# Towards Understanding and Improving the Transferability of Adversarial Examples in Deep Neural Networks

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### Abstract

Currently it is well known that deep neural networks are vulnerable to adversarial examples, constructed by applying small but malicious perturbations to the original inputs. Moreover, the perturbed inputs can transfer between different models: adversarial examples generated based on a specific model will often fool other unseen models with a significant success rate. This allows the adversary to leverage it to attack the deployed systems without any query, which could raise severe security issue particularly in safety-critical scenarios. In this work, we empirically investigate two classes of factors that might influence the transferability of adversarial examples. One is about model-specific factors, including network architecture, model capacity and test accuracy. The other is the local smoothness of loss surface for generating adversarial examples. More importantly, relying on these findings on the transferability of adversarial examples, we propose a simple but effective strategy to improve the transferability, whose effectiveness is confirmed through extensive experiments on both CIFAR-10 and ImageNet datasets.

**Keywords:** Adversarial example, transferability, black-box attack, deep learning, neural network.

# 1. Introduction

Recently (Szegedy et al., 2013) showed that an adversary is able to fool deep neural network models into producing incorrect predictions by manipulating the inputs maliciously. The corresponding manipulated samples are called adversarial examples. More severely, it was found that adversarial examples have cross-model generalization ability, i.e., that adversarial examples generated from one model can fool another different model with a high probability. We refer to such property as transferability. By now, more and more deep neural network models are applied in real-world applications, such as speech recognition, computer vision, etc.. Consequently, an adversary can employ the transferability to attack those deployed models without querying the systems (Papernot et al., 2016b; Liu et al., 2016), inducing serious security issues.

Understanding the mechanism of transferability could potentially provide various benefits. Firstly, for the already deployed deep neural network models in real systems, it could help to design better strategies to improve the robustness against the transfer-based attacks. Secondly, revealing the mystery behind the transferability of adversarial examples could

also expand the existing understandings on deep learning, particularly the effects of model capacity (Madry et al., 2017; Fawzi et al., 2015) and model interpretability (Dong et al., 2017; Ross and Doshi-Velez, 2017). Therefore, studying the transferability of adversarial examples in the context of deep networks is of significant importance.

In this paper, we empirically investigate two classes of factors that might influence the adversarial transferability, and further provide a simple but rather effective strategy for enhancing the transferability. Our contributions are summarized as follows.

- We find that adversarial transfer is not symmetric, which means that adversarial examples generated from model A can transfer to model B easily does not implies the reverse is also natural. This finding is consistent with (Su et al., 2018). Second, we find that adversarial examples generated from a deep model appear less transferable than those from a shallow model. We also explore the impact of the non-smoothness of the loss surface. Specifically, we find that the local non-smoothness of loss surface harms the transferability of generated adversarial examples.
- Inspired by previous investigations, we propose a novel algorithm, the *smoothed gradient* attack, to improve the adversarial transferability significantly. It employs the locally averaged gradient instead of the original one to generate adversarial examples, which implicitly help to smooth the loss landscape. Extensive numerical validations justify the effectiveness of our method.

Related work The phenomenon of adversarial transferability was first studied by (Szegedy et al., 2013). By utilizing the transferability, (Papernot et al., 2016b,a) proposed a practical black-box attack by training a substitute model with limited queried information. (Liu et al., 2016) first studied the targeted transferability and introduced the ensemble-based attacks to improve the transferability. More recently, (Dong et al.) showed that the momentum can help to boost transferability significantly.

Meanwhile, there exist several works trying to explain the adversarial transferability. (Papernot et al., 2016a) attributed the transferability to the similarity between input gradients of source and target models. (Liu et al., 2016) proposed to use the visualization technique to see the large-scale similarity of decision boundaries. However, our empirical investigations imply that these similarity-based explanations have their intrinsic limitation that they cannot explain the non-symmetric property of adversarial transferability.

Our smoothed gradient attacks that enhance the transferability by utilizing the information in the small neighborhood of the clean example is inspired by the works on shattered gradients (Balduzzi et al., 2017) and model interpretability (Smilkov et al., 2017). Recently, similar strategies are also explored by (Athalye et al., 2018) and (Warren He, 2018) for white-box attacks, whereas we focus on the black-box settings. Our method is also related to the work by (Athalye and Sutskever, 2017), which introduced the expectation over transformation (EOT) method to increase robustness of adversarial examples. The EOT formulation is similar to our objective (7), but they did not study the transferability. Also our motivations are totally different from theirs.

# 2. Background

### 2.1. Adversarial attack

Let  $f(x): \mathbb{R}^d \to \mathbb{R}^K$  denote the classifier. In deep learning, it is found that for almost any sample x and its label  $y^{\text{true}}$ , there exists a small perturbation  $\eta$  that is nearly imperceptible to human such that

$$\underset{i}{\operatorname{argmax}} f_i(x) = y^{\text{true}}, \quad \underset{i}{\operatorname{argmax}} f_i(x+\eta) \neq y^{\text{true}}. \tag{1}$$

We call  $\eta$  the adversarial perturbation and correspondingly  $x^{adv} := x + \eta$  the adversarial example. The attack (1) is called a non-targeted attack since the adversary has no control over which class the input x will be misclassified to. In contrast, a targeted attack aims at fooling the model to produce a wrong label specified by the adversary, i.e.  $argmax_i f_i(x+\eta) = y^{target}$ .

In this paper, we consider pure black-box attacks. This means the adversary has no knowledge of the target model (e.g. architecture and parameters) and is also not allowed to query any input-output pair from the target model. On the contrary, the *white-box* attack means that the target model is available to the adversary.

## 2.2. Generating adversarial examples

**Modeling** In general, crafting adversarial examples can be modeled as the following optimization problem,

$$\begin{array}{ll}
\text{maximize}_{x'} & J(x') := J(f(x'), y^{\text{true}}) \\
\text{s.t.} & \|x' - x\|_{\infty} \le \varepsilon,
\end{array} \tag{2}$$

where J is some loss function measuring the discrepancy between the model prediction and ground truth; the  $\ell_{\infty}$  norm  $\|\cdot\|_{\infty}$  is used to quantify the magnitude of the perturbation. Other norm is also possible, but we focus on  $\ell_{\infty}$  norm in this paper. To improve the strength of adversarial transferability, instead of using a single model, (Liu et al., 2016) proposed the *ensemble attack*, which generates adversarial examples against a model ensemble  $f^{1}(x), f^{2}(x), \dots, f^{M}(x)$ :

**Optimizer** We use the following iteration to solve (2) and (3),

$$x_{t+1} = \operatorname{proj}_{\mathcal{D}} \left( x_t + \alpha \operatorname{sign}(\nabla_x J(x_t)) \right). \tag{4}$$

where  $\mathcal{D} = [0, 255]^d \cap \{x' | \|x' - x\| \le \varepsilon\}$  and  $\alpha$  is the step size. We call the attack that evolves (4) for T steps iterative gradient sign method (IGSM) attack (Kurakin et al., 2016; Madry et al., 2017). Furthermore, the famous fast gradient sign method (FGSM) is a special case with  $\alpha = \varepsilon, T = 1$ .

(Dong et al.) recently proposed the momentum-based attack as follows

$$g_{t+1} = \mu g_t + \nabla J(x_t) / \|\nabla J(x_t)\|_1 x_{t+1} = \text{proj}_D (x_t + \alpha \text{ sign}(g_t)),$$
 (5)

where  $\mu$  is the decay factor of momentum. By using this method, they won the first-place in NIPS 2017 Non-targeted Adversarial Attack and Targeted Adversarial Attack competitions. We will call it mIGSM attack in this paper.

# 3. Evaluation of Adversarial Transferability

**Datasets** To evaluate the transferability, three datasets including MNIST, CIFAR-10 and ImageNet are considered. For ImageNet, evaluations on the whole ILSVRC2012 validation dataset are too time-consuming. Therefore, in each experiment we randomly select 5,000 images that can be correctly recognized by all the examined models to form our new validation set.

Models (i) For MNIST, we trained fully connected networks (FNN) of *D* hidden layers, with the width of each layer being 500. (ii) For CIFAR-10, we trained five models: lenet,resnet20, resnet44, resnet56, densenet. The test accuracies of them are 76.9%, 92.4%, 93.7%, 93.8% and 94.2%, respectively. (iii) For ImageNet, the pre-trained models provided by PyTorch are used. The Top-1 and Top-5 accuracies of them can be found on website<sup>1</sup>. To increase the reliability of experiments, all the models have been tested. However, for a specific experiment we only choose some of them to present since the findings are consistent among all the tested models.

Criteria Given a set of adversarial pairs,  $\{(x_i^{adv}, y_i^{\text{true}})\}_{i=1}^N$ , we calculate the *Top-1 success rate* (%) fooling a given model f(x) by  $\frac{100}{N}\sum_{k=1}^N 1[\operatorname{argmax}_i f_i(x_k^{adv}) \neq y_k^{\text{true}}]$ . For targeted attacks, each image  $x^{adv}$  is associated with a pre-specified label  $y^{\text{target}} \neq y^{\text{true}}$ . Then we evaluate the performance of the targeted attack by the following Top-1 success rate:  $\frac{100}{N}\sum_{k=1}^N 1[\operatorname{argmax}_i f_i(x_k^{adv}) = y_k^{\text{target}}]$ . The corresponding Top-5 rates can be computed in a similar way.

# 4. Model-specific Factors for Transferability

Previous studies on adversarial transferability mostly focused on the influence of attack methods (Liu et al., 2016; Dong et al.; Tramèr et al., 2017; Kurakin et al., 2016). However it is not clear how the choice of source model affects the success rate transferring to target models. In this section, we investigate this issue from three aspects including architecture, test accuracy and model capacity.

### 4.1. Architecture

We first explore how the architecture similarity between source and target model contributes to the transferability. This study is crucial since it can provide us guidance to choose the appropriate source models for effective attacks. To this end, three popular architectures including ResNet, DenseNet and VGGNet are considered, and for each architecture, two networks are selected. Both one-step and multi-step attacks are performed on ImageNet dataset. Table 1 presents the experiment results, and the choice of hyper-parameters is detailed in the caption.

<sup>1.</sup> http://pytorch.org/docs/master/torchvision/models.html

Table 1: Top-1 success rates(%) of FGSM and IGSM attacks. The row and column denote the source and target models, respectively. For each cell, the left is the success rate of FGSM ( $\varepsilon=15$ ), while the right is that of IGSM ( $T=5, \alpha=5, \varepsilon=15$ ). The dashes correspond to the white-box cases, which are omitted.

	resnet18	resnet101	vgg13_bn	vgg16_bn	densenet121	densenet161
resnet18	-	36.9 / 43.4	51.8 / 58.0	45.1 / 51.7	41.1 / 49.2	30.0 / 35.8
resnet101	48.5 / 57.2	-	38.9 / 41.6	$33.1 \ / \ 40.0$	33.2 / 46.9	28.7 / 43.2
vgg13_bn	35.5 / 26.8	14.8 / 10.8	-	58.8 / 90.7	19.1 / 15.9	13.8 / 11.7
$vgg16_bn$	35.2 / 26.1	15.6 / 11.1	61.9 / 91.1	-	21.1 / 16.8	15.8 / 13.2
densenet121	49.3 / 63.8	34.4 / 50.7	47.6 / 58.7	41.0 / 57.8	-	38.5 / 73.6
densenet161	45.7 / 56.3	$33.8 \ / \ 54.6$	48.6 / 56.0	$41.3\ /\ 55.9$	43.4 / 78.5	-

We can observe that the transfer between two models are non-symmetric, and this phenomenon is more obvious for the models with different architectures. For instance, the success rates of IGSM attacks from densenet121 to vgg13\_bn is 58.7%, however the rate from vgg13\_bn to densenet121 has only 15.9%. The lack of symmetry implies that previous similarity-based explanations of adversarial transferability are quite limited.

Another interesting observation is that success rates between models with similar architectures are always much higher. For example the success rates of IGSM attacks between vgg13\_bn and vgg16\_bn are higher than 90%, nearly twice the ones of attacks from any other architectures.

# 4.2. Model Capacity and Test Accuracy

We first study this problem on ImageNet dataset. A variety of models are used as source models to perform both FGSM and IGSM attacks against vgg19\_bn and resnet152, and the results are displayed in Figure 1. The horizontal axis is the Top-1 test error, while the vertical axis is the number of model parameters that roughly quantifies the model capacity.

We can see that the models with powerful attack capability concentrate in the bottom left corner, whereas for those models with either large test error or large number of parameters, the fooling rates are much lower.

We suspect that the impact of test accuracy is due to that the decision boundaries of high-accuracy models are similar, since they all approximate the ground-truth decision boundary of true data very well. On the contrary, the model with low accuracy has a decision boundary relatively different from the high-accuracy models. Here the targeted network vgg19\_bn and resnet152 are high-accuracy models. Therefore it is not surprising to observe that high-accuracy models tend to exhibit stronger attack capability. This phenomenon implies a kind of data-dependent transferability, which is different from the architecture-specific transfers observed in the previous section.

It is somewhat strange that adversarial examples generated from deeper model appear less transferable. To further confirm this observation, we conduct additional experiments on MNIST and CIFAR-10. Table 2 shows the results, which is basically consistent. This

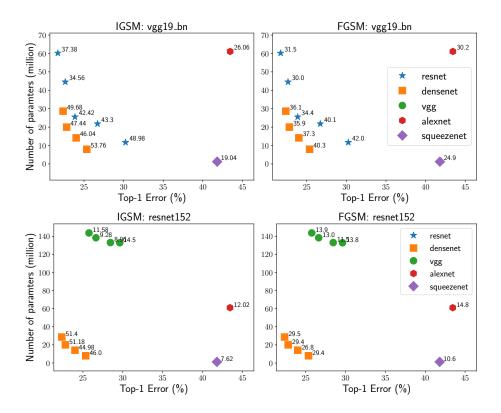


Figure 1: Top-1 success rates of IGSM  $(T=20,\alpha=5,\varepsilon=15)$  and FGSM $(\varepsilon=15)$  attacks against vgg19\_bn (the first row) and resnet152 (the second row) for various models. The annotated value is the success rate transferring to the target models. Here, the models of vgg-style or resnet-style have been removed to exclude the influence of architecture similarity. For the same color, the different points corresponds networks of different depths.

Table 2: Top-1 success rates (%) of attack from the source model (row) to the target model (column). (Left) FGSM attack for MNIST with  $\varepsilon=40$  and D denotes the depth of the fully connected network. (Right) FGSM attack for CIFAR-10 with  $\varepsilon=10$ .

	D = 0	D=2	D=4	D=8					
D=0						resnet20	resnet44	resnet56	densenet
		62.9	62.9	64.4	resnet20	_	70.4	64.0	71.6
D=2	52.9	-	48.3	49.4	resnet44		.0.1	57.1	er o
D=4	47.3	43.1	_	44.8			-	57.1	65.8
				11.0	resnet56	66.2	62.9	-	40.3
D=8	31.2	29.2	29.0	-					

observation suggests us not use deep models as the source models when performing transferbased attacks, although we have not fully understand this phenomenon.

# 5. Non-smoothness of the Loss Surface

In this section, we consider that how the smoothness of loss surface J(x) affects the transferability. For simplicity, let  $g(x) = \nabla_x J(x)$  denote the gradient. (Smilkov et al., 2017) showed that gradient g(x) is very noisy and uninformative for visualization, though the model is trained very well. (Balduzzi et al., 2017) studied a similar phenomenon that gradients of deep networks are extremely shattered. Both of them imply that the landscape is locally extremely rough. We suspect that this local non-smoothness could harm the transferability of adversarial examples.

### 5.1. Intuition

For simplicity, assume model A and B are the source and target models which are well trained with high test accuracies, respectively. Previous methods use  $g_A(x)$  to generate adversarial perturbations, so the transferability mainly depends on how much sensitivity of  $g_A$  for model A can transfer to model B. As illustrated in Figure 2, where three curves denote the level sets of three models, we can see that the non-smoothness can hurt the transferability, since both model A and B have very high test accuracy, their level sets should be similar globally, and  $J_B(x)$  is probably unstable along  $g_A$ . As illustrated, the local oscillation of  $g_A$  makes the sensitivity less transferable. One way to alleviate this is to smooth the landscape  $J_A$ , thereby yielding a more transferable gradient  $G_A$ , i.e.  $\langle \hat{G}_A, \hat{g}_B \rangle > \langle \hat{g}_A, \hat{g}_B \rangle$ . Here we use G denote the gradient of the smoothed loss surface.

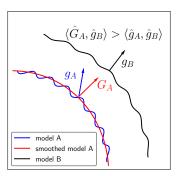


Figure 2: How the non-smoothness of the loss surface hurts the transferability. For any a, let  $\hat{a}$  denote the unit vector  $a/\|a\|_2$ .

#### 5.2. Justification

To justify the previous arguments, we consider smoothing the loss surface by convolving it with a Gaussian filter, then the smoothed loss surface is given by  $J_{\sigma}(x) := \mathbb{E}_{\xi \sim \mathcal{N}(0,I)}[J(x+\sigma\xi)]$ . The corresponding gradient can be calculated by

$$G_{\sigma}(x) = \mathbb{E}_{\xi \sim \mathcal{N}(0,I)}[g(x+\sigma\xi)]. \tag{6}$$

The extent of smoothing is controlled by  $\sigma$ , and we show the smoothing effect by visualizing the smoothed gradient.

Visualization of smoothed gradient we provide an visualization for understanding how the local average has the smoothing effect. We choose densenet121 as the model and visualize the saliency map of gradient  $\nabla_x J(x)$  and the smoothed versions for varying m. Two images are considered, and the results are shown in Figure 3. We can easily observe that local average can smooth the gradient significantly and capture the most important semantic information. Please refer to the work by (Smilkov et al., 2017) for more details.

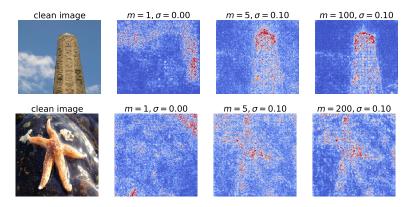


Figure 3: Visualization of gradients. The leftmost is the examined image. The second corresponds to the original gradient, whereas the remaining two images corresponds to the smoothed gradients estimated by different m.

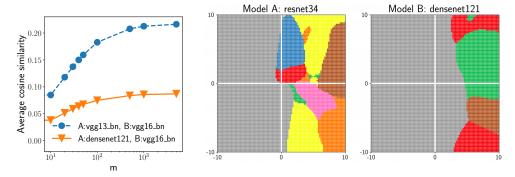


Figure 4: (Left) Cosine similarity between the gradients of source and target models. (Middel, Right) Visualization of decision boundaries. The origin corresponds to the clean image. The colors denote the different class categories, and the gray color corresponds to the ground truth label.

In the following, we will quantitatively show that  $G_{\sigma}(x)$  is more transferable than g(x).

Gradient similarity We first quantify the cosine similarity between gradients of source and target models, respectively. Two situations are considered:  $vgg13\_bn \rightarrow vgg16\_bn$ , densenet121 $\rightarrow vgg13\_bn$ , which correspond the within-architecture and cross-architecture transfers, respectively. We choose  $\sigma = 15$ , and the expectation in (6) is estimated by using  $\frac{1}{m} \sum_{i=1}^{m} g(x + \xi_i)$ . To verify the averaged gradients do transfer better, we plot the cosine

similarity against the number of samples m. In Figure 4 (Left), as expected, we see that the cosine similarity between  $G_A$  and  $g_B$  are indeed larger than the one between  $g_A$  and  $g_B$ . Moreover, the similarity increases with m monotonically, which further justifies that  $G_A$  is more transferable than  $g_A$ .

Visualization In Figure 4 (Middle) and (Right), we visualize the transferability by comparing the decision boundaries of model A (resnet34) and model B (densenet121). The horizontal axis represents the direction of  $G_A$  of resnet34, estimated by  $m=1000, \sigma=15$ , and the vertical axis denotes orthogonal direction  $h_A:=g_A-\langle g_A,\hat{G}_A\rangle\hat{G}_A$ . This process means that we decompose the gradient into two terms:  $g_A=\alpha G_A+\beta h_A$  with  $\langle G_A,h_A\rangle=0$ . Each point in the 2-D plane corresponds to the image perturbed by u and v along each direction, clip $(x+u\,\hat{G}_A+v\,\hat{h}_A,0,255)$ , where x is the clean image. The color corresponds to the label predicted by the target model.

It can be easily observed that for model A, a small perturbation in both directions can produce wrong classification. However, when applied to model B, the sensitivities of two directions dramatically change. The direction of  $h_A$  becomes extremely stable, whereas to some extent  $G_A$  preserves the sensitivity, i.e.  $G_A$  do transfer to model B better than  $h_A$ . This suggests that for gradient  $g_A$ , the noisy part  $h_A$  is less transferable than the smooth part  $G_A$ . We also tried a variety of other models shown in Supplementary Material, and the results are consistent.

## 6. Smoothed Gradient Attack

**Method** Inspired by the previous investigations, enhancing the adversarial transferability can be achieved by smoothing the loss surface. Then our objective becomes,

maximize 
$$J_{\sigma}(x') := \mathbb{E}_{\xi \sim \mathcal{N}(0,I)}[J(x' + \sigma \xi)]$$
  
s.t.  $||x' - x|| \le \varepsilon$ . (7)

Intuitively, this method can also be interpreted as generating adversarial examples that are robust to Gaussian perturbation. Expectedly, the generated robust adversarial examples can still survive easily in spite of the distinction between source and target model.

If use the iterative gradient sign method to solve (7), we have the following iteration:

$$G_t = \frac{1}{m} \sum_{i=1}^{m} \nabla J(x_t + \xi_i), \quad \xi_i \sim \mathcal{N}(0, \sigma^2 I)$$

$$x_{t+1} = \operatorname{proj}_{\mathcal{D}} (x_t + \alpha \operatorname{sign} (G_t)),$$
(8)

where  $G_t$  is a mini-batch approximation of the smoothed gradient (6). Compared to the standard IGSM, the gradient is replaced by a smoothed version, which is endowed with stronger transferability. Therefore we call this method sg-IGSM. The special case  $T = 1, \alpha = \varepsilon$ , is accordingly called sg-FGSM. Any other optimizer can be used to solve the (7) as well, and we only need to replace the original gradient with the smoothed one.

Choice of Hyperparameters We first explore the sensitivity of hyper parameters  $m, \sigma$  when applying our smoothed gradient technique. We take ImageNet dataset as the testbed,

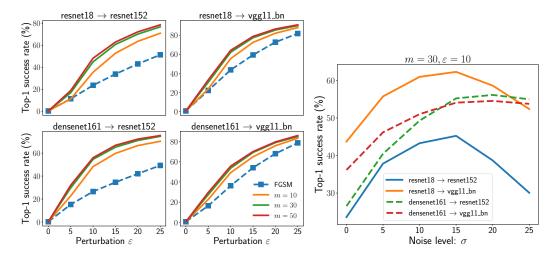


Figure 5: (Left) Success rates (%) for sg-FGSM attacks with different m. Here we use  $\sigma = 15$ . (Right) The sensitivity of the hyper parameter  $\sigma$ .

and sg-FGSM attack is examined. To increase the reliability, four attacks are considered here. The results are shown in Figure 5.

We see that sg-FGSM consistently outperforms FGSM for all distortion levels, although the improvement varies for different  $\varepsilon$ . Furthermore, larger m leads to higher success rate due to the better estimation of the smooth part of gradient, and the benefit starts to saturate after  $m \geq 30$ . For the smoothing factor  $\sigma$ , we find neither too large nor too small value can work well, and the optimal  $\sigma$  is about 15. Overly large  $\sigma$  will introduce a large bias in (8), and extremely small  $\sigma$  is unable to smooth the landscape enough.

Therefore, in the subsequent sections, we will use  $m = 20, \sigma = 15$  to estimate the smoothed gradient and only report the result for one distortion level.

#### 6.1. Effectiveness of Smoothed Gradient Attack

**Single-model attack** We first test the effectiveness of our method for single-model based attacks on non-targeted setting. To make a fair comparison, we fix the number of gradient calculation per sample at 100. Specifically, for sg-IGSM we have T=5 due to m=20, whereas T=100 for IGSM. The results are shown in Table 3. It can be observed that smoothed gradients do enhance the transferability dramatically for all the considered attacks. Especially note those bold rates, where the improvements have reached about 30%.

Ensemble attack In this part, we examine the ensemble-based attack on the targeted setting<sup>2</sup>. For targeted attacks, generating an adversarial example predicted by target models as a specific label is too hard, resulting in a very low success rate. We instead adopt both Top-1 and Top-5 success rates as our criteria to better reflect the improvement of our method. A variety of model ensembles are examined, and the results are reported in Table 4 and 5.

<sup>2.</sup> Compared to non-targeted attack, we find that a larger step size  $\alpha$  is necessary for generating strong targeted adversarial examples. The fully understanding of this observation will be left as future work. Therefore a much larger step size than the non-target attacks is used in this experiment.

Table 3: Top-1 success rates(%) of non-targeted IGSM and sg-IGSM attacks. The row and column denote the source and target models, respectively. For each cell, the left is the success rate of IGSM ( $T=100, \alpha=1$ ), while the right is the that of sg-IGSM ( $T=5, \alpha=5$ ). In this experiment, distortion level  $\varepsilon=15$ .

	densenet121	resnet152	resnet34	vgg13_bn	$vgg19_bn$
densenet121	-	50.1 / 80.6	59.9 / 87.2	62.2 / 82.2	56.5 /84.3
resnet152	52.5 / 81.3	-	$57.2 \ / \ 85.6$	47.7 / 71.1	42.9 / 72.6
resnet34	51.5 / 76.4	$46.5 \ / \ 73.1$	-	53.8 / 74.8	49.1 / 74.5
$\overline{\mathrm{vgg13\_bn}}$	24.1 / 49.2	14.3 / 33.5	25.1 / 54.1	-	90.6 / 96.4
$vgg19_bn$	27.1 / 57.5	16.7 / 41.6	27.6 / 60.7	92.0 / 96.1	-

Table 4: Top-1 success rates (%) of ensemble-based targeted IGSM and sgd-IGSM attacks. The row and column denote the source and target model, respectively. The left is the success rate of IGSM ( $T=20, \alpha=15$ ), while the right is that of sg-IGSM ( $T=20, \alpha=15$ ). The distortion  $\varepsilon=20$ .

Ensemble	resnet152	resnet50	vgg16_bn
resnet101+densenet121	11.6 / 37.1	11.9 / 34.5	2.6 / 14.1
$\overline{\text{resnet} 18 + \text{resnet} 34 + \text{resnet} 101 + \text{densenet} 121}$	30.3 / <b>55.2</b>	36.8 / <b>57.3</b>	12.8/35.0
vgg11_bn+vgg13_bn+resnet18+	10.1 / 25.1	22.2 / 47.9	49 1 / <b>79 1</b>
resnet34+densenet121	10.1 / 33.1	22.2 / 41.9	42.1/12.1

Table 5: Top-5 success rates (%) of ensemble-based approaches for IGSM  $(T=20,\alpha=15)$  versus sg-IGSM  $(T=20,\alpha=15)$ . The row and column denote the source and target models, respectively.

Ensemble	resnet152	resnet50	$vgg16_bn$
resnet101+densenet121	28.1 / 56.8	26.2 / 52.4	8.1 / 29.7
resnet 18 + resnet 34 + resnet 101 + densenet 121	50.4 / 70.4	54.7 / 72.4	28.1 / 52.6
$ m vgg11\_bn+vgg13\_bn+resnet18 \\ +resnet34+densenet121$	24.3 / 55.8	36.9 / 65.9	62.2 / 83.5

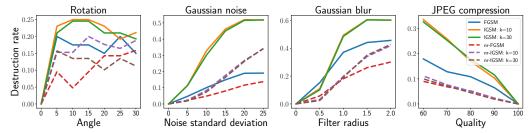


Figure 6: Destruction rates of adversarial examples for various methods. For smoothed gradient attacks, we choose  $m = 20, \sigma = 15$ . The distortion  $\varepsilon = 15$ .

Table 6: Top-1 success rates (%) of momentum attacks and smoothed gradient attacks. The row and column denote the source and target model, respectively. Each cell contains three rates corresponding to mIGSM, sg-IGSM and sg-mIGSM attacks, respectively.

	resnet18	densenet121	$vgg13_bn$	
resnet18	-	65.6 / 73.1 / 86.5		
	72.7 / 84.5 / 91.1		68.7 / 80.3 / 86.7	
$vgg13_bn$	43.1 / 58.6 / 74.3	28.4 / 44.7 / 60.9	-	

As we can see, it is clear that smoothed gradient attacks outperform the corresponding normal ones by a remarkable large margin. More importantly, the improvement never be harmed compared to single-model case in Table 3, which implies that *smoothed gradient can be effectively combined with ensemble method without compromise*.

Momentum attack In this experiment, three networks of different architectures are selected. As suggested in (Dong et al.), we choose  $\mu=1$ , and all attacks are iterated for T=5 with step size  $\alpha=5$ . In Table 6, we report the results of non-targeted attacks of three attacks including mIGSM, sg-IGSM and mIGSM with smoothed gradient (sg-mIGSM). It is shown that our method clearly outperforms momentum-based method for all the cases. Moreover, by combining with the smoothed gradient, the effectiveness of momentum attacks can be further improved significantly.

### 6.2. Robustness

Smoothed gradient attacks can be viewed as generating adversarial examples robust against Gaussian noise perturbations. Therefore, we are interested in how robust is the adversarial example against other image transformations. To quantify the influence of transformations, we use the notion of destruction rate defined by (Kurakin et al., 2016). The lower is this rate, the more robust are the adversarial examples.

Densenet121 and resnet34 are chosen as our source and target model, respectively; and four image transformations are considered: rotation, Gaussian noise, Gaussian blur and JPEG compression. Figure 6 displays the results, showing that adversarial examples generated by our methods are more robust than those generated by vanilla methods. This numerical result is interesting, since we only explicitly increase the robustness against

Gaussian noise in generating adversarial examples. We speculate that the robustness can also transfer among different image transforms.

## 7. Conclusion

In this paper, we first investigated the influence of model-specific factors on the adversarial transferability. It is found that the model architecture similarity plays a crucial role. Moreover models with lower capacity and higher test accuracy are endowed with stronger capability for transfer-based attacks. we second demonstrate that the non-smoothness of loss surface hinders the transfer of adversarial examples. Motivated by these understandings, we proposed the smoothed gradient attack that can enhance the transferability of adversarial examples dramatically. Furthermore, the smoothed gradient can be combined with both ensemble and momentum based approaches rather effectively.

Combining our smoothed gradient strategy with other type of adversarial attacks and rigorous theoretical analysis are left as future works.

### References

- Anish Athalye and Ilya Sutskever. Synthesizing robust adversarial examples. arXiv preprint arXiv:1707.07397, 2017.
- Anish Athalye, Nicholas Carlini, and David Wagner. Obfuscated gradients give a false sense of security: Circumventing defenses to adversarial examples. 2018. URL https://arxiv.org/abs/1802.00420.
- David Balduzzi, Marcus Frean, Lennox Leary, JP Lewis, Kurt Wan-Duo Ma, and Brian McWilliams. The shattered gradients problem: If resnets are the answer, then what is the question? arXiv preprint arXiv:1702.08591, 2017.
- Yinpeng Dong, Fangzhou Liao, Tianyu Pang, Hang Su, Xiaolin Hu, Jianguo Li, and Jun Zhu. Boosting adversarial attacks with momentum. arXiv preprint arXiv:1710.06081.
- Yinpeng Dong, Hang Su, Jun Zhu, and Fan Bao. Towards interpretable deep neural networks by leveraging adversarial examples. arXiv preprint arXiv:1708.05493, 2017.
- Alhussein Fawzi, Omar Fawzi, and Pascal Frossard. Fundamental limits on adversarial robustness. *ICML 2015*, 2015.
- Alexey Kurakin, Ian Goodfellow, and Samy Bengio. Adversarial examples in the physical world. arXiv preprint arXiv:1607.02533, 2016.
- Yanpei Liu, Xinyun Chen, Chang Liu, and Dawn Song. Delving into transferable adversarial examples and black-box attacks. arXiv preprint arXiv:1611.02770, 2016.
- Aleksander Madry, Aleksandar Makelov, Ludwig Schmidt, Dimitris Tsipras, and Adrian Vladu. Towards deep learning models resistant to adversarial attacks. arXiv preprint arXiv:1706.06083, 2017.

- Nicolas Papernot, Patrick McDaniel, and Ian Goodfellow. Transferability in machine learning: from phenomena to black-box attacks using adversarial samples. arXiv preprint arXiv:1605.07277, 2016a.
- Nicolas Papernot, Patrick McDaniel, Ian Goodfellow, Somesh Jha, Z Berkay Celik, and Ananthram Swami. Practical black-box attacks against deep learning systems using adversarial examples. arXiv preprint arXiv:1602.02697, 2016b.
- Andrew Slavin Ross and Finale Doshi-Velez. Improving the adversarial robustness and interpretability of deep neural networks by regularizing their input gradients. arXiv preprint arXiv:1711.09404, 2017.
- Daniel Smilkov, Nikhil Thorat, Been Kim, Fernanda Viégas, and Martin Wattenberg. Smoothgrad: removing noise by adding noise. arXiv preprint arXiv:1706.03825, 2017.
- Dong Su, Huan Zhang, Hongge Chen, Jinfeng Yi, Pin-Yu Chen, and Yupeng Gao. Is robustness the cost of accuracy?—a comprehensive study on the robustness of 18 deep image classification models. In *Proceedings of the European Conference on Computer Vision (ECCV)*, pages 631–648, 2018.
- Christian Szegedy, Wojciech Zaremba, Ilya Sutskever, Joan Bruna, Dumitru Erhan, Ian Goodfellow, and Rob Fergus. Intriguing properties of neural networks. arXiv preprint arXiv:1312.6199, 2013.
- Florian Tramèr, Nicolas Papernot, Ian Goodfellow, Dan Boneh, and Patrick McDaniel. The space of transferable adversarial examples. arXiv preprint arXiv:1704.03453, 2017.
- Dawn Song Warren He, Bo Li. Decision boundary analysis of adversarial examples. *International Conference on Learning Representations*, 2018. URL https://openreview.net/forum?id=BkpiPMbA-.