan, and E. Vangioni, "Precision big bang nucleosynthesis with improved Helium-4 predictions," Phys. Rept. **754** (2018) 1–66, arXiv:1801.08023 [astro-ph.CO].

[35] Interestingly, recent measurements of the primordial helium abundance by the EMPRESS survey [75] could be interpreted as a hint towards a positive asymmetry in electron neutrinos [75, 76].

[36] S. Pastor, T. Pinto, and G. G. Raffelt, "Relic density of neutrinos with primordial asymmetries," Phys. Rev. Lett. **102** (2009) 241302, arXiv:0808.3137 [astro-ph].

[37] G. Mangano, G. Miele, S. Pastor, O. Pisanti, and S. Sarikas, "Constraining the cosmic radiation density due to lepton number with Big Bang Nucleosynthesis," JCAP **03** (2011) 035, arXiv:1011.0916 [astro-ph.CO].

[38] G. Mangano, G. Miele, S. Pastor, O. Pisanti, and S. Sarikas, "Updated BBN bounds on the cosmological lepton asymmetry for non-zero θ_{13}," Phys. Lett. B **708** (2012) 1–5, arXiv:1110.4335 [hep-ph].

[39] E. Castorina, U. Franca, M. Lattanzi, J. Lesgourgues, G. Mangano, A. Melchiorri, and S. Pastor, "Cosmological lepton asymmetry with a nonzero mixing angle θ_{13}," Phys. Rev. D **86** (2012) 023517, arXiv:1204.2510 [astro-ph.CO].

[40] G. Barenboim, W. H. Kinney, and W.-I. Park, "Resurrection of large lepton number asymmetries from neutrino flavor oscillations," Phys. Rev. D **95** (2017) no. 4, 043506, arXiv:1609.01584 [hep-ph].

[41] L. Johns, M. Mina, V. Cirigliano, M. W. Paris, and G. M. Fuller, "Neutrino flavor transformation in the lepton-asymmetric universe," Phys. Rev. **D94** (2016) no. 8, 083505, arXiv:1608.01336 [hep-ph].

[42] V. Domcke, K. Kamada, K. Mukaida, K. Schmitz, and M. Yamada, "A new constraint on primordial lepton flavour asymmetries," arXiv:2208.03237 [hep-ph].

[43] M. Joyce and M. E. Shaposhnikov, "Primordial magnetic fields, right-handed electrons, and the Abelian anomaly," Phys. Rev. Lett. **79** (1997) 1193–1196, arXiv:astro-ph/9703005.

[44] A. D. Linde, "Infrared Problem in Thermodynamics of the Yang-Mills Gas," Phys. Lett. B **96** (1980) 289–292.

[45] O. Gould and T. V. I. Tenkanen, "On the perturbative expansion at high temperature and implications for cosmological phase transitions," JHEP **06** (2021) 069, arXiv:2104.04399 [hep-ph].

[46] D. Curtin, J. Roy, and G. White, "Gravitational waves and tadpole resummation: Efficient and easy convergence of finite temperature QFT," arXiv:2211.08218 [hep-ph].

[47] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, "Theoretical uncertainties for cosmological

first-order phase transitions," JHEP **04** (2021) 055, arXiv:2009.10080 [hep-ph].

[48] A. Gynther, "Electroweak phase diagram at finite lepton number density," Phys. Rev. D **68** (2003) 016001, arXiv:hep-ph/0303019.

[49] F. Gao, J. Harz, C. Hati, I. M. Oldengott, Y. Lu, and G. White, "To appear soon,".

[50] L. Carson, X. Li, L. D. McLerran, and R.-T. Wang, "Exact Computation of the Small Fluctuation Determinant Around a Sphaleron," Phys. Rev. D **42** (1990) 2127–2143.

[51] An amount of Y_{L_e} will be generated once the neutrino oscillations are in equilibrium shortly before BBN, however, it has no impact on our analysis.

[52] T. Moroi and L. Randall, "Wino cold dark matter from anomaly mediated SUSY breaking," Nucl. Phys. B **570** (2000) 455–472, arXiv:hep-ph/9906527.

[53] S. D. Thomas, "Baryons and dark matter from the late decay of a supersymmetric condensate," Phys. Lett. B **356** (1995) 256–263, arXiv:hep-ph/9506274.

[54] T. Moroi, M. Yamaguchi, and T. Yanagida, "On the solution to the Polonyi problem with 0 (10-TeV) gravitino mass in supergravity," Phys. Lett. B **342** (1995) 105–110, arXiv:hep-ph/9409367.

[55] R. Allahverdi and M. Drees, "Production of massive stable particles in inflaton decay," Phys. Rev. Lett. **89** (2002) 091302, arXiv:hep-ph/0203118.

[56] T. Moroi and T. Takahashi, "Cosmic density perturbations from late decaying scalar condensations," Phys. Rev. D **66** (2002) 063501, arXiv:hep-ph/0206026.

[57] A. B. Lahanas, "Dilaton dominance in the early Universe dilutes Dark Matter relic abundances," Phys. Rev. D **83** (2011) 103523, arXiv:1102.4277 [hep-ph].

[58] M. Fujii and K. Hamaguchi, "Nonthermal dark matter via Affleck-Dine baryogenesis and its detection possibility," Phys. Rev. D **66** (2002) 083501, arXiv:hep-ph/0205044.

[59] F. Gao, J. Chen, Y.-X. Liu, S.-X. Qin, C. D. Roberts, and S. M. Schmidt, "Phase diagram and thermal properties of strong-interaction matter," Phys. Rev. D **93** (2016) no. 9, 094019, arXiv:1507.00875 [nucl-th].

[60] F. Gao and J. M. Pawlowski, "QCD phase structure from functional methods," Phys. Rev. D **102** (2020) no. 3, 034027, arXiv:2002.07500 [hep-ph].

[61] F. Gao and J. M. Pawlowski, "Chiral phase structure and critical end point in QCD," Phys. Lett. B **820** (2021) 136584, arXiv:2010.13705 [hep-ph].

[62] The QCD EoS applied in this work and derived in [29] assumes negligible super cooling. Indeed, Ref. [77] indicates that the surface tension may be relatively small (compared to T_c^3). This means Δp remains small and very little vacuum energy is converted into radiation. In such a case there is negligible change to $Y_B^{\rm ini}, Y_{L_\alpha}^{\rm ini}$ and $Y_Q^{\rm ini}$ during the first-order QCD phase transition.

[63] T. Dore, J. M. Karthein, I. Long, D. Mroczek, J. Noronha-Hostler, P. Parotto, C. Ratti, and Y. Yamauchi, "Critical lensing and kurtosis near a critical point in the QCD phase diagram in and out of equilibrium," Phys. Rev. D **106** (2022) no. 9, 094024, arXiv:2207.04086 [nucl-th].

[64] C. Caprini, R. Durrer, and X. Siemens, "Detection of gravitational waves from the QCD phase transition with pulsar timing arrays," Phys. Rev. D **82** (2010) 063511, arXiv:1007.1218 [astro-ph.CO].

[65] J. Garcia-Bellido, H. Murayama, and G. White, "Exploring the early Universe with Gaia and Theia," JCAP **12** (2021) no. 12, 023, arXiv:2104.04778 [hep-ph].

[66] A. Kobakhidze, C. Lagger, A. Manning, and J. Yue, "Gravitational waves from a supercooled electroweak phase transition and their detection with pulsar timing arrays," Eur. Phys. J. C **77** (2017) no. 8, 570, arXiv:1703.06552 [hep-ph].

[67] Y. Hochberg, E. Kuflik, T. Volansky, and J. G. Wacker, "Mechanism for Thermal Relic Dark Matter of Strongly Interacting Massive Particles," Phys. Rev. Lett. **113** (2014) 171301, arXiv:1402.5143 [hep-ph].

[68] M. J. Ramsey-Musolf, "The electroweak phase transition: a collider target," JHEP **09** (2020) 179, arXiv:1912.07189 [hep-ph].

[69] C. Caprini et al., "Detecting gravitational waves from cosmological phase transitions with LISA: an update," JCAP **03** (2020) 024, arXiv:1910.13125 [astro-ph.CO].

[70] M. Hindmarsh, S. J. Huber, K. Rummukainen, and D. J. Weir, "Gravitational waves from the sound of a first order phase transition," Phys. Rev. Lett. **112** (2014) 041301, arXiv:1304.2433 [hep-ph].

[71] M. Hindmarsh, S. J. Huber, K. Rummukainen, and D. J. Weir, "Numerical simulations of acoustically generated gravitational waves at a first order phase transition," Phys. Rev. D **92** (2015) no. 12, 123009, arXiv:1504.03291 [astro-ph.CO].

[72] H.-K. Guo, K. Sinha, D. Vagie, and G. White, "Phase Transitions in an Expanding Universe: Stochastic Gravitational Waves in Standard and Non-Standard Histories," JCAP **01** (2021) 001, arXiv:2007.08537 [hep-ph].

[73] J. Ellis, M. Lewicki, and J. M. No, "Gravitational waves from first-order cosmological phase transitions: lifetime of the sound wave source," JCAP **07** (2020) 050, arXiv:2003.07360 [hep-ph].

[74] J. R. Espinosa, T. Konstandin, J. M. No, and G. Servant, "Energy Budget of Cosmological First-order Phase Transitions," JCAP **06** (2010) 028, arXiv:1004.4187 [hep-ph].

[75] A. Matsumoto et al., "EMPRESS. VIII. A New Determination of Primordial He Abundance with Extremely Metal-poor Galaxies: A Suggestion of the Lepton Asymmetry and Implications for the Hubble Tension," Astrophys. J. **941** (2022) no. 2, 167, arXiv:2203.09617 [astro-ph.CO].

[76] M. Escudero, A. Ibarra, and V. Maura, "Primordial Lepton Asymmetries in the Precision Cosmology Era: Current Status and Future Sensitivities from BBN and the CMB," arXiv:2208.03201 [hep-ph].

[77] F. Gao and Y.-x. Liu, "Interface Effect in QCD Phase Transitions via Dyson-Schwinger Equation Approach," Phys. Rev. D **94** (2016) no. 9, 094030, arXiv:1609.08038 [hep-ph].