Nonlinear Stability of Curved Multi-phase Composite Panels: Influence of Agglomeration in Randomly Distributed Carbon Nanotubes with Non-uniform In-plane Loads

Abstract

The nonlinear stability characteristics of doubly curved panels made of three-phase composites with randomly dispersed carbon nanotubes (RD-CNTRFC) subjected to practically-relevant non-uniform in-plane loads are investigated in this study. Carbon nanotubes, when mixed with resin polymer, may give rise to bundles, termed as agglomerations, which can have a profound impact on the effective material properties. There exists a strong rationale to investigate the influence of such agglomeration on the nonlinear equilibrium path of panels, which can subsequently be included in the structural stability design process to enhance operational safety. A multi-stage bottom-up numerical framework is developed here to probe the nonlinear stability characteristics. The effective material properties of RD-CNTRFC panels are determined using the Eshelby-Mori-Tanaka approach and the Chamis method of homogenization. By considering von-Kármán non-linearity and Reddy's higher-order shear deformation theory, strain-displacement relations are established for the non-linear stability analysis. The governing partial differential equations are simplified into nonlinear algebraic relations using Galerkin's method. Subsequently, by reducing the stiffness matrix neglecting the non-linear terms and solving the Eigenvalue problem, we obtain critical load and non-linear stability path of shell panels based on arc-length approach. In the present study, various shell geometries such as cylindrical, elliptical, spherical and hyperbolic shapes are modeled along with the flat plate-like geometry to investigate the non-linear equilibrium paths, wherein a geometry-dependent programmable softening and hardening behavior emerges.

Keywords: Doubly curved shells, Randomly distributed carbon nanotubes, Post-buckling analysis of composites, Three-phase composites, Programmable softening and hardening behaviour

1. Introduction

Shell panels are a critical structural component in the design of aircraft wings, forming the flaps and membranes. These panels are typically thin-walled structures and are often supported with stiffeners or stringers to enhance structural stiffness. However, this additional support increases the overall weight of the structure significantly. Carbon nanotubes (CNTs) offer a promising solution by improving the stiffness of composite panels without the need for additional stringers, providing high flexural resistance under both static and dynamic loads. Past experiences have shown that these thin-walled structures can become vulnerable under in-plane compression loads (Reddy, 2003), potentially leading to catastrophic failures during operation. Moreover, the in-plane loading applied to these components can be non-uniform, depending on the stiffness of neighboring elements, complexity of the adjacent structural configuration and surrounding loading conditions. Therefore, addressing the stability criteria of thin-walled structures becomes crucial. In this context, the present study delves into the stability performance of randomly distributed carbon nanotube-reinforced composite shell panels when subjected to non-uniform in-plane loadings. Investigating the influence of CNT aggregation on stability performance is of great significance for aircraft designers, as it aids in accurately predicting the stability performance of CNT-based composite panels considering real-world manufacturing complications. The current study focuses on investigating the non-linear stability of carbon nanotube (CNT) reinforced shell panels like Cylindrical, Spherical, Elliptical, and Hyperbolic in geometry, considering the random distribution of nanotubes in the polymer matrix.

In general, thin-walled structures are susceptible to losing stability when subjected to a critical amount of in-plane compressive load (Garg et al. 2022; Dey et al. 2018b). This phenomenon is known as buckling and can result in the structure undergoing significant deformation during the post-buckling regime or even collapsing under further loads. As a result, researchers need to consider the post-buckling behavior when designing and analyzing thin-walled structures to ensure they can withstand the expected loading conditions and maintain stability. The members may lose one stable configuration and attain another stable configuration. The load at which it changes from one stable configuration to another is known as critical load or buckling load, while the member may withstand

a higher amount of load and undergo large deformation. Such reserve strength is gained through a post-buckling path or non-linear equilibrium path (Reddy, 2003). The current study probes the post-buckling strength of different types of shell panels, including Cylindrical, Spherical, Elliptical, and Hyperbolic, where randomly distributed carbon nanotubes are investigated as reinforcement. Apart from aerospace applications, these thin-walled structures are widely used as structural members in the broader field of engineering, such as Automobile, Marine, Civil, and Mechanical Engineering, due to their combination of strength and lightweight characteristics. Despite their advantages, it is important to consider the post-buckling behavior of thin-walled structures to ensure they maintain stability under the expected loading conditions. Thus, enhancement of the buckling behavior has been a center of focus of the research community. To this end, different reinforcements have been used including CNTs and different forms of graphene derivatives (Chandra et al., 2022).

Following the discovery of carbon nanotubes (CNTs) by Sumio Lijima in 1991, it was observed that CNTs exhibit exceptional performance under mechanical load, thermal environment, and electric-magnetic loading (Fard, 2021; Fard and Pensky, 2023; Khaniki and Ghayesh, 2020; Roy et al., 2015, 2014). Numerous studies predicted CNT reinforced composites with functional graded distribution profiles perform well under bending but random distribution is considered to be a more practical distribution profile in terms of manufacturing of laminates. The present study examines the stability performance of shell panels reinforced with CNTs that are randomly distributed while considering the agglomeration impact of nanotubes in the polymer matrix. To improve the strength, fibers are added in the hybrid matrix (CNT and Polymer) that results in a multiphase composite. The behavior of CNT-reinforced flat panels under transverse and in-plane loads have been studied by several researchers and it can be observed that functional gradation with an X distribution pattern provides the maximum stiffness against bending (opposite for the O distribution pattern). This is because the density of nanotubes is highest at the surface and lowest near the neutral axis for the X distribution (Ansari et al., 2018; Chakraborty et al., 2021; Kiani and Mirzaei, 2018; Shen and Zhang, 2010; Shen, 2012). Ansari et al. (2016) investigated the stability of functionally graded conical shell panels by using the Variational Differential Quadrature (VDQ) technique. First-order shear

deformation theory (FSDT) was used to develop the strain-displacement model of the shell panel, and the Hamilton variational approach was used to construct the governing equations. The stability of Piezoelectric CNT-reinforced cylindrical panels was analyzed by Nasihatgozar et al. (2016). The effective material properties of functionally graded CNT composites were determined using the Eshelby-Mori-Tanaka approach and the Donnel shell theory was used to model the panel. The kp-Ritz meshless technique along with the arc-length method was used to trace the non-linear equilibrium path of CNT-reinforced cylindrical shell panels by Liew et al. (2014). It has been observed that the panel follows a non-linear equilibrium path with a softening phase initially and then hardening in the later stage. A finite element approach was presented by Gracia et al. (2017a, 2017b) to obtain the stability characteristics of the FG-CNTRC cylindrical shell panels subjected to axial loadings and the effective material properties of composites were obtained using the Eshelby-Mori-Tanaka technique. Zghal et al. (2018) presented a finite element shell model considering the higher-order shear theory to investigate the FG-CNTRC curved panel stability characteristics. The authors observed significant improvement in buckling load due to the gradation of nanotubes and presented the stability response for various parametric properties. Chakraborty et al. (2019), using a semi-analytical solution, investigated the non-linear stability of a functional graded cylindrical shell panel subjected to non-uniform and localized loadings. It has been observed that the panel will deform continuously due to the tensile stresses developed in the transverse direction of the applied load. FG-CNTRC cylindrical shell panels' buckling stresses under an in-plane load were estimated using the stable space approach by Liew and Alibeigloo (2021). The authors observed, that the panel with FG-X pattern exhibits minimum in-plane stresses and vice-versa for FG-O profile. The stability behavior of a cylindrical panel with an internal cutout was studied by Reddy and Ram (2021) using the finite element method. The finding reveals that the FG- X and FG-O pattern performs well as compared to other distributions and with an increase in cutout size the buckling load decreases.

The investigations concerning the stability of CNT-reinforced composites is limited, particularly focusing on the effects of some of the practical and inevitable phenomenon like agglomeration. Shi et al. (2004) discussed the procedure to estimate the resulting material properties

of randomly dispersed CNT-reinforced composites through the Eshelby-Mori-Tanaka micromechanical technique. Forougi et al. (2013) presented the stability performance of a randomly dispersed nanotube-reinforced polymer-fiber composite resting on an elastic foundation. The effective material properties were obtained using Mori-Tanaka micromechanical modeling and it was observed that with an increase in the volume fraction of nanotubes the non-dimensional buckling load increases. Arani et al. (2011) presented a finite element model to investigate the stability performance of randomly distributed CNT in a matrix considering the agglomeration effect. It was observed that an increase in agglomeration leads to a decrease in buckling strength when the panel is subjected to axial load. The influence of CNTs waviness and agglomeration with the inclusion of polymer matrix was considered in the design parameters of CNT-polymer composites by Georgantzinous et al. (2021) using FEM. Recent research has focused on the stability of RD-CNTRC plate and shell panels by considering sinusoidal shear deformation theory to model these panels (Daghigh et al., 2020; Georgantzinos et al., 2021). Dastjerdi and Malek-Mohammadi (2017) proposed a higher-order displacement theory along with Navier's approach to model sandwich plates reinforced with RD-CNTRC face sheets subjected to bi-axial loadings. Sandwich CNTRC plate stability under a hygro-thermal environment was studied by Kiarasi et al. (2020) using Classical Laminated Plate Theory (CLPT). The influence of agglomeration was considered by obtaining the effective material properties using the Mori-Tanaka Approach. Safaei et al. (2019) modeled material properties of CNTRC sandwich plates by considering temperature sensitivity and obtained buckling response using a meshfree technique. The authors designed porosity in the core by considering gradation along the thickness of the plate. In a subsequent article (Moradi-Dastjerdi et al., 2020) stability of porous CNTRC sandwich plates was studied with two piezo-electric facesheets and subjected to thermo-mechanical loading.

Fantuzzi et al. (2020) and Tornabene (2019) presented homogenization techniques to define the resulting material properties of three-phase randomly dispersed CNT-reinforced composite plate and shell panels. Following these homogenization techniques, Bacciocchi (2020) obtained the critical load of the multi-phase CNT-reinforced fiber composites. Based on experimentally obtained elastic

properties of multi-walled CNT-reinforced fiber composites, Kamarian et al. (2020) investigated stability characteristics using the general differential quadrature (GDQ) approach. Thermal non-linear stability analysis was carried out for multi-phase shape memory alloy (SMA) fiber composites reinforced with CNTs by Mehar et al. (2021a) using the finite element approach. Some recent studies investigated the stability characteristics of randomly dispersed CNT-reinforced fiber composite structures like beam, plate, and shell panels using semi-analytical approaches (Chakraborty et al., 2022b, 2022a; Dash et al., 2022).

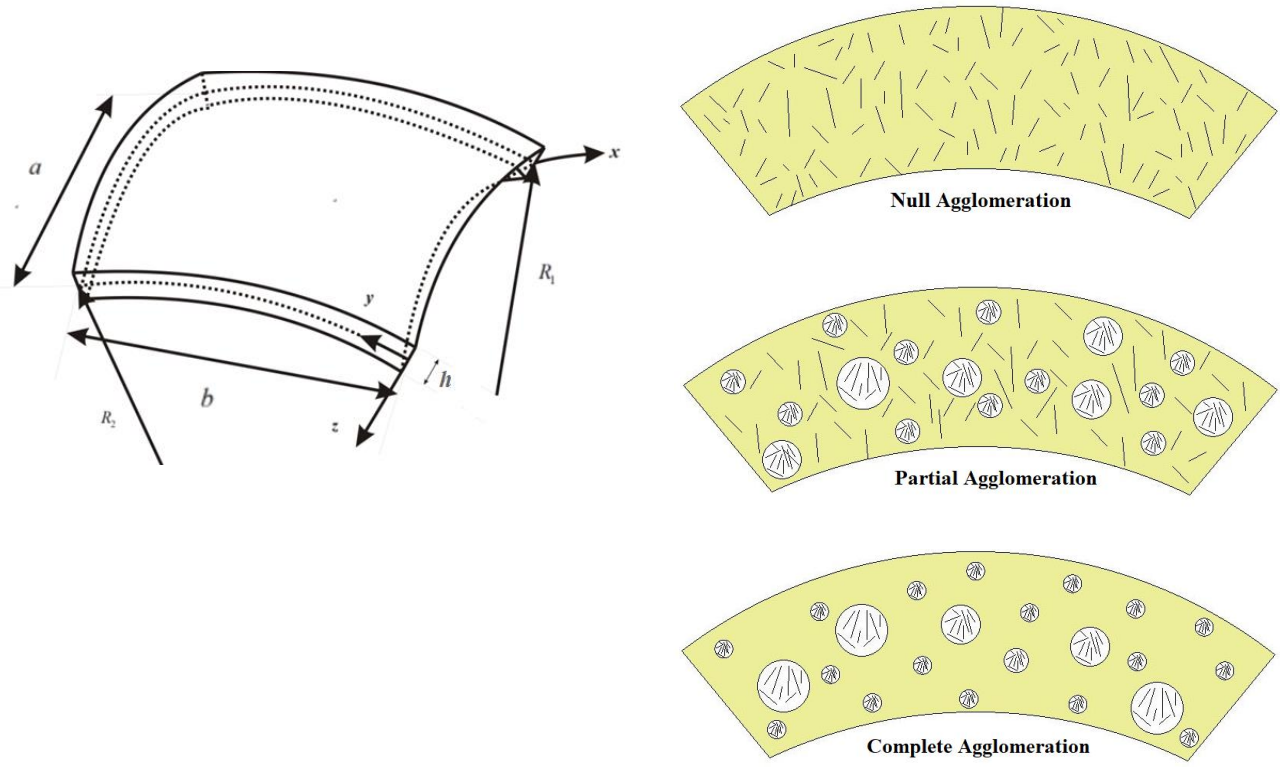
The stability of flat and cylinder-shaped CNTRC panels was primarily covered in the above-mentioned studies, whereas the current study's focus is to investigate the nonlinear stability of various shell panels, followed by the possibility of exploiting geometry-dependent programmable features. In this section, a detailed review of the literature on doubly curved panels is framed to understand the behavioral pattern of doubly curved panels and broadly define the gap of research. Arefi and Amabili (2021) investigated the buckling characteristics of doubly curved nano-shell panels by incorporating the sinusoidal transverse shear effect under electro-magneto-elastic loadings. Post-buckling strength of nanocomposite doubly curved FG-CNTRC shell panels lying on an elastic foundation in a thermal environment was examined by Duc et al. (2019). The Galerkin approach was used along with von-Kármán type nonlinearity. The influence of functionally graded doubly curved shallow shell panels were studied by Dung and Dong (2016) for post-buckling analysis. The shell panels were modeled using third-order shear deformation theory, which takes geometrical non-linearities into account. Imperfection in the panel was added for a more realistic portrayal of the shell panel’s non-linear equilibrium path. Huang et al. (2021) modeled micro doubly curved FG-CNTRC shell panels by considering the transverse shear effect with a higher-order model. It was observed by the authors that the spherical panels predict maximum buckling strength compared to other panels and the hyperbolic panel possesses the lowest strength. Investigation of doubly curved shell panel’s stability under a hygro-thermal environment was studied by Karimiasl et al. (2019) using a multi-scale perturbation technique. The authors carried out the post-buckling analysis for CNT-based, Graphene-based and Shape memory alloy-based three-phase composites. Panda and co-authors (Kar

et al., 2016; Kar and Panda, 2016; Mahapatra et al., 2016; Mehar et al., 2021b; Panda and Singh, 2013) have made an effort to analyze the non-linear stability and vibration of curved panels in thermal environment. Kumar et al. (2003) studied the buckling and parametric instability of doubly curved panels including tensile patch loading. They observed that spherical panels have high buckling strength compared to other curved panels. The non-linear buckling of doubly curved shallow shell panels under hygrothermal loadings was studied using the finite element approach by Kundu and Han (2009a). The authors obtained the large deformation in panels with temperature increment and multiple deformation modes was observed due to change in moisture content. In a subsequent article (Kundu and Han, 2009b), the authors also studied the vibration and stability characteristics of doubly curved panels under in-plane loads. To trace the non-linear equilibrium path of the shell panel Kundu and Sinha (2007) opted for the incremental iterative arc-length approach. The authors compared the approach with the published works and found a good correlation for both snap-through and snapback behavior. The authors (Kundu et al., 2007) modelled piezo composite doubly curved panels and studied the non-linear equilibrium path using a finite element approach. The non-linear static instability of anisotropic laminated doubly curved panels was studied by Li et al. (Li et al., 2021; Liu et al., 2021). A closed-form solution was presented to obtain the non-linear equilibrium path of doubly curved shell panels under out-of-plane loading.

The stability of thin-walled structures has attracted many researchers' interest as discussed in the concise review presented here. The problem gets more difficult for composite materials because of their stacking sequences, complex reinforcement and matrix elements, specific properties of the reinforcements (including irregularities such as agglomeration) along different loading circumstances and boundary conditions. Most of the published works on non-linear stability for curved panels are presented using the finite element approach which normally takes a large computation time. For different structural analyses where multiple realizations of the deterministic model are necessary (such as uncertainty quantification and reliability analysis, sensitivity analysis and optimization), it becomes practically impossible to deal with intensive simulation models (Dey et al., 2018a; Dey et al., 2019; Kumar et al., 2019; Trinh et al., 2020). To reduce such computational time a semi-

analytical approach may be an efficient alternative with adequate physical insights (Haldar et al., 2018). Further, the nonlinear stability of doubly curved three-phase composite shells panels with randomly distributed carbon nanotubes has not been investigated considering the effect of agglomeration and non-uniform in-plane loading. Based on these considerations and identified research gaps, we aim to address the following issues in this article: (a) doubly curvature geometry-dependent (considering spherical, hyperbolic, elliptical and cylindrical panels) modulation of the softening and hardening behavior in the nonlinear stability analysis of three-phase randomly distributed CNT reinforced composite panel, (b) influence of different non-uniform in-plane linear and non-linear loads on non-linear stability of RD-CNTRFC doubly curved panels, (c) influence of CNTs agglomeration on non-linear stability of doubly curved panels, (d) coupled influence of transverse shear stresses (for moderately thick panels) and double curvatures on developing strain-displacement relations, and subsequently analysing the stability behavior. In the context of broader applications, the insights provided in this article, particularly regarding the softening and hardening behaviour of shell panels, would hold significant implications for engineering systems across the length scales. For example, in the realm of energy harvesting, understanding how the structural characteristics of shell panels change under various conditions can be instrumental in optimizing the design of energy harvesting devices. By harnessing these insights, it would become possible to generate optimized voltage output that can be utilized to power sensors and actuators efficiently (Yan et al., 2020), making this knowledge highly valuable for practical applications in the field.

For developing the semi-analytical computational framework of nonlinear stability analysis, the strain-displacement relations would be developed here considering the geometrical non-linearities of von-Kármán type. The governing equations would subsequently be obtained by minimizing the variation of total energy of the system and the equations would be simplified into algebraic relations using Galerkin's approach. The non-linear equilibrium path of the curved panels would be obtained using Riks-Arc length approach to trace the actual path for load and deflection. The numerical results would explore the aims outlined in the preceding paragraph based on the developed semi-analytical framework.



(a) (b)

Fig. 1: (a) Coordinate system of doubly curved panels; (b) Type of agglomerations.

2. Mathematical Formulation

Fig. 1(a) shows the doubly curved panel under consideration having radius of curvature R_1 and R_2 along y and x axes, respectively, with plan length of a, width of b, and thickness of h. The panel is constructed with layers of lamina having an identical thickness and bonded together with polymer matrix. The origin of coordinate system is located at the edge of panel mid-layer. Four different geometries of panel are modelled for the present investigation; Spherical Panel $\left(R_1 = R, R_2 = R\right)$, Cylindrical Panel $\left(R_2 = R, R_1 = \infty\right)$, Elliptical Panel $\left(R_2 = R, R_1 = 2R\right)$, and Hyperbolic Panel $\left(R_2 = R, R_1 = -R\right)$. In the present study, three-phase Fiber/CNT/Polymer composite panel is modelled by assuming the dispersion of CNTs to be random within polymer matrix, resulting in a composition of hybrid matrix where the polymer is further reinforced with straight fibers. Due to improper mixing of nanotubes in matrix polymer, the nanoparticles may form

bundles which are known as agglomerations (Shown in Fig. 1(b)). These can be categorized into three types: Null agglomeration $(\alpha = 1; \beta = 1)$, Partial agglomeration $(\alpha = 0.5; \beta = 0.75)$, and Complete agglomeration $(\alpha = 0.5; \beta = 1)$. Here, α and β are the agglomeration coefficients.

2.1 Determination of effective elastic properties of RD-CNTRFC

Effective material properties of CNT-reinforced doubly curved composite panels are estimated in two stages. Initially, hybrid matrix effective material properties are determined through Eshelby-Mori-Tanaka approach, and further Chamis approach is utilized to assess the material characteristics of three-phase composites.

If $\bar{V}_{cnt}$ is the CNT fraction in terms of volume present in the entire hybrid matrix volume ($\bar{V}$), then the nanotubes can exist both inside and outside of aggregated form of CNT-matrix sphere, termed as inclusion. Consequently, the overall CNT volume percent of the hybrid matrix can be calculated as (Shi et al., 2004),

$$\bar{V}_{cnt} = \bar{V}_{cnt}^{in} + \bar{V}_{cnt}^{m} \tag{1}$$

Here, $\bar{V}_{cnt}^{in}$ and $\bar{V}_{cnt}^{m}$ denote the volume of CNTs inside the inclusion, and the volume of CNTs outside the inclusion, respectively. If $\bar{V}^{in}$ is the total volume of inclusion in the hybrid matrix then the agglomeration coefficients (α and β) can be expressed as (Shi et al., 2004),

$$\alpha = \frac{\bar{V}^{in}}{\bar{V}}; \beta = \frac{\bar{V}_{cnt}^{in}}{\bar{V}_{cnt}} \tag{2}$$

2.1.1 Eshelby-Mori-Tanaka approach for hybrid matrix

Using the Eshelby-Mori-Tanaka (E-M-T) method, the elastic moduli of a hybrid matrix can be calculated (Tornabene et al., 2019) as

$$K_{hm} = K_{out}\left[1 + \frac{\alpha\left(\frac{K_{in}}{K_{out}} - 1\right)}{1 + \eta_1(1-\alpha)\left(\frac{K_{in}}{K_{out}} - 1\right)}\right] \tag{3}$$

$$G_{hm} = G_{out}\left[1+\frac{\alpha\left(\frac{G_{in}}{G_{out}}-1\right)}{1+\eta_2(1-\alpha)\left(\frac{G_{in}}{G_{out}}-1\right)}\right] \quad (4)$$

$$\nu_{hm} = \frac{3K_{hm}-2G_{hm}}{6K_{hm}+2G_{hm}} \quad (5)$$

$$E_{hm} = \frac{9K_{hm}G_{hm}}{3K_{hm}+G_{hm}} \quad (6)$$

Here, $\eta_1 = \frac{1+\nu_{out}}{3(1-\nu_{out})}$, $\eta_2 = \frac{2(4-5\nu_{out})}{15(1-\nu_{out})}$. The parameters E_{hm}, ν_{hm}, G_{hm}, and K_{hm} represent the hybrid matrix elastic properties like Elastic modulus, Poisson's ratio, Shear modulus, and Bulk modulus, respectively. Shear moduli (G_{in} and G_{out}) and Bulk moduli (K_{in} and K_{out}) of inclusion and matrix are presented in Appendix-A.

2.1.2 Chamis method for estimation of resulting material properties of randomly distributed CNT reinforced three-phase composites

The obtained hybrid matrix is further fortified with straight fibers, resulting in a three-phase fiber polymer composites. The effective material properties are estimated in the present study using Chamis technique. The volume proportion of fibres and hybrid matrix can be crucial variables to estimate the effective properties. Fibers volume fraction is assumed as $\bar{V}_f = 0.8$ (Lee, 2018) and the volume proportion of hybrid matrix can be determined using rule of mixture $\left(V_{hm} + V_f = 1\right)$. Fibers are the reinforcing agent in polymer matrix and they improve the material stiffness. It has been observed in the published study (Lee, 2018) that the in-plane shear modulus can play an important role for the stability of CNT reinforced composite panels. Therefore for accurate non-linear stability analysis of CNT reinforced fiber composites, the volume fraction of fibers should be calibrated for a higher percentage. Considering the influence of in-plane shear modulus in CNT reinforced composites proposed by Lee, 2018, we have considered a higher volume fraction of fibers ($\bar{V}_f = 0.8$) in the present study. The resulting expressions of three-phase composites' elastic constants are

discussed in Appendix-A.

2.2 Kinematic relations

Incorporating the transverse shear effect of higher order, proposed by Reddy (HSDT), displacement variables can be expressed as (Reddy, 1984),

$$u(x,y,z,t)=\left(1+\frac{z}{R_1}\right)u^0(x,y,t)-zw^0_{,x}(x,y,t)+\wp(z)\phi_1(x,y,t) \tag{7}$$

$$v(x,y,z,t)=\left(1+\frac{z}{R_2}\right)v^0(x,y,t)-zw^0_{,y}(x,y,t)+\wp(z)\phi_2(x,y,t) \tag{8}$$

$$w(x,y,z,t)=w^0(x,y,t) \tag{9}$$

Here, mid-plane displacements are represented as $u^0, v^0,$ and w^0. The total rotational components can be expressed as, $\phi_1=\varphi_x+w^0_{,x}; \phi_2=\varphi_y+w^0_{,y}$. $\wp(z)=z\left[1-\left(\frac{4}{3}\right)\left(\frac{z}{h}\right)^2\right]$ represents the function of transverse shear. φ_x and φ_y are rotation components of cross-section referring to *y* and *x*-directions, respectively. Subsequently, the strain-displacement relationship for doubly curved panels can be expressed as (Reddy, 1984),

$$\varepsilon_{xx}=\varepsilon^0_{xx}-z\left(w_{,xx}-\frac{u^0_{,x}}{R_1}\right)+\wp(z)\phi_{1,x} \tag{10}$$

$$\varepsilon_{yy}=\varepsilon^0_{yy}-z\left(w^0_{,yy}-\frac{v^0_{,y}}{R_2}\right)+\wp(z)\phi_{2,y} \tag{11}$$

$$\gamma_{xy}=\gamma^0_{xy}-z\left(2w^0_{,xy}-\frac{u^0_{,y}}{R_1}\right)-z\left(2w^0_{,xy}-\frac{v^0_{,x}}{R_2}\right)+\wp(z)\phi_{1,y}+\wp(z)\phi_{2,x} \tag{12}$$

$$\gamma_{xz}=\wp'(z)\phi_1 \tag{13}$$

$$\gamma_{yz}=\wp'(z)\phi_2 \tag{14}$$

Here $\varepsilon^0_{xx}, \varepsilon^0_{yy}$, and γ^0_{xy} are the mid-plane strains and expressed by considering von-kármán geometrical non-linearities as (Reddy, 1984),

$$\boldsymbol{\varepsilon}^0 = \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{,x}^0 + \frac{w^0}{R_1} + \frac{1}{2}\left(w_{,x}^0 - \frac{u^0}{R_1} \right)^2 \\ v_{,y}^0 + \frac{w^0}{R_2} + \frac{1}{2}\left(w_{,y}^0 - \frac{v^0}{R_2} \right)^2 \\ u_{,y}^0 + v_{,x}^0 + \left(w_{,x}^0 - \frac{u^0}{R_1} \right)\left(w_{,y}^0 - \frac{v^0}{R_2} \right) \end{Bmatrix} \tag{15}$$

The curved panels are composed of multi-layered laminates where local coordinate of each lamina is transformed to global system. The resulting constitutive relations can be presented as (Reddy, 1984),

$$\boldsymbol{\sigma}_{\mathbf{k}} = \mathbf{Q}_{\mathbf{k}} \boldsymbol{\varepsilon}_{\mathbf{k}} \tag{16}$$

$\boldsymbol{\sigma}_k$ and $\boldsymbol{\varepsilon}_k$ are the vector representation of k^{th} layer stress and strain vectors which can be illustrated as $\boldsymbol{\sigma}_k^T = \{\sigma_{11}, \sigma_{22}, \sigma_{23}, \sigma_{13}, \sigma_{12}\}_k$, $\boldsymbol{\varepsilon}_k^T = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{23}, \varepsilon_{13}, \varepsilon_{12}\}_k$. $\mathbf{Q}_{\mathbf{k}}$ is the reduced stiffness matrix for k^{th} layer of panel and each elements of the matrix can be represented as $Q_{ij}(i, j = 1, 2, 6)$ and along the thickness direction it can be represented as $Q_{ij}(i, j = 4, 5)$ (Reddy, 1984).

$$\begin{aligned} &Q_{11} = \frac{E_{11}^c}{1 - \nu_{12}\nu_{21}}; Q_{12} = \frac{E_{22}^c \nu_{12}^c}{1 - \nu_{12}^c \nu_{21}^c}; Q_{22} = \frac{E_{22}^c}{1 - \nu_{12}^c \nu_{21}^c}; \\ &Q_{44} = G_{23}^c; Q_{55} = G_{13}^c; Q_{66} = G_{12}^c \end{aligned} \tag{17}$$

The constitutive equations are then represented in terms of resultant force/moment-strain relations as (Reddy, 1984),

$$\aleph = \mathrm{A}\,\boldsymbol{\varepsilon}^0 + \mathrm{B}\,\boldsymbol{\kappa} + \mathrm{C}\,\boldsymbol{\phi} \tag{18}$$

$$\mathbf{M} = \mathrm{B}\,\boldsymbol{\varepsilon}^0 + \mathrm{C}\,\boldsymbol{\kappa} + \mathrm{D}\,\boldsymbol{\phi} \tag{19}$$

$$\boldsymbol{\mu} = \mathrm{D}\;\boldsymbol{\varepsilon}^0 + \mathrm{E}\,\boldsymbol{\kappa} + \mathbf{F}\,\boldsymbol{\phi} \tag{20}$$

$$\mathrm{V} = \mathrm{H}\,\vartheta \tag{21}$$

$\aleph, \mathbf{M}, \boldsymbol{\mu}$, and V are the resultant forces, Moments, Additional moments, and Shear force. Here **A** matrix refers to in-plane stiffness, **B** is the coupling of in-plane and flexure stiffness, **D** matrix represents the flexure stiffness, **C** , **E** , **F** are the stiffness contribution due to higher order terms and

328 **H** is the stiffness representing the influence of transverse shear stress (defined in Appendix-B). $\boldsymbol{\kappa}, \boldsymbol{\phi}$,

329 and $\boldsymbol{\vartheta}$ are the strains due to bending, additional bending and shear, respectively, as

$$\boldsymbol{\kappa}^T = \left\{-w^0_{,xx} + \frac{u_{,x}}{R_1}, -w^0_{,yy} + \frac{v_{,y}}{R_2}, -2w^0_{,xy} + \frac{u_{,y}}{R_1} + \frac{v_{,x}}{R_2}\right\}; \boldsymbol{\phi}^T = \left\{\phi_{1,x}, \phi_{2,y}, \phi_{1,y} + \phi_{2,x}\right\};$$

330 $$\boldsymbol{\vartheta}^T = \left\{\phi_2 - \frac{v}{R_2}, \phi_1 - \frac{u}{R_1}\right\} \tag{22}$$

331

In terms of stresses, the resultant forces, and moments for k^{th} layer of laminates can be evaluated as

332 (Reddy, 1984),

$$\left(\begin{Bmatrix}\aleph_{xx}\\ \aleph_{yy}\\ \aleph_{xy}\end{Bmatrix}, \begin{Bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix}, \begin{Bmatrix}\mu_{xx}\\ \mu_{yy}\\ \mu_{xy}\end{Bmatrix}\right) = \int_{-h/2}^{h/2} \begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix} (1, z, \wp(z)) dz = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix} (1, z, \wp(z)) dz$$

333 $$\begin{Bmatrix}V_{xz}\\ V_{yz}\end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix}\tau_{xz}\\ \tau_{yz}\end{Bmatrix} \wp'(z) dz = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{Bmatrix}\tau_{xz}\\ \tau_{yz}\end{Bmatrix} \wp'(z) dz \tag{23}$$

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335 *2.3 Governing partial differential equations*

336 The governing partial differential equations can be derived by minimizing the variation of

337 energy of the system (Soldatos, 1991; Xiao-Ping, 1996) referring to displacement variables. The

obtained partial differential equations for doubly curved RD-CNT reinforced composite panels are

338 $$\delta u : \hat{\aleph}_{xx,x} + \hat{\aleph}_{xy,y} + \left(\frac{M_{xx,x} + M_{xy,y}}{R_1}\right) = 0 \tag{24}$$

339

340

$$\delta v : \hat{\aleph}_{yy,x} + \hat{\aleph}_{xy.x} + \frac{(M_{yy,y} + M_{xy,x})}{R_2} = 0 \tag{25}$$

341 $$\delta w : M_{xx,xx} + 2M_{xy,xy} + M_{yy,yy} - \frac{\hat{\aleph}_{xx}}{R_1} - \frac{\hat{\aleph}_{yy}}{R_2} + \left[\hat{\aleph}_{xx}(w_{,x}) + \hat{\aleph}_{xy}(w_{,y})\right]_{,x} + \left[\hat{\aleph}_{xy}(w_{,x}) + \hat{\aleph}_{yy}(w_{,y})\right]_{,y} = 0 \tag{26}$$

342 $$\delta\phi_1 : \ \mu_{xx,x} + \mu_{xy,y} - V_{xz} = 0 \tag{27}$$

343 $$\delta\phi_2 : \ \mu_{xy,x} + \mu_{yy,y} - V_{yz} = 0 \tag{28}$$

Here, $\hat{N}_{xx} = (N_{xx} - n_{xx})$, $\hat{N}_{yy} = (N_{yy} - n_{yy})$, $\hat{N}_{xy} = (N_{xy} - n_{xy})$ and n_{xx}, n_{yy}, *and* n_{xy} are the in-plane membrane stresses that the panel may experience due to axial non-uniform compression loads. These membrane stresses can be determined by satisfying compatibility and edge conditions (the detailed determination procedure is discussed in Appendix – C). The present study would investigate non-linear stability of doubly curved panels under linear and non-linear in-plane compression loads as shown in Fig. 2. The load distribution profile for linearly varying in-plane loads is expressed as: $\bar{N}_{xx} = N_0\left(1-\eta\left(\frac{y}{b}\right)\right)$, and Parabolic in-plane load can be expressed as (Kumar et al., 2016): $\bar{N}_{xx} = 4N_0\left(\frac{y}{b}\right)\left(1-\frac{y}{b}\right)$. Here η is the load factor that represents the linear distribution of in-plane loads. It may be noted for linear in-plane loads that the panel may not experience pre-buckling stresses as linear in-plane loads match with the neutral axis layer. Thus the investigation of non-uniform loading conditions may be critical for stability analysis.

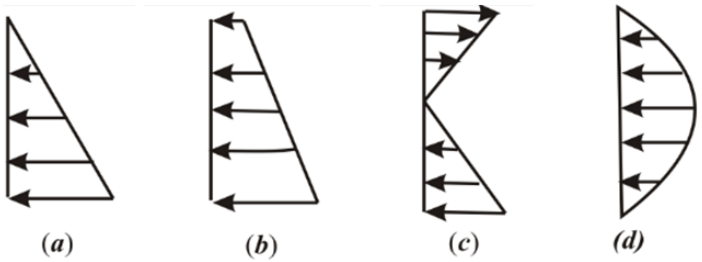

Fig. 2: Linear in-plane loads (*a*) Triangular $(\eta = 1.0)$; (*b*) Trapezoidal $(\eta = 0.5)$; (*c*) Partial Tension $(\eta = 1.5)$; and (*d*) Parabolic in-plane load.

2.4 Boundary conditions

The present methodology can solve for different boundary conditions like all edges are clamped (CCCC), alternate clamped (SCSC) and simply-supported edges (CSCS) (Chakraborty et al., 2019). For numerical investigation, here the non-linear stability analysis of doubly curved panels is performed considering all of the edges as simply supported, and it can be expressed as (Chakraborty et al., 2019a),

For ***x* = 0, *a***: $n_{xx} - \hat{N}_{xx} = -\bar{N}_{xx};\ v^0 = w^0 = M_{xx} = \mu_{xx} = \varphi_y = 0$ (29)

For ***y* = 0, *b***: $n_{yy} - \hat{N}_{yy} = 0;\ u^0 = w^0 = M_{yy} = \mu_{yy} = \varphi_x = 0$ (30)

The assumed displacement field variables can be obtained by satisfying the above boundary conditions (Chakraborty et al., 2019a),

$$\hat{u} = \sum_{p=1}^{P}\sum_{q=1}^{Q} U_{pq} \cos\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi y}{b}\right) \tag{31}$$

$$\hat{v} = \sum_{p=1}^{P}\sum_{q=1}^{Q} V_{pq} \sin\left(\frac{p\pi x}{a}\right)\cos\left(\frac{q\pi y}{b}\right) \tag{32}$$

$$\hat{w} = \sum_{p=1}^{P}\sum_{q=1}^{Q} W_{pq} \sin\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi y}{b}\right) \tag{33}$$

$$\hat{\varphi}_{xx} = \sum_{p=1}^{P}\sum_{q=1}^{Q} K_{pq} \cos\left(\frac{p\pi x}{a}\right)\sin\left(\frac{q\pi y}{b}\right) \tag{34}$$

$$\hat{\varphi}_{yy} = \sum_{p=1}^{P}\sum_{q=1}^{Q} L_{pq} \sin\left(\frac{p\pi x}{a}\right)\cos\left(\frac{q\pi y}{b}\right) \tag{35}$$

Here (ˆ) represents the approximated displacement variables which depend on finite terms p and q along x and y directions respectively.

The above displacement field variables [equations (31) – (35)] are substituted in the governing relations [equations (24) - (28)] and further these partial differential equations (PDEs) are simplified into non-linear algebraic equations using Galerkin's approach. Next, the algebraic relations are rearranged in the form of $\mathbf{K}\boldsymbol{\chi} = 0$. Here, $\boldsymbol{\chi}$ is the displacement vector, and $\mathbf{K}$ is the stiffness matrix. Further $\mathbf{K}$ can be represented in terms of linear stiffness matrix $\mathbf{K_l}$ by neglecting non-linear displacement terms, in-plane axial load, and the geometrical stiffness matrix $\mathbf{K_g}$ containing participation of in-plane load. To obtain these critical buckling loads, the non-linear terms are omitted, and a standard Eigenvalue solution is employed. Furthermore, for the evaluation of non-linear equilibrium paths, the Riks arc-length approach (Vasios, 2015) is utilized, taking into account the non-linear terms in the strain-displacement relationship. It can be noted that neglecting the non-linear terms may not yield very accurate results in non-linear stability analysis for doubly curved panels, while the important behavioral trends can be predicted.

3. Result and Discussion

3.1 Comparative analysis and validation

Geometry-dependent non-linear stability of doubly curved panels such as spherical, elliptical, cylindrical, and hyperbolic panels is the focus of this study. In order to trace non-linear equilibrium path of the panel accurately, geometrical non-linearities are introduced when designing the strain-displacement model and transverse shear deformation theory of higher order is taken into account. The accuracy of the current model is established by comparing the critical load of CNTRC panels (Khdeir et al., 1989; Lei et al., 2016; Shadmehri et al., 2012), and non-linear stability of laminated composite shell panels (Girish and Ramachandra, 2006), as available in literature. The effective material properties of hybrid matrix obtained using E-M-T approach are compared with Shi et al. (2004).

In Table 1, the non-dimensional critical load of a flat polymer composite panel reinforced with CNTs subjected to uniaxial and biaxial compressive in-plane loads are compared with the literature. By assuming that nanoparticles are evenly distributed throughout the polymer matrix and the edges are simply supported, the non-dimensional buckling parameter is obtained for various volume fractions of CNTs and compared with the kp-Ritz solution approach based on transverse shear theory of first order (Lei et al., 2016). One can notice that the current method closely matches with the existing literature, wherein the critical load increases as the % CNTs fraction increases. Here, N_{xcr} represents the instability load for CNT based composite flat panels.

Table 1. Non-dimensional critical load $\left(\frac{N_{xcr}b^2}{E_m h^3}\right)$ of uniformly dispersed CNT flat panel with simply supported edges $(a/b=1, b/h=10, [0/90/90/0])$.

$\bar{V}_{cnt}$	Uniaxial Compression		Biaxial Compression	
	Present	Lei et al. (2016)	Present	Lei et al. (2016)
0.11	20.12	19.84	10.06	9.93
0.14	23.21	22.64	11.60	11.36
0.17	31.29	30.95	15.64	15.49

Table 2. Non-dimensional buckling load $\left(\frac{N_{xcr}a^2}{100E_{22}h^3}\right)$ of simply supported laminated cylindrical shell panel subjected to uniform in-plane compression loads (R/h = 10, R/a = 1, a = b =1).

Ply Layup	Present	Shadmehri et al. (2012)	Khdeir et al. (1989)
[*0/90/0*]	0.2754	0.2765	0.2813
[*0/90*]	0.1703	0.1525	0.1670

Table 2 shows the non-dimensional buckling load parameter for a laminated cylindrical shell panel under in-plane compressive force with simply supported edges. With the help of the current method, non-dimensional parameters are obtained for various ply layups and compared with the solution obtained using Ritz method considering transverse shear of first order as presented by Shadmehri et al. (2012) and higher order transverse shear effect as presented by Khdeir et al. (1989), respectively. The current results are in good agreement with the results of published works.

Further, the non-linear stability of a laminated cylindrical composite shell is traced and compared with the published literature of Girish and Ramachandra (2006), as shown in Fig. 3. It can be noticed that the present non-linear stability results show close agreement with the available literature. The cylindrical shell panel's non-linear equilibrium response exhibits softening behavior in the early stages of deformation and hardening characteristics as the deformation progresses. Such a trend is accurately captured by the current computational framework.

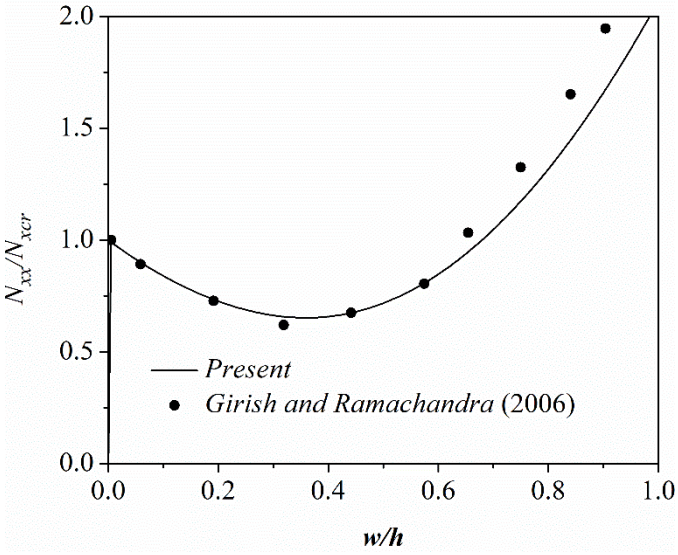

Fig. 3: Post-buckling path of simply supported cylindrical shell panel [0/90/90/0] subjected to in-plane compression load $(a/b = 1, a/R = 0.38)$.

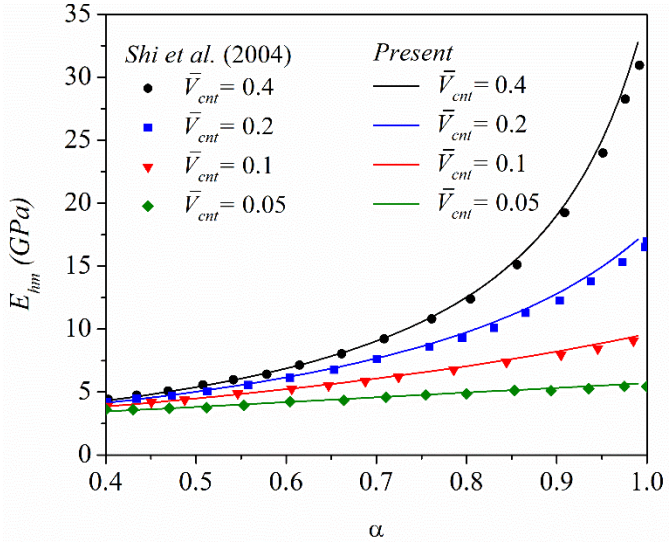

Fig. 4: Comparison of effective elastic property of hybrid matrix with an increase in agglomeration of CNTs $(\beta = 1)$.

After establishing the validations for stability behavior of the shell panels based on the availability of literature (refer to Table 1 – 2 and Fig. 3), we concentrate on the accuracy of the models for obtaining effective material properties of RD-CNT based polymer composites. Fig. 4 shows the comparative results for effective material properties relation given by Shi et al. (2004) for RD-CNT based polymer composites. The material properties are modelled by considering the CNTs agglomeration effect and the elastic modulus of CNT reinforced hybrid polymer composites are seen to decrease with an increase in aggregation of CNTs in polymer, while they increase with % increase of volume content of nanoparticles. To improve the effective elastic properties of CNT reinforced composites, fiber reinforcement has been added to CNT reinforced hybrid matrix which can significantly improve the stiffness property of the composite. In general, a good agreement of the trends can be noticed with respect to published literature.

In this context, it can be noted that a random distribution of nanotubes leads to a heterogeneous mixture, which was also predicted by Shi et al. (2004). In the discussed study by Shi et al. (2004), the authors compared the characteristics of nanotube composites with isotropic and anisotropic properties of nanotubes. They observed that the isotropic characteristics of CNTs may overestimate the tensile modulus of elasticity, which is more pronounced when CNTs are uniformly dispersed in the matrix. Building upon these conclusions, the authors of the present study considered the material properties of nanotubes to be transversely isotropic. Effective material properties are

estimated here using the Mori-Tanaka approach for its simplicity and high accuracy, even at a high volume fraction of inclusions, and also validated with the experimental results by Shi et al., (2004). This approach also takes into account the heterogeneous nature of nanotubes in the matrix. To demonstrate the presence of inhomogeneous behaviour of nanotubes in the polymer matrix, the present micromechanical model considered the agglomeration and dispersion characteristics of nanotubes in the polymer matrix, as shown in Figure 4.

Having validated the computational model from different aspects like the stability behavior and evaluation of effective material properties, we present numerical results concerning the nonlinear stability of doubly curved three-phase composite shells reinforced with randomly distributed carbon nanotubes including the effect of agglomeration in the following subsections. The material properties of CNTs, Polymer and Glass fibers used in the present investigation are mentioned in Table 3.

Table 3. Material properties of CNT, matrix, and fiber

SWCNT (10,10)		
Hill's Modulus (Singh et al., 2022)	**Glass Fiber properties** (Chamis, 1983)	**Matrix Properties (PmPV)** (Singh et al., 2022)
$k_{cnt} = 271\ GPa$	$E_{11}^f = E_{22}^f = E_{33}^f = 73.08 GPa$	$E_m = 2.11 Gpa$
$l_{cnt} = 88\ GPa$	$G_{12}^f = G_{13}^f = G_{23}^f = 30.13 GPa$	$\nu_m = 0.34$
$m_{cnt} = 17\ GPa$	$\nu_{12}^f = \nu_{13}^f = \nu_{23}^f = 0.22$	
$n_{cnt} = 1089\ GPa$		
$p_{cnt} = 442\ GPa$		
Elastic of SWCNT (10,10) (Singh et al., 2022)		
$E_{11}^{cnt} = 5.646 TPa;\ \nu_{12}^{cnt} = 0.175$		

3.2 Influence of CNTs agglomeration

The current study examines the non-linear stability of composite shell panels with randomly dispersed CNT reinforcement. The impact of CNT agglomeration on the non-linear stability of several forms of shell panels, such as cylindrical, spherical, elliptical, and hyperbolic, are covered in this section. In Fig. 5, numerical results are presented for panels subjected to uniform in-plane compression loads with simply supported edges. The non-linear equilibrium path of each panel is normalised with the critical load corresponding to null agglomeration of RD-CNTRFC cylindrical

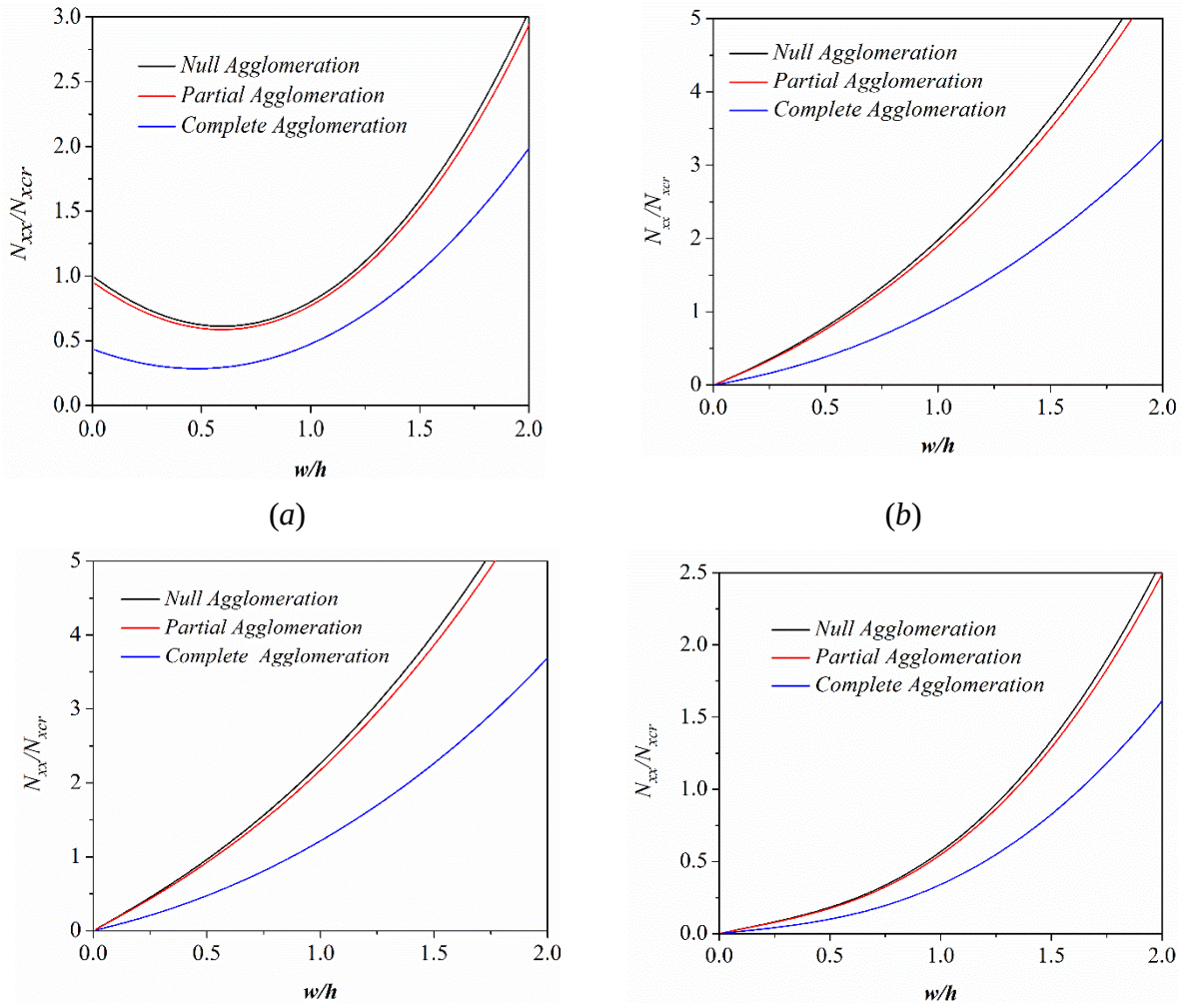

Fig. 5: Influence of agglomeration on RD-CNTRFC panels $[0/90/0/90/0]$ $(a/b=1, b/h=100, R=10a, w_{cnt}=0.25, \alpha=1., \beta=1)$ (a) Cylindrical; (b) Spherical; (c) Elliptical; (d) Hyperbolic.

shell panels. The numerical results reveal that with an increase in CNTs agglomeration, effective material properties decrease, resulting in a decrease of the panel stiffness. Therefore, with reference to null agglomerations the panel exhibits maximum buckling strength, while the minimum value is attained with respect to complete agglomerations. It can be observed that the cylindrical shell panel's non-linear equilibrium path starts with bifurcation point and follows a softening behaviour, and further deformation leads to hardening behavior. Whereas Spherical, Elliptical and Hyperbolic shell panels follow a continuous deformation non-linear equilibrium path due to the tensile stress developed in the transverse direction of the panel with the application of in-plane compressive load. Such non-unique mechanical characteristics give scope for programming (i.e. application-specific

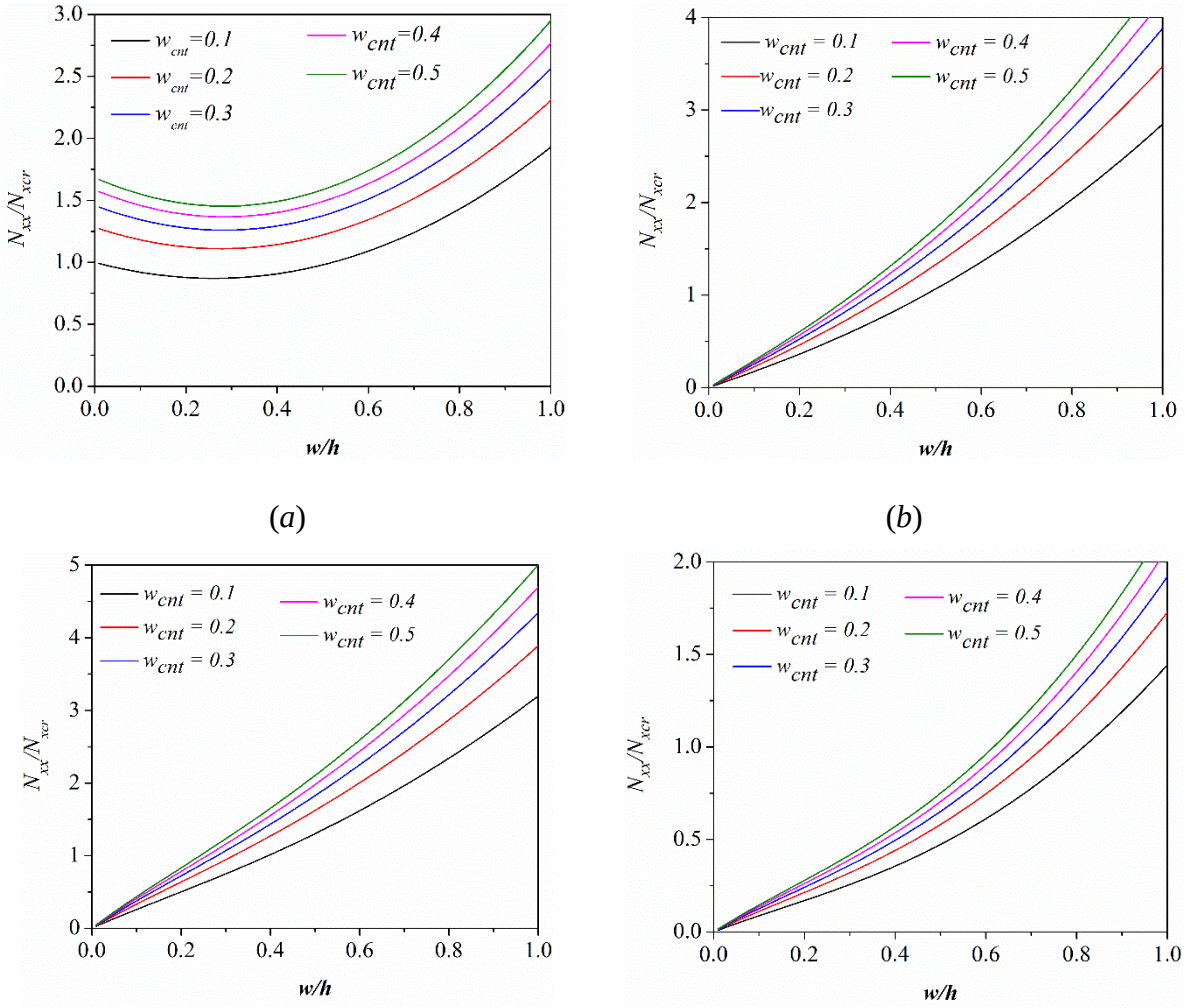

Fig. 6: Non-linear stability path of the RD-CNTRFC shell panels $[0/90/0/90/0]$ for different mass fraction of CNTs $(a/b=1, b/h=10, R/a=2, \alpha=0.50, \beta=0.75)$ (*a*) Cylindrical; (*b*) Spherical; (*c*) Elliptical; (*d*) Hyperbolic.

designing) the constitutive behaviour of softening and hardening based on the shell geometry for application-specific demands.

3.3 Influence of CNTs mass fraction

The influence of CNTs mass fraction on non-linear stability analysis of different shell panels is studied in Fig. 6. The panels are simply supported considering the CNTs to be partially agglomerated in the hybrid matrix and subjected to uniform in-plane compressive load. The non-linear equilibrium path of different panels is normalised with the critical load corresponding to mass fraction (w_{cnt} = 0.1) of the cylindrical shell panel. It can be observed from Fig. 6 that with an increase in mass fraction of CNTs the stiffness of the panels improves, resulting in higher buckling strength, but the rate of increase in post-buckling strength with an increase in mass fraction of

nanotubes decreases. This happens because the increment of nanotubes in a constant volume of polymer matrix tends the nanotubes distribution to agglomerated stage which further reduces the effective stiffness of composites and results in a decrease in the rate of enhancing buckling strength with an increase of mass fractions of nanotubes. Similar observations are reported in published literature (Thirugnanasambantham et al., 2021).

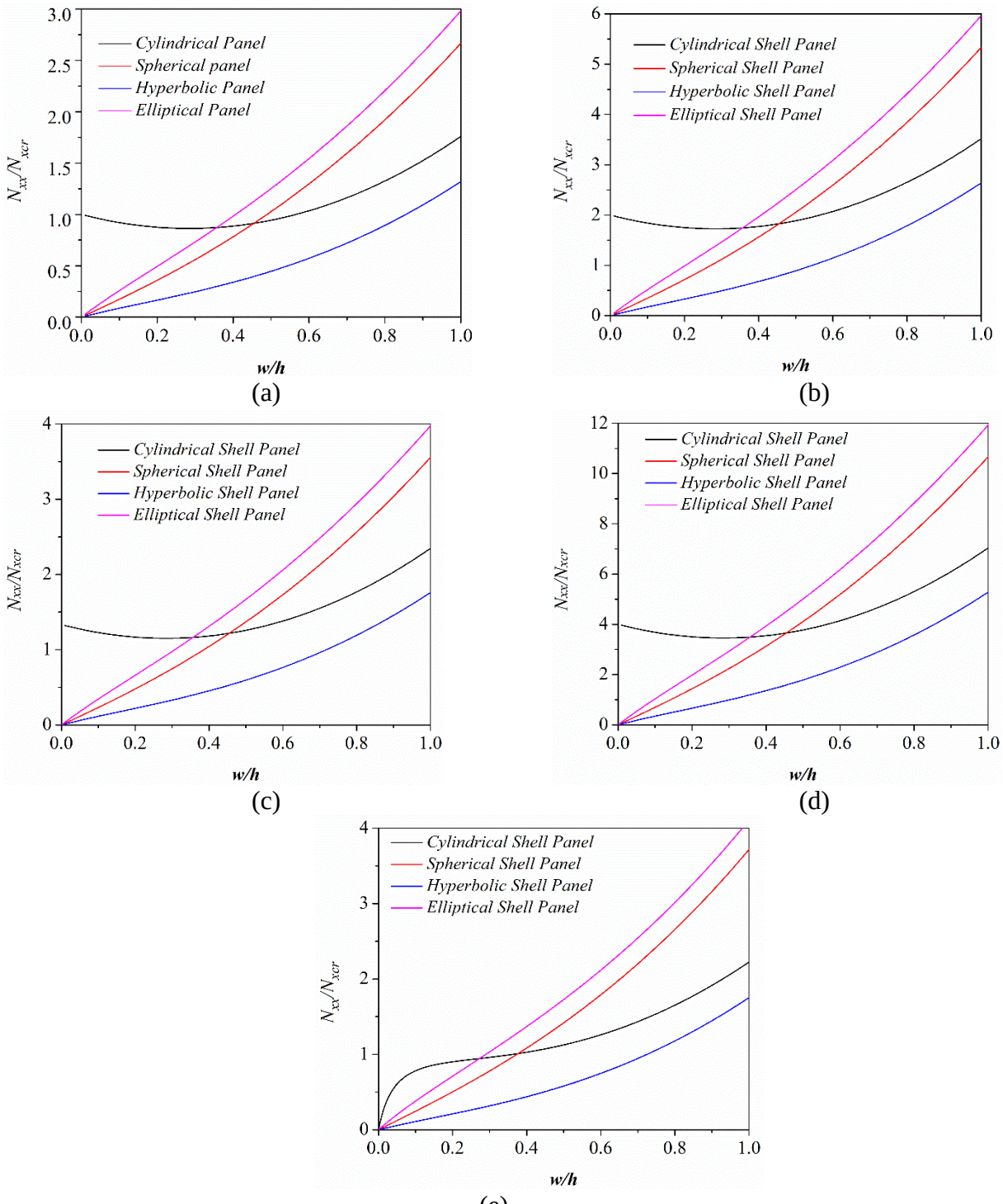

Fig 7: Influence of various in-plane compression loads on post-buckling path of different RD-CNTRFC shell panels $[0/90/0/90/0]$ $(a/b=1, b/h=10, R/a=2, \alpha=1, \beta=1, w_{cnt}=0.25)$ (*a*) Uniform; (*b*) Triangular; (*c*) Trapezoidal; (*d*) Partial Tension; (*e*) Parabolic.

3.4 Influence of non-uniform in-plane loads

In a practical scenario, it is often unlikely that a shell panel will always be subjected to uniform compressive load. To investigate such effects, in this section, the panels are subjected to uniform and non-uniform in-plane loads like triangular, trapezoidal, partial tension, and parabolic load variations (refer to Fig. 7). For uniform and linear in-plane loads panel does not experience pre-buckling stresses as the loads coincide with the panels' neutral axis. But for a parabolic in-plane load, the panel will experience the membrane stresses and therefore it has been incorporated for estimation of exact stress distribution. The non-linear equilibrium path of each panel is normalised with the critical load corresponding to cylindrical shell panel of uniform load distribution. The CNTs in the panel are assumed to be properly mixed in the hybrid matrix. It can be noticed that for cylindrical shell panels subjected to uniform and linear in-plane loads, the non-linear equilibrium path follows bifurcation point. Whereas for cylindrical panels with parabolic in-plane loads, it follows the continuous deformation similar to doubly curved panels. This is due to the nature of tensile stress developed in the transverse direction to applied in-plane load. It can also be observed that panels subjected to partial tensile in-plane load show maximum buckling strength, and the minimum corresponds to uniform in-plane loads. It is also noticed that Elliptically curved panels exhibit maximum buckling strength, followed by Spherical panel, Cylindrical panel, and minimum corresponds to Hyperbolic shell panels (as shown in Fig. 7). In essence, we show in this section that the stability behaviour including softening and hardening depends on the coupled effect of shell geometry and nature of in-plane loading, which can be programmed based on application-specific demands for most optimal performances.

3.5 Influence of Radius of curvature of shell panels

Fig. 8 represents the influence of radius of curvature of shell panels on non-linear stability path of different shell panels. The non-dimensional ratio for equilibrium paths for the shell panel is normalised with critical load corresponding to flat panel when subjected to uniform in-plane compressive load. Thus the presented numerical results give a clear perspective on the stability behaviour of different shell geometries with respect to the flat plate structure. CNTs are assumed to

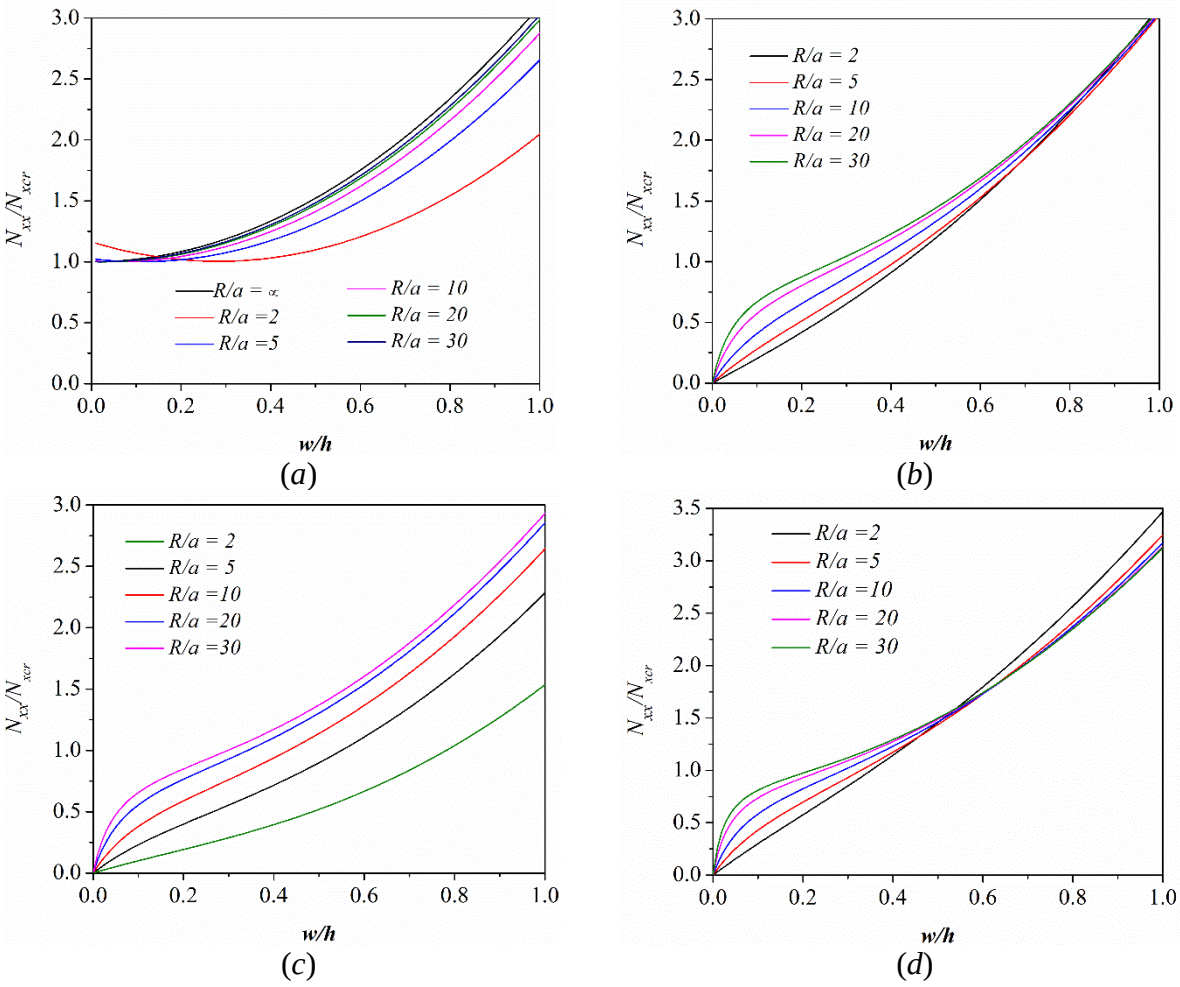

Fig. 8: Influence of *R/a* ratio on non-linear stability path of different RD-CNTRFC shell panels [0/90/0/90/0] configuration (a) Cylindrical; (b) Spherical; (c) Hyperbolic; (d) Elliptical $(a/b=1, b/h=10, \alpha=1, \beta=1, w_{cnt}=0.25)$.

be well mixed with the polymer matrix here. For cylindrical shell panels bifurcation point is observed to be on the higher side with the least value of radius-to-span (*R/a*) ratio. With an increase of *R/a* ratio, the minimum non-dimensional value is observed corresponding to flat panels. For lower *R/a* value, cylindrical panels follow the softening behaviour for the initial non-dimensional displacement parameter (*w/h*) and as deformation progresses hardening behaviour can be seen. But with an increase in *R/a* value softening behaviour in cylindrical panel vanishes. It is also observed in cylindrical panels that, with decreasing *R/a* value buckling strength also decreases with an increase in deformation. Similar behaviour can be noticed for Spherical and Hyperbolic panels, whereas Elliptical panels follow the same characteristic for initial *w/h* value and in a later stage reverse behaviour can be observed. Interestingly, in Spherical panels, the difference in non-linear equilibrium path for different *R/a* values is found to be insignificant at higher values of *w/h*.

Whereas, a significant difference can be observed for Elliptical and Hyperbolic shell panels. At the transition point of softening and hardening behavior of shell panels, stiffness is minimum and corresponding to this stiffness, displacement amplitude can be maximum when a panel is subjected to static and dynamic loads. As an application of the knowledge presented in this article, such softening and hardening behavior can be of significant importance for obtaining the resulting voltage output in energy harvesting devices to operate sensors and actuators (Yan et al., 2020).

In essence, along with the discussions of the preceding subsections, we establish here that a non-linear programming of hardening and softening constitutive behavior is achievable as a function of shell geometry including the radius of curvature and nature of non-uniform loading to address application-specific functional demands optimally. The distribution of CNTs and agglomeration further influence the stability behaviour, albeit not affecting the general trends significantly.

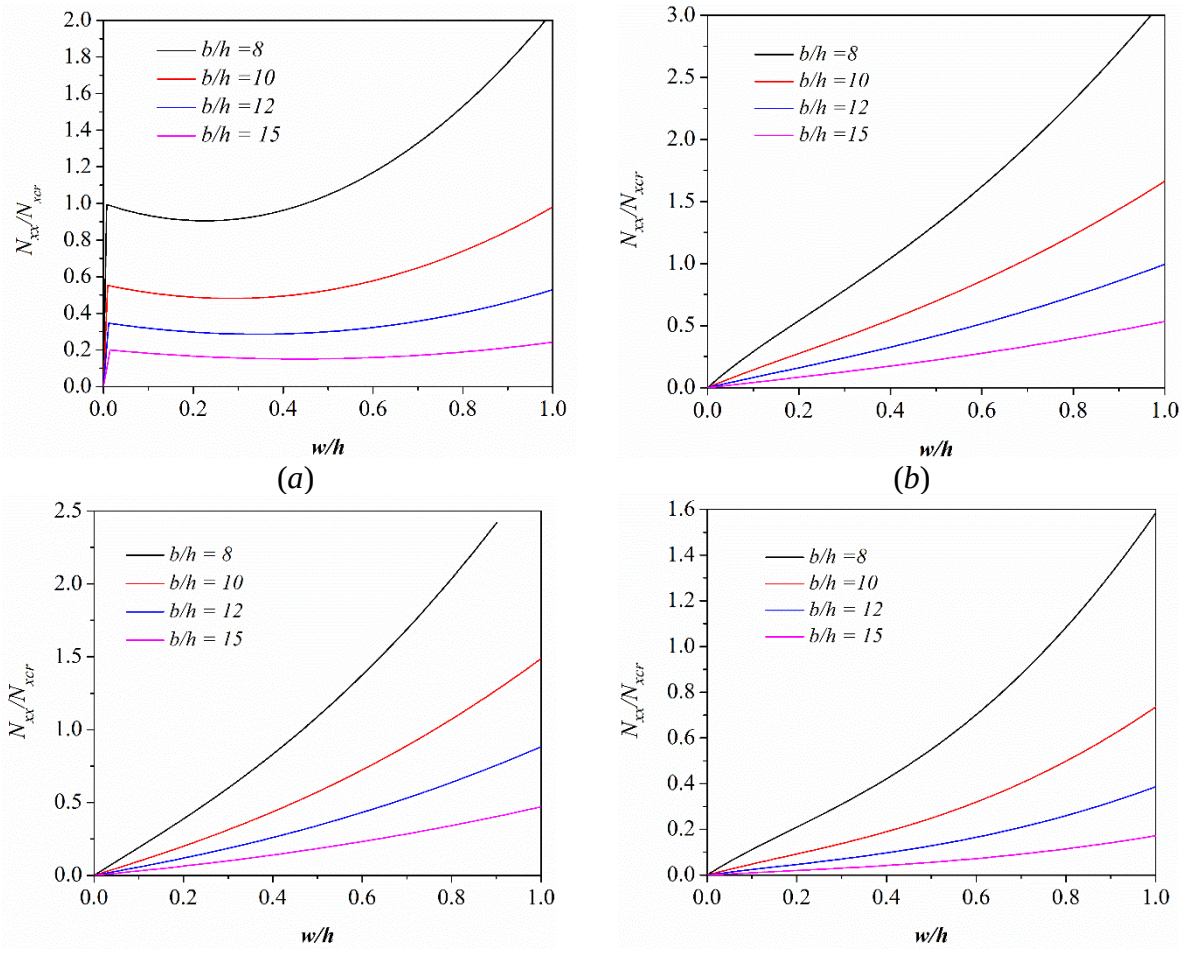

Fig. 9: Influence of thickness of shell panels [0/90/0/90/0] subjected to uniform compressive load (a) Cylindrical Panel; (b) Elliptical Panel; (c) Spherical Panel; (d) Hyperbolic Panel $(a/b=1, R/a=2, \alpha=1, \beta=1, w_{cnt}=0.25)$.

549 *3.6 Influence of thickness of shell panels*

550 Due to the influence of transverse shear effect, thickness of panels is an important parameter
551 to explore the non-linear stability of shell panels. Therefore, the present model is developed to
552 include the higher-order shear theory. The width-to-thickness ratio (*b*/*h*) of the shell panels is taken
553 into account while modelling them by assuming that the CNT distribution is evenly mixed with the
554 polymer matrix. The non-dimensional parameter is then obtained by normalising the non-linear
555 equilibrium route with the critical load. It can be observed from Fig. 9 that with an increase in width-
556 to-thickness ratio (*b/h*) non-linear equilibrium path of shell panels decreases and this is due to the
557 reduction of panels' stiffness with the decrease of shell panel thickness.

558

4. Conclusions

559 In the present work, the non-linear equilibrium path is investigated for different doubly
560 curved panel geometries reinforced with randomly distributed CNTs in the polymer matrix. Due to
561 random distribution, the effective material properties and the stability behaviour are significantly
562 influenced which have been investigated including the effect of agglomeration. Along with the
563 parameters involved with CNT distribution and shell geometry, the effect of non-uniform edge
564 loading is explored in this study to understand their coupled influences on the non-linear equilibrium
565 path. Some of the salient conclusions drawn from the numerical analysis are mentioned here.

566 • Panels with complete agglomeration of CNTs show minimum buckling strength compared to null
567 agglomeration. This is due to the reduction of effective material properties of CNT-reinforced
568 fiber composites that result in a reduction of stiffness of composite with an increase of CNTs
569 agglomeration in the polymer matrix.
570 • Higher stability of the panel is achieved with an increase in mass fraction of CNTs due to an
571 improvement in the stiffness of shell panels. The non-linear equilibrium path of elliptical shell
572 panels exhibits a higher load-deformation path compared to hyperbolic panels.
573 • It is observed that with an increase of mass fraction of nanotubes, keeping the volume of polymer
574 constant, the non-linear stability of panels improves. However, the rate of increase in non-linear

stability decreases subsequently due to the change of state from null agglomerated to agglomerated phase with an increase in mass fraction of CNTs in polymer matrix resulting in deterioration of materials properties.

- Cylindrical panels follow softening type of non-linear behaviour at the initial non-dimensional deformation (*w/h*) and successive hardening characteristics can be observed at higher value of *w/h*. However, the initial softening behaviour is dependent on the radius of curvature and it vanishes with an increase of *R/a* value.
- For doubly curved panels non-linear equilibrium path follows continuous deformation with the application of in-plane loads. This is due to the tensile stress developed in the transverse direction with respect to the in-plane applied load.
- Thickness of shell panels significantly influences the non-linear stability characteristics, albeit without altering the general trends significantly. With an increase of shell panel thickness, the panel stiffness also improves resulting in higher stability compared to thin panels.

In summary, multiple shell geometries such as cylindrical, elliptical, spherical and hyperbolic shapes are modelled here along with the flat plate-like geometry, wherein a geometry-dependent programmable softening and hardening behavior emerges with a quantifiable coupled dependence on the non-uniform nonlinear loading condition, shell thickness, radius of curvature and the CNT parameters. Such control on the nonlinear stability behavior of thin-walled structures would lead to enhancing the performance of a multitude of critical engineering applications including energy harvesting, soft robotics, lightweight structural systems and the design of sensors and actuators. Future work in the field of post-buckling for doubly curved panels reinforced with randomly distributed Carbon Nanotubes (CNTs) in the polymer matrix could involve developing new analytical and numerical models, conducting experimental studies to validate theoretical predictions, exploring the potential of using other types of nanoparticles or hybrid reinforcements, investigating the impact of various boundary conditions and loading scenarios, accessing the influence of different defects and progressive damages such as layer-wise debonding (including quantification of the subsequent effect of reduced stiffness), exploring the effect of operating

environment and evaluating the potential applications of these panels in various critical engineering fields.

Appendix

Appendix-A, Appendix-B and Appendix-C are provided in a supplementary document.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Ansari, R., Torabi, J., Hassani, R., 2018. In-plane and shear buckling analysis of FG-CNTRC annular sector plates based on the third-order shear deformation theory using a numerical approach. Comput. Math. with Appl. 75, 486–502. https://doi.org/10.1016/j.camwa.2017.09.022

Ansari, R., Torabi, J., Shojaei, M.F., Hasrati, E., 2016. Buckling analysis of axially-loaded functionally graded carbon nanotube-reinforced composite conical panels using a novel numerical variational method. Compos. Struct. 157, 398–411. https://doi.org/10.1016/j.compstruct.2016.08.028

Arani, A.G., Maghamikia, S., Mohammadimehr, M., Arefmanesh, A., 2011. Buckling analysis of laminated composite rectangular plates reinforced by SWCNTs using analytical and finite element methods. J. Mech. Sci. Technol. 25, 809–820. https://doi.org/10.1007/s12206-011-0127-3

Arefi, M., Amabili, M., 2021. A comprehensive electro-magneto-elastic buckling and bending analyses of three-layered doubly curved nanoshell, based on nonlocal three-dimensional theory. Compos. Struct. 257. https://doi.org/10.1016/j.compstruct.2020.113100

Bacciocchi, M., 2020. Buckling analysis of three-phase CNT/polymer/fiber functionally graded orthotropic plates: Influence of the non-uniform distribution of the oriented fibers on the critical load. Eng. Struct. 223, 111176. https://doi.org/10.1016/j.engstruct.2020.111176

Balakrishna Reddy, A., Sai Ram, K.S., 2021. Buckling of functionally graded carbon nanotube reinforced composite cylindrical shell panel with a cutout under uniaxial compression. Mater. Today Proc. https://doi.org/10.1016/j.matpr.2021.08.059

Chakraborty, S., Dey, T., Kumar, R., 2021. Instability characteristics of damped CNT reinforced laminated shell panels subjected to in-plane excitations and thermal loading, in: Structures. Elsevier, pp. 2936–2949.

Chakraborty, S., Dey, T., Kumar, R., 2019. Stability and vibration analysis of CNT-Reinforced functionally graded laminated composite cylindrical shell panels using semi-analytical approach. Compos. Part B Eng. 168, 1–14. https://doi.org/10.1016/j.compositesb.2018.12.051

Chakraborty, S., Dey, T., Mahesh, V., Harursampath, D., 2022a. Post-buckling and vibration analysis of randomly distributed CNT reinforced fibre composite plates under localised heating. Mech. Adv. Mater. Struct. 1–20.

Chakraborty, S., Dey, T., Singh, V., Kumar, R., 2022b. Thermal Stability Analysis of Three-Phase

CNTRFC Cylindrical Shell Panels. J. Aerosp. Eng. 35, 4022074.
Chamis, C.C., 1983. Simplified Composite Micromechanics equations for Hygral , Thermal and Mechanical Properties. NASA TM-83320.
Chandra, Y., Adhikari, S., Mukherjee, S., Mukhopadhyay, T., 2022. Unfolding the mechanical properties of buckypaper composites: nano- to macro-scale coupled atomistic-continuum simulations, Engineering with Computers. Springer London. https://doi.org/10.1007/s00366-021-01538-w
Daghigh, H., Daghigh, V., Milani, A., Tannant, D., Lacy, T.E., Reddy, J.N., 2020. Nonlocal bending and buckling of agglomerated CNT-Reinforced composite nanoplates. Compos. Part B Eng. 183, 107716. https://doi.org/10.1016/j.compositesb.2019.107716
Dash, S., Chakraborty, S., Dey, T., Kumar, R., 2022. Buckling and free vibration analysis of randomly distributed CNT reinforced composite beam under thermomechanical loading. Eur. J. Mech. 104749.
Dey, S., Mukhopadhyay, T., Adhikari, S., 2018a. Uncertainty quantification in laminated composites: a meta-model based approach. CRC Press.
Dey S., Mukhopadhyay T., Naskar S., Dey T. K., Chalak H. D., Adhikari S., 2019. Probabilistic characterization for dynamics and stability of laminated soft core sandwich plates, Journal of Sandwich Structures & Materials, 21(1) 366 - 397. https://doi.org/10.1177/1099636217694229
Dey S., Mukhopadhyay T., Sahu S. K., Adhikari S., 2018b. Stochastic dynamic stability analysis of composite curved panels subjected to non-uniform partial edge loading, European Journal of Mechanics / A Solids, 67 108–122. https://doi.org/10.1016/j.euromechsol.2017.09.005
Duc, N.D., Nguyen, P.D., Cuong, N.H., Van Sy, N., Khoa, N.D., 2019. An analytical approach on nonlinear mechanical and thermal post-buckling of nanocomposite double-curved shallow shells reinforced by carbon nanotubes. Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci. 233, 3888–3903. https://doi.org/10.1177/0954406218802921
Fantuzzi, N., Bacciocchi, M., Agnelli, J., Benedetti, D., 2020. Three-phase homogenization procedure for woven fabric composites reinforced by carbon nanotubes in thermal environment. Compos. Struct. 254, 112840. https://doi.org/10.1016/j.compstruct.2020.112840
Fard, M.Y., 2021. Carbon nanotube network and interphase in buckypaper nanocomposites using atomic force microscopy. Int. J. Mech. Sci. 212, 106811.
Fard, M.Y., Pensky, A., 2023. Weibull multiscale interlaminar fracture analysis of low-weight percentage CNT composites. Int. J. Mech. Sci. 250, 108300.
Foroughi, H., Askariyeh, H., Azhari, M., 2013. Mechanical Buckling of Thick Composite Plates Reinforced with Randomly Oriented, Straight, Single-Walled Carbon Nanotubes Resting on an Elastic Foundation using the Finite Strip Method. J. Nanomechanics Micromechanics 3, 49–58. https://doi.org/10.1061/(asce)nm.2153-5477.0000060
García-Macías, E., Rodriguez-Tembleque, L., Castro-Triguero, R., Sáez, A., 2017a. Buckling analysis of functionally graded carbon nanotube-reinforced curved panels under axial compression and shear. Compos. Part B Eng. 108, 243–256. https://doi.org/10.1016/j.compositesb.2016.10.002
García-Macías, E., Rodríguez-Tembleque, L., Castro-Triguero, R., Sáez, A., 2017b. Eshelby-Mori-Tanaka approach for post-buckling analysis of axially compressed functionally graded CNT/polymer composite cylindrical panels. Compos. Part B Eng. 128, 208–224. https://doi.org/10.1016/j.compositesb.2017.07.016
Garg A., Chalak H. D., Sahoo R., Mukhopadhyay T., 2022. Vibration and buckling analyses of sandwich plates containing functionally graded metal foam core, Acta Mechanica Solida Sinica, 35, 1–16. https://doi.org/10.1007/s10338-021-00295-z
Georgantzinos, S.K., Antoniou, P.A., Giannopoulos, G.I., Fatsis, A., Markolefas, S.I., 2021. Design of laminated composite plates with carbon nanotube inclusions against buckling: Waviness and agglomeration effects. Nanomaterials 11, 1–23. https://doi.org/10.3390/nano11092261
Girish, J., Ramachandra, L.S., 2006. Thermomechanical postbuckling analysis of cross-ply laminated cylindrical shell panels. J. Eng. Mech. 132, 133–140. https://doi.org/10.1061/(ASCE)0733-9399(2006)132
Haldar, A., Reinoso, J., Jansen, E., Rolfes, R., 2018. Thermally induced multistable configurations of

variable stiffness composite plates: Semi-analytical and finite element investigation. Compos. Struct. 183, 161–175.
Huang, Y., Karami, B., Shahsavari, D., Tounsi, A., 2021. Static stability analysis of carbon nanotube reinforced polymeric composite doubly curved micro-shell panels. Arch. Civ. Mech. Eng. 21, 1–15.
Kamarian, S., Bodaghi, M., Isfahani, R.B., Song, J. Il, 2020. Thermal buckling analysis of sandwich plates with soft core and CNT-Reinforced composite face sheets. J. Sandw. Struct. Mater. 0, 1–39. https://doi.org/10.1177/1099636220935557
Kar, V.R., Panda, S.K., 2016. Post-buckling behaviour of shear deformable functionally graded curved shell panel under edge compression. Int. J. Mech. Sci. 115–116, 318–324. https://doi.org/10.1016/j.ijmecsci.2016.07.014
Kar, V.R., Panda, S.K., Mahapatra, T.R., 2016. Thermal buckling behaviour of shear deformable functionally graded single/doubly curved shell panel with TD and TID properties. Adv. Mater. Res. 5, 205–221. https://doi.org/10.12989/amr.2016.5.4.205
Karimiasl, M., Ebrahimi, F., Akgöz, B., 2019. Buckling and post-buckling responses of smart doubly curved composite shallow shells embedded in SMA fiber under hygro-thermal loading. Compos. Struct. 223, 110988. https://doi.org/10.1016/j.compstruct.2019.110988
Khaniki, H.B., Ghayesh, M.H., 2020. A review on the mechanics of carbon nanotube strengthened deformable structures. Eng. Struct. https://doi.org/10.1016/j.engstruct.2020.110711
Khdeir, A.A., Reddy, J.N., Frederick, D., 1989. A study of bending, vibration and buckling of cross-ply circular cylindrical shells with various shell theories. Int. J. Eng. Sci. 27, 1337–1351.
Kiani, Y., Mirzaei, M., 2018. Rectangular and skew shear buckling of FG-CNT reinforced composite skew plates using Ritz method. Aerosp. Sci. Technol. 77, 388–398. https://doi.org/10.1016/j.ast.2018.03.022
Kiarasi, F., Babaei, M., Dimitri, R., Tornabene, F., 2020. Hygrothermal modeling of the buckling behavior of sandwich plates with nanocomposite face sheets resting on a Pasternak foundation. Contin. Mech. Thermodyn. 1–22. https://doi.org/10.1007/s00161-020-00929-6
Kumar, R., Dutta, S.C., Panda, S.K., 2016. Linear and non-linear dynamic instability of functionally graded plate subjected to non-uniform loading. Compos. Struct. 154, 219–230. https://doi.org/10.1016/j.compstruct.2016.07.050
Kumar R. R., Mukhopadhyay T., Pandey K. M., Dey S., 2019. Stochastic buckling analysis of sandwich plates: The importance of higher order modes, International Journal of Mechanical Sciences, 152 630-643. https://doi.org/10.1016/j.ijmecsci.2018.12.016
Kundu, C.K., Han, J.H., 2009a. Nonlinear buckling analysis of hygrothermoelastic composite shell panels using finite element method. Compos. Part B Eng. 40, 313–328. https://doi.org/10.1016/j.compositesb.2008.12.001
Kundu, C.K., Han, J.H., 2009b. Vibration and post-buckling behavior of laminated composite doubly curved shell structures. Adv. Compos. Mater. 18, 21–42. https://doi.org/10.1163/156855108X385320
Kundu, C.K., Maiti, D.K., Sinha, P.K., 2007. Post buckling analysis of smart laminated doubly curved shells. Compos. Struct. 81, 314–322. https://doi.org/10.1016/j.compstruct.2006.08.023
Kundu, C.K., Sinha, P.K., 2007. Post buckling analysis of laminated composite shells. Compos. Struct. 78, 316–324. https://doi.org/10.1016/j.compstruct.2005.10.005
Lee, S.Y., 2018. Dynamic instability assessment of carbon nanotube/fiber/polymer multiscale composite skew plates with delamination based on HSDT. Compos. Struct. 200, 757–770. https://doi.org/10.1016/j.compstruct.2018.05.121
Lei, Z.X., Zhang, L.W., Liew, K.M., 2016. Buckling analysis of CNT reinforced functionally graded laminated composite plates. Compos. Struct. 152, 62–73. https://doi.org/10.1016/j.compstruct.2016.05.047
Li, Z.M., Liu, T., Qiao, P., 2021. Buckling and Postbuckling of Anisotropic Laminated Doubly Curved Panels under Lateral Pressure. Int. J. Mech. Sci. 206, 106615. https://doi.org/10.1016/j.ijmecsci.2021.106615
Liew, K.M., Alibeigloo, A., 2021. Predicting bucking and vibration behaviors of functionally graded carbon nanotube reinforced composite cylindrical panels with three-dimensional flexibilities.

Compos. Struct. 256, 113039. https://doi.org/10.1016/j.compstruct.2020.113039
Liew, K.M., Lei, Z.X., Yu, J.L., Zhang, L.W., 2014. Postbuckling of carbon nanotube-reinforced functionally graded cylindrical panels under axial compression using a meshless approach. Comput. Methods Appl. Mech. Eng. 268, 1–17. https://doi.org/10.1016/j.cma.2013.09.001
Liu, T., Li, Z.M., Qiao, P., 2021. The closed-form solutions for buckling and postbuckling behaviour of anisotropic shear deformable laminated doubly-curved shells by matching method with the boundary layer of shell buckling. Acta Mech. 232, 3277–3303. https://doi.org/10.1007/s00707-021-02952-3
Mahapatra, T.R., Panda, S.K., Kar, V.R., 2016. Geometrically nonlinear flexural analysis of hygro-thermo-elastic laminated composite doubly curved shell panel. Int. J. Mech. Mater. Des. 12, 153–171. https://doi.org/10.1007/s10999-015-9299-9
Mehar, K., Mishra, P.K., Panda, S.K., 2021a. Thermal post-buckling strength prediction and improvement of shape memory alloy bonded carbon nanotube-reinforced shallow shell panel: a nonlinear finite element micromechanical approach. J. Press. Vessel Technol. 143, 61301.
Mehar, K., Mishra, P.K., Panda, S.K., 2021b. Thermal buckling strength of smart nanotube-reinforced doubly curved hybrid composite panels. Comput. Math. with Appl. 90, 13–24. https://doi.org/10.1016/j.camwa.2021.03.010
Moradi-Dastjerdi, R., Behdinan, K., Safaei, B., Qin, Z., 2020. Buckling behavior of porous CNT-reinforced plates integrated between active piezoelectric layers. Eng. Struct. 222. https://doi.org/10.1016/j.engstruct.2020.111141
Moradi-Dastjerdi, R., Malek-Mohammadi, H., 2017. Biaxial buckling analysis of functionally graded nanocomposite sandwich plates reinforced by aggregated carbon nanotube using improved high-order theory. J. Sandw. Struct. Mater. 19, 736–769. https://doi.org/10.1177/1099636216643425
Nasihatgozar, M., Daghigh, V., Eskandari, M., Nikbin, K., Simoneau, A., 2016. Buckling analysis of piezoelectric cylindrical composite panels reinforced with carbon nanotubes. Int. J. Mech. Sci. 107, 69–79. https://doi.org/10.1016/j.ijmecsci.2016.01.010
Panda, S.K., Singh, B.N., 2013. Large amplitude free vibration analysis of thermally post-buckled composite doubly curved panel embedded with SMA fibers. Nonlinear Dyn. 74, 395–418. https://doi.org/10.1007/s11071-013-0978-5
Ravi Kumar, L., Datta, P.K., Prabhakara, D.L., 2003. Tension buckling and dynamic stability behaviour of laminated composite doubly curved panels subjected to partial edge loading. Compos. Struct. 60, 171–181. https://doi.org/10.1016/S0263-8223(02)00314-8
Reddy, J.N., 2003. Mechanics of laminated composite plates and shells: theory and analysis. CRC press.
Reddy, J.N., 1984. Exact solutions of moderately thick laminated shells. J. Eng. Mech. 110, 794–809.
Roy, S., Das, T., Ming, Y., Chen, X., Yue, C.Y., Hu, X., 2014. Specific functionalization and polymer grafting on multiwalled carbon nanotubes to fabricate advanced nylon 12 composites. J. Mater. Chem. A 2, 3961–3970. https://doi.org/10.1039/c3ta14528j
Roy, S., Das, T., Zhang, L., Li, Y., Ming, Y., Ting, S., Hu, X., Yue, C.Y., 2015. Triggering compatibility and dispersion by selective plasma functionalized carbon nanotubes to fabricate tough and enhanced Nylon 12 composites. Polymer (Guildf). 58, 153–161. https://doi.org/10.1016/j.polymer.2014.12.032
Safaei, B., Moradi-Dastjerdi, R., Behdinan, K., Qin, Z., Chu, F., 2019. Thermoelastic behavior of sandwich plates with porous polymeric core and CNT clusters/polymer nanocomposite layers. Compos. Struct. 226, 111209. https://doi.org/10.1016/j.compstruct.2019.111209
Shadmehri, F., Hoa, S. V., Hojjati, M., 2012. Buckling of conical composite shells. Compos. Struct. 94, 787–792. https://doi.org/10.1016/j.compstruct.2011.09.016
Shen, H.-S., Zhang, C.-L., 2010. Thermal buckling and postbuckling behavior of functionally graded carbon nanotube-reinforced composite plates. Mater. Des. 31, 3403–3411.
Shen, H.S., 2012. Postbuckling of functionally graded fiber reinforced composite laminated cylindrical shells, Part I: Theory and solutions. Compos. Struct. 94, 1305–1321. https://doi.org/10.1016/j.compstruct.2011.11.034

Shi, D.L., Feng, X.Q., Huang, Y.Y., Hwang, K.C., Gao, H., 2004. The effect of nanotube waviness and agglomeration on the elastic property of carbon nanotube-reinforced composites. J. Eng. Mater. Technol. Trans. ASME 126, 250–257. https://doi.org/10.1115/1.1751182

Singh, V., Vescovini, R., Kumar, R., Patel, S.N., Watts, G., 2022. Nonlinear vibration and instability of a randomly distributed CNT-reinforced composite plate subjected to localized in-plane parametric excitation. Appl. Math. Model. 101, 453–480.

Soldatos, K.P., 1991. A refined laminated plate and shell theory with applications. J. Sound Vib. 144, 109–129.

Thirugnanasambantham, K.G., Sankaramoorthy, T., Karthikeyan, R., Kumar, K.S., 2021. A comprehensive review: Influence of the concentration of carbon nanotubes (CNT) on mechanical characteristics of aluminium metal matrix composites: Part 1. Mater. Today Proc. 45, 2561–2566. https://doi.org/10.1016/j.matpr.2020.11.267

Tornabene, F., Michele Bacciocchi, Fantuzzi, N., Reddy, J.N., 2019. Multiscale Approach for Three-Phase CNT/Polymer/ Fiber Laminated Nanocomposite Structures. Polym. Compos. 40, E102–E126. https://doi.org/10.1002/pc

Trinh, M.C., Mukhopadhyay, T., Kim, S.E., 2020. A semi-analytical stochastic buckling quantification of porous functionally graded plates. Aerosp. Sci. Technol. 105, 105928. https://doi.org/10.1016/j.ast.2020.105928

Van Dung, D., Thuy Dong, D., 2016. Post-buckling analysis of functionally graded doubly curved shallow shells reinforced by FGM stiffeners with temperature-dependent material and stiffener properties based on TSDT. Mech. Res. Commun. 78, 28–41. https://doi.org/10.1016/j.mechrescom.2016.09.008

Vasios, N., 2015. The Arc length method: Formulation, Implementation, and Applications, Harvard education.

Xiao-Ping, S., 1996. An improved simple higher-order theory for laminated composite shells. Comput. Struct. 60, 343–350.

Yan, Z., Sun, W., Hajj, M.R., Zhang, W., Tan, T., 2020. Ultra-broadband piezoelectric energy harvesting via bistable multi-hardening and multi-softening. Nonlinear Dyn. 100, 1057–1077.

Zghal, S., Frikha, A., Dammak, F., 2018. Mechanical buckling analysis of functionally graded power-based and carbon nanotubes-reinforced composite plates and curved panels. Compos. Part B Eng. 150, 165–183. https://doi.org/10.1016/j.compositesb.2018.05.037