# Uncertainty Shocks, Precautionary Pricing, and Optimal Monetary Policy<sup>\*</sup>

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## Abstract

Existing studies show that, in standard New Keynesian models, uncertainty shocks manifest as costpush shocks due to the precautionary pricing channel. We study optimal monetary policy in response to uncertainty shocks when the precautionary pricing channel is operative. We show that, in the absence of real imperfections, the optimal monetary policy fully stabilizes the output gap and inflation, implying no policy trade-offs. Our result suggests that precautionary pricing matters only insofar as expected inflation is volatile. Thus, a simple Taylor rule that places high weight on inflation leads to a stabilized output gap, thereby attaining the "divine coincidence."

**Keywords:** Uncertainty shocks, Precautionary pricing, Optimal monetary policy **JEL Classification:** E12, E52

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## 1 Introduction

Time-varying uncertainty has recently received considerable attention from policymakers and academics, spurring the burgeoning literature on identifying transmission mechanisms of uncertainty shocks. It has been shown that a precautionary pricing motive is an important mechanism that amplifies uncertainty shocks. This mechanism is present in New Keynesian models when sticky prices are modeled according to Calvo (1983) and monetary policy follows an empirical Taylor rule. Due to the presence of a precautionary pricing motive, uncertainty shocks behave like cost-push shocks; a rise in uncertainty causes an increase in inflation but a fall in the output gap. A classic and important question for policymakers is whether these shocks generate the well-known output-inflation trade-off that emerges in response to cost-push shocks.

This paper studies optimal monetary policy when the precautionary pricing channel is present. Our main result is that the output gap and inflation are both stabilized under optimal monetary policy, meaning that policy trade-offs do not emerge. Our finding suggests that the precautionary pricing channel is operative only in an environment in which inflation is volatile. Therefore, monetary policy that fully stabilizes inflation eliminates the inefficiencies related to the precautionary pricing channel, thus allowing policymakers to attain efficient allocation.

Our conclusion is drawn from comparing allocations under optimal monetary policy in two popular price-setting approaches. The first is Calvo (1983) pricing, under which firms face a constant probability of not being allowed to reoptimize their price in every period. The second is Rotemberg (1982) pricing, under which firms can always adjust their price upon payment of a quadratic price adjustment cost. While the precautionary saving motive is operative under both Calvo and Rotemberg pricing, precautionary pricing is only operative with Calvo pricing (Oh, 2020). Accordingly, comparing allocations under the optimal monetary policy in Calvo and Rotemberg allows us to evaluate the extent to which precautionary pricing matters for a monetary policy prescription.

Specifically, under Rotemberg pricing, uncertainty shocks act as negative demand shocks; a rise in uncertainty increases the households' precautionary savings motive, which causes a decrease in both inflation and the output gap. In contrast, under Calvo pricing, a rise in uncertainty triggers firms' precautionary pricing motives along with households' precautionary saving motives. A precautionary pricing motive stems from firms' exposure to the risk of not being able to set their desired price level in the future. Under Calvo, as long as the future expected inflation is volatile, this risk is always present. Price-resetting firms that are exposed to such risk raise prices today to hedge against an uncertain future profit stream. This causes a rise in inflation and a sharper fall in the output gap, as the resulting inflation increase further compresses aggregate demand. Therefore, because of the precautionary pricing channel, uncertainty shocks are more amplified in Calvo than in Rotemberg. We find that, under optimal monetary policy, the differences between allocations in Calvo and Rotemberg disappear. This implies that the precautionary pricing channel is not operative under optimal monetary policy. Moreover, the joint stabilization of the output gap and inflation in Rotemberg suggests that a precautionary saving motive does not pose any policy trade-off, which is consistent with the property of demand shocks in textbook New Keynesian models.

The joint stabilization of the output gap and inflation under optimal monetary policy suggests that the divine coincidence holds in the case of uncertainty shocks; inflation stabilization also brings about the output gap stabilization. Thus, a simple rule that places extremely high weight on inflation (i.e., the strict inflation targeting rule) closes the output gap. It is worth noting that the divine coincidence does not emerge in response to uncertainty shocks in all models. As discussed in Blanchard and Galí (2007), the divine coincidence emerges only in the absence of nontrivial real imperfections. We confirm their results by showing that a trade-off between the output gap and inflation arises in response to uncertainty shocks when real wage rigidities are introduced.

**Related Literature** Our paper is related to three main streams of literature. The first focuses on the transmission of uncertainty shocks in New Keynesian models. Leduc and Liu (2016) and Basu and Bundick (2017) focus on the demand channel of uncertainty shocks due to precautionary saving behavior. Moreover, Born and Pfeifer (2014), Fernández-Villaverde et al. (2015), and Mumtaz and Theodoridis (2015) study the supply channel of uncertainty shocks engendered by precautionary pricing. We contribute to this literature by studying the implications of the demand and supply channels of uncertainty shocks in designing optimal monetary policy.

Our paper is also related to the extensive body of literature on optimal monetary policy in New Keynesian models such as Khan et al. (2003), Yun (2005), Blanchard and Galí (2007), and Faia (2008) among many others. These papers study how monetary policy should be implemented in the presence of frictions in the real and monetary sectors. All of the papers in this literature focus on first-moment shocks, whereas our interest lies in second-moment shocks.

Finally, our paper is related to the literature that compares normative results under the Calvo and Rotemberg pricing assumptions. Nisticó (2007) and Lombardo and Vestin (2008) compare the welfare implications of the Calvo and Rotemberg models. Leith and Liu (2016) compares the inflation bias. All of these papers present an environment in which monetary policy is suboptimal. In contrast, our work compares the dynamics in response to uncertainty shocks when monetary policy is optimal.

The rest of the paper is structured as follows. Section 2 gives a brief overview of the optimality conditions of a textbook New Keynesian model under the Calvo and Rotemberg pricing schemes. Section

3 describes the problem of the Ramsey optimal monetary policy. Section 4 describes the calibration and solution method. Section 5 discusses both the analytical and numerical results on optimal monetary policy in response to uncertainty shocks. In Section 6, we discuss the optimal responses in the presence of real wage rigidities. Section 7 concludes.

# 2 Textbook New Keynesian Models

We describe the equilibrium conditions of a basic New Keynesian model under the Calvo (1983) and Rotemberg (1982) price rigidities. The model features a utility-maximizing household, perfectly competitive final good firms, monopolistically competitive intermediate good firms, and exogenous productivity subject to second-moment shocks.

The optimal labor supply and consumption of a representative household are characterized by:

$$\frac{\chi N_t{}^\eta}{C_t{}^{-\gamma}} = w_t,\tag{1}$$

$$C_t^{-\gamma} = \beta E_t C_{t+1}^{-\gamma} \frac{R_t}{\pi_{t+1}},$$
(2)

where  $C_t$  indicates consumption,  $N_t$  is labor supply, and  $w_t$  is the real wage.  $\pi_t$  is the gross inflation rate, while  $R_t$  is the gross nominal interest rate.  $\gamma$  is the risk-aversion parameter,  $\chi$  is the labor disutility parameter, and  $\eta$  is the inverse of the Frisch labor supply elasticity.

Differentiated goods are produced by a continuum of intermediate good firms, indexed by  $i \in [0, 1]$ , according to:

$$Y_t(i) = A_t N_t(i). \tag{3}$$

 $A_t$  is the exogenous productivity following:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_t^A \varepsilon_t^A, \quad 0 \le \rho_A < 1, \quad \varepsilon_t^A \sim N(0, 1).$$
(4)

 $\sigma^A_t$  is the time-varying volatility of productivity and follows:

$$\log \sigma_t^A = (1 - \rho_{\sigma^A}) \log \sigma^A + \rho_{\sigma^A} \log \sigma_{t-1}^A + \sigma^{\sigma^A} \varepsilon_t^{\sigma^A}, \quad 0 \le \rho_{\sigma^A} < 1, \quad \varepsilon_t^{\sigma^A} \sim N(0, 1), \tag{5}$$

where  $\sigma^A$  indicates the steady state of  $\sigma_t^A$ .

Intermediate goods are aggregated into final goods using a CES technology with the elasticity of sub-

stitution  $\epsilon > 1$ . The average real marginal cost is given by:

$$mc_t = \frac{w_t}{A_t}.$$
(6)

The efficient output  $Y_t^e$ , which is the level of output that would prevail under flexible prices and perfect competition, is determined by:

$$\chi \left(\frac{Y_t^e}{A_t}\right)^{\eta} = Y_t^{e-\gamma} \frac{\epsilon - 1}{\epsilon \left(1 - \tau\right)} A_t,\tag{7}$$

where  $\tau = \frac{1}{\epsilon}$  indicates the rate at which firms' production is subsidized and ensures the efficient steady state. In this case, the natural output, i.e., output under flexible prices, is exactly equal to the efficient output. In this paper, we define the output gap as the distance between the actual and efficient output:

$$\widetilde{Y}_t = \frac{Y_t}{Y_t^e}.$$
(8)

**Calvo** Under Calvo pricing, only a fraction  $1 - \theta$  of intermediate good firms, are allowed to reset their price in a given period. Denoting the optimal reset price by  $P_t^{\star}$ , the optimal relative reset price,  $p_t^{\star} = \frac{P_t^{\star}}{P_t}$ , solves:

$$p_t^{\star} = \frac{\epsilon \left(1 - \tau\right)}{\epsilon - 1} \frac{p_t^n}{p_t^d},\tag{9}$$

$$p_t^d = C_t^{-\gamma} Y_t + \theta \beta E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon-1} p_{t+1}^d,$$
(10)

$$p_t^n = C_t^{-\gamma} m c_t Y_t + \theta \beta E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon} p_{t+1}^n, \tag{11}$$

where  $P_t$  indicates the aggregate price level. Inflation evolves according to:

$$\theta\left(\frac{\pi_t}{\pi}\right)^{\epsilon-1} = 1 - (1-\theta) p_t^{\star 1-\epsilon}.$$
(12)

The aggregate production function and resource constraint are given by:

$$\Delta_t Y_t = A_t N_t,\tag{13}$$

$$Y_t = C_t, \tag{14}$$

where  $\Delta_t$  is a measure of price dispersion, which evolves according to:

$$\Delta_t = (1 - \theta) p_t^{\star - \epsilon} + \theta \left(\frac{\pi_t}{\pi}\right)^{\epsilon} \Delta_{t-1}.$$
(15)

**Rotemberg** Under Rotemberg pricing, firms can reset their price in every period upon payment of a quadratic price adjustment cost, controlled by the parameter  $\psi \ge 0$ . In equilibrium, all intermediate good firms are symmetric and charge the same price. The inflation rate,  $\pi_t$ , is determined by the following firms' optimal pricing condition:

$$\psi C_t^{-\gamma} \left(\frac{\pi_t}{\pi} - 1\right) \frac{\pi_t}{\pi} Y_t = \psi \beta E_t C_{t+1}^{-\gamma} \left(\frac{\pi_{t+1}}{\pi} - 1\right) \frac{\pi_{t+1}}{\pi} Y_{t+1} + (1-\epsilon) C_t^{-\gamma} Y_t + \epsilon \left(1 - \tau\right) C_t^{-\gamma} m c_t Y_t.$$
(16)

The aggregate production function and resource constraint are given by:

$$Y_t = A_t N_t, \tag{17}$$

$$Y_t = C_t + \frac{\psi}{2} \left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t.$$
 (18)

# 3 Ramsey Optimal Monetary Policy

Following Schmitt-Grohé and Uribe (2005), we assume that, in every period, the Ramsey planner honors commitments made in the very distant past, i.e.,  $t = -\infty$ , in choosing optimal policy. This means that the constraints that the planner faces at date  $t \ge 0$  are the same as those at date t < 0. This is referred to as an optimal policy from the timeless perspective (Woodford, 2003).

**Calvo** We now describe a Ramsey equilibrium, starting with the Calvo price-setting approach. Let  $\lambda_{1,t}$ ,  $\lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}, \lambda_{8,t}, \lambda_{9,t}$ , and  $\lambda_{10,t}$  be Lagrangian multipliers on constraints (1), (2), (6), (9), (10), (11), (12), (13), (14), and (15), respectively. Given  $\{C_t, N_t, w_t, R_t, mc_t, \pi_t, Y_t, p_t^\star, p_t^d, p_t^n, \Delta_t\}_{t=-\infty}^{-1}$ ,  $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}, \lambda_{8,t}, \lambda_{9,t}, \lambda_{10,t}\}_{t=-\infty}^{-1}$ , and stochastic processes  $\{A_t, \sigma_t^A\}_{t=0}^{\infty}$ , a Ramsey equilibrium consists of a set of control variables  $\{C_t, N_t, w_t, R_t, mc_t, \pi_t, Y_t, p_t^\star, p_t^d, p_t^n, \Delta_t\}_{t=0}^{\infty}$  and a set of co-state variables  $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}, \lambda_{8,t}, \lambda_{9,t}, \lambda_{10,t}\}_{t=0}^{\infty}$  that solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right),$$
(19)

subject to (1), (2), (6), (9), (10), (11), (12), (13), (14), and (15). The predetermined price dispersion  $\Delta_{-1}$  is set equal to 1. The predetermined Lagrangian multipliers  $\lambda_{2,-1}$ ,  $\lambda_{5,-1}$ , and  $\lambda_{6,-1}$  are set equal to their steady state.

**Rotemberg** We then describe a Ramsey equilibrium for the Rotemberg price-setting approach. Let  $\lambda_{1,t}$ ,  $\lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}$ , and  $\lambda_{6,t}$  be Lagrangian multipliers on constraints (1), (2), (6), (16), (17), and (18), respectively. Given  $\{C_t, N_t, w_t, R_t, mc_t, \pi_t, Y_t\}_{t=-\infty}^{-1}$  and  $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=-\infty}^{-1}$ , and stochastic processes  $\{A_t, \sigma_t^A\}_{t=0}^{\infty}$ , a Ramsey equilibrium consists of a set of control variables  $\{C_t, N_t, w_t, R_t, mc_t, \pi_t, Y_t\}_{t=0}^{\infty}$  and a set of co-state variables  $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^{\infty}$  that solve (19), subject to (1), (2), (6), (16), (17), and (18). The predetermined Lagrangian multipliers  $\lambda_{2,-1}$  and  $\lambda_{4,-1}$  are set equal to their steady state.

The Ramsey optimal equilibrium conditions under the Calvo and Rotemberg price-setting approaches are outlined in Appendix A. Appendix B describes the steady-state values of all variables in the Ramsey equilibrium.

# 4 Calibration and Solution Method

The models are calibrated to a quarterly frequency. Table 1 provides a summary of the key parameters. The discount factor  $\beta$  is calibrated to 0.99 to match a steady-state annual real interest rate of 4%. The risk-aversion parameter  $\gamma$  is 2. The inverse of labor supply elasticity  $\eta$  is set to 1. The labor disutility parameter  $\chi$  is calibrated to match the steady-state hours worked of 1/3. The elasticity of substitution between differentiated intermediate goods  $\epsilon$  is fixed to 11, implying a steady-state markup of 10%. We assume zero steady-state inflation. We parameterize  $\theta = 0.75$  to match an average price duration of four quarters. The Rotemberg price adjustment cost  $\psi$  is 116.50 so that the slopes of the linearized Phillips curve in Rotemberg and Calvo are the same. We follow Leduc and Liu (2016) to parameterize the shock processes. For the productivity shock, we set A = 1,  $\sigma^A = 0.01$ , and  $\rho_A = 0.95$ . For the uncertainty shock, we set  $\sigma^{\sigma^A} = 0.392$  and  $\rho_{\sigma^A} = 0.76$ .

### [Table 1 about here.]

To show the effects of uncertainty shocks, we solve the models using a third-order approximation around the deterministic steady state (Adjemian et al., 2011) with the pruning scheme (Andreasen et al., 2018). When constructing impulse responses, we compute them in percent deviations from the stochastic steady state, which we construct following Born and Pfeifer (2014) and Basu and Bundick (2017). In particular, we set the exogenous shocks in the models to zero and search for a fixed point by iterating the thirdorder solution. This fixed point is the stochastic steady state and conceptually the same concept as what Coeurdacier et al. (2011) label as the risky steady state: the point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is zero at this point.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>We compare how the deterministic and stochastic steady states differ numerically in Appendix B.2.

## 5 Results

To illustrate the presence of the precautionary pricing channel, we start by showing the impulse responses to uncertainty shocks under a simple Taylor rule:

$$\log R_t - \log R = \phi_\pi \log \pi_t,\tag{20}$$

where  $\phi_{\pi} = 1.5$ , in line with the empirical literature.

## [Figure 1 about here.]

Figure 1 shows the impulse responses to a one standard deviation increase in uncertainty when monetary policy follows Equation (20). An increase in uncertainty induces risk-averse households to cut consumption and engage in precautionary savings. When prices are flexible, the fall in consumption is offset by a precautionary labor supply. That is, increased hours worked by households put upward pressure on output, eventually leading to zero consumption and output. When prices are sticky, the precautionary labor supply is dominated by a fall in labor demand that arises from a countercyclical markup. Accordingly, under the Rotemberg pricing approach, a precautionary savings motive leads to a drop in output and inflation. The joint decline in prices and quantities implies that uncertainty shocks act as negative demand shocks. As analyzed by Oh (2020) and Oh and Rogantini Picco (2020), under the Calvo pricing approach, there is an additional propagation channel, activated by the precautionary pricing behavior of firms. When uncertainty increases, firms that are allowed to reset their price raise it to self-insure against the risk of being stuck at the price level below the desired level in the future. Because the increase in prices engendered by the precautionary pricing motive is dominant, inflation increases after a positive uncertainty shock. Hence, under Calvo pricing, uncertainty shocks act as cost-push shocks: inflation rises, and the output gap drops. As monetary policy follows Equation (20), the nominal interest rate falls in Rotemberg, whereas it rises in Calvo. In this section, we show that the precautionary pricing channel disappears under optimal monetary policy. In particular, we show that the output gap and inflation in Calvo and Rotemberg are identical and fully stabilized under the optimal monetary policy.

## 5.1 Analytical Results

We first derive the analytical expressions for the output gap and inflation under optimal monetary policy. These analytical expressions are derived under two simplifying assumptions. First, the uncertainty shock hits the economy in period 1, after which the economy goes back to a steady state. Second, we remove the relative price dispersion term in the production for the Calvo model and the price adjustment cost term in the resource constraint for the Rotemberg model. For brevity, we only show the final expressions in the main text. The full detailed derivations for both price-setting approaches are shown in Appendix C.

In both Calvo and Rotemberg, the output in period 1 obtained from the first-order condition of the Ramsey planner's problem with respect to  $Y_1$  is:

$$Y_1 = \left(\frac{1}{\chi} A_1^{1+\eta}\right)^{\frac{1}{\gamma+\eta}},$$
(21)

which is equal to the efficient output  $Y_1^e$ . Accordingly, we obtain the fully stabilized output gap in period 1:

$$\tilde{Y}_1 = \frac{Y_1}{Y_1^e} = 1.$$
(22)

Using the expression for output in period 1, it can be shown that the real marginal cost in period 1 is equal to 1. From the equations that represent the relationship between real marginal costs and inflation, it follows that inflation in period 1 is equal to its steady state:

$$\pi_1 = \pi. \tag{23}$$

As can be seen, the output gap and inflation are fully stabilized under optimal monetary policy and invariant to uncertainty shocks.

## 5.2 Numerical Results

We now discuss the results obtained from our infinite-horizon models described in Section 2. Figure 2 displays the impulse responses to an increase in uncertainty under optimal monetary policy. In this case, the output gap and inflation are fully stabilized in both Calvo and Rotemberg pricing models.<sup>2</sup> This result is different from the responses under our empirical Taylor rule shown in Figure 1, in which full stabilization is not attained. Joint stabilization under optimal monetary policy in Calvo implies that the familiar cost-push effect, i.e., a rise in inflation associated with a fall in the output gap, which arises in response to cost-push shocks, is no longer present when it comes to uncertainty shocks.

#### [Figure 2 about here.]

The contrasting results under our empirical Taylor rule and optimal monetary policy in Calvo suggest that the precautionary pricing channel is present only when agents expect future inflation to be volatile.

 $<sup>^{2}</sup>$ We have verified that joint stabilization emerges in the case of optimal monetary policy under discretion as well. Also, the joint stabilization is robust to different planner's objective functions, such as the ad-hoc loss function consistent with the Fed's dual mandate.

Under our empirical Taylor rule, as monetary policy is not so responsive to changes in inflation, expected inflation is volatile, and thus the precautionary pricing channel is at play. In contrast, under optimal monetary policy, agents expect stable inflation, and thus the precautionary pricing channel is not operative. The joint stabilization result under optimal monetary policy in Rotemberg is consistent with the view that uncertainty shocks manifest themselves as demand shocks. In the textbook New Keynesian models, it is well known that demand shocks, often modeled as preference shocks, lead to joint stabilization under optimal monetary policy.

## 5.3 Strict Inflation Targeting

The joint stabilization result under optimal monetary policy is tightly linked to what Blanchard and Galí (2007) call the divine coincidence; stabilization of inflation leads to stabilization of the output gap. Therefore, a simple implementable rule that replicates the allocation under optimal monetary policy is the strict inflation targeting rule, which corresponds to a Taylor rule with  $\phi_{\pi} = \infty$ .<sup>3</sup> The divine coincidence is confirmed in Figures 3(a) and 3(b), which show that the strict inflation targeting rule leads to the joint stabilization in both Calvo and Rotemberg.

## [Figure 3 about here.]

In the Calvo model, as monetary policy places higher weight on stabilizing inflation, the volatility of inflation decreases. In this environment, firms expect a less volatile markup and profits stream. Accordingly, a precautionary pricing motive is dampened, and thus inflation increases by less, inducing a smaller fall in the output gap. In Rotemberg, recall that households' precautionary saving motives decrease the output gap and inflation. The higher the value of  $\phi_{\pi}$ , the bigger the drop in the real interest rate is realized with a fall in inflation. A large drop in the real interest rate on savings weakens precautionary saving motives and works to stabilize aggregate demand and thus inflation.

# 6 Optimal Monetary Policy with Real Wage Rigidity

In our baseline model, the distance between the natural level of output and the efficient level output is constant and invariant to uncertainty shocks. In this scenario, a policy that fully stabilizes inflation closes the gap between the actual and natural output and so closes the gap between the actual and efficient output, i.e., the output gap. However, when the distance between the natural and efficient output is time varying, a policy that closes the gap between the actual and natural output, such as strict inflation targeting, is no

<sup>&</sup>lt;sup>3</sup>As inflation is invariant under the strict inflation targeting rule, price level is invariant as well. We have verified that the strict price level targeting rule,  $\log R_t - \log R = \phi_p (\log p_t - \log p)$  with  $\phi_p = \infty$ , leads to the joint stabilization.

longer optimal, because what is relevant for the welfare is the gap between the actual and efficient output. In this case, closing the output gap requires some fluctuations in inflation, so a trade-off emerges between stabilizing the output gap and stabilizing inflation.

## [Figure 4 about here.]

One ingredient that causes the distance between the natural and efficient level of output endogenously to be time varying is the real wage rigidity introduced by Blanchard and Galí (2007). In the presence of real wage rigidities, real wages are rigid even in the flexible price equilibrium. Thus, the efficient output, the output level that prevails under flexible real wages and prices, does not coincide with the natural output, the output level that prevails in the flexible price equilibrium. They show that, in the presence of real wage rigidities, the divine coincidence does not emerge in response to productivity shocks, and so the optimal policy is to allow some fluctuations in the output gap and inflation. A natural question that arises is what the allocation in response to uncertainty shocks looks like under optimal monetary policy in the presence of real wage rigidities. As in Blanchard and Galí (2007), we allow some inertia in real wages  $w_t$ :

$$w_t = w_{t-1}^{\ \omega} \left( \chi N_t^{\ \eta} C_t^{\ \gamma} \right)^{1-\omega}, \tag{24}$$

where  $\omega$  is the degree of real wage rigidity and is set equal to 0.8. We include Equation (24) as a constraint in the planner's optimization problem and solve for the welfare-maximizing allocation as we did in Section 3.

Figure 4 shows that, under Ramsey optimal monetary policy with full commitment, the joint stabilization of the output gap and inflation is not achieved in response to uncertainty shocks. This feature emerges in both Calvo and Rotemberg price-setting approaches. The intuition on the lack of joint stabilization is as follows. As discussed in the previous section, in response to increased uncertainty, households' increased precautionary saving motive leads to a drop in the output gap. When real wages are flexible, the planner can stabilize the output gap by lowering the real interest rate and boosting real wages. However, when real wages are rigid, it is costly to adjust real wages to the extent that they were adjusted under flexible real wages. Accordingly, the only way to stabilize the output gap is to lower the real interest rate further. A further drop in real interest rates can be attained by raising inflation further. In Appendix C.2, we show analytically that the stabilization of the output gap is associated with increased inflation under the one-period assumption.

From the figure, one can observe that a trade-off between the output gap and inflation stabilization is more pronounced in Calvo than in Rotemberg. As discussed, in the presence of real wage rigidities, the full stabilization of the output gap requires an increase in inflation. Under Calvo pricing, this increases the distance between the price level at which firms are stuck and the desired price level, thereby strengthening firms' precautionary pricing motives. As a result, inflation increases further. Therefore, the volatility of the output gap and inflation under optimal monetary policy is larger in Calvo. Even though we have focused on real wage rigidities, there are other realistic ingredients that generate policy trade-offs. For example, Alves (2014) shows that a New Keynesian model with a positive steady-state inflation rate gives rise to a policy trade-off in response to preference and technology shocks. We expect his result to apply to uncertainty shocks as well.

## 7 Conclusion

Existing studies document that, when monetary policy follows an empirical Taylor rule, the precautionary pricing channel is key to the propagation of uncertainty shocks. In contrast, we have shown that the precautionary pricing channel is not operative when monetary policy is optimal, and real imperfections are absent. We have illustrated this result by comparing allocations under optimal monetary policy in the presence and absence of precautionary pricing. We have found that allocations under optimal monetary policy are identical in Calvo and Rotemberg, and joint stabilization of inflation and the output gap is attained in both price-setting approaches. Our result implies that precautionary pricing matters for the propagation of uncertainty shocks only in an environment in which inflation is expected to be volatile.

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Parameter	Description	Value
β	Discount factor	0.99
$\gamma$	Risk-aversion coefficient	2.00
$\eta$	Inverse of labor supply elasticity	1.00
$\chi$	Labor disutility parameter	27.00
$\epsilon$	Elasticity of substitution between goods	11.00
$\pi$	Steady-state inflation	1.00
$\theta$	Calvo price stickiness	0.75
$\psi$	Rotemberg price stickiness	116.50
$ ho_A$	Persistence of a productivity shock	0.95
$\sigma^A$	Steady-state volatility of a productivity shock	0.01
$ ho_{\sigma^A}$	Persistence of an uncertainty shock	0.76
$\sigma^{\sigma^{A}}$	Volatility of an uncertainty shock	0.392

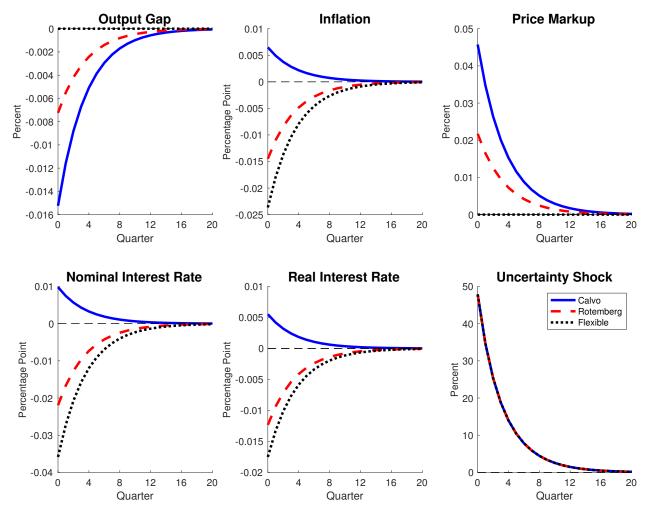


Figure 1: Impulse Responses to Uncertainty Shocks: Empirical Taylor Rule

Note: Impulse responses are in percent deviation from the stochastic steady state.

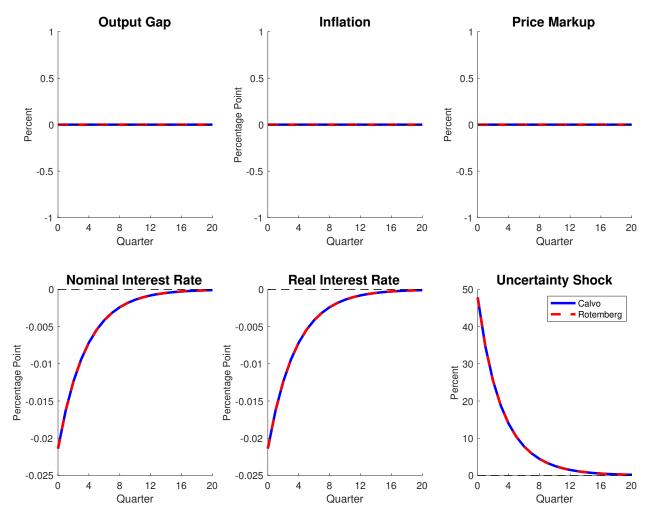


Figure 2: Impulse Responses to Uncertainty Shocks: Ramsey Optimal Monetary Policy

Note: Impulse responses are in percent deviation from the stochastic steady state.

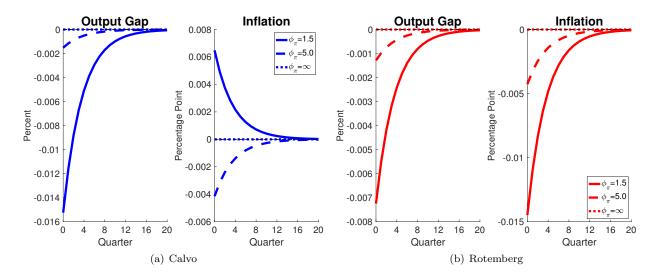


Figure 3: Impulse Responses to Uncertainty Shocks under Different Taylor Rules Note: Impulse responses are in percent deviation from the stochastic steady state.

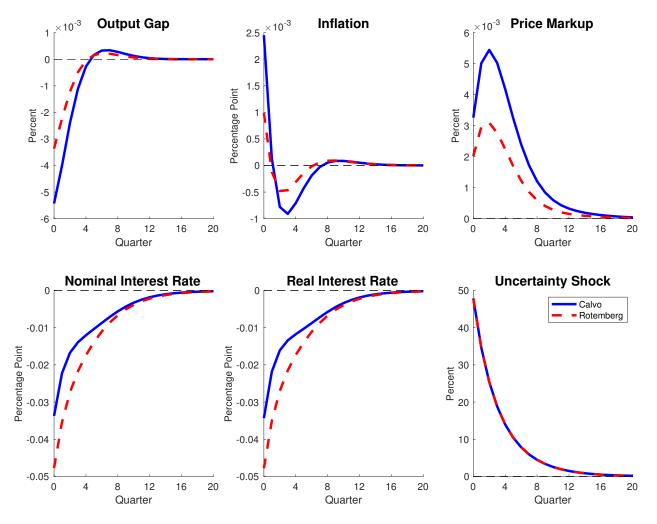


Figure 4: Impulse Responses to Uncertainty Shocks under Real Wage Rigidity: Ramsey Optimal Monetary Policy

Note: Impulse responses are in percent deviation from the stochastic steady state.