# AUTOMATIC GAIN CONTROL FOR PARAMETRIC ARRAY LOUDSPEAKERS

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## ABSTRACT

Parametric array loudspeaker (PAL) modulates the audio input on an ultrasonic carrier and relies on airborne nonlinear acoustic effects to generate the audible sound output. The sound output is mainly confined in the beam of the ultrasonic carrier and thus shows a pronounced directivity. There are three parameters that together influence the output volume of a PAL. They are the input level, modulation index, and ultrasound level. In existing PALs, the volume knob is associated with the ultrasound level, while the modulation index is either fixed in the circuit or rarely adjustable by another knob. In this paper, an automatic gain control is proposed to improve the sound quality of the PAL by minimizing the modulation index, maintaining the output-to-input ratio, and ensuring the ultrasound level within the safety range. Simulation and measurement results validate that the proposed approach leads to a reduction in the average total harmonic distortion (THD) level by more than one third for all the tested modulation methods.

*Index Terms*— Parametric array loudspeaker, automatic gain control, modulation index, total harmonic distortion

#### 1. INTRODUCTION

PAL is an application of parametric sound generation with the interesting ability to transmit a directional sound beam in air. Nonlinearity of air works in a similar way to a product demodulator, but the oscillator required by the product demodulator is provided by the modulated signal itself. The envelope of a pulsed ultrasound is therefore recovered without external interference [1]. As early as 1965, Berktay referred to the same underwater phenomena as self-demodulation and presented a simple equation widely known as the Berktay's far-field solution [2]. Hence, self-demodulation in air was experimentally validated by Bennett and Blackstock in 1975 [3] and then the first prototype of the PAL was established by Yoneyama *et. al.* in 1983 [4].

Nowadays, there have been an ever-increasing number of prototypes and commercial products of the PAL. The common design includes a processor to carry out modulation and an amplifier to drive an ultrasonic emitter, which is shown in Fig. 1. There are three components that are applicable for



Fig. 1. An illustration of the parametric array loudspeaker.

gain control. They are the gain to audio input  $g_a$ , modulation index m, and gain to ultrasound output  $g_u$ . The input-output relation of a PAL is thus formulated as

$$y(t) = N\{g_u M[g_a x(t), m]\},$$
(1)

where x(t) and y(t) are the input and output amplitudes at time t, respectively;  $N(\cdot)$  models nonlinearity of air; and  $M(\cdot, m)$  denotes the modulation method.

Since 1983, various modulation methods have been extensively examined for PALs. The double sideband (DSB) modulation method is the first and still widely applied modulation method [4]. The second harmonic distortion level of the DSB modulation method is proportional to the modulation index. Hence, Kamakura et al. proposed the square root (SRT) modulation method to preprocess the envelope of the DSB modulation method [5]. The SRT modulation method deals with the square operation in the Berktay's far-field solution, but the preprocessed audio input has an infinite bandwidth that can only be reproduced by an ideal ultrasonic emitter. Moreover, the second-order derivative in the Berktay's far-field solution implies a high-pass behavior of the PAL. Therefore, Kite et al. and Pompei respectively proposed and examined to carry out the double integral to equalize the frequency response of the PAL [6, 7].

As another trend, the single sideband (SSB) modulation method, has been studied since 1991 [8, 9]. The SSB modulation method is essentially a quadrature modulation method. It introduces a quadrature path that cancels nonlinear distortion occurred in the DSB modulation method. Similarly, a class of modified amplitude modulation (MAM) methods have been developed [10]. The quadrature path in the SSB modulation method is usually implemented with a Hilbert filter, while that in the MAM method is calculated by a polynomial equation. The SSB modulation and MAM methods have similar theo-

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retical performance. However, in practice, the SSB modulation method outperforms the MAM methods in THD tests [11, 12].

Recently, there have been two hybrid modulation methods proposing to combine the DSB and SSB modulation methods. The weighted DSB modulation method proposed by Ikefuji *et al.* emphasized on the louder sound output of the DSB modulation method at the low frequency band as compared to the SSB modulation method [13]. On the other hand, the asymmetrical amplitude modulation (AAM) method explored the feasibility of bandwidth extension contributed by the second harmonic distortion of the DSB modulation method [14]. These two hybrid modulation methods are coincidentally opposite. Furthermore, as a complement to the AAM method, a psychoacoustical method has also been carried out to make use of the missing fundamental effect to compensate for the relatively weak bass output of the PAL [15].

In every modulation method, a key affecting factor is the modulation index. A higher modulation index leads to a larger sound pressure level but also more severe distortion. For this reason, careful selection of the modulation index contributes to a good design of the PAL. But research interests in the modulation index have been marginalized. In existing PALs, the modulation index is either fixed in the circuit or manually adjustable by another knob. In the former case, users tend to increase the ultrasound level without knowing the hazard of ultrasound exposure, when they are trying to adjust the volume. In the later case, users are confused by too many knobs.

In order to coordinate the modulation index and gain to ultrasound output, this paper presents an automatic gain control for PALs. It targets to maintain the ultrasound level but minimize the modulation index with a given or estimated input level. By maintaining the ultrasound level, the guideline of ultrasound exposure and amplifier saturation are taken into account. Minimizing the modulation index has a significant advantage in improving the sound quality of the PAL. Last but not the least, the linear relation between the input and output levels, which was previously neglected, is also achieved.

# 2. THEORETICAL ANALYSIS

Berktay's far-field solution is one of the most widely applied model equations for PALs [2]. Although its accuracy is sometimes criticized, the explicit expression of the Berktay's farfield solution allows the theoretical analysis of modulation methods to be concise and straightforward as compared to other complicated model equations [16, 17] and Volterra filter representations [18, 19]. Therefore, nonlinearity of air in (1) adopts the Berktay's far-field solution in this section, which is written as

$$N\{\cdot\} = KP_0^2 \frac{\partial^2}{\partial t^2} \{\cdot\}^2,\tag{2}$$

where K is an acoustic coefficient combining many parameters of air, speed of sound, size of the ultrasonic emitter and so

on;  $P_0$  is the largest ultrasound output permitted by the safety regulation and amplifier design.

Without loss of generality, the input level is assumed to be able to be estimated or even known by the PAL for simplicity. Therefore, if x(t) is treated to have an unit amplitude, the input gain  $g_a$  becomes equivalent to the input level. In the following analysis,  $g_a$  is assumed to be a known factor, subject to  $0 \le g_a \le 1$ .

### 2.1. DSB Modulation Method

The DSB modulation method gives

$$M_{\rm DSB} \left[ g_a x \left( t \right), m \right] = 1 + m g_a x \left( t \right). \tag{3}$$

Substituting (3) into (2) yields

$$y(t) = KP_0^2 g_u^2 \frac{\partial^2}{\partial t^2} \left[ 2mg_a x(t) + m^2 g_a^2 x^2(t) \right].$$
 (4)

The ratio between the second term and the first term in square brackets, proportional to  $mg_a$ , reflects the THD level. Since  $g_a$  is a known factor, the goal of the automatic gain control is to change the output level linearly with respect to  $g_a$  and minimize m to improve sound quality. Automatic gain control of the DSB modulation method is thus subject to two constraints, *i.e.* 

$$2g_u^2 m g_a = k g_a \tag{5}$$

and

$$g_u \left( 1 + m g_a \right) \le 1. \tag{6}$$

In (5), k is the only adjustable parameter in the automatic gain control that influences the maximum output level of the PAL. The possible range of k will be discussed for every modulation method in the following paragraphs of this paper. Furthermore, (6) reflects the limitation of the ultrasound level.

Combining (5) and (6) leads to a quadratic inequality of  $g_u$  and gives the analytical solution as

$$\frac{1 - \sqrt{1 - 2kg_a}}{2} \le g_u \le \frac{1 + \sqrt{1 - 2kg_a}}{2}.$$
 (7)

The upper bound of  $g_u$  in (7) leads to the minimized m, which gives

$$m_{\rm AGC} = \frac{2k}{\left(1 + \sqrt{1 - 2kg_a}\right)^2}.$$
 (8)

Since  $0 \le g_a \le 1$ ,  $k \le 0.5$  ensure a real value of  $\sqrt{1-2kg_a}$ . Hence, k = 0.5 is the upper bound for the DSB modulation method. A smaller k leads to a higher upper bound of  $g_u$  and subsequently a lower  $m_{AGC}$ .

The above analysis is based on the assumption of an ideal ultrasonic emitter. Furthermore, we introduce a special ultrasonic emitter that retains the carrier frequency but scales all the other frequencies by a factor of  $\delta$ . In this case, the DSB modulation method becomes

$$M_{\rm DSB} \left[ g_a x \left( t \right), m \right] = (1 - \delta) + \delta \left[ 1 + m g_a x \left( t \right) \right].$$
(9)

Changing the ultrasonic emitter has no effect on the constraint in (6). However, the constraint in (5) is modified to

$$2\delta g_u^2 m g_a = k g_a. \tag{10}$$

This is equivalent to scale k in (8) by  $\delta$ , which can be understood by multiplying  $\delta$  on both sides of (5).

## 2.2. SRT Modulation Method

When an ideal ultrasonic emitter is adopted, the SRT modulation method gives

$$M_{\text{SRT}}\left[g_a x\left(t\right), m\right] = \sqrt{1 + m g_a x\left(t\right)}.$$
 (11)

By substituting (11) into (2), the output level is given by

$$y(t) = KP_0^2 g_u^2 \frac{\partial^2}{\partial t^2} \left[ mg_a x(t) \right].$$
(12)

Similar to the analysis of the DSB modualtion method, the automatic gain control is subject to two constraints, which are

$$g_u^2 m g_a = k g_a \tag{13}$$

and

$$g_u \sqrt{1 + mg_a} \le 1. \tag{14}$$

Substituting (13) into the square of (14) yields

$$g_u^2 \le 1 - kg_a,\tag{15}$$

and the minimized m is given by

$$m_{\rm AGC} = \frac{k}{1 - kg_a}.$$
 (16)

Eq. (11) requires  $mg_a \leq 1$  to ensure a real square root when x(t) = -1. Therefore, multiplying  $g_a$  on both sides of (16) yields  $kg_a \leq 0.5$ , which leads to  $k \leq 0.5$  for the SRT modulation method.

In fact, (16) has nothing to do with sound quality, because the SRT modulation method has perfect performance under assumptions of an ideal ultrasonic emitter and Berktay's farfield solution. However, if the aforementioned special ultrasonic emitter is considered, the SRT modulation method becomes

$$M_{\text{SRT}}[g_a x(t), m] = (1 - \delta) + \delta \sqrt{1 + m g_a x(t)}.$$
 (17)

Applying the Taylor's expansion yields an approximation, *i.e.* 

$$\hat{M}_{\text{SRT}}\left[g_{a}x\left(t\right),m\right] = 1 + \frac{1}{2}\delta m g_{a}x\left(t\right) - \frac{1}{8}\delta m^{2}g_{a}^{2}x^{2}\left(t\right).$$
(18)

By substituting (18) into (2), the resultant output level is

$$\hat{y}(t) = KP_0^2 g_u^2 \frac{\partial^2}{\partial t^2} \left[ \delta m g_a x\left(t\right) - \frac{\delta - \delta^2}{4} m^2 g_a^2 x^2\left(t\right) \right].$$
(19)

The ratio between the second term and the first term in square brackets is an increasing function of  $(1 - \delta) mg_a$ . The ideal ultrasonic emitter provides  $\delta = 1$ . Any ultrasonic emitter providing  $\delta \neq 1$  results in the THD level of the SRT modulation method to be proportional to the modulation index, which is similar to the DSB modulation method. Automatic gain control of the SRT modulation method has already been given by (16), but k needs to be scaled by  $\delta$  when  $\delta \neq 1$ .

### 2.3. SSB Modulation Method

There are two types of the SSB modulation method, namely the upper single sideband (USB) and lower single sideband (LSB) modulation methods. If a perfect Hilbert filter could be implemented, the quadrature path eliminates nonlinear distortion in the DSB modulation method. The SSB modulation method provides a similar envelope to that of the SRT modulation method, which is expressed by

$$M_{\rm SSB}\left[g_a x\left(t\right), m\right] = \sqrt{1 + m^2 g_a^2 + 2m g_a x\left(t\right)}.$$
 (20)

By substituting (20) into (2), the output level is given by

$$y(t) = KP_0^2 g_u^2 \frac{\partial^2}{\partial t^2} \left[2mg_a x(t)\right].$$
 (21)

Automatic gain control of the SSB modulation method subjects to two constraints, which are

$$2g_u^2 m g_a = k g_a \tag{22}$$

and

$$g_u \sqrt{1 + 2mg_a + m^2 g_a^2} = g_u \left(1 + mg_a\right) \le 1$$
 (23)

They are same as (5) and (6). The minimized m is therefore given by (8), and  $k \le 0.5$  remains valid too.

However, when the Hilbert filter is not perfect, the output of the quadrature path may decompose into two components.  $\beta_1$  denotes the portion that is perpendicular to the distortion component in the DSB modulation method and  $\beta_2$  denotes the overall residual distortion component. In this case, the SSB modulation method becomes

$$M_{\rm SSB} = \sqrt{1 + \beta_1 m^2 g_a^2 + 2m g_a x \left(t\right) + \beta_2 m^2 g_a^2 x^2 \left(t\right)}.$$
(24)

Substituting (24) into (2) yields the output level as

$$y(t) = KP_0^2 g_u^2 \frac{\partial^2}{\partial t^2} \left[ 2mg_a x(t) + \beta_2 m^2 g_a^2 x^2(t) \right].$$
 (25)

It is observed from (25) that the THD level of the SSB modulation method is also proportional to  $mg_a$ . To minimize m for a given  $g_a$ , the constraint in (23) is modified to

$$g_u \sqrt{1 + 2mg_a + (\beta_1 + \beta_2) m^2 g_a^2} \le 1.$$
 (26)

The solution to (26) is derived as

$$g_u^2 \le \frac{1 - kg_a + \sqrt{1 - 2kg_a + (1 - \beta_1 - \beta_2)k^2g_a^2}}{2}.$$
 (27)

When  $\beta_1 + \beta_2 = 1$ , (27) results in the same upper bound of  $g_u$  as (7).

### 3. SIMULATION AND MEASUREMENT

THD tests were carried out using a sine sweep from 1 kHz to 8 kHz as the excitation [20]. The averaged THD level over the frequency range of the sine sweep was recorded in the simulation and measurement. The sampling frequency was set at 192 kHz and carrier frequency was generated at 40 kHz. Eqs. (8) and (16) were adopted as the automatic gain control formulas. The modulation indexes with respect to the input level were plotted in Fig. 2(a), where k = 0.5,  $\delta = 1$ ,  $\beta_1 = 0.9$  and  $\beta_2 = 0.1$  were set in the simulation for simplicity.

In a sound proof room  $(2.9 \text{ m} \times 3.1 \text{ m} \times 2.1 \text{ m})$ , the microphone was placed 3 meters away from the ultrasonic emitter for the measurement. The frequency response of the ultrasonic emitter was plotted in Fig. 2(b). The frequency response was further approximated by a 300-tap finite impulse response (FIR) filter to be used in the simulation. The simulation and measurement results were plotted in Figs. 3(a) and 3(b), respectively.

The DSB modulation method has relatively high level of THD. Reduction in the THD level is remarkable, after the automatic gain control is applied. It is common for the SRT modulation method to have lower level of THD as compared to the DSB modulation method. After the automatic gain control is applied, the THD level of the SRT modulation method is still reduced by about 5%. However, the DSB modulation method achieves similar level of THD when both modulation methods are implemented with the automatic gain control, especially in the measurement. This demonstrates that the proposed automatic gain control is a better approach to improve the DSB modulation method, since the performance of the DSB modulation method relies less on the ultrasonic emitter.

The SSB modulation method has the least THD among the three modulation methods. The THD level is even lower than 5% in the measurement, since a perfectly quadrature path is feasible to be carried out for the sine sweep. However, due to the settings,  $\beta_1 = 0.9$  and  $\beta_2 = 0.1$ , the THD level of the SSB modulation method can exceed 10% in the simulation. Nevertheless, reductions in the THD levels are observed in both the simulation and measurement results, after the automatic gain control is applied. Hence, the proposed automatic gain control proves its effectiveness to help different modulation methods to achieve better sound quality.

#### 4. CONCLUSIONS

Automatic gain control has been derived for the PAL for the first time. The strategy is to keep the ultrasound level for the sake of safety regulation and amplifier saturation, and meanwhile to minimize the modulation index. This is motivated by the fact derived in this paper that THD levels of different modulation methods are all proportional to the modulation index. Simulation and measurement results have proved the effectiveness and advantage of the proposed approach in the PAL.



Fig. 2. Simulation and measurement conditions.



**Fig. 3**. Validation of the automatic gain control (AGC) for the PAL.

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