Preface to Resurgent Asymptotics, Painlevé Equations and Quantum Field Theory Focus Issue

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December 11, 2023

1 Background

Asymptotic analysis has been a powerful and versatile technique within mathematics and physics for over a century, but recent breakthroughs have led to radical changes in the mathematical understanding of these methods. By exploiting the idea of "resurgence", researchers have been able to extend asymptotic analysis beyond the boundaries of the traditional asymptotic analysis of Poincaré. This is an exciting area where with significant current transfer of technology between mathematical analysts, applied mathematicians and theoretical physics. This new mathematical landscape was the focus of the 2022 Newton Institute Programme "Applicable resurgent asymptotics: towards a universal theory". Much of the work contained in this issue was conceptualised or developed during this programme.

This focus issue collects recent results on resurgent asymptotics, and highlights ongoing advances in the study of Painlevé equations and quantum field theory. The interplay between these two subjects has seen considerable progress in recent years, with major advances such as the discovery of the so-called "Kyiv formula" [6]. This formula relates tau-functions of the Painlevé equations to partition functions supersymmetric quantum field theories, or equivalently, $c = 1$ Liouville conformal blocks. This new connection unleashed a wave of enthusiasm, as it opens up exciting avenues for reaching new insights and making significant discoveries in both fields. This is just one example of the many recent results whose impact spans across both applied mathematics and theoretical physics. The aim of this focus issue is to present other new results from across mathematics and physics that can generate such enthusiasm, and open up their own new avenues of further study.

2 Focus Issue Contents

In "Painlevé I and exact WKB: Stokes phenomenon for two-parameter transseries", van Spaendonk and Vonk combine isomonodromic deformation and transseries to obtain a full description of Stokes phenomenon for the most general two-parameter family solutions to the Painlevé I nonlinear ordinary differential equation [8]. Painlevé equations

play a significant role in many of the other papers included in this collection: Deaño uses a Riemann-Hilbert formulation to compute exponentially small corrections to the tronquée solution of Painlevé I [4], and Clarkson classifies solutions to the degenerate fifth Painlevé equation and studies appplications of these solutions, including deriving solutions to the complex sine-Gordon equation [3].

In addition to the work of Clarkson, other articles presented here study integrable equations. Bonelli, Globlek & Tanzini propose a generalisation of the Kyiv formula to a non-autonomous Toda chain with general simple gauge group [2], and Barnes et al. study similarity reductions two different integrable partial differential equations, the modified Camassa-Holm equation and the Novikov equation, obtaining Painlevé equations through these reductions [1]. They use these reductions to construct travelling wave solutions to the PDE in terms of elliptic functions.

Resurgent asymptotic methods are currently playing an important role in the study of Fuchsian singularities in special functions, in addition to the nonlinear Painlevé equations. This role is showcased in the paper by Lisovyy and Naiduk, who provide a rigorous derivation for a formula expressing coefficients of Heun equation using classical Virasoro conformal blocks, allowing these coefficients to be computed to arbitrary order [7]. Del Monte, Desiraju & Gavrylenko explore the monodromy dependence of isomonodromic tau functions, and provide a detailed discussion of the behaviour of these functions on the torus [5].

These articles provide a broad sample of the exciting work in physics and mathematics currently being performed using new tools made available due to recent developments in the theory of resurgence.

References

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