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Perception of Area, Volume, and Weight

by

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Abstract

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Countless everyday actions require us to estimate the size and weight of objects that come in various combinations of shape, material, and density. Humans are highly skilled at using different sources of perceptual information, as well as prior experience, to estimate size and weight, and regularly use this information to prepare action. For example, visual estimation of an object's size and material are used to infer its weight, and thus to prepare the correct forces to pick the object up. However, several aspects of these interrelated human perceptual abilities are subject to poorly understood biases. For example, in size perception, surfaces and objects appear larger or smaller depending on their shape. In weight perception, smaller objects feel heavier than same-weight larger objects (the 'size-weight illusion'). In this thesis, I explore biases in size and weight perception to broaden the understanding of these fundamental features of human perception. In Chapter 1, I (i) review the literature on biases in area, volume, and weight perception; (ii) highlight how we still lack an accurate understanding of how we perceive the area of surfaces, or volume of objects of different shape, and (iii) how the relationship between perceived volume, weight, and density is still unclear; (iv) identify key questions in each of these areas, addressed in the empirical work presented in the following chapters (2–4). In Chapter 2, I provide a comprehensive account of shape-related biases in area perception and propose a model that quantifies known biases (previously described in qualitative terms), and extends to novel stimuli. In Chapter 3, I provide a quantitative model that predicts biases in perceived weight in the size-weight illusion and generalises to other variations in object size / weight / density. In Chapter 4, I provide an account of the influence of shape in volume and weight perception (the 'shape-weight illusion'), and provide the first report of the sensorimotor correlates (finger forces and torques for grasping and lifting the objects) of these biases. Findings from the three empirical chapters are discussed together in Chapter 5 within the broader context of the literature reviewed in Chapter 1.

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Research Thesis: Declaration of Authorship

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I declare that this thesis and the work presented in it are my own and has been generated by me as the result of my own original research.

I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;
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3. Where I have consulted the published work of others, this is always clearly attributed;
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7. None of this work has been published before submission

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Chapter 1 Literature review

1.1 Introduction

Every day we effortlessly interact with a wide range of objects of different size, shape, and material. Unbeknownst to us, actions as simple as drinking a cup of tea entail fine-tuned interactions between current sensory cues and previous experience. Visual cues to haptic features are used in a predictive fashion to select appropriate parameters for action: Information about the cup's size, shape, and material is used to choose an adequate grip type and infer object weight to scale finger forces in prevision of grasping and lifting; more or less force is applied depending on how much the object is expected to weigh (Johansson & Flanagan, 2009; Johansson & Westling, 1988). Visually-estimated size (volume) is a fundamental cue guiding motor preparation (Cole, 2008; Gordon et al., 1991c, 1991a): we expect a larger cup to be heavier than a same-material smaller cup; we expect the same cup to be heavier or lighter if full or half empty. Once contact is made, online adjustments based on haptic information about the object weight can be made to correct any erroneous predictions from visual cues (Gordon et al., 1991b; Johansson & Flanagan, 2009; Johansson & Westling, 1987, 1988) to produce a smooth movement, making it possible to enjoy a hot drink without risk of tipping or spillage.

Despite numerous examples of everyday perception in action, it is often found in controlled studies that humans systematically misperceive both size (area, volume) and weight. In size perception, objects or surfaces appear larger or smaller in size depending on their shape (see, e.g., Kahrimanovic et al., 2010b; Krider et al., 2001; Krishna, 2006): for example, the volume of more elongated / less compact objects tends to be overestimated. Whilst these perceptual phenomena are still not well explained (as I will review below, 1.2), they are well-known to marketing professionals: A drink sold in a sleek, elongated can may appear better value-for-money than the same drink sold in a classic, more compact, can of the same volume, even if the label states '330 ml' in both cases (Raghubir & Krishna, 1999). In weight perception, same-weight objects are perceived as lighter or heavier depending on their size, shape, or material (Buckingham, 2014; Kahrimanovic et al., 2010c). One well-studied example is the size-weight illusion (SWI, Charpentier, 1891; Nicolas et al., 2012): where the observation is that the smaller of two same-weight stimuli of different size (and, at least apparently, same material) is systematically and persistently perceived as heavier than the larger one (Figure 1.1). In our environment, larger objects are usually heavier than smaller objects made of the same material, therefore, a real-life example of the SWI might be difficult to encounter. Indeed, as I will discuss below, the illusion might arise from a 'contrast' between expectations derived from previous experience (that the smaller object should be lighter, and the larger object should be heavier), and the sensed weight (the same for the two objects).

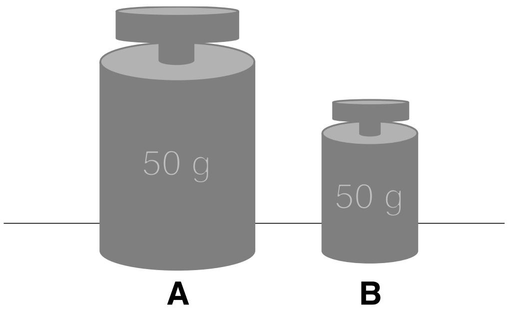

Figure 1.1 The size-weight illusion. A and B have the same physical weight, but A is perceived as lighter, and B as heavier.

Biases such as the SWI have long been investigated as a means to understand human perception. However, despite more than a century of research, there is still no consensus as to how the illusion occurs or, as a consequence, a comprehensive quantitative model of heaviness perception. Although size and / or related expectations are taken into account when predicting or perceiving weight, biases in area and volume perception have received much less attention, with few attempts to describe these biases in quantitative terms.

Ideally, a model of size and heaviness perception should be able to explain these biases, and integrate them in the same framework. This work seeks to address some open questions in the dominion of size and heaviness perception (summarised below, 1.4) using computational modelling to provide quantitative descriptions of each investigated phenomenon. In this chapter, I will first review shape-related biases in size (area, volume) perception, then move to biases in heaviness perception. In both cases, after a description of the phenomenon, I will summarise the current state of the literature highlighting discrepancies and open questions. At the end of the chapter, I will summarise the research questions which have informed the empirical chapters of the present work.

1.2 What do we talk about when we talk about size? Shape-related biases in size perception

The term ‘shape-related bias’ reflects the systematic over- or underestimation of the area of a surface or the volume of an object depending on its shape. For example, in area estimation, triangles are systematically perceived as larger in area than area-matched disks and squares (Anastasi, 1936; Dresslar, 1894; Fisher & Foster, 1968; Martinez & Dawson, 1973; Warren & Pinneau, 1955), as shown in Figure 1.2 A. Similarly, in volume perception, tetrahedrons are perceived as larger in volume than volume-matched cubes and spheres (Frayman & Dawson, 1981; Kahrimanovic et al., 2010a, 2010b; Vicovaro et al., 2019). Additionally, elongated stimuli are perceived as larger than less elongated stimuli of the same shape (Frayman & Dawson, 1981; Krishna, 2006; Raghubir & Krishna, 1999; Wansink & van Ittersum, 2003) – a phenomenon also known as ‘elongation bias’ (Krishna, 2006). Similar shape-related biases have been reported in visual, haptic, and visuo-haptic volume estimation tasks (Dresslar, 1894; Kahrimanovic et al., 2010b; Krishna, 2006).

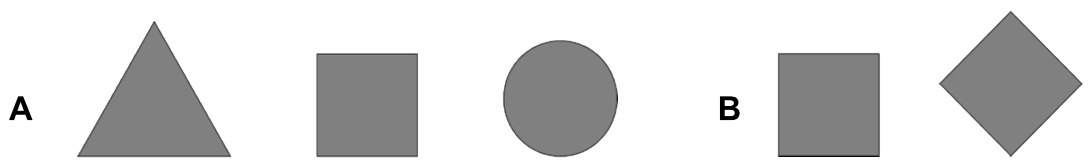

Figure 1.2 Shape-related biases in area perception. All the shapes in the figure have the same-area. (A) Triangles appear larger than same-area squares and disks. (B) The ‘square-diamond illusion’, illustrating the effect of orientation on perceived area: The same square appears larger when presented with the diagonal vertically oriented.

Although these biases have been reported since the 19th Century (Dresslar, 1894), a relative paucity of in-depth description exists in the literature. Several often-cited studies stem from marketing research, where shape-related biases have been investigated in applied contexts (mostly in relation to product packaging: see, e.g., Krider et al., 2001; Raghubir & Krishna, 1999), therefore with little interest in identifying their underlying causes. As I will detail in the next paragraphs, very few quantitative models are available in the literature, methodological limitations apply to a number of studies, and several inconsistencies are still unresolved.

1.2.1 Shape-related biases in area perception

The first, indirect, account of shape-related biases in area perception is that of Dresslar (1894), who reported that triangles and rectangles created from sheet-lead, explored in full visuo-haptic conditions, were perceived as lighter in weight than same-surface-area, same-weight disks and squares. The author's interpretation of these results was that triangles and rectangles were perceived as larger than more 'compact' shapes, thus perceived as lighter – analogous to the size-weight illusion (Charpentier, 1891), whereby smaller objects are perceived as heavier than same-weight larger objects. Independent of the validity of this interpretation (see 1.3.2 below for further discussion of shape-related biases in weight perception), Dresslar's (inferred) perceived area biases have been replicated, at least in part, in studies focussing on area perception.

Many of the subsequent studies on biases in area perception have focused on the visual modality only. Consensus indicates that triangles (in most studies, equilateral) are reported to be perceived as larger in area than disks and squares across several experiments and tasks (Anastasi, 1936; Dresslar, 1894; Fisher & Foster, 1968; Martinez & Dawson, 1973; Warren & Pinneau, 1955). However, results for other shapes are mixed: disks have been reported to be perceived as both smaller (Anastasi, 1936; Di Maio & Lansky, 1990) and larger (Fisher & Foster, 1968; Warren & Pinneau, 1955) than same-area squares.

Additionally, rectangles have been reported to be perceived as either larger (Dresslar, 1894; Holmberg & Holmberg, 1969, as cited in Martinez & Dawson, 1973; Mates et al., 1992), not different (Holmberg and Wahlin, 1969, as cited in Martinez & Dawson, 1973), or smaller (Martinez & Dawson, 1973) than less elongated rectangles and / or squares. The former studies indicate the presence of an 'elongation bias', but taking the literature as a whole, whether an 'elongation bias' happens in area perception is not clear. Part of the challenge in interpreting results is that most studies do not specify, or do not systematically manipulate, the rectangles' elongation (i.e., aspect ratio) and orientation[1]. Orientation, and related changes in shape features (e.g., height, as the length along the y axis), do play a role in perceived area: Squares appear larger when they are presented with their diagonal to be vertical (like a 'diamond'; Mach, 1897), as shown in Figure 1.2 B.

Whilst most studies have considered small sets of shapes, a few have broadened this to consider a wider range of regular shapes. For example, compared to triangles, six-point stars and rhombuses (measures not specified) have been reported to be perceived as larger (Anastasi, 1936), while symmetrical crosses have been reported either as larger (Greek cross: Anastasi, 1936) or smaller (unspecified 'symmetrical cross': Warren & Pinneau, 1955). The perceived area of three- to eight-point stars was negatively correlated with the number of sides (points), and positively correlated for five- to ten-sided polygons (Martinez & Dawson, 1973). Note that all these results feature in single studies and would thus benefit from replication.

1.2.2 Shape-related biases in volume perception

Due to daily experience with three-dimensional objects of different shape, and the availability of haptic cues to both size and mass, it would be tempting to imagine perceived volume not to be biased. Instead, numerous biases have been reported in visual volume perception, analogous to biases in area perception; similar biases also occur in haptic volume perception.

[1] To provide an example: (i) Mates and colleagues (1992) compared rectangles of different elongation and orientation (longer side vertically or horizontally oriented) with squares, but only report results averaged across different elongations and orientations; (ii) Martinez & Dawson (1973) compared shapes (triangles, quadrilaterals, ovals) of different elongation, each reported in seemingly random orientations (e.g., most quadrilaterals were presented with their longer linear length to be vertical, with the exception of one vertically oriented and one slightly tilted with respect to the y axis).

A widely reported phenomenon in volume perception is the 'elongation bias' (Krishna, 2006). Here, taller shapes (e.g. cuboids, cylinders) or containers (e.g. glasses, boxes) are perceived as larger in volume (or containing more liquid), than same-volume shorter objects (Frayman & Dawson, 1981; Holmberg, 1975, as cited in Raghubir & Krishna, 1999; Krishna, 2006; Raghubir & Krishna, 1999; Stanek, 1968, 1969; Wansink & van Ittersum, 2003). The bias is consistent in visual conditions; a reversal of the bias – where wider rather than taller glasses were perceived as larger in volume – has been reported in haptic (but not in visuo-haptic) conditions (Krishna, 2006). Differences in perceived volume for cuboids presented with their longer axis vertical vs. horizontal suggests that, within volume perception, elongation may interact with orientation (Krishna, 2006).

Further biases have been described in studies where participants are asked to compare other regular solid shapes. Analogous to triangle / disk, square comparisons in area perception, tetrahedrons were consistently reported as visually larger than same-volume spheres and cubes (Frayman & Dawson, 1981; Kahrimanovic et al., 2010a, 2010c; Mehraeen et al., 2021; Vicovaro et al., 2019). Kahrimanovic and colleagues (2010a) compared perceived volume for solid brass spheres, cubes, and tetrahedrons in visual, haptic, and visuo-haptic conditions. Different from the elongation bias, these biases were similar across modalities: In each condition, tetrahedrons were perceived as larger than spheres and cubes. Biases were smaller in visuo-haptic than in unimodal conditions, and in haptic conditions when mass information was available (i.e., when stimuli where hand-held as opposed to fixed on a stand). This latter result might indicate that mass information helps disambiguate volume biases; however, consistent with Dresslar's (1894) early findings, volume biases appear in turn to influence weight perception (Kahrimanovic et al., 2011; Vicovaro et al., 2019; see 1.3.2 and Chapter 4 below).

As in the area case, data for shapes beyond the small set described above are scarce. Frayman & Dawson (1981) compared platonic solids, spheres, and cylinders (1:2 aspect ratio) in a visual task; cylinders were judged as smaller than tetrahedrons; octahedrons, dodecahedrons, icosahedrons (8- 12- 20-face regular solids) were all judged as larger than spheres and smaller than tetrahedrons, but not significantly different from each other.

1.2.2.1 How to explain these biases?

Area perception

A range of explanations have been proposed for the biases. Most authors identified one geometric feature – or combination of features – of the shape as the source of the bias. Observers would be either (1) judging one other shape feature rather than area, or (2) combining shape features following an incorrect rule or heuristic to infer area (Carbon, 2016; Krider et al., 2001; Yousif & Keil, 2019).

Geometric features include: longest linear length (Anastasi, 1936); height ('maximum vertical dimension': Warren & Pinneau, 1955), compactness[2] (Dresslar, 1894; Owen, 1970; Smets, 1970; Smith, 1969); elongation / aspect ratio (Holmberg & Holmberg, 1969, as cited in Krider et al., 2001; Owen, 1970; Smith, 1969); perimeter (Di Maio & Lansky, 1990); isoclines of the image function (defined by the gaussian error along the shape perimeter, i.e. image blurring; Mates et al., 1992). Note that several of these factors are correlated (as acknowledged by some authors, e.g., Anastasi, 1936; Owen, 1970; Smith, 1969), although they do not completely overlap: for example, a decrease in compactness corresponds to increases in the shape's height and / or width (depending on the shape's elongation and presence of local concavities) and perimeter; increases in elongation correspond to decreases in compactness (but decreasing compactness does not necessarily entail changes in elongation). The picture is further complicated by the fact that most studies have dealt with small subsets of shapes, which might give rise to non-generalisable explanations. Across different shape sets, compactness was correlated both negatively (Dresslar, 1894; Owen, 1970) and positively with perceived area (Foster, 1976; Smith, 1969); the same applies to elongation (positively: Holmberg & Holmberg, 1969, as cited in Krider et al., 2001; negatively: Martinez & Dawson, 1973).

[2] Different metrics of compactness exist (see Appendix A, A.3), the simplest being the perimeter-to-area ratio. The disk is by definition the most compact shape, as it encloses any given area in the shortest perimeter possible.

Volume perception

As for biases in area perception, several authors have identified different geometric features that correlate with biases in volume perception, and proposed that observers use these features to estimate volume. These include, for visual biases, object height (Raghubir & Krishna, 1999), elongation (Holmberg, 1975, as cited in Raghubir & Krishna, 1999; Vicovaro et al., 2019), and compactness (Frayman & Dawson, 1981). Kahrimanovic and colleagues (2010a) proposed that surface area, i.e., the extent of cutaneous stimulation, would underlie haptic and bimodal biases; however, similar-magnitude biases occurred when participants judged the 'volume' of wireframe stimuli (i.e., in absence of surface area information), in which case volume judgements were biased towards the maximum distance between two vertex points.

Krishna (2006) interpreted the elongation bias, and its reversal in haptic conditions, in terms of 'perceptual saliency' (Krider et al., 2001): Height would be more salient in presence of visual size information, and width in haptic-only conditions. Note that this reversal of the bias might depend on the experimental conditions, as participants were not able to fully enclose the stimuli (standard-size lightweight plastic glasses) to appraise their volume.

According to Bennette and colleagues (2021), judgements of volume would follow the same 'additive heuristic' already proposed for area perception (Yousif & Keil, 2019); as in the case of 'additive area', only displays made of multiple objects (multiple cubes or spheres) were tested, and it is not clear whether this heuristic would extend to the estimation of simple shapes / objects.

1.2.3 Overview

To summarise, both area and volume perception are biased, given characteristics of the shape of the surface or object being estimated. Biases for a small number of simple shapes (e.g., triangles vs. disks, tetrahedrons vs. spheres) have been replicated across several experiments and tasks, but human judgements relating to broader range of shape configurations are limited. No proposed feature-based explanation adequately explains the whole range of the biases, possibly due to the limited number of shapes in the majority of studies, that can lead to local, non-generalisable, explanations. Discrepancies are made more difficult to resolve by the lack of formalised descriptions of the stimuli in some of the studies (e.g., Krishna, 2006). Testing the biases in wider sets of shapes could help disambiguate the contribution of different shape features to the bias: For example, whilst height can explain how rectangles are usually perceived as larger in volume than squares, it cannot explain how rectangles are also perceived as smaller than equilateral triangles (similar applies to their three-dimensional counterparts). The need for further testing applies to the 'additive heuristic' too, as pointed out above.

It has been argued that a specialised mechanism for perceiving area and / or volume might not exist, or be needed (Carbon, 2016; Morgan, 2005; Nachmias, 2011). However, the question is ill-posed: Independent of the existence of such a specialised mechanism, the consistency of the biases suggests that common features might be taken into account to approximate a shape's area or volume. Furthermore, shape-related biases appear to translate into weight perception (see 1.3.2), making their understanding valuable besides that of size perception per se.

In summary, despite the number of studies, it is still not known how why shape-related biases in size perception biases occur, and whether biases in area and volume perception can be ascribed to a shared mechanism. Furthermore, no quantitative models are available to predict perceived area or volume across different shapes. In Chapter 2 of this work I will focus on shape-related biases in area perception, present an extensive account of these biases across a wide set of stimuli which included previously-tested and novel shapes, and propose a quantitative model that predicts biases in perceived area for stimuli of various shape and orientation.

1.3 Is mass all there is (to weight)? Biases in heaviness perception

How do humans perceive an object's weight? It would be tempting to assume that perceived weight is simply a function of the object mass (and, being on Earth, gravity) – in other words, that heaviness perception works like a set of scales, measuring the force needed to lift or hold an object in our hands. Instead, a wealth of studies has shown that concurrent sensory information, other than that directly correlated with weight, is taken into account alongside the object's mass to produce the conscious perception an object's weight. Various factors, including fatigue (Burgess & Jones, 1997), grip configuration (Flanagan & Bandomir, 2000), the object's surface texture (Flanagan et al., 1995) and temperature (Kuhtz-Buschbeck & Hagenkamp, 2020) have been shown to influence heaviness perception. Moreover, several documented weight illusions – where objects systematically feel heavier or lighter than they actually are due to prior expectations – have been reported (for reviews, see Buckingham, 2014; Dijker, 2014; Saccone et al., 2019; Saccone & Chouinard, 2019b). These illusions occur independent of the factors above, across a wide range of grasping styles and experimental conditions.

In the size-weight illusion (SWI, Charpentier, 1891; Nicolas et al., 2012) – the most studied and one of the longest-known weight illusions – same-weight stimuli are perceived to weight differently depending on their volume: smaller objects (expected to be lighter) feel heavier, and larger objects (expected to be heavier) feel lighter. As I will review in the next section, the SWI has inspired a wealth of research which has identified numerous aspects of heaviness perception, although none of the proposed hypotheses is able to fully explain the SWI and reconcile it with heaviness perception as a whole. Furthermore, of numerous factors which have been shown to influence weight perception, the object's shape has been comparatively overlooked. At the end of the current section, I will summarise the open questions which have informed Chapter 3 and Chapter 4 of the present work.

1.3.1 The size-weight illusion

In our environment, size and weight are positively correlated for objects made of the same material (that is, of same density): on average, a lemon will weigh less than a grapefruit, and a teaspoon will weigh less than a tablespoon from the same set. Many people will lift one brick easily, but superpowers (or a forklift) would be needed to lift a pallet of the same bricks. Due to these real-world associations, we expect larger objects to weigh more – thus, require more force to grasp and lift – than same-material smaller objects (Gordon et al., 1991c). The SWI occurs when this association is violated: when lifting two objects of different volume, but equal weight, the smaller is perceived as heavier, and the larger as lighter. This perceived heaviness difference increases in magnitude with size difference (Cross & Rotkin, 1975; Ellis & Lederman, 1993; Peters et al., 2016; J. Ross & Di Lollo, 1970), and persists despite the lifter being aware of the objects being same-weight (it is cognitively impenetrable; Buckingham, 2014; Flournoy, 1894).

The SWI occurs for stimuli presented using haptic, visual, and visuo-haptic cues to size. Haptic cues to size (obtained, for example, when grasping an object with two fingers, or enclosing an object in the hand) are sufficient to elicit a full-strength SWI; the addition of visual size cues does not increase the magnitude of the illusion (Ellis & Lederman, 1993; Saccone et al., 2019). Size cues can be garnered through vision only (e.g., when objects are lifted via handles or strings, so that grip aperture is kept constant; Buckingham & Goodale, 2010; Ellis & Lederman, 1993; Flanagan & Beltzner, 2000; Kawai et al., 2007; Masin & Crestoni, 1988; Mon-Williams & Murray, 2000), however the magnitude of the illusion is reduced compared to when it is elicited by (visuo-) haptic cues to size (Ellis & Lederman, 1993; Kawai et al., 2007; Masin & Crestoni, 1988; Saccone et al., 2019). Yet, visual size still influences perceived weight when stimuli are only shown prior to lifting (and participants actually lift identical objects; Buckingham & Goodale, 2010; but see Masin & Crestoni, 1988), and a short glimpse of the object size (200 ms; before and up to a short time after lifting the object) is sufficient to elicit the illusion (Plaisier et al., 2019). Size cues obtained through echolocation are sufficient for blind individuals to experience the illusion in absence of haptic size cues (Buckingham et al., 2015).

Interestingly, perception of object weight – as obtained by lifting the object to counter gravity – is not necessary to experience the illusion, that also occurs when perceiving the acceleration of the object – as obtained by pushing the object suspended from a wire. Both gravitational and inertial cues to mass produce a SWI of similar magnitude (Plaisier & Smeets, 2015; Platkiewicz & Hayward, 2014), despite mass being underestimated when only inertial cues are available (Bergmann Tiest & Kappers, 2010; Plaisier & Smeets, 2015; Ross & Reschke, 1982).

1.3.2 Do biases in size perception also affect weight perception? The shape-weight illusion

The lesser-known shape-weight illusion (Kahrimanovic et al., 2011) describes a phenomenon where objects of same size and weight are perceived as heavier or lighter depending on their shape. The shape-weight illusion was first described by Dresslar (1894), who compared perceived weight for a set of shapes made of lead sheet, explored in visuo-haptic conditions; they report that disks were perceived to be heavier than area-matched triangles and rectangles of same weight. Later studies building on Dresslar's early work have reported that spheres feel heavier than tetrahedrons of same volume and weight, both when volume cues are obtained haptically (by enclosing the objects; Kahrimanovic et al., 2011) or visually (when stimuli are lifted using strings; Vicovaro et al., 2019).

As already suggested by Dresslar (1894), the shape-weight illusion appears broadly related to differences in perceived size for stimuli of different shape (described above, 1.2.2) suggesting that shapes whose size is underestimated or overestimated are also perceived as respectively heavier and lighter. However, the size bias was not sufficient to accurately predict the weight bias in either of the two more recent studies (Kahrimanovic et al., 2010c; Vicovaro et al., 2019), suggesting that factors besides perceived size might contribute to the shape-weight illusion.

1.3.3 What is the weight of expectations? The material-weight illusion and the brightness-weight illusion

Several other factors have been shown to contribute to the (mis)perception of weight in absence of size or shape manipulations. One widely-reported example is the material-weight illusion (MWI) (Buckingham et al., 2009; Harshfield & DeHardt, 1970; Seashore, 1899; Wolfe, 1898), occurring when lifting same-volume, same-mass objects that appear made of different materials. Similar to the case of associations between weight and size, different weight expectations are related to different materials, based on real-world associations between material and density (mass / volume): we expect objects made of a denser material to be heavier than similar objects made of a less dense material; a brass coin will weigh more than a chocolate coin of the same size. When lifting MWI stimuli, participants report the object which appears to be made of a less dense material (e.g.

polystyrene) as heavier, and the object which appears to be made of a denser material (e.g. metal) as lighter. As in the case of the SWI, heaviness expectations appear contrasted, rather than integrated, with current sensory cues.

In the brightness-weight illusion, lighter objects are perceived as heavier than darker objects of equal shape, size, and weight (De Camp, 1917; Walker et al., 2010; but see Vicovaro et al., 2019 for non-significant results). As darker objects are expected to be heavier, and vice versa (Walker et al., 2010, 2017), the brightness-weight illusion has been related to the MWI (Buckingham, 2014). On the other hand, darker objects also tend to be perceived as smaller in size (Oyama & Nanri, 1960; Vicovaro et al., 2019; Wallis, 1935), inconsistent with the SWI.

1.3.4 How to explain biases in heaviness perception?

Several explanations have been proposed for the SWI and other weight illusions, commonly labelled as either 'top-down' or 'bottom-up' explanations (for reviews, see Buckingham, 2014; Dijker, 2014; Saccone et al., 2019; Saccone & Chouinard, 2019b). In 'top-down' explanations, mis-perception of weight would arise from the contrast between (learned) heaviness expectations and the current sensory information; in 'bottom up' explanations , physical features associated with mass would erroneously contribute to perceived weight. Buckingham (2014) further distinguished sensorimotor hypotheses, which focus on the mismatch between sensorimotor expectations (e.g., the forces for grasping and lifting the objects programmed prior to the lift) and the incoming sensory feedback. It should be noted that these labels, customary in the SWI literature, are umbrella terms encompassing hypotheses that belong to different frameworks, as I will detail below. In the next paragraphs, I will review the main hypotheses to highlight their strengths and limitations and unsolved aspects that still need to be addressed.

1.3.4.1 (Not) what I expected: The role of previous experience in the SWI

As mentioned previously, through day-to-day experience with a wide range of objects, humans learn associations that reflect real-world statistical regularities between object features such as size, weight, and material; these learnt associations, in turn, inform expectations about object weight. From this perspective, weight illusions have been described in terms of a ‘contrast’ or ‘repulsion’ between expected and actual weight, i.e., they would arise when expectations are not met.

Explanations that focus on the role of previous experience have been broadly labelled as ‘top-down’. These include accounts focussing on generic ‘cognitive expectations’ (described below; Dijker, 2008; Ellis & Lederman, 1998; Saccone & Chouinard, 2019a) as well as Bayesian models that formalise expectations in terms of priors that reflect statistical regularities of the environment.

Bayesian decision theory has provided an overarching framework to describe perception in terms of a combination of current stimulus information and expectancies (Ernst, 2006; Langer & Bülthoff, 2001; Weiss, Simoncelli, & Adelson, 2002; Yuille & Bülthoff, 1996). In the Bayesian framework, prior knowledge (the ‘prior’) is encoded as the expected probability distribution of distinct object properties, that is, the stimulus values or combinations that observers expect to encounter. These learnt expectations are integrated (generally, by means of a weighted averaging) with current sensory cues to produce the final percept (Adams et al., 2004; Ernst, 2006, 2007; Ernst & Bülthoff, 2004; Kersten et al., 2004). In our environment, weight and size are broadly positively correlated: larger objects are usually heavier than smaller objects (especially in the case of objects made of the same material). A plausible prior over size and weight will correspond to expectations for the smaller object to be lighter than a same-material larger object (this will be discussed in more detail in Chapter 3, 3.1). In the SWI, therefore, the integration of prior and current information should result in the perception of the smaller object as lighter and the larger as heavier: an averaging instead of a ‘contrast’, or ‘repulsion’ effect. For this reason, the SWI has been labelled as ‘anti-Bayesian’ (Brayanov & Smith, 2010; for studies challenging this, see Bays, 2023; Peters et al., 2016; Wolf et al., 2018). Other authors have proposed that expectancies might not play a role in the SWI (see below, 1.3.4.3); however, evidence that the illusion can be reversed following extensive practice with objects in which the size-weight relationship was inverted (i.e., where mass decreases with size; Flanagan et al., 2008) has provided compelling evidence for the role of prior expectations (for examples of how well-learnt priors are recalibrated with experience see Adams et al., 2004, 2010).

Furthermore, the MWI can only arise from expectations related to the object's material (therefore density), providing another example of an 'anti-Bayesian' effect (together with other expectancy-based weight illusions described below; Saccone & Chouinard, 2019b).

Different kinds of expectations have been explored in relation to weight illusions. A first broad distinction can be made between conscious / explicit expectations, and implicit expectations. The latter are independent of cognitive understanding and refer to learnt, long-term implicit knowledge of size-weight (or material-weight) relationships (Buckingham, 2014; Ernst, 2009) which do not necessarily match explicit expectations (therefore, the term 'implicit expectations' found in the literature largely corresponds to the 'prior' described above). Only implicit expectations appear to influence weight perception. As mentioned above, the knowledge of the objects being the same weight does not affect the SWI; short-term explicit weight expectations obtained by labelling identical stimuli as 'light' or 'heavy' do not result in a perceptual bias (Naylor et al., 2020).

A few studies have shown that heaviness perception can indeed be influenced by various implicit expectations, also independent of manipulations of size and / or material. For example, novel weight biases can arise from the observers' expertise. Ellis and Lederman (1998) asked golfers and naïve participants to report the weight of same-size, same-mass standard and practice golf balls. Unknown to the naïve participants, practice golf balls are usually lighter. Consistent with their respective expectations, experienced golfers judged the practice golf balls to be heavier, whereas naïve participants reported no difference. Two different studies (Dijker, 2008; Saccone & Chouinard, 2019a) reported that 'female' dolls are perceived as heavier than 'male' dolls, which was interpreted by the authors as showing that even socially-reinforced gender biases can contribute to perceived weight (more simply, different weight expectations might be related to the slender vs. muscular appearance of the 'female' and 'male' dolls used in the experiment).

Implicit expectations also appear (at least partly) independent of sensorimotor performance. In a widely-cited study, Flanagan and Beltzner (2000) measured fingertip forces for grasping (grip force, i.e., the force perpendicular to the grasping surface to hold the object) and lifting (load force, i.e., the force exerted vertically to lift the object) SWI objects. The rates of change of grip and load force, pre-emptively scaled on expected weight (that is, programmed before making contact with the object), have been widely used as a measure of sensorimotor predictions based on implicit weight expectations (Flanagan & Beltzner, 2000; Gordon et al., 1991c; Johansson & Flanagan, 2009; Johansson & Westling, 1988). Force rates for grasping and lifting SWI objects were initially scaled consistently with expectations based on stimulus size (higher rates for the larger object, and vice versa), then rapidly (in a few lifts) adapted to the actual object weight, while the illusion remained unaffected. This shows that sensorimotor predictions for individual objects can be quickly updated independent of long-term expectancies (see also 1.3.4.2).

The aforementioned 'cognitive expectations' studies (Dijker, 2008; Ellis & Lederman, 1998; Saccone & Chouinard, 2019a) have described expectancies in the SWI / perceived heaviness as causing biases in heaviness perception via 'contrast' effects. Other authors have proposed that, despite it appears to be a 'contrast' effect, the SWI can be explained within the Bayesian framework. Peters and colleagues (2016) have proposed a Bayesian causal inference model featuring competing priors over the relationship between the objects' size and density ratio. According to this model, the continuum of all possible density relationship between two stimuli of different size is represented by three categorical priors: (i) both objects have the same density; (ii) the smaller object is denser; (iii) the smaller object is less dense. The perceived weight ratio between the two stimuli depends on which prior dominates perception given the incoming sensory evidence about the size and weight relationships of the two stimuli. It is unclear whether this model generalises to heaviness perception more broadly, as it was only evaluated on classic SWI stimuli (equal weight, density negatively correlated with volume). It should also be noted that competitive prior models (Yuille & Bülthoff, 2012) are suited to describe categorical world structures. The reduction of the real-world continuum of size-density relationships to three categories, assumed by Peters and colleagues' model, is therefore difficult to justify at the conceptual level. Recently (Bays, 2023), it has been proposed that the SWI can be predicted by a Bayesian model incorporating efficient coding (predicting that perception can be biased away from the prior; Wei & Stocker, 2015). As Bays' model has only been evaluated on Peters and colleagues' (2016) averaged data, its generalisability is also unclear.

To summarise, several studies have shown that implicit expectations / prior knowledge play a role in weight illusions, usually in the form of a ‘contrast effect’ between the expected weight and actual weight of the object. This contrast effect has proven difficult to describe with available models. However, it has been proposed that Bayesian models might in fact account for the SWI (Bays, 2023; Peters et al., 2016; see also Wolf et al., 2018, reviewed below, 1.3.4.3). Other theories of weight illusions, reviewed next, have proposed that the contrast would correspond to a ‘mismatch’ between forces applied according to the lifter’s expectations and the forces needed to lift the object; however, further limitations apply.

1.3.4.2 I was not ready for that: The role of motor output in the SWI

‘Sensorimotor’ (Buckingham, 2014) explanations propose that the SWI (and MWI) stem from a mismatch between efferent and afferent signals, i.e., between the forces programmed prior to the lift and the sensory feedback signalling that forces were programmed in error. This hypothesis was first proposed by Davis & Roberts (1976) who observed that, in SWI pairs, large objects are lifted with a greater acceleration than small objects, consistent with a higher rate of force programmed to lift the large object. Consistent with this, muscle tension before lift-off of SWI objects is higher for larger stimuli (Davis & Brickett, 1977). As mentioned above, the observation that force rates are initially consistent with distinct weight expectations for the two objects has since been widely reported (Buckingham et al., 2011a; Buckingham & Goodale, 2010b, 2010a; Flanagan & Beltzner, 2000; Gordon et al., 1991a; Grandy & Westwood, 2006). However, the observation that the perceptual illusion persists after sensorimotor memory has adapted to the actual weight of the objects has ruled out sensorimotor mismatch as a sole cause for the SWI (Buckingham, 2014; Flanagan & Beltzner, 2000; Grandy & Westwood, 2006; Mon-Williams & Murray, 2000).

On the other hand, consistent with the sensorimotor hypothesis, efferent signals have been shown to influence weight perception. It has been commonly observed in experimental setups that heaviness estimates tend to increase with repeated lifting (Buckingham et al., 2009; Flanagan & Beltzner, 2000; Grandy & Westwood, 2006); indeed, perceived heaviness increases with muscle fatigue, anaesthesia, or partial paralysis, i.e., when more force needs to be generated to obtain the desired output (Burgess & Jones, 1997; Gandevia et al., 1980; Jones & Hunter, 1983). Objects with a smooth surface, which require a greater grip force to prevent slipping, are perceived as heavier compared to those with a textured surface (Flanagan et al., 1995). The style of lifting also contributes: lifters report objects to be heavier when instructed to lift them gently, and as lighter when instructed to lift them with more force (Davis et al., 1977); objects are perceived as lighter when grasped with a wider grip, a larger contact area, or with five digits compared to two (Flanagan & Bandomir, 2000). Lastly, although the SWI remains unaltered with subsequent lifts, effects of short-term sensorimotor memory on perceived heaviness have been reported (Maiello et al., 2018; Uznadze, 1939, as cited in H. E. Ross, 1969), showing that heaviness perception in subsequent lifts is influenced by the stimulus lifted first: the same weight will feel lighter if lifted after a heavier stimulus, and heavier after lifting a lighter stimulus.

It should also be noted that not all sensorimotor errors appear to be corrected as quickly as those in the programming of force rates: persistent errors in peak grip and load force have been reported (Buckingham & Goodale, 2010b; Gordon et al., 1991a), making it difficult to exclude that efferent signals might have a role in weight illusions.

1.3.4.3 Not what I thought it was: The role of other object physical properties in weight perception

Different from those previously reviewed, ‘bottom-up’ explanations disregard expectations to propose that other object features related to the object’s mass (e.g., density, rotational inertia) would be perceived as weight, or confounded / conflated with weight, thus bias heaviness perception.

Density

With mass kept constant, volume differences entail differences in the object’s density; thus, the SWI has occasionally been labelled as ‘density-weight illusion’ (Peters et al., 2016; Thouless, 1932; Wolf et al., 2018). In its simplest formulation, if density is the object property being utilized by the perceiver, and subsequently interpreted by the perceiver as weight, then it follows that the smaller, denser object will be perceived as heavier (Saccone & Chouinard, 2019b). Weight and density might indeed be difficult to disentangle for the perceiver: (i) increases in density difference are perceived as increases in weight difference (J. Ross & Di Lollo, 1970); (ii) heavier objects are also perceived as denser (Merken & van Polanen, 2020); (iii) weight difference discrimination is better when density varies with weight (Kawai, 2002).

Perception (or inference) of object density might also reconcile the SWI within the Bayesian framework. Wolf and colleagues (2018; Wolf & Drewing, 2020) have proposed a cue-integration model where perceived heaviness is a weighted average of one heaviness estimate derived from the object's mass and one other estimate derived from the object's (inferred) density, with weights assigned according to each estimate's reliability. In other words, humans would conflate weight and density when judging an object's heaviness. It should be noted that this is not a 'bottom-up' model in the strict sense, as it suggests that the integration of mass and density is adaptive (therefore, Bayesian) as it is biased towards environment statistics (denser objects are generally heavier). Thus, this model predicts that density would generally influence weight estimates, provided that the perceiver has access to size information, necessary to infer density. The strength of Wolf and colleagues' model is that it accounts for weight perception in single objects, across SWI and non-SWI stimuli. On the other hand, it is not clear why the perceptual system should integrate weight and density as a general rule. First, density is in itself a poor cue to weight (unless conditioned on size). Second, as weight and density will have correlated noise (density being inferred from weight and volume), the integration of weight and density negates the normal benefits of integration (i.e. obtaining a less noisy / more precise, final percept by combining two noisy estimates; Ernst & Banks, 2002). The seemingly paradoxical implication of the model is that the higher the quality of perceptual information (full vision, unconstrained haptic exploration of the stimuli), the more biased towards density the heaviness estimate.

As pointed out elsewhere (H. E. Ross, 1969a; Saccone & Chouinard, 2019b), whilst perception / inference of object density might explain the SWI, it cannot explain the perceived weight difference that occurs in the MWI (described above, 1.3.3), where the two objects have the same physical density (i.e., they both have the same size and weight, and differ only in their surface material).

Rotational inertia

Rotational inertia, which varies with the mass distribution in the object, has been proposed as a potential candidate in ‘bottom-up’, ‘ecological’ accounts. According to Amazeen (2014), the aim of weight perception would be to perceive the object’s resistance to lifting forces, rather than mass. Amazeen & Turvey (1996) asked participants to sway rods of identical mass and volume whose distribution of mass was manipulated by attaching weights in different locations; they reported a SWI-analogue effect where participant’s perceived heaviness varied with the object’s rotational inertia. In another study (Amazeen, 1997), participants were asked to compare the weight of a standard stimulus to that of a same-mass stimulus whose volume increased either along its height or the width. In absence of vision, increases in height produced an increase in perceived weight, whilst increases in width resulted in a decrease in perceived weight, that is, a reversal of the size-weight illusion. Interestingly, when visual information about stimulus size was available, both styles of volume change resulted in a decrease in perceived weight, although the decrease was smaller for increases in height. Consistent results have been reported by the same author across a variety of task, including individual and team lifting of heavy boxes varying in height and width (Amazeen, 2014; Amazeen & Jarrett, 2003).

Results consistent with a (partial) contribution of rotational inertia to the SWI have also been reported more recently by Plaisier & Smeets (2015) who measured the SWI using a cube, a cuboid, and ‘spacer’ object where the cube was split in two equal halves connected by a thin rod. On the other hand, Zhu and colleagues (2013) reported no differences in perceived weight for hand-held spheres whose centre of mass was manipulated[3].

Although the evidence summarised here indicates that an object's distribution of mass might contribute to heaviness perception, it is difficult to understand how rotational inertia might alone explain the classic SWI (as pointed out by Buckingham, 2014; Saccone & Chouinard, 2019b) or the MWI. Furthermore, the SWI still occurs in absence of actual changes in haptic volume (therefore in mass distribution), when participants grasp and lift identical objects whose apparent size has been manipulated with the use of distorting lenses (Koseleff, 1957) or virtual reality (Buckingham, 2019).

1.3.5 The more, the merrier: Arguments in favour of the contribution of multiple factors to the SWI

In summary, none of the main proposed hypotheses appears fully successful in describing the SWI and heaviness perception more broadly. This suggests that weight illusions might arise from a combination of expectations and factors that have been labelled 'bottom-up' and 'sensorimotor' – a view that has gained consensus in recent years (Buckingham, 2014; Freeman et al., 2019; Saccone & Chouinard, 2019b).

As mentioned, the finding that the SWI can be reversed following extensive training with stimuli where the size-weight relationship had been inverted (Flanagan et al., 2008) has provided compelling evidence in favour of the involvement of prior assumptions in the SWI. Whilst this argument disproves 'bottom-up' (in the strict sense, e.g., Amazeen & Turvey, 1996) hypotheses as a sole explanation for the SWI, expectancies are not the whole story. The strongest argument in favour of a contribution of 'bottom-up' factors to the SWI comes from descriptions of illusion magnitude. A

[3] In a further 'bottom-up', 'ecological' account, Zhu & Bingham (2011) proposed that the SWI would be a direct consequence of the human ability, selected in prehistoric times, to identify objects based on their 'throwability', i.e. their aptitude to be thrown the farthest. However, as pointed out by Buckingham (2014), it is not clear which object feature would ultimately produce the heaviness estimate.

meta-analysis by Saccone and colleagues (2019) has shown that the haptic and visuo-haptic SWI are significantly larger than the MWI – that is, the perceived heaviness difference in SWI pairs is larger; interestingly, the known to be smaller (Ellis & Lederman, 1993; Plaisier & Smeets, 2015) magnitude of the visual-only SWI was instead comparable to that of the MWI. Similarly smaller is the SWI obtained with expectations alone (where participants lift identical stimuli which they believe to be larger or smaller; Buckingham & Goodale, 2010a). In all these cases, cues to (but possibly not limited to) the object's density or rotational inertia are available in the full-magnitude SWI, but absent or greatly reduced in the visual-only (or expectations-only) SWI and MWI. Note that, although the MWI is usually considered to be largely expectation-driven (and indeed still occurs without touching the objects; Ellis & Lederman, 1999), haptic feedback is required for a full-magnitude illusion (Buckingham et al., 2011a; Ellis & Lederman, 1999). Lastly, Freeman and colleagues (2019) have reported that while the SWI is not impaired by increasing cognitive load, it is reduced with lower-quality sensory information. The latter observation is consistent with Wolf and colleagues' (2018) report of a positive correlation between the SWI magnitude and the quality of sensory information, that the authors interpret as due to the concurrent increase in reliability of the size cues used to infer density.

1.3.6 Overview

Even after accepting that several factors that have been shown to influence heaviness perception might contribute to the SWI alongside heaviness expectations, the identity, and relative importance of these factors are still not known. As reviewed above, density and / or rotational inertia appear likely to contribute; however, it is still not clear how and under which circumstances they contribute the final heaviness percept. On the other hand, the role of prior expectations is also far from clear: The description of weight illusions in terms of a 'contrast' effect being a description rather than an explanation (Ernst, 2009).

Some aspects which have been underexplored in the literature can provide new hints. First, the SWI has not been tested at very small size / density differences. It is known that integration occurs for small conflicts, but not large conflicts (Knill, 2007; Landy et al., 1995). Therefore, it is possible that small conflicts between expected and actual weight might be integrated, rather than contrasted. Second, it is not known whether weight and density are conflated generally (Wolf et al., 2018; Wolf & Drewing, 2020), or only when size-weight expectations are not met, as in the specific (and unusual) case of the SWI. Third, it is still unexplained how object shape contributes to heaviness perception. Whilst perceived size cannot fully explain the bias (Kahrimanovic et al., 2011; Vicovaro et al., 2019), the suggestion that biases might be correlated to surface area remains untested (Vicovaro et al., 2019), and it seems unlikely that weight (density) expectations can vary across shapes (as suggested by Buckingham, 2014).

The first two questions will be addressed in Chapter 3, where I will present a model of heaviness perception that predicts perceived weight in the SWI and across a range of variations in stimulus volume, weight, and density. The third question will be addressed in Chapter 4, where I will provide an account of perceived weight, finger forces and torques in the shape-weight illusion alongside corresponding biases in perceived volume.

1.4 Overview of the present work

In the previous sections of this chapter, I have reviewed existing reports about biases in the perception of area, volume and weight, and indicated numerous gaps that still exist in the literature. The amount of available knowledge varies greatly across these topics. Whilst the SWI has been well-described, less is known about (biases in) heaviness perception outside of same-weight stimulus pairs. Biases in area and volume perception remain underexplored. Common, however, is the scarcity of formalised models that can successfully predict human perceptual performance.

This work contributes to our understanding of size and weight perception, and their interplay, by addressing the following outstanding questions (previewed above in this chapter):

1) *Can we predict biases in area perception from shapes' geometric features?*

In Chapter 2, I will provide an account of known and novel shape-related biases in area perception and propose a model that predicts variations in perceived area as a function of several geometric features of the shape.

2) *Can we develop a generalised model of weight perception for varied combinations of size, weight and density?*

   In Chapter 3, I will provide novel evidence to characterise heaviness perception in a range of stimuli that varied in size and / or weight and / or density. I will propose a model that predicts perceived heaviness in the SWI and across several combinations of size, weight, and density, and compare it with current models of the SWI.

3) *Can we better characterise the influence of shape on perceived heaviness?*

   In Chapter 4, I will provide an account of the visuo-haptic shape-weight illusion in a novel set of shapes, and the first report of the sensorimotor correlates of the illusion, obtained by measuring finger forces and torque when grasping and lifting the stimuli.

Results from the three studies presented in Chapter 2–4 will be discussed in Chapter 5 within the broader context of the literature, together with limitations and future directions.

Chapter 2 Biases in the perceived area of different shapes: A comprehensive account and model

Abstract

Common daily tasks require us to estimate surface area. Yet, area judgements are substantially and consistently biased: For example, triangles appear larger than same-area squares and disks. Previous work has explored small subsets of shapes, and related biases in area perception to one or two geometric features, such as height or compactness. However, a broader understanding of shape-related biases is lacking. Here we quantify biases in area perception for a wide variety of shapes and explain them in terms of geometric features. In four online experiments (each N = 35), typical adult observers made 2AFC judgements ('which stimulus has larger area?') for pairs of stimuli of different shape, orientation, and / or area. We found clear shape-related biases, that replicate known biases and extend them to novel shapes. We provide a multi-predictor model (R^2 = .98) that quantitatively predicts biases in perceived area across 22 shape / orientation combinations.

Notes

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2.1 Introduction

Estimating surface area is integral to many daily tasks, such as choosing a pizza slice for lunch (Krider et al., 2001), or perhaps more critically, judging the surface area / volume of a bodily structure when performing surgery (Schuld et al., 2012). As in the pizza case, judging area is an integral step in estimating volume and inferring weight in preparation for grasping and lifting. It is somewhat surprising, therefore, that humans exhibit substantial and systematic biases in area perception. For example, triangles appear larger than same-area squares or disks (Anastasi, 1936; Dresslar, 1894; Fisher & Foster, 1968b; Martinez & Dawson, 1973; Warren & Pinneau, 1955), and rectangles appear larger than same-area squares (Krider et al., 2001). However, little is known about shape-related biases beyond a small number of simple shapes. Moreover, there has been little effort to quantify biases in area perception and no models exist to generate predictions of perceived area for novel shapes.

In previous studies, triangles are consistently perceived as larger than same-area disks and squares. This suggests that compactness (e.g., the ratio of a shape's perimeter to its area) may be negatively associated with perceived area (Dresslar, 1894). Various metrics of compactness (see Figure 2.1; Appendix A, Table A1) all produce the same ordering across these shapes: disks > squares > triangles. However, inconsistent findings muddy the waters: disks have been reported as perceptually smaller (Anastasi, 1936; Di Maio & Lansky, 1990) or larger (Fisher & Foster, 1968b; Warren & Pinneau, 1955) than same-area squares. Across other sets of shapes, compactness has been reported to correlate either negatively (Dresslar, 1894; Owen, 1970) or positively (Foster, 1976; Smith, 1969) with perceived area.

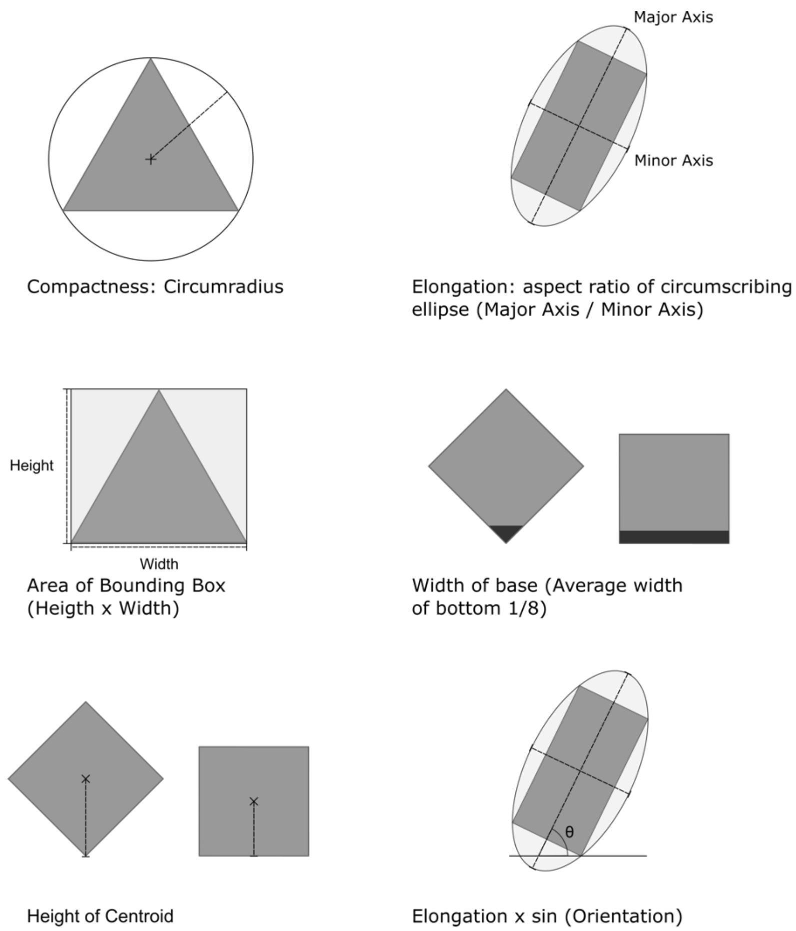

Figure 2.1 Shape metrics featured in the best model (described below, 2.8). See Appendix A, Table A1 for the full list of predictors with definitions. Elongation corresponds to the aspect ratio of the ellipse circumscribing the shape; orientation corresponds to the angle between the x axis and the major axis of the circumscribing ellipse.

In volume perception, elongation is associated with increased perceived volume (the 'elongation bias', Krishna, 2006). Whether this generalises to area perception is unclear, as more elongated rectangles have been reported as perceptually larger (Holmberg & Holmberg, 1969, as cited in Krider et al., 2001; Mates et al., 1992), smaller than (Martinez & Dawson, 1973), or not different from (Holmberg & Wahlin, 1969, as cited in Krider et al., 2001) less elongated rectangles and squares. Orientation also plays a role in perceived area: squares appear larger when they are presented in a 'diamond' orientation (Mach, 1897). However, the wider effects of orientation, and their interaction with elongation, are unclear.

As described above, many authors have identified a single geometric feature (e.g., height, elongation or compactness) as the source of shape-related biases. Others have suggested that observers use multiple geometric features, but combine these via an incorrect rule to infer area

(Carbon, 2016; Krider et al., 2001; Yousif & Keil, 2019). For example, Krider and colleagues (2001) propose that observers compare shapes according to the ratio of their most and least salient dimensions. However, saliency is ill-defined, and the model gives no quantitative predictions. Other authors have suggested that observers sum stimulus height and width ('additive area') to estimate area (Yousif et al., 2020; Yousif & Keil, 2019).

In summary, our understanding of biases in area perception is limited to a small number of simple shapes; few studies have included polygons with more than four sides or concave polygons such as stars and crosses (Anastasi, 1936; Martinez & Dawson, 1973; Owen, 1970; Warren & Pinneau, 1955). In addition, discrepancies are hard to reconcile due to limitations such as the absence of formalised descriptions of geometric features (e.g., Anastasi, 1936; Martinez & Dawson, 1973; Owen, 1970). We lack a quantitative account of area perception that characterises biases across a varied set of shapes and provides testable predictions for novel shapes.

The current work seeks to address these limitations: Across four experiments, we measured biases in area perception for a wide range of shapes / orientations. To preview the key results: Shape related biases are substantial; the perceptually largest shape (3-point star) was perceived to be around 41%, or 2 JNDs (Just Noticeable Differences) larger than the perceptually smallest shape (disk). We present a quantitative model that captures variations in area perception (R^2 = .98), with a combination of shape metrics including compactness, elongation, and orientation. The model captures previously reported area biases (e.g., disks < squares < triangles; squares < rectangles; squares < 'diamonds') but extends to 22 different shapes/orientations.

2.2 General methods

2.2.1 Participants

Effect sizes for shape-related biases could only be calculated from Warren & Pinneau (1955). These were large (*d* = 5.07, 4.57, 1.06 for disks vs. triangles, squares vs. triangles, disks vs. squares, respectively) suggesting 10 observers to give .8 power with alpha = .05. In order to quantify biases across a larger set of shapes, we chose a sample of 35. Different sets of participants were recruited via Prolific (www.prolific.co) for each experiment; all were financially compensated. Mean ages for experiments 1–4 were 29.2 (*7.5*), 27.7 (*7.4*), 31.5 (*11.6*), 29.4 (*9.0*) years (*SD*). The study was approved by the University of Southampton Psychology Ethics Committee; all participants gave informed prior consent.

2.2.2 Setup

Experiments were conducted online, hosted on a JATOS (Lange et al., 2015) server at the University of Southampton and accessed via Prolific. The experimental software was written in HTML, CSS, and JavaScript using JsPsych (de Leeuw, 2015). The experiment was rendered in a fixed screen partition (width: 800 px) and required a physical keyboard. To control stimulus size across different displays, we used the jsPsych 'resize' plugin: participants adjusted a rectangle on the screen to match the size of a credit card. Participants were asked to sit at a viewing distance of approximately 57 cm. Some variation in viewing distance was deemed acceptable; as expected, shape related area biases proved to be broadly scale-invariant.

2.3 Stimuli

Each experiment employed a different subset of stimulus conditions (geometric shape-orientation combinations) from a total of 22 (Figure 2.2 A). Stimuli were presented in four different sizes (5–10cm^2 in equal steps). All stimuli were pre-rendered in white on a grey background using MATLAB (The MathWorks Inc., 2020).

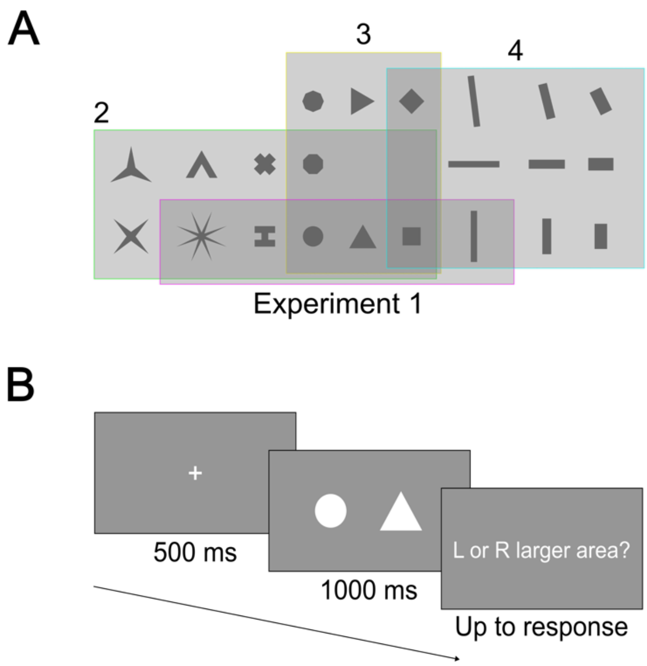

Figure 2.2 (A) Stimuli for all four experiments, presented here with equal area. (B) Trial structure (not to scale) for all four experiments.

2.3.1 Procedure

On each trial, participants were presented with a central fixation cross (500 ms), followed by two stimuli, ± 5cm from fixation (1000 ms, see Figure 2.2 B). Participants reported which of the two stimuli appeared larger in area, via arrow keys (left vs. right). An opportunity to take a break was given every 20 trials. The number of trials varied across experiments, see below.

Each experiment was divided into two blocks (each included one repetition of the full trial set), separated by a break. Stimulus left and right positions were assigned randomly in block 1 and swapped in block 2. Trial order was randomised within blocks. The first experimental block was preceded by a variable number of practice trials (2 × number of shapes) which were identical to the experimental trials except that feedback was given (‘Correct!’ or ‘Wrong.’). To avoid providing information about shape-related biases, stimulus pairs in practice trials had very different sizes (5 and 10 cm^2). The pre-task instructions included a definition of area. Average completion times in minutes (*SD*) for experiments 1–4 were 40 (*15*), 56 (*14*), 48 (*17*), 49 (*9*).

2.3.2 Data analysis

All analyses were performed in MATLAB. We screened participants' data for control trial errors (same stimulus, size 1 vs. 4) and all exceeded threshold performance (85% correct). Control trials were excluded from subsequent analyses.

Each participant's 2AFC judgements were converted to estimates of relative perceived area in just noticeable differences (JNDs) using Thurstone case V scaling (Thurstone, 1927; see Adams et al., 2018; Perez-Ortiz & Mantiuk, 2017). Each unique stimulus (combination of shape, orientation, and size) is assumed to invoke perceived area with a unimodal mean (μ), perturbed by Gaussian noise with standard deviation σ. Figure 2.3 A illustrates the method: For every pair of unique stimuli (s_1, s_2), the distance between the corresponding means gives the probability of perceiving s_1 as larger than s_2. In the examples given in Figure 2.3 A, p($\hat{s}_2 > \hat{s}_1) = 0.75$, p($\hat{s}_3 > \hat{s}_2) = 0.83$, p($\hat{s}_3 > \hat{s}_1) = 0.99$. Thus, for each stimulus pair we calculate the probability of the observed responses (number of trials, number of $s_1 > s_2$ responses), given values of μ_1 and μ_2. Using gradient descent (*fminsearch*, MATLAB), we find the set of μ values for all unique stimuli that maximises the probability of each participant's complete dataset.

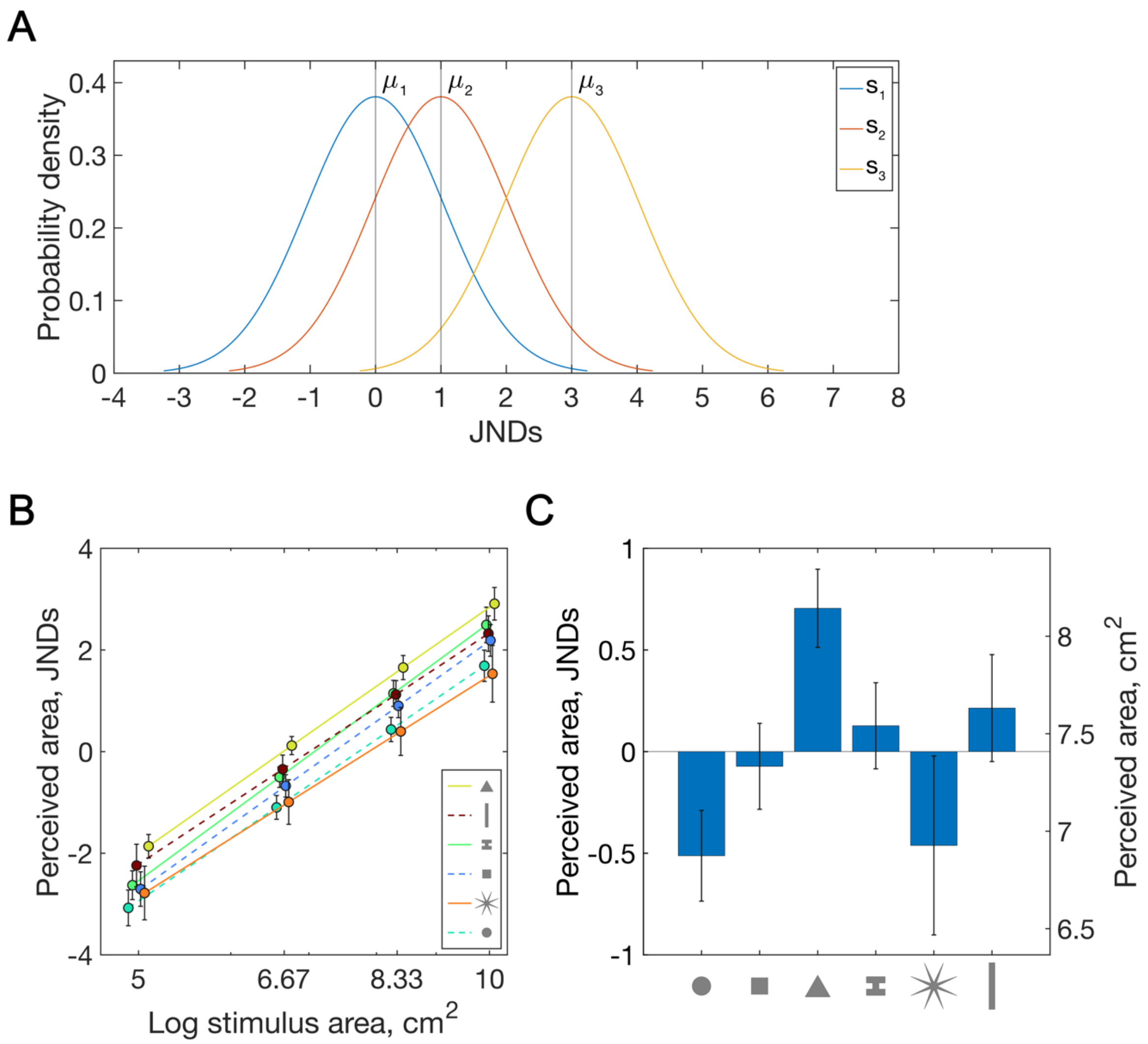

Figure 2.3 Thurstonian scaling and data for Experiment 1. (A) A simplified scenario with only three unique stimuli. Perceived area for each stimulus is represented by a Gaussian with σ = 1.05 JNDs. (B) Perceived area in JNDs for each condition as a function of stimulus size. (C) Data summarised by averaging across stimulus size. Error bars give 95% confidence intervals.

Although the method assumes a common noise parameter for all stimuli, estimates of mean bias (averaged across size, as in Figure 2.3 C) are minimally affected even by substantial deviations from this assumption (see Appendix A, A.1 Simulations).

We compared two nested models of perceived area. In the first, JNDs were fitted independently to all unique stimuli (degrees of freedom = $n_{conditions} \times n_{sizes}$ - 1). The second model assumes Weber's law within each condition, i.e. that perceived area in JNDs increases linearly with log(area) (degrees of freedom = $n_{conditions} \times 2 - 1$); see the straight line fits in Figure 2.3 B. For all experiments, the second model was clearly preferable (likelihood-ratio tests, all $p > .99$).

2.4 Experiment 1: Common shapes

In Experiment 1 we quantified relative biases in perceived area for shapes commonly used in the literature (disk, triangle, square, 8:1 ratio rectangle). In addition, we included two concave shapes (i.e., at least one interior angle greater than 180°): an H-shape and an 8-pointed star. This set decoupled potential correlates of perceived area such as compactness, elongation, height and perimeter length. For example, the H-shape and square have similar height and width but different compactness. The star and rectangle are both low in compactness with similar height but differ in elongation.

2.4.1 Trials

Stimuli were presented in all four sizes. Experimental trials (N = 516) excluded the least informative stimulus pairings (same condition and size; same condition with size difference greater than one step; we expected the latter to be unambiguous). All other possible pairings were included, in two repetitions. Control trials (N = 12) were intermingled with experimental trials. Twelve practice trials preceded the first experimental block.

2.4.2 Results

Figure 2.3 B shows perceived area in relative JNDs as a function of true area. Our method gives relative, not absolute JNDs; the mean JND value (across conditions and sizes) was set to 0 for each participant. The fitted lines for each condition are roughly parallel, suggesting that biases in area perception are broadly scale-invariant; triangles were perceived as 19% larger than same-area disks, irrespective of absolute size. Figure 2.3 C summarises the same data, averaged across size. To provide a more intuitive representation of the area biases, we converted JNDs to perceived area in cm^2 (righthand y-axis) under the simple assumption that squares (common to all experiments) were perceived veridically. Selecting a different shape would simply scale the values up or down, without affecting the relative biases between shapes in percentage terms.

Close inspection of Figure 2.3 B reveals that two shapes (rectangle, 8-point star) have slightly shallower slopes than the others. There are two possible explanations: First, biases may not be entirely scale invariant: at one viewing distance, a rectangle might appear larger than an equal area 'H', but on moving closer, this relationship is reversed. Alternatively, the rectangle and star might be associated with greater uncertainty than other shapes, i.e., a violation of the equal noise assumption of our fitting method (see, Appendix A, A.1 Simulations, Figure A1). The latter presents a more likely explanation for the observed small slope variations; note that the star and rectangle are also associated with the largest inter-observer variation (error bars in Figure 2.3 C). For this reason, we focus our analyses and interpretation on the mean shape-related biases (averaged across size, treated as scale invariant), whose estimation is negligibly affected by noise variations across stimuli. Standardized effect sizes for these biases are reported in Appendix A, Figure A3.

2.4.3 Interim discussion

As expected, triangles were perceived as larger than squares or disks (Anastasi, 1936; Dresslar, 1894; Fisher & Foster, 1968a; Martinez & Dawson, 1973; Warren & Pinneau, 1955). In accordance with the elongation bias (Krishna, 2006), rectangles were perceptually larger than squares. That triangles were perceived to be the largest shape suggests that height, elongation, or compactness alone cannot explain the biases. Surprisingly, the 8-point star – the widest, tallest and least compact shape in the set – was perceived as one of the smallest. This is at odds with suggestions that less compact or taller shapes are perceived as larger (Dresslar, 1894; Owen, 1970; Smets, 1970). One possibility is that shapes with a circular, or near circular, convex hull (such as the star) are underestimated, in a similar way to disks.

However, perception of the star was more variable across observers than other shapes (see error bars in Figure 2.3 C), suggesting that observers may use different strategies to assess this shape.

2.5 Experiment 2: Convex hull and compactness

In Experiment 2 we explore how the form of a shape's convex hull affects perceived area and how this interacts with compactness. The stimuli included four convex shapes (disk, equilateral triangle, square, octagon) plus two subsets of concave shapes whose convex hulls matched the polygons (triangle, square, octagon). The two subsets had moderate (rotated 'V-' and 'H-shape', and 'X-shape') and low compactness (3-, 4-, 8-pointed star shapes).

2.5.1 Trials

Stimuli were presented in sizes 1–3 in the experimental trials. Participants completed 870 trials (850 experimental, 20 control) in 2 blocks. Selection criteria for trials were identical to those of Experiment 1.

2.5.2 Results

Figure 2.4 shows the results of Experiment 2. We uncovered larger relative biases than in Experiment 1. The perceptually largest shape (3-point star) was perceived to be considerably larger (52%) than the perceptually smallest shape (disk). For shapes common to Experiments 1 and 2 (disk, triangle, square, 8-point star), the biases are broadly consistent except that the 8-point star was perceived as smaller than the square in Experiment 1, but this was reversed in Experiment 2. However, the star was again associated with the most variability.

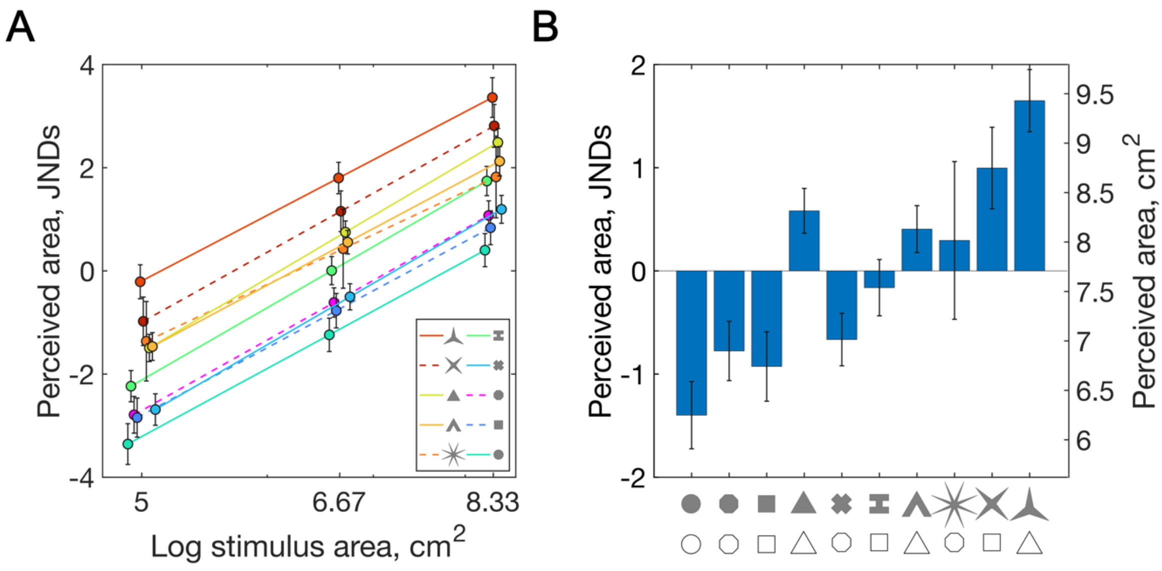

Figure 2.4 Data for Experiment 2. (A) Perceived area in JNDs for each condition as a function of stimulus size. (B) Data summarised by averaging across stimulus size. Error bars give 95% confidence intervals. Open symbols on the x axis bottom row represent the shape of the convex hull for each stimulus.

2.5.3 Interim discussion

As for Experiment 1, simple models of area perception based on height, width or their combination (Krider et al., 2001; Yousif et al., 2020; Yousif & Keil, 2019) cannot explain the pattern of biases.

Within both subsets of concave stimuli, shapes with a triangular convex hull were perceived as larger than their square and octagonal counterparts (i.e., 3-point stars larger than 4- and 8-point stars, and ‘v’ shapes larger than ‘h’ and ‘x’ shapes). This pattern replicates the bias observed for convex shapes, and is consistent with previous reports for star-shaped stimuli (Martinez & Dawson, 1973). Within each convex hull shape, perceived area increased with decreasing compactness (e.g. triangle < V-shape < 3-point star). Thus, compactness negatively correlates with perceived area but is not the whole story; the 3-point star is more compact, but perceptually larger than the 8-point star.

2.6 Experiment 3: Orientation

Experiments 1 and 2 suggest that height, width or their combination cannot fully explain area biases. Nonetheless, changes in orientation (with corresponding changes to height and width) do affect perceived area, as when rotating a square by 45° (Mach, 1897). Here we explore the effects of orientation by presenting stimuli in their canonical orientation or with their longest linear length vertical to maximise stimulus height. Stimuli included the four convex shapes from Experiment 2, with the three polygons presented in both orientations (Figure 2.2 A).

2.6.1 Trials

Stimuli were presented in all four sizes. Participants completed 704 trials (690 experimental, 14 control) across two blocks. Experimental trials included all possible pairings except (i) same shape and orientation: only stimuli separated by one size step were compared, (ii) same shape and different orientation: same-size and one size step away were compared. The 14 practice trials featured stimuli only in their canonical orientations (sizes 1 and 4).

2.6.2 Results

The results of Experiment 3 are shown in Figure 2.5. Rotated triangles (perceptually largest) were perceived as 33% larger than disks (perceptually smallest). The relative biases for shapes in their canonical orientations are consistent with those observed in Experiments 1 and 2.

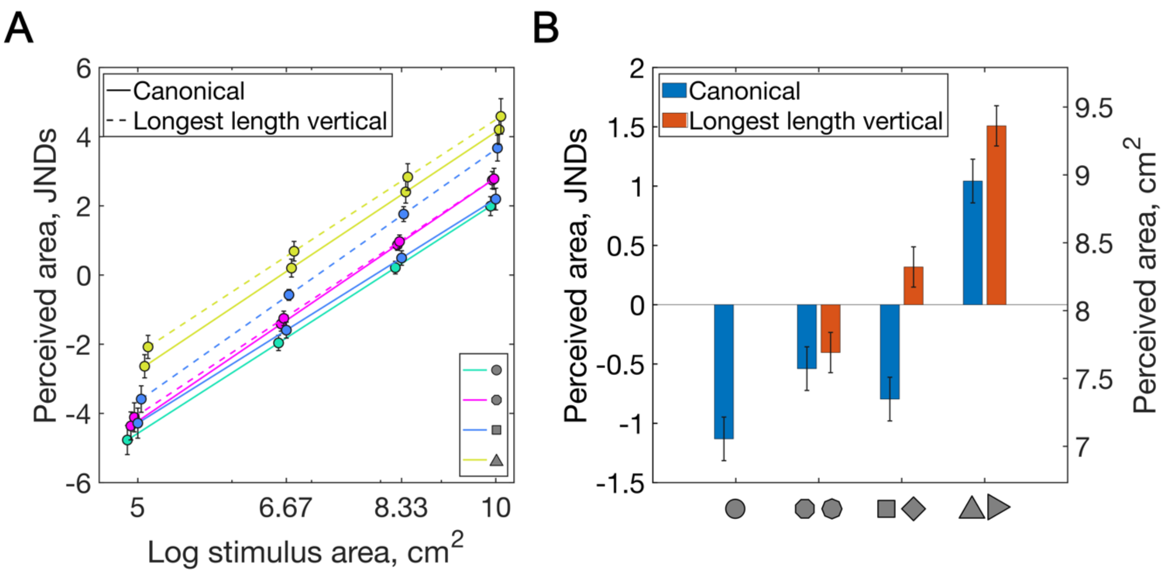

Figure 2.5 Data for Experiment 3. (A) Perceived area in JNDs for each condition as a function of stimulus size. Line colours represent stimulus shape; line types represent stimulus orientation. (B) Data summarised by averaging across stimulus size. Bar colours represent shape orientation. Error bars give 95% confidence intervals.

2.6.3 Interim discussion

There was a clear effect of orientation: all shapes were perceived as larger when presented with the longest length vertical. The orientation effect was largest for squares, with a smaller difference for triangles, and a negligible difference for octagons, i.e. a greater change in height was associated with a greater change in perceived area.

2.7 Experiment 4: Orientation and elongation

Here we further probe the effects of orientation and determine how this interacts with elongation. The stimulus set (Figure 2.2 A) included rectangles in four aspect ratios (1:1, 1:2, 1:4, 1:8), presented in three orientations. These were (i) longest edge vertical, (ii) longest length vertical and (iii) longest edge horizontal.

2.7.1 Trials

Experimental trials employed stimulus sizes 1–3. Participants completed 800 trials (778 experimental, 22 control) across two blocks. Comparisons of identical stimuli (same size and condition) were excluded. All remaining condition pairs were compared up to one size away. The 12 practice trials employed stimuli in their canonical orientation (sizes 1 and 4).

2.7.2 Results

Experiment 4 results are shown in Figure 2.6. Perceived area followed the pattern predicted by the elongation bias: Holding orientation constant, perceived area increased with elongation. Vertical 1:8 rectangles were perceived as 22% larger than canonical squares. Similar to Experiment 3, orientation also affected perceived area: all rectangles were perceived as smaller when presented horizontally. The difference between the two vertical orientations (longest edge vertical versus longest length vertical) was small and inconsistent.

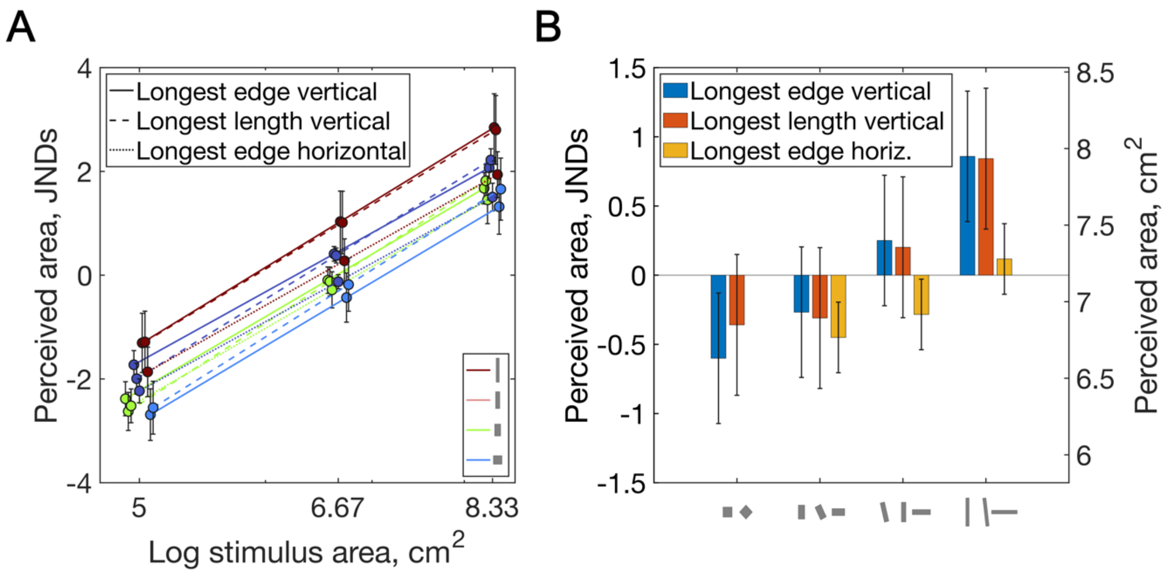

Figure 2.6 Data for Experiment 4. (A) Perceived area in JNDs for each condition as a function of stimulus size. Line colours represent stimulus shape; Line types represent stimulus orientation. (B) Data summarised by averaging across stimulus size. Bar colours represent shape orientation. Error bars give 95% CIs.

2.7.3 Interim discussion

The effect of orientation increased with elongation, consistent with the idea that perceived area is positively correlated (albeit imperfectly) with height (or negatively correlated with width). This is broadly consistent with the relative overestimation of vertical line length seen in the vertical-horizontal illusion and bisection illusion (Valentine, 1912; U. Wolfe et al., 2005). In summary, perceived area increases with elongation and this effect interacts with orientation.

2.8 Modelling biases in area perception

Our four experiments demonstrate that biases in perceived area are substantial and are associated with various shape metrics including compactness, elongation, orientation and width or height. Here we determine how different shape metrics combine to predict perceived area, across all stimulus conditions.

Raw data (2AFC responses) for stimulus sizes 1–3 (common to all four experiments) were pooled across observers and experiments. This allowed us to determine the maximum likelihood set of JND values (i.e. perceived area for each condition in units of discrimination), given all data. The results are shown in Figure 2.7.

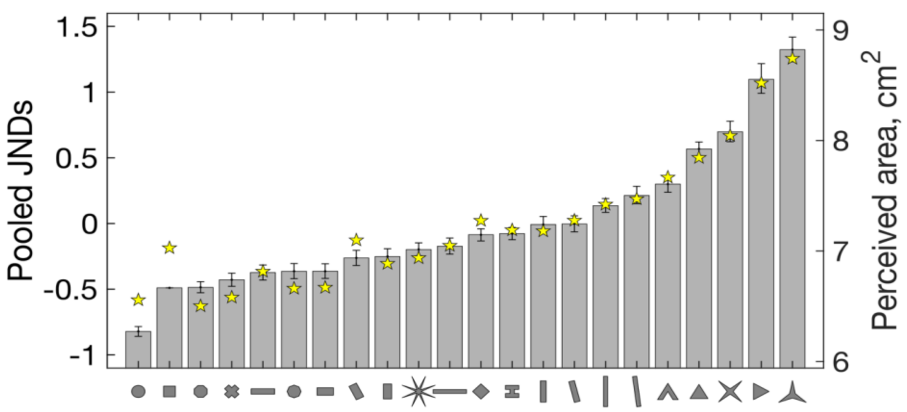

Figure 2.7 JNDs for each condition, for data pooled across observers and experiments, converted to perceived area and averaged across size. Error bars give 95% CIs from bootstrapping (10,000 samples). Yellow stars show the model fit for each condition.

Note that the range of JND values is compressed relative to the JNDs derived from individual observers (Figure 2.3–Figure 2.6). This is because inter-observer variation is conflated with uncertainty within the pooled data. Pooling, therefore, underestimates relative biases in units of discrimination (JNDs). However, relative biases in percentage terms (A is perceptually X% larger than B) are preserved. As above, JNDs were converted into perceived area in cm^2 by assuming that the common shape (canonical square) is perceived veridically (see righthand y-axis of Figure 2.7).

We compared linear regression models that characterise perceived area as a function of various shape metrics. Our set of candidate predictors included shape descriptors previously proposed in the literature (e.g. compactness, height, aspect ratio, convexity) in addition to some of our own design (e.g. width of object base, height of centroid, see Figure 2.1). To capture scale invariant biases, all predictors were proportional to stimulus area: linear measures (e.g. height) were multiplied by the diameter of the equal-area disk, and ratio measures (e.g. aspect ratio) were multiplied by stimulus area. The full list of candidate predictors, with definitions, is given in Appendix A, Table A1.

Regression models were evaluated and compared via leave-one-out cross validation (leaving out each of the 22 stimulus conditions, in turn). This approach prevents overfitting by quantifying how well each model generalises to novel stimuli. Whilst many candidate predictors are partially correlated, cross validation only rewards the addition of predictors that explain additional systematic variance in the data.

The selected model includes 7 predictors (Figure 2.1), and accounts for 98% of the variance in perceived area across the 66 unique stimuli (combinations of shape, orientation, and size), see Figure 2.8. The influence of each predictor on every condition is given in Appendix A, Figure A3.

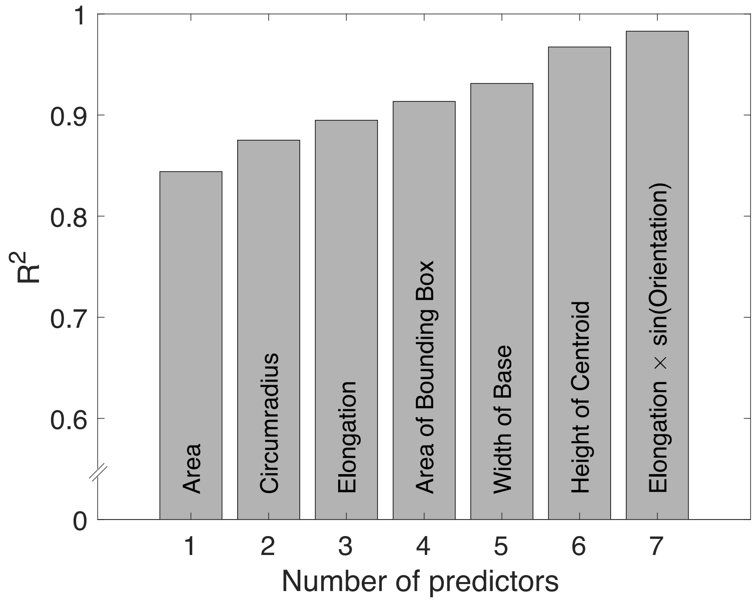

Figure 2.8 Variance in perceived area (cm^2) explained by the addition of each predictor (see Figure 2.1) in the model. The order of predictors follows the maximum increase in R^2 with each addition.

Clearly, perceived area increases with stimulus area, accounting for 84% of the variance in perceived area (β = 0.90, 95% CI [0.84, 0.96]). The second predictor, circumradius, is a measure of compactness. This is positively correlated with perceived area; less compact shapes are perceived as larger (β = 2.35, 95% CI [2.09, 2.61]). The third and fourth predictors are perhaps best interpreted as modulating the relationship between compactness and perceived area (both are positively correlated with compactness but have negative coefficients). Elongation (the aspect ratio of the circumscribing ellipse, β = -0.05, 95% CI [-0.06, -0.03]) reflects the observation that the most elongated shapes (1:8 and 1:4 rectangles) were perceived as smaller than shapes of a comparable compactness but with aspect ratio close to 1:1. The area of the bounding box is also negatively associated with perceived area in the model (β = -0.38, 95% CI [-0.49, -0.28]), such that some of the least compact shapes (the stars) and in particular the 8-pointed star are associated with reduced perceived area than predicted by compactness alone.

The remaining predictors (5–7) are all orientation dependent, and together capture the observation that more bottom-heavy / wide shapes are perceived as smaller than tall / top-heavy ones. Base width (average width of bottom 1/8 of stimulus' convex hull) has a negative coefficient (β

= -0.77, 95% CI [-0.88, -0.66]), reflecting the observation that the triangle and square both appear larger when presented with their longest edge vertical (and resting on a corner). It also captures the orientation effect for rectangles, i.e. that they are perceived as smaller when presented horizontally. The 7th predictor is elongation × sin(orientation), see Figure 2.1, reflecting the larger orientation effect for elongated stimuli (β = -0.38, 95% CI [-0.49, -0.28]). Predictor 6, height of centroid, can be thought of as a suppressor variable: it is negatively correlated with base width (r = -0.30), and with elongation × sin(orientation) (r = -0.38). It thus suppresses some of the (over-inflated) effects introduced by predictors 5 and 7, while reflecting the fact that the star is perceived as smaller than other shapes of low compactness (β = -1.43, 95% CI [-1.65, -1.21]).

2.9 Discussion and conclusions

We quantified biases in area perception for a wide range of shapes and orientations. These biases were substantial and highly consistent across participants and experiments, with the exception of one shape: the 8-pointed star. We used an assumption-free method, cross validation, to derive a model that provides an excellent account of these biases, explaining 98% of response variance in terms of simple shape metrics. Our model quantifies biases that have previously been reported in qualitative terms (e.g. triangles > disks). Simultaneously, the model accounts for biases demonstrated here with novel shapes.

Consistent with previous suggestions (Dresslar, 1894; Owen, 1970), compactness is strongly correlated with perceived area, with less compact shapes appearing to be larger. Within squares and rectangles, perceived area also increases with elongation, as previously suggested for volumetric judgments of cuboids (Krishna, 2006). However, elongation has a negative coefficient in the model: shapes that are concentrated along one axis (i.e., rectangles) are not perceptually enlarged to the same degree as stimuli with similar compactness but an aspect ratio close to one. Elongation interacts with orientation in its effects on perceived area: when very elongated rectangles are rotated there is a bigger change in perceived area than for less elongated stimuli. More generally, shapes that are wider or more bottom-heavy appear to be smaller.

Previous work has, in general, focussed on a small set of objects and identified a single feature that correlates with perceived area. For example, one might assume that triangles look larger because they are taller (and wider) than squares and disks (Krider et al., 2001), but this rule fails when comparing equilateral triangles and rectangles. Similarly, 'additive area' (Yousif et al., 2020) captures the difference in perceived area for squares presented in different orientations, but fails to explain the biases across our larger set of shapes.

A big unanswered question remains: why do these biases occur? In general, our sensorimotor system recalibrates in order to reduce perceptual biases (Adams et al., 2001; Burge et al., 2008). However, this mechanism appears to fail in the case of area perception. There is some evidence that biases in area perception have correlates in volume perception: Tetrahedrons are perceived to be larger than cubes and spheres (Kahrimanovic et al., 2010b), and elongated cuboids appear larger than cubes (Krishna, 2006). This apparent translation of area biases into erroneous volume and weight perception (Kahrimanovic et al., 2011) makes it all the more peculiar that these perceptual biases are not corrected during everyday object handling. On the other hand, it is well known that grasping forces adapt rapidly in the size-weight illusion, while the perceptual illusion persists (Flanagan & Beltzner, 2000).

Although we have no clear explanation of why these systematic biases in area perception occur, we can try to mitigate their effects. When selecting a partner for a wife-carrying contest (https://en.wikipedia.org/wiki/Wife-carrying), go for a tall, skinny one rather than a deceptively compact stout one. When subsequent hunger strikes, if in doubt, choose the circular pizza.

Chapter 3 A new model of heaviness perception as a function of weight, size, and density

Abstract

In the size-weight illusion (SWI), the smaller of two same-weight, same apparent material objects is perceived as heavier. The SWI has proved difficult to explain via Bayesian models, in which expected weight from size (smaller = lighter) is integrated with felt weight, predicting the opposite effect. Other models have thus proposed that weight and density are perceptually confounded. Here we ask whether the SWI occurs when two same-weight objects differ in volume by a small amount, or whether, given this small conflict between weight and expected weight, these estimates are integrated, rather than causing a repulsion effect. More broadly, we seek to establish a general model of perceived weight for all pairs of objects, that differ in weight and / or density (and / or size) by varying amounts. To this end, we measured perceived weight for pairs of stimuli (cubes) which varied in size (difference range: 50–500 cm^3), and / or weight (25–400 g), and / or density (0.02–0.36 g/cm^3). In a visuo-haptic task, participants (N = 30) grasped and lifted pairs of cubes, and reported their perceived heaviness via magnitude estimation. The SWI still occurred at very small size / density differences, and increased with density difference. Across all object pairs, perceived weight was well explained by a model (R^2 = .98) that included a positive influence of the target and non-target weights and the target density, but a negative influence of the non-target density. Critically, the influence of both densities on perceived weight was strongly modulated by weight difference, being three times as large for zero / small weight differences than large differences. Thus, it is only under the non-accidental (highly unusual) conditions of typical SWI studies that we confuse / confound weight with density to any substantial extent. Our model provides a quantitative, accurate and generalised account of weight perception for pairs of objects across a variety of weight and size conditions.

Notes

Alisha Andrews and Michaela Treščáková helped collecting data for the experiment pilot (N = 6) which was submitted for their BSc (A. A.) and MSc (M. T.) dissertations at the University of Southampton. Data from the three non-naïve participants from the pilot were re-analysed separately as part of the participant sample for this chapter.

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CRediT (Contributor Roles Taxonomy)
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administration, Software, Visualization, Writing – original draft, Writing – review and editing
Erich W. Graf: Conceptualization, Project administration, Resources, Supervision, Writing – review & editing
Wendy J. Adams: Conceptualization, Formal analysis, Funding acquisition, Methodology, Project administration, Resources, Supervision, Writing – review & editing

3.1 Introduction

In the well-known size-weight illusion (SWI) (Charpentier, 1891), participants are typically presented with one large and one small object, apparently of the same material, but in fact of equal weight. On lifting, the smaller object is perceived as heavier than the larger one. The illusion is robust, persisting even when the lifter knows that the objects have equal weight (Buckingham, 2014; Flournoy, 1894), and after grasping forces have adapted to the actual weight of the objects (Flanagan & Beltzner, 2000), see below.

Although the SWI has long been investigated as a means to understanding human heaviness perception, its mechanism remains largely unexplained (for reviews, see Buckingham, 2014; Dijker, 2014; Saccone & Chouinard, 2019). Proposed explanations differ on the hypothesised causes of the illusion. Firstly, the SWI has been explained as a 'contrast' or 'repulsion' effect between expected and actual weight. Second, the lifter confounds / combines felt weight with other physical features (generally density: Thouless, 1932; Wolf et al., 2018; Wolf & Drewing, 2020; but also rotational inertia: Amazeen, 1997; Amazeen & Turvey, 1996). Finally, the SWI has been modelled using a causal inference model in which different priors over size and density compete (Peters et al., 2016).

In 'contrast' or 'violated expectation' accounts it is noted that object size is used as a predictive feature for object weight (Cole, 2008; Gordon et al., 1991c, 1991a). Smaller objects are expected to be lighter than larger objects, particularly when they are apparently made of the same material – that is, of equal density – consistent with real-world associations between size and weight. Upon lifting, the smaller object is heavier than expected, and the suggestion is that this disparity between expectation and sensory evidence causes the illusion. Evidence for this conflict between expected and actual weight comes from measuring the forces used to grip and lift the two objects. On initial lifts, subjects use more force to lift the larger (expected to be heavier) object than to lift the smaller one (Davis & Roberts, 1976; Flanagan & Beltzner, 2000). These misapplied forces have thus been suggested to drive the illusion: the larger object, being lighter than expected, is lifted faster, and vice versa, producing in a mismatch between the expected and actual sensory feedback (Davis & Roberts, 1976; Dijker, 2014). However, it has been widely reported that grasping forces adjust to the weight of the object after a few lifts, while the illusion persists (Buckingham, 2014; Flanagan & Beltzner, 2000; Grandy & Westwood, 2006; Mon-Williams & Murray, 2000).

Because the illusion is at odds with the perceiver's prior expectations (that the smaller object should be lighter) the SWI has been labelled 'anti-Bayesian' (Brayanov & Smith, 2010; Ernst, 2009). The Bayesian framework has successfully described numerous perceptual phenomena in terms of the integration (generally a weighted averaging) of prior expectations with current sensory cues (Ernst, 2006; Ernst & Bülthoff, 2004; Kersten et al., 2004; Knill & Richards, 1996). In the SWI, however, the Bayesian prediction is that the smaller object will be perceived as lighter than the larger object. Figure 3.1 (A, B) illustrates this: Figure 3.1 A represents the stimulus space, with each stimulus combination (weight, volume, density log ratio) indicated by the labels in each quadrant: Figure 3.1 B shows three hypothetical priors on the same axes: The red lines show priors corresponding to the expectation for the larger object to be heavier and same-density (solid line), or heavier and slightly less dense (dashed line) (see Peters et al., 2015); both these priors predict the opposite of the SWI (shown by the green dots; black dots show the stimuli), hence the 'anti-Bayesian' label. The green line shows one hypothetical prior that would predict the SWI (smaller = heavier); however, this is an implausible prior, as it would not be consistent with environmental statistics (where size and weight are positively correlated).

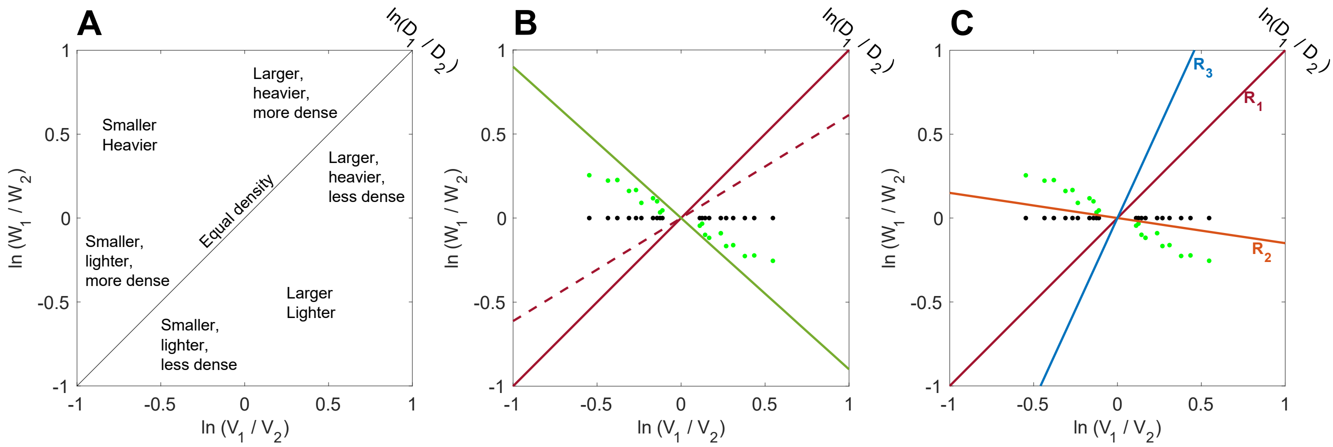


Figure 3.1 (A) Stimulus space for pairs of stimuli in log-log coordinates, showing weight ratio and volume ratio; the diagonal line indicates equal-density. The labels in each quadrant indicate possible stimulus combinations relating object 1 to object 2. (B) SWI and three hypothetical priors over volume and weight ratio, plotted on the same axes. The black dots show SWI stimuli, green dots show perceived weight for the same stimuli (data from the current experiment, see below). The red lines show priors corresponding to the expectation for the larger object to be heavier and same density (solid line), or heavier and slightly less dense (dashed line) (as observed by Peters et al., 2015 for man-made, liftable objects). The green line represents one hypothetical prior which would predict the SWI (smaller = heavier). (C) Peters and colleagues' (2016) model priors, each corresponding to a different density / volume relationship (R); R_1 = equal-density prior; R_2 = smaller is denser and heavier; R_3 = smaller is less dense and lighter.

Other models suggest that we are unable to fully separate density and weight (J. Ross & Di Lollo, 1970), or proactively combine them (Wolf et al., 2018). In these models, weight perception is biased, such that denser objects are systematically perceived as heavier. Indeed, heaviness and density estimation appear to be closely related: perceived weight difference increases with increasing density difference (J. Ross & Di Lollo, 1970); objects of greater weight are reported as denser (Merken & van Polanen, 2020), and weight difference discrimination is improved when density varies in the same direction as weight (Kawai, 2002). Within typical SWI stimuli, weight is kept constant such that density is negatively correlated with volume. Thus, size might contribute to the illusion as it is used to infer density ('density-weight-illusion', Harshfield & DeHardt, 1970; Thouless, 1932; Wolf et al., 2018).

Models that have challenged the 'anti-Bayesian' label (Peters et al., 2016; Wolf et al., 2018) have thus focussed on the role of object density (inferred from volume and weight) rather than size per se in the genesis of the illusion. Wolf and colleagues (2018; Wolf & Drewing, 2020) proposed a cue-integration model where perceived heaviness is a weighted average of two heaviness estimates, one derived from the object's weight and one derived from the object's inferred density. The argument is that, because denser objects tend to be heavier, it is adaptive / Bayesian to combine estimates of weight and density in estimating weight. In the model, the weight (relative influence) assigned to density depends on its relative reliability; in accordance with this, the lower the reliability of (visual or haptic) size information, the less influence density has in the final heaviness estimate (Wolf et al., 2018). In the limiting case, of course, when size information is completely absent (objects lifted via strings / a handle, and no vision), the illusion disappears (Ellis & Lederman, 1993; Plaisier et al., 2019; Wolf et al., 2018). It is not clear, however, why we should combine weight and density information in this way, when volume information is available to condition the density-weight relationship. Not only is density a poor cue to weight (when not conditioned on size) but the two estimates will also have correlated noise (thus negating the normal benefits of integration).

Peters and colleagues (2016) proposed a Bayesian model that includes three competing priors (see Figure 3.1C) over the relationship between the size ratio of two objects and their density ratio: Objects (i) have equal density, irrespective of size ratio or (ii) smaller objects are denser and heavier, or (iii) smaller objects are less dense (much lighter) than larger objects. When typical SWI objects are presented, the 'smaller is denser and heavier' prior dominates, resulting in the illusion. However, it is not clear whether their model can account for weight perception more broadly as it was only evaluated on pairs of equal-weight objects (i.e., typical SWI stimuli). In this model, the predicted effect of relative density on relative weight perception depends on the volume and density ratios of the stimuli and thus which prior dominates perception.

It is well known that the SWI magnitude increases with volume (density) difference (Cross & Rotkin, 1975; Ellis & Lederman, 1993; Peters et al., 2016; J. Ross & Di Lollo, 1970; Saccone et al., 2019) and this relationship is predicted by all of the above models. SWI demonstrations tend to use fairly large, obvious differences in object volume[4], in order to obtain a reliable illusion. However, under small conflicts between size-based expectations of weight and direct sensory evidence, estimates of heaviness might be integrated: It is known that integration occurs for small conflicts but breaks down under conditions of large conflict (Knill, 2007; Landy et al., 1995). If integration (the opposite of the SWI) occurs under small conflicts, this would be broadly in line with the idea that the SWI is driven by violated expectations, and would also be consistent with the competing priors Bayesian model (Peters et al., 2016), but would contradict models in which weight and density are confounded / integrated in general (Wolf et al., 2018; Wolf & Drewing, 2020).

Furthermore, it is unclear whether humans conflate weight and density in all situations (such as when a smaller object is slightly less dense), or only in the specific and highly unusual / non-accidental conditions of SWI stimuli, where size-weight expectations are violated, i.e., when weight is held constant and volume varies (as in the SWI).

Here we asked: Can we develop a generalised model of weight perception for varied combinations of size, weight and density? To this end we measured perceived weight for pairs of cubes from a large set in which volume, and / or weight, and / or density were varied in small steps. Size and density were decorrelated in the full set to prevent learning of unusual size - weight associations. To preview the main findings: 1) The SWI still occurred at very small size (density) differences, and thus does not rely on large conflicts; 2) Perceived weight was positively correlated

[4] Density ratios (less dense / denser) for the minimal density differences reported in the literature are: 0.67 (Plaisier & Smeets, 2015; Experiment 4); 0.64 (Stevens & Rubin, 1970); 0.52 (Ellis & Lederman, 1993); 0.51 (Walker et al., 2017; Experiment 1); larger density ratios were tested by Wolf and colleagues (2018): 0.87, in a set of same-volume (increasing density) stimuli. The largest density ratio tested by Peters and colleagues (2016) was 0.42. To our knowledge, only Kawai and colleagues (2007) investigated smaller differences (density ratio: 0.98), by manipulating the visual size of stimuli lifted via strings in an augmented-reality setting; however the authors suggest that heaviness perception might be veridical at very small (but still detectable) volume differences, it should be noted that the magnitude of the SWI is expected to be smaller in absence of haptic size cues (Ellis & Lederman, 1993; Masin & Crestoni, 1988) and in virtual environments (Buckingham, 2019; Rohrbach et al., 2021).

with the true weight of both the target and non-target stimulus, in addition to the target object's density; 3) However, the effect of both densities on perceived weight was strongly modulated by weight difference: three times larger at small / zero weight differences, than at large weight differences. The non-target object's density was negatively associated with the target's perceived weight, but only had an effect when the two objects had similar weight. Thus, the perceptual system does conflate weight and density, but this effect is much larger in the specific and peculiar conditions of typical studies of the SWI.

3.2 Methods

3.2.1 Participants

Thirty naïve participants (age *M* 21.4, *SD* 5.6 years) from the University of Southampton Psychology student community took part in the experiment; undergraduate students were compensated for their participation with course credits; all reported normal or corrected-to-normal eyesight and no upper limb impairment. The experiment was approved by the University of Southampton Psychology Ethics Committee; all participants gave prior informed consent.

3.2.2 Stimuli

Stimuli were 15 plastic cubes (3D-printed in black polylactic acid) of variable volume, weight and density (Figure 3.2 A–B). Each cube was uniformly filled with sand, polystyrene pellets, and / or brass; a panel concealed at the bottom allowed adjustments to weight while keeping the appearance as homogeneous. Weight was measured with a 0.1 g resolution.

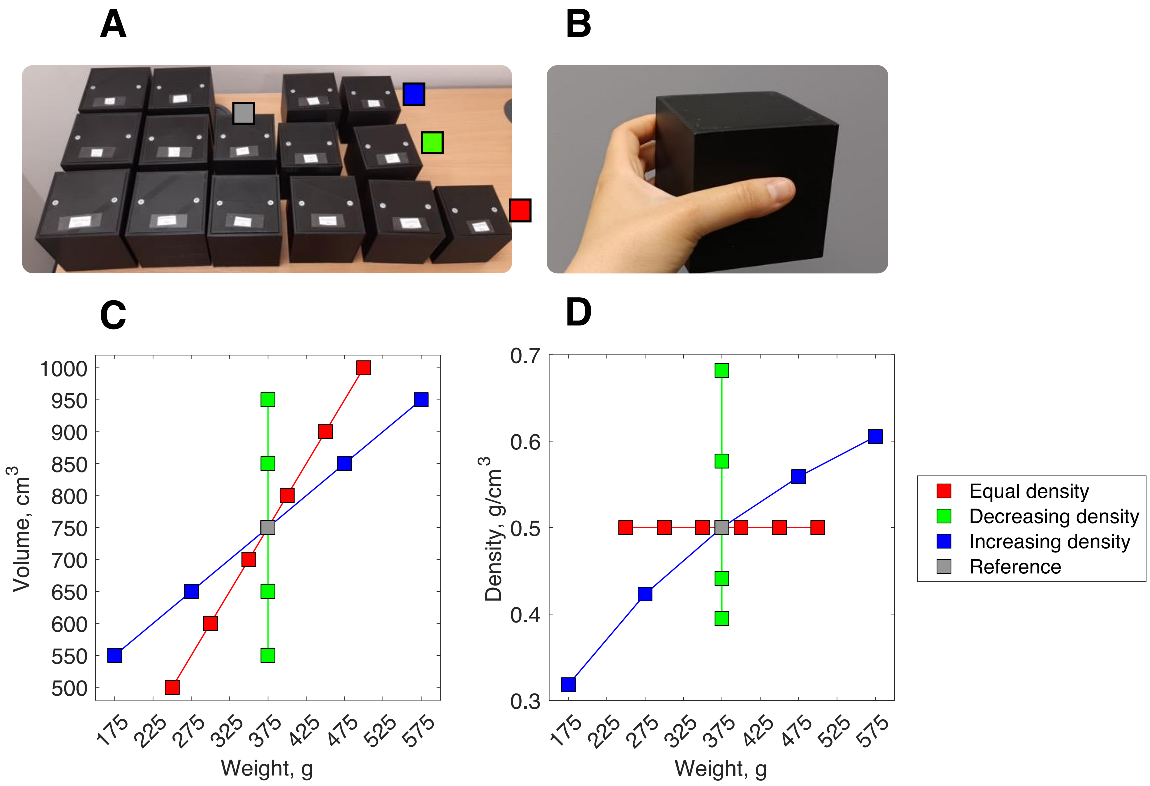


Figure 3.2 Stimulus set. (A) Full stimulus set, see legend in (D). Note that the stimuli are shown upside-down in the picture. (B) 500 cm^3 stimulus for scale. (C) Stimulus volume as a function of weight. (D) Stimulus density ad a function of weight.

The complete stimulus set included three subsets (Figure 3.2, C–D), corresponding to different volume / weight relationships: 1) Equal density (red): weight increased linearly with volume (as for ‘normal’ same-material objects); 2) Decreasing density with increasing size (green): equal weight (typical SWI stimuli); 3) Increasing density with increasing size (blue): both weight and density increased with size). The middle cube (volume, weight, density: 750 cm^3, 375 g, 0.5 g / cm^3) was common to all three subsets and also served as the reference stimulus. Volumes within each set increased in 100 cm^3 steps, with the addition of 50 cm^3 steps below and above the reference stimulus in the fixed density set. Minimum (maximum) volume, weight, density differences across any stimulus pair were: 50 (250) cm^3, 25 (200) g, 0.02 (0.18) g / cm^3, respectively. Density ratios for stimulus pairs ranged between 0.47–0.97 (less dense / denser stimulus), 0.58–0.89 for equal weight (SWI) stimulus pairs.

3.2.3 Task and trials

Stimuli were presented in pairs. In each trial, participants simultaneously grasped and lifted both cubes in full visuo-haptic conditions, and verbally reported their weights using magnitude estimation. Experimental trials included all possible stimulus pairs (same- and different-subset pairs), excluding pairs of identical stimuli. This gave 133 pairs in total. Each stimulus pair was presented twice, with left and right position switched between the two trials (266 trials in total). Trials were randomised for each participant.

3.2.4 Setup

The participant and the experimenter sat at opposite sides of a table covered in thick felt cloth to muffle auditory cues to the objects' weights. Two numbers (1, 2) marked the position of the stimuli on the table (centre-to-centre distance between stimuli, centre of stimuli to table edge: both 20 cm) and served to remind participants the order in which to report their estimates. A frame-mounted black curtain was used to control stimulus presentation, such that the participant only ever saw the two stimuli of the current trial. A desktop computer running MATLAB (The MathWorks Inc., 2020) was used to randomise stimulus presentation and record participant responses.

3.2.5 Procedure

The experiment was completed in two identical laboratory sessions on separate days, each lasting between 60–90 minutes. At the beginning of each session, participants were presented with the reference stimulus, and told that this had a weight of '50' (arbitrary units). The reference was presented again every 20 trials. Participants were instructed to look at the stimuli while lifting. Exploration mode was not restricted: Participants were free to grasp and lift the cubes as they thought best (e.g., they could place their fingers on the top of the object, or on the sides; they were not restricted to use index and thumb only). This was done to obtain the most reliable density estimates (Wolf et al., 2018). Trials were not timed, but participants were encouraged to give an answer if they waited more than one minute (approx.).

3.3 Results

Weight estimates for within-subset pairs (i.e., both objects belonged to the same subset) are shown in Figure 3.3, alongside model fits (see 3.4 below). Data have been collapsed across repetitions within participants and *SE* bars show inter-observer variation. We present the perceived weight of the 'target object' (i.e., the object within each pair whose weight is being reported) in terms of its own properties (target weight / density) and the properties of the 'non-target' (weight / density of the other object in the pair). Each object can in turn be considered the 'target'.

For equal density pairs (Figure 3.3 A, B), weight estimates were highly correlated with target weight, as expected. However, they were also positively correlated with the weight of the non-target object (denoted by symbol size in Figure 3.3 A); note that this is an averaging, rather than a contrast effect. Perceived weight difference (perceived target weight – perceived non-target weight) was linearly related to stimulus weight difference (Figure 3.3 B; data are symmetrical in all difference plots (B, D, F) as each object in each stimulus pair serves as 'target').

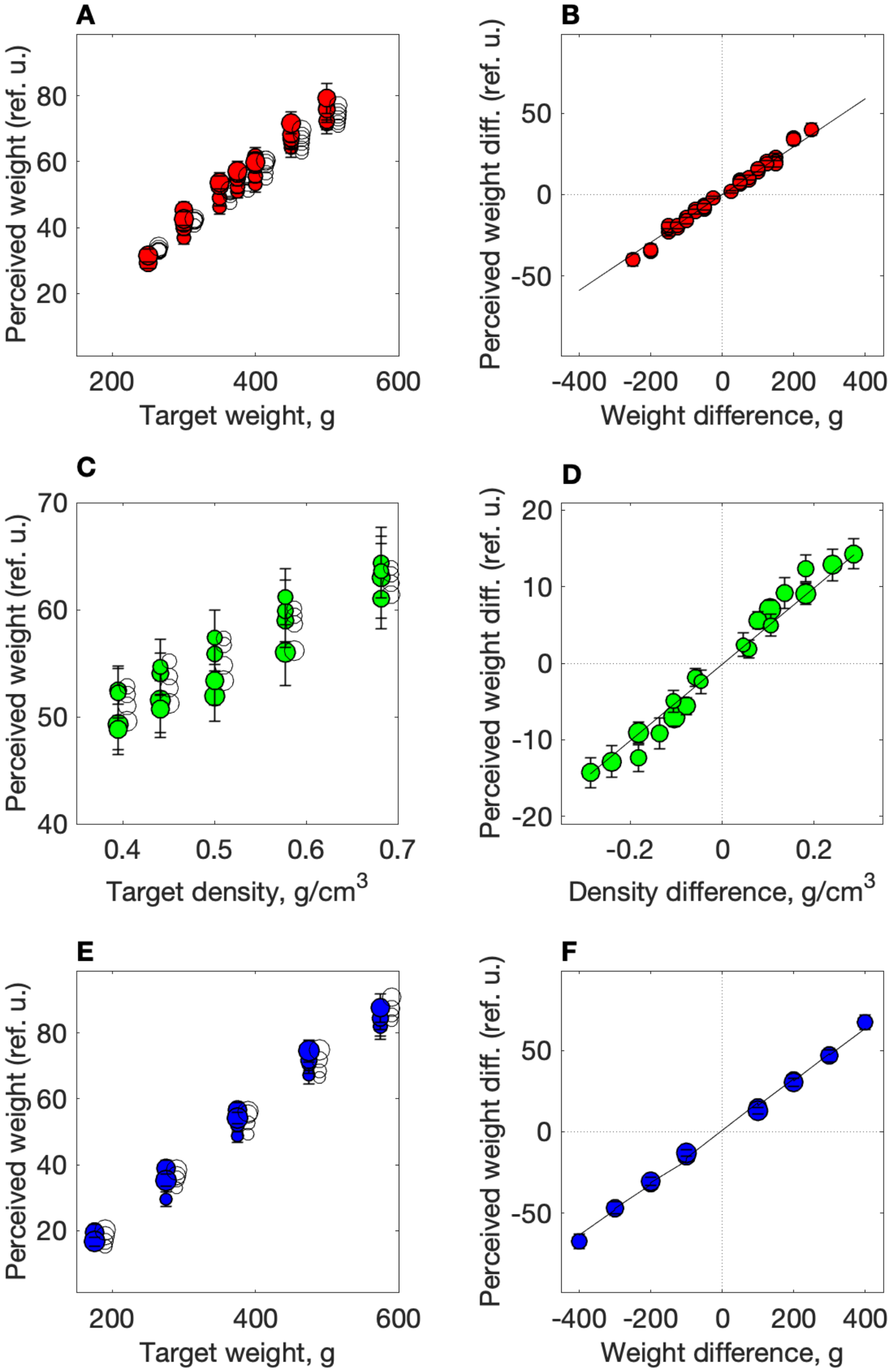


Figure 3.3 Perceived weight (A, C, E) / weight difference (B, D, F) for within-subset stimulus pairs, averaged across participants; error bars show ± 1 SEM; open symbols (A, C, E) / lines (B, D, F) show the model predictions (described below, Equation 3-1, Equation 3-2). (A–B) Equal density pairs: (A) Weight estimates (in reference units) for the target stimulus, as a function of stimulus weight; symbol size indicates the weight of the non-target stimulus. (B) Weight estimates re-expressed as perceived weight difference. (C, D) Equal weight (SWI) pairs: (C) Weight estimates for the target stimulus, as a function of stimulus density; symbol size indicates the density of the non-target stimulus. (D) Perceived weight difference, as a function of density difference; symbol size indicates the average density. (E, F): Increasing density pairs: (E) See A; (F) see B; symbol size indicates the average density.

Weight estimates for the equal weight (SWI) pairs are shown as a function of target density (x-axis) and non-target density (symbol size, Figure 3.3 C). As expected, perceived weight was positively correlated with target density: denser (and smaller) stimuli were perceived as heavier. Perceived weight was negatively correlated with non-target density, the target appearing heavier when judged next to less dense (larger) non-targets. The data are re-expressed in terms of perceived weight difference as a function of density difference in Figure 3.3 D. It is clear that the magnitude of the SWI increases with density difference, and still occurs with small density (or volume) differences. Thus, we find no evidence for integration of expected and sensory weight estimates under small conflicts.

Lastly, in the increasing density pairs (Figure 3.3 E), weight estimates were positively correlated with the target's weight, but also correlated with the non-target weight (as in the equal-density pairs). Figure 3.3 F shows that perceived weight difference increases linearly with true weight difference, which, in this set, also corresponds to a correlated density difference. Comparison of the similar slopes in Figure 3.3 B and F indicates that density difference has a positive but small effect on perceived weight difference in this set, as captured by our model.

Figure 3.4 A summarises results for all trials, including across-subset pairs, in a format that illustrates the variable influence of density on perceived weight. For every stimulus pair, perceived weight difference is presented as a function of stimulus weight difference (x-axis) and stimulus density difference (colour). Lines show the model fit (see Equation 3-2 below). The overall slope of the data shows the strong relationship between true and perceived weight difference. The vertical spread of the different colours shows the effect of density difference on perceived weight difference. The two are not independent: the central 'bulge' clearly shows that the effect of density (or density difference) on perceived weight (or perceived weight difference) is strongly modulated by stimulus weight difference, being much greater for small / zero weight differences. In other words, the conflation of weight and density is dramatically increased under the conditions of typical size-weight stimuli, where weight difference is zero.

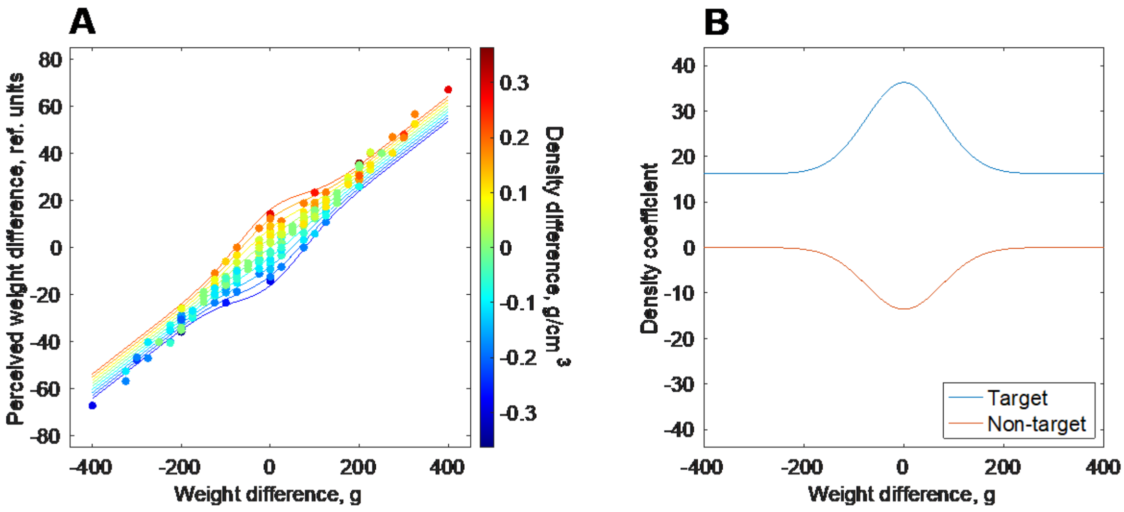


Figure 3.4 (A) Perceived weight difference as a function of weight difference and density difference in the full dataset. Markers show the data, lines show the model predictions (see Equation 3-2) at equally-spaced density differences. (B) Density coefficients as a function of weight difference in the model.

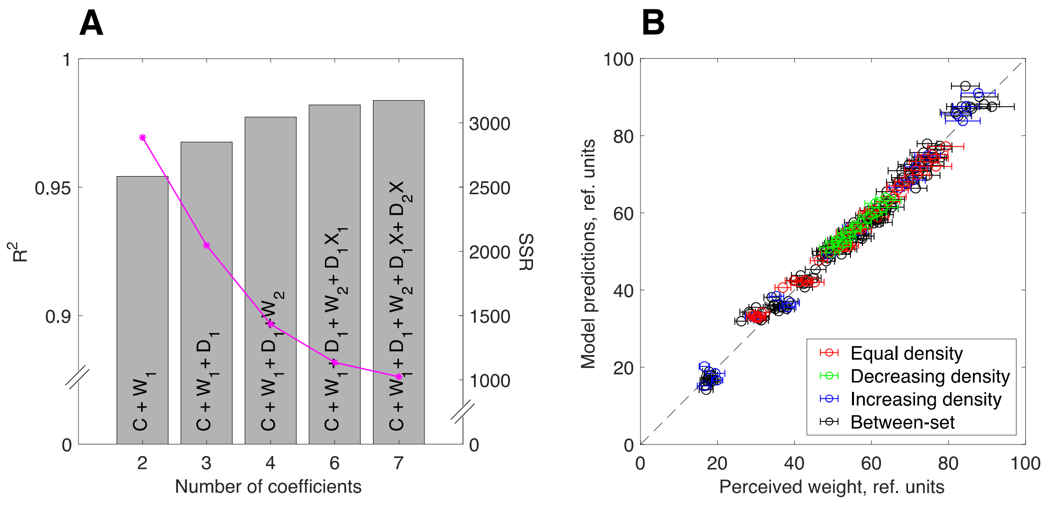
3.4 Model

We developed a model to capture perceived weight ($\widehat{W_1}$) for all stimulus combinations, as a function of the weight and density of the target (W_1, D_1) and non-target (W_2, D_2). All analyses were done in MATLAB and optimal coefficients (minimising the summed squared residuals) were identified via gradient descent. Alternative candidate models are presented in Appendix B, Table B1 and Table B2, alongside model comparison statistics. The best model (Equation 3-1) explains 98% of the variance in the full dataset.

$$\widehat{W}_1 = C + \beta_1 W_1 + \beta_2 D_1 + \beta_3 W_2 + \beta_4 D_1 \mathcal{N}(|W_1 - W_2|, 0, \sigma^2) + \beta_5 D_2 \mathcal{N}(|W_1 - W_2|, 0, \sigma^2)$$

Equation 3-1

Model predictions for $\widehat{W}_1$ as a function of the observed values are shown in Figure 3.5 B.



Note: $X = \mathcal{N}(|W_1 - W_2|, 0, \sigma^2)$

Figure 3.5 (A) Variance in perceived weight explained by the addition of each predictor in the model (grey bars) and corresponding SSR (magenta lines and markers). The order of predictors follows the maximum increase in R^2 with each addition (see Appendix B, Table B1). (B) Model predictions (see Equation 3-1) for perceived weight as a function of participants' estimates for each stimulus, across all object pairs. Error bars give ± 1 SEM.

Accordingly, the perceived weight difference ($\widehat{W}_{\text{diff}}$) between two stimuli can be predicted via the following equation:

$$\widehat{W}_{\text{diff}} = (\beta_1 - \beta_3)(W_1 - W_2) + \beta_2(D_1 - D_2) + \beta_4(D_1 - D_2)\mathcal{N}(|W_1 - W_2|, 0, \sigma^2) + \beta_5(D_2 - D_1)\mathcal{N}(|W_1 - W_2|, 0, \sigma^2)$$

Equation 3-2

Perceived weight was positively correlated with the target weight (β_1 = 0.17) and density (β_2 = 16.22), and with the non-target weight (β_3 = 0.02); the influence of density on perceived weight varied as a function of weight difference, as captured by the coefficients for the two Gaussians: this influence was positive for target density (β_4 = 3790.9; σ = 75.34), and negative for the non-target density (β_5 = -2566.2; σ = 75.34). Figure 3.4 B shows the influence of target and non-target density as a function of weight difference: the influence of density on perceived weight was approximately three times larger for zero weight differences than for large weight differences.

3.5 Discussion

We investigated perceived heaviness in a set of stimuli which varied in weight and / or volume, and / or density in small steps. We reported three novel findings: 1) The SWI still occurred at very small density (size) differences; 2) Across a wide range of stimuli, weight and density are confounded / integrated to estimate weight; 3) However, the influence of object density on perceived weight was strongly modulated by weight difference. Additionally, we provided a model (R^2 = .98) of perceived heaviness that accounts for the SWI as well as capturing biases in perceived weight across a range of variations in stimulus weight and density. Here we will discuss our findings within the broader context of heaviness perception and models of the SWI.

The magnitude of the SWI increased, as expected, with increasing density (size) difference (Cross & Rotkin, 1975; Ellis & Lederman, 1993; Peters et al., 2016; J. Ross & Di Lollo, 1970). We extended this, by measuring the SWI with smaller density differences than those previously tested. We hypothesised that under small conflicts, the SWI might reverse (with integration instead: smaller stimuli would be perceived as lighter). This is also predicted by Peters and colleagues (2016) model; others suggested that the SWI might disappear, as size information is ignored (Kawai et al., 2007). However, the SWI was clearly present for all equal-weight pairs. This is in line, therefore, with Wolf and colleagues (2018; Wolf & Drewing, 2020) previous suggestion that weight is confounded / integrated with density information, and is somewhat at odds with the idea that the SWI is driven by a (substantial) conflict between expected and perceived weight.

Consistent with previous literature, perceived weight was positively correlated with both the object's weight and its density. Across all pairs, irrespective of weight difference, perception of the denser object was positively biased, whereas the less dense object was perceived as even lighter, compared to a pair with the same weight difference but equal densities. Are our results therefore, entirely consistent with Wolf and colleagues (2018)?

This model provides a fairly good fit to our data (see Appendix B, Table B3). However, there are a number of systematic discrepancies, that can also be seen by comparing Figure 3.3 with Figure B1 (Appendix B), showing Wolf and colleagues' model predictions for within-subset stimulus pairs alongside the data. First, Wolf and colleagues suggest that the contribution of density to heaviness perception is dependent solely on the relative reliability of size information (either haptic or visual), used to infer density. As participants in our experiment had full access to visual and haptic cues, the contribution of density should have been constant across our stimuli. However, in our experiment the influence of density was strongly modulated by weight difference, with even the largest density differences having little impact on weight estimates when the weight difference was also large (as shown in Figure 3.4 A). In addition, heaviness estimates in our experiment were positively correlated with the non-target weight, suggesting that the two weights were assimilated, rather than contrasted. This assimilation happened both in the equal- and increasing-density set (see Figure 3.3 A, C). Predicted weight in Wolf and colleagues' model is independent of the contextual object(s). It is possible that our weight assimilation effect might reflect a response bias (i.e., an anchoring of second estimate to the first) related to the magnitude estimation method used in our experiment (Tversky & Kahneman, 1974); nevertheless, this would not invalidate our findings (as weight difference estimates in the SWI would have been hindered, not enhanced). Finally, when the two weights were similar, we found a negative correlation between perceived weight and the non-target density (see Figure 3.4 B), also not predicted by Wolf and colleagues.

One strength of Wolf and colleagues' model is that, being reliant on the stimulus weight and density only, it accounts for the misperception of weight that occurs in single-object judgements (see, e.g., H. E. Ross, 1969). On the other hand, perception typically occurs in a context: Our model too could be generalised to judgements of single objects, for example by replacing the non-target object by a weighted combination of the previous stimuli.

Although we found that the SWI persists even under small conflicts between the actual stimulus weight and the weight expected under equal density, our results are otherwise qualitatively consistent with the suggestion that the SWI is a repulsion or contrast illusion, driven by the difference between expected and perceived weight. The negative influence of the non-target density is also consistent with the 'contrast' effect. The observation that density has a greater impact for small weight differences, might also be broadly consistent with this explanation of the SWI: it is plausible that density differences are more detectable / salient in the absence of weight differences, increasing the experienced conflict between expected and felt heaviness. Finally, the ‘contrast’ account also gains plausibility, due to the fact that it can also explain the material-weight-illusion (described above, 1.3.3), where perceived weight appears contrasted with weight expectations evoked by the objects surface material / apparent density (Buckingham et al., 2009; Harshfield & DeHardt, 1970; Seashore, 1899; Wolfe, 1898). Our model could be seen as a quantitative description providing predictions for perceived weight that is broadly consistent with this account, i.e. that perceived weight is modulated by density when expectations about weight are violated.

Are our results consistent with Peters et and colleagues’ competitive priors model? No: First, this model predicts a reversal of the size-weight illusion under small to moderate size / density differences, in same-weight pairs. Second, the model also predicts a negative influence of density for other combinations of size and weight differences; this depends upon where the stimuli sit relative to the three priors over size and density ratios. Their model also makes unintuitive predictions when different size but equal density pairs are presented: the model suggests that the volume difference between objects is substantially underestimated. When this is combined with the equal density prior, the model predicts that the weight difference is similarly underestimated. In contrast, within our equal density pairs the weight ratio is perceived nearly veridically: in fact, the weight difference is slightly overestimated. These discrepancies can all be observed in Figure B2 (Appendix B), showing Peters and colleagues’ model predictions against our data. At a conceptual level, the Peters and colleagues model seems peculiar, given that it divides the continuous world of size and density into three relatively narrow categories.

Historically, explanations for the SWI have either proposed that the illusion would be due to a ‘contrast effect’ between expected and sensed weight, or to other physical properties of the stimuli – especially, density – which would be confounded, or combined, with weight (see above, 3.1). Truth, however, might be somewhere in the middle. Our results add to evidence that inferred density influences heaviness estimates in the SWI (Wolf et al., 2018), but suggest that the contribution of density is limited to zero (or very small) weight differences, and greatly reduced as the weight difference between the two stimuli increases. Therefore, the SWI doesn't appear to be a simple by-product of a mechanism which generally integrates / confounds density and weight estimates. A third account of the SWI has been proposed (Peters et al., 2016), suggesting that competing categorical priors over the objects' size and density ratio (stimuli are of equal density, or smaller / larger stimulus is denser and heavier) would ultimately cause the illusion. This model, however, predicts a reversal of the illusion at small size / density differences, at odds with our finding that the illusion still occurs under these conditions. The role of prior expectations in the SWI, therefore, remains an open question. Our results are broadly compatible with a 'contrast' effect: it appears that perceived weight is modulated by density when the sensed weight is different from the weight predicted under an equal-density prior. Bayesian models constrained by efficient coding (Wei & Stocker, 2015), that can predict biases away from the prior, might provide a mechanistic, quantitative account of the 'contrast' or 'repulsion' in the SWI.

3.6 Conclusions

We reported two novel findings to complement the current debate on heaviness perception in the SWI and more broadly. First, we extended the well-known observation that the SWI magnitude increases with size / density difference to include previously untested very small differences. Second, we showed that the influence of density on weight perception is not constant, but rather depends on weight difference. We proposed a simple model that accounts for heaviness perception in the SWI as well as in a range of fine-grained variations in stimulus weight and density. Our model is broadly consistent with the idea that the SWI occurs when sensory information contradicts expectations for the two stimuli to be different weight under equal density priors.

Chapter 4 Shape-related biases in volume and weight perception

Abstract

A number of recent studies have reported that objects are perceived as lighter or heavier depending on their shape: the shape-weight illusion. For example, spheres feel heavier than tetrahedrons of same weight and volume. Analogous but opposite biases occur in size perception: for example, spheres look smaller than volume-matched tetrahedrons. Therefore, it has been suggested that perceptually smaller objects might feel heavier, consistent with the well-known size-weight illusion, where the smaller of two same-weight objects feels heavier. However, perceived size was not sufficient to predict the shape-weight illusion, and surface area was proposed as an alternative predictor. Here we tested the shape-weight illusion on a set of four shapes (at three volumes) which varied greatly in surface area: sphere, tetrahedron, cuboid, and H-shape. Participants (N = 30) reported the object's perceived volume in a visual task, and the object's weight in a visuo-haptic task. Additionally, we recorded finger forces and torques during grasping and lifting to investigate the sensorimotor correlates of the illusion. We found clear shape-related biases in both tasks: cuboids and spheres were perceived as the largest and smallest shape; spheres and tetrahedrons were perceived as the heaviest and lightest shape. As expected, there were discrepancies between size and weight biases; yet, these were independent from surface area. Instead, we found systematic differences in torque for lifting the different shapes. We propose that differences in rotational inertia might contribute to the shape-weight illusion alongside perceived volume.

Notes

CRediT (Contributor Roles Taxonomy)
Veronica Pisu: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Software, Visualization, Writing – original draft, Writing – review and editing
Erich W. Graf: Conceptualization, Project administration, Resources, Supervision, Writing – review & editing
Wendy J. Adams: Conceptualization, Funding acquisition, Methodology, Project administration, Resources, Supervision, Writing – review & editing

4.1 Introduction

Everyday actions such as drinking a cup of tea require fine-tuned interactions between vision and touch. Before making contact with an object, forces for grasping and lifting are planned based on visual cues to the object's size and material properties: We expect larger objects to weigh more (thus, require more force to grasp and lift) than same-material smaller objects (Cole, 2008; Flanagan & Beltzner, 2000; Flanagan & Johansson, 2010; Gordon et al., 1991c). After lifting is initiated, cues relating to the object's weight and friction become available and forces are automatically and seamlessly adjusted (Flanagan & Johansson, 2010; Gordon et al., 1991a; Johansson & Westling, 1988). Interestingly, both lifting forces and perceived weight can be affected by perceptual biases. In the well-known size-weight illusion (SWI), observers are presented with two objects of apparently the same material but differing in size and nonetheless of equal weight. When lifted, the smaller (expected to be lighter) object is gripped and lifted with less force (Flanagan & Beltzner, 2000), and perceived as heavier than the larger (expected to be heavier) object (Charpentier, 1891). Grip and load forces quickly adapt to the true weight after a few lifts, while the perceptual bias persists (Flanagan & Beltzner, 2000). Thus, while misapplied forces, based on expected weight, have been hypothesised to cause the SWI (Davis & Roberts, 1976; Dijker, 2014; Granit, 1972; H. E. Ross, 1969b), they cannot be the whole story.

Here we investigate the lesser-known shape-weight illusion (Kahrimanovic et al., 2011; Vicovaro et al., 2019). In this illusion, objects that differ in shape, but are matched in physical size and weight are perceived to differ in weight. Dresslar (1894) reported that disks made of lead sheet feel heavier than triangles or rectangles of equal mass and surface area, and suggested that perceived differences in weight might relate to differences in perceived size. Indeed, disks are perceived as smaller in area than rectangles or triangles (Fisher & Foster, 1968b; Krider et al., 2001; Martinez & Dawson, 1973; Pisu et al., 2021; Warren & Pinneau, 1955). Similarly, spheres feel heavier than same-mass, same-volume cubes and tetrahedrons, and this effect occurs when size and shape are perceived haptically, directly (Kahrimanovic et al., 2011) or visually (with stimuli lifted via strings; Vicovaro et al., 2019). Again in this case, the shape-weight illusion may be partially correlated with biases in size perception: spheres are perceived as smaller in volume than cubes or tetrahedrons (Frayman & Dawson, 1981; Kahrimanovic et al., 2010a, 2010c; Mehraeen et al., 2021).

In the SWI, objects that are (correctly) perceived as smaller are erroneously perceived as heavier than same-mass, larger objects. Within the shape-weight illusion, however, the relationship between perceived size and perceived weight is less consistent. Kahrimanovic and colleagues (2011) found that biases in weight perception did not match predictions derived from haptically perceived volume (Kahrimanovic et al., 2010a). Similarly, Vicovaro and colleagues (2019) reported discrepancies between volume and weight biases such that spheres were perceived as both larger and heavier than cubes. They suggested that biases in weight perception (but not in volume perception) are mediated by surface area (greater surface area corresponding to lower perceived weight). In both studies, discrepancies were mainly related to sphere / cube comparisons, whose perceived size difference is small (compared to the perceived difference between either shape and tetrahedrons), and further reduced when mass information is available (Kahrimanovic et al., 2010a).

Existing studies of the shape-weight illusion have used a small number of shapes that invoke relatively small biases in perceived size and weight (e.g., spheres and cubes). As seen in Chapter 2 for area perception, wider sets of shapes / more complex shapes can help reveal the correlates of these biases. In addition, different participants have been used to measure volume vs. weight biases. Individual differences, which can be large (Kahrimanovic et al., 2010b, 2011), may have confounded the reported pattern of biases. Furthermore, if perceived differences in size are causally related to perceived differences in weight, it is unknown whether this is mediated by misapplied forces: we may use less force to grip and lift perceptually smaller, spherical objects, than perceptually larger ones. Finally, while it is apparent that perceived weight is modulated by expectations related to an object's material (the material-weight illusion, Seashore, 1899; H. K. Wolfe, 1898) or size (within the SWI, Saccone et al., 2019), other physical properties such as rotational inertia and torque also affect perceived weight (Amazeen, 1997; Amazeen & Turvey, 1996; Plaisier & Smeets, 2015). Thus, the shape-weight illusion may be explained by a combination of these top-down and bottom-up factors, whose influence may differ across participants.

In this study, we address these limitations to provide a more thorough understanding of the shape-weight illusion. Our set of shapes is selected to more broadly sample volume space and thus invoke the substantial variation in perceived volume seen in recent work (Mehraeen et al., 2021). Within a single participant sample, we measured perceived volume in a visual-only task, and perceived weight in a visual-haptic grasping task, in which we also measured grip and load forces (i.e., the forces applied horizontally and vertically at the fingertips, to respectively grasp and lift the object) and torque.

4.2 Methods

4.2.1 Participants

Thirty naïve participants were recruited from the University of Southampton student community (mean age: 20.5 years, *SD* 3.78; five left-handed, self-reported). All were compensated for their participation, either with course credits or monetarily. The experiment was approved by the University of Southampton Psychology Ethics Committee; all participants gave prior informed consent.

4.2.2 Stimuli

The 12 experimental stimuli comprised of four different shapes: sphere, tetrahedron, cuboid (3:1 height-to-width ratio), H-shape – each in three volumes (510, 600, 690 cm^3; see Figure 4.1 A–B). The cuboid and H-shape were chosen to decouple surface area from other shape features that are reportedly correlated with perceived volume and / or weight (elongation, compactness; Dresslar, 1894; Krishna, 2006; Vicovaro et al., 2019). A 600 cm^3 cylinder (height-to-width ratio: 1.3:1, weight 300 g; see Figure 4.1 B) was used as a reference stimulus, and an additional pair of cubes (510, 690 cm^3) was used in practice trials before the grasping task (see Setup and Procedure below). All stimuli were 3D-printed in black polylactic acid (PLA), with a removable panel concealed at the bottom or back, and uniformly filled using sand, polystyrene pellets, and brass to give a density of 0.5 g / cm^3. Weights were measured with a 0.1 g resolution. At the top of each stimulus a small steel disk was housed within a PLA disk (Figure 4.1 A–C). This magnetic attachment enabled a force sensor handle to be attached in grasping trials (see below, Setup and procedure).

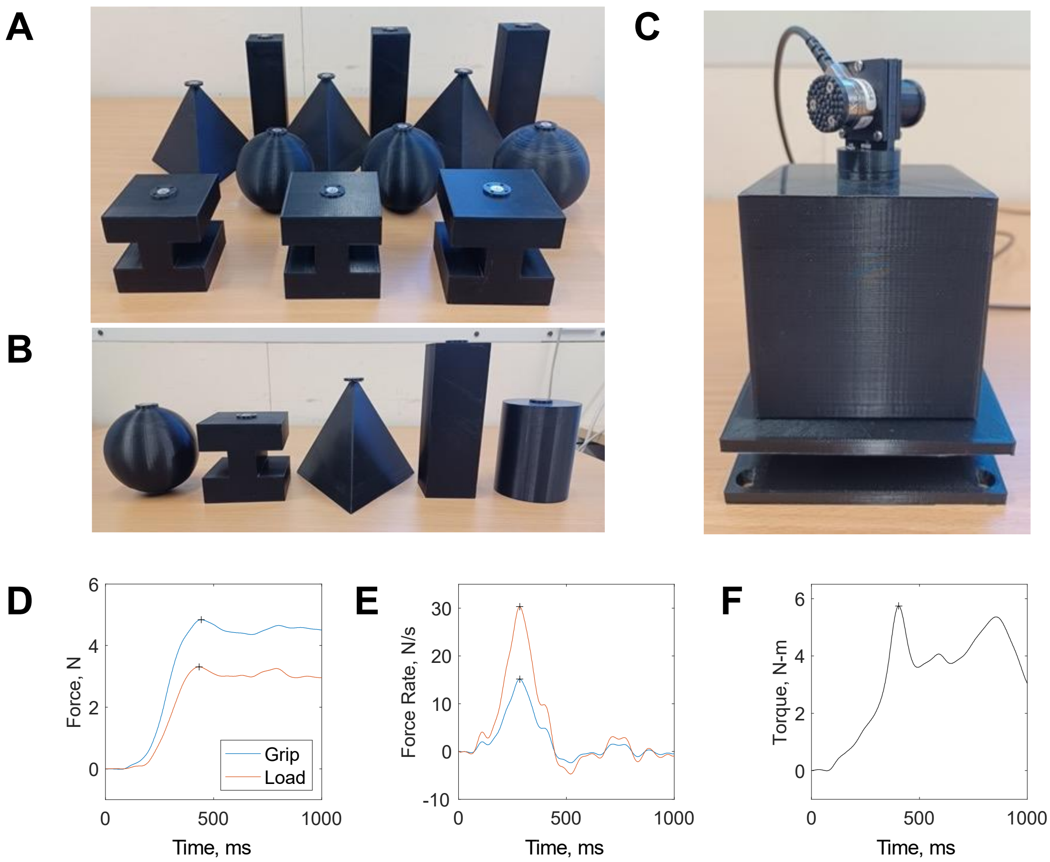


Figure 4.1 (A) Full stimulus set. Each shape, left to right: 510, 600, 690 cm^3. (B) Middle-size (600 cm^3) stimuli including the reference cylinder. (C) Practice cube (510 cm^3), with the handle attached, presented on one of the supports used in the grasping task; Supports and handles were 3D-printed in the same material as the stimuli. (D-E) Force profiles for one example lift. (D) Grip and load forces from handle transducer (E) Grip force rate from handle, load force rate from platform transducer. (F) Torque from handle transducer. Crosses indicate the maximum values used in subsequent analyses.

4.2.3 Task and trials

Participants reported either perceived volume (visual task) or perceived weight (grasping task) of stimuli using magnitude estimation; in both tasks, they were given the cylindrical reference stimulus and told that it had a volume / weight of 50 units. In the grasping task, forces were recorded for each lift (see below, Setup and procedure).

Stimuli were presented in pairs in both tasks. Visual trials (N = 528) included four repetitions of all possible stimulus pairs, except pairs of identical stimuli. Each unique stimulus was thus presented 44 times in total. Trials were divided in two blocks, each including two repetitions of each stimulus pair, in random order. Left and right position for each stimulus within each pair were randomly assigned in the first block, and reversed in the second. The cylindrical reference stimulus was presented at the beginning of each block of trials; participants were allowed to lift the reference, but they were not allowed to touch the experimental stimuli.

Grasping trials (N = 52) included all different-shape, same-size pairs, and same-shape pairs one size step away; each stimulus was presented eight (lightest, heaviest) or 10 (middle) times in total. Each trial consisted of 10 lifts (five / stimulus) and corresponding weight estimates. Trials were equally divided across two blocks completed in separated sessions. Each session included all stimulus combinations, with left and right positions randomly assigned in the first, and switched in the second session. The reference stimulus was presented at the beginning of each session; participants grasped and lifted the reference stimulus to assess its weight first freely, then using the handle.

4.2.4 Setup and procedure

The three experimental sessions (one visual, followed by two visual-haptic / grasping) were completed on different days; the second and third session were both completed within two weeks from the first. Each session lasted 50–90 minutes. Participants and experimenter sat at opposite sides of a table. A frame with a black felt curtain in the middle of the table controlled stimulus presentation. The full stimulus set was kept out of the participant's view throughout the experiment.

Stimuli were presented on top of supports that had a concave centre to keep the spherical stimuli in place, at a distance of approx. 20 cm from the edge of the table (centre-to-edge), and at 20 cm from each other (centre-to-centre); dimpled silicone grip tape was applied on the support to dampen auditory cues to object weight on contact. Supports in the grasping trials (shown in Figure 4.1 C) each concealed one six-axis force-torque (F/T) transducer (Nano17; ATI Industrial Automation, Garner, NC) to measure load force up to stimulus lift-off (Baugh et al., 2012, 2016). Participants lifted the objects using one of two identical handles (Figure 4.1 C), each one including one Nano17 transducer on one side, and a mock transducer of equal weight on the opposite side (for studies using a single transducer, see Flanagan & Beltzner, 2000; Grandy & Westwood, 2006; Naylor et al., 2020). Forces were sampled at 1,000 Hz. The total weight of each handle was 32.7 grams. Textured grip pads on each side of the handle allowed for a comfortable grasp using the thumb and index finger (distance between contact points: 45 mm). A magnet housed at the bottom of the handle allowed quick attachment to (detachment from) the stimuli between trials. Due to the handle configuration, forces were recorded from the index finger for right-handed participants and from the thumb for left-handed participants.

A desktop computer running MATLAB (The MathWorks Inc., 2020) with the Psychophysics Toolbox (Brainard, 1997; Pelli, 1997) was used to control stimulus randomisation, timings, and go signals, and to record the transducer outputs and participant responses.

Instructions at the beginning of the first block of the visual task included a simple definition of volume as ‘the amount of space that an object occupies’. Participants reported volume estimates for both stimuli verbally, starting from the left-hand object. The curtain was kept down for the duration of the block; participants briefly saw the hands of the experimenter placing and removing the stimuli between trials, minimising the availability of kinematic cues to weight.

In the grasping task, participants grasped and lifted both stimuli in turn, using their dominant hand, starting with the stimulus on the left-hand-side. They were instructed to grasp the handle using their index finger and thumb, and to lift each object to the same height (approx. 10 cm above the platform), taking care not to tilt the object (to limit cues to rotational inertia; Vicovaro et al., 2019). Participants started each lift with the hand resting on a spot marked on the table. Beeps of different pitch signalled the start and end of each lift within a trial: on hearing the first beep, participants grasped and lifted the current object to the designated height and held it still until the second beep prompted them to put it back down, return to the start position, and report the perceived weight. There was a two-second interval between the two beeps, corresponding to the transducer registration time window. The first lift within each trial differed from the other four, in that participants lifted both objects before reporting their estimates. Experimental trials (in both sessions) were preceded by one practice trial which was identical to the experimental trials except for the stimuli used (510, 690 cm^3 cubes). The curtain was lowered between trials to allow the experimenter to select the stimuli out of the participant's view.

4.2.5 Preliminary analyses, outliers, and exclusions

The force normal to the handle's transducer surface provided our estimate of grip force. Our set-up provided two sources of load force information. From the handle transducer, the vector sum of the forces orthogonal to grip force provides an estimate of load force throughout the trial, including peak load force (see Figure 4.1 D). Although this approach is customary (see e.g. Buckingham & Goodale, 2010; Flanagan & Beltzner, 2000), it assumes that deviations from a purely vertical lift are negligible. The transducer below the object gives a direct estimate of load force (the force orthogonal to the transducer / platform surface) including the peak load force rate, as this occurs before lift-off (Figure 4.1 E). Peak load force rate values obtained from each transducer where nevertheless very highly correlated, $r = .98$, 95% CI [.97, .98]. We use the handle transducer for peak load force, and the platform transducer to estimate load force rate in subsequent analyses.

All F/T signals were filtered using a 14-Hz, 4th-order Butterworth filter. Force rates were obtained by differentiating the force signals using a 5-point central difference equation (see e.g., Buckingham & Goodale, 2010). Torque (Figure 4.1 F) was calculated as the vector sum of the torque across all three axes of the handle transducer. Our five F/T variables of interest were extracted from each lift using a custom MATLAB program; these were: peak load force (PLF), peak grip force (PGF); peak load force rate (PLFr), peak grip force rate (PGFr) and peak torque (PT) (Figure 4.1 D–F). Based on findings with the SWI, we hypothesised that shape-related biases in perceived volume would be reflected in shape-related differences in PLFr and PGFr, at least in initial lifts (Buckingham et al., 2011b; Flanagan & Beltzner, 2000; Johansson & Flanagan, 2009). We expected participants to produce smaller force rates for perceptually smaller shapes, before possibly adjusting force rates to reflect the true weights on subsequent lifts. In contrast, PLF and PGF can be adjusted online according to the true object characteristics, and are therefore less likely to reflect erroneous expectations (Johansson & Flanagan, 2009). We thus expected peak load and grip forces to be independent of shape. Finally, if participants performed perfectly vertical lifts, we expected no relationship between shape and torque.

Practice trials were excluded from the analyses. Anticipatory lifts, where participants lifted the objects before the go-signal, were discarded. Force and torque data for all remaining lifts were plotted and visually checked by the experimenter. Peak forces / force rates were extracted from the 1.5s following the go signal. In the case of late lifts, where forces peaked after this window the correct time window was manually selected where possible, unless the full lift was not recorded. 1250 lifts (7.99 %) were discarded in this phase.

For each F/T variable, extreme outliers (± 5 SD) were then identified to check for any lifts with artefacts (e.g., where participants inadvertently hit the support, producing a huge spike in load force). When clearly identifiable, correct peak values were selected by the experimenter otherwise data for the lift were discarded. The same procedure was repeated with the remaining outliers (± 3SD). This removed further 3 (0.02%) lifts.

4.3 Results

4.3.1 Perceived volume and weight

Figure 4.2 A shows perceived volume as a function of true volume (re-expressed in reference units) for each stimulus. We found substantial shape-related biases: cuboids were perceived as the largest, and spheres as the smallest shape, with the smallest cuboid perceptually equivalent to the largest sphere (35% larger in true volume). As expected, tetrahedrons were also judged as much larger than spheres. The H-shapes were perceived as slightly larger than spheres. Regression coefficients for perceived volume as a function of shape and volume are reported in Appendix C (Table C1).

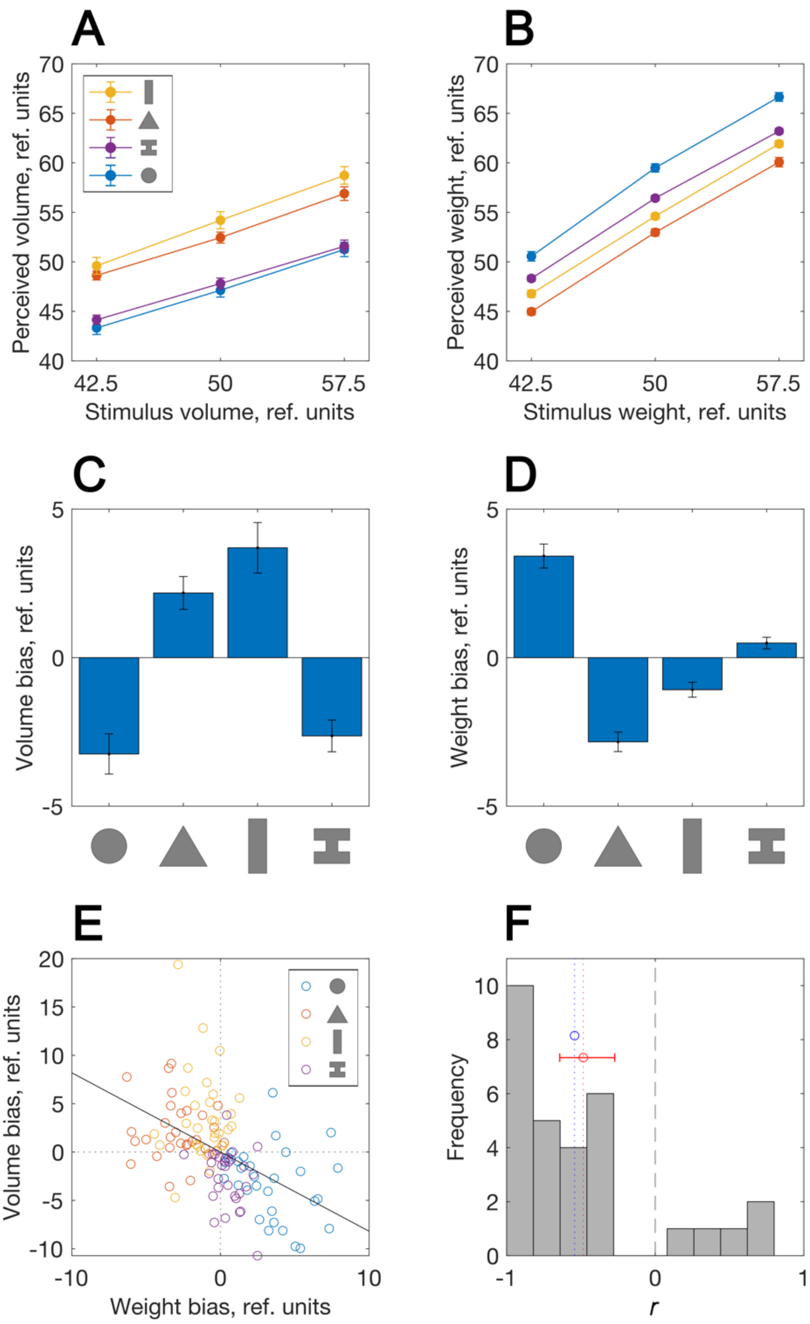


Figure 4.2 Perceptual data. (A) Perceived volume as function of true volume. (B) Perceived weight as a function of true weight. Perceptual data were normalised to remove inter-observer variation in how the reference scale was applied: for each participant, we subtracted the difference between the participant's mean estimate across conditions and the mean response across participants and conditions before averaging across participants (N = 30). Stimulus volume and weight have been rescaled to match the reference stimulus (arbitrary) units. Error bars ± 1 SEM. (C–D) The perceptual data re-expressed as biases for the two perceptual tasks, averaged across participants. Error bars ± 1 SEM. (E) Scatterplot showing the correlation between volume and weight bias. Each point represents one shape for one participant. Black line: least squares fit. (F) Histogram of individual observer correlations between volume and weight biases (Pearson's *r*). The red circle represents the average correlation across participants; error bars give bootstrap 95% CIs. The blue circle represents the average correlation for random participant pairings.

Figure 4.2 B shows perceived weight as a function of true weight[5] (see Appendix C, Table C2, for regression coefficients for perceived weight as a function of shape and weight). As in the case of perceived volume, we found clear shape-related biases: spheres were judged as the heaviest, and tetrahedrons as the lightest shape, as expected (Kahrimanovic et al., 2011; Vicovaro et al., 2019). Also consistent with previous reports, perceived weight did not perfectly map onto perceived volume across shapes: on average, cuboids were perceived as (marginally) larger than tetrahedrons, although they were not perceived as lighter; H-shapes appeared close in volume to spheres, but they did not feel as heavy. Results appear unrelated to the shapes' surface area, contrary to Vicovaro and colleagues' (2019) proposal, according to which H-shapes (having the largest surface area in the set) should have been perceived as the lightest.

To quantify shape-related biases in volume and weight (independent of stimulus volume / weight), we fitted linear regression models to each participant's perceived volume (or weight) data using stimulus volume (or weight) and shape (3 dummy variables, sphere as intercept) as predictors. The addition of further predictors to the model was not supported for the majority of participants, indicating that there were no significant effects of context (likelihood-ratio tests, $p > 0.05$, N/30; volume task: non-target volume: 24/30; non-target shape: 22/30; non-target shape + volume: 27/30; weight task: non-target weight: 30/30; non-target shape: 27/30; non-target shape + weight: 27/30). More complex models treating each condition as independent were also not supported for the majority of participants: 19/30 for the volume task; 29/30 for the weight task. These shape coefficients (normalised to have mean zero) were taken as a measure of the shape biases and are shown in Figure 4.2 C–D, averaged across participants.

Volume and weight biases were negatively correlated, both across (*r* -.50, 95% *CI* [-.62, -.35]; Figure 4.2 E) and within participants; Figure 4.2 F shows the distribution of correlations for individual

[5] Note that we considered the actual object weight without the handle and cable in the analyses; all stimuli including the reference were lifted using the handle.

participants. The average correlation was low-to-moderate, *M* = -.49, 95% *CI* [-.64, -.26]. To test whether individual participants' patterns of volume biases predict their pattern of weight biases, we correlated volume biases from one participant with weight biases from another participant, for 10,000 random participant pairs. The average correlation for random pairings (*M* -0.54, 95% *CI* [-.61, -.48], see the blue circle in Figure 4.2 F) was comparable to the correlations within participants, providing no evidence for a causal relationship between volume and weight biases at the individual level.

To summarise, we found a negative correlation between volume and weight biases, as expected. However, the magnitude of the correlation was only moderate. Thus, if volume biases cause weight biases, there may be additional factors also modulating perceived weight.

4.3.2 Forces and torque

Force and torque variables are shown in Figure 4.3. Figure 4.3 A–E show F/T variables as a function of stimulus weight and shape. Data were first collapsed across lifts and trials, then averaged across participants.

To quantify shape-dependent differences in forces (torque), we calculated shape-dependent force 'biases' (Figure 4.3 F–J) , following the same procedure described above for perceptual data. The linear regression models for each force variable can be found in Appendix C, Tables Table C3–Table C7.

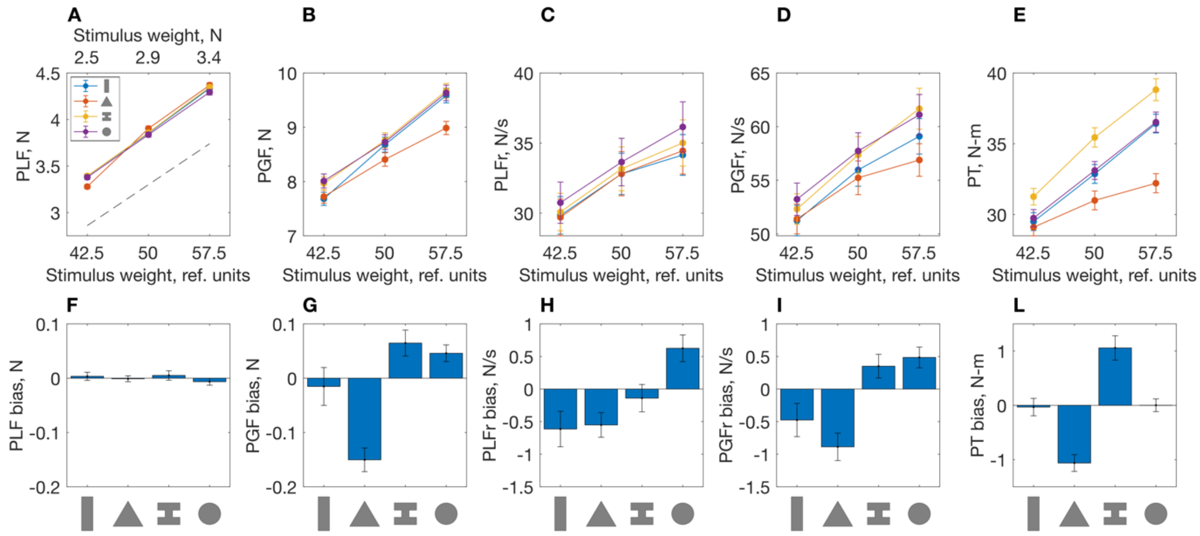


Figure 4.3 (A–E) Forces and torque variables as a function of stimulus weight in reference units (and Newtons in (A)) averaged across participants (N = 30). Error bars ± 1 SEM. The dashed line in (A) shows the load force needed to lift the object, handle and cables (weight of object + approx. 0.35 N). (F–J) F/T variables re-expressed as bias, averaged across participants. Error bars ± 1 SEM.

Peak load force and peak grip force are known to predominantly reflect the physical properties of the lifted object (and resultant friction with fingers in the case of grip force) (Flanagan & Johansson, 2010, but see Buckingham & Goodale, 2010 for evidence that peak load force can reflect the lifter's expectations when applied in considerable excess). In contrast, peak load force rate and peak grip force rate are pre-programmed based on the actor's (possibly erroneous) implicit expectations about the object's properties (Flanagan & Beltzner, 2000; Gordon et al., 1991c; Johansson & Flanagan, 2009; Johansson & Westling, 1988).

Peak load force (Figure 4.3 A, F), the vertical force used to lift the object, was closely related to stimulus weight (slightly in excess, as expected; Flanagan & Johansson, 2010) and largely independent of shape. The dashed line in Figure 4.3 A gives the exact force required to counteract the objects' weights. In contrast, peak grip force did vary significantly with shape (Figure 4.3, B, G). Participants applied less force to grasp tetrahedrons than the other shapes. One possible explanation for this shape-dependent effect is that, although the shapes were matched in weight, they differed in rotational inertia and indeed the measured torque (see Figure 4.3 E, J, and below).

Peak load force rate and peak grip force rate (related to sensorimotor predictions of weight) (Figure 4.3 C, D) both varied significantly with object shape. However, both of these followed more or less opposite patterns from the predictions based on biases in perceived size. PLFr was higher for the (perceptually smaller) spheres compared to the other shapes (Figure 4.3 H). Larger peak grip force rates were also observed for spheres and (perceptually second smallest) H-shapes (Figure 4.3 D, 2I). Thus, these shape-related force effects are not explained by size-based weight expectations. In the SWI, force rates on initial lifts reflect expectations about object weight (i.e., higher rates are produced to lift the larger object, and lower rates to lift the smaller object), then they are rapidly adjusted to the object's actual mass (i.e., equivalent rates are produced to lift each object) (Buckingham, 2014; Flanagan & Beltzner, 2000; Grandy & Westwood, 2006; Mon-Williams & Murray, 2000). Due to the averaging here we are not able to distinguish possible differences between early and subsequent lifts (see Figure 4.3 A–E). Thus we can ask whether, in our data, force rates reflect size-based expectations on initial lifts. On average, no clear pattern in force rates between early and subsequent lifts was detectable, as shown in Appendix C, Figure C1–Figure C2 (although it cannot be excluded that any learning might have happened in the first lifts, instead of trials). It should be noted, however, that a clear pattern might be difficult to detect in our paradigm, due to the number of stimuli and randomised order of presentation.

Lastly, peak torque values (Figure 4.3 E), depending on the objects' rotational inertia and angular acceleration, varied significantly across shapes. Peak torque was the highest for H-shapes, and the lowest for tetrahedrons (Figure 4.3 J). Note that these differences in torque were found independent of the instructions given to participants (i.e., to lift the objects vertically, and minimise any swaying / lateral movements). However, the objects did differ in mass distribution / rotational inertia, and this, combined with deviations from the ideal lifts created shape-related differences in torque.

To understand potential relationships between the different force variables (e.g., between grip forces and torque), and to identify sensorimotor correlates of the perceptual biases, we computed participant-level correlations between biases for each variable pair. As above, we also compared these correlations to those from randomly paired participants.

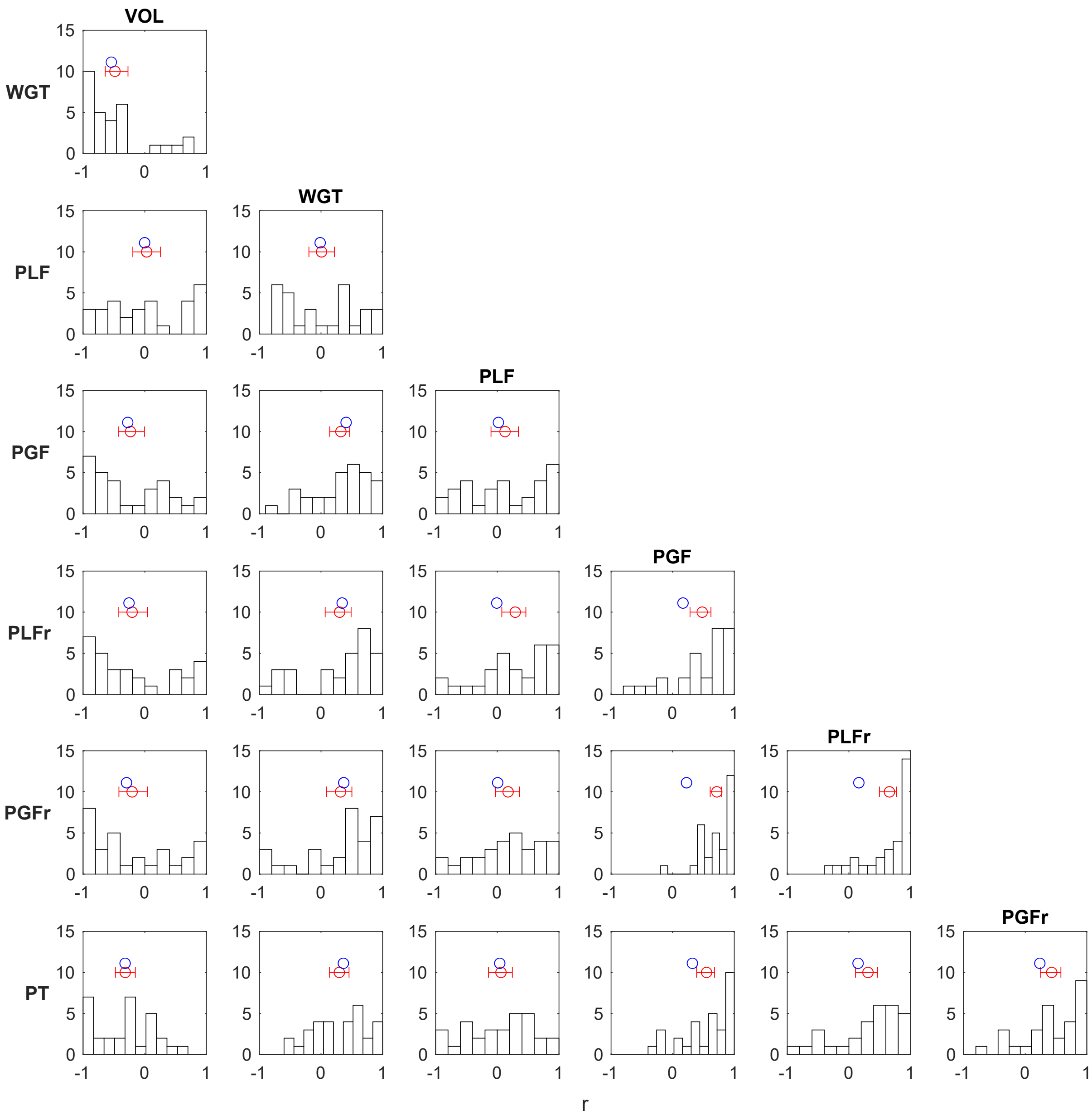


Figure 4.4 Histograms of correlations (Pearson's r) for each bias. Red circles represent the average correlation across participants; error bars give bootstrap 95% CIs. Blue circles represent the average correlation for random participant pairings.

The correlations confirm that PLF was unrelated to perceptual biases and only weakly related to other force variables. In contrast, PGF was related to perceived volume and weight: participants applied greater grip force to perceptually smaller and heavier shapes. Importantly, PGF was strongly correlated with peak torque; it is known that greater grip force is required to counteract greater torque (Kinoshita et al., 1997) and that increased torque causes objects to be perceived as heavier

(Amazeen, 1997). In addition, PGF was strongly and positively correlated with PGFr, suggesting that these modulations of the applied grip force matched expectations. Together, these results suggest that some shapes were associated with greater torque (due to differences in weight distribution combined with horizontal and / or rotational acceleration) and this may (at least partially) explain some of the shape-related biases in perceived weight, in grip force and in grip force rate. Participants may have learnt the relationship between shape and grip force requirements, thus also driving the pre-emptory effects seen in PGFr.

PLFr was positively correlated with perceived weight. There is no reason to expect that differences in torque should impact load force, or load force rate. Indeed, peak load force was independent of shape. Thus it seems likely that shape-related biases in perceived weight (that were maintained across trials) directly caused shape-related differences in PLFr, with PLF then corrected online in accordance with true weight.

4.4 Discussion

We found large shape-related biases in both volume and heaviness perception: tetrahedrons were perceived to be 11.5% larger and 11% lighter than spheres. Our results extend previous work (Kahrimanovic et al., 2011; Vicovaro et al., 2019) by the inclusion of novel shapes: cuboids and H-shapes. Broadly, perceptually smaller shapes (that are expected to be lighter) were perceived as heavier. Although participants in the current study did not directly report expected weight, visual judgements of perceived volume and expected weight were strongly positively correlated in our previous study with a similar set of shapes (Mehraeen et al., 2021). Yet, as in previous reports of the shape-weight illusion, the bias in perceived volume did not perfectly account for the bias in perceived weight, as determined by the moderate correlation between the biases.

Vicovaro and colleagues (2019) suggested that surface area, rather than perceived volume, might underlie the shape-weight biases, given the negative association between surface area and perceived heaviness when considering only spheres, cubes, and tetrahedrons. Our set of shapes decouples surface area from perceived volume, allowing a direct test of this hypothesis. Critically, H-shapes, with the largest surface area in the set, were perceived as relatively small (close to spheres) and heavy (second to spheres), thus contradicting the surface area hypothesis.

Our study is the first to explore the relationship between perceptual biases in volume and weight, and the forces associated with grasping and lifting. Perceptual biases in weight were positively correlated with shape-related differences in grip force, torque and grip force rate. This suggests that cues to the objects' rotational inertia (i.e., resistance to rotational forces during lifting) might have contributed to shape-related differences in perceived weight, grip force and grip force rates. The amount of rotational inertia depends on the object's mass and its distribution, increasing with the distance between the point of rotation and the centre of mass. It has been shown that perceived heaviness varies as a function of mass distribution (Amazeen, 1997; Plaisier & Smeets, 2015). For example, taller objects feel heavier (higher rotational inertia), than wider (lower rotational inertia) but otherwise similar objects (Amazeen, 1997, 2014). In our experiment, cuboids were perceptually largest, yet were not perceived as the lightest, and instead heavier than tetrahedrons. However, they did produce more torque than tetrahedrons, and thus weight biases may be explained by a combination of biases in perceived volume and differences in rotational inertia. However, this is fairly speculative; it is not clear why the H-shape, which induced the greatest torque, was not perceived as heavier.

As mentioned above (see 4.1), in cases of sensorimotor mismatch between expected and actual weight, such as in the SWI, force rates are quickly adapted to correspond to true weight (Flanagan & Johansson, 2010). Here, instead, average force rates roughly matched perceived weight differences. This mismatch between grip force, grip force rate and true weight in the current study might be at least partially due to shape-based torque differences – we found relatively strong correlations between these forces / force rates, that were stronger within than between participants. Thus, in a sense grip forces are relatively well adapted to what is required in the presence of torque. On the other hand, increased torque does not require an increase in load force. While load forces were indeed invariant to shape (and biases in weight perception, and torque), shape-related differences in load force rate perseverated.

4.5 Conclusions

Heaviness perception is consistently biased, and depends on the object's shape. Similar to the size-weight illusion (perceptually) smaller objects feel heavier. However, discrepancies between biases in perceived volume and weight suggest the contribution of additional factors in the final heaviness percept. We ruled out the contribution of surface area to perceived heaviness, and suggest that differences in rotational inertia across shapes might instead be involved. Future research should clarify the role of rotational inertia in weight biases to understand how these cues interact with perceived size. Although a quantitative model of the shape-weight-illusion is still lacking, our results offer a step towards a better understanding of this phenomenon, and heaviness perception more broadly.

Chapter 5 General discussion

Skills such as estimating the size of a surface or an object, or using size and material to predict or estimate an object's weight are an integral part of humans' remarkable ability to effortlessly interact with an environment made of objects that come in a myriad of combinations of size, shape, and material. Nevertheless, the perception of area, volume, and weight have long been known to be biased depending on several stimulus features such as size, shape, and material. These biases have generally proven difficult to describe in quantitative terms, and several questions are still outstanding. This thesis sought to address a number of gaps in the literature (summarised above, 1.4), specifically regarding: 1) Shape-related biases in area perception; 2) Size-related biases in heaviness perception – the size-weight illusion; 3) Shape-related biases in volume and heaviness perception – the 'shape-weight illusion'. In the three studies here presented, I sought to provide falsifiable, replicable, quantitative descriptions of each investigated phenomenon.

In this final chapter, I will summarise the key findings from the studies presented in the three empirical chapters (Chapter 2–4) and review them in the context of the broader literature presented in Chapter 1; I will highlight the studies' limitations, and indicate directions for future research stemming from the evidence here provided.

5.1 Summary of the key findings

Question 1: Can we predict biases in area perception from shapes' geometric features?

In Chapter 2, I presented results that replicate and extend previous findings about biases in area perception, and proposed a quantitative model that describes the biases across a wide range of shapes and orientations. I reported that: 1) Biases in area perception are substantial and highly consistent across observers; 2) Discrepancies in the findings and proposed explanations reported in the literature appear due to the use of small subsets of shapes; 3) No single geometric feature can successfully predict the biases, but it is possible to predict these biases from a combination of various features.

Question 2: Can we develop a generalised model of weight perception for varied combinations of size, weight and density?

In Chapter 3, I described novel findings that provide an account of heaviness perception in the SWI (same-weight pairs) and across a range of fine-grained variations in stimulus volume and / or weight and / or density. I reported that: 1) The SWI still occurs at very small size (density) differences, indicating that small conflicts are not integrated, and extending the observation that the magnitude of the illusion increases with density difference; 2) The influence of density on perceived weight was modulated by weight difference, with the largest influence at zero or very small differences; 3) Results were not consistent (Peters et al., 2016) or only partly consistent (Wolf et al., 2018) with recent models of the SWI.

Question 3: Can we better characterise the influence of shape on perceived heaviness?

Lastly, in Chapter 4, I provided an account of the visuo-haptic shape-weight illusion in a novel set of shapes, together with a first report of finger forces and torque for grasping and lifting illusion-inducing stimulus pairs. I showed that: 1) Volume and weight perception are consistently biased depending on the object's shape, confirming and extending previous findings; 2) Shape-related biases in volume and weight perception are broadly consistent, however, perceived volume is not sufficient to predict the shape-weight illusion (consistent with previous findings); 4) Different from previous suggestions, surface area was not correlated with perceived heaviness; I speculate that differences in rotational inertia across shapes might contribute to the bias.

5.2 Contributions to the understanding of area and volume perception

In Chapter 1, I described how area and volume perception have long been known to be fundamentally biased by the shape of the surface / object being judged, but nevertheless remain relatively underexplored; I identified several gaps in the literature, and conflicting reports in the description and proposed explanations of these biases.

As raised in the Introduction (1.2), 1) accounts of these biases were sometimes inconsistent. Inconsistencies regard both the reported perceptual ordering of some shapes (e.g. disks vs. squares, more vs. less elongated shapes) and the geometric features (e.g. elongation, compactness) which have been identified as correlated with the biases. It is not possible to resolve these discrepancies, which might only be apparent, due to insufficient (or lacking) descriptions of the stimuli in several studies. Furthermore, 2) proposed explanations do not generalise beyond small sets of shapes, and 3) it is not known, nor has been hypothesised, why these biases occur. The findings presented in Chapter 2 and Chapter 4 of this thesis demonstrate that area and volume judgements are fundamentally biased by the shape of the surface / object being estimated, in line with previous studies (reviewed above, 1.2). In the next paragraphs, I will discuss the contributions of the present work with regards to the points listed above.

5.2.1 Shape matters: size judgements are consistently biased

The work presented in Chapter 2 provides an account of shape-related biases in area perception in a set of shapes (shown in Figure 2.2) that included variations in several geometric features known to contribute to the biases (e.g., compactness, elongation, orientation). We reported systematic and substantial biases, which were consistent across participants and experiments, with the exception of only one shape (the 8-point star, see Figure 2.3 C, Figure 2.4 B). Additionally, in Chapter 4, biases in volume perception were explored in the context of the shape-weight illusion. Both shape-related biases in area and volume perception extended to previously-untested shapes.

Area perception

It was hypothesised (see above, 1.2.3) that (apparent) discrepancies in the literature might be largely due to the use of small subsets of shapes, which might have resulted in local, non-generalisable explanations (but which have, nevertheless, sometimes been generalised to explain area perception as a whole). This was confirmed in Chapter 2 (and also suggested by the study described in Chapter 4, see below): for example, whilst elongation might describe how (‘vertically’ oriented, elongated) rectangles are perceived to be larger than squares (in their canonical orientation), elongation cannot explain variations in the perceived area of the same shapes induced by orientation. Whilst orientation-dependent variations in perceived area might be correlated with the shape's height, height would not predict that triangles are perceived as smaller than (taller) rectangles, and so on (see Figure 2.7). The same limitations apply to 'heuristic' accounts, which can be reduced to different combinations of the shape's height and width (see above, 1.2.2.1). For example, both 'perceptual saliency' (Krider et al., 2001) and 'additive area' (Yousif et al., 2020) can easily describe – at least qualitatively – differences in perceived area for squares presented in different orientations, but both fail to explain differences in perceived area for rectangle / triangle comparisons (see Figure 2.7). Therefore, the 'additive area' heuristic does not generalise to area perception more broadly, different from what hypothesised by its proponents (Yousif et al., 2020).

In Chapter 2 (see above, 2.8) I presented a quantitative model of biases in area perception that accounts for known biases as well as those here reported with novel shapes. The model – selected using an assumption-free method, cross-validation – includes several predictors previously featured in the literature, and helps to disambiguate their association with area biases. The interplay between elongation and compactness – features having been reported to be positively and negatively correlated with perceived area (see 1.2.2.1) – can provide an example. As I observed above, increases in elongation correspond to decreases in compactness, but decreases in compactness do not necessarily entail changes in elongation. Consistent with previous reports (Dresslar, 1894; Owen, 1970), and qualitatively illustrated by disk / triangle comparisons, the model predicts a negative influence of compactness on perceived area: less-compact shapes are perceived as larger in area. Whilst elongation was positively correlated with perceived area in subsets made of squares and rectangles (see Figure 2.6 B) the model predicts a negative influence of elongation on perceived area, reflecting the lesser increase in perceived area for elongated shapes compared to that of shapes of similar compactness but lesser (or no) elongation (e.g., triangles: triangles appear larger than rectangles). Although it cannot be guaranteed, it can be expected that the model will generalise to novel stimuli, as it was (i) tested on a wide range of shapes (N = 22 shape / orientation combinations, see 2.8), and (ii) selected using leave one-out cross-validation over shapes (i.e., leaving out each shape / orientation combination, in turn).

Volume perception

In Chapter 4, I reported shape biases in perceived volume in a set of real objects that included three-dimensional H-shapes and cuboids alongside the often-compared tetrahedrons and spheres (Figure 4.1 A). Although it includes only four shapes, this set decouples compactness, surface area, and elongation / height (known to be correlated to biases in perceived volume; see above, 1.2.2.1). Thus, it allows to replicate known biases (tetrahedron vs. sphere), extend them to novel shapes (cuboid and H-shape), and test previous explanations.

Findings reported in Chapter 4 indicate that previous studies on biases in volume perception suffer from limitations that are similar to those discussed above with regards to the area perception literature. For example, the elongation bias has been widely regarded as a feature of volume perception – and, therefore, pragmatically exploited in product packaging (Raghubir & Krishna, 1999). How well this would generalise to other shapes (as it has been proposed; Stanek, 1969; Vicovaro et al., 2019), however, is not clear. Indeed, cuboids were perceived as the largest shape in the set (Figure 4.2 A). Cuboids, however, were perceived as only slightly larger than tetrahedrons, suggesting that elongation might not be the (only) factor underlying the bias (consistent with observations by Kahrimanovic et al., 2010b). Similarly, these results are at odds with the 'additive volume' heuristic (Bennette et al., 2021), according to which observers would sum the shape height and width to approximate volume. Tetrahedrons, whose sum of height and width largely exceeds that of cuboids, should have been perceived as the largest shape. Therefore, the 'additive' heuristic fails to generalise to single shapes in a similar way as its two-dimensional counterpart (discussed above).

Obtaining a quantitative model of biases in volume perception was out of the scope of Chapter 4. To this aim, future studies could investigate biases in volume perception using a similar methodology to that used here to investigate area perception. Preliminary results obtained in another study (in collaboration with colleagues at the University of Ulm, Germany; Mehraeen et al., 2021) are promising. In an online experiment (see Chapter 2, General methods), we compared a set of three-dimensional stimuli that matched those stimuli used in Experiment 1: Common shapes from Chapter 2, thus including the same shapes used in Chapter 4 (see Figure 5.1; note that the cuboids in the two experiments have different elongations). Biases for these common shapes were consistent with those reported in Chapter 4 (perceived volume: sphere < H-shape < tetrahedron < cuboid). This suggests that biases in volume perception, similar to biases in area perception, are consistent across participants and experiments, and extend to novel shapes.

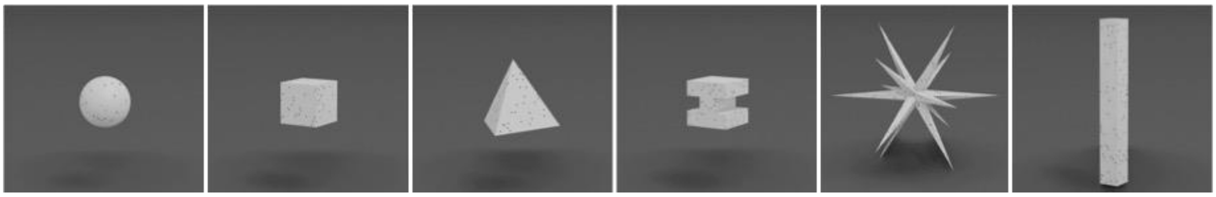


Figure 5.1 Stimulus set from Mehraeen and colleagues (2021) experiment. Stimuli were short clips, showing the objects rotating around the y-axis (3 s, one full rotation / trial). Methods matched those of Chapter 2, Experiment 1: Common shapes (see Methods).

Results from Mehraeen and colleagues' study also suggest that a similar multi-predictor model as that presented in Chapter 2 might describe biases in volume perception; whether the same factors (or their three-dimensional counterparts) might predict biases in volume perception is a question that still needs to be addressed.

As an aside, these results validate the online paradigm, with which we obtained results that mirror those obtained with real objects in a laboratory setting (both in Chapter 4 and in previous studies, e.g. Kahrimanovic et al., 2010c). This method offers some advantages, although it is necessarily limited to visual stimuli. First, it allows to collect data in a shorter time, from a greater number of participants. Second (not limited to the online setting), it overcomes limitations related to creating complex stimuli (availability of a 3D printer, long printing times etc.), making it easier to compare shapes that systematically vary along several geometric features.

5.2.2 'Why do these biases occur?' is still an open question

Chapter 2 offers a substantial contribution to the literature on shape-related biases in area perception. However, although the proposed model gives a quantitative description (and prediction) of area biases, it does not provide an answer to the question: 'Why do these biases occur?'. Heuristic explanations have proposed that shape biases arise from the difficulty encountered by the perceptual system in comparing objects / shapes on more than one dimension at once (Carbon, 2016; Krider et al., 2001; Yousif & Keil, 2019). For example, Carbon (2016) showed that observers are not able to perceive an A4 paper sheet as twice the size of an A5 sheet unless they align the two sheets on the side of same length. Thus, observers would compare the shapes along only one linear dimension (e.g., height) to simplify the task. Yousif and colleagues (2020) have proposed that observers would approximate area by summing the height and width of the object for the same reason. However, results from Chapter 2 show that no simple combination of height and width (nor any of the two taken alone) can explain the biases in area perception across a wide range of shapes.

Given the consistency of shape-related area biases across participants and experiments, as well as qualitative similarities between biases in area and volume perception (reported in the literature, and confirmed in Chapter 2 and Chapter 4 here: Triangles look larger than disks; tetrahedrons look larger than spheres; also see Mehraeen et al., 2021 discussed above), it would be difficult to argue that biases are due to idiosyncratic strategies used to overcome the puzzle of size (area, volume) perception along more than one dimension. Instead, these biases appear a general feature of size perception, with further ramifications. As pointed out above (2.9), and demonstrated in Chapter 4 with regards to biases in volume perception, these biases are not corrected during daily interactions with objects. It has been reported that biases in volume perception are reduced (but not zero) when participants can also feel the object's weight (Kahrimanovic et al., 2010b), but despite this, the perceptual system does not recalibrate to reduce biases more generally. Evidence – including results presented in Chapter 4 in this work – suggest that erroneous perception of size contributes to biases in weight perception (Amazeen, 2014; Kahrimanovic et al., 2011; Vicovaro et al., 2019). Furthermore, these biases extend to daily behaviour: for example, both children and adults tend to pour and drink more liquid (e.g., fruit juice), and underestimate the amount consumed, when given shorter / wider glasses (Wansink & van Ittersum, 2003).

Work reported in Chapter 2 warns against generalising results obtained in small, often purposefully chosen, sets of stimuli as all-encompassing principles: Whilst elongation might explain area (volume) perception in stimuli varying only along their height, it doesn't hold as a general rule. This limitation still applies to the studies presented here: It is not possible to know how well the proposed model would generalise to an even larger set of randomly chosen shapes. Although the model was built using an assumption-free method, there is no guarantee that all possible shape features have been taken into account, as the list of predictors and tested shapes, though carefully chosen, had necessarily to be limited. The proposed model probably does not offer a one-size-fits-all solution to the problem of area perception; yet, it can be used to generate testable predictions of perceived area for novel shapes, as well as falsifiable hypotheses to help broaden our understanding of size perception.

To conclude, although an explanation for shape-related biases in area and volume perception is still lacking, Chapter 2 and Chapter 4 provide a valuable contribution to the literature, overcome previous limitations and suggest directions for future studies. Chapter 2 offers a first quantitative description of area perception taking into account the contribution of several features (compactness, elongation, orientation) which have been so far treated separately, resulting in models that do no generalise to broader shape sets.

5.3 Contributions to the understanding of heaviness perception

In the Introduction of this work (see 1.3), I discussed how several aspects of heaviness perception are still unclear. I reported how well-known biases such as the SWI are still fundamentally unexplained, and how other phenomena, such as the influence of the object's shape on its perceived weight (the 'shape-weight illusion') have remained relatively underexplored. In Chapter 3, I provided a description and quantitative model of heaviness perception that predicts perceived weight in the SWI and across a wide range of size / weight / density differences. I showed that current models of the SWI fail to predict perceived heaviness in the whole set. In Chapter 4, I investigated the effect of shape on perceived weight, for the first time reporting finger forces and torque alongside perceptual measures; I extended and clarified previous findings, and proposed directions for future research. Evidence presented in both chapters will be summarised and discussed in the next paragraphs.

5.3.1 On the SWI and beyond: Perceived heaviness as a function of object weight, volume, and density

As detailed in Chapter 1 (see also 3.1), heaviness perception in the SWI has proven difficult to describe with available models and several questions remain open. Furthermore, these models have generally been tested on limited sets of stimuli (usually equal-weight stimuli). Chapter 3 of the present work reports two novel findings: 1) The SWI still occurs at (previously untested) very small size / density differences; this shows that small conflicts are not integrated; 2) The (previously reported) influence of density on weight perception is modulated by weight difference. Critically, 3) we tested a wider range of size / weight combinations and showed that no recent model of the SWI predicts perceived weight across the whole stimulus set. Here I will discuss these findings within the current debate and in light of recent models of the SWI.

The SWI still occurs at minimal size / density differences

The SWI has routinely been described as a ‘contrast’, or ‘repulsion’ effect, where weight expectations (for the smaller object to be lighter, and the larger to be heavier) are contrasted with incoming sensory cues indicating that the two (different-size) objects weigh the same, leading to the illusion being labelled as ‘anti-Bayesian’ (Brayanov & Smith, 2010). The Bayesian prediction (see 3.1, Figure 3.1 A) is that prior expectations will be integrated / averaged with sensory cues, therefore that the smaller object will be perceived as lighter, and the larger as heavier.

Models that have challenged the ‘anti-Bayesian’ label have either proposed that expectations do not contribute to the SWI (discussed in the next paragraph; Wolf et al., 2018) or that the SWI can be predicted by competing priors over the size and density ratio of the objects (Peters et al., 2016). Whilst the competing prior hypothesis is appealing, this model, which had previously only been tested on a small set of classic SWI stimuli, does not generalise to heaviness perception more broadly.

It has been reported that integration does not occur under large conflicts (see, e.g., Knill, 2007; Landy et al., 1995): Therefore, in Chapter 3 it was hypothesised that integration might occur in the case of small conflicts (very small size / density differences). This would be evidenced by a reversal of the SWI: the smaller, expected lighter, object would be perceived as slightly lighter than the larger. Results reported in Chapter 3, however, show that the SWI occurred in all same-weight pairs, including the smallest density differences (and increased with density difference, consistent with previous reports). These results are not consistent with Peters and colleagues (2016) competing-prior Bayesian model of the SWI, which also predicts a reversal of the illusion at small conflicts. It has been proposed that seemingly anti-Bayesian perceptual phenomena might be predicted by Bayesian models incorporating efficient coding (Wei & Stocker, 2015). Efficient coding (of stimulus properties) reflects the distribution of properties in the environment, resulting in systematically skewed likelihoods that predict that perception can be biased away from prior expectations. Recently, it has been proposed that efficient coding is broadly consistent with the SWI (Bays, 2023), accounting for the 'contrast' effect. Although this model was not tested in the present work, it should be noted (as pointed out above, 3.5) that it does not qualitatively predict some of the systematic effects seen in Chapter 3, such as the effect of the non-target stimulus, and the increase in the bias for stimuli of similar weight. Nevertheless, the incorporation of efficient coding should be considered in future attempts to model the 'contrast' effect in the SWI.

Density affects weight estimates, but its influence is modulated by weight difference

It has been proposed that the SWI is due to a conflation of weight and density: human observers confound these two correlated physical object features, such that heaviness estimates are routinely biased towards the object's density (denser objects of same weight are perceived as heavier) (Wolf et al., 2018; Wolf & Drewing, 2020). Results presented in Chapter 3 are somewhat consistent with this account: Heaviness estimates were positively correlated with both objects' weight and density. However, this is only part of the story, as the effect of density on perceived weight was modulated by weight difference, which was larger at very small / zero weight differences, and greatly reduced at larger differences (see Figure 3.4). This is a critical difference from Wolf and colleagues' proposal, where the influence of density on perceived weight is only modulated by the availability and quality of size information (necessary to infer density): the better the quality of density information, the greater the influence of density on heaviness estimates. The integration of weight and density is deemed adaptive, although as noted above this is not entirely consistent with the usual benefits of cue-integration (see 3.1). Wolf and colleagues' model, therefore, does not theoretically entail violated expectations, nor a 'contrast' effect.

Results reported in Chapter 3 are more compatible with a contrast effect: it appears that the effect of density on perceived weight (the conflation of weight and density) is larger when there is a larger conflict between sensed weight and the weight predicted under an equal-density prior. This last interpretation remains speculative; however, it is appropriate to assume that the perceptual system expects same-material objects to be same-density. Such expectation would be consistent with the ability to correctly scale fingertip forces prior to lifting (Gordon et al., 1991b, 1991c; this can also be seen in the MWI, where greater forces are applied in the first trials to lift objects that appear made of a denser material; Buckingham et al., 2009; Paulun et al., 2019): it has been proposed that the sensorimotor system would rely on the object's size, together with density expectations for classes of objects / materials (Cole, 2008). It should be noted that Wolf and colleagues' model predictions are nevertheless broadly consistent with a 'contrast' effect: The model predicts a larger bias for larger deviations from equal density. However, being only reliant on the target's mass and density, it predicts a smaller-magnitude SWI at small density differences than that reported in Chapter 3 (see Appendix B, Figure B1 C–D).

To recap, in Chapter 3 I reported evidence that partially supports Wolf and colleagues' (2018) hypothesis that the SWI is due to a conflation of weight and density. However, I have shown that this conflation of weight and density is larger (therefore, density has a larger influence on perceived weight) when weight difference is zero or very small, not consistent with Wolf and colleagues' predictions.

Incidentally, the finding that the perceptual system conflates weight and density, and that this conflation is modulated by weight difference is also compatible with previous findings showing that expectations related to the object material (such as those commonly assumed to give rise to the MWI; see above, 1.3.3) are 'overridden' by size-related expectations in producing weight illusions (Buckingham et al., 2016; Buckingham & Goodale, 2013; Buckingham & MacDonald, 2016; Vicovaro & Burigana, 2017). For example, Buckingham and MacDonald (2016) presented participants with two sets of three different-material (different expected weight) balls; the first set included three similar-weight balls of different size (golf ball, toy football, beach ball); the second included three similar-size balls, adjusted to weight the same (polystyrene, golf, cricket ball); In both sets, results were at odds with (violated) material-related expectations: In the first set, participant judgements were consistent with their expectations (perceived weight: golf ball > toy football > beach ball), whilst in the second set perceived weight differences were minimal, but also independent from (the small, but still detectable) weight difference. The results from the first set have generally been interpreted in terms of the SWI: implicit expectations for larger objects to be heavier would be independent of the material of the object, and the violation of these implicit expectations would produce the impression that the beach ball is lighter than the (approximately) same-weight toy football and golf ball. One other interpretation, qualitatively consistent with the model presented in Chapter 3, is that perceived heaviness in both sets was instead biased by the objects' density at (unexpected for the perceiver) zero / very small weight differences. One advantage of this interpretation is that it is easier to reconcile with prior expectations that allow for differential fingertip force scaling across variations in size and material (as discussed above in this paragraph).

5.3.2 Shape-related biases in weight perception

Weight perception is consistently biased by the object's shape, but perceived size cannot fully explain the bias

In Chapter 4, I provided a description of the visuo-haptic shape-weight illusion. In the same participant sample, I reported perceived volume and weight, as well as finger forces and torque for grasping and lifting different-shaped stimuli. Consistent with previous reports, both volume and heaviness perception were biased by the object's shape: spheres were perceived as the smallest and heaviest shape; cuboids were perceived as the largest shape, and tetrahedrons (second to cuboids in terms of perceived volume) were perceived as the lightest shape. Critically, weight biases extended to shapes (Figure 4.1 B) which were previously untested (H-shape) or not tested in the context of the shape-weight illusion (cuboid).

As mentioned in Chapter 1 (1.3.2), perceived weight in the shape-weight illusion has been reported (Kahrimanovic et al., 2011; Vicovaro et al., 2019) to be only roughly consistent with predictions based on the perceived size of the stimuli (perceived smaller = feels heavier), and it has been proposed (Vicovaro et al., 2019) that perceived weight, but not perceived volume, depends on the object's surface area (larger surface area = feels lighter). Similar to previous reports, in Chapter 4 I reported that the variance in perceived weight was not fully explained by the bias in perceived volume measured in the same participant sample (see Figure 4.2 A–B, C–D). However, a direct test of this (previously untested) hypothesis ruled out surface area as the predictor of the weight bias (H-shapes, having the largest surface area in the set, were not perceived as the lightest shape).

The results discussed in Chapter 4 offer a valuable contribution to the literature by replicating, extending, and disambiguating previous findings; however, they do not offer an explanation regarding the origin of these biases. It should be noted that, similar to the case of analogue biases in size perception, shape-related biases in heaviness perception appear on the whole fairly consistent across participants and experiments (although large individual differences have been reported by Kahrimanovic et al., 2011). Thus, the suggestion that the shape-weight illusion might be due to individual prior experience with objects of different shape (Buckingham, 2014) does not appear supported (as an aside, it is not clear how prior experience with given shapes should result in a bias in perceived weight unless different-shaped objects have different average densities). Interestingly, if the shape-weight illusion cannot be explained via the same mechanism as the SWI (i.e., if perceived size and related expectations are not the only factor governing the bias) investigation of the shape-weight illusion might help identify or describe the role of further factors involved in biases in heaviness perception. In Chapter 4, I proposed that mass distribution and related changes in rotational inertia / torques during lifting might be involved. I will discuss this proposal, which remains speculative, in the next paragraph.

Finger forces and torque in the shape-weight illusion

In Chapter 4, alongside perceptual biases, I investigated finger forces and torques when grasping and lifting different-shaped stimuli. A wealth of data is available about these variables in the SWI (as well as in non-illusory contexts; Baugh et al., 2016; Buckingham & Goodale, 2010b; Flanagan & Beltzner, 2000; Flanagan & Johansson, 2010; Gordon et al., 1991b, 1991c), but no reports existed about forces and torques in the context of the shape-weight illusion. Thus, an exploratory approach was deemed justifiable.

In the context of the SWI (and, more generally, in situations in which forces are initially grossly misapplied), it has widely been reported (Baugh et al., 2016; Buckingham & Goodale, 2010b; Flanagan & Beltzner, 2000) that the rates of change of grip and load force, pre-emptively scaled on the expected object weight, adjust to the actual object weight within a few trials. No clear pattern signalling a change in force rates between early and subsequent lifts was found in Chapter 4, which might be due to the adopted paradigm, which included several shape / volume combinations, different from classic SWI grasping paradigms involving two same-shape stimuli. A lack of adaptation has been reported by Platkiewicz & Hayward (2014) in a SWI study using a large set of uncorrelated mass-size stimulus pairs. Results from Chapter 4, therefore, add to previous research indicating that erroneously programmed force rates do not appear to contribute to weight biases (Buckingham et al., 2009; Flanagan & Beltzner, 2000; Grandy & Westwood, 2006; but see Buckingham & Goodale, 2010b; Dijker, 2014) and further suggest that rapid adaptation of sensorimotor memory might be context-dependent.

As expected, shapes varied in the amount of torque produced during lifting (see Figure 3.4 E), partly due to the different weight distribution in each shape. Interestingly, the pattern of correlations between force / torque variables and the perceptual biases revealed positive correlations between the weight bias and each force / torque variable with the exception of load force, which was independent of shape; in Chapter 4, I interpreted these correlations as showing adaptation of grip force in the presence of torque (see 4.3.2), and speculated differences in torque might contribute to the shape-weight illusion. Previous studies have shown that perceived heaviness varies as a function of mass distribution (Amazeen, 1997, 2014; Amazeen & Turvey, 1996; Plaisier & Smeets, 2015): For example, perceived heaviness is positively correlated with increases in height, and negatively correlated with increases in width that entail identical changes in the object's volume (Amazeen, 1997, 2014). The perceived weight for cuboids (higher than expected based on perceived size alone; compare Figure 4.2 A–B, C–D) might depend on both perceived size (negatively correlated with perceived weight) and torque / rotational inertia / distribution of mass along the vertical axis (positively correlated with perceived weight; Amazeen, 1997, 2014). Limitations apply: 1) One shape (the H-shape; see 4.4) does not fit this qualitative pattern; 2) It is not immediately clear how this account could extend to the shape-weight illusion that occurs when stimuli are placed on the participant's hand and not lifted (Kahrimanovic et al., 2011); 3) Chapter 4 does not provide a quantitative account for the biases as a function of perceived volume and torque. Existing evidence shows that the influence of rotational inertia (or related cues) on perceived weight is greatly reduced in presence of visual size cues (Amazeen, 1997), however, this has not been quantified. Future studies should investigate the influence of shape on perceived weight by systematically manipulating rotational inertia (e.g., by presenting the same cuboid vertically and horizontally oriented) and availability of size (i.e., with and without vision) and inertial cues (i.e., different exploration modes) to clarify the role of lifting dynamics in the shape-weight illusion and in heaviness perception more broadly.

5.3.3 The more, the merrier? Towards a comprehensive account of heaviness perception

In Chapter 1 (1.3.5), I reviewed evidence that suggests that several factors, including prior weight expectations and the perception of physical object features such as density and rotational inertia, are likely to contribute to the SWI and other biases in weight perception. Evidence provided in Chapter 3 and Chapter 4 of the present work offers several contributions to the debate on biases in heaviness perception, as discussed in the previous paragraphs.

Although offering a comprehensive account of biases in heaviness perception is beyond the scope of the present work, it is worth mentioning that a unifying account for the SWI and MWI is still lacking. The account presented in Chapter 3 cannot clearly explain the MWI and other 'conceptually-driven' weight illusions (Dijker, 2008; Ellis & Lederman, 1998; Saccone & Chouinard, 2019a) described in the Introduction of this thesis (1.3.3), nor the widely-cited finding that the illusion can be reversed after extensive training with objects where the usual size-weight relationship has been reversed (i.e. smaller = heavier; Flanagan et al., 2008). In all these cases, it appears that the bias in perceived weight is driven by the contrast between expected and actual weight, either because both objects are equal density (MWI), or because the smaller stimulus is also denser (reversed SWI). It should be noted that whilst Peters and colleagues have proposed that their competing prior model can extend to the MWI (Peters et al., 2016, 2018), the same model fails to explain data presented in Chapter 3 of the present work. Therefore, an explanation for these 'contrast' effects remains elusive. The model proposed in Chapter 3 provides an accurate description of perceived weight in the SWI and more broadly. However, it does not provide an explanation for the bias. A Bayesian model incorporating efficient coding (predicting biases away from the prior; Wei & Stocker, 2015) could broadly explain the SWI and MWI (as suggested by Bays, 2023). Future work should aim at modelling the effects captured in the model proposed in Chapter 3 within the Bayesian framework.

Furthermore, consistent with previous suggestions and as hinted by results presented in Chapter 4 (with the limitations discussed above), models of heaviness perception might have to take into account the rotational dynamics of the stimuli to extend to wider sets of objects.

5.4 Conclusions

The empirical work presented in this thesis makes several contributions to the understanding of how human observers estimate and utilise surface area, volume and weight. First, Chapter 2 provides a comprehensive account and model for biases in area perception that quantifies known (previously described in qualitative terms) and novel biases in terms of shape geometric features. Second, Chapter 3 provides an account of biases in perceived weight in a wider range of size / weight / density combinations than those classically utilised to test the SWI. It highlights limitations of recent models of the SWI, and proposes a model that describes perceived heaviness in the SWI and across a wide variety of variations in weight and density. Last, Chapter 4 provides a description of shape-related biases in weight perception that extends to novel shapes, and a first account of finger forces and torques in the context of these biases. Taken together, these results overcome several limitations from previous literature, generate new research questions, and highlight the importance of obtaining quantitative, replicable models of human perception.

Appendix A Chapter 2 Supplementary material

A.1 Simulations

Our Thurstonian scaling analysis method assumes that perception of stimulus area is perturbed by noise and that the spread (σ) of the Gaussian noise distribution is the same for every stimulus. Here we investigate the effect on estimated parameters (i.e. perceived relative area) when this assumption is violated, i.e., if shapes differ in noise/uncertainty with respect to their perceived area. To this end, we simulated data for three different variants of observers performing Experiment 1: Common shapes.

The first variant (V1) corresponds to an idealised observer completing Experiment 1, for whom all shape-related biases are perfectly scale invariant, and all stimuli are associated with the same noise (σ = 1.05, see Figure A1 A). The mean shape-related simulated biases (JNDs, averaged across stimulus size) were fixed to match those measured in Experiment 1: Common shapes. Discriminability within shapes was also matched to the average of those measured in Experiment 1: Common shapes.

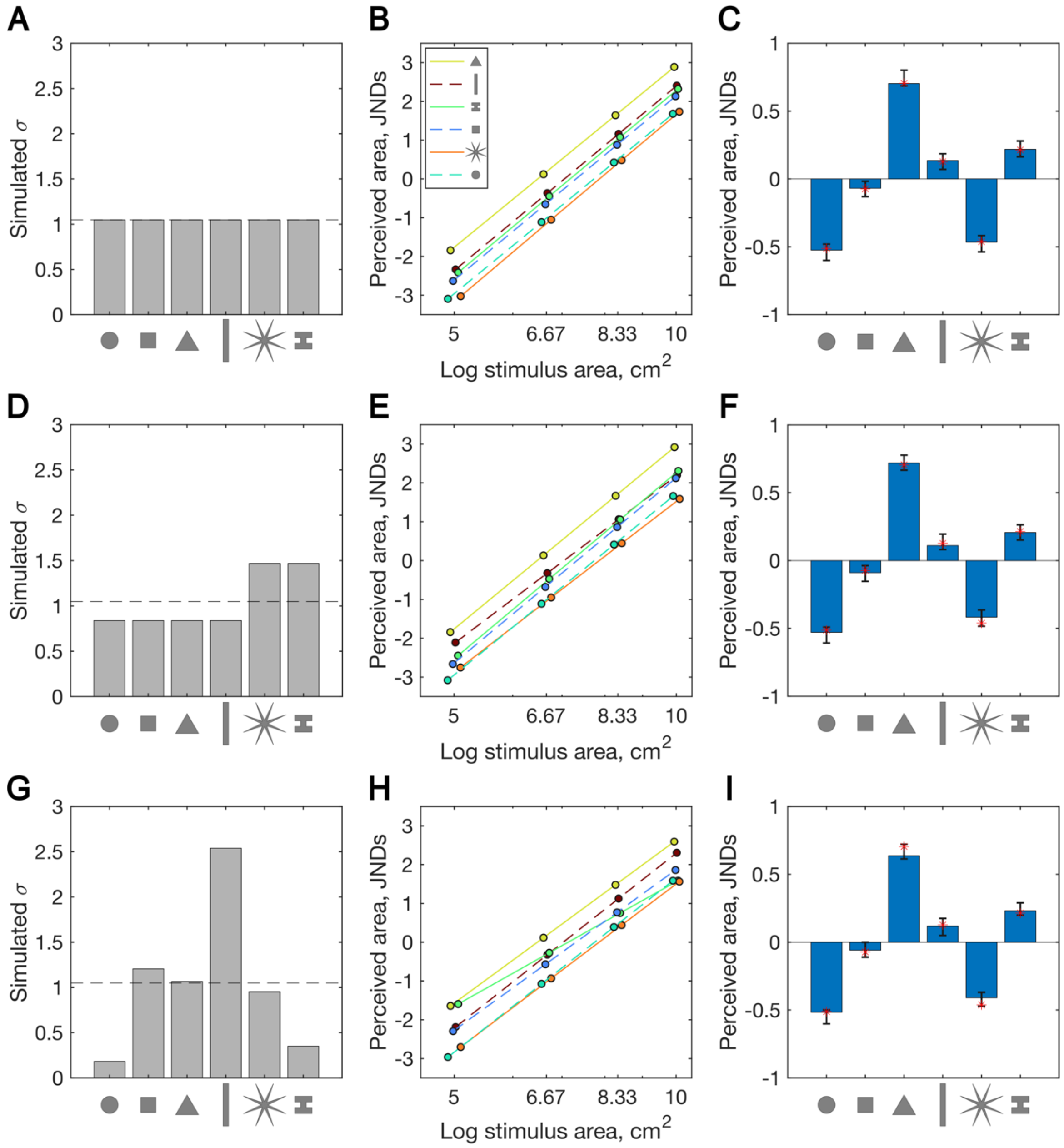


Figure A1 Data from simulations showing the effects of different noise distributions. Each row corresponds to one of the three observer variants. (A, D, G) Simulated noise parameters (σ) for each shape. (B, E, H) Perceived area in JNDs for each shape as a function of stimulus size for one observer completing 1000 trials per condition. (C, F, I) Perceived area in JNDs, averaged across stimulus size. Bars show data for the same single observer. Error bars give 95% CIs for the mean biases estimated from 35 simulated observers performing 2 trials per condition, from bootstrapping. Red stars give the mean simulated biases for each shape.

For the second and third observer variants (V2, V3) the simulated means of perceived area (i.e., the simulation JNDs) were unchanged, but we adjusted the noise parameters, violating the equal-noise assumption of our analysis. In V2, two shapes (rectangle, 8-pointed star) are associated with more uncertainty than the others (see Figure A1 D). Finally, in V3, the noise parameter for each shape was randomly assigned by sampling from a Gaussian distribution (Figure A1 G), resulting in huge variation across shapes.

For each variant, we first simulated a single observer performing the experiment, with 1000 trials for each stimulus pairing featured in the original experiment. This revealed the effects of different simulation parameters, with minimal sampling noise. The results are shown in Figure A1 B, E, H (all stimulus sizes) and also by the bars in Figure A1 C, F, I (biases averaged across size). For comparison, the simulated biases are shown by red asterisks.

For variant one, as expected, the simulated data reflects the simulation parameters (alignment of bars and stars in Figure A1 C). Under V2, the mismatched noise parameters result in small, spurious deviations from scale-invariant biases: the lines representing the rectangle and star in Figure A1 E have smaller slopes. For these shapes, perceived area is slightly overestimated for smaller stimuli and underestimated for larger stimuli (compare Figure A1 D with the actual data from Experiment 1: Common shapes in Figure 2.3 B). However, examination of Figure A1 F reveals that, after averaging across stimulus size, the resultant errors in the estimates of perceived relative area for each shape are minimal. Similarly, even the extreme range of values of σ simulated in V3 produce very small errors in estimated bias, when averaged across stimulus size Figure A1 I.

Next, we investigated the reliability of estimates of perceptual biases for the three simulated observer variants, given the true number of observers and trials in Experiment 1: Common shapes. To this end, for each variant, we simulated 1000 independent observers, each of whom performed the same number of stimulus comparisons as in the real experiment. Then we simulated experimental data collection by randomly selecting 35 of the 1000 candidate observers, and recording the mean shape-related biases, averaged across observers and stimulus size. This was repeated 1000 times, to derive 95% confidence intervals for the mean estimated area biases; these CIs are shown by the error bars in Figure A1 C, F, and I. It is apparent that large deviations from the equal noise assumption are well tolerated by our method; such deviations do not introduce meaningful systematic errors in estimates of perceptual bias, nor do they alter the reliability of those estimates. As noted above, deviations from equal noise do introduce small systematic (and opposite) errors in estimation of perceived area for small and large stimuli (the different slopes in Figure A1 E and H). In other words, our method can produce spurious scale dependencies in area biases. For this reason, our quantification and modelling of shape-related biases ignores the apparent small deviations from scale invariance.

A.2 Effect sizes

For each experiment, we calculated effect sizes for biases for each condition (shape / orientation combination, averaged across sizes 1–3 for each participant) compared against the reference condition (square, canonical orientation), i.e. perceived $area_{condition}$ - perceived $area_{reference}$. Figure A2 shows Cohen's d for each comparison.

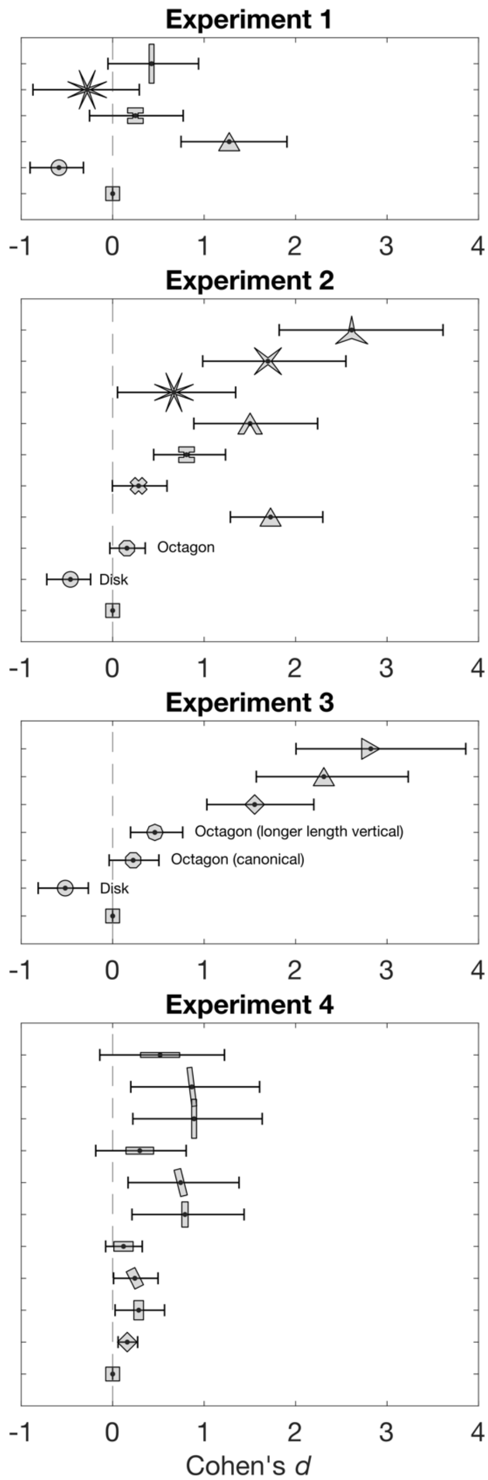


Figure A2 Effect sizes for all experiments. Marker shapes show the condition being compared to the reference stimulus (square, canonical orientation; vertical dashed line). Error bars show 95% *CIs* for paired samples (*meanEffectSize*, MATLAB).

A.3 Predictors

Measure	Description	References
Surface measures (in cm²)		
Area		
Area of base (1)	Area of the bottom 1/8 portion of the shape	
Area of base (3)	Area of the bottom 1/8 portion of the shape's convex hull	
Area of bounding box	Area of the smallest rectangle, with horizontal and vertical edges, that encloses the shape	
Area of convex hull (convex area)	Area of the shape's convex hull	
Area of circumscribing disk	Area of the smallest disk enclosing the shape	
Area of circumscribing ellipse	Area of the smallest ellipse enclosing the shape; equals to the area of circumscribing disk for shapes of aspect ratio of 1 and triangular shapes	
Linear measures (in cm)	All linear measures were multiplied by radius of same area disk	
'Additive area'	Sum of the shape's height and width (see below)	Yousif & Keil (2019)
Height (maximum vertical distance)	Longest distance between two boundary points on the y axis	Warren & Pinneau (1955)
Height of centroid	Distance between the shape's base and centroid on the y axis	
Width (maximum horizontal distance)	Longest distance between two boundary points on the x axis	
Width of base (1)	Average width calculated at the bottom 1/8 portion of the shape	
Width of base (2)	Average width calculated at the bottom 1/8 portion of the shape convex hull	
Perimeter		Anastasi (1936)
Convex Perimeter	Perimeter of the shape's convex hull	
Compactness: circumradius	Radius of the smallest disk enclosing the shape	

Measure	Description	References
Ratios	All ratio measures were multiplied by stimulus area	
Compactness (1): Area-to-area of circumdisk ratio	Ratio of the shape area to that of the smallest circumscribing disk	
Compactness (2): Convex area-to-area of circumdisk ratio	Ratio of the shape convex area (area of convex hull) to that of the smallest circumscribing disk (circumdisk)	
Compactness (3): Circularity	Ratio of the shape area to that of a circle of the same convex perimeter (perimeter of convex hull); equals IPQ (below) for convex shapes	
Compactness (4): Isoperimetric quotient (IPQ)	Ratio of the shape area to that of a circle of same perimeter	
Compactness (5): P2A	Squared perimeter-to-area ratio	Smets (1970)
Compactness (6): Roundness	Indisk-to-circumdisk ratio: Ratio of the area of the maximum inscribed disk (indisk) to the circumdisk of the shape	
Convexity	Ratio of the shape perimeter to the perimeter of the shape's convex hull	
Elongation	Aspect ratio of the ellipse enclosing the shape; orientation-invariant	
Elongation × sin(orientation)	Elongation × sin(θ); θ is the angle from the x axis to the major axis of the ellipse circumscribing the shape	
Elongation × cos(orientation)	Elongation × cos(θ)	
Height-to-width ratio	Ratio of the maximum vertical distance (height) to the maximum horizontal distance (width); orientation-dependent	Holmberg & Holmberg (1969), as cited in Krider et al., (2001)
Extent	Ratio of the shape's area to the area of the shape's bounding box	
Height of centroid-to-height ratio	Ratio of the shape's height of centroid (see above) to the shapes' height	
Solidity	Ratio of the shape area to the area of the shape's convex hull	

Table A1 List of predictors evaluated in the linear regression models. Predictors featuring in the final model are listed in italics.

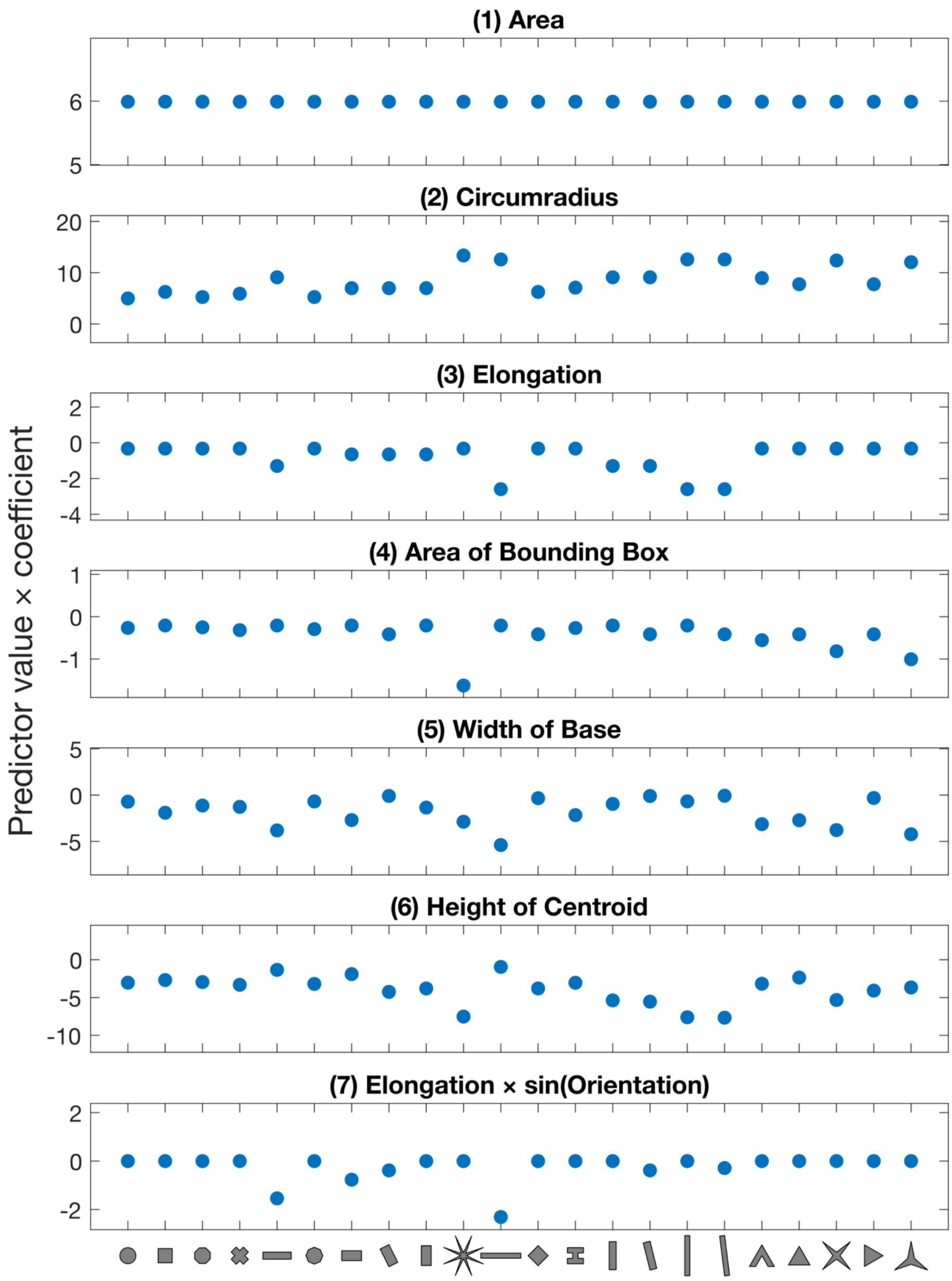


Figure A3 Influence of each predictor in each condition (shape / orientation combination). Conditions reported in order of perceived size (smaller to larger).

Appendix B Chapter 3 Supplementary material

B.1 Alternative candidate models: Model selection

Table B1 and B2 show alternative candidate models for perceived weight ($\widehat{W}$) as a function of the weight and density (Table B1) / volume (Table B2) of the target (W_1, D_1, V_1) and non-target (W_2, D_2, V_2). in the full dataset. Models with 2 to 5 coefficients were selected via stepwise-forward regression; at each step, the predictor which produced the largest significant increase to the model was selected. Statistical significance of each addition / exclusion was checked using F-tests.

Number of coefficients	2	3	4	5	6	7*	7	7	8**	9
Intercept	-10.75	-19.84	-26.87	-24.88	-26.41	-24.05	-23.68	-23.25	-24.25	-23.84
W_1	0.17	0.16	0.16	0.16	0.16	0.17	0.16	0.16	0.17	0.16
D_1		28.87	28.87	28.45	22.82	16.22	21.66		16.74	17.57
W_2			0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
D_2				-5.88			-8.01			-2.09
D_1X_1					1.6806	3790.9	1911.6	20210	3411.4	3050.8
D_2X_2						-2566.2		-7230.8	-2137.2	-1794.8
σ_1					95.67	75.34	101.1	287.28	65.05	62.59
σ_2						(σ_1)		315.82	58.2	54.26
SSR	2886.6	2047	1434.4	1399.8	1135.6	1025.1	1072.4	1119.1	1016.2	1013.7
R^2	0.95	0.97	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
$F(DoF_1, DoF_2)$		84.9	87.98	5.07	47.47	21.87	11.96	2.99	1.77	0.51
p		< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	<0.001	>.99

Note: $X_1 = \mathcal{N}(|W_1 - W_2|,\ 0,\ \sigma_1^2)$; $X_2 = \mathcal{N}(|W_1 - W_2|,\ 0,\ \sigma_2^2)$

Table B1 Fitted coefficients and model comparison statistics for each candidate model. W_1, W_2 = target, non-target weight; D_1, D_2 = target, non-target density. For each addition, models were compared with the (best) model with N-1 predictors; * denotes the best 7-predictor model; ** indicates the best model.

Number of coefficients	2	3	4	5
Intercept	-10.75	-19.62	-14.63	-14.63
W_1	0.17	0.17	0.2	0.2
V_1		0.01	0.01	0.01
V_2			-0.02	-0.02
W_2				0.01
SSR	2886.7	2156.2	1843.8	1575.4
R^2	0.95	0.96	0.97	0.98

Table B2 Fitted coefficients and model comparison statistics for the stepwise regression using target and non-target weight (W_1, W_2) and volume (V_1, V_2) as candidate predictors.

B.2 Alternative candidate models: Other models

We fit (1) Wolf and colleagues' (2018) cue integration model and (2) Peters and colleagues' (2016) Bayesian model to our data. Fitted coefficients for model (1) are reported in Table B3 alongside model comparison statistics. Model (1) predictions for the current experiments' data are shown in Figure B1 (1) (compare with Figure 3.3 in Chapter 3); model (2) predictions for the current experiments' data are shown in Figure B2 B alongside our model predictions (Figure B2 A). Note that Peters and colleagues' model (2) predicts the perceived weight ratio between the two objects in a pair, but does not predict perceived weight for each object.

Number of coefficients	4
b_1	0.04
x	1.19
b_2	41.56
y	3.59
SSR	2375
R^2	0.96

Table B3 Fitted coefficients and model comparison statistics for Wolf's (2018) model. As availability of size information was not manipulated in our experiment, the model does not include coefficients for density weights (see Chapter 3 for a description of the model).

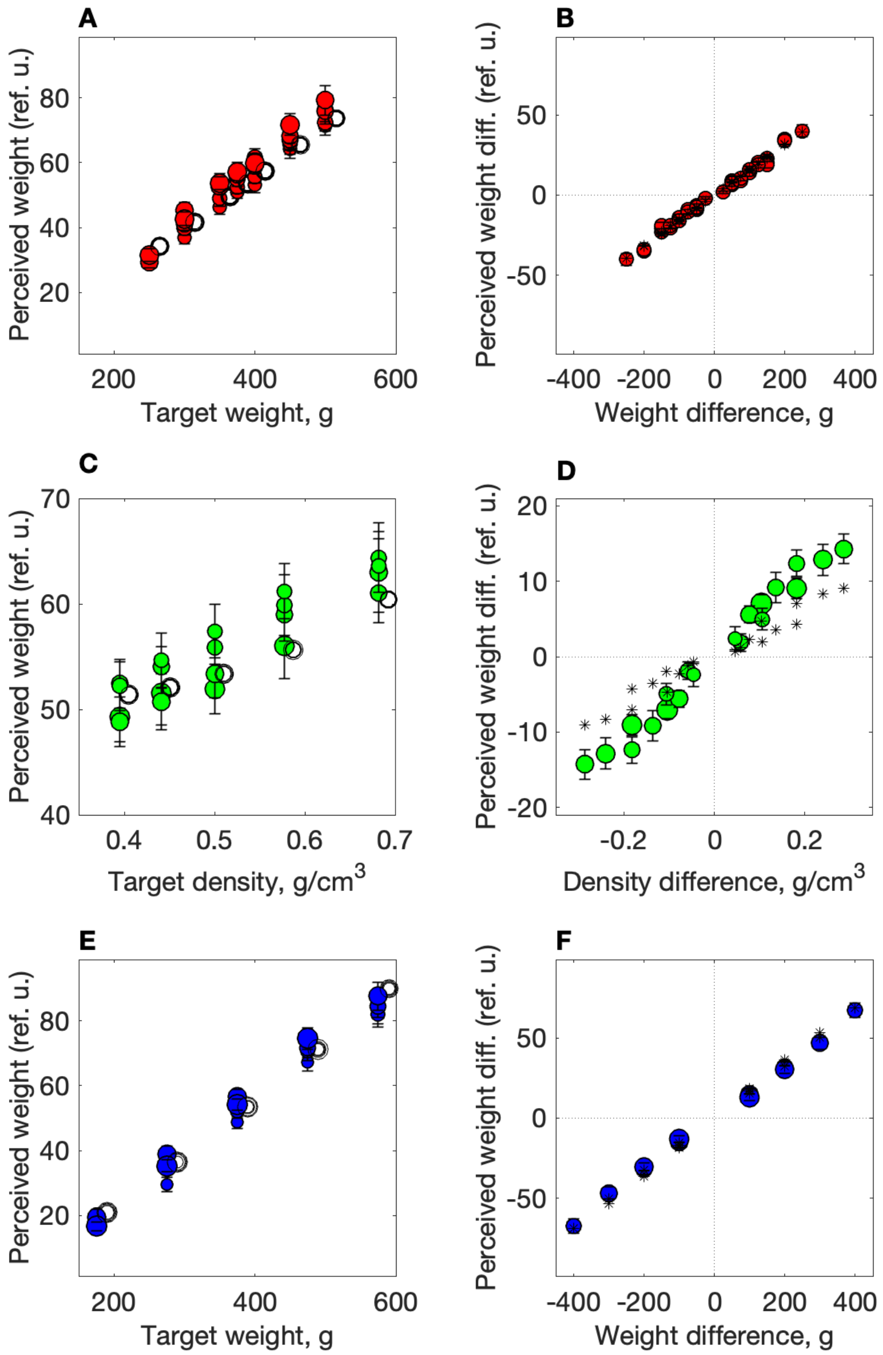


Figure B1 Experiment data from Chapter 3 (see Figure 3.3) with Wolf and colleagues (2018) model predictions; (A, C, E) black circles, (B, D, F) black asterisks show the model predictions.

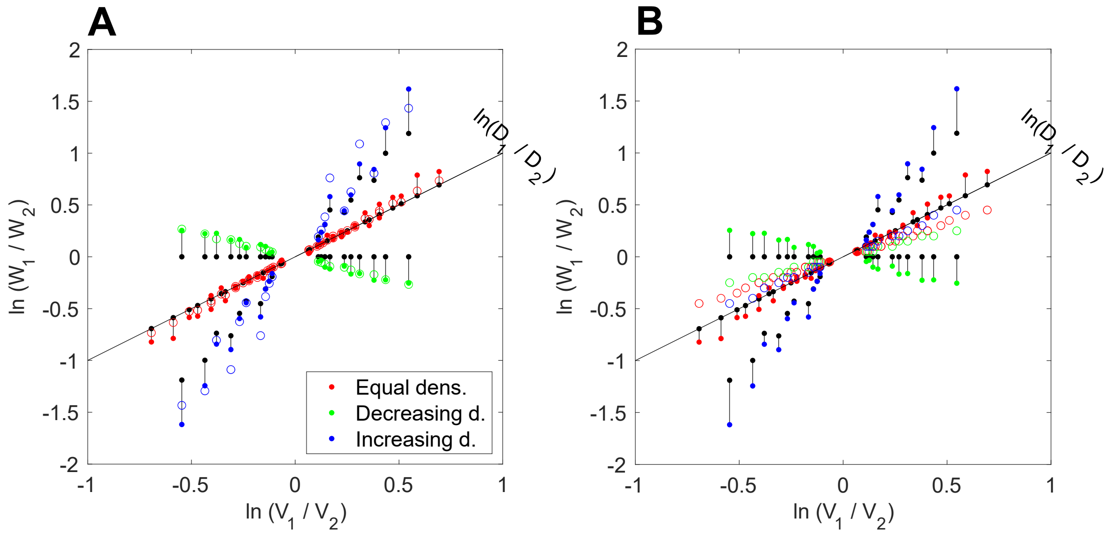


Figure B2 Stimuli (black dots), data (coloured dots), and model predictions (circles) for each within-subset stimulus set in the current experiment. Plotting conventions as in Figure 3.1. (A) Predictions from best model (see Equation 1 in main text); Data and model predictions re-expressed as log ratios to match Peters and colleagues' (2016) model predictions. (B) Peters and colleagues' (2016) model predictions.

Appendix C Chapter 4 Supplementary material

C.1 Force rates across trials

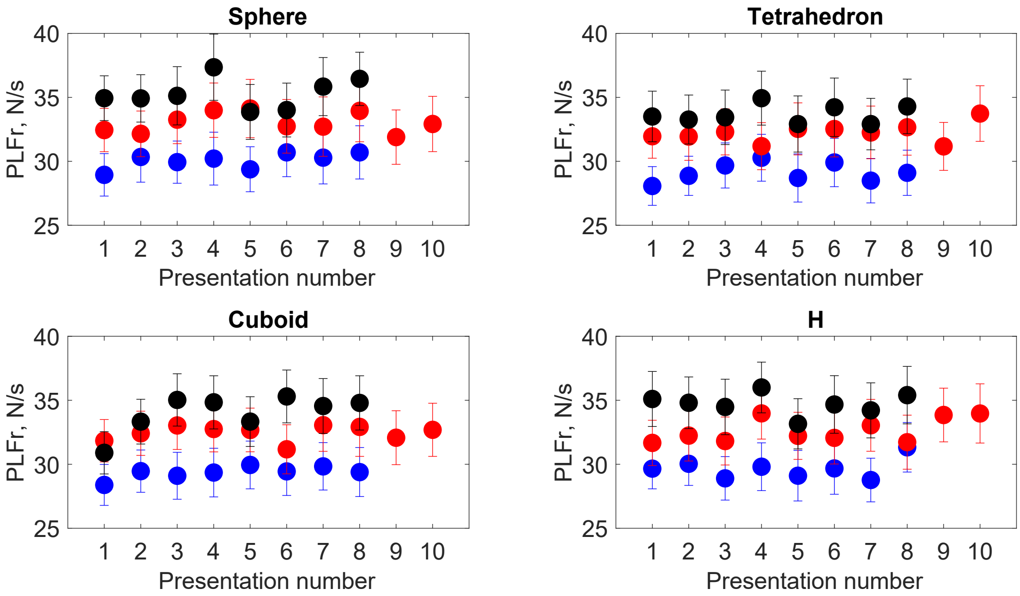


Figure C1Peak load force rate (PLFr) for each shape and volume as a function of the stimulus number (i.e. trial number for a given stimulus), averaged across lifts and participants. Colours show the stimulus volume; lines show the averages for each volume.

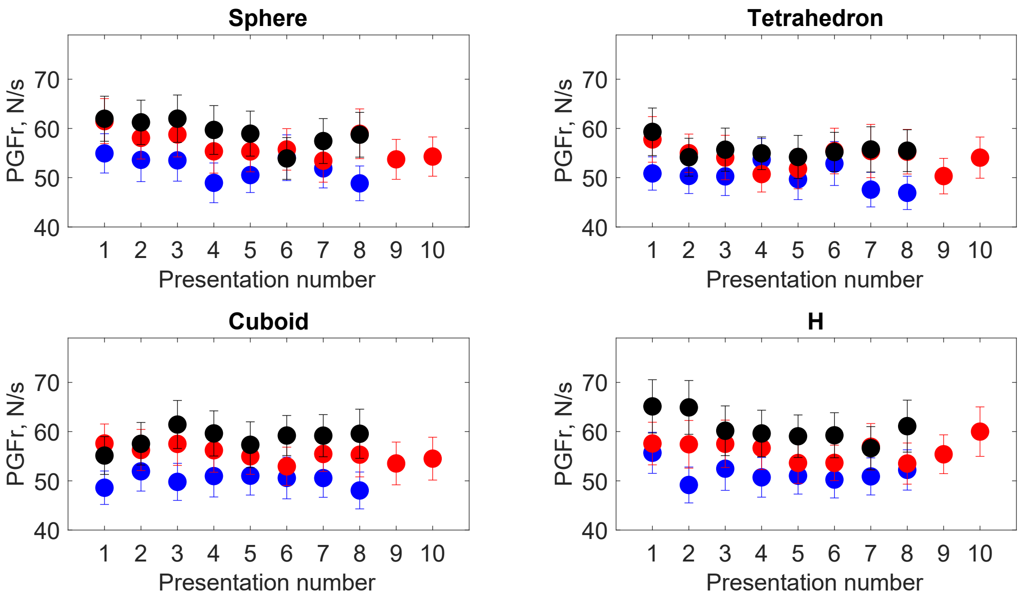


Figure C2 Peak grip force rate (PGFr) for each shape and volume as a function of the stimulus number (i.e. trial number for a given stimulus), averaged across lifts and participants. Colours show the stimulus volume; lines show the averages for each volume.

C.2 Regression models

In each table are reported regression coefficient for Y (averaged across participants; see table captions) as a function of shape (3 dummy variables, sphere as intercept) and volume (or weight; both in reference units).

	Estimate	95%CI	
		LL	UL
Intercept (Shape = sphere)	47.24	46.98	47.50
Shape = tetrahedron	5.42	5.05	5.79
Shape = cuboid	6.94	6.57	7.31
Shape = H	0.61	0.24	0.98
Volume	0.05	0.04	0.05

Table C1 Perceived volume (reference units) as a function of shape and volume.

	Estimate	95%CI	
		LL	UL
Intercept (Shape = sphere)	58.95	58.24	59.65
Shape = tetrahedron	-6.25	-7.25	-5.25
Shape = cuboid	-4.50	-5.50	-3.50
Shape = H	-2.93	-3.93	-1.93
Weight	0.17	0.16	0.18

Table C2 Perceived weight (reference units) as a function of shape and weight.

	Estimate	95%CI	
		LL	UL
Intercept (Shape = sphere)	3.84	3.81	3.87
Shape = tetrahedron	0.01	-0.03	0.06
Shape = cuboid	0.02	-0.02	0.06
Shape = H	0.03	-0.02	0.07
Weight	0.07	0.07	0.07

Table C3 Peak load force (N) as a function of shape and weight.

	Estimate	95%CI	
		LL	UL
Intercept (Shape = sphere)	8.74	8.61	8.89
Shape = tetrahedron	-0.38	-0.58	-0.19
Shape = cuboid	-0.11	-0.31	0.08
Shape = H	0.06	-0.14	0.25
Weight	0.11	0.095	0.12

Table C4 Peak grip force (N) as a function of shape and weight.

	Estimate	95%CI	
		LL	UL
Intercept (Shape = sphere)	33.43	33.05	33.80
Shape = tetrahedron	-1.06	-1.59	-0.52
Shape = cuboid	-1.13	-1.67	-0.60
Shape = H	-0.54	-1.08	-0.01
Weight	0.32	0.29	0.35

Table C5 Peak load force rate (N/s) as a function of shape and weight.

	Estimate	95%CI	
		LL	UL
Intercept (Shape = sphere)	57.03	56.19	57.87
Shape = tetrahedron	-2.52	-3.71	-1.34
Shape = cuboid	-1.73	-2.91	-0.54
Shape = H	0.15	-1.04	1.33
Weight	0.50	0.43	0.57

Table C6 Peak grip force rate (N/s) as a function of shape and weight.

	Estimate	95%CI	
		LL	UL
Intercept (Shape = sphere)	32.97	32.24	33.70
Shape = tetrahedron	-2.15	-3.19	-1.11
Shape = cuboid	-0.09	-1.13	0.95
Shape = H	2.07	1.04	3.11
Weight	0.40	0.34	0.46

Table C7 Peak torque (N-m) as a function of shape and weight.

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