Sub-array equalization technique for the parametric array loudspeaker to reduce nonlinear distortion

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ABSTRACT

The parametric array loudspeaker (PAL) is a directional loudspeaker which uses the nonlinear acoustic effect, namely the parametric array, to produce an audio beam from narrow ultrasonic beams. The PAL can efficiently deliver audible information, without generating noise to surroundings. One significant drawback of the PAL is nonlinear distortion. Therefore, many sophisticated methods have been proposed to preprocess the input signal of the PAL. However, those methods usually request a flat frequency response of the ultrasonic transducer array (UTA). In the past, equalization has been tried out for the whole UTA, but the performance was sometimes unsatisfactory due to the inconsistent productions of ultrasonic transducers. This paper proposes to group the ultrasonic transducers by their impedances. Several sub-arrays are resultantly created and equalized individually. The comparison results demonstrate the possibility of the proposed sub-array equalization technique is successful in suppressing the nonlinear distortion of the PAL.

1. INTRODUCTION

Parametric array loudspeaker (PAL) can transmit a narrow audio beam based on the nonlinear acoustic effect of ultrasonic beams [1, 2]. In the 1960s, Westervelt found that two ultrasonic beams can generate a low-frequency sound beam, which is at the difference frequency of the two primary frequencies [3, 4]. The directivity of the difference frequency can be described as an end-fire array, such that the beamwidth of the difference frequency is similar to those of the ultrasonic beams [5]. This nonlinear acoustic effect is now known as the parametric array. The PAL, as an application of the parametric array in air, is now widely implemented in audio projection, active noise control, virtual/mixed reality, and even in contemporary artworks [6–8].

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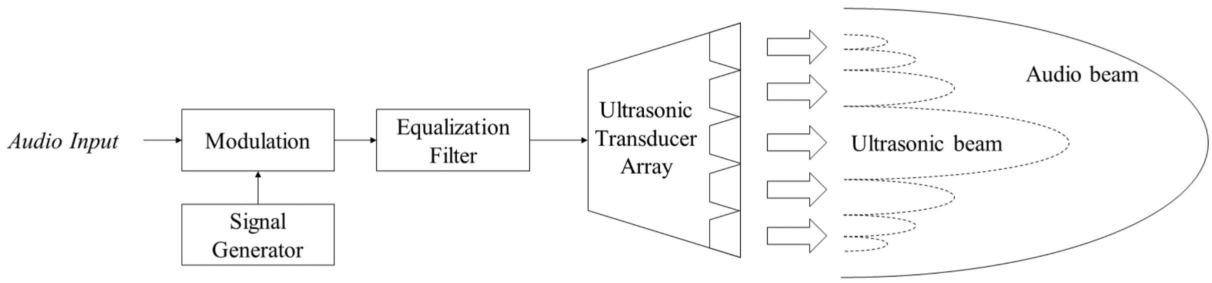


Figure 1: Block diagram of a PAL with an equalization filter for the whole UTA.

Figure 1 shows a conventional structure of the PAL, except for the equalization filter. The audio input is modulated onto an ultrasonic carrier by a specific modulation method. The first PAL created uses the double sideband (DSB) modulation, whose second harmonic distortion is proportional to the modulation index [1]. Subsequently, the square root (SQRT) modulation is proposed to reduce the second harmonic distortion of the PAL and a double integral is further introduced to deal with the higher order harmonic distortion [9, 10]. Since nonlinear distortion is still recognized as a significant drawback of the PAL, many sophisticated modulation methods have been proposed to resolve this issue [11–13].

After the modulation block, the modulated signal is amplified and transmitted into the air through the UTA. In the air, the sideband frequency wave and the carrier frequency wave interact with each other, resulting in their difference frequencies becoming similar to the envelope of the modulated signal. This process is also known as self-demodulation and mathematically described by the Berktay's far-field solution [14]. An early work indicates that the Berktay's far-field solution is a very useful model in the development of the PAL, as long as the effect of the UTA is taken into account [15]. Therefore, an equalization filter is inserted before the UTA in Figure 1. This equalization filter is designed based on the overall frequency response of the whole UTA. Inconsistent productions of ultrasonic transducers sometimes result in large variations in their frequency responses. In order to improve the equalization performance, this paper proposes a sub-array equalization technique for the PAL to reduce nonlinear distortion.

2. NEAR-FIELD AND FAR-FIELD EXTENSIONS OF BERKTAY'S SOLUTION

The one-dimensional acoustic model of the PAL is established from the primary sound pressure, which is written as

$$p_1 = P_0 E(t)\exp(-\alpha_0 z)\cos(\omega_0 t), \tag{1}$$

where P_0 is the initial sound pressure level; $E(t)$ is the envelope of the modulated signal; α_0 is the attenuation rate; ω_0 is the frequency of the ultrasonic carrier; z is the distance; and t is the retarded time. Therefore, the sound strength density is derived as

$$q_d = \frac{\beta {P_0}^2}{{\rho_0}^2 {c_0}^4}\exp(-2\alpha_0 z)\frac{\partial}{\partial t}[\frac{E^2(t)}{2}], \tag{2}$$

where β is the nonlinear coefficient; ρ_0 is the density of air; and c_0 is the speed of the sound in air. The difference frequency sound pressure is given by

$$p_d = \frac{\rho_0 S_0}{4\pi}\int_0^{z'}\frac{1}{z'-z}\frac{\partial q_d}{\partial t}\mathrm{d}z = \frac{\beta {P_0}^2 S_0}{8\pi\rho_0 c_0^4}u(l_A)\frac{\partial^2}{\partial t^2}[E^2(t)], \tag{3}$$

where S_0 is the intersection area of source; z' is the observation point; l_A is the length of the parametric array; and

$$u(l_A) = \int_0^{l_A} \frac{\exp(-2\alpha_0 z)}{z' - z} \mathrm{d}z. \tag{4}$$

When the limited absorption conditions and the far-field are assumed, Equation 3 is simplified to the Berktay's far-field solution, which can be written as

$$p_d = \frac{\beta {P_0}^2 S_0}{16\pi\rho_0 c_0^4 \alpha_0 z'} \frac{\partial^2}{\partial t^2}[E^2(t)]. \tag{5}$$

The UTA often consists of hundreds of ultrasonic transducers. Assuming that the impulse response of the ith transducer is $s_i(t)$, the superposition of the self-demodulated sound pressure generated in the near field is expressed as

$$p_n = \frac{\beta {P_0}^2 S_0}{8\pi\rho_0 c_0^4 M} u(l_N) \times \sum_{i=1}^{M} \left\{ s_a(t) * \frac{\partial^2}{\partial t^2} \{s_i(t) * [E(t)\cos(\omega_c t)]\}^2 \right\}, \tag{6}$$

where M is the total number of ultrasonic transducers; $*$ denotes the convolution;$s_a(t)$ is the impulse response of a low-pass filter to cut off inaudible frequencies of human beings; and l_N is the near-field range of an individual ultrasonic transducer.

Moreover, the self-demodulated sound pressure is generated in the far field by the whole UTA as

$$p_f = \frac{\beta {P_0}^2 S_0}{8\pi\rho_0 c_0^4} [u(l_A) - u(l_N)] \times \left\{ s_a(t) * \frac{\partial^2}{\partial t^2} \left\{ \sum_{i=1}^{M} s_i(t) * [E(t)\cos(\omega_c t)] \right\}^2 \right\}. \tag{7}$$

Equation 6 and 7 can be combined into a generalized formula, *i.e.*

$$p_d = s_a(t) * \sum_{i=1}^{\hat{M}} \left\{ K_i \{\hat{s}_i(t) * [E(t)\cos(\omega_c t)]\}^2 \right\}, \tag{8}$$

where K_i is an acoustic parameter associated with a certain group of ultrasonic transducers that forms the i-th sub-array; $\hat{s}_i(t)$ is the impulse responses of the i-th sub-array; and $\hat{M}$ is the number of sub-arrays in the whole UTA.

3. SUB-ARRAY EQUALIZATION

Due to the limited bandwidth of the ultrasonic transducer, the PAL typically exhibits a band-passing frequency response. Moreover, the magnitude response of the ultrasonic transducer is not flat within its bandwidth, well-designed modulation methods still result in notable nonlinear distortion. For example, the SQRT method implements the envelope function in Equation 8 as

$$E(t) = \sqrt{1 + mx(t)}, \tag{9}$$

where m is the modulation index and $x(t)$ is the audio input. This envelope function eventually requests the ultrasonic transducer to have both an unlimited bandwidth and a flat frequency response. Therefore, the audio input should be band-passed before the modulation method catering to the bandwidth of the ultrasonic transducer. Meanwhile, an equalization filter is added in, as shown in Figure 1. In this case, the self-demodulated sound pressure can be written as

$$p_d = \left\{ s_a(t) * \sum_{i=1}^{\hat{M}} \left\{ K_i \{\hat{s}_i(t) * h(t) * [E(t)\cos(\omega_c t)]\}^2 \right\} \right\}, \tag{10}$$

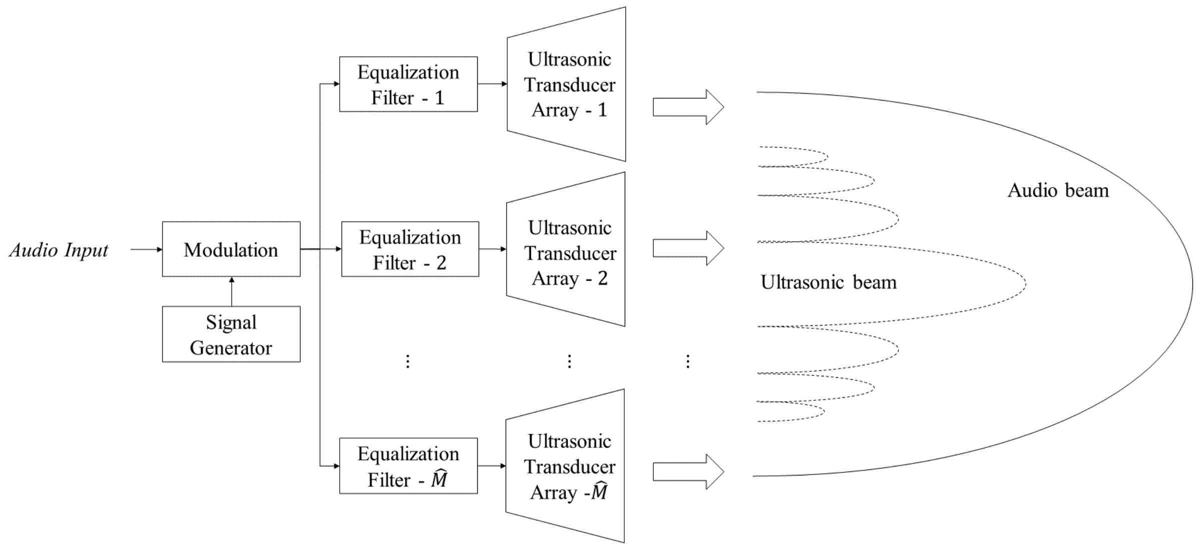


Figure 2: The block diagram of a PAL with sub-array equalization.

where $h(t)$ is the impulse response of the equalization filter.

When the ultrasonic transducers are consistently produced, they have very similar frequency responses. Therefore, letting $\hat{M} = 1$ in Equation 8 gives a sufficiently accurate description of the self-demodulated sound pressure of the PAL. A single equalization filter $h(t)$ can adequately compensate for the uneven frequency response of the whole UTA. However, it is more likely that the ultrasonic transducers are produced in batches. Each batch may share a close impendence. For this reason, a sub-array equalization technique is proposed, as shown in Figure 2. The ultrasonic transducers are grouped based on their impedances. Each group of ultrasonic transducer forms a sub-array. The sub-arrays are equalized individually, so as to achieve more accurate equalization. Similarly, the self-demodulated sound pressure can be written as

$$p_d = s_a(t) * \sum_{i=1}^{\hat{M}} h_i(t) * \left\{ K_i \left\{ \hat{s}_i(t) * [E(t)\cos(\omega_c t)] \right\}^2 \right\}, \tag{11}$$

where $h_i(t)$ is the impulse response of the i-th equalization filter corresponding to the i-th sub-array. For the ease of implementation, $\hat{M} = 3$ is recommended.

4. VALIDATIONS

In this section, the proposed sub-array equalization in Figure 2 is compared with the "one-array" equalization filter in Figure 1 by two simulations. The frequency response of an ultrasonic transducer is randomly generated by changing the resonance frequency based on a measured frequency response. In the first simulation, resonance frequencies are set to be centered at 39 kHz, 40 kHz or 41 kHz, which are shown in Figure 3. Three sub-arrays are formed by selecting ultrasonic transducers with the same center frequency. The least mean square (LMS) algorithm is used to design the equalization filters for the sub-arrays and the whole UTA. By changing the bandwidth of the input band-limited gaussian noise, effectiveness bandwidths of the equalization filters are also adapted. The total harmonic distortion (THD) level is simulated to demonstrate the nonlinear distortion of the PAL under the SQRT method with modulation indices of $m = 0.4$ and $m = 0.8$.

Figure 4 shows the THD levels when the equalization bandwidth is 50 kHz. The THD performance of the one-array and sub-array equalizations remarkably outperform the PAL without equalization. However, the advantage of the sub-array equalization over the one-array equalization is not obvious,

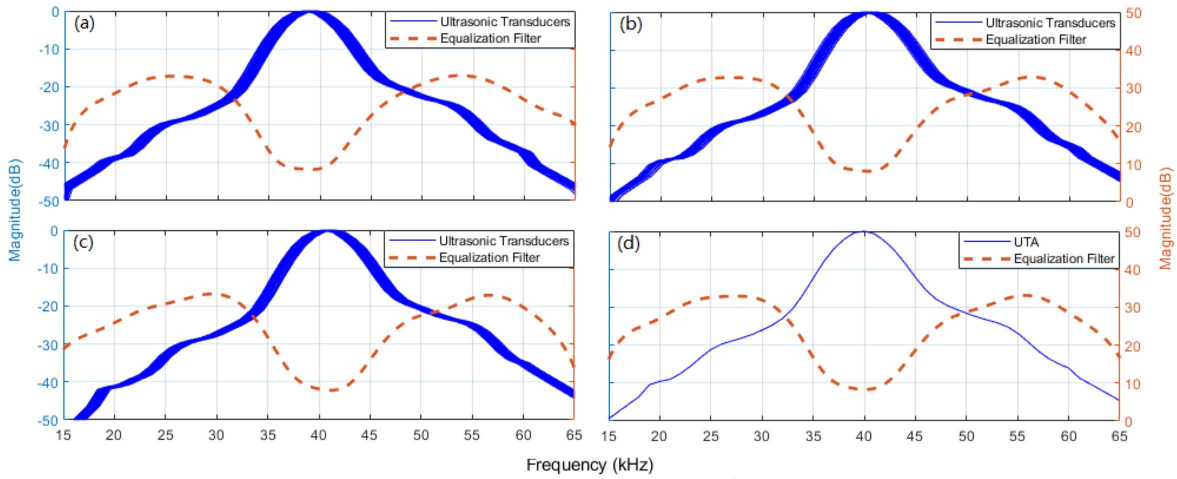
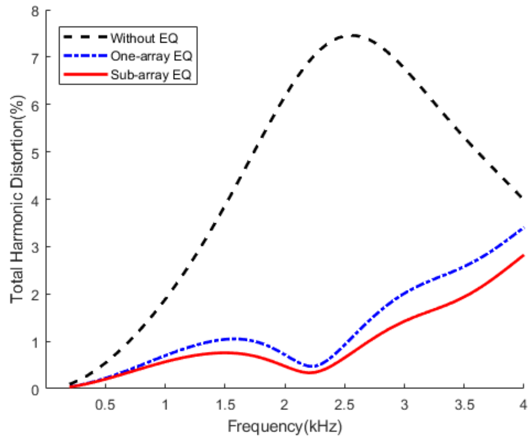
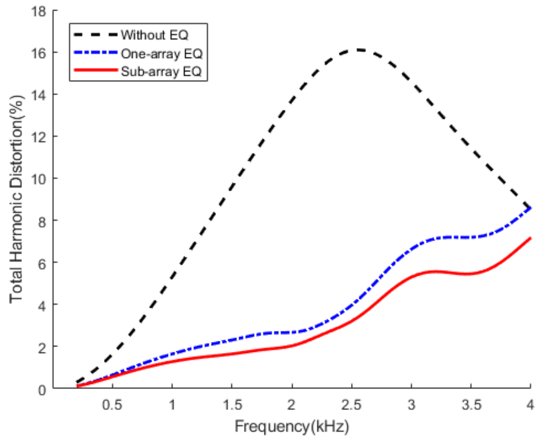


Figure 3: Frequency responses of ultrasonic transducers, whose resonance frequencies are randomly generated to be centered at (a) 39 kHz, (b) 40 kHz and (c) 41 kHz, and (d) presents the frequency response of the whole UTA.

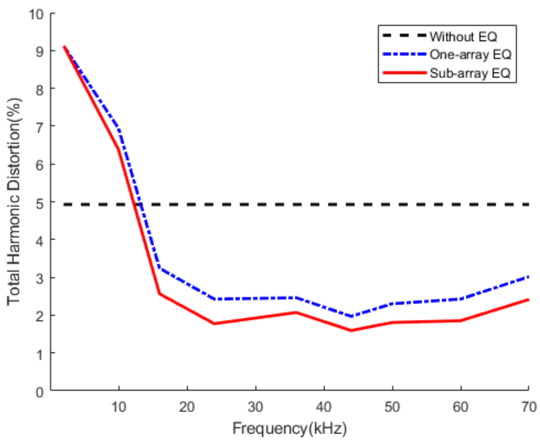


(a) m=0.4

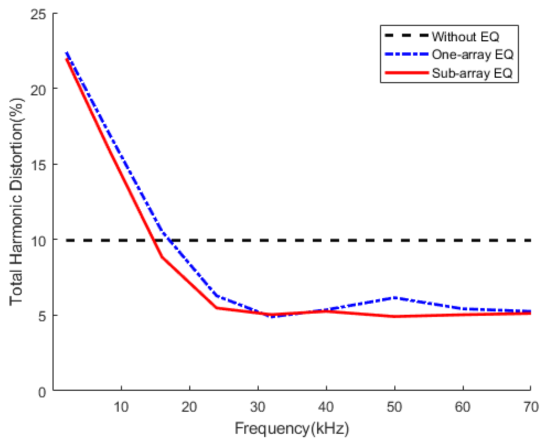


(b) m=0.8

Figure 4: THD performance when the equalization bandwidth is 50 kHz.



(a) m=0.4



(b) m=0.8

Figure 5: THD performance under different equalization bandwidths.

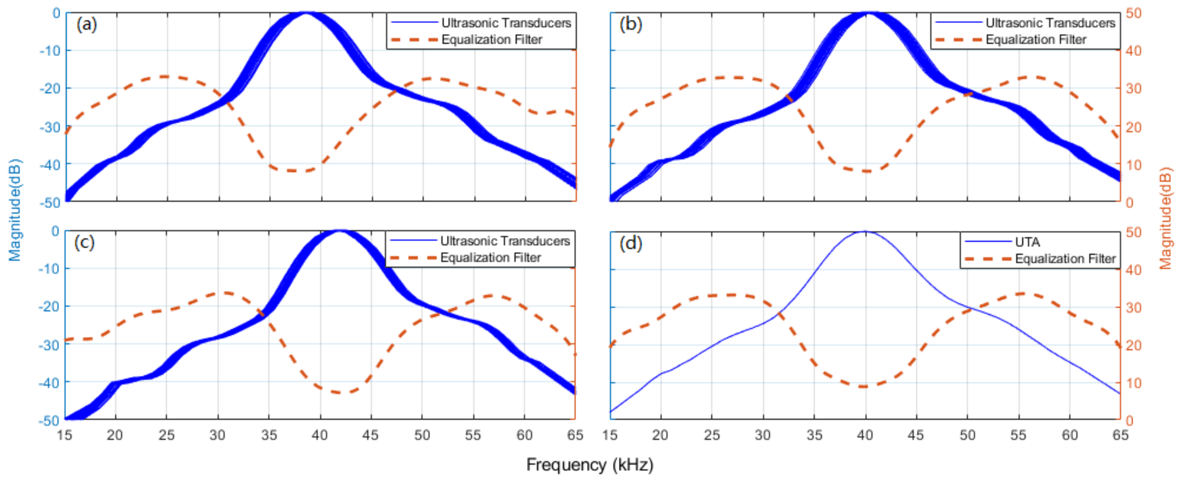
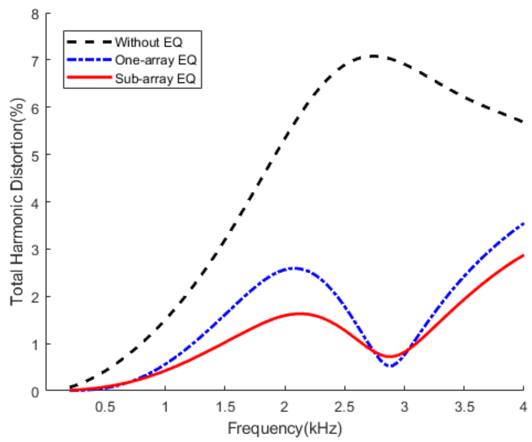
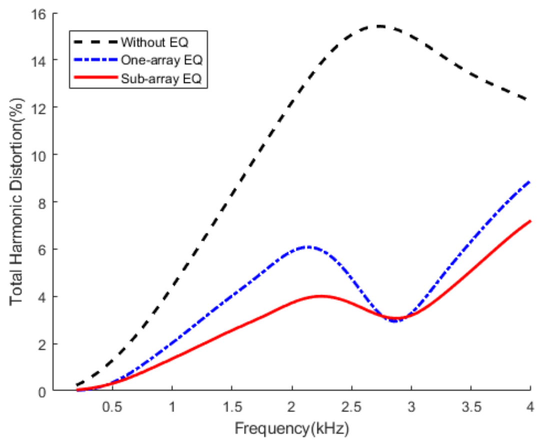


Figure 6: Frequency responses of ultrasonic transducers, whose resonance frequencies are randomly generated to be centered at (a) 38 kHz, (b) 40 kHz and (c) 42 kHz, and (d) presents the frequency response of the whole UTA.

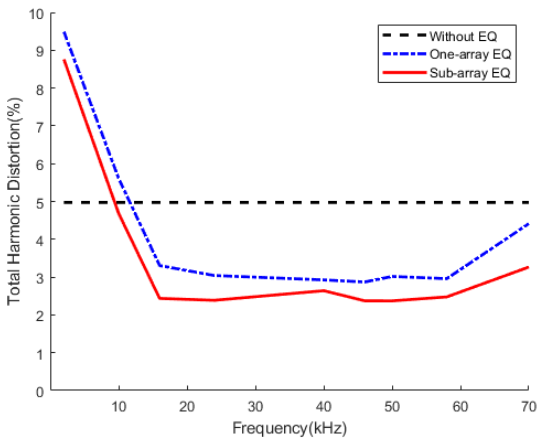


(a) m=0.4

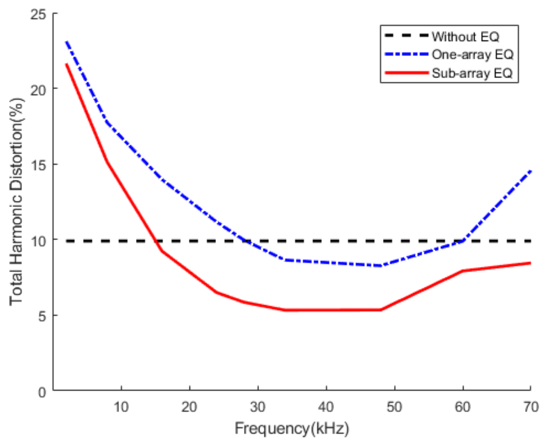


(b) m=0.8

Figure 7: THD performance when the equalization bandwidth is 50 kHz.



(a) m=0.4



(b) m=0.8

Figure 8: THD performance under different equalization bandwidths.

due to the fact that the variant of the simulated resonance frequencies of ultrasonic transducers is mild. Figure 5 shows the averaged THD levels when the equalization bandwidth varies up to 70 kHz. Since the frequency range of the THD test is merely up to 4 kHz, once the equalization bandwidth exceeds 8 kHz, the second harmonic distortion reduces significantly. Therefore, the selection of the equalization bandwidth should follow the rule of thumb, *i.e.* two times the frequency range of the audio input.

In the second simulation, resonance frequencies are set to be centered at 38 kHz, 40 kHz or 42 kHz. Their corresponding frequency responses and equalization filters are shown in Figure 6. The rest of simulation settings are kept the same as those in the first simulation. Figure 7 shows the THD levels when the equalization bandwidth is 50 kHz. The THD performance of the one-array and sub-array equalizations still outperform the PAL without equalization. The advantage of the sub-array equalization over the one-array equalization is more obvious, with the exception of specific frequency bands. In those frequency bands, the one-array equalization has already achieved very low THD level. Figure 8 shows the averaged THD levels when the equalization bandwidth varies up to 70 kHz. When the equalization bandwidth is 70 kHz, there is an adverse effect caused by the one-array equalization, and the effectiveness of the sub-array equalization is degraded. The sound pressure level of the PAL is also traded off when the equalization bandwidth is inappropriately selected. When the modulation index is higher, the sub-array equalization can further reduce the THD level of the PAL as compared to the one-array equalization. This suggests that the sub-array equalization is better at achieving higher sound pressure levels than the one-array equalization.

5. CONCLUSIONS

This paper proposes to group the ultrasonic transducers in a PAL by their impedances. Several sub-arrays are thereafter formed and equalized individually. By doing so, the PAL can achieve relatively low THD and high sound pressure levels by using the SQRT method with a large modulation index. The equalization bandwidth is suggested to be two times greater than the frequency range of the input audio, i.e. sufficiently wide enough for its optimal performance and not too wide which may result in performance degradation. In practice, grouping the ultrasonic transducers requires a large number of manpower which would be preferably automated in future works.

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