Identifying Critical Links in Urban Transportation Networks Based on Spatio-Temporal Dependency Learning

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***Abstract*—The urban transportation network is crucial for societal development, but it is prone to failures like congestion caused by accidents or disasters. In particular, often network-wide failure is the result of a series of cascading failures originating from a small set of individual links. To prevent such failures, it is essential to identify these critical links and take early action. However, most existing approaches in the literature for evaluating the importance of each link rely on manually designed metrics (e.g., the Network Robustness Index). These methods are time-consuming and not suitable for large-scale urban networks. Additionally, these metrics fail to accurately capture the dynamic traffic interactions influenced by vehicle movement. In this paper, we present a novel method for identifying critical links by learning effective traffic interaction representation (the spatio-temporal dependencies) among roads. By representing the network as an un-directed graph and abstracting the road links as the nodes, we introduce a temporal graph attention model to capture spatial and temporal dependence between nodes. This model combines a graph attention network and a long short-term memory neural network and produces an attention matrix, which represents traffic interactions among links. Furthermore, we propose a traffic influence propagation model to evaluate the influence of each link for the entire road network based on the traffic interaction representation. We rank the importance of links based on their influence and then identify the critical links. A real-world case study in the city of Hangzhou, China is conducted to test our method and we use the network efficiency ratio to quantify its performance. The results suggest that our method can effectively identify the critical links at different periods.**

***Index Terms*—Critical links, graph neural networks, LSTM, network propagation dynamics, urban transportation network**

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I. Introduction

The urban transportation network has a pivotal role in maintaining social and economic development, which facilitates the daily movement of people and goods [1]. With the rapidly growing demand for mobility, the transportation network has suffered from colossal pressure and has become highly congested in many cities [2]. As a complex and dynamic system, the transportation network is extremely vulnerable to failures caused by anomalous perturbations, such as traffic accidents, natural disasters, etc [1]. In particular, a small number of individual links may cause a wide-ranging network collapse if they are interrupted, and this phenomenon is also called 'cascading failure' [3], [4]. This indicates that when a few critical links fail, the whole transportation network's performance will significantly degrade. At the same time, it is impossible to constantly monitor all of the road network's segments because of limited resources and data [5], [6]. Therefore, accurately identifying the critical road links in the transportation network is of great importance and practical relevance to traffic management and control, which can maintain and improve transportation efficiency [7].

Many studies have attempted to identify critical links in the transportation network. Most of them rely on manually designed metrics to evaluate the importance of each road link. For example, the Network Robustness Index [8] has been proposed to assess the importance of each link in a transportation network, based on the change in total travel time when removing certain road links from the network. Combinations of metrics, such as link traffic flow and path betweenness can also be used to identify the critical road links [9]. While these studies made some important contributions, there are some limitations that should be noted:

1) Most of these methods rely on metrics, such as total travel time, betweenness centrality (which measures the number of

times a node appears on the shortest path between any two other nodes in a network, and can be used to identify important nodes that act as bridges in the network), *etc*. These methods can accurately identify the links that have a large impact. However, to calculate variations, a disruption on each link needs to be simulated at a time and a new traffic assignment needs to be carried out, which can be a very time-consuming and impractical process for large transportation networks [10], [11]. In addition, these methods usually require full Origin-Destination (OD) data for the traffic assignment, which may be difficult to obtain in reality.

2) Some of the methods use manually defined local metrics, which are derived from the field of complex network analysis, to identify the critical links. However, these methods are not generalizable or transferable [2] because the road network layout may vary considerably from city to city.

3) The traffic impact among neighbouring links is a complex process and existing methods cannot accurately quantify it, regardless of the multi-index evaluation system (different traffic assignment models may also lead to biased critical link identification results).

In recent years, many studies have revealed that the phenomenon of cascading failures exists in transportation network [3], [12], [13]. Failures in transportation networks can spread as a result of the traffic impact between road links, as caused by the movement of vehicles [14]. This impact includes spatial and temporal dependencies [15], and we use the term "traffic interaction" to describe it as in previous work [14]. Therefore, an effective representation of traffic interactions is key to identifying the critical links in the transportation network. In transportation, many studies utilize Graph Neural Networks (GNNs) to capture interactions between traffic units and predict traffic states. For example, Zhao *et al.* [16] propose a Temporal Graph Convolution Network (T-GCN) combining the Graph Convolution Network (GCN) and the Gated Recurrent Unit (GRU) model for predicting traffic speed, and showing that T-GCN can capture the spatial-temporal dependencies in transportation network very well.

In this paper, we propose a novel method for identifying critical links based on learning effective traffic interaction by employing GNNs to model the spatial-temporal dependencies between links. First of all, we transform the transportation network into an undirected weighted graph. In this process, the links are abstracted as the nodes in the graph. Then, we apply a Temporal Graph Attention (T-GAT) model to learn the traffic interaction representation between nodes, which is a combination of the Graph Attention (GAT) network [17] and the Long Short-Term Memory neural (LSTM) network [18]. We use the attention matrix to represent traffic interactions between neighbouring nodes, which is trained in the temporal graph attention model. Furthermore, we propose a traffic influence propagation model, which can evaluate the influence of each node on the entire network based on the traffic interaction. We rank the importance of nodes in order of their influence. As nodes in the graph correspond to links in the real-world, we can ultimately identify critical links in the transportation network. Our main contribution can be summarized as follows:

- We apply T-GAT to learn the traffic interaction representation, which integrates GAT and LSTM. The GAT is used to capture the spatial dependencies and the LSTM is used to capture the temporal dependencies between links.
- We design a traffic influence propagation model to calculate the influence of links on the entire network based on traffic interaction representation, which utilizes complex network propagation dynamics.
- We utilize the parameters of the T-GAT model to represent the interactions between traffic nodes and verify the validity and accuracy of our proposed method through an experiment, thereby enhancing the interpretability of the GNN model and expanding its applications in the field of transportation. Additionally, our method is superior in terms of computation complexity and is suitable for the large-scale road network.

Our paper is organized as follows. Section II reviews related work. Section III presents our method. The experimental setups are introduced in Section IV and Section V shows the experimental results. We conclude this paper in Section VI.

II. Related Work

In this section, we review the related works on critical link identification and graph deep learning in transportation.

A. Identification of critical links

Identifying critical links in transportation networks is a classical problem in traffic and a large number of studies have attempted to tackle it.

In the earlier research, the importance of links is assumed to be positively related to the degree of congestion. The Volume-to-Capacity (V/C) ratio [19] is proposed to identify the critical links, which can indicate road congestion. Scott *et al.* [8] propose the Network Robustness Index (NRI) to rank the importance of road links, which is calculated by the change in total travel time for all the travellers after removing each road link iteratively, with the link with the maximum NRI value being considered the most critical in each iteration. Sullivan *et al.* [20] build on the NRI and propose an improved indicator, the Network Trip Robustness (NTR), which is calculated by summing the NRI values across all links and dividing that sum by the total trip demand; this makes the indicator suitable for road networks of different sizes. Oliveira *et al.* [21] determine critical links based on each of the two attributes of vulnerability and congestion, and point out that using a congestion indicator only may lead to a biased result. Zhou *et al.* [7] argue that existing methods mostly mix up the concept of the criticality of links with vulnerability, and propose a critical links identification method based on two aspects: vulnerability and potential. Li *el al.* [9] propose a traffic flow betweenness index to identify the critical links, considering path betweenness, traffic flow and OD demand. Gokalp *et al.* [22] develop a bidirectional search heuristic with customized pruning and branching strategies to determine the priority for road restoration during post-disaster reconstruction. Hamedmoghadam *et al.* [23] propose a percolation-based network analysis framework underpinned by flow heterogeneity to identify bottleneck links. This study also demonstrates that alleviating congestion on critical links can effectively improve the overall performance of the transportation network.

Most methods in this research area can be divided into two categories: 1) link importance ranking based on network-disruption analysis, which mostly relies on system-level metrics; and 2) link importance ranking based on local metrics, such as traffic flow, degree, and V/C ratio.

The work in the first category generally includes three steps: i) reduce link capacity in the transportation system; ii) execute traffic assignment model to reassign traffic flow on each link; and iii) measure the decline rate of the system performance. Although these methods can accurately identify critical links for relatively small transportation networks, they are difficult to implement on a large scale due to time and computational constraints. The work in the second category identifies the critical links that rely on some manually defined local metrics, such as degree centrality, traffic flow and so on, which are simpler and intuitive. However, these methods are non-universal because of heterogeneous topological features and traffic distributions in different regions.

In recent years, some scholars have introduced the idea of machine learning into the field. For example, Liu $et~al.$ [14] propose an approach called Road2Vec to quantify the implicit traffic interaction among roads based on large-scale taxi trajectory data employing the Word2Vec model. Saffari $et~al.$ [24] apply Principal Component Analysis (PCA) to identify the main traffic features and regard the links which are associated with these features as critical. Dai $el~al.$ [25] utilize graph representation learning models to learn the representation of links and use classifier models to identify the critical ones. However, there is still limited research on deep learning-based critical road link identification. Moreover, these existing studies often do not effectively explore the spatio-temporal dependencies in the traffic data. For example, [24] mainly focuses on the individual features of road links and [25] mainly focuses on the spatial structure of the road network.

In this paper, we attempt to use a deep learning approach to learn the traffic interaction representation to identify the critical links.

B. Graph Neural Networks in transportation

GNNs attempt to generalize neural networks to apply in arbitrarily structured graphs [26], and have been successfully applied in many fields.

In the transportation field, a great deal of research on the application of GNNs has focused on the problem of traffic state prediction. Cui $et~al.$ [26] propose a deep learning framework named Traffic Graph Convolutional Long Short-Term Memory neural network (TGC-LSTM) to forecast traffic speed, and finds that, on the basis of visualization results, the road segments with higher graph convolution weights appear to play a more important role in the transportation network. Zhang $et~al.$ [27] propose a GAT convolution network to capture the intrinsically spatio-temporal dependencies between roadways, combining the GAT and temporal convolution operation. The graph attention heatmap reveals that the attention weights are related to road structural features and some sophisticated roads have higher spatial attention weights. Zhang $et~al.$ [28] propose a graph convolution sequence-to-sequence model to predict multi-step speed on traffic networks by combining the Seq2Seq model and graph convolution network; their results indicate that graph convolution weight values are positively correlated with the congestion on the links. Huo $et~al.$ [29] design a hierarchical traffic flow forecasting network based on the newly designed long-term temporal transformer network and the spatio-temporal graph convolution networks, which can capture the short-term and long-term temporal relations on traffic data. Xu $et~al.$ [30] propose a deep learning framework called Dynamic Traffic Correlation-based Spatio-Temporal Graph Convolutional network to predict the urban traffic, which utilizes GCN and LSTM to capture the spatial and temporal dependencies based on the dynamic adjacency matrix. Bao $et~al.$ [31] propose a traffic prediction model called Spatial-Temporal Complex Graph Convolution Network (ST-CGCN). This model constructs and integrates the distance matrix, the data correlation matrix and the comfort measurement matrix, improving the joint modelling ability of spatio-temporal features and external factors. Guo $et~al.$ [32] propose a graph convolution neural network with an attention mechanism, which can achieve excellent performance in traffic prediction problems by learning the interaction of different road links in the spatial and temporal dimensions. Wu $el~al.$ [33] propose a encoding-forecasting structure combining with GAT and LSTM to predict traffic flow. Fang $el~al.$ [34] propose a L-GAT (LSTM-Graph Attention Network) framework to forecast traffic speed in urban road networks. GAT is employed to capture spatial dependency relationships between nodes and exhibit excellent performance in these works.

So far, a number of studies have used GNNs and Recurrent Neural Networks (RNNs) to solve traffic prediction problems. Taken together, these studies support the notion that GNNs can effectively capture the potential traffic interaction between units (road links or intersections) in the transportation network, and that there is a relationship between model weights and traffic interactions [35], [36], [37], [38].

Based on the previous studies, we adopt GNNs as the basic framework for our proposed temporal graph attention model to learn the traffic interaction representation.

III. METHODOLOGY

A. Notations

1) Road Network G: We use an undirected graph $G = (V, E, W)$ to represent the topological structure of the road network, where $V = \{v_1, v_2, \cdots, v_n\}$ is a set of nodes, E is a set of edges, W is a set of edge weights and n is the number of the nodes. In this paper, we treat each road link as a node in the graph.

2) Feature Matrix X_t: We use the feature matrix $X_t = \{x_t^1, x_t^2, \cdots, x_t^n\}$ to represent the node feature in graph G, where x_t^i denotes the feature on v_i at time t. In this paper, the node feature is the vehicle's average speed on each road link.

3) The k-hop neighborhood Nbh_i^k: Nbh_i^k represents the k-hop neighborhood for node v_i (i.e. the set of nodes within distance k of node v_i) in graph G, as defined in equation 1:

$$Nbh_i^k = \{v_j \in V | d(v_i, v_j) \leqslant k + 1\} \tag{1}$$

where $d(v_i, v_j)$ denotes the number of nodes on the shortest path from node v_i to v_j.

4) Attention Coefficient Matrix $M^{n\times n}$: This matrix $M^{n\times n}$ is trained by our temporal graph attention model. The value $m_{ij} \in M$ represents the attention coefficient that indicates the traffic interaction between v_i and v_j in graph G.

B. Construction of Road Network

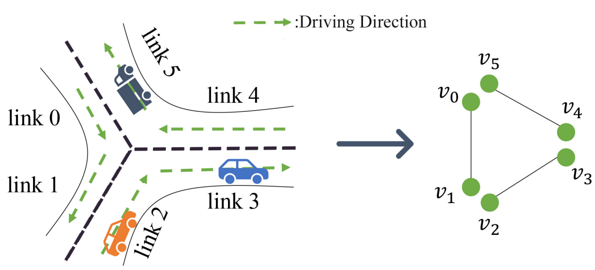

Fig. 1. Formation of the road network G.

In this section, we construct an undirected weighted graph $G(V, E, W)$ based on the geographic road network. The process is shown in Fig 1. We abstract the road links as the nodes in G, so we get the node-set V. For every two nodes v_i and $v_j \in V$, if v_i and v_j are connected , there will be an edge (e_{ij}) between v_i and v_j. To model the distinct interactions brought by bidirectional traffic flow on the road, we utilize two distinct edges to represent the traffic interactions. Then, we get the edge set E. Due to the fact that the impact of traffic congestion will influence both upstream and downstream of the traffic network [39], G is set to be an undirected graph, as in previous studies [26], [16].

For the edge e_{ij}, the weight w_{ij} is defined as:

$$w_{ij} = (length_i + length_j)/2, \tag{2}$$

where $length_i$ and $length_j$ denote the length of links i and j, respectively.

C. Overall Architecture

Fig. 2 illustrates the overall structure of our proposed method. Once the road network G is obtained, we utilize it along with the traffic feature data as inputs to train the temporal graph attention model, aiming to extract the spatio-temporal dependencies. The learned attention coefficient matrix serves as the representation of traffic interaction and is fed to the traffic influence propagation model. By calculating the influence of each road link, we ultimately obtain the results for identifying critical links. In the subsequent sections, we provide a detailed introduction of the temporal graph attention and traffic influence propagation models.

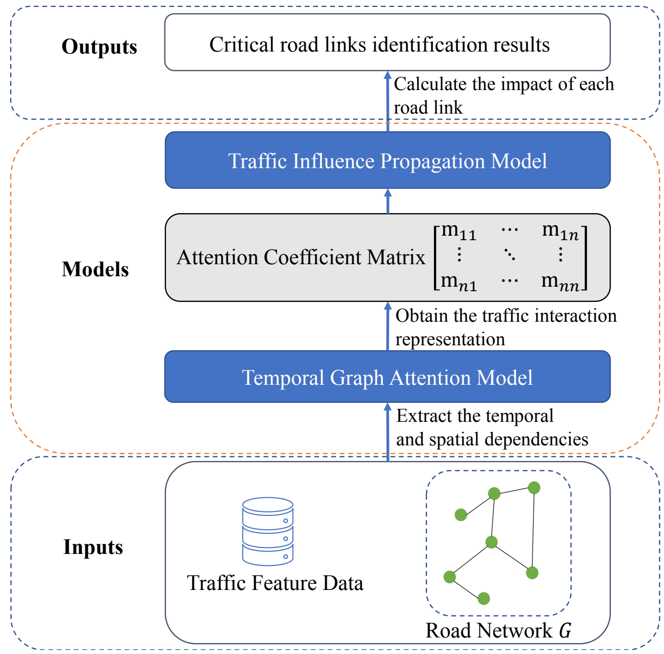

Fig. 2. The overall architecture of our proposed method.

D. T-GAT Model

In this section, we apply a temporal graph attention model to effectively learn the spatial-temporary dependence among links in the transportation network. The framework of the T-GAT model is shown in Fig 3. The inputs of this model are the road network and the node feature matrix. Then, we use GAT and LSTM to learn the spatial and temporal dependencies of the nodes. The outputs are the subsequent one-step feature X_{t+1} and the attention coefficient matrix $M^{n\times n}$.

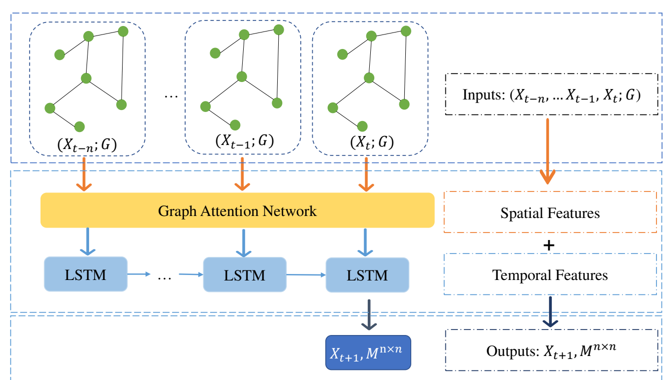

Fig. 3. The overall process of temporal graph attention model

In the next sections, we introduce the graph attention network and the LSTM in detail.

1) graph attention network: We use GAT [17] to learn the spatial dependencies among road links. GAT is a classic model in graph neural networks, known for its simplicity and effectiveness. Compared to Convolution Neural Networks (CNNs) and frequency-domain-based GCNs [40], GAT excels at handling graphs with varying structures by dynamically adjusting attention weights for each node during information aggregation rather than relying on a fixed convolutional computation. Meanwhile, GAT's attention mechanism can naturally scale to large graphs without the need for complex approximations or sampling techniques required by some GCN variants for scalability. Furthermore, GAT can assign distinct attention weights to nodes within the same neighborhood, offering a more intuitive understanding of node interactions compared to other models. Hence, we adopt GAT as our spatial dependency extraction model. GAT consists of graph attention layers, which aggregate the features of neighbouring nodes to the central node by attention mechanisms. Fig 4 shows how GAT uses the attention mechanism to calculate the attention coefficient between nodes. The inputs are node features (x_t^i, x_t^j) and the output is the attention coefficient m_{ij}.

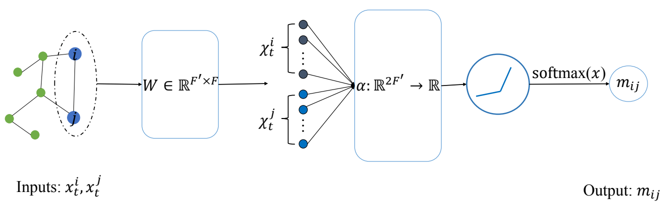

Fig. 4. The process of calculating the attention coefficient between nodes

Firstly, as shown in equation 3, a shared transformation with a learnable weighting matrix $W \in \mathbb{R}^{F' \times F}$ is applied to each node to obtain sufficient representation power.

$$\chi_t^i = W x_t^i, \tag{3}$$

where $x_t^i \in \mathbb{R}^F$ denotes the feature of node $v_i (v_i \in V)$ when time t; F and F' denote the dimensions of the input output features, respectively; and $\chi_t^i \in \mathbb{R}^{F'}$ denotes the high-dimensional feature after linear transformation.

Then, GAT calculates the attention coefficient by a shared attention mechanism as shown in equation 4:

$$e_{ij} = \mathrm{LeakyRelu}(\alpha[\chi_t^i \| \chi_t^j]), \tag{4}$$

$$\mathrm{LeakyRelu}(x) = \begin{cases} x, & \text{if } x \geq 0 \\ x/0.2, & \text{if } x < 0 \end{cases}, \tag{5}$$

where e_{ij} denotes the attention coefficient between node v_i and v_j $(v_i, v_j \in V)$; χ_t^i and χ_t^j denote the features defined by equation 3; $(\cdot \| \cdot)$ denotes the splicing operator; and α denotes a trainable single-layer feed forward neural network.

In order to incorporate the structural information of the graph and reduce the computational complexity, we only calculate the attention coefficients of nodes with other nodes in the 1^{st} order neighbourhood. We normalize the coefficients as per equation 6:

$$m_{ij} = \mathrm{softmax}(e_{ij}) = \frac{\exp(e_{ij})}{\sum_{q \in Nbh_i^1} \exp(e_{iq})}, \tag{6}$$

where $m_{ij} \in M^{n \times n}$ denote the normalized attention coefficients between node v_j and node v_i; e_{ij} denotes the attention coefficient defined by equation 4; and Nbh_k^1 denotes the set of nodes in the 1^{st} order neighborhood of node v_i.

After obtaining the attention coefficient matrix $M^{n \times n}$, GAT calculates the linear combination of node features as the output feature, as shown in equation 7:

$$x_t^{i'} = \varphi\Big(\sum_{v_j \in Nbh_i^1} m_{ij} \chi_t^j\Big), \tag{7}$$

where $x_t^{i'} \in \mathbb{R}^{F'}$ denotes the output feature of node v_i and φ is the activation function(e.g. $\mathrm{Relu}(x)$).

2) LSTM: LSTM is a type of recurrent neural network. Compared to Standard RNNs, LSTM can mitigate the problem of gradient disappearance or explosion based on the design of the gating units, while wildly successful in practice [41]. Compared to 'Transformer' model, LSTM requires less data and fewer computational resources and has been found to be superior in capturing the temporal dependencies in traffic data [26] [42] [43]. Therefore, we utilize LSTM to model the temporal dependencies between nodes.

We use $X_t' = \{x_t^{1'}, x_t^{2'}, \cdots, x_t^{n'}\}$ to represent the node features transformed by the graph attention network, and the LSTM model is applied to learn the temporal dependencies between road links. Firstly, LSTM defines the forget gate f_t for time step t as follows:

$$f_t = \sigma(W_f \cdot [X_t' \| h_{t-1}] + b_f), \tag{8}$$

where $(\cdot \| \cdot)$ is the splicing operator; $\cdot$ is the matirx multiplication operator; W_f is the trainable weight matrix; b_f is the bias vector; and σ is the gate activation function. Then, LSTM uses the input gate i_t and the state update unit C_t to update the information h_{t+1}:

$$i_t = \sigma(W_i \cdot [X_t' \| h_{t-1}] + b_i), \tag{9}$$

$$C_t = \tanh(W_C \cdot [X_t' \| h_{t-1}] + b_C), \tag{10}$$

$$h_t = f_t \times h_{t-1} + i_t \times C_t, \tag{11}$$

Lastly, LSTM generates the final output features through the output gate o_t and the updated information as follows:

$$o_t = \sigma(W_o \cdot [X_t' \| h_{t-1}] + b_o), \tag{12}$$

$$\hat{X}_t = o_t \times \tanh(h_t), \tag{13}$$

where $\hat{X}_t$ is the output feature of the LSTM model.

3) Loss Function: In the training process, we learn the spatio-temporal dependencies among road links by minimizing the error between the real-world traffic features and the predicted value. Thus, the loss function of our temporal graph attention model is defined as equation 14:

$$loss = \mathrm{MSE}(X_{t+1}, \hat{X}_t), \tag{14}$$

where MSE denotes Mean Squared Error; and X_{t+1} and $\hat{X}_t$ denote the true and predicted values, respectively.

Finally, we use the attention matrix $M^{n \times n}$ obtained from the temporal graph attention model as the **traffic interactions representation** between adjacent road links. It is worth noting that the impact of upstream on downstream and the impact of downstream on upstream are distinct. Our model also can learn and capture these asymmetric influences. For example, we used m_{12} to represent the interaction from v_2 to v_1 and used m_{21} to represent the interaction from v_1 to v_2. m_{12} is not equal to m_{21}.

E. Traffic Influence Propagation Model

In this section, we propose a traffic influence propagation model that extends the traffic interaction between adjacent road links to the whole transportation network. This model can calculate the influence of nodes on higher-order neighbours. The propagation of traffic impacts is very complex and will have simultaneous impacts upstream and downstream [44]. Taking traffic congestion as an example, generally, in the upstream section of the congestion point the capacity decreases and the traffic density keeps increasing; in the downstream section, the traffic density decreases due to the absence of vehicles entering. In past research, Li $et\ al.$ [3] pointed out that the phenomenon of distance-dependent cascading failure exists in road networks and the correlation of nodes obeys power law distribution. Similarly, in the complex network propagation dynamics there is an exponentially negative power-law relationship between the propagation of nodes and the distance between nodes (propagation is a metric that describes the influence of a node on neighbouring nodes in a network). Based on that, we define the propagation capacity of the transportation nodes as equation 15:

$$C(ij) \sim e^{-\beta r_{ij}}, \tag{15}$$

where $C(ij)$ denotes the propagation between nodes v_i and v_j; β is a hyper-parameter that relates to the distribution of nodes in the network; and r_{ij} denotes the distance between v_i and v_j, which is the weight between the nodes and can be calculated by equation 2.

Since the influence of the links propagates based on the relative link locations and direction of travel, we design the propagation formula to calculate the impact of node v_i on node v_j:

$$p_{ij} = C(ij) \sum_{path_k \in Path_{ij}} \prod_{s_1 s_2 \in path_k} m_{s_2 s_1}, \tag{16}$$

where p_{ij} denotes the impact of node v_i on node v_j; $Path_{ij}$ denotes the set of all simple paths (the nodes on the path are all different) from node v_i to v_j, with $path_k$ being one of the paths in this set; $s_1 s_2$ denotes all pairs of neighbouring nodes on $path_k$; and $m_{s_2 s_1}$ denotes the traffic interactions representation outputted by the temporal graph.

Therefore, we can calculate the impact of node v_i on the whole road network:

$$I_i = \sum_{v_j \in V} p_{ij} \tag{17}$$

Considering that the impact between road links will decay with increasing distance, in order to reduce the computational complexity, we only compute the influence within the k-th order neighbourhood of node i:

$$I_i = \sum_{v_j \in Nbh_i^k} P_{ij} \tag{18}$$

where Nbh_i^k denotes the k-hop neighborhood for v_i; p_{ij} denotes the impact of node v_i on node v_j.

The greater the influence I of the road link, the more critical it is in the road network.

IV. Experiments Setup

In this section, we evaluate the performance of our proposed method. To achieve it, we apply it to a real urban road network and compare it with some alternative methods of identifying critical links.

A. Experimental Datasets

In this paper, we use two real-world datasets of the city of Hangzhou, China to conduct our experiments, which are provided by a Chinese online mapping and navigation service called 'AMAP', which is similar to Google Maps in the U.S.. These datasets provide the road information and five-minute average vehicle speed data of road sections in the Xiaoshan District, Hangzhou, collected on Monday, July 11, 2017. Table I and Table II illustrate the specifics of the road network dataset and the average vehicle speed dataset, respectively. The geographical coverage of the dataset is shown in Fig 5 (the purple lines of this figure). For the first dataset, we can construct the road network G following the method introduced in Section III-B (excluding the isolated road links). The resulting graph G comprises 397 nodes and 1024 edges. For the second dataset, the feature matrix is obtained, whereby missing or outlier values are completed using the average value from the previous and subsequent time intervals X_t.

TABLE I
ROAD NETWORK DATASET

Road ID	Road Name	St Road	Ed Road	Length(m)
10001	Jiangsi Road	Chenghe Street	Renmin Road	184.41
10007	Jiangsi Road	Wenhua Road	Xiaoshao Road	383.24
10022	Mingxing Road	Jianshe 2nd Road	Qidi Road	296.22

B. Baseline Methods

We consider the following methods that are often used for critical link identification for comparison.

- **Betweenness Centrality:** Betweenness centrality is a common metric to determine the importance of a node in a graph [45]. The betweenness centrality of node i is defined in equation 19:

$$BC_i = \sum_{j,k \in N} \frac{n_{jk}(i)}{n_{jk}}, \tag{19}$$

TABLE II
AVERAGE VEHICLE SPEED DATASET

Time	Road ID	Road Name	Speed(km/h)
0:00	10005	Wenhua Road	38.8
0:05	10005	-	36.2
0:10	10005	-	39.6
0:15	10005	-	39.6
0:20	10005	-	33.8

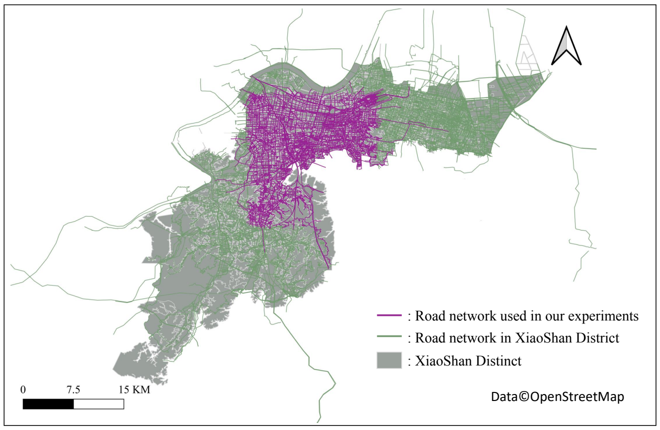

Fig. 5. The geographical area of the dataset.

where n_{jk} denotes the number of shortest paths between nodes j and k; and $n_{jk}(i)$ denotes the number of shortest paths between nodes j and k that pass through node i.

- **Congestion Rate:** We consider the time when the average speed of the road is below 35km/h as the congestion time and calculate the all-day congestion rate of the road. We rank the road by congestion rate.
- **VoteRank:** VoteRank [46] is a vote-based method for ranking the importance of nodes. In VoteRank, the main idea is to choose a set of spreaders one by one according to the voting scores of nodes obtained from their neighbours. The node getting the most votes in each turn is regarded as the most influential node in that turn and will be elected as one of top-r influential spreaders.
- **PCA method:** This method is proposed in [24], and uses Principal Component Analysis (PCA) to identify the main traffic features from the traffic dataset. The links that are associated with these features are regarded as critical.
- **Road2vec method:** Road2Vec [14] utilizes the Word2Vec model to obtain the vector representation of road links. The traffic interaction is calculated by the cosine similarity of the road vectors. However, due to the limitations of the experimental data, we substitute the Word2Vec with Node2Vec [47], which is an extension of the Word2Vec model specifically designed for graph-based applications.
- **HTC method:** The Heterogeneous Traffic Correlations (HTC) method proposed in [48] measures the traffic correlation between two road segments with Pearson's correlation coefficient and uses p-value tests to assess the validity of these correlations.

C. Evaluation Metric

As in previous studies [49] [2], we use **network efficiency** to measure the transmission efficiency of the road network. The definition of network efficiency is:

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in V} \frac{1}{cost_{ij}}, \tag{20}$$

where $cost_{ij}$ is the actual average travel cost from node i to node j; and n is the total number of nodes in the network. Each node represents a real-world roadway.

We use different methods to obtain different results for ranking the importance of road links. The decline rates of $E(G)$ can directly demonstrate the accuracy of the results when we delete the nodes in the graph in turn. We define the decline rates of $E(G)$ after removing node i as equation 21:

$$Eff_i = \frac{E(G)'}{E(G)}, \tag{21}$$

where $E(G)'$ is the network efficiency after removing node i and $E(G)$ is the initial network efficiency.

We iteratively remove nodes from the graph and compute Eff metric. A higher Eff value indicates that the node has a greater impact on the road network, enabling us to obtain the ranking of node importance as the ground truth. We compare the differences between the ranking results obtained from various methods and the ground truth using the following three metrics.

- **Spearman's Correlation Coefficient:** Spearman's Correlation Coefficient (SCC) measures the extent to which two ranking sequences are arranged in a similar order, with values ranging from -1 to 1. A value closer to 1 indicates a higher degree of similarity between the rankings. The definition of SCC is:
$$SCC = 1 - \frac{6 \sum_{i=1}^{n} {d_i}^2}{n(n^2-1)}, \tag{22}$$
where d_i denotes the difference in the ordering of node v_i and n denotes the number of nodes.
- **Kendall's Rank Correlation Coefficient:** Kendall's Rank Correlation Coefficient (KRCC) is used to measure the correlation between two ranking sequences. It takes into account not only the monotonicity but also the concordance of different arrangements between the two sequences. KRCC ranges from -1 to 1 with a value closer to 1 indicating a higher degree of similarity between the rankings. The definition of KRCC is:
$$KRCC = \frac{(\zeta - \varpi)}{(\zeta + \varpi)}, \tag{23}$$
where ζ and ϖ denote the number of matched pairs (the node pairs have the same order in the two sorted sequences) and of unmatched pairs in the two ranking sequences, respectively.
- **Manhattan Distance:** Manhattan Distance (MD) is the sum of the absolute differences between the corresponding values at each position in two ranking sequences. It is used to measure the distance between two sequences, with a smaller MD value indicating a higher level of similarity. The definition of MD is:
$$MD = \sum_{i=1}^{n} |d_i| = \sum_{i=1}^{n} |x_i - y_i|, \tag{24}$$
where d_i denotes the difference in the ordering of node v_i; and x_i and y_i denote the ranking of node v_i in the two sequences, respectively.

D. Model Settings

1) Temporal Graph Attention Model: For the GAT model, we use two graph attention layers to capture the spatial dependencies. We manually adjust and set the attention head to 8, the channel (output feature) to 8, and the activation function to $\mathrm{elu(x)}$ in the first layer. We set the attention head to 1 and the activation function to $\mathrm{Relu(x)}$ in the second layer. For the LSTM model, we set the time step to 3. The learning rate is set as 0.005 and we choose adam [50] as the algorithm optimizer. We set the batch size to 5 and the training epoch to 500 and the number of early stops to 10.

2) Traffic Influence Propagation Model: We use the exact formulation as shown in equation 25 to replace the equation 15

$$C(ij) = e^{-\beta r_{ij}} \tag{25}$$

and the hyperparameters of the traffic influence propagation model include the node distribution β, and the neighbourhood order k, defined in equation 18. In the experiment, we set $\beta = 1$ and set $k = \{1, 2, 3\}$ to analyze the effects of different parameters.

V. Experimental Results

In this section, we report the results of the identification of the critical links and perform a comparative analysis against baseline methods. Keras and Spektral [51] are used to code our model. Keras is an API designed for deep learning based on Python and Spektral is a Python library for graph deep learning, based on the Keras API and TensorFlow 2. We also perform a complexity analysis of our method.

A. Results of Identified Critical Links

We select different time periods of the traffic feature data as the prediction target.

Tables III, IV and V show the results of the critical link identification (Top 15) at 18:00 and Tables VI, VII and VIII show the results at 24:00. It should be noted that in the tables some road links have the same name (e.g., Shixin Road), but they have different road IDs, because they represent different sections of the same road. For these two periods, the MSE at the completion of model training was 5.331 and 6.739, respectively. The latest traffic prediction model may outperform ours in terms of prediction accuracy. However, Our model is still able to deliver good enough predictive accuracy, and its main advantage lies in its simplicity, making it much more straightforward to identify the model parameters corresponding to the interactions between nodes.

By comparing critical rankings with different parameters k, we can find that these can be quite different, which indicates that the neighbourhood parameter can affect the results significantly. Theoretically, the larger this parameter, the wider the range of the road network that the road link will affect. We also find there are some road links that are equally important under different parameters, such as road 10258 in table III and table V. Fig 6 and Fig 7 show the locations of the top 4 road segments on the map in table V. With the help of online maps, we can analyze the land-use situation around critical road links. Critical links share several common characteristics. Taking road 10267 as an example, it belongs to a central road in the Xiaoshan District called "Shixin Road," and it is connected to another important road, "Xiaoran South Road." This road segment is equipped with two traffic signals, and its surroundings are characterized by significant infrastructure, such as large hospitals, hotels, residential areas, and shopping centres. Due to its complex traffic conditions and high traffic volume, any capacity decrease caused by congestion, accidents, or other factors could have a severe impact on the overall traffic network. Such critical links are characterized by their proximity to a multitude of facilities, as well as high pedestrian and traffic flows. Moreover, these road links often adjoin a considerable number of other roads, rendering them more susceptible to exerting a significant impact on the overall road network.

Fig. 6. Location of Critical Links in the Road Network

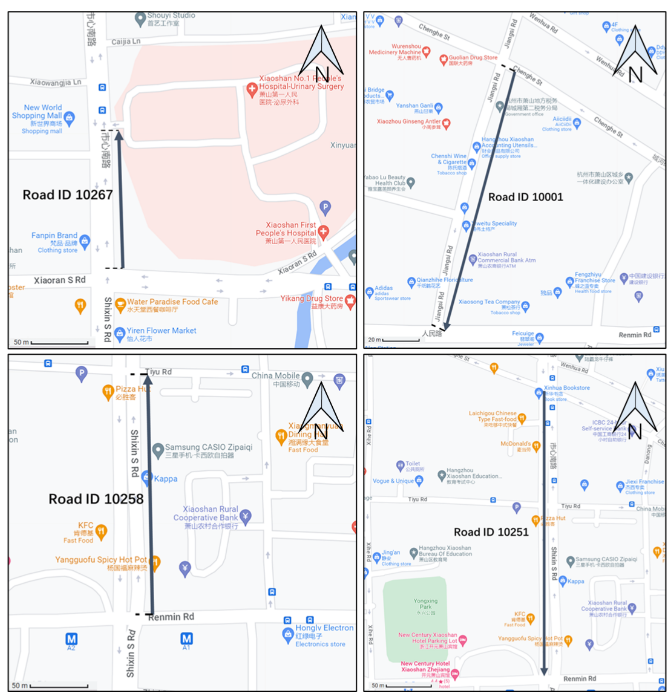

Fig. 7. Visualization of Critical Links

With a comparison of critical ranking between different time periods, it can be seen that the same road link has different impacts on the road network at different times. For example, road 10251 is the most critical link when $k = 1$ at 18:00, but it is the 8^{th} most important link at 24:00. The real-world data shows that this road link is congested at 18:00, but uncongested at 24:00. This phenomenon suggests that our method can effectively capture the spatio-temporal variability of traffic features.

B. Performance Comparison

Table IX shows the performance comparison of critical links results between our method and other baseline methods at different time periods. We have marked the methods that perform best under different scenarios with an asterisk (*). 'Top15 nodes' and 'All nodes' refer to the evaluation of the identification performance of the top 15 critical links and that of all links, respectively. As shown in Table IX, when $k = 1$, our method performs similarly to most benchmark methods. However, as we increase the value of k, the performance of our method improves accordingly. When $k = 3$, our method significantly outperforms the baseline methods in almost all metrics (due to the presence of distance decay, increasing the value of k beyond $k = 3$ has little impact on the critical links identification results). The experimental results validate the effectiveness of our proposed method. 'Our method_ablation' represents the critical links identification model with the LSTM module removed while keeping the rest of the parameters unchanged. It is evident that the identification performance is significantly inferior to the complete model. This observation demonstrates that the inclusion of the LSTM module contributes to the model's ability to learn the dependencies within the traffic data effectively.

Compared to topology-based baseline methods (Betweenness Centrality, VoteRank and Road2Vec), our approach can capture dynamic features between traffic nodes, enabling the identification of critical road links during different time periods. Compared to traffic feature-based methods (Congestion Rate, PCA and HTC), our approach addresses the limitation of not considering the traffic network's underlying structure. The position of the nodes within the network also plays a significant role in their influence.

Fig. 8 shows the cumulative distribution of the network efficiency decline ratio at $k = 3$. As can be seen, in both time periods the curve remains above the diagonal line, indicating that links with higher rankings have a greater impact on the efficiency of the road network.

To illustrate the results more intuitively, we present the links in groups of 25 according to their rankings. Fig. 9 and Fig. 10 show the variation of the network efficiency decline ratio when removing links in groups at 18:00 and 24:00, respectively. As shown in the figures, the network efficiency ratio curves follow a similar decreasing trend under different values of k, which indicates that our method can rank the links with high impact on the road network at the top. If we only need to know which the critical links are and not their importance, we can set $k = 1$, which can significantly reduce the computational complexity.

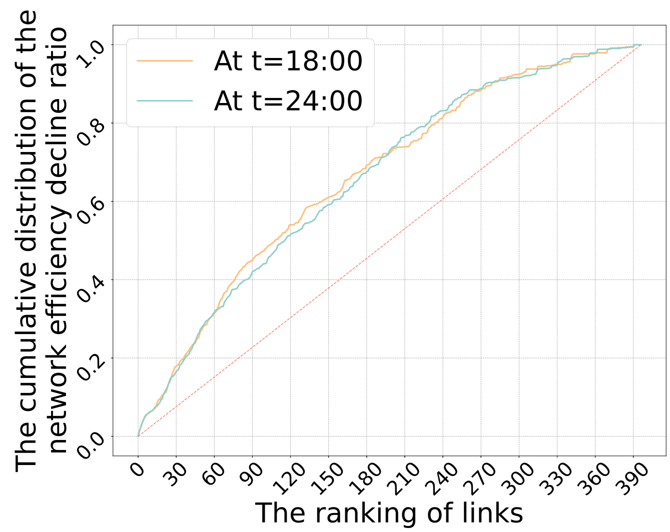

Fig. 8. The cumulative distribution of the network efficiency decline ratio at $k = 3$

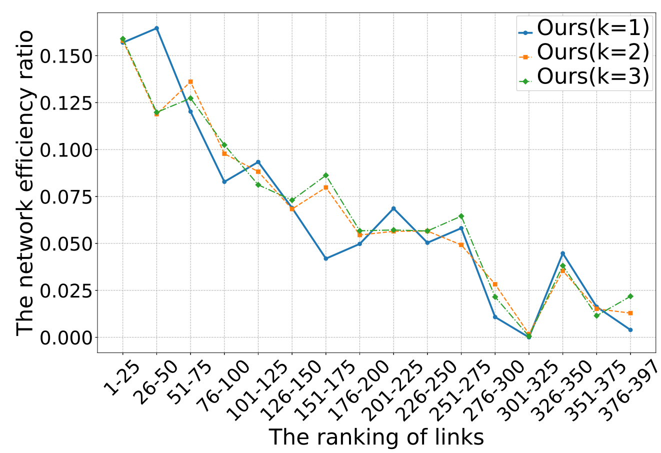

Fig. 9. Variation of the network efficiency ratio at 18:00.

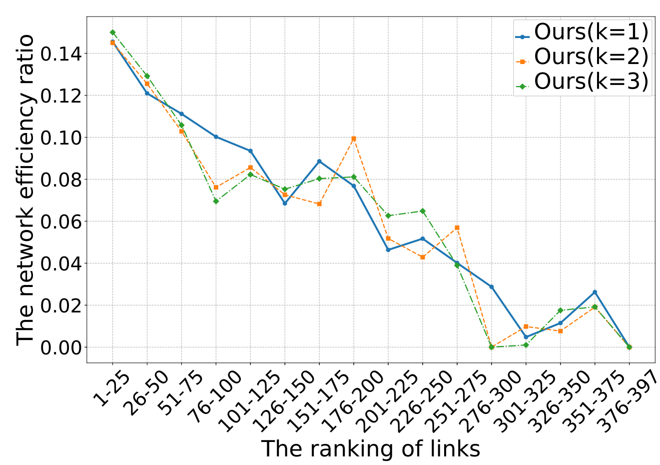

Fig. 10. Variation of the network efficiency ratio at 24:00.

TABLE III
RESULTS OF CRITICAL LINKS WHEN K=1 AT 18:00.

	Road ID	Road name	Influence	length(m)	St Road	Ed Road
k=1	10251	Shixin Road	1.63	153.15	Tiyu Road	Renmin Road
	10258	Shixin Road	1.629	204.6	Renmin Road	Tiyu Road
	10668	Xiaoran East Road	1.622	47.76	Renmin Road	Xiaoran East Road
	10666	Xiaoran East Road	1.618	109.82	Wenhua Road	Renmin Road
	10665	Xiaoran East Road	1.617	41.76	Tiyu Road	Renmin Road
	10234	Xihe Road	1.616	212.71	Tiyu Road	Renmin Road
	10250	Shixin Road	1.615	204.6	Tiyu Road	Renmin Road
	10267	Shixin Road	1.613	115.16	Xiaoran South Road	West Gate of the People's Hospital
	11280	Xiaoran East Road	1.607	206.04	Xiaoshao Road	Jinjia Qiao
	10001	Jiangsi Road	1.599	184.41	Chenghe Street	Renmin Road
	10232	Xihe Road	1.595	211.63	Chenghe Street	Tiyu Road
	10527	Tonghui Road	1.594	109.08	Bus East Station	Gongxiu Road
	10562	Renmin Road	1.593	195.48	Baichilou Road	Jiangsi Road
	10675	Yucai Road	1.592	40.91	Renmin Road	Wenhua Road
	10004	Jiangsi Road	1.591	45.03	Wenhua Road	Chenghe Street

TABLE IV
RESULTS OF CRITICAL LINKS WHEN K=2 AT 18:00.

	Road ID	Road name	Influence	length(m)	St Road	Ed Road
k=2	10258	Shixin Road	1.912828	204.56	Renmin Road	Tiyu Road
	10267	Shixin Road	1.881679	115.16	Xiaoran South Road	West Gate of the People's Hospital
	10004	Jiangsi Road	1.867957	45.03	Wenhua Road	Chenghe Street
	10001	Jiangsi Road	1.866491	184.41	Chenghe Street	Renmin Road
	10251	Shixin Road	1.857974	153.15	Tiyu Road	Chenghe Street
	10666	Xiaoran East Road	1.857244	109.82	Renmin Road	Xiaoran South Road
	10234	Xihe Road	1.853	212.71	Tiyu Road	Renmin Road
	10665	Xiaoran East Road	1.85099	41.76	Wenhua Road	Renmin Road
	10252	Tiyu Road	1.843896	460.37	Shixin Road	Xihe Road
	10244	Shixin Road	1.834372	153.15	Chenghe Street	Tiyu Road
	10232	Xihe Road	1.830406	211.63	Chenghe Street	Tiyu Road
	10255	Xihe Road	1.829569	212.71	Renmin Road	Tiyu Road
	10250	Shixin Road	1.818033	204.6	Tiyu Road	Renmin Road
	10668	Xiaoran East Road	1.817909	41.76	Renmin Road	Wenhua Road
	10233	Tiyu Road	1.814371	460.37	Xihe Road	Shixin Road

C. A Case Study Based on a Large-scale Dataset

We also conduct a case study with a large-scale dataset. This dataset includes the average road speed data on Friday, December 20, 2019, on over 50,000 roads in the Xiaoshan District, Hangzhou, China. Table X shows the top 10 road links that have the greatest impact on the road network.

With the help of online maps, we find that some of the roads in table X are traffic arteries, which play an important role in the network, such as QingLiu South Road and Qunyu Line. Additionally, some of the roads with dense surrounding infrastructure are prone to congestion, such as Shixin Road and Tonghui Road. Our method is able to identify the critical roads in the network on the large-scale dataset.

D. Computational Complexity Analysis

In this section, we analyze the computation complexity of our algorithm. Our method consists of two modules and the computational complexity is determined by the module with the highest complexity of the two. For the first experimental dataset, all experiments are conducted on a single NVIDIA 3060 GPU with 16GB Ram and the average program run time is about 15 minutes with different parameters.

1) Temporal Graph Attention Model: This model consists of a GAT model and an LSTM model. For the GAT model, the computational complexity of the graph attention layer is $O(|V| \times F \times F') + O(|E| \times F')$, where $|V|$ is the number of nodes in the graph, F and F' are the dimensions of the input

TABLE V
RESULTS OF CRITICAL LINKS WHEN K=3 AT 18:00.

	Road ID	Road name	Influence	length(m)	St Road	Ed Road
k=3	10267	Shixin Road	2.022365	115.16	Xiaoran South Road	West Gate of the People's Hospital
	10258	Shixin Road	1.985112	204.56	Renmin Road	Tiyu Road
	10001	Jiangsi Road	1.969537	184.41	Chenghe Street	Renmin Road
	10251	Shixin Road	1.963567	153.15	Tiyu Road	Chenghe Street
	10665	Xiaoran East Road	1.953198	41.76	Wenhua Road	Renmin Road
	10666	Xiaoran East Road	1.951184	109.82	Renmin Road	Xiaoran South Road
	10198	Jiangsi Road	1.944905	184.41	Chenghe Street	Renmin Road
	10234	Xihe Road	1.939758	211.63	Chenghe Street	Tiyu Road
	10233	Tiyu Road	1.937655	460.37	Xihe Road	Shixin Road
	10244	Shixin Road	1.936493	153.15	Chenghe Street	Tiyu Road
	10232	Xihe Road	1.928332	211.63	Chenghe Street	Tiyu Road
	10255	Xihe Road	1.928116	212.71	Renmin Road	Tiyu Road
	10250	Shixin Road	1.907883	204.6	Tiyu Road	Renmin Road
	10253	Renmin Road	1.89474	443.09	Xihe Road	Shixin Road
	10252	Tiyu Road	1.894676	460.37	Shixin Road	Xihe Road

TABLE VI
RESULTS OF CRITICAL LINKS WHEN K=1 AT 24:00.

	Road ID	Road name	Influence	length(m)	St Road	Ed Road
k=1	10011	Yucai Road	1.649437	181.74	Beiganshan South Road	Yudong Road
	10666	Xiaoran East Road	1.637625	109.82	Renmin Road	Xiaoran South Road
	10233	Tiyu Road	1.6315	460.37	Xihe Road	Shixin Road
	10665	Xiaoran East Road	1.631349	41.76	Wenhua Road	Renmin Road
	10258	Shixin Road	1.63036	204.6	Wenhua Road	Tiyu Road
	10267	Shixin Road	1.628533	115.16	Xiaoran South Road	West Gate of the People's Hospital
	10675	Yucai Road	1.626796	40.91	Renmin Road	Wenhua Road
	10251	Shixin Road	1.625993	153.15	Tiyu Road	Chenghe Street
	11283	Shixin Road	1.62199	460.37	Xihe Road	Shixin Road
	10004	Jiangsi Road	1.621451	45.03	Wenhua Road	Chenghe Street
	11280	Xiaoran East Road	1.621168	206.04	Xiaoshao Road	Jinjia Qiao
	10255	Xihe Road	1.619253	212.71	Renmin Road	Tiyu Road
	10234	Xihe Road	1.615633	212.71	Tiyu Road	Renmin Road
	10673	Yucai Road	1.613447	40.91	Wenhua Road	Renmin Road
	11161	Tonghui Road	1.609025	174.2	Shangcheng West Road	Zhanqian Road

and output feature, respectively, and $|E|$ is the number of edges in the graph. Because we use two graph attention layers in our model and the attention head and the input feature are both set to 8 in the first layer, the complexity of the GAT model is $(O_{layer1} + O_{layer2}) = O((8 \times (8|V| + 8|E|)) + 64|V| + |E|)$. For the LSTM model, the computational complexity is determined by the number of parameters in the network [52]. In our case, the LSTM complexity is $O(|V|^2)$. In summary, the computational complexity of the temporal graph attention model is $O(|V|^2 + 128|V| + 65|E|)$.

2) Traffic influence propagation model: In this model, the complexity lies mainly in finding the paths of the nodes to the surrounding nodes. We do not consider loops that we can obtain the paths from the adjacency matrix of the graph. Supposing that the average degree of the nodes in the graph is d, the complexity of the traffic influence propagation model is $O((d \times |V|)^k)$, where k denotes the neighbourhood order.

In summary, the computational complexity of the whole model is $O((d \times |V|)^k + |V|^2 + 128|V| + 65|E|)$. As can be seen in Fig 9 and Fig 10, when $k = 1$, our approach already distinguishes well between critical links and normal links. Additionally, we also can use the GRU [53] model to replace the LSTM model, which can reduce the computation complexity. Compared with existing methods for critical links identification based on traffic assignment (assuming the simplest form, i.e. all-or-nothing [54] and using the global travel time as the evaluation metric [8]), the computational complexity for the importance of a link is $O(|V|(|V| + |E|\log|E|)|E|$ [55].

TABLE VII
RESULTS OF CRITICAL LINKS WHEN K=2 AT 24:00.

	Road ID	Road name	Influence	length(m)	St Road	Ed Road
k=2	11283	Shixin Road	1.934272	115.1575	West Gate of the People's Hospital	Xiaoran South Road
	10267	Shixin Road	1.924428	115.1575	Xiaoran South Road	West Gate of the People's Hospital
	10004	Jiangsi Road	1.906783	45.03426	Wenhua Road	Chenghe Street
	10666	Xiaoran East Road	1.906063	109.8198	Renmin Road	Xiaoran South Road
	10665	Xiaoran East Road	1.89625	41.75527	Wenhua Road	Renmin Road
	10258	Shixin Road	1.896035	204.5957	Renmin Road	Tiyu Road
	10251	Shixin Road	1.863965	153.1451	Tiyu Road	Chenghe Street
	10232	Xihe Road	1.860526	211.6322	Chenghe Street	Tiyu Road
	11161	Tonghui Road	1.857738	174.1998	Shangcheng West Road	Zhanqian Road
	10011	Yucai Road	1.857468	181.7377	Beiganshan South Road	Yudong Road
	10234	Xihe Road	1.854803	212.7081	Tiyu Road	Renmin Road
	10255	Xihe Road	1.853764	212.7081	Renmin Road	Tiyu Road
	10233	Tiyu Road	1.844009	460.3711	Xihe Road	Shixin Road
	11280	Xiaoran East Road	1.842102	206.0378	Xiaoshao Road	Jinjia Qiao
	10559	Renmin Road	1.83226	630.3295	Huilan Road	Yucai Road

TABLE VIII
RESULTS OF CRITICAL LINKS WHEN K=3 AT 24:00.

	Road ID	Road name	Influence	length(m)	St Road	Ed Road
k=3	10267	Shixin Road	2.044846	115.1575	Xiaoran South Road	West Gate of the People's Hospital
	10258	Shixin Road	2.032875	204.5957	West Gate of the People's Hospital	Xiaoran South Road
	10666	Xiaoran East Road	1.999239	109.8198	Renmin Road	Xiaoran South Road
	10004	Jiangsi Road	1.998949	45.03426	Wenhua Road	Chenghe Street
	10251	Shixin Road	1.996855	153.1451	Tiyu Road	Chenghe Street
	10665	Xiaoran East Road	1.995081	41.75527	Wenhua Road	Renmin Road
	10559	Renmin Road	1.967993	630.3295	Huilan Road	Yucai Road
	10011	Yucai Road	1.955475	181.7377	Beiganshan South Road	Yudong Road
	10255	Xihe Road	1.948567	212.7081	Renmin Road	Tiyu Road
	10234	Xihe Road	1.941989	212.7081	Tiyu Road	Renmin Road
	10233	Tiyu Road	1.938182	460.3711	Xihe Road	Shixin Road
	11161	Tonghui Road	1.936363	174.1998	Shangcheng West Road	Zhanqian Road
	10257	Renmin Road	1.929267	443.0885	Shixin Road	Xihe Road
	10232	Xihe Road	1.927284	211.6322	Chenghe Street	Tiyu Road
	11280	Xiaoran East Road	1.921785	206.0378	Xiaoshao Road	Jinjia Qiao

Since all the links need to be traversed, the total computational complexity is $O(|V|^3|E| + |V|^2|E|^2\log|E|))$, and therefore, our method is superior in terms of computational complexity and is more suitable for large-scale urban road networks.

VI. CONCLUSION AND FUTURE WORK

In this paper, we propose a novel method to identify critical links in transportation networks, which improves upon previous approaches. Our method consists of a temporal graph attention model and a traffic influence propagation model. Specifically, the temporal graph attention model is used to learn the traffic interaction representation, which is combined with the GAT and LSTM network. The traffic influence propagation model, then, is used to calculate the influence of each road link. We tested our method using real data from Hangzhou, China, and the results validate our method. We discovered that certain road links have varying importance at different times. However, some roads, like Shixin Road and Xiaoran East Road, consistently play a significant role due to high traffic and complex conditions. This information can guide authorities in implementing targeted traffic control measures to prevent congestion. We also conduct experiments using a large-scale dataset and give the results of the ranking of the critical links. However, our study also has some limitations. For example, due to the lack of traffic origin-destination data, we have not been able to select the critical links identification model based on the traffic assignment model as the comparison model. In the traffic influence propagation model, we have also

TABLE IX
PERFORMANCE COMPARISON OF DIFFERENT METHODS

Model	t=18:00;Top15 nodes			t=18:00; All nodes			t=24:00;Top15 nodes			t=24:00; All nodes		
	SCC	KRCC	MD	SCC	KRCC	MD	SCC	KRCC	MD	SCC	KRCC	MD
Our method(k=1)	0.325	0.238	1421	0.606	0.436	30034	0.110	0.085	984	0.593	0.429	30768
Our method(k=2)	0.379	0.257	1500	0.644	0.459	**25061***	0.264	0.142	1031	0.663	0.476	30644
Our method(k=3)	**0.45***	**0.354***	**1005***	**0.682***	**0.486***	28684	**0.406***	**0.372***	**783***	**0.698***	**0.523***	**26982***
Betweenness Centrality	0.368	0.257	2435	0.278	0.187	42328	-0.010	0.009	2647	0.268	0.184	42698
Congestion Rate	0.029	0.010	1953	0.527	0.375	34818	0.107	0.180	923	0.540	0.383	34046
VoteRank	-0.830	-0.692	3981	0.061	0.040	27516	-0.292	-0.200	1003	0.073	0.049	28401
PCA method	0.186	0.124	3196	-0.082	-0.067	55126	-0.070	-0.104	3411	-0.109	-0.079	55870
Road2vec method(k=1)	-0.596	-0.467	1279	0.573	0.384	33448	-0.103	-0.085	934	0.581	0.390	32660
Road2vec method(k=2)	-0.104	-0.029	1383	0.609	0.420	32306	0.064	0.047	1179	0.631	0.438	30802
Road2vec method(k=3)	0.164	0.124	1638	0.627	0.434	31262	0.142	0.123	1373	0.655	0.455	29844
HTC method(k=1)	-0.075	-0.067	1534	0.432	0.293	38122	-0.053	-0.028	1139	0.452	0.306	37874
HTC method(k=2)	0.139	0.086	1785	0.512	0.355	35240	0.060	0.047	1184	0.535	0.369	34648
HTC method(k=3)	0.118	0.067	1858	0.540	0.375	33590	0.089	0.104	1260	0.569	0.399	32464
Our method_ablation(k=3)	0.157	0.143	2209	0.406	0.278	38152	0.035	0.028	1591	0.474	0.324	35946

TABLE X
RESULTS OF CRITICAL LINKS BASED ON THE LARGE-SCALE DATASET

Ranking	Road
1	QingLiu South Road
2	Qunyu Line
3	Xinru Road
4	Xinyi Road
5	Shixin Road
6	Wenming Road
7	Tonghui Road
8	Dongling South Road
9	Xinhe East Road
10	Dongrui Second Road

simplified the process of propagation. These limitations can be further addressed in future studies.

In terms of future avenues for research, these could include optimizing traffic management based on critical road links and optimizing road network design to make the transportation system more robust.

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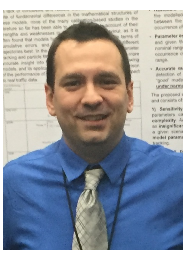

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