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Programmable multi-physical mechanics of mechanical metamaterials

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ABSTRACT

Mechanical metamaterials are engineered materials with unconventional mechanical behavior that originates from artificially programmed microstructures along with intrinsic material properties. With tremendous advancement in computational and manufacturing capabilities to realize complex microstructures over the last decade, the field of mechanical metamaterials has been attracting wide attention due to immense possibilities of achieving unprecedented multi-physical properties which are not attainable in naturally-occurring materials. One of the rapidly emerging trends in this field is to couple the mechanics of material behavior and the unit cell architecture with different other multi-physical aspects such as electrical or magnetic fields, and stimuli like temperature, light or chemical reactions to expand the scope of actively programming on-demand mechanical responses. In this article, we aim to abridge outcomes of the relevant literature concerning mechanical and multi-physical property modulation of metamaterials focusing on the emerging trend of bi-level design, and subsequently highlight the broad-spectrum potential of mechanical metamaterials in their critical engineering applications. The evolving trends, challenges and future roadmaps have been critically analyzed here involving the notions of real-time reconfigurability and functionality programming, 4D printing, nano-scale metamaterials, artificial intelligence and machine learning, multi-physical origami/kirigami, living matter, soft and conformal metamaterials, manufacturing complex microstructures, service-life effects and scalability.

1. Introduction

The recent advancement in material development has largely been focused on altering the material composition to generate functional material properties. Materials with desired properties can be engineered to cater to the application-specific demands [1]. The inherent mechanical and other physical properties are constraints in the development of novel materials since these conventional naturally occurring materials have specific properties of their own [2]. The common practice is to design mechanical and structural systems based on the material properties that are available. Such conventional materials have fundamental limits in terms of various physical properties, which also constrain the subsequent design process. The limitations become more evident in multi-functional systems where multiple goals need to be attained simultaneously, or extreme properties are required well beyond the limits of natural materials. The emerging concepts of mechanical metamaterials can address such objectives to a significant extent, pushing the boundaries of achieving individual properties in an extraordinary regime and multi-functional system designs for modern structures across length scales. Metamaterials are artificially engineered materials with periodic (or quasi-periodic, aperiodic) microstructure or nanostructure comprised of unique tailor-made (application-specific, based on functional demands) geometry and patterns that produce unprecedented, extraordinary and unusual bulk properties not achievable in conventional materials. Due to their ability to manifest remarkable acoustic, optical, electromagnetic or mechanical characteristics in desirable ranges, these novel classes of architected materials have attracted the attention of the scientific community significantly in the recent past [3–5].

The term ‘metamaterial’ is derived from the Greek language meaning beyond matter, indicating its transcending character. Metamaterials may not qualify as ‘materials’ going by the conventional definition of traditional materials. These materials often possess a (user-)defined microstructure at a much lower length scale than the one in which bulk properties are measured. Thus, even though it is a structure at the micro or nanoscale, it appears as a material from a macro-scale perspective. These are a result of human creativity, showcasing physical properties which are rare or not observable in naturally occurring materials. Their

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constituting components have distinct spatial variations. These artificially-engineered materials can be designed at the micro or even nano-scale to exhibit unprecedented properties at the macro-scale [6–8] (refer to Fig. 1(B)). Instead of intrinsic material properties solely, the unconventional properties of metamaterials arise from their micro-structural geometry (refer to Fig. 1(D-F)). Thus, the material and physical properties can be defined at two different length-scales in metamaterials: one at the lower scale that corresponds to the material of

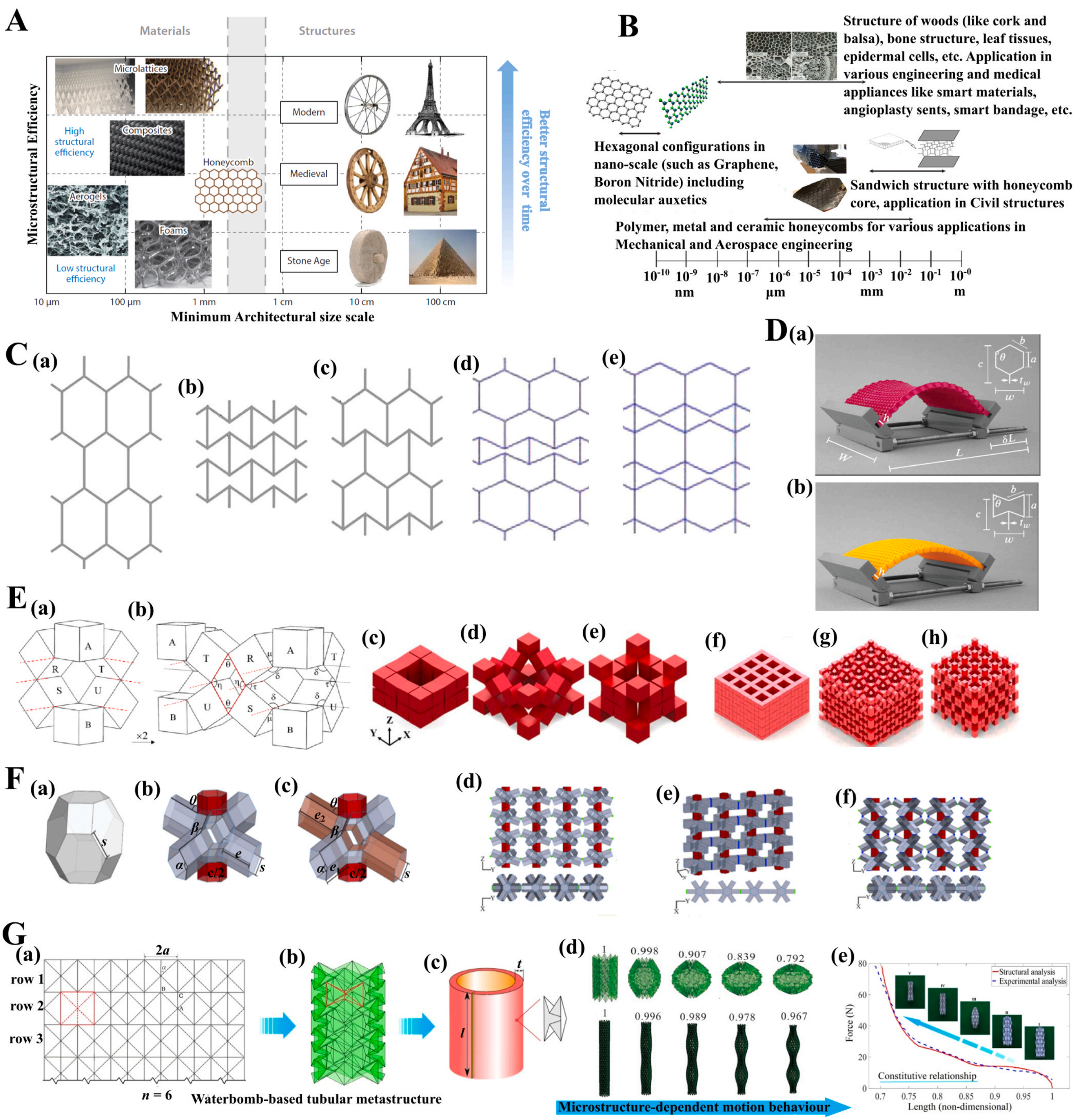


Fig. 1. Materials and structures interplay in metamaterials. (A) Utilization of mechanical design and artificial microstructuring at material scale for enhanced performance [20]. As technology progresses over time, the ability of defining functional architectures gets more intricate and moves towards finer scales, from macro to micro and nano. **(B)** Availability and application of lattices across the length scales [17]. **(C)** Demonstration of Poisson's ratio programming in hexagonal lattice microstructures with (a) positive Poisson's ratio. (b) negative Poisson's ratio (c-e) zero Poisson's ratio [21]. **(D)** Programming Gaussian curvatures (a) negative (b) positive, of meta-plates upon bending [22]. **(E)** Sarrus modular origami design. (a) Cubical units depicting Sarrus mechanisms. (b) Assembly of Sarrus linkages. (c-e) Transformation sequence of the module. (f-h) Deformation configurations of Sarrus metamaterial [23]. **(F)** Kirigami-based modular materials. (a) A typical truncated octahedron. (b - c) Symmetric and asymmetric unit-cells derived from the truncated octahedron. (d) A 2D meta-sheet obtained by tessellation of symmetric unit-cell. (e) Meta-sheet configuration when subjected to compressive loading. (f) Meta-sheet configuration when subjected to tensile loading [24]. **(G)** Waterbomb-based tubular metamaterial (a) The waterbomb based crease pattern. (b) Waterbomb tube in 3D view. (c) Macro-scale illustration of tubular meta-structure. (d) Micro-structure and far-field actuation dependent shape morphing. (e) Microstructure-dependent programming of constitutive relationship [25].

the constituting elements that form a unit cell (referred to as intrinsic material properties) and the effective properties at the macro scale that is a resultant effect of intrinsic properties and microstructural geometry. Metamaterials normally adopt a periodic microstructural form, where a unit cell is designed based on functional demands and the unit cell can be tessellated in 1D, 2D or 3D space to create the material microstructure [9–14]. However, depending on the functional goals, metamaterials can have graded, quasi-periodic, or aperiodic microstructures as well [15–19].

In nature, cellular materials exhibit useful specific combinations of antagonistic mechanical properties of high stiffness and low-density [26]. But this is only an exception, and a philosophical inspiration for developing a large section of mechanical metamaterials (refer to Fig. 1 (B)). It is difficult to find natural materials in which such desired properties coexist within a single material. Moreover, the properties of natural materials can be stretched to a limited short range. In general, the physical properties of naturally-occurring materials like mass density are coupled with their mechanical properties. For example, conventional materials with high strength and stiffness would also have high mass density and vice-versa. In many technologically demanding sectors like aerospace, robotics and mechanical engineering, there exists a strong demand for breaking such correlations and developing materials that are strong and stiff, but lightweight at the same time. Significant efforts have been put in by researchers to develop unit cells at the micro-level to get desired specific stiffness and strength at the macro-level (refer to Fig. 1(A - B)). The reduction in material scale in design essentially amplifies the mechanical characteristics of material [27,28]. The ability of small-scale design results in the realization of novel materials with tailorability of the microstructure at an expanded design space, which allows the realization of mechanical properties based on application-specific requirements [29,30]. Recent advancements in 3D printing techniques have enabled the physical realization of complex micro-structures for different materials [31,32].

Cellular Structures, as discussed in the preceding paragraph, are such entities in which the structural and material response are dependent on microstructural geometry rather than on their chemical constitution. These are available in plenty in both natural (bone tissue) as well as artificial systems (truss bridges), across varying length-scales (refer to Fig. 1(B)). In nature, materials like wood and bone have high stiffness-to-weight ratios owing to their internal configuration which help these light materials carry heavy loads without failure. The history of mechanics of cellular architecture can be traced back to cellular hierarchy obsereved in natural systems [33]. Recent research work includes that on stochastic polymeric foams, the cellular structure of which was altered by Lakes, resulted in the material with negative Poisson's ratio [34]. The mechanics of cellular solids, as a network of beams, was described in the work of Gibson and Ashby [35,36], wherein the macroscopic properties were shown to be controlled by tailoring the geometric parameters of the constituent beam elements. As a demonstration, Fig. 1(C) shows hexagonal lattice microstructures with positive, negative and zero Poisson's ratios. Such cellular designs enabled the design of architected materials [37] that have a greater stiffness than foams with better geometric connectivity [38]. Decades later, with the advancement in additive manufacturing techniques, the physical forms of these complex structures got the momentum with increased enthusiasm from the research community [39]. The emerging trends in this field involve multi-physical metamaterials where the microstructures, besides being able to carry mechanical stresses with high specific strength and stiffness properties, are also able to achieve multi-physical performance such as active property modulation under external stimuli (for example, electrical and magnetic fields), vibration and wave control, energy harvesting etc. The recent developments in the field of mechanical metamaterials covering different classes of microstructures (refer to Fig. 1(C - F)) and their exploitation in the design of multi-functional structures make it crucial to present an overview of these developments with critical analyses of the present trends and future roadmaps.

The primary objective of this review paper is to provide a critical perspective on the recent developments concerning artificially engineered mechanical metamaterials with a focus on the active, reconfigurable and multi-physical behavior. While we would start with a description of the emerging classes of metamaterials, this paper intends to emphasize on mechanical metamaterials which is one of the youngest members in the family of metamaterials. Besides conventional unit cell-based periodic network design, we will discuss the emerging concepts of bilevel design where the elementary constituent members are further exploited to achieve extreme mechanical properties and active behavior in an expanded design space. Hereafter in this paper, we first introduce emerging classes of metamaterials based on functionality and microstructure, followed by which we shift our focus primarily on mechanical metamaterials with bi-level passive and active behaviour. The evolving concepts of multi-functionality are discussed further, wherein the mechanics of microstructure is coupled with the multi-physical behaviour in conjunction with active microstructural components of a metamaterial. Subsequently, we shift our focus to the pertinent issue of physical realization through manufacturing complex material microstructures which are computationally designed. We conclude the paper by summarizing the critical issues that need immediate attention from the scientific community along with a moderately long-term roadmap and perspective in the domain of mechanical metamaterials.

2. Emerging classes of metamaterials based on functionality and microstructure

Metamaterials can be classified based on functionality and microstructural configurations (refer to Fig. 2). In this section, we briefly discuss different emerging classes of metamaterials. In this context, it is noteworthy that the emerging trends involve multi-functionality, where the metamaterials are designed to achieve objectives across different domains of physics.

2.1. Functionality based classification

The objective of metamaterials is essentially based on the output properties they intend to achieve, i.e. the intended functionality. Metamaterials are engineered at the lower length scale (such as micro or nano scale) to achieve such functionalities at a higher length scale (such as macroscale). Metamaterials can be broadly classified into four types based on the effective parameters which are obtained using the homogenization theory [40,41], namely, *Electromagnetic* metamaterials, Optical metamaterials, Acoustic metamaterials, and Mechanical metamaterials. To set a proper perspective, we give a brief account of these classifications before moving towards detailed discussions on mechanical metamaterials which is the main focus of this paper.

2.1.1. Electromagnetic metamaterials

Electromagnetic metamaterials are designer materials that are obtained by tailoring microscopic parameters resulting in exceptional command over light-matter interactions. These metamaterials give distinct responses to the electric and magnetic components of light (refer to Fig. 3(A)). Electromagnetic metamaterials can have unusual properties like negative magnetic permeability and negative electric permittivity. Negative magnetic permeability implies that the magnetic dipoles, under an applied magnetic field, are formed in opposite direction to the applied magnetic field. Similarly, negative electric permittivity implies that the electric field vector and the electric displacement vector point in directions opposite to each other. Such metamaterials have various applications such as antennas [42,43], invisible cloaks [44,45], perfect lenses [46,47] and different kinds of sensors [48–50].

2.1.2. Optical metamaterials

Optical metamaterials [54–56] are artificially engineered (micro-)

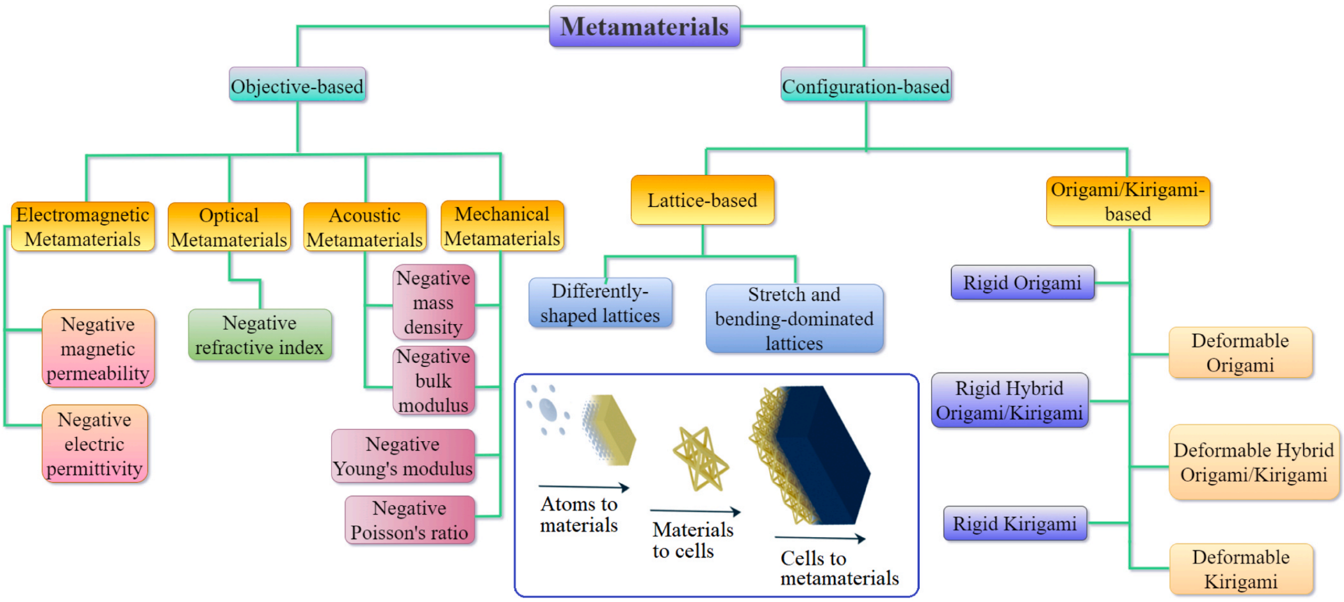


Fig. 2. Emerging classes of metamaterials with their current practical applications. Different types of metamaterials are explained based on their intended functionality and microstructural configurations. As shown in the inset, it can be noted that the intrinsic materials (such as metals, rubbers, ceramics, etc.) are made of atoms and molecules. Metamaterials are designed by introducing architectures (often through designing unit cells) in such materials to obtain application-specific functional properties at higher length scales. Thus, in a metamaterial, the material properties are normally defined at two different length scales: (1) properties of the intrinsic materials at a relatively lower length scale, derived based on the atomic or molecular arrangements, (2) properties of the metamaterials at a relatively higher length scale, derived based on the unit cell architecture. The difference between the two length scales are considered to be significant in metamaterials. Note that the effective properties at the higher length scales are further influenced by external stimuli in case of active metamaterials.

structures with unprecedented optical functionalities which are rare in natural materials such as negative refraction [57–59], hyperbolic dispersion [60,61] and super-resolution imaging [62–64]. In nature, there are no negative index materials. Negative refractive index means that when a ray of light travels from one medium to another, it reflects instead of refracting (refer to Fig. 3(B)(a,b)). This requires tailoring the properties at local level such as impedance and phase velocity of an electromagnetic wave passing through the material (refer to Fig. 3(B)(c-f)). Earlier optical metamaterials were either metasurfaces or meta-films. At present, however, different 3D optical metamaterial structures have been realized (refer to Fig. 3(B)(g)). Similarly, near-field version of the superlens has also been identified which allows imaging resolution such that the wavelength is not the limiting aspect but the quality of the material.

2.1.3. *Acoustic metamaterials*

The sound waves can be controlled and manipulated at sub-wavelength scales using acoustic metamaterials. Propagation of sound fields can be controlled by coupling theory of transformation acoustics and highly anisotropic acoustic metamaterials [65]. The acoustic metamaterial can be one with negative mass density [66] or with negative bulk modulus [53]. When both the effective density and bulk modulus are simultaneously negative, a double negative acoustic system is obtained [67]. Note that these unique properties are not common in naturally-occurring materials. Negative mass density implies that for negative mass the acceleration is oppositely directed. Negative bulk modulus results in an increase in volume with the application of pressure. These unprecedented properties can be put to use in acoustic superlensing [68,69], cloaking [70] and the design of compact ultrasound imaging elements [71]. Recent studies in this field include soft 3D acoustic metamaterials [72] that can be put to use in sub-wavelength imaging and transformation acoustics for which the devices should have negative indices (refer to Fig. 3(C)).

2.1.4. *Mechanical metamaterials*

Inspired by the predecessors in the family of metamaterials, a new class of architected metamaterials has taken a prominent shape that deals with effective mechanical properties over the last decade or so. Initially, the field of mechanical metamaterials was limited to achieving unconventional (zero or negative) values for some familiar physical properties, like Poisson's ratio [73–75], density [76], or compressibility [77]. However, recent research in this area has brought forward this relatively new class of mechanical metamaterials — including topological [78,79], shape-morphing [80–82] and nonlinear metamaterials [83,84]. These materials show unprecedented functionalities like programmable stiffness [85], shape and pattern transformations [86], dissipation [87–89], or unidirectional guiding of motion and waves [90,91]. The mechanical metamaterials can have negative Young's modulus and negative Poisson's ratio. Negative Young's modulus implies that the deflection of an element is in opposite direction to that of the applied load [92]. Materials with negative Poisson's ratio compress laterally when longitudinal compression is applied to them, or vice-versa [24,93,94] (refer to Fig. 3(D)). A detailed literature review on the recent progress of different passive and active mechanical metamaterials along with their multi-physical behavior is presented later in this paper.

Since the focus of this review paper is mechanical metamaterials, we further present a perspective on their current and potential applications in Fig. 4. Note that the applications discussed here is only a small set of the endless engineering examples for this class of metamaterials. The lattices can be used as stealth cloak so as to escape deliberate attempt to detect a body. Such cloaks can find crtical applications in defence sectors (refer to Fig. 4(A)). The progress in 3D printing has led to manufacturing of complex microstructures with controllable flexibility that result in metamaterials for soft robotics, electronic skin, bionic grippers, flexible batteries, etc. (refer to Fig. 4(B, C, D, I)). Recent applications of mechanical metamaterials include mechanical computing and logic gates (refer to Fig. 4(E)). The property of auxetic mechanical metamaterials can be exploited in the development of impact resistant structures (refer to Fig. 4(F)). Recently auxetic mechanical metamaterials have found application in the footwear industry as well (refer to Fig. 4(H)). The auxetic property gives safety advantage while driving car when they are used in the seat belt material. The seat belt expands laterally in case of its

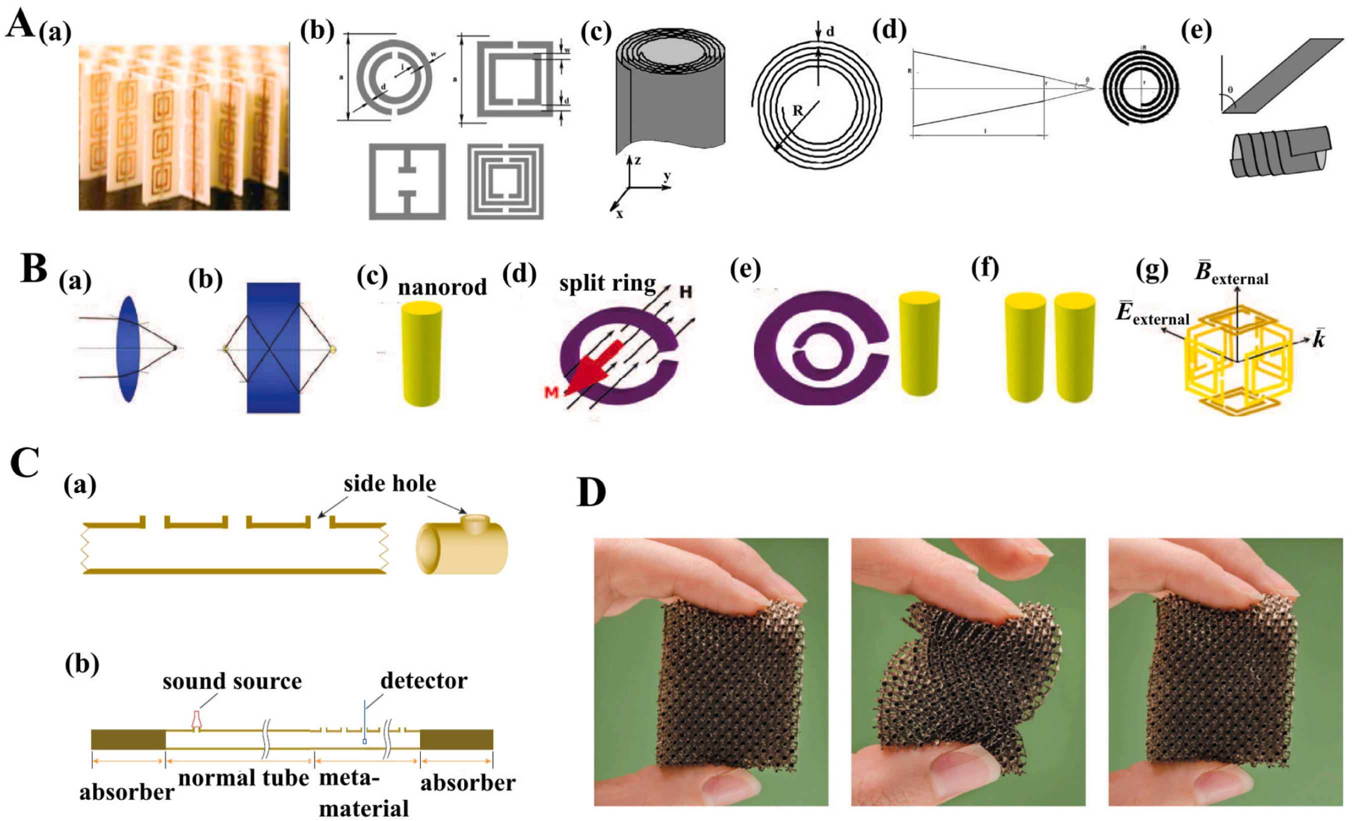


Fig. 3. Overview on multiple functionalities of metamaterials. (A) Electromagnetic metamaterials: (a) A left-handed material (LHM) depicted by a 2D periodic array of copper split ring resonators and wires, exhibiting negative refraction [7]. (b) Split ring resonator of circular, squared, single ring and multiple ring structures. A split ring resonator (SRR) is a highly conductive structure, the inductance of which is balanced by the capacitance between the two rings. (c) Perspective and top view of a Swiss roll. The current in a Swiss roll flows due to the self-capacitance of the structure which permits the alternative current circuit to be completed. (d) Lateral and top view of a conical Swiss roll. A conical Swiss roll helps in transmission of electromagnetic waves at relatively large distances and with reduced damping. (e) The unfolded conducting sheet which is used to make a chiral Swiss roll and top view of a chiral Swiss roll. Each layer of the conductive sheet is filled with a dielectric material [51]. **(B)** Optical metamaterials: Focusing of light using (a) conventional lens. (b) a negative refractive index metamaterial. (c) plasmonic nanorods that give negative electric response (d) split nanorings that provide negative magnetic resonance, (e) nanorods paired with double split nanorings to produce negative magnetic and electric response. (f) coupled nanorods can also give negative magnetic and electric responses in specific situations, (g) a 3D optical metamaterial unit cell made up of double split rings [52]. **(C)** Acoustic metamaterials: (a) A tube structure with a pattern of side holes on it showing negative effective modulus. The unit cell is depicted on the side. (b) Experimental set-up for measuring the phase and transmission velocity [53]. **(D)** Mechanical metamaterials: A microlattice showing auxeticity and reversible deformation [20].

longitudinal extension and covers a larger part of human chest which is not possible in conventional materials (refer to Fig. 4(K)). Similar property makes them applicable in bandages to help heal wounds by covering a larger wound area in case of expansion along one direction (refer to Fig. 4(M)). Auxetic lattices have shown advantageous properties in truss core aerofoils (refer to Fig. 4(J)). Further, honeycomb core sandwich structures are widely used in lightweight structural applications like the aerospace industry [95]. Assembled honeycombs with appropriate unit cell geometry have tremendous potential in protective marine structures (refer to Fig. 4(L)). In the field of medical applications, the mechanical metamaterials are being used as cardio-vascular stents that help keep the hearts of patients healthy by maintaining proper blood flow (refer to Fig. 4(G)). Note that the applications discussed here are primarily passive in nature. However, with the emergence of active mechanical metamaterials, it will be possible to modulate the shape and mechanical properties for on-demand functionalities. This essentially adds a whole new dimension to the prospective engineering applications with significantly improved performances.

2.2. *Microstructural configurations*

Based on architecture at the microscale, metamaterials can broadly be lattice-based or Origami/ Kirigami-based. Such microstructures are primarily periodic in nature, but they can also be of graded, quasi-periodic and aperiodic nature [15–17,19]. A geometrically periodic microstructure may have spatially varying material properties of the unit cells, or in a periodic microstructural unit cell, multiple materials may be present (referred to as multi-material lattices) [108,109]. In this section, we provide a brief description of different classes of metamaterial microstructures highlighting the emerging trends.

2.2.1. *Lattice-based metamaterials*

On the basis of constituting elements of the unit cells, lattices may be classified as: plate based lattices, strut/ beam based lattices and triply periodic minimal surfaces (TPMS) based lattices (refer to [110] for more details on different types of lattices). The lattice-based metamaterials can be further discussed following two different categories of differently shaped lattices and stretch/bending-dominated lattices.

2.2.1.1. Differently shaped lattices. Planar lattices are obtained by tessellating a polygon that fills the entire plane. Classically, honeycombs can be regular, semi-regular or irregular [1]. The regular honeycombs are formed by the periodic repetition of a single regular polygon, or by the three traditional shapes of lattices which are hexagonal, triangular and square (refer to Fig. 5(A)). The semi-regular lattices are obtained by tessellation of two or more different regular shapes [111,112]. Irregular

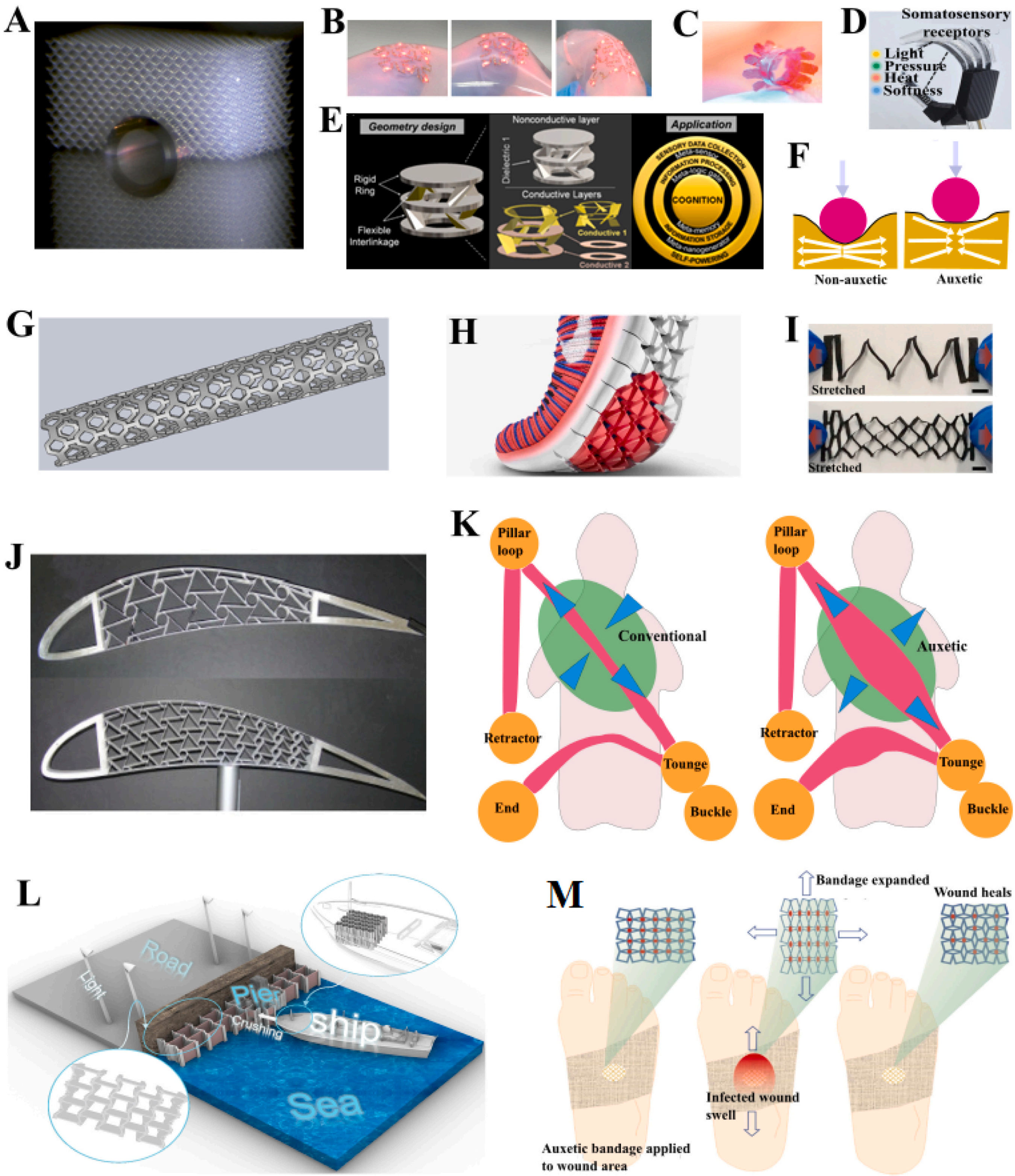


Fig. 4. Practical applications of mechanical metamaterials. **(A)** Stealth cloak [96] **(B)** Electronic skin [97] **(C)** Soft robot [98] **(D)** Bionic Gripper [99] **(E)** Mechanical computing through metamaterials [100] **(F)** Impact resistant structures [101] **(G)** Vascular stent [102] **(H)** Running shoes [103] **(I)** Flexible batteries [104] **(J)** Truss-core airfoils [105] **(K)** Auxetic seat belt [106] **(L)** Assembled honeycomb structures in marine structural applications [107] **(M)** Auxetic bandage [106]. Note that we have mentioned only a few representative applications here and the list is endless in reality.

lattices like Voronoi lattice [113] or Penrose tiling [114] are formed by repeating two or more differently sized irregular polygons. While most of the proposed metamaterials conventionally follow periodic architectures, non-periodic lattices are rapidly making inroads to the design of mechanical metamaterials due the scope of introducing more localized spatial features [115,116].

In addition to this classification, the lattice-based materials can be categorized as chiral honeycombs and anti-chiral honeycombs. Chiral

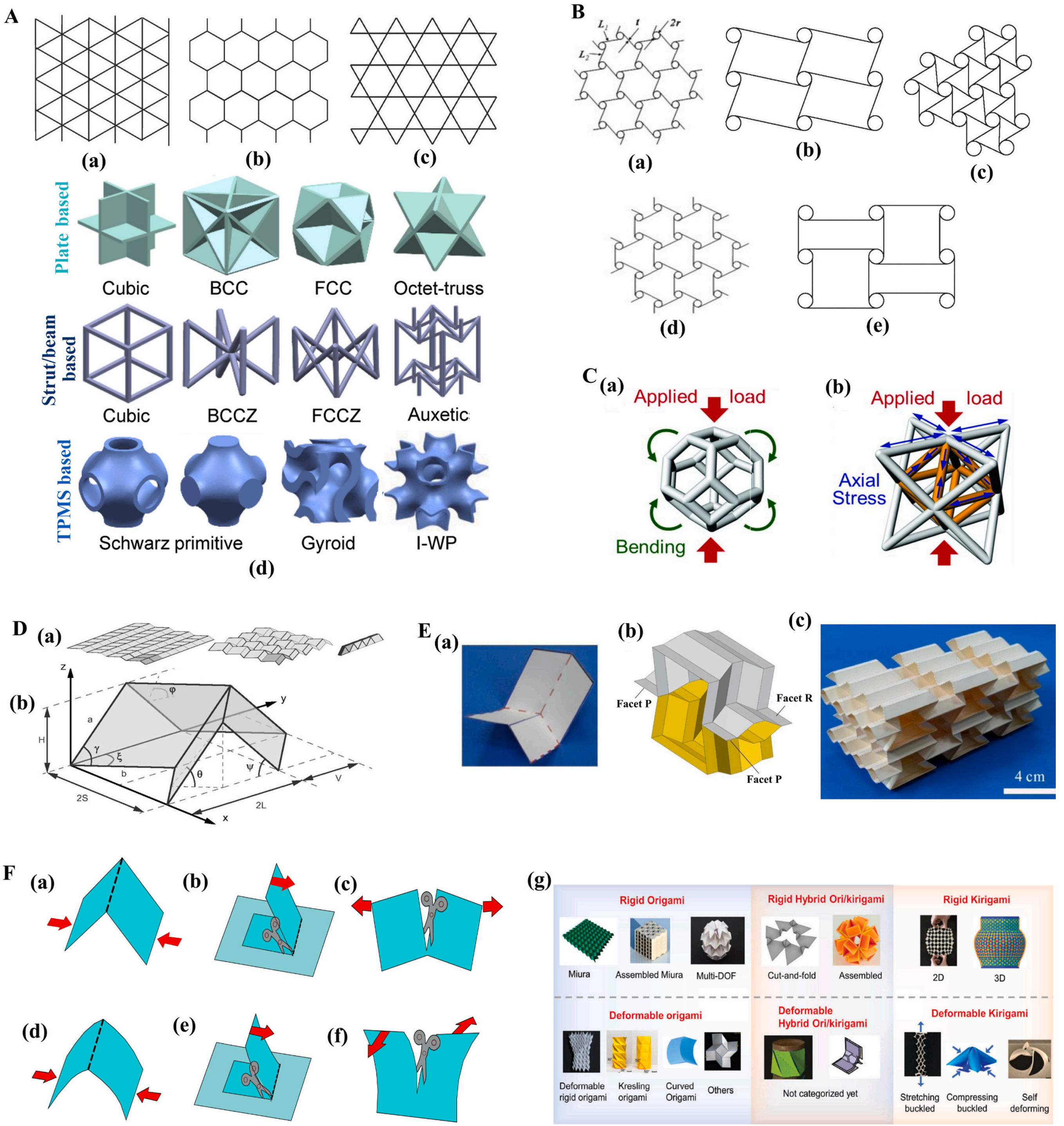


Fig. 5. Microstructural configuration of metamaterials. **(A)** Differently shaped lattices. (a) a regular triangular lattice. (b) a regular hexagonal lattice. (c) a semi-regular triangular-hexagonal lattice, called Kagome [117]. (d) Lattice metamaterials with different types of constituent elements such as plate based, strut/ beam based and TPMS based members [110]. **(B)** Chiral and anti-chiral honeycombs [117]. (a) Trichiral honeycomb [118]. (b) Tetrachiral honeycomb. (c) Hexachiral honeycomb. (d) Anti-trichiral honeycomb [118]. (e) Anti-tetrachiral honeycomb. **(C)** Mode of primary deformation. (a) A bending-dominated lattice unit cell. (b) A stretch-dominated lattice unit cell[119]. **(D)** Origami based mechanical metamaterials. (a) Motion behavior of a Miura-ori sheet. (b) Unit cell geometry of a Miura-ori origami structure [120]. **(E)** Hybrid origami-based metamaterial. (a) A standard Miura-ori patterned unit cell that can exhibit in-plane auxeticity. (b) 3D view of a unit cell of hybrid origami composed of combining the geometry of the unit cell of a conventional re-entrant hexagonal honeycomb and that of a conventional Miura-ori pattern. (c) Metamaterial microstructure obtained by periodic repetition of the unit cell [121]. **(F)** Different categories of origami and kirigami-based metamaterials. (a) Rigid origami. (b) Rigid hybrid ori/kirigami. (c) Rigid kirigami. (d) Deformable origami. (e) Deformable hybrid ori/kirigami. (f) Deformable kirigami. (g) Subcategories of origami and kirigami-based metamaterials [122].

honeycombs exhibit some unconventional properties like negative Poisson's ratio [34,123] or negative dynamic bulk modulus [124]. Chiral structures are those that cannot be superimposed on its mirror image [125,126]. Chiral structures are comprised of constant radius circular elements that act as nodes which are linked to each other by straight tangentially attached ligaments. By changing the spatial arrangement of the ligaments and nodes, chiral lattices of different types can be obtained. For instance, trichiral, tetrachiral and hexachiral lattices can be derived from the classical hexagonal, rectangular and triangular lattice by changing the number of ligaments connecting each node to 3, 4, and 6, respectively. Further, the anti-chiral lattices can be obtained by connecting adjacent nodes by the same side of the ligament. Also based on the number of linking elements, i.e., 3, 4 and 6, these can be anti-trichiral, anti-tetrachiral and anti-hexachiral, respectively (refer to Fig. 5(B)). Other emerging classes of periodic lattice-like geometries involve fractal metamaterials and hierarchical metamaterials [127,128].

2.2.1.2. Stretch and bending-dominated lattices. Bending-dominated cellular materials (like foams) deform by bending of their elements (like beams and plates) while stretching-dominated structures deform through the uniaxial tension or compression of their interconnecting members. The stretching-dominated structures are typically represented by a triangulated arrangement of interconnecting members (refer to Fig. 5(C)). The bending and stretching-dominance can be differentiated by their topological configuration. The specific strength and specific stiffness of the stretch-dominated structures are found to be higher than those of the bending-dominated structures [37], while bending-dominated lattices normally show more energy absorption capability. The scaling laws proposed by Gibson and Ashby [1,113] to study the mechanical behavior of lattice materials by applying standard beam theory, show that the deformation mechanism depends on the topology as well as the material distribution. The scaling ratios for the relative stiffness E/E_s and relative strength σ/σ_s of the bending-dominated and stretch-dominated architecture with respect to relative density ρ/ρ_s are given as

$$\text{stretch - dominated}: (E/E_s)\propto(\rho/\rho_s), (\sigma/\sigma_s)\propto(\rho/\rho_s) \tag{1}$$

$$\text{bending - dominated}: (E/E_s)\propto(\rho/\rho_s)^3, (\sigma/\sigma_s)\propto(\rho/\rho_s)^2 \tag{2}$$

Here the relative stiffness E/E_s is the ratio of effective Young's modulus E of the lattice to Young's modulus E_s of the constituent material. Similarly, relative strength σ/σ_s is the ratio of effective strength σ of the lattice to the strength σ_s of the constituent material. The relative density ρ/ρ_s is the ratio of density ρ of the lattice to the density ρ_s of the constituent solid material. Since the relative density ρ/ρ_s of a lattice material is generally less than 0.2 [113], we can conclude that the stretch-dominated structures are stronger and stiffer than the bending-dominated ones.

2.2.2. Origami/ Kirigami-based metamaterials

Origami is derived from the Japanese words 'ori' (meaning 'to fold') and 'kami' (meaning 'paper'). Thus this is an art of paper folding which has attracted the attention of the scientific community lately. The concept of origami has been extensively used in deployable space structures over the decades due to the ability of compact storage. Lately, with the advancement of manufacturing capabilities, origami is finding increasing attention in the field of developing material microstructures with unprecedented effective mechanical characteristics such as programmable stiffness, negative Poisson's ratio and multi-directional auxeticity, static morphing, extreme deformation and multistability [129–134]. They primarily follow the plate-and-hinge mechanism, wherein plates are connected by compliant hinges, which results in complex geometries with tunable deformation mechanisms. The origami-based metamaterials find potential applications in different shape morphing systems, medical stents, electronic devices, DNA nanofabrication and mechanical parts (like levers, pulleys, gears, etc.) [135–142]. Origami-based metamaterials can result in complex 3D structures by using the principles of folding and assembling 2D planar materials (refer to Fig. 5(D-E)). The geometries of such metamaterials are dependent essentially on two parameters which are creases and vertices, and the resulting shape is based on the sequence, orientation, magnitude and quantity of the folds [143,144].

Apart from origami-based metamaterials, Kirigami, which includes paper cutting, can also be employed to obtain mechanism-based metamaterial. The elastic characteristics of materials can be tailored using this art to obtain extreme strains or topological changes [145–147]. Sheets patterned using Kirigami show buckling and out-of-plane deformation, thereby resulting in intricate 3D structures at multiple length scales for structural and mechanical applications [148–152]. Also, the Kirigami design principles can be coupled with origami-based materials to result in complex folded structures with different cut patterns [153,154]. This essentially results in expanded design space for mechanism-based metamaterials.

The metamaterials based on paper folding and cutting can be broadly classified as origami-based, hybrid ori/kirigami-based and kirigami-based metamaterials. These metamaterials can be both rigid and deformable (refer to Fig. 5(F)(a-f)). The subsequent classification of origami metamaterials can be based on different patterns such as Miura [155], assembled Miura [156], waterbomb [25], eggbox [157,158], kresling [159], square-twist [160] and different other multi-DOF rigid origami [161]. The rigid hybrid ori/kirigami can be cut-and-fold [162] and assembled to obtain new architectures [163]. Further, rigid kirigami can be subcategorized as 2D [164] and 3D rigid kirigami [165]. Based on the nature of creases, origami can be classified to have straight [166] and curved creases [167]. The deformable kirigami can be subcategorized as stretching buckled [168], compressing buckled [169] and self-deforming kirigami [170] (refer to Fig. 5(F)(g)). The deformable hybrid ori/kirigami metamaterials [171,172] can be categorized based on the nature of adopted origami or kirigami patterns.

3. Multi-physical property modulation in mechanical metamaterials

Mechanical metamaterials are a class of metamaterials that are obtained by aligning different microstructures to realize the application-specific mechanical properties at macro-scale [173]. The effective properties of these artificially engineered microstructures are dependent not only on the intrinsic characteristics of the constituent elements but also on the geometric and structural configurations [21,174,175]. Such metamaterials can offer various unprecedented, unconventional, extreme and beneficial properties by the intelligent design of microstructures [93,176–179], such as negative stiffness [180], vanishing shear modulus [181,182], negative compressibility [183–185], negative Poisson's ratio [34,123,186–188] and singularly non-linear behaviour [189,190]. Compared to conventional composites (such as fiber-reinforced laminates) [191–194], mechanical metamaterials essentially provide an extended design space with a significantly improved scope of multi-functional property modulation [195]. Thus mechanical metamaterials use topological optimization (structured or unstructured) rather than material composition to obtain the resultant properties [196].

Intensive investigation on the unit cell level topology to derive different effective mechanical properties has led to some level of saturation in metamaterial development by solely unit cell geometry. To push the boundaries further, one of the evolving trends is to adopt a bi-level design, wherein the elementary constituting components of the unit cell (such as beam/ plate-like elements) are further designed [11,197–201]. The interaction of the mechanics involved at the elementary constituting element level and at the unit cell level leads to the realization of extreme properties through an expanded design space.

Through this bi-level framework, it is possible to involve the passive and active physics of deformation at the beam level (such as the effect of intrinsic beam-level pre-existing stresses [202], viscoelasticity of the intrinsic material [16], or multi-physical behavior like electro [200,201] and magneto [199] active deformation) along with the unit cell level mechanics. It may be noted that the inclusion of active elements (where the deformation can be controlled by non-mechanical external stimuli like magnetic or electric fields) in metamaterials leads to the active on-demand control of the effective constitutive behavior of metamaterials including fundamental properties like elastic moduli. In the following subsections, we discuss the recent developments in active and passive mechanical metamaterials with a focus on bi-level design and multi-physical behavior.

3.1. Passive property modulation

The mechanical metamaterials in which property modulation is not possible after manufacturing are referred to as passive metamaterials. In

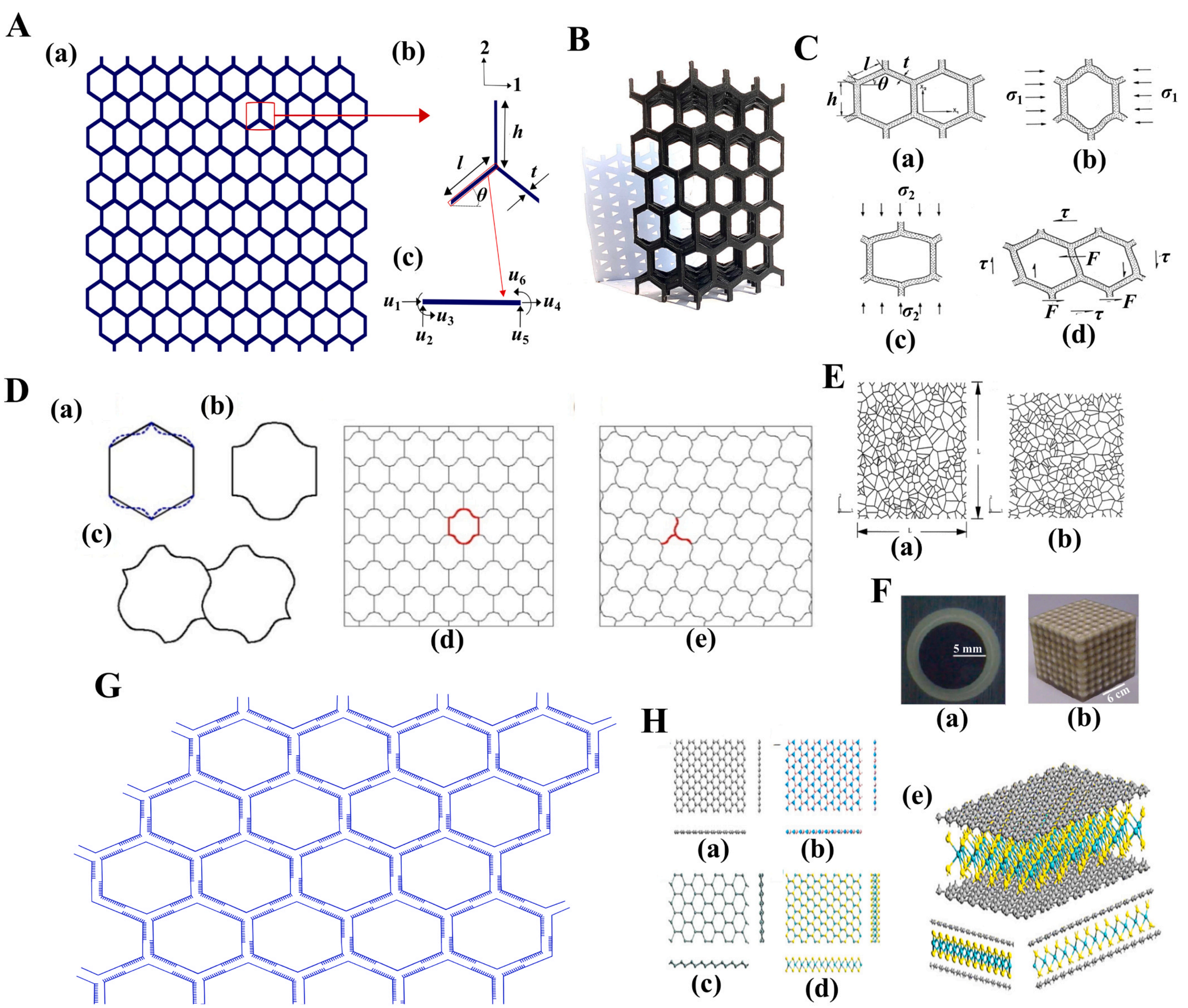


Fig. 6. Passive property modulation of mechanical metamaterials. (A) Analysis of honeycomb lattices based on unit cell based bottom-up approach. (a) Regular 2D honeycomb lattice. (b) Unit cell of honeycomb which forms the entire lattice, when tessellated. (c) A cell wall of honeycomb considering it as a beam element with degrees of freedom [202]. **(B)** Regular 3D honeycomb lattice [203]. **(C)** Mechanics of 2D hexagonal honeycomb. (a) An undeformed honeycomb. (b) Unit cell subjected to in-plane loading along X_1 direction. (c) Unit cell subjected to in-plane loading along X_2 direction. (c) Unit cell subjected to in-plane shear loading [204]. For other 2D and 3D lattices, the mechanics of appropriate unit cell needs to be analyzed. **(D)** Anticurvature effect in lattices under different stress conditions. (a) Unit cell of a regular hexagonal lattice. (b) Unit cell having curved cell walls obtained when subjected to compressive stress against tensile loading along X_1. (c) Unit cell having curved cell walls obtained when subjected to shear stress. (d) Anti-curvature in cell walls of honeycomb lattice against tensile normal stress acting in X_1. (e) Anti-curvature in cell walls of honeycomb lattice against anti-clockwise shear stress. [205]. **(E)** Cell irregularity in 2D Voronoi honeycombs. (a) A random voronoi honeycomb in undeformed configuration. (b) Deformed voronoi honeycomb structure with periodic boundary conditions [206]. **(F)** Wave propagation in mechanical metamaterials (a) Cross-section of the spherical atom with resonance. (b) Acoustic metamaterial with unit cell in sonic crystal [207]. **(G)** A typical non-invariant honeycomb lattice with domain discontinuity [197]. **(H)** Concept of nano-scale metamaterials based on nano-heterostructures. (a) Side and top view of a typical nanostructure wherein a single type of atom forms the entire structure in a single plane. (b) Side and top view of a typical nanostructure wherein different types of atoms form the entire structure in a single plane. (c) Side and top view of a typical nanostructure wherein an atom forms the entire structure in multiple planes. (d) Side and top view of a typical nanostructure wherein different types of atoms form the entire structure in multiple planes. (e) Multi-layered heterostructure composed of different 2D materials. [208].

this class of metamaterials, it is possible to achieve target properties (including multi-functional features) based on the microstructural design [113] (refer to Fig. 6). In this subsection, we discuss some of the most critical and prominent functionalities for which material microstructures have been developed.

3.1.1. Effective elastic properties and mechanical constitutive behavior

The elastic properties of the passive mechanical metamaterial rely on the intrinsic material properties and microstructural geometry. The characterization of effective elastic properties is crucial to utilize these metamaterials in different structural and mechanical applications. Honeycomb lattices are one of the most typical configurations of mechanical metamaterials and thus have been extensively investigated for different useful effective properties. Study in this field includes the work on derivation of effective elastic properties of regular hexagonal honeycomb structures [113,209,210] considering beam-level bending, axial and shear deformations. Periodic hexagonal honeycombs (including their derivatives like rhombic, rectangular and auxetic configurations) were numerically and analytically analyzed considering different cell geometries and relative densities [211]. The impact of imperfections (like misaligned or fractured cell walls, variation in cell-size, non-uniformly thick cell walls and missing cells) on the mechanical properties of honeycombs were studied [212,213]. The influence of non-periodic voronoi microstructure (refer to Fig. 6(E)) on the effective elastic properties of honeycomb was studied [214]. The effect of different microstructural geometric parameters such as cell aspect ratio, relative thickness and cell angle on the Young's moduli and Poisson's ratios of honeycomb structures were experimentally and numerically studied [215]. Isogeometric analysis and model order reduction were used for 3D topology optimization of auxetic metamaterials [216]. In recent works, the equivalent in-plane elastic properties were obtained for the irregular honeycomb lattice structures using an analytical framework [206,217]. Further, the viscoelastic effect was studied along with structural irregularity [16]. Nano-scale analysis of the elastic properties of lattice-like 2D materials has been presented analytically and computationally including the mechanical characterization of nano-heterostructures that essentially bring the notion of architected design at nano-scale [208,218–224] (refer to Fig. 6(H)).

The effect of inevitable intrinsic stresses that may be regarded as a form of manufacturing irregularity in lattice structures was investigated considering 2D honeycomb lattices through a bi-level analytical framework [202] (refer to Fig. 6(A)). This analysis was further extended to 3D lattices where the effective elastic properties were investigated through an analytical framework [203] (refer to Fig. 6(B)). It is noted that such beam-level intrinsic stresses can also be exploited as a beneficial effect to improve and modulate the lattice-level effective properties. Non-invariant elastic moduli in normal and shear modes have been achieved in the linear deformation regime by introducing domain discontinuities in the cell walls of the honeycombs [197] (refer to Fig. 6 (G)). The effective elastic moduli in tension and compression or clockwise and anti-clockwise modes can be programmed to be different as a function of the placement and severity of such domain discontinuities. The concept of anti-curvature in lattices was introduced recently and subsequent characterization of effective elastic moduli and failure strength was carried out based on a bi-level semi-analytical non-linear framework [205,225,226] (refer to Fig. 6(D)). The strength and stiffness of these lattices can be significantly enhanced depending on the degree of anti-curvature. Programmed out-of-plane curvature was proposed to enhance multimodal stiffness of bending-dominated composite lattices [227], while maintaining their conventional multifunctional advantages such as high energy absorption capacity. Extreme specific stiffness can be achieved through bi-level cellular networks in micro-topology architected metamaterials [228], wherein the constituting beam-like elements in a lattice were further topology-optimized in addition to unit cell level geometry. A recent study proposes kirigami-inspired modular metamaterials for contact-induced stiffness modulation and programming of constitutive laws [24]. It is further shown that mixed-mode multi-directional auxeticity (including the transition from a non-auxetic to auxetic behavior, and vice-versa) and programmable stiffness can be achieved through origami and kirigami-inspired metamaterials without external non-mechanical stimuli [24,25,93]. The above-presented literature review of passive property modulation concerning effective elastic moduli and constitutive behavior shows that the effect of unit cell level geometry has been extensively investigated along with the recent trends of exploiting material physics of the intrinsic material and bi-level design frameworks. It can be noted that the elastic moduli and the mechanical constitutive behavior of metamaterials are fundamental to analyzing the static and dynamic characteristics of structures.

Apart from the discussion on elastic moduli and constitutive behavior, mechanical metamaterials can further be classified as extremal materials and negative materials. Extremal materials are very stiff in certain deformation modes and very compliant in other modes [229]. The extremal materials can be further subcategorized as penta-mode metamaterials and dilational metamaterials. The penta-mode metamaterials [181] have a very large bulk modulus in comparison to their shear modulus. High value of bulk modulus results in no volume change on deformation and very less value of shear modulus makes them similar to fluids. Thus, penta-mode materials are also known as 'meta-fluids'. The penta-mode metamaterials are used in diverting the elastodynamic waves in desired directions resulting in optical cloaking for acoustic waves. The dilational materials [186], on the other hand, have very large shear modulus compared to their bulk modulus. This implies that on deformation, only their size changes while the shape remains the same, which results in improved damage resistance. Negative metamaterials can be those with negative elastic modulus or negative bulk modulus [230]. Negative elastic modulus implies negative stiffness. Negative stiffness metamaterials deflect in a direction opposite to that of the external force applied. Similarly, negative bulk modulus results in negative compressibility. This means when hydrostatic pressure is applied, these materials expand. Along with the extremal and negative metamaterials, there are mechanical metamaterials that exhibit ultra-property characteristics, i.e., the rationally designed mechanical metamaterials can show ultra- high stiffness, high toughness, high strength and at the same time can have low mass density [231,232]. The fatigue behavior of passive mechanical metamaterials has recently caught attention of the researchers with a focus on the prospective enhancement in fatigue tolerance through designing [233]. The characteristics as discussed above are rare, unusual and are not available in conventional materials.

3.1.2. Wave propagation and vibration

Wave propagation and vibration are important features in the design of mechanical metamaterials under dynamic environment [234,235]. The dynamics of periodic structures has been investigated since the mid 60's [236]. Note that the term mechanical metamaterial was not popular during that period. The investigation of wave propagation in metamaterials for the majority of studies is based on the Floquet-Bloch theorem which is a computational method considering the unit cell boundary condition [237]. The wave propagation response of a unit cell is essentially a dynamic feature and the concept of band gaps [234,238] is used to understand it. Thus recent studies have focussed on the evaluation of band gaps of metamaterials using numerical and analytical methods [239–242]. Undamped metamaterials were studied using classical wave propagation methods in a large number of such literature. Some researchers considered internal damping in unit cell for damped metamaterials [243,244]. A significant number of recent investigations have employed machine learning to study the wave propagation behavior [245,246].

Vibration is a crucial aspect of metamaterial analysis and design since it may result in resonance and fatigue, thereby leading to structural failure. Thus it becomes necessary to control, suppress, or mitigate the undesired vibrations [247]. The use of a wave barrier in the path of transmission can reduce the unwanted disturbance by disturbing the source-receiver vibration path [248,249]. The passive control of vibration includes relocating the natural frequency of the system away from the working span. This is done by adding layers of damping material, varying stiffness and adding mass. This passive control does not require any complex system and the vibration is absorbed by changing the geometric parameters of the system. Research in this field includes the use of mechanical metamaterials for vibration suppression. It combines the idea of periodic structures with the control of mechanical properties of structures along with the use of smart materials. The phononic crystals that use filtering of elastic waves work on basis of periodic structures. The wave dispersion properties are changed due to the diffraction caused when wave travels in periodically aligned media (refer to Figure 6(F)). The spatial crystal alignment decides the Bragg bandgap position. Vibration control can be achieved by using thin metamaterial plates, micro-structured metamaterial, quasi-zero-stiffness metamaterial and by using metamaterial beams with embedded absorber [250–253]. Passive metamaterials have been proposed involving smart beams coupled with resonant shunt circuits including random impedance disorder for wave propagation and vibration control [254,255]. Such Passive control techniques rely on modification in the basic structure without the requirement of a source of external stimuli. Recent studies have investigated the frequency dependence of the effective elastic moduli of lattice metamaterials under a dynamic environment [12,256,257]. It is shown that the effective elastic moduli can increase significantly at higher frequencies, which can be exploited for optimal structural design under operational conditions with ambient vibration.

3.1.3. Energy absorption

The design of a regular standard energy-absorbing mechanical metamaterial generally involves a bending-dominated microstructure [258]. The unit cell is designed with cell walls having energy dissipating characteristics and then the metamaterial is obtained by periodic repetition of the unit cell [259]. Both 2D and 3D lattice metamaterials have been investigated for their energy absorption capability [260,261]. An innovative energy-absorbing mechanical metamaterial has been propounded [262] using bi-material components with negative stiffness by combining rigidity with softness. The emerging capabilities of machine learning have been exploited in a recent study proposing failure pattern-driven honeycomb lattice materials for strength and energy absorption capability enhancement [115]. Origami patterns have been used to develop new metamaterials with enhanced energy absorption features [263].

With the advancement in manufacturing capability, different forms of energy-absorbing structures have drawn tremendous attention of researchers [264]. Buckling-induced energy-absorbing mechanical metamaterials have been proposed for enhanced performance [265]. A significant amount of investigations on beam-based energy-absorbing mechanical metamaterials have been reported [266,267] that analyze the relation between beam parameters and their connectivity with the energy absorption capability [268–270]. A multistable mechanical metamaterial was designed using bistable curved beam model, wherein the energy absorption could be controlled [271]. Thus by intelligently configuring the cell walls and their orientations of a unit cell, the energy absorption capacity of mechanical metamaterials can be improved.

3.1.4. Energy harvesting

Mechanical metamaterials can be used in energy harvesting applications by controlling the magnitude of energy flow. For instance, in acoustically reflecting surfaces, acoustic waves are absorbed and converted into other forms of energy [272,273]. Planar structures inspired from metamaterial have been shown to harvest mechanical wave energy by wave guiding, wave focusing and localization of energy [274,275]. The locally resonant metamaterials can have resonators randomly distributed throughout the material as opposed to Bragg scattering-type phononic crystals [276,277]. The membrane-type metamaterials can attenuate waves and harvest energy through the energy harvesting devices introduced in them [278]. A novel idea of combining energy harvesting features in a periodic structure with resonant cavity has been proposed [279,280]. A piezoelectric material with defective acoustic metamaterial can act as an acoustic energy harvester [272] by introducing resonant defects that trap the strain energy resulting from an acoustic incidence. Topological surface wave metamaterials were proposed recently for the dual functionality of robust vibration attenuation and energy harvesting [281].

3.2. Active property modulation

The mechanical metamaterials in which on-demand effective property modulation can be realized even after manufacturing are the ones with active property modulation. This can be achieved by using active materials in the unit cells and activating them through external stimuli like magnetic or electric fields (refer to Figure 7). Recent studies in this field include the coupling of stimuli-responsive materials with the design of unit cell based microstructural configuration of mechanical metamaterials. Under the external stimulus, these metamaterials can show unconventional characteristics and thus can be put to use as per different application-specific requirements based on live operational demands. The external stimuli can be pressure action [282,283], heat [284,285], light [286], magnetic field [287–289], electric current [290,291] and chemicals [292,293]. Further, shape memory alloys can be used in metamaterials for achieving active behavior including shape morphing [294]. Note that such active property modulation often entails the multi-physical behaviour of mechanical metamaterials including the mechanics of deformation under mechanical load and non-mechanical stimuli. This multi-physical behavior at the elementary beam-level and the micro-scale unit cell level can thus manifest unprecedented active properties at the macro-scale metamaterial-level through a bi-level design paradigm. In the following subsections, we discuss the recent progress in the multi-physical behavior of active metamaterials under different external stimuli.

3.2.1. Thermal-responsive active mechanical metamaterials

Thermal actuation approach works on the principle of controlling the exchange of heat to modify the behavior of thermal-responsive materials. Broadly, thermal actuation involves photo-thermal effects, magneto-thermal effects, electro-thermal effects and thermo-chemical reactions, which are one of the most extensively used actuation methods for active mechanical property modulation in metamaterials. The actuation can be provided either by heating the metamaterials directly or heat transfer to metamaterials by altering the temperature of the surroundings. Using the most widely available typical mechanical metamaterial microstructures, i.e., lattices and origami structures, thermal-responsive active property modulation has been achieved. Lightweight active lattices were developed in which significant variation in stiffness was observed with variation in temperature [285]. Under impact loading, these structures could absorb the shock significantly and could also restore back to their initial shape even in case of large deformation. 4D printing was used to produce auxetic thermally-activated origami-based metamaterials [299], wherein the deformation behavior and mechanical properties were explored. Metamaterials based on chiral configurations that responded to external thermal stimulus were developed [300,301]. Simple structures that could achieve different shapes like shell, tube, sector, rod, etc., from a plane state under thermal stimulus were developed [302]. Metamaterials with robust stiffness, extensible aspects and recoverable deformation at particular temperatures were prepared [298] (refer to Fig. 7(I)). The thermal expansion coefficient was made controllable in the design of multi-stable structures from 1D to 3D [284]. Mechanical metamaterials with adjustable coefficients of thermal expansion were designed [303], wherein the creases of the origami panels were arranged

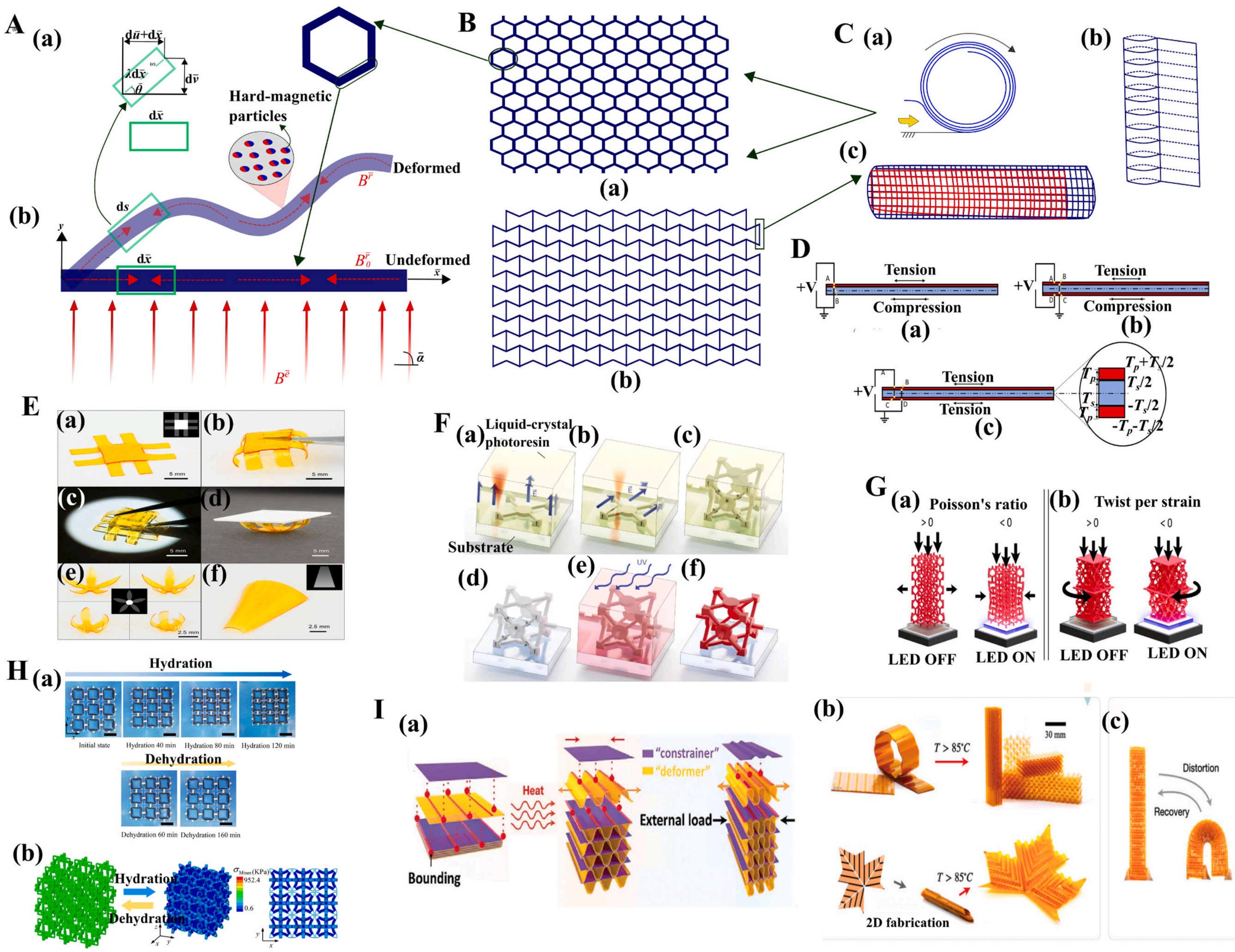


Fig. 7. Active property modulation of mechanical metamaterials. (A - D) Bi-level active lattice metamaterials. The lattices (auxetic and non-auxetic) shown in Fig. B are made of different active beam components as shown in Figures A, C and D. Magneto-driven active mechanical metamaterial is formed based on the Hard Magnetic Soft (HMS) beam shown in Figure A. A HMS beam in both undeformed and deformed configurations under external magnetic actuation is shown [11]. Pressure-driven active mechanical metamaterials are formed based on the inflatable beam shown in Figure C. Coiling and compact storage is possible in such inflatable lattices [198]. Electro-driven active mechanical metamaterials are formed based on piezoelectric composite beams as shown in Figure D, wherein pure bending, pure stretching and combined bending/ stretching modes are possible through unimorph and bi-morph configurations [200]. **(E)** Light-driven active mechanical metamaterial. (a) Polymer sheet right after photopolymerization. (b) Free bending of spatial, differently cured sheet. (c) Shape fixing of bending structures under a uniform light by post-curing. (d) Stiff sample after post-curing. (e) Different opening degrees in flower structures. (f) Continuous variation of curvature in Polymer sheet [295]. **(F)** 3D optomechanical metamaterial. (a) Liquid-crystal director of photoresin is oriented using an external quasi-static electric field, $\vec{E}$, and fixed locally by voxel printed by two-photon polymerization (TPP). (b) Different director alignment for another part of the structure resulting in changed alignment of electric field. (c) Polymerized and unpolymerized regions of the structure. (d) Optically transparent polymerized structure comprising a targeted 3D liquid-crystal-director field. (e) Sample immersed in a dye solution which diffuses into it and acts as an absorber to couple to the stimulus light. (f) Final metamaterial [296]. **(G)** Mechanical metamaterial stimulated by light. Lattice-based structure showing positive and negative (a) Poisson's ratio and (b) twist per strain when external LED is turned off and on, respectively [296]. (H) Chemical-driven active mechanical metamaterial. (a) Deformation pattern of a 2D metamaterial when in hydration and in dehydration. (b) Negative hydration expansion deformation of 3D metamaterial [297]. (I) Thermal-driven active mechanical metamaterial. (a) Deformation of a thermally actuated super-elastic metamaterial. (b) Transformation of elastic thin laminates into volumetric materials. (c) Large deformation of the metamaterial that can regain its shape on unloading [298].

at certain locations. The thermal expansion coefficients achieved positive, zero and negative values with a change in temperature. Structures with different glass transition temperatures were made using modular 4D printing, wherein the structural deformation due to change in temperature could be configured arbitrarily [304]. Thus we see that thermal energy has been exploited widely as an external stimulus in the design of active materials with the capability of on-demand shape and mechanical property modulation.

3.2.2. *Light-responsive active mechanical metamaterials*

The mechanical behavior of light-responsive materials is controlled by the external light stimulus. The properties of materials responsive to light can be obtained by combining light-responsive functional groups in polymer agents (refer to Fig. 7(F-G)). Thus the light actuation can be photo-thermal and photochemical [305]. The photo-thermal actuation is a type of thermal stimulus and thus similar concepts are applied in the design of microstructure of mechanical metamaterial, while the photochemical stimulus is free from the effect of temperature. The photochemical effect is complex when compared to the photothermal effect. The design of these mechanical metamaterials actuated by light uses the multiphysical mechanics of the light-stimulated materials coupled with the geometry of microstructure. A deployable 3D origami structure was developed using the concept of volume shrinkage effect in photopolymerization [295] (refer to Fig. 7(E)), wherein the light time controlled the structure deformation. A rapid shape-changing and adaptable 4D printing material system was designed [306]. The structural deformation occurred when light volatilized the volatile substances in polymer. After volatalization, light curing was done for the residual nonvolatile materials in the polymer network. The deformation mechanics of square-twist origami structures under external light stimulus was analysed [286]. Photosensitive hinges, that shrinked in repsonse to light, were used in the design of metamaterials [307]. Owing to their fast, accurate and controllable response, these metamaterials find extensive use in soft robotics. The idea of crawling of caterpillars was used in the design of microrobot that activated on light actuation [305]. A tensegrity robot was constructed using rigid plastic rods [308]. Similarly, the configuration of a kirigami structure was used in designing a rolling robot [309].

3.2.3. Chemical-responsive active mechanical metamaterials

In addition to the above-discussed active property modulation, chemicals can also add to the multiphysical aspect in the design of mechanical metamaterials. The liquid environment is a crucial factor upon which chemical-responsive materials depend. Hydrogels can quickly modify their shapes when actuated by external chemicals like pH, salinity, humidity or temperature. They are soft hydrophilic materials that have water-absorbing and discharging potential. Flexible deformability and programmability were shown by patterned chemically actuated hydrogels [310]. Printing technology with ion transfer property was developed which printed simple hydrogel structures that would modify into more complex forms at larger dimensions [311]. Programmable composite hydrogel sheets based on kirigami design were fabricated [312]. Auxetic metamaterials with negative hydration expansion function were designed [297] (refer to Figure 7(H)). Active metamaterials with water-induced shape memory effect were proposed [313]. The physical and chemical expansion effects were shown to affect the water-driven shape memory process. Auxetic metamaterials were designed using plates in two layers with different swelling characteristics [314]. The plates underwent out-of-plane bending when they absorbed solvent leading to negative swelling.

Insertion of metal cations results in a different type of active metamaterials. These cations enhance the physical properties and are useful in controlling deformation. An origami with multi-physical mechanics was proposed involving a combination of mechanical and chemical effects [292]. The origami bending depended on the stiffness mismatch of the cations. The deformation of periodic hydrogel 2D sheets was controlled by the concentration of cations in the solution [315]. Also, the pH-actuated metamaterials are attracting researchers due to the programmable behaviour of active metamaterials under different pH environments. The pH triggered the change in configuration from 2D to 3D in pH-actuated metamaterial [316]. The chemical-actuated structures find application in sensors, soft robotics and different biomedical devices [317].

3.2.4. Electro-responsive active mechanical metamaterials

Electrically actuated metamaterials work on the principle of change in mechanical behavior induced by electric field [318]. Thus the multiphysical aspect of these metamaterials gets involved since electrical, thermal and mechanical forces are coupled to give unconventional deformation characteristics. They can be broadly of two types as follows. In the first category, mechanical deformation occurs by the use of thermal energy produced by current in conductors. Thus these are essentially thermally actuated metamaterials where the temperature is generated through electrical fields. The other category gets triggered by external electrical actuation caused by chemical or physical reactions. These include electroactive polymers, electrochemically stimulated materials and ionic polymer-metal composite materials.

The works in the electro-thermal driven materials include the concept of heat generated by electrification being used as a stimulus for controlling deformation. Origami structures actuated by electro-thermal stimulus were designed which could undergo swift and reversible folding based on the input voltage [319]. A metamaterial robot was created that could modify its shape in an unconfined manner obtained by merging functional fibres with the fabric [320]. In a recent work concerning piezoelectric lattice metamaterials, it is shown through a bi-level framework that the effective elastic properties can be actively modulated as a function of electric field [200,201] (refer to Fig. 7(D) (a-c)). The Poisson's ratio showed both positive and negative values at specific applied voltages and the geometric parameters of the microstructure.

The electrochemical effects were utilized to report a new class of active metamaterials [318,321–324]. The microstructure of the metamaterials was found to align electrochemically by alloying and dealloying reactions [322]. A nanoscale electrochemical actuator surface was prepared wherein the actuator was combined to obtain origami robots [321]. Another work used metal liquid in the unit cells of metamaterials [323], wherein the resistance of the metal liquid changed with the application of force that resulted in the deformation of the microfluidic channel. The electro-stimulated metamaterials were used in the filed of botany wherein the mechanical signals were converted to electrical signals, resulting in conductive metamaterials with extreme adhesion property [324].

3.2.5. Magneto-responsive active mechanical metamaterials

The requirement of shape morphing materials in different structural and industrial applications has resulted in the development of novel mechanical metamaterials that get actuated by the application of magnetic fields like the magneto-mechanical foams or magneto-elastic lattices [325,326]. A study in this field demonstrated that under a constant magnetic field an auxetic magneto-mechanical polyurethane foam had varying acoustic absorption coefficient [325,326]. Research on magneto-mechanical metamaterial showed that under an external magnetic field these active metamaterials can undergo reconfiguration [327–331]. 3D printed auxetic structures obtained by embedding magnetisable inclusions in rubber matrix shrank when subjected to an external magnetic field [332]. In another study, magnets were inserted into elastomers which showed coupled twist-buckling behaviour in response to large magnetic fields [333]. Magnetic inclusions were shown to control the deformation of semi-rigid as well as rigid mechanical lattices [328,334–336].

Magneto-mechanical metamaterials showed improved impact resistance [337] and wave attenuation [338,339]. This can lead to their application in protective and damping devices with a notion of on-demand control. A mechanical metamaterial made with hexagonal units embedded with magnetic particles was shown to have negative stiffness and negative Poisson's ratio simultaneously [340]. This work on quasi-2D metamaterials was then applied to 3D structures [336]. At nano-scale, these magneto-mechanical metamaterials were found to alter the magnetic domain evolution [341,342]. A recent study proposed an innovative magneto-mechanical metamaterial that can be miniaturized and showed shape programming and shape recovery [343]. It has been shown that an active on-demand programming of effective elastic moduli is possible following a contactless framework in magnetostrictive lattices [199]. It was demonstrated that the same metamaterial can behave like stiff metals or soft polymers, depending on the applied magnetic field.

Over the years, hard-magnetic soft active materials have drawn extensive attention of researchers as they show reversible and rapid shape changes [332,344]. In these materials, hard-magnetic particles were embedded in a soft elastomer matrix, leading to high residual magnetic flux and high coercivity. These materials being soft can undergo large deformation and the application of an external magnetic field in conjunction with remnant magnetic field results in non-linear deformation. A recent study in this area has utilized the multi-physical behaviour of the magneto-driven metamaterials at the elementary beam level and obtained the effective elastic properties at lattice level [11] under the combined effect of mechanical load and magnetic field (refer to Fig. 7(A-B)). These active mechanical metamaterials have extensive potential in biomedical devices [345] and soft robots [344,346,347].

3.2.6. Pressure-responsive active mechanical metamaterials

The pressure-driven approach for property modulation has been one of the most extensively used actuation methods. The shapes of these metamaterials change under the effect of pressure that can be induced either by inflation or deflation. A recent study in this field observed the change in mechanical behaviour of a soft metamaterial when subjected to a pressure gradient. In this work, the concept of pneumatic actuation was combined with the design pattern of the metamaterial to result in adjustment effects [348]. A pressure-sensitive soft bending metamaterial was reported with auxetic properties [349]. Metamaterials with holes arranged periodically were designed, wherein the shape of holes could be tailored pneumatically which resulted in variation of the mechanical behaviour [350]. A metamaterial with adjustable negative stiffness was designed by the introduction of cavities that could inflate like balloons [351,352]. The pneumatic actuation approach is apt for origami and kirigami structures due to the advantage of folding and

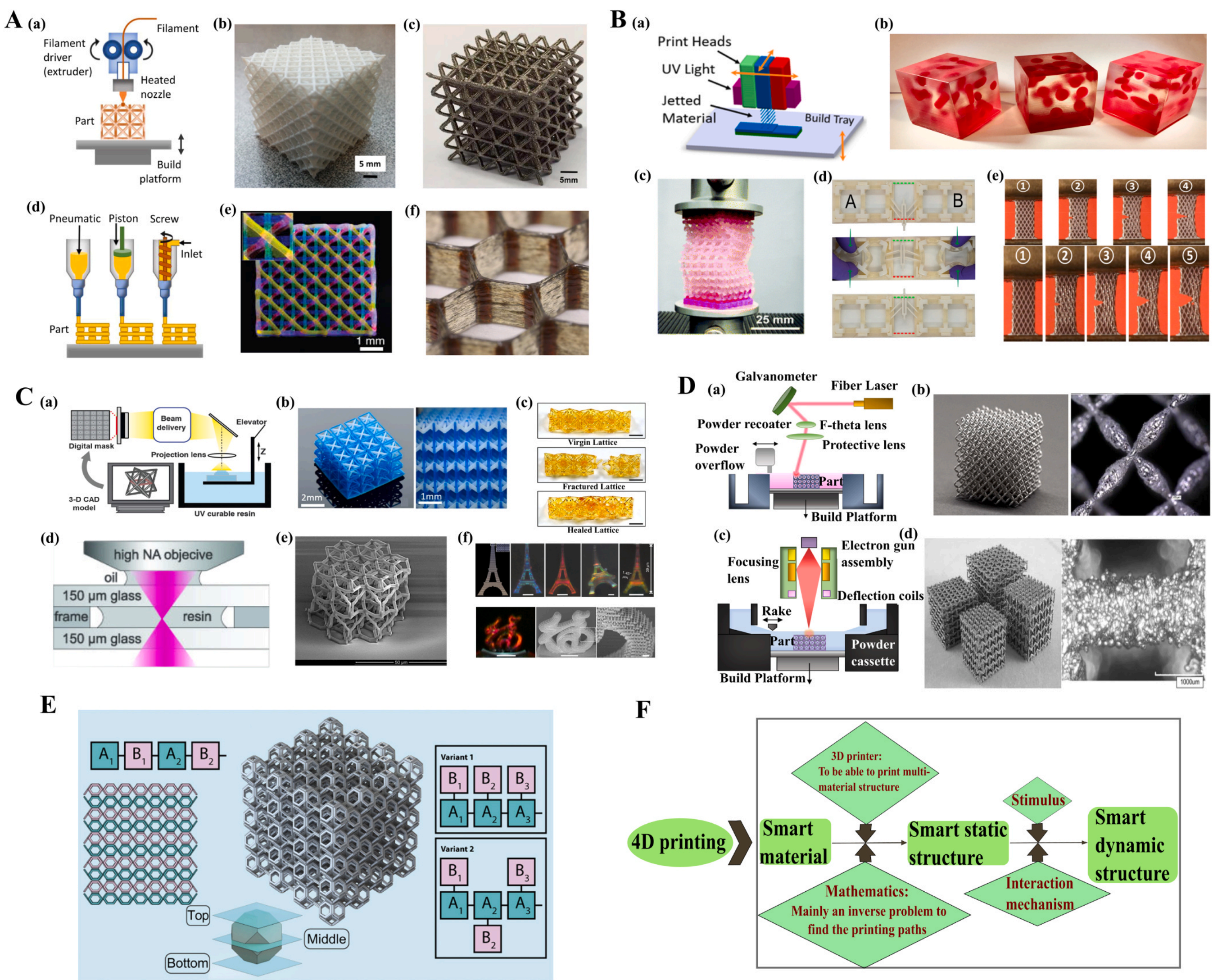


Fig. 8. Physical realization of of mechanical metamaterials. (A) Material extrusion process. (a) Demonstration of the setup and working methodology of Fused Deposition Modeling (FDM) [375]. (b) A FDM based polymer octet-truss lattice [376]. (c) A composite lattice having Ni outer shell and polymer core fabricated by electroless plating and FDM [377]. (d) Illustration of the basic working methodology for Direct Ink Writing (DIW) [375]. (e) A DIW based 3D micro-lattice [378]. (f) A fiber-filled epoxy honeycomb composite fabricated using DIW [379]. **(B)** Inkjet printing. (a) Schematic illustration of setup of inkjet printing (IJP) [380]. (b) Particle composites with tunable size and shape printed using Polyjet [381]. (c) Buckling in a periodic lattice due to insertion of rigid defects in it using Polyjet printing [382]. (d) Physical AND gate that allows for logically tunable properties, manufactured using PolyJet printing [383]. (e) Controlling fracture properties of a 2D composite material (PolyJet printing) by changing ratio of digital materials [384]. **(C)** Vat photopolymerization. (a) Illustration of digital light processing (DLP) technology [385]. (b) Metamaterials with tailorable Poisson's ratio created using multimaterial μDLP method [385]. (c) Healing in lattices as a result of the flexibility of technology in vat polymerization [386]. (d) Illustration of two-photon polymerization (TPP) technology [387]. (e) TPP can result in microlattices with tunable buckling properties [388]. (f) TPP process can lead to greater lattice spacings since it allows for the structure to shrink which leads the structure to show photonic properties [389]. **(D)** Powder bed fusion. (a) Demonstration of setup for Selective Laser Melting (SLM) [375]. (b) Pentamode metamaterial printed using SLM [390]. (c) Demonstration of setup for Electron Beam Melting (EBM) [375]. (d) Lattices fabricated using EBM [391]. **(E)** The motion kinematics for folding hexagonal lattice structures which is used for folding it from a flat configuration. The design of the lattice structure is based on the repetitive unit cell [392]. **(F)** Manufacturing of active metamaterials using 4D printing and realization of active time-dependent and programmable response [393–395]. This is essentially achieved though coupling 3D printing and active materials to manufacture time-dependent and external stimuli-sensitive metamaterials.

deployability [353,354]. Recently extreme specific stiffness was achieved in inflatable lattices through a bi-level framework. Here the elementary-level mechanics of inflatable beams was utilized to get lattice-level effective elastic properties. These lattices could be put to use in different structures with the requirement of deployability, storage and portability [198] (refer to Fig. 7(B-C)) along with active control of elastic moduli. Other studies used the viscosity of fluids to design hydraulic-driven active metamaterials which can thus be utilized in all metamaterial designs which are pneumatic actuated [355]. One such work presented a fluidic origami whose stiffness could be tailored by the volume of fluid [356]. The pressure-actuated metamaterials when combined with other physical parameters give unprecedented novel properties such as autonomic perspiration in hydrogel actuators [357].

4. Physical realization of mechanical metamaterials

The recent advancement in design tools based on extensive computational capabilities has led to more complex microstructures of mechanical metamaterials [358–362]. Since the geometric tailoring is done at micro or even nano-scale, the conventional subtractive manufacturing methods do not serve the interest. Further, the microstructures could be made of multiple materials [108,363]. Thus researchers have started to use 3D printing or additive manufacturing to realize complex microstructural geometries (refer to Fig. 8). Progress in additive manufacturing has made possible the fabrication of submicron parameters that was not possible earlier [364–366]. In addition, multi-material additive manufacturing [367–371] essentially provides an expanded design space than that by conventional fabrication techniques. Recent advancements in 3D printing have also enabled the fabrication of stimuli-actuated materials resulting in structures with transformable functionality, geometry or property. This is commonly referred to as 4D printing [372–374] for manufacturing active mechanical metamaterials (refer to Fig. 8(F)). In the following subsections, we briefly discuss different 3D printing techniques used for manufacturing mechanical metamaterials.

4.1. Material extrusion

This manufacturing approach includes both Fused Deposition Modeling (FDM) and Direct Ink Writing (DIW). It is one of the most popular methods of manufacturing mechanical metamaterials owing to its simplicity. FDM is used extensively to print engineering thermoplastics [396–398] (refer to Fig. 8(A)(a-c)) and particle-embedded composites, where high accuracy is not required. In this method, a 2D slice is drawn using solid polymer filament which gets layered upon a build plate. Each layer gets deposited one by one till the complete 3D part is obtained. DIW uses a viscoelastic or viscoplastic ink which gets deposited on the application of pressure to a nozzle to draw the 2D slice [399,400] (refer to Fig. 8(A)(d-f)). DIW is widely used for printing different porous microstructures including beam or strut. The material extrusion process is a slow method since each layer gets deposited one at a time which starts from 1D lines. Fig. 8(A)(b) shows the printed sample of a 3D arrayed truss structure [376] using this method. The thickness of struts of these structures was kept to the thinnest printable size possible to obtain the smallest features. Similarly, Ni-coated polymer meso-lattice composites (refer to Fig. 8(A)(c)) were obtained by continuously depositing a thin layer of nickel-phosphorus alloy on the polymer-only lattices using electroless plating method [377].

4.2. Inkjet printing

Inkjet printing (IJP) is used for multi-material fabrication [401]. In this method the ink drops are deposited on the build plate using piezoelectric nozzles of size varying from 20 μm to 40 μm (refer to Fig. 8 (B)(a)). The desired structure is obtained with the print head panning the entire build plate in quick passes. IJP uses photocurable ink. An advanced form of IJP is PolyJet technology which is used for printing digital materials [402] (refer to Fig. 8(B)(b)). This technology uses rigid and soft resin inks which get deposited in a prefixed ratio. It is instrumental in determining the effective properties of the finally fabricated material (refer to Fig. 8(B)(c-e)). IJP is not suitable to print porous structures or lattices since the ink droplets get individually deposited resulting in the requirement of support materials. The ink should also have low viscosity leading to the limitation in printing new materials with embedded particles. Fig. 8(B)(c) demonstrates the experiment done using a cellular structure (manufactured using PolyJet printing) to anticipate the buckling modes [382]. A geometrical imperfection was created by the summation of the deformations corresponding to the buckling modes that was used to pre-dispose the cellular structure. The PolyJet printing has also been used to create physical logic gates [383] and composites with tunable toughness [384] (refer to 8(B)(d-e)).

4.3. Vat photopolymerization

Vat photopolymerization uses light of different wavelengths to cure polymer resins that are photo-sensitive. This method includes two-photon polymerization (TPP), digital light processing (DLP), and stereolithography (SLA). SLA is an additive manufacturing method that cures the resin by scanning through a UV laser beam. It can print a large area but the speed can be slow. DLP offers a high printing speed with a simple low-cost setup [385] (refer to Fig. 8(C)(a-c)). Curing is done for one layer at a time. A projector that employs micromirror device is used to shine a pattern of 2D light into a resin vat. TPP method draws shapes in resin using a laser beam [387] (refer to Fig. 8(C)(d-f)). In this method, curing is done using a very high-intensity focal point of the laser resulting in the absorption of two photons. This allows the laser beam to pierce through the resin vat leaving the upper regions uncured. Fig. 8(C)(b) shows 3D printed microlattice architecture with distinct material [385], printed using μDLP method. Fig. 8(C)(e) shows a microlattice with unit cell height 18 μm and beam diameter 0.5 μm [388] printed using TPP.

4.4. Powder bed fusion

Powder bed fusion (PBF) is a method of 3D printing that encompasses different other methodologies like electron beam melting (EBM) (refer to Fig. 8(D)(c-d)) [403,404], selective laser melting (SLM) [405, 406] (refer to Fig. 8(D)(a-b)) and Multijet Fusion (MJF) [407]. In these methods, a light source or a high-powered laser heats the print bed which has a thin powder layer deposited on it [408]. The powder layer is heated depending on the shape of 2D pattern desired, leading to frittage of materials. PBF helps in the fabrication of complex structures such as hollow lattices, tapered-walled lattices, etc., that are difficult to be injection-molded or machined. Fig. 8(D)(b) shows a pentamode metamaterial manufactured using SLM. The lattice has 5 unit cells in each direction with nominal strut size being 3.464 mm [390]. Fig. 8(D)(d) presents the lattices fabricated using EBM. The lattices were designed to have $4 \times 4 \times 4$ number of unit cell repetitions [391].

5. Evolving trends and future roadmaps in mechanical metamaterials

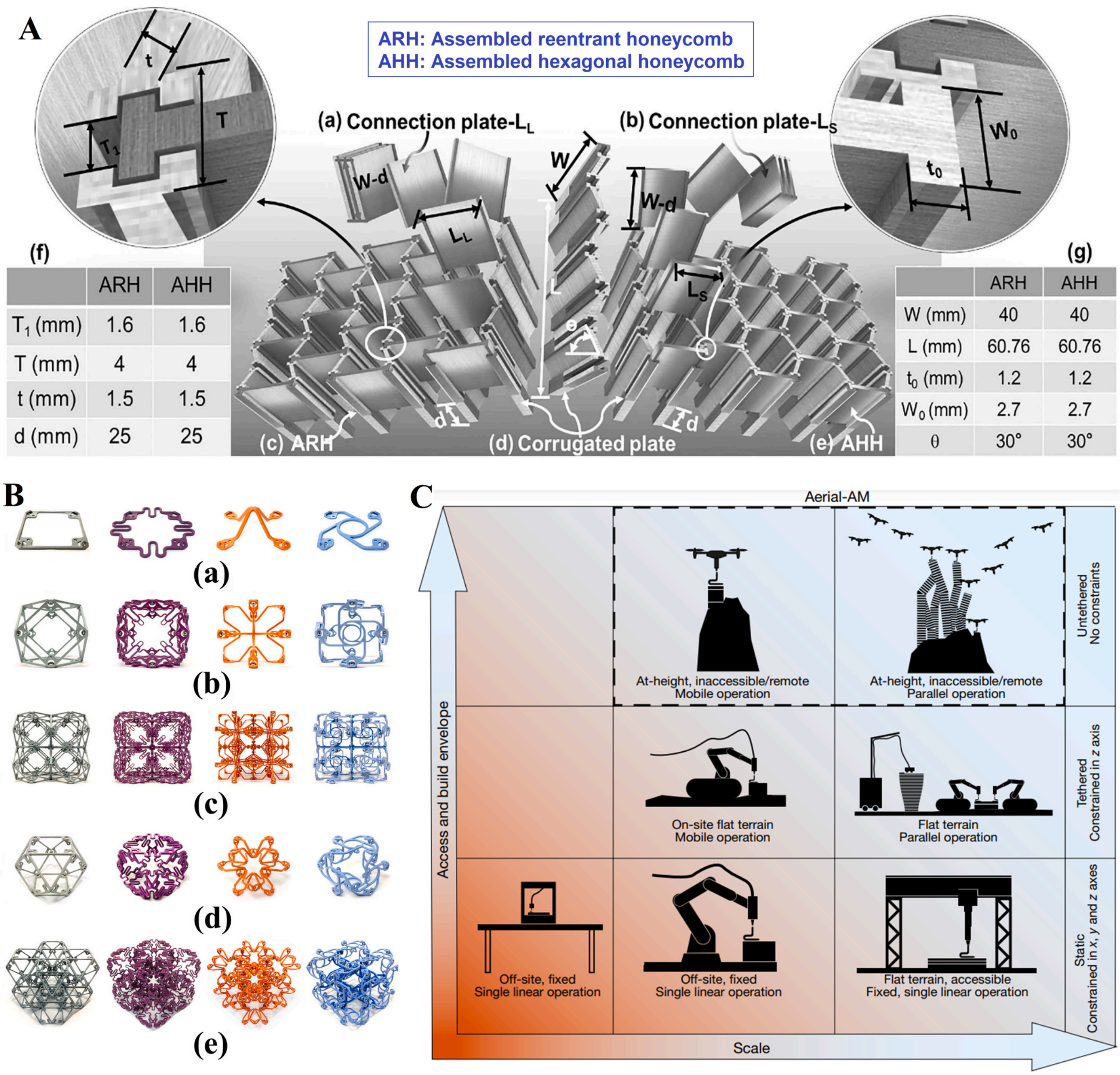
Even though mechanical metamaterial as a recognized scientific field is not older than a decade or two, and it is continuously evolving, intense research over this period has brought forth few clear trends and directions for future research. In this section, we would discuss such trends and the areas which need further attention from the metamaterial community in the near future.

5.1. 4D printing of complex microstructures

The extensive research in the field of mechanical metamaterials has made it possible to push the boundaries of conventional material properties. 3D printing techniques have made it possible to realize complex computationally designed microstructures physically which was earlier not possible using conventional fabrication methods. One of the emerging trends in mechanical metamaterials is the on-demand active modulation of physical properties and shape. This is achieved through 4D printing involving active materials. 4D printing is still in an infant stage of development, where more attention is needed to improve the precision and functionality of different active materials with multiple components involved. 4D printing with living matter is an upcoming research direction, where the presence of living organisms would be exploited for active control of physical properties including damage repair and growth of the structure.

5.2. Real-time reconfigurability and functionality programming

Passive mechanical metamaterials do not allow for property modulation after manufacturing. However, active mechanical metamaterials permit the control of static and dynamic parameters after fabrication as a function of external stimuli like electrical or magnetic fields. On-demand shape morphing as well as programmable effective property modulation can be achieved in real-time as per operational conditions. Origami and kirigami-based metamaterials allow large-scale shape changes along with contact-induced programming of the effective constitutive behavior. The notion of real-time reconfigurability and functionality programming are quite new and a significant amount of research is necessary in this direction to explore the interactive space of



(f)	ARH	AHH
T_1 (mm)	1.6	1.6
T (mm)	4	4
t (mm)	1.5	1.5
d (mm)	25	25

(g)	ARH	AHH
W (mm)	40	40
L (mm)	60.76	60.76
t_0 (mm)	1.2	1.2
W_0 (mm)	2.7	2.7
θ	30°	30°

Fig. 9. Scalability of mechanical metamaterials and pathways for additive manufacturing with large build volume. **(A)** Fabrication of metamaterials through assembly process [107]. Here each of the metamaterial unit cells (or a group of few unit cells) can be additively manufactured and subsequently assembled to achieve large build volume. **(B)** Discretely assembled mechanical metamaterials. Four different types of metamaterials, namely, rigid, compliant, auxetic and chiral are shown from left to right. (a) Face parts (b) Front view of single voxel (c) Front view of a 2 × 2 × 2 cube (d) Oblique view of single voxel (e) A 2 × 2 × 2 oblique view [409] **(C)** Conventional, robotic and drone-assisted (aerial) additive manufacturing for achieving large build volume and scale [410].

active deformation and unit cell-level mechanics along with application-specific product development. In this context, multi-physical behavior of metamaterials would need further attention to meet the multi-functional demands of modern structural systems. For example, energy-harvesting metamaterials should also be able to carry mechanical loads efficiently and store energy for future usage, leading to the most optimum material utilization.

5.3. Nano-scale metamaterials

Most studies on metamaterials have primarily focused on micro-scale architecturing for achieving effective physical properties at the macro scale. However, there exists an immense possibility of replicating such concepts at the nano-scale with the recent advancements in nano-manufacturing. The scientific community has recently started exploring the nano-scale architectures and this field has a long way ahead both for computational and experimental innovations.

5.4. Scalability

One of the pressing challenges of adopting metamaterials at the industry-scale is the issue of scalability and mass production with adequate quality control. Most experimental research on metamaterials relies on physical samples of relatively small dimensions. While theoretically there would not be any change in the effective properties of periodic microstructures if the dimensions are increased, it becomes a challenge to manufacture large structures (i.e. large build volume) using metamaterials. Recent progress in robotic additive manufacturing and drone-assisted additive manufacturing show prospective routes for solving such problems (refer to Fig. 9(C)). Further, the possibility of attaching multiple segments (or unit cells) of metamaterials through mechanical bonds could be another prospective panacea for building large structures (refer to Fig. 9(A-B)). Since additive manufacturing has emerged as one of the most suitable manufacturing techniques for complex metamaterial microstructures, other challenges besides achieving large build volume in industry-scale adoption of metamaterials remain to be addressed like slow production speed, inconsistencies in material properties, automation of post-processing after additive manufacturing and the requirement of initial investment.

5.5. Artificial intelligence and machine learning

As the design of mechanical metamaterials has started to become more and more complicated, the need for their response to be pre-programmed has come to the fore. The design of artificial microstructures with the aid of artificial intelligence (AI) and machine learning (ML) has attracted the scientific community in the recent past. Through AI and ML, we can carry out geometrical optimization in the design of metamaterials to achieve multi-functionality. An inverse design approach can be adopted in the topology optimization of a unit cell to achieve multiple objectives with given constraints, which would have been impossible in the traditional approach of designing unit cells in the forward intuitive framework. In the recent past, research in the field of mechanical metamaterials has evolved from the analysis of rational microstructures to developing efficient computational methods [411, 412]. To achieve this objective, AI has been deployed for designing optimal materials and microstructures [413–418]. AI and ML offer high accuracy and convenience in the rational design of microstructures, that leads to catering application-specific multi-functional mechanical metamaterials [419–422].

Over the years, research in the field of mechanical metamaterials have involved the use of analytical and experimental investigations or the use of intricate and subtle finite element simulations. The finite element methodology is computationally expensive and time-consuming while the experimental studies involve strenuous exercise. Considering the vast possibility of unit cell architectures and their dimensions, it becomes practically impossible to carry out finite element simulations by modeling each microstructural configurations individually or experimental testing by manufacturing them. The analytical investigations are generally preferred for lattices involving lesser structural complications. Thus, researchers are now focusing on ML so as to apply them as surrogate models to make various complicated metamaterial configurations easier to deal with. These models can relate the input (such as microstructural design space) and output parameters (such as effective mechanical properties) of a metamaterial by establishing efficient computational mappings [115]. The recent advancements in ML has led to progress in different inter-related fields, which not only include design of metamaterials and advanced composites [423–429] but pave way for optimized manufacturing methods [109, 430,431]. Image based machine learning approaches [432] can further enhance the capability of generalizing the prediction of effective mechanical properties by capturing the unit cell architectures more conveniently.

In the context of multi-scale design of metamaterials, as described in the inset of Figure 2, material properties are defined at two different length scales. Machine learning can be useful at both the levels: (1) for defining the properties of intrinsic materials at the lower length scales through developing interatomic potentials [433–435] and (2) the unit cell level design at a higher length scale as discussed in the preceding paragraph.

An interesting and novel domain of research in mechanical metamaterials is neuromorphism [436]. This means that the metamaterial can be structurally and functionally replicated to perform as biological neural networks. Two characteristics of mechanical metamaterials that make them useful for neuromorphism are hierarchical connectivity and weighted coupling of every nodal pair within the network. This field, when explored more, will open vast research avenues with the progress of artificial intelligence.

5.6. Multi-physical origami/ kirigami

Origami and kirigami-based metamaterials have attracted significant attention lately due to their programmable features. The research community has also started to realize the prospective advantages of involving active components in such metamaterials for enhanced programmability. These multi-physical origami / kirigami-based metamaterials will result in active property modulation based on external stimuli, including contactless actuation.

5.7. Micro-architecturing with living matter

Metamaterials with living matter as the constituting elements is an upcoming research direction, where the presence of living organisms would be exploited for active control of physical properties and programmed growth of the structure. Note that programmed growth not only would solve the problems of scalability, but it will also help achieve different curvature and predefined shapes. Further, such metamaterials would have inherent self-healing and damage repair capabilities.

5.8. Soft and conformal metamaterials

One of the most rapidly evolving fields in mechanical metamaterials is soft metamaterials for their anticipated applications in a range of engineering systems including soft robotics and biomedical devices. In such analysis, the aspect of nonlinearity and large deformations become apparent, which the research community is increasingly appreciating. Further, it has been recently shown that a soft mechanical metamaterial can be converted into a stiff one actively by involving multi-physical mechanics. Such multifunctional on-demand property modulations need more attention. The aspect of shape conformability in soft metamaterials along with fitting to a predefined shape actively would open up a plethora of innovative applications.

5.9. Influence of manufacturing imperfections

Additive manufacturing has emerged as the most prominent way of realizing complicated metamaterial microstructures. However, additive manufacturing often has a range of defects such as unwanted prestress, void and irregularity in the manufactured geometry. Such effects can significantly influence the effective properties of metamaterials. Moreover, the intrinsic material properties are dependent on the type of additive manufacturing and different other calibration parameters during manufacturing. Detailed investigations are required in these directions to quantify the influence of manufacturing defects and other uncertainties concerning additive manufacturing.

5.10. Service-life effects: environmental and operational conditions

Besides manufacturing uncertainties, the service-life conditions of mechanical metamaterials need significant attention. Effects of the surrounding environment, material degradation, and accumulation of damage over time should be included in the design of metamaterials in line with the notion of digital twins. Moreover, there are a few critical long-term mechanical properties such as fatigue, effects of viscoelasticity and creep, and the effect of additive manufacturing methods in the failure modes, which have not received adequate attention yet. Further research is required in such directions for a reliable adoption of these complex microstructures in industry-scale structures.

6. Concluding remarks

Extensive research in the field of mechanical metamaterials has made it possible to engineer materials with unconventional behaviour, having conflicting and often uncorrelated multi-objective goals, thereby expanding the material design space to push the limits of physical properties. In this paper, we have reviewed the recent progress in mechanical metamaterials with a particular emphasis on active and multi-physical behavior (involving external electrical or magnetic fields, and stimuli like temperature, light or chemical reactions) coupled with the mechanics of bi-level architectures to expand the scope of actively programming on-demand mechanical responses. We have started with a brief overview of different emerging classes of metamaterials based on functionality and microstructural configurations, followed by a critical review of passive and active mechanical metamaterials. It is noted that with the emergence of multifunctionality and on-demand property modulation in the design of mechanical metamaterials, the division among different classes of metamaterials becomes interconnected and the microstructural design needs more and more interactive spaces. The evolving trends and future roadmaps in the field of mechanical metamaterials have been critically analyzed involving the notions of real-time reconfigurability and functionality programming, 4D printing, nano-scale metamaterials, scalability, artificial intelligence and machine learning, neuromorphism, multi-physical origami/ kirigami, living matter, soft and conformal metamaterials, manufacturing and service-life effects. The current paper with a comprehensive review of literature and practical perspectives on various emerging aspects of mechanical metamaterials would contribute to myriad possibilities for the researchers and engineers to explore the trends, patterns, and multi-physical design space for developing novel metamaterials with improved and unprecedented functionalities.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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