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Measurement, modelling, and analysis of the dynamic properties of resilient elements used for vibration isolation

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Abstract: Resilient elements are widely applied for vibration and noise control in many areas of engineering. Their complex dynamic stiffness gives fundamental information to describe their dynamic performance and is required for predicting structure-borne sound and vibration using dynamic modelling. Many laboratory measurement methods have been developed to determine the dynamic properties of resilient elements. This paper presents a review of recent developments of the measurement methods from the perspective of force-displacement relations of the resilient element assembly rather than of their material properties. To provide context, the review begins with an introduction to modelling methods for resilient elements, especially for rubber and rubber-like isolators, and three standardized measurement methods are introduced. Recent developments are then discussed including methods to extend the frequency range, which are mainly developments of the indirect method. Mobility methods, modal-based methods, recent active frequency-based substructuring (FBS) and inverse substructuring (IS) methods to study the dynamic properties of resilient elements are also described. Laboratory test rigs and the corresponding identification methods are outlined. Methods to evaluate nonlinear dynamic properties of resilient elements by laboratory measurements are also discussed. Finally, the review is concluded by discussing advantages and limitations of the existing methods and giving suggestions for future research.

Keywords: Resilient elements; Vibration isolation; Measurement methods; Dynamic stiffness; Nonlinearity

1. Introduction

Resilient elements are widely used in vibration isolation systems in many engineering fields. In automotive applications, vehicle suspension systems include coil springs and hydraulic dampers, which are connected to the chassis through rubber bushings [1]. Moreover, engines are attached to the chassis using flexible engine mounts [1],[2]. In railway applications, as well as vehicle suspensions [3], resilient elements are used in the track, for example [4]: rail pads installed between rails and sleepers; under-sleeper pads attached beneath sleepers; and under-ballast mats installed beneath the ballast layer. For aerospace applications, auxiliary dampers and isolators are deployed throughout the structure, providing a significant increase in energy dissipation and reduction of motion [5],[6]. They are also adopted for civil engineering structures such as tall buildings [7]. In each application, resilient elements commonly contain rubber components. Moreover, they may have complex shapes or include further methods to provide additional damping, such as hydroelastic engine mounts [1],[2], hydrobushings [1] and specialised cab mounts such as the Hystec mount [8]. They are useful for controlling vibration and reducing structure-borne noise by isolating vibration sources from receiving structures or from structural components with a significant radiating area.

The stiffness and damping of resilient elements are important parameters required for dynamic modelling of vibration isolation or the dynamic simulation of complex structures, e.g. [9],[10]. This paper mainly focuses on the concept of complex stiffness, which is commonly used to describe the dynamic properties of resilient elements, although an alternative is the use of four-pole parameters [11],[12], the measurement of which is restricted to a single direction. The complex stiffness has been used to describe the frequency-dependent dynamic behaviour of materials and vibration control devices since Kimball and Lovell first suggested the concept of solid damping in 1927 [13]. Many technical articles, as well as some operator manuals for elastomeric test machines, refer to the complex stiffness as dynamic stiffness. Nevertheless, there are different definitions used for dynamic stiffness. Commonly, as used here, the dynamic stiffness is characterised by the modulus and phase (loss angle) of the complex stiffness. However, as pointed out by Lewitzke and Lee [1], SAE

Recommended Practice J1085 defines dynamic stiffness as the real part of the complex stiffness (storage stiffness).

Much effort has been made to study various dynamic characteristics of resilient elements using laboratory experiments and theoretical analysis. Many of these publications, however, deal with detailed analyses of particular situations or highly idealized systems and with measurements intended to demonstrate the importance of the various phenomena or to provide data on practical systems [14]. There are also some reviews in certain areas, such as modelling methods [15]-[19], applications in different engineering industries [1]-[5] and advances in nonlinear passive vibration isolators [20].

Three standardized measurement methods [21]-[24] have been established by ISO 10846 to obtain the dynamic stiffness of resilient elements in laboratory conditions. With the development of testing technology, commercial test machines are increasingly used to assess the dynamic properties of resilient elements. Some test machines, e.g. [25],[26], can give results related to the stiffness and damping of test elements directly. However, interpretation of the results relies on an understanding of the relationships among the various parameters used for stiffness and damping. Modelling resilient elements and accurately obtaining the parameter information are still very challenging tasks. The requirements can be very different according to the purpose of the analysis. For example, the stiffness data from rail fastening systems required for use in ground-borne noise models are different from those to be used in rolling noise studies [27]; the elastomeric mounts in electric vehicles carrying motors experience loads with small amplitudes that can be applied at high frequencies, while those in conventional vehicles with internal combustion engines are exposed to vibrations with large amplitudes in the lower frequency range [28],[29]. Moreover, normally only information in one, or at most three, translational directions can be obtained from such test systems. Resilient elements often work in three-dimensional states and require six degrees of freedom to describe them fully. Not only the axial and shear stiffnesses are important, but also the rotational [30] and cross-coupling [31] stiffnesses are significant contributors to the vibration and structure-borne sound transmission.

The aim of this paper is to give a detailed review of measurement and assessment methods for the dynamic characteristics of resilient elements. To provide the context for the review, it begins by summarising the various measurement quantities in Section 2 and the conventional laboratory measurement methods in Section 3. More recent developments are then introduced, including methods to extend the frequency range of validity of these methods in Section 4, and various alternative approaches using a dynamic substructuring framework in Section 5. Section 6 gives an overview of measurement and evaluation methods for nonlinear dynamic properties. Finally, conclusions are drawn in Section 7. Although the corresponding material properties are very important for the design and modelling of vibration mitigation devices, nevertheless, from the practical viewpoint of structural dynamic simulations, more 'global' force-displacement relations for the whole resilient element are required, rather than internal stress-strain relations. The scope of the paper is therefore focused on these more global dynamic properties rather than the material properties. The review focuses mainly on the dynamic properties of resilient elements from vehicle suspensions, engine mounts and railway tracks, although some relevant examples from civil engineering and aerospace applications are also included.

2. Theory

2.1. Dynamic stiffness of a resilient element

Resilient elements are introduced between two structures to provide a low impedance connection. Representing the interface at each end of the element by a point connection, the resilient element can be represented in terms of six vibrational degrees of freedom (DOF) at each end, as shown in Fig. 1. The displacement vectors (including three linear and three angular displacement components) on the two ends of the resilient element are written as $\mathbf{u}_1 = [u_1\ v_1\ w_1\ \alpha_1\ \beta_1\ \gamma_1]^{\mathrm{T}}$ and $\mathbf{u}_2 = [u_2\ v_2\ w_2\ \alpha_2\ \beta_2\ \gamma_2]^{\mathrm{T}}$, where the direction of w_i is along the axis of the element. In the frequency domain, these can be expressed by complex amplitudes $\tilde{\mathbf{u}}_1$ and $\tilde{\mathbf{u}}_2$, which are functions of circular frequency ω, assuming a harmonic time dependence of $e^{\mathrm{j}\omega t}$.

The forces and moments acting on the two ends of the resilient element in the positive coordinate directions are represented by two vectors of complex amplitudes, $\tilde{\mathbf{f}}_1$ and $\tilde{\mathbf{f}}_2$, respectively, each of which contains three orthogonal forces and three orthogonal moments. The relationship between the forces and displacements can be described by the matrix

$$\begin{Bmatrix} \tilde{\mathbf{f}}_1 \\ \tilde{\mathbf{f}}_2 \end{Bmatrix} = \underbrace{\begin{bmatrix} \tilde{\mathbf{K}}_{1,1} & -\tilde{\mathbf{K}}_{1,2} \\ -\tilde{\mathbf{K}}_{2,1} & \tilde{\mathbf{K}}_{2,2} \end{bmatrix}}_{\tilde{\mathbf{K}}} \begin{Bmatrix} \tilde{\mathbf{u}}_1 \\ \tilde{\mathbf{u}}_2 \end{Bmatrix}, \quad (1)$$

Consequently, $\widetilde{\mathbf{K}}$ is a 12×12 dynamic stiffness matrix, which can be decomposed into four 6×6 submatrices, as shown. $\widetilde{\mathbf{K}}_{1,1}$ and $\widetilde{\mathbf{K}}_{2,2}$ contain the driving-point stiffnesses; $\widetilde{\mathbf{K}}_{1,2}$ and $\widetilde{\mathbf{K}}_{2,1}$ are the transfer stiffness matrices. Reciprocity implies $\widetilde{\mathbf{K}}_{1,1}$ and $\widetilde{\mathbf{K}}_{2,2}$ are symmetric, $\widetilde{\mathbf{K}}_{1,1} = \widetilde{\mathbf{K}}_{1,1}^{\mathrm{T}}$ and $\widetilde{\mathbf{K}}_{2,2} = \widetilde{\mathbf{K}}_{2,2}^{\mathrm{T}}$, with T denoting a transpose, while $\widetilde{\mathbf{K}}_{1,2} = \widetilde{\mathbf{K}}_{2,1}^{\mathrm{T}}$. If the mass of the resilient element can be neglected, which is the case at low frequency, the forces acting at its two ends are equal but opposite in direction, $\mathbf{f}_1 = -\mathbf{f}_2$, and the point and transfer stiffnesses will be equal. The dynamic stiffness matrix can then be simplified as a 6×6 one.

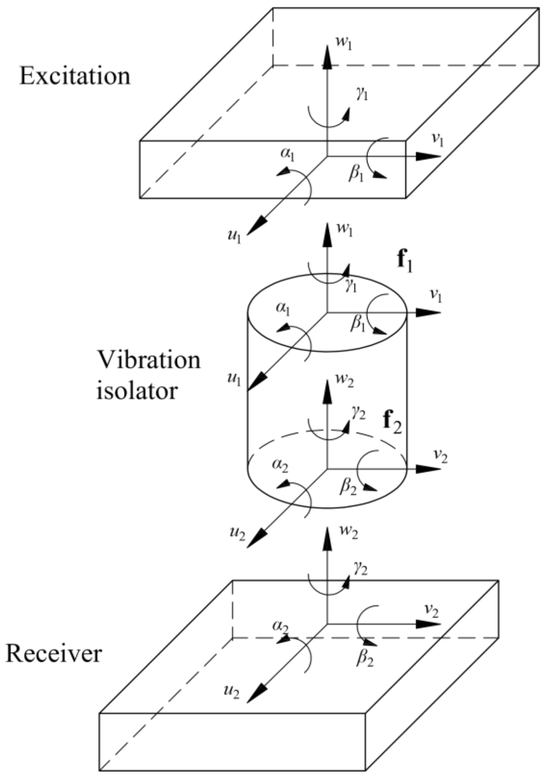

Fig. 1. The coordinate system and displacement components of a resilient element

Considering vibration in a single direction, the complex dynamic transfer stiffness can be written as

$$\tilde{k}_{2,1}(\omega) = \frac{-\tilde{f}_2(\omega)}{\tilde{u}_1(\omega)}\bigg|_{\tilde{u}_2=0} = \left|\tilde{k}_{2,1}(\omega)\right|\exp(\mathrm{j}\delta) = k^{'}(\omega) + \mathrm{j}k^{''}(\omega), \qquad (2)$$

where $k^{'}(\omega)$ is known as the storage stiffness, $k''(\omega)$ is the loss stiffness and δ is the loss angle. It can also be rewritten as

$$\tilde{k}_{2,1}(\omega) = k'(\omega)(1 + \mathrm{j}\eta), \qquad (3)$$

where $\eta = k^{''}(\omega)/k^{'}(\omega) = \tan\delta$ is the loss factor, which in general is also frequency dependent.

The dynamic transfer stiffness is applied more commonly than the point stiffness in engineering practice. The transmissibility, which is a performance index of a vibration isolation system, is related to the dynamic transfer stiffness under the assumption of sufficient impedance mismatch. Moreover, the knowledge of the dynamic transfer stiffness of resilient elements is a key part in the important test-based methodologies to study the transmission of mechanical vibration, known as transfer path analysis (TPA). The first exploration of techniques denoted as classical TPA is often attributed to the work of Verheij [32] who studied the sound transmission of ship machinery via resilient mountings [33]. Nevertheless, there are situations in which the driving-point stiffness is more relevant, such as, for example, for the stiffness of rail supports in calculating railway rolling noise [34].

2.2. Modelling methods

Tests are designed and performed to identify the parameters such as dynamic stiffness and provide justification for the use of a chosen model. A number of thorough reviews have been published covering modelling methods for rubber and rubber-like materials, for example [15]-[19]. Here, modelling methods used for the dynamic stiffness of resilient elements are briefly reviewed to assist understanding of the measurement methods.

2.2.1. Constitutive relations and global behaviour

Normally, two different approaches can be distinguished in the development of models that represent the behaviour of resilient elements [35]: constitutive models of materials, that can be used for example in finite element models, and lumped models that represent the global behaviour of the components. Constitutive models describe the stress-strain relationships of materials, while lumped parameter models describe the force-displacement relationships of resilient elements. As the former can be used as a basis for the latter, the force-displacement relationships of resilient elements are often written in forms that are analogous to the stress-strain relationships of their materials [36]-[38]. Often in the literature, storage and loss moduli of materials are not clearly distinguished from storage and loss stiffnesses of resilient elements, e.g. in [39].

2.2.2. Rubber and rubber-like isolators

Vibration isolators based on rubber and rubber-like materials have many special dynamic features which make the modelling work difficult. A linear approximation is made in the small amplitude region in engineering practice, which is sufficiently accurate for high frequency analyses [40]. The classical Kelvin-Voigt, Maxwell and Zener models, and the corresponding fractional derivative models, are often used to describe frequency-dependent behaviour. The dynamic stiffness can conveniently be calculated by taking Fourier transforms, e.g. in [41],[42]. Nevertheless, it is known that rubber and rubber-like materials are nonlinear and exhibit different types of behaviour, including hyperelasticity, viscoelasticity and hysteresis. Especially filled vulcanized rubbers have highly nonlinear viscoelastic behaviour [19],[43]-[45]. There are two well-known effects occurring in filled rubber materials, the Mullins effect [45] and the Payne (or Fletcher-Gent) effect [19]. Test objects are often mechanically conditioned prior to measurements to minimise the influence of the Mullins effect. The Payne effect is more pronounced with higher filler content. Other nonlinear viscoelastic phenomena are present, including hysteresis, and dependence on pre-strain [46] and strain rate [45]. Consequently, the dynamic stiffness of rubber elements depends on preload, amplitude of excitation and strain history as well as frequency and temperature.

2.2.3. Modelling including wave motion

The internal resonances (also called wave effects) at high frequency, introduced by the mass of a resilient element cause further frequency dependence in the dynamic stiffness. Lumped-parameter models can be used to approximate the effect of the internal resonance of resilient elements, such as in [29],[47]. However, more accurate results are obtained using continuum models.

A resilient element can often be represented as a finite cylinder. For example, rubber elements can be treated as a finite rod in the axial direction [48]-[50], or a finite beam [51]-[54] in the transverse direction. Each of these models was limited to one or a few wave types to determine the dynamic stiffness in certain DOFs. Fredette and Singh [55] developed a coupled, 6-DOF dynamic stiffness matrix of a cylindrical rubber isolator, resembling a short beam, assuming longitudinal and torsional waves in a rod alongside the Timoshenko beam theory for shear and flexure. Moreover, in [56]-[61], waveguide models were presented for the dynamic stiffness of cylindrical isolators in the audible frequency range. Finite element [62] and boundary element [63] methods are also used to study the vibration of finite cylinders, which allows wave effects to be included. These models assume linearity. The damping is introduced by expressing the elastic, bulk and shear moduli as complex using a loss factor, similar to Eq. (3).

Lee and Thompson [64] modelled a helical spring from an automotive suspension using an analytical approach and found that the dynamic stiffness increases sharply at frequencies as low as about 40 Hz due to internal resonances or wave effects.

3. Three standardized measurement methods

To identify appropriate values of stiffness and damping of resilient elements, suitably designed measurements are required, which may be based on either field or laboratory experiments. Laboratory experiments allow more controlled measurements and are the focus of the discussion in this section. A series of international standards [21]-[24] have been formulated to make recommendations for laboratory measurements of vibro-acoustic transfer properties of resilient elements. These are divided into direct, indirect and driving-point methods and will be described below. These methods can be categorized as non-resonant approaches, since they operate over a broad range of frequencies and are not limited to resonance frequencies of the configuration.

Preload dependence can be measured by applying gravitational loading or through a hydraulic actuator or an electro-magnetic shaker. The actuator or shaker may also be used to provide the dynamic excitation, or this may be provided separately. The most common examples are harmonic sines/cosines [47],[65]-[67] and linear, logarithmic or octave sine sweep [32],[68]-[71] excitations. In [36] and [48], pseudo-random excitation has been used with a shaker. Moreover, excitation by an impact hammer is also used [48],[72]-[75]. Both vertical and horizontal setups can be used [7],[23],[76], with the test element mounted in a way which is representative of its use in practice.

The dynamic stiffness parameters can be obtained based on Eq. (1) by imposing certain boundary conditions. For uni-directional vibration, a two-terminal dynamic stiffness matrix can be obtained as [32]

$$\tilde{k}_{1,1}(\omega) = \left.\frac{\tilde{f}_1}{\tilde{u}_1}\right|_{u_2=0}, \tilde{k}_{1,2}(\omega) = -\left.\frac{\tilde{f}_1}{\tilde{u}_2}\right|_{u_1=0},$$
$$\tilde{k}_{2,1}(\omega) = -\left.\frac{\tilde{f}_2}{\tilde{u}_1}\right|_{u_2=0}, \tilde{k}_{2,2}(\omega) = \left.\frac{\tilde{f}_2}{\tilde{u}_2}\right|_{u_1=0}, \qquad (4a\text{-}d)$$

in which the displacement at each terminal in turn is constrained. If the isolator is connected to a receiver with relatively large dynamic stiffnesses compared with the isolator, the forces at the receiver approximate the blocked forces:

$$\tilde{\mathbf{f}}_2 \approx -\tilde{\mathbf{K}}_{21}\tilde{\mathbf{u}}_1. \qquad (5)$$

This can facilitate the measurement of multi-directional properties.

3.1. Direct method

The direct method described in ISO 10846-2 [22] is a uniaxial cross-point method (i.e. for the transfer stiffness) and is commonly used. A dynamic displacement is applied at one terminal and the resulting blocked force at the other terminal is measured using load cells. The method is often used at low frequency, typically from 1 Hz up to between 300 Hz and 500 Hz [22]. Many commercial test machines are based on the principle of this method, e.g. [25]. There are many studies for automotive applications, normally using commercial test machines based on the standardized direct method [47],[65]-[68],[77].

In the example shown in Fig. 2, the test element is attached to a distribution plate (mass) on the input side, through which a static preload is applied, and to a rigid termination on the output side. A harmonic excitation F_1 is applied on the input side and the displacement u_1 on the input side is measured as well as the blocked force F_2 on the output side, where it is assumed that the displacement u_2 is zero. The dynamic transfer stiffness is computed by Eq. (4c).

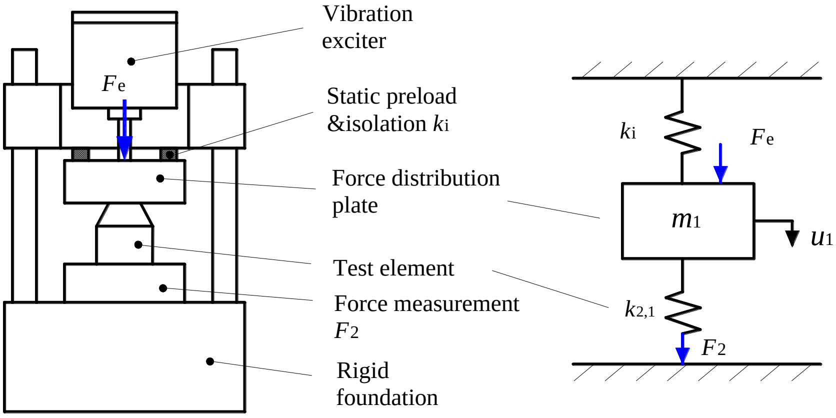

Fig. 2. Model of the direct method

3.2. Indirect method

To measure at higher frequency and in more than one direction, the indirect method described in ISO 10846-3 [23] has been developed. As shown in Fig. 3, it consists of two blocks between which the resilient element is mounted. This whole arrangement is itself mounted on very soft auxiliary rubber springs under the lower block (and above the upper block). The transmitted force F_2 is derived indirectly by measuring the acceleration of the lower block on the output side of the resilient element and combining it with the mass of the lower block, m_2, assumed to behave as a rigid body.

That is, only input and output accelerations are measured. The vertical dynamic transfer stiffness can be calculated by

$$\tilde{k}_{2,1}(\omega) \approx \frac{\tilde{F}_2}{\tilde{u}_1} = -m_2\omega^2\frac{\tilde{u}_2}{\tilde{u}_1} \quad \text{for } \omega > 3\omega'_2. \tag{6}$$

where ω'_2 represents the resonance frequency of the lower block with the upper block held rigidly [32],[69].

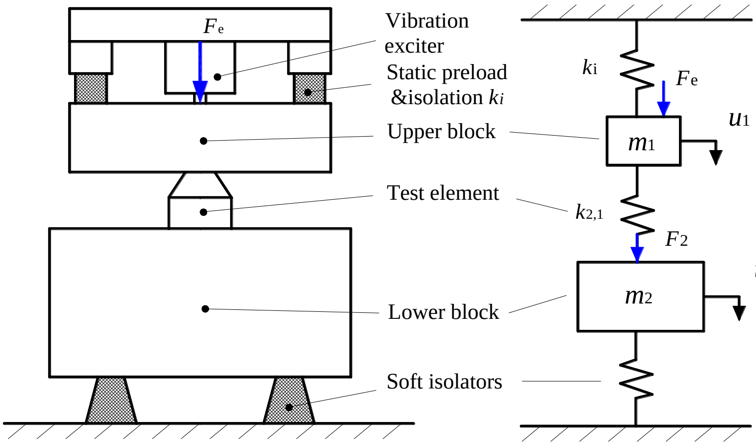

Fig. 3. Model of the indirect method

The indirect method was first developed by Verheij [32]. Through appropriate choice of the two block masses and the auxiliary mountings, the measurements can be carried out over a large frequency range, say 30-2000 Hz. The method has been used for example by Liu et al. [29], who used a commercial test machine [26], based on the standardized indirect method, to measure the translational-direction dynamic stiffness of the rubber isolators for electric vehicles in the frequency range from 50-1400 Hz.

For measuring multi-directional properties of a resilient element, the dynamic transfer stiffness components are defined by [32]

$$\tilde{k}_{j,i}(\omega) = \left.\frac{\tilde{F}_{2,j}}{\tilde{u}_{1,i}}\right|_{\substack{\text{other 11} \\ \text{displacements zero}}}. \tag{7}$$

From Eq. (1), $\tilde{k}_{j,i}$ is a component of the matrix $\tilde{\mathbf{K}}_{21}$ and the 12 displacements are the components of $(\tilde{\mathbf{u}}_1 \quad \tilde{\mathbf{u}}_2)^{\mathrm{T}}$. The requirement for velocities to be zero is sufficiently met if $|\tilde{k}_{j,i}\tilde{u}_{1,i}|$ is much larger than the modulus of the sum of all other terms. Accordingly, Verheij [32] proposed a refinement to allow rotational and lateral components to be separated reliably, as rotational stiffnesses or mobilities cannot usually be identified directly unless they are measured in combination with a lateral motion

[78]. Kari [56],[71] showed that an indirect measurement setup could be established, with the test object mounted between a block and the moving table of an electro-dynamic vibration generator. This enabled measurement of the axial dynamic transfer stiffness under the condition $|u_b/u_{\mathrm{mt}}| \ll 1$, where u_{b} and u_{mt} were the displacements of the block and the moving table.

3.3. Driving-point method

The driving-point method described in ISO 10846-5 [24] can be implemented on the same test rig as the direct method. However, only the force and displacement on the input side are required. An example application is shown in Fig. 4. The dynamic driving-point stiffness can be obtained by Eq. (4a). Li et al. [79] performed laboratory tests on a full-size 750 mm long sample of an embedded rail system and used the driving-point method to determine the vertical dynamic stiffness at frequencies up to 20 Hz.

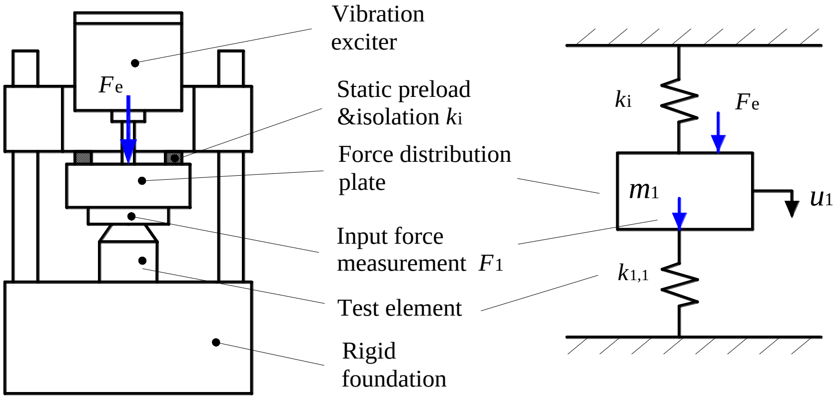

Fig. 4. Model of the driving-point method

At low frequencies (typically f<200 Hz [24]), the dynamic transfer stiffness is approximately equal to the dynamic point stiffness,

$$\tilde{k}_{2,1}(\omega) \approx \tilde{k}_{1,1}(\omega), \tag{8}$$

because the inertial forces are negligible compared with the elastic and damping forces. For a two-stage resilient rail fastener, Herron [72] alternatively measured the driving-point stiffness of the component parts and used them to determine the transfer stiffness of the complete assembly by developing simple models for the response of the assembly.

4. Extended frequency measurements

Although the methods described in Section 3 have been standardized, care is needed because of practical experimental difficulties or complications due to the presence of system resonances within the specified measurement range. To allow more accurate estimates of the dynamic transfer stiffness of highly resilient rail fastenings over wider frequency ranges, Morison et al. [80] suggested a method for correcting for the inertial forces due to the rail in an improved driving-point method. Kari [70] extended the indirect method to increase the accuracy by adopting a series of techniques, including an improved excitation and termination arrangement, source correlation, stepped sine excitation and effective mass calibration. At frequencies approaching and above the natural resonance of the system the contribution of inertial forces becomes significant for the method. Moreover, much effort has been made by various authors to improve the indirect method to obtain a wider frequency range.

4.1. Higher frequency

4.1.1. Need for high frequency measurements

The frequency range of interest may extend to much higher frequencies in many applications related to vibration isolation or acoustic comfort. In certain applications, for example, rail fasteners [75] and rubber engine mounts [81] may be required to operate at frequencies as high as 5000 Hz; the excitation frequency generated by the powertrain of the electric vehicles can exceed 2000 Hz [29] and may be up to 3000 Hz [28]. In fact, "high frequency" is a relative concept depending on the size, stiffness and mass of the element under test. For many elastomeric components, at frequencies above 100 Hz both transfer and driving-point dynamic stiffness results differ significantly from the low frequency ones [47] and from each other.

4.1.2. Allowing for lower block dynamics

To achieve measurements over a wider frequency range by the indirect method, at least two sets of measuring blocks are required. Large blocks are needed to achieve results at low frequency, and consequently they no longer behave as rigid bodies at higher frequencies. Li et al. [36] investigated the preload- and frequency-dependence of the vertical dynamic stiffness behaviour of a rail fastener by improving the indirect method of [69]. Two modification factors were defined which were derived from a finite element model of the test rig to allow for the local vibration of the blocks in the high frequency range and extend the upper frequency limit. In this case the upper frequency was extended from 500 to 1000 Hz.

Based on the standard indirect method, Gejguš et al. [28] designed a high-frequency test bench to perform the dynamic stiffness measurements for an elastomeric component used for an electric motor over the frequency range of 50-3000 Hz. A large shaker was used and attached to a 500 kg stone table through rubber air springs. The upper block (seismic mass) was suspended ‘freely’ above the exciter using elastic support cords to ensure that its rigid body resonances were outside the measurement range.

4.1.3. Using a lower adjustment frame

To overcome resonances of the test rig in the frequency range of 1000-5000 Hz, Vahdati and Saunders [81] described a uniaxial high frequency test machine that consisted of an indirect-method test frame with a lower test frame capable of vertical adjustment, as shown in Fig. 5. The large output-side block, suspended on four soft auxiliary mounts, was placed between the test element and the adjustment frame. The output-side mass is isolated from the high frequency resonances of the adjustment frame. It allowed study of the performance of resilient elements at frequencies up to 5000 Hz through appropriately designing resonance frequencies of the output-side mass and the test fixture.

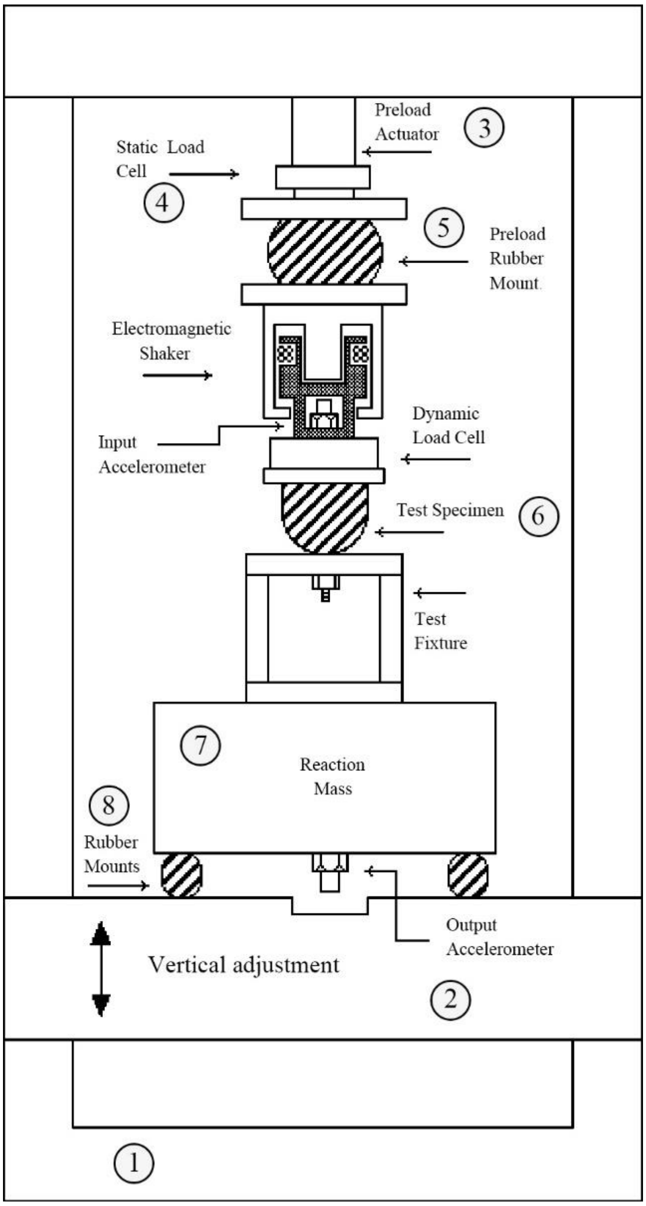

Fig. 5. High frequency test machine schematic from Vahdati and Saunders [81]: 1- a conventional test frame; other labels as shown in the figure. (Reproduced from N. Vahdati, L.K.L. Saunders, High frequency testing of rubber mounts, ISA Transactions 41 (2002) 145-154, https://doi.org/10.1016/S0019-0578(07)60074-3, used with permission from Elsevier).

4.2. Lower frequency

4.2.1. Using standardized measurement methods

Unlike the direct and driving-point methods, the indirect method has a lower frequency limit, see Eq. (6). Some refinements to the indirect methods have been reported through improving measured variables or measurement setups. Thompson et al. [69] refined the indirect method to allow it to be extended to lower frequencies, in this case from 100 Hz down to about 40-50 Hz. This was achieved by measuring the dynamic compression of the resilient element ($\tilde{u}_1 - \tilde{u}_2$) and the response of the lower block $\tilde{u}_2$ instead of the responses of the two blocks $\tilde{u}_1$ and $\tilde{u}_2$. This improved method was then used in [48],[72],[82],[83]. As well as the high frequency corrections described in Section 4.1.2, Li et al. [36] further improved the method [69] at low frequencies by including an estimate of the support stiffness in the formulation. Gao et al. [84] combined the direct and indirect methods to measure the vertical and transverse dynamic stiffness and loss factor of a rail fastener in the range 5-1250 Hz. Later, they used this comprehensive test system to obtain the vertical dynamic stiffness of a rail fastener under temperature- and frequency- dependent conditions in the frequency range 10-1250 Hz [85].

4.2.2. Using a floating mass

Dickens [49] and Dickens and Norwood [86] used a floating mass to reduce the lower frequency limit of the indirect measurement, as shown in Fig. 6. It is capable of measuring the properties of vibration isolators over the frequency range 5-2000 Hz, with static loads over the range 1-30 kN.

5. Methods based on dynamic substructuring framework

Although, in many practical situations, the number of terms of the dynamic stiffness matrix that should be considered can be reduced because of factors such as isolator shape symmetries, or simplifications that are made depending on the purpose of the investigation [31],[87], in general all six DOF may be required to characterise the vibration transmitted into the receiver structure through a resilient element. Rotational stiffnesses will become increasingly important with increasing frequency in certain practical structures [30],[88]. However, of the measurement methods described in the previous two sections, only the indirect method can be used for multi-dimensional dynamic measurements [23],[69]. A method for reliably separating lateral and rotational stiffnesses should be used. Normally, several translational accelerometers can be placed close to each other to approximate the rotational vibration. Moreover, the use of rotational accelerometers to measure the rotations directly has recently attracted attention in structural dynamics [89]-[91]. However, these sensors are usually heavy and may only be suitable for bulky structures [92],[93].

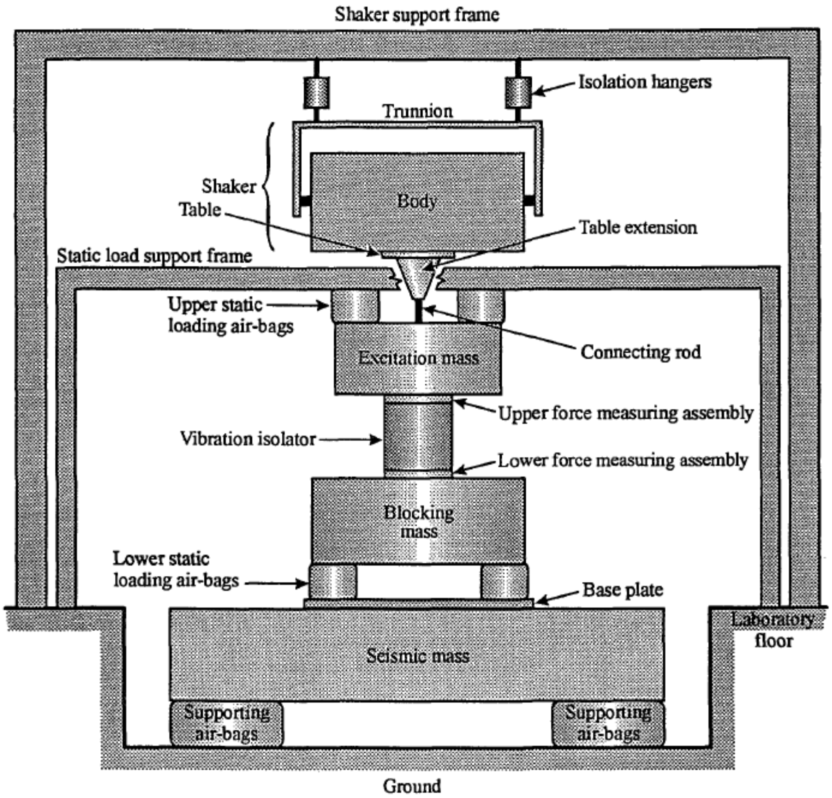

Fig. 6. Schematic diagram of vibration isolator test rig from Dickens [49] (Reproduced from J.D. Dickens, Dynamic characterisation of vibration isolators, University of New South Wales, Australia, (Ph.D. thesis), 1998, https://doi.org/10.26190/unsworks/18020, under Creative Commons Licence CC BY-NC-ND 3.0)

To solve the issue of characterizing multi-dimensional stiffnesses of resilient elements, mobility methods, modal-based methods, frequency-based substructuring (FBS) and inverse substructuring (IS) methods, have been developed. The dynamic model can be based on data measured on an experimental setup, e.g. [107], or on a combination of both experimental data and simulation data, e.g. [118]. According to Klerk et al. [94], these methods all belong to the dynamic substructuring framework. Resilient elements can be treated as a form of joint connecting the substructures in a dynamic system.

5.1. Mobility method

The mobility and impedance methods [95], are commonly used to study the dynamic behaviour of resilient elements, especially for multi-dimensional properties. These are sometimes known as linear multi-terminal network methods. The assemblies containing vibration isolators are suspended in approximately free-free conditions and are usually excited by hammer impacts to excite all directions at the connection to the resilient elements. Responses are normally measured with accelerometers. A free-free arrangement does not allow any preload to be applied, but does make it possible readily to examine the dynamics in all directions and check for coupling [96].

As shown in Fig. 7, Kim and Singh [51],[97] used a suspended experimental system with the isolator located between two known inertial elements, similar to the indirect measurement setup. The mobilities of the assembled system were measured and the frequency-dependent multi-dimensional transfer dynamic stiffnesses of the isolator were extracted using a mobility synthesis method [98], assuming rigid body properties of the inertial elements.

To consider the effect of the preload, Huang et al. [99] improved Kim and Singh's measurement set-up to obtain the axial, transverse and bending mobilities of a rubber isolator. The loading part was composed of several air springs, which supported the system with a very low natural frequency simulating the free-free condition and provided preloads for the isolator by varying pressure in the air bags. The whole system was hung by elastic cords.

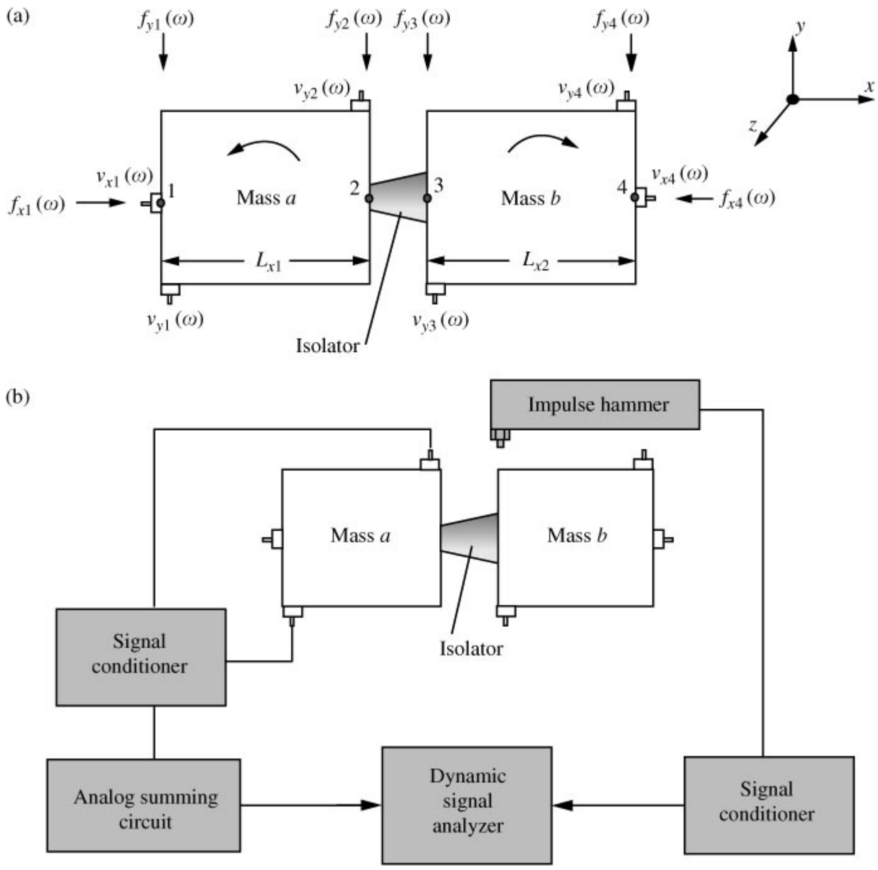

Fig. 7. Experimental study used to identify mobilities of an isolator: (a) simplified model; (b) experimental schematic. [51] (Reproduced from S. Kim, R. Singh, Multi-dimensional characterization of vibration isolators over a wide range of frequencies, Journal of Sound and Vibration 245(5) (2001) 877-913, https://doi.org/10.1006/jsvi.2001.3617, used with permission from Elsevier)

5.2. Modal-based method

Forrest [100] discussed the measured free-free dynamics of three small-scale vibration isolator models. The modal properties were extracted from FRFs measured in the three principal directions and accordingly the isolator parameters were calculated in the frequency range up to 250 Hz. Only translational motion in the three principal directions was considered and the assumption was made that coupling between them was negligible.

In reality, more than one resilient element is normally used to connect the source and receiver structures in a vibration isolation system, and these may also be inclined at different angles. Experiments on multi-isolator mounted systems are designed to extract the stiffness and damping properties of the vibration isolators, e.g. in [101]-[106]. Identification methods that utilize computational and experimental modal analysis, including direct and inverse methods, are integrated into these applications. A common feature of these studies is that the resilient element is treated as a joint at a certain point, defined in the driving point [101]-[105] or the elastic centre [106], so that only a point dynamic stiffness matrix is applied to describe the dynamic properties.

5.3. Frequency-based substructuring (FBS) method

Compared with the modal-based methods, FBS methods provide a great advantage due to the fact that the directly measured FRFs are utilized without any modal parameter estimation [92]. The virtual point transformation method [107] is proposed to transform experimental FRF measurements into a model that is compatible for coupling with other models, which describes every connection point of a component by three both translations and rotations.

Haeussler et al. [108] recently investigated the performance of FBS to identify the 12-DOF dynamic properties of isolator elements on a freely suspended mass-isolator-mass assembly. They designed two cross-shaped structures for use as the rigid bodies. The dynamic stiffness of a freely suspended, rigid cross could be given by

$$\mathbf{Z} = -\omega^2 \mathbf{M} \tag{9}$$

where $\mathbf{M}$ is the mass matrix of the cross-plus-sensors, for the six rigid body DOF around a virtual point, which does not need to correspond to the centre of mass. The FBS method is theoretically equivalent to the mobility method of Kim and Singh [51]. Using the symbols in Fig. 7, the dynamic stiffness matrix of the vibration isolator is calculated by

$$\begin{bmatrix} \tilde{\mathbf{Z}}_{22}^{\mathrm{I}} & \tilde{\mathbf{Z}}_{23}^{\mathrm{I}} \\ \tilde{\mathbf{Z}}_{32}^{\mathrm{I}} & \tilde{\mathbf{Z}}_{33}^{\mathrm{I}} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{Z}}_{22}^{\mathrm{a}} + \tilde{\mathbf{Z}}_{22}^{\mathrm{I}} & \tilde{\mathbf{Z}}_{23}^{\mathrm{I}} \\ \tilde{\mathbf{Z}}_{32}^{\mathrm{I}} & \tilde{\mathbf{Z}}_{33}^{\mathrm{b}} + \tilde{\mathbf{Z}}_{33}^{\mathrm{I}} \end{bmatrix} - \begin{bmatrix} \tilde{\mathbf{Z}}_{22}^{\mathrm{a}} & 0 \\ 0 & \tilde{\mathbf{Z}}_{33}^{\mathrm{b}} \end{bmatrix} \tag{10}$$

where $\tilde{\mathbf{Z}}_{22}^{\mathrm{a}}$ and $\tilde{\mathbf{Z}}_{33}^{\mathrm{b}}$ are the coupling interface impedance matrices of the masses a and b. In [108], $\tilde{\mathbf{Z}}_{22}^{\mathrm{a}}$ and $\tilde{\mathbf{Z}}_{33}^{\mathrm{b}}$ were determined from Eq. (9). Compared with the IS method, described below, the results suggested that it was possible to identify valid stiffness magnitudes up to the kilo-Hertz frequency range with both methods. Nevertheless, it was found that the multi-dimensional dynamic stiffness properties of the rubber isolator obtained with the FBS decoupling could provide better results when used in an assembly. Later, they used the FBS experimental method to obtain the 12-DOF rubber isolator models and performed parametric NVH design optimization of complex industry-relevant assemblies by the combination of FBS and blocked force TPA [109]. A similar problem was discussed in the design of a particle damper using the FBS method [110].

5.4. Inverse substructuring (IS) method (or in-situ method)

The IS method assumes the isolator or joint has negligible mass. This allows for a way to decouple the joint dynamic stiffness from that of the total system without knowing the dynamics of the connected substructures, i.e., only the connection dynamics of substructures are required [92]. As opposed to the FBS method, a specific stiffness matrix topology of the rubber isolator is assumed, $\tilde{\mathbf{Z}}_{22}^{\mathrm{I}} \approx -\tilde{\mathbf{Z}}_{23}^{\mathrm{I}} = -\tilde{\mathbf{Z}}_{32}^{\mathrm{I}} \approx \tilde{\mathbf{Z}}_{33}^{\mathrm{I}}$ in Eq. (10), as in the in-situ substructure decoupling method [111],[112].

Although there are some in-situ decoupling methods that have been investigated for mechanical systems with resilient links, the links often only allow translational vibrations [113][114]. Besides Haeussler's paper [108], the in-situ method has been developed to determine translational and rotational dynamic transfer stiffness matrices of resilient elements by measuring FRFs of a source and receiver subsystem over a wider frequency range (up to 1000 Hz) [111],[115],[116]. The axial dynamic transfer stiffnesses of resilient elements are obtained through measuring the contact interface mobility matrices of the laboratory test and the rotational components are obtained via the application of a finite difference approximation [117] to separate the translational, rotational and cross mobility terms. When the isolator is placed between the two flexible substructures, an over-determined remote mobility measurement returns an interface mobility with minimal error. This is attributed to the fact that no spatial averages are required and that this consequently reduces the error in the matrix inversion.

Recently, Meggitt and Moorhouse [118] proposed an in-situ updating strategy through a numerical example, considering FE beam structures, in the presence of an arbitrary or unknown boundary condition. Taking a resilient isolator as an example, they managed to extend the measurement frequency range of the transfer stiffnesses up to 3 kHz with the aid of the updated isolator FE model. Besides, the proposed in-situ updating has the potential to provide a convenient means of determining the point stiffness of a vibration isolator from in-situ measurements alone. The in-situ methods proposed in [111],[115],[116],[118] provide an alternative to freely suspending the substructures and can consider the effect of the preload. Similar to the free-free measurement, the isolator assembly is also excited by an impact force hammer and responses are measured with accelerometers. This method is suitable for measurements either on a working installation or on a test bench.

The IS method is the easiest FBS method to carry out and the least time-consuming one in terms of measurements. It has been introduced in modular vehicle designs [119] and the measurement setup proposed by Haeussler [108] has been used to characterized the rubber mounts in NVH design of electric vehicles [120],[121].

6. Measurements for nonlinear dynamic properties

Nonlinear dynamic properties of resilient elements are of interest, not only because they may have important effects on the performance in vibration isolation and structure-borne noise transmission, for both passive and active control (e.g. [122]), but also because the nonlinear properties can often be utilised when designing isolators [20],[123]. Besides simple rubber isolators, and coil and leaf springs, there is interest in the design of more complex isolators. These include rubber coil springs [43], hydraulic engine mounts [1],[2], laminated rubber-metal springs [124], silicone oil-filled rubber isolators for earthmoving machinery cab isolation [125], vibration isolation mounts with wide quasi-zero-stiffness range [123],[126] and other novel vibration isolators, e.g. [127]. These isolators are liable to present more advantageous nonlinear behaviour.

For nonlinear dynamic systems, the superposition principle is no longer applicable and such dynamic systems are sensitive to the characteristics of the excitation, including its level and type. Most analytical techniques available are limited to the steady-state response of nonlinear vibration isolation systems [128]. Nevertheless, under transient conditions, resilient elements may present different nonlinear properties. Most nonlinear studies of resilient elements are still limited to unidirectional dynamic properties. Parameter identification procedures sometimes rely on the selected modelling methods.

6.1. Dependence on static preload and amplitude from standardized measurement

The nonlinear dynamic properties are mainly apparent under low frequency, high amplitude conditions. The standardized methods [21]-[24] are used to determine frequency-dependent properties and, for measurement at high frequencies, they are usually carried out at small amplitudes. They are limited to characterizing linear or linearised behaviour of resilient elements. These measurements can be applied at different amplitudes and preloads, by assuming approximately linear behaviour. In this regard, it is important that the test method replicates the expected working conditions of the resilient element.

Changes in dynamic stiffness due to static preload are mainly caused by significant nonlinear changes of isolator geometry, e.g. a stiffening due to increased cross-sectional area of the preloaded rubber isolator [108]. For example, the preload dependence of rubber elements used in railway tracks [36],[69],[75],[79],[129] and vehicles [70],[71],[130] have been examined, assuming the amplitude dependence could be neglected.

The dynamic characteristics of hydraulic engine mounts [66],[67],[77] and hydraulic bushings [131] are highly frequency- and amplitude- dependent. These nonlinear characteristics have been studied by using the direct measurement method at given excitation frequencies and different displacement amplitudes.

6.2. Non-standardized measurements to determine nonlinear properties

Apart from small amplitude frequency domain measurements, it may not be assumed in general that nonlinear systems can be adequately and sufficiently characterized by means of conventional dynamic stiffness and phase angle measurements. To assess intrinsic nonlinear behaviour of resilient elements, the force-displacement hysteresis loops and frequency content can be studied. Additionally, transmissibility, normally obtained by resonant vibration tests, is often used under different types of excitation. This allows nonlinear characteristics, especially amplitude-dependent behaviour, to be captured more easily. Some examples can be found for rubber isolators [43],[132]-[135], resilient elements composed of rubber elements and other damping materials [125],[136] and a quasi-zero-stiffness mount [126]. Nevertheless, transmissibility describes the performance of a vibration isolation system, not the resilient element itself.

6.2.1. Force-displacement hysteresis loops and overtones

An indication of the nonlinearity present in a resilient element can be obtained from the degree of distortion that occurs in the force-displacement hysteresis loops for sinusoidal excitation [43]. For evaluating the behaviour of nonlinear resilient elements, the complex stiffness magnitude may be computed using a force-displacement hysteresis loop as [137]-[140]

$$\left|\tilde{k}(\omega)\right| = \frac{F_\mathrm{a}}{U_\mathrm{a}} \text{ or } \frac{F_\mathrm{max}}{U_\mathrm{max}}, \tag{11}$$

where F_a is the peak-peak force amplitude and U_a is the peak-peak displacement amplitude; alternatively, F_max and U_max are the maximum values during a cycle. The damping may be obtained from the energy loss, given by the area enclosed by the ellipse. For example, Campolina et. al. [7] obtained the loss factor of an elastomeric isolator using a hysteresis test at 2 Hz. The loss factor was given by:

$$\eta = \frac{\Delta E}{2\pi E_\mathrm{max}} \tag{12}$$

where $\Delta E = \int U(t)F(t)dt$ is the energy dissipated per cycle and E_max is the maximum potential energy of the system during a cycle, given by $E_\mathrm{max} = \frac{k_0 U_\mathrm{max}^2}{2}$, where k_0 is a stiffness calculated from the average slope of the hysteresis curve, similar to Eq. (11). Alternatively, the dynamic stiffness for an angular frequency ω_0 is defined as [141],[142]

$$\tilde{k}(\omega) = \left.\frac{\tilde{F}(\omega)}{\tilde{u}(\omega)}\right|_{\omega=\omega_0} = \frac{F_\mathrm{max}}{U_\mathrm{max}} \exp(\mathrm{j}\delta). \tag{13}$$

However, the responses of a nonlinear dynamic system to a harmonic excitation will include overtones at multiples of the excitation frequency. If the stiffness magnitude is defined as in Eq. (11), a measurement with a harmonic displacement excitation could give an underestimation of the stiffness if only the force amplitude at the fundamental frequency is used, e.g. by Fourier analysis; on the other hand the damping evaluation will not introduce systematic errors due to force overtones [44].

As well as conventional sinusoidal dynamic testing, non-sinusoidal excitations [44],[143], such as triangular- and square-wave, dual-sine and random inputs, may be used to capture nonlinear properties of resilient elements, where these correspond to practical service conditions. Hysteresis loops can be measured and dynamic stiffness and phase angles are calculated using Fourier analysis to obtain the frequency content of the force and deflection.

6.2.2. Transient measurements

Elastomeric materials behave entirely differently under transient and harmonic loadings [144], so it is important to understand their behaviour under typical transient operational conditions, e.g. travel on bumpy roads, abrupt accelerations or decelerations, braking, and cornering. Meram [145] carried out a series of low velocity drop tests to characterize the dynamic behaviour of an elastomer buffer. The impact force time histories for the samples were measured for varying initial impact velocities. Curves of impact force versus deflection were obtained. Adiguna et al. [146] studied nonlinear transient responses of a typical hydraulic engine mount to design and diagnose the transient characteristics. Bench experiments were constructed using a commercial test system, and step up, step down and triangular waveforms were applied. Nonlinear compliances and resistances were identified, with parameters estimated from measured dynamic stiffness data under 50 Hz. Furthermore, He and Singh [147] identified discontinuous compliance nonlinearities of hydraulic engine mounts.

7. Discussion and conclusion

Advances have been presented in laboratory measurement methods for examining the dynamic properties of resilient elements used for vibration isolation. The complex dynamic stiffness is one of the main parameters used to describe resilient elements. It is inevitable that laboratory measurements are required, as practical difficulties may be expected in field measurements. Of the three standardized measurement methods recommended by ISO 10846, the direct and driving-point methods are normally used in one translational direction at low frequencies, from several Hertz to hundreds of Hertz, whereas the indirect method can be used for translational and rotational vibration in multiple directions over a broader frequency range, from tens of Hertz up to 2000 Hz or more. Compared with the direct method, the standardized indirect measurement method avoids the need for direct measurement of the force by estimating it from the acceleration of the block on the output side of the resilient element. However, all these methods require cumbersome test rigs. For measurements over an extended frequency range, the literature is mainly concerned with the refinement of the indirect methods from the point of view of identification methods or test rigs.

Multi-dimensional dynamic properties of resilient elements are difficult to obtain. Approaches based on a dynamic substructuring framework, including mobility methods, modal-based methods, FBS and IS (or in-situ) methods, have been used to measure the translational and rotational dynamic properties of resilient elements. These methods primarily use an approximately free suspension assembly. A modal-based method through designed experiments on multi-isolator mounted systems is mainly limited to certain point dynamic stiffnesses [101]-[106]. For the frequency-dependent properties of resilient elements and where more damping is introduced to the structures, the FBS method will be superior to the modal-based method. An in-situ measurement assembly may be constrained, so the effect of the preload can be considered, but only dynamic transfer stiffnesses are measured [111],[115],[116]. Although cumbersome test rigs may not be needed in these approaches, more complicated identification methods are required. The frequency range of these methods is normally limited from several Hertz to 1-2 kHz, while some in-situ updating approaches can extend the measurement of the transfer stiffnesses up to 3 kHz [118]. Because of the simplicity and efficiency of the IS method, it has been introduced in modular vehicle designs using the measurement setup proposed by Haeussler [108].

Many resilient elements have intrinsic nonlinear dynamic properties, which are mainly apparent under low frequency, high amplitude conditions. Nevertheless, the standardized measurement methods can be used to evaluate the dependence on static preload and excitation amplitude. Various non-standardized measurements to determine nonlinear dynamics have also been discussed.

Overall, although the standardized methods are commonly used to examine the dynamic stiffness of resilient elements, including both dynamic transfer and driving-point stiffnesses in a wide frequency range, often some improvements and corrections are required, for example to allow for the nonlinear properties of resilient elements at low frequencies as well as for measurements at high frequencies. Further work is needed to generalize these developments.

Further research is needed into hybrid methods that combine experimental measurement methods and numerical simulation methods, such as finite element models, to help make corrections and improvement to extend the frequency range, e.g. in [36],[118]. By means of finite element models, it should be possible to use more convenient test rigs without some of the limitations of more cumbersome ones. Such hybrid approaches should then be combined with suitable parameter identification and optimisation methods.

As well as developments involving free-free arrangements of the experimental assembly, the modelling of constraints is an important factor that influences the results. For example, Zucchini et al. [148] recently compared two different boundary conditions in the context of dynamic substructuring for extracting the dynamic stiffness of rubber bushings belonging to the rear drivetrain of an electric car. One was a free-free measurement procedure and the other used fixed boundary conditions created by clamping the rubber mounts to the ground. Besides, although substructuring approaches have gained more attention in the multi-dimensional dynamic parameter identification of resilient elements in the last few years, there are still challenges for the examination of multi-dimensional dynamic behaviour of resilient elements, including both measurement and modelling. A lot of attention should still be paid to the measurement and modelling of rotational stiffness components and separation methods of the translational, rotational and cross-coupling terms. In addition, due to length constraints in this article, the discussion of evaluation methods of nonlinear dynamic properties of resilient elements has been limited to fundamental but effective approaches. This is an active research area and quite a few measurement and identification methods have been developed for characterizing nonlinear systems [128],[149], including dynamic substructuring techniques [94],[150], although they are still less mature for accurately identifying parameters. There is still much work to be done in the future.

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