Continuous Space-Time Crystal State Driven by Nonreciprocal Optical Forces

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Abstract: An ensemble of thermally driven oscillators that are nonreciprocally coupled and inhomogeneously broadened exhibits a spontaneous transition to the continuous space-time crystal state.

A Space-time crystal (STC) is an eagerly sought non-equilibrium state of matter where space and time translational symmetries are spontaneously broken. Recently, the continuous STC state has been experimentally observed in a thermally driven array of nanowires decorated with π-shaped plasmonic metamolecules [1]. It is argued that non-reciprocal optical forces trigger high-amplitude synchronized oscillation across the array [2]. STCs offer valuable insights into the dynamics of classical many-body states within the realm of strongly correlated phenomena, and understanding the criteria for systems to become STCs is essential. Here, we show that noise-driven oscillators must be nonreciprocally coupled and inhomogeneously broadened to unlock the STC state.

Noise-driven coupled oscillators can be described by the Langevin equation: $\ddot{x}_i + \gamma\dot{x}_i + \omega_{0i}^2 x_i + \beta x^3 + \sum\xi_{ij}(x_i - x_j) = \sqrt{2k_B T\gamma/m_i}\eta_i(t)$, where ω_{0i} are the natural angular frequencies of individual oscillators with effective mass m_i, ξ_{ij} is the coupling coefficient describing their interaction, k_B is the Boltzmann constant, T is temperature, γ is the dissipation factor, β signifies the geometric nonlinearity, and $\eta_i(t)$ is a normalized white noise term. We consider a pair of oscillators with natural frequencies $\omega_0 \pm \delta\omega$; $\omega_0 = 2\pi \times 2MHz$, effective mass $m_1 = m_2 = 1pg$, $\gamma = 10^4$, and $\beta = 10^9\omega_{0i}^2$ (Fig 1). Coupling coefficients are expressed as $\xi_{ij} = n\omega_{0i}^2$, where n is a real number. The non-linearity, β is present only to constrain the exponential growth of oscillation amplitude in the synchronized state and does not play any crucial role in triggering synchronization. We numerically solve the equation for various values of $\delta\omega$.

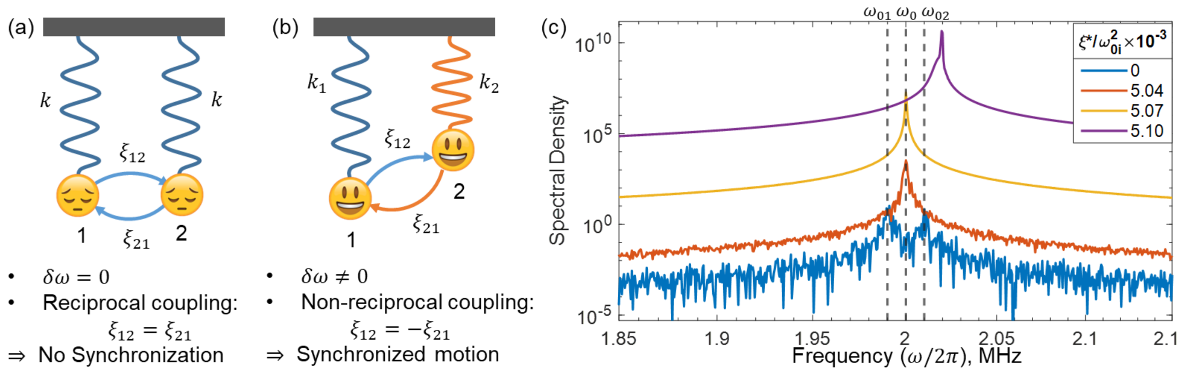

Fig. 1 (a) Schematic of coupled oscillators illustrating scenarios of (a) no synchronization and (b) synchronization (CTC state). (c) Spectral density of relative position for a pair of nonreciprocally coupled oscillators at various coupling strengths.

The STC state is characterized by synchronization of the oscillators – at a threshold value of coupling strength, they undergo a sharp 'phase transition' to a state in which they oscillate at the same frequency. In the case of identical ($\delta\omega = 0$) and reciprocally coupled oscillators, they do not synchronize; incoherent oscillation persists regardless of coupling strength (Fig. 1a). When the coupling is nonreciprocal, the oscillators move in phase, but synchronized state is not accompanied by sharp phase transition. Hence, it can be not characterized as an STC state. The sharp phase transition assisted synchronization and, hence STC state is achieved by introducing inhomogenous frequency broadening ($\delta\omega \neq 0$) in nonreciprocally coupled oscillators (Fig. 1b). The critical coupling, ξ_c decides the onset of synchronization where individual peaks merge into a single peak which grows with coupling (Fig. 1c for $\delta\omega = 10$ KHz). This ξ_c increases/decreases linearly with increasing/decreasing detuning, except for small values of $\delta\omega$.

In conclusion, we have demonstrated that a space-time crystal state among noise-driven oscillators can be driven by nonreciprocal coupling between and inhomogeneous frequency broadening.

References

[1] T. Liu, J. Y. Ou, K. F. MacDonald, N. I. Zheludev, Photonic metamaterial analogue of a continuous time crystal. Nat. Phys. 19, 986–991 (2023).

[2] V. Raskatla, T. Liu, J. Li, K. F. MacDonald, N. I. Zheludev, Continuous Space-Time Crystal State Driven by Nonreciprocal Optical Forces. arXiv:2310.10747 (2023).