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Revealing preference discovery: a chronological choice framework

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Abstract

We propose a framework for the analysis of choice behaviour when the latter is made explicitly in chronological order. We relate this framework to the traditional choice theoretic setting from which the chronological aspect is absent, and compare it to other frameworks that extend this traditional setting. Then, we use this framework to analyse various models of *preference discovery*. We characterise, via simple revealed preference tests, several models that differ in terms of (1) the priors that the decision-maker holds about alternatives and (2) whether the decision-maker chooses period by period or uses her knowledge about future menus to inform her present choices. These results provide novel testable implications for the preference discovery process of myopic and forward-looking agents.

Keywords Preference discovery · Chronological choice · Revealed preferences · Myopic agents · Forward-looking agents

1 Introduction

An important accomplishment of modern economic theory is the precise identification of its behavioural implications. A rich and now classical tradition of research, initiated by Samuelson (1938) and pursued by Houthakker (1950), Chernoff (1954), Arrow (1959), Richter (1966), Sen (1971), among many others, has formulated these implications in terms of a *choice function* (sometimes generalised to a *choice correspondence*), that assigns to every set of alternatives (or menu) in some universe a unique element of it, interpreted as the chosen alternative in that menu. By now, the behavioural implications of a significant variety of theories have been examined

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through the formalism of choice functions or correspondences (e.g., Richter 1966; Masatlioglu et al. 2012; Manzini and Mariotti 2007, 2012; Apesteguia and Ballester 2013; Cuhadaroglu 2017).

Flexible and amenable to formulations of testable implications of many choice models as it is, a choice function may still be considered unduly abstract for many applications. One of the important and easily observable features of reality that it neglects is the (time) *period* at which the menu is made available to the decision-maker. Indeed, as used in the literature above, a choice function describes a timeless process that only specifies the chosen alternatives in every admissible menu. It does not record (nor use information on) the periods at which the menus are available. Yet, in most choice data that we can think of, menus of alternatives present themselves to the decision-maker one after the other, and this chronological information is known. For instance, most firms record the time at which their consumers make their purchases, and economic experiments record the chronological order of choices made by their subjects.

In this paper, we extend the traditional choice-theoretic setting to analyse choice behaviour as an explicit function of *both* the period at which the choice takes place *and* the menu available at that period. Doing this enables us to identify the revealed preference implications of many empirically relevant theories of choice that could not be analysed in the standard timeless framework. The theories of choice to which we focus in this paper concern *preference discovery*. The hypothesis that decision-makers behave inconsistently when first faced with unfamiliar tasks but that with "sufficient" practice they *discover* stable preferences is influential in behavioural and experimental economics (see the seminal papers by Smith 1994, Plott 1996, and Binmore 1999). While the empirical evidence for this hypothesis is mixed, several experiments show that subjects' choices change and converge towards rationality benchmarks with experience (e.g., Cubitt et al. 2001; van de Kuilen and Wakker 2006; van de Kuilen 2009; Birnbaum and Schmidt 2015). Here, we characterise–via simple revealed preference axioms–five models of preference discovery. In all these models, the decision-maker chooses as if she discovers her "definite" preference over two alternatives only after having chosen (tried) them. The models differ in terms of (1) the priors that the decision-maker holds about alternatives before trying them and (2) whether or not the decision-maker uses her knowledge about future menus to inform her present choices.

We view our paper as making two contributions to the literature. First, we introduce a revealed preference framework for the analysis of models of choice behaviour that depend on the chronological order of menus.[1] We are certainly not the first to extend the revealed preference methodology from the classical abstract choice theoretic setting. A prominent extension is provided by Salant and Rubinstein (2008) who analyse a choice framework in which every menu of alternatives

[1] To the best of our knowledge, Cerigioni (2017) is the only other paper that has–independently–used an analogous framework of chronological choice. However, while we focus on the characterisation of individual behaviour by means of revealed-preference axioms in a deterministic (non-stochastic) setting, Cerigioni (2017) is interested in the probabilities of choosing alternatives that can be inferred from the choices of a collection of decision-makers.

is supplemented with a *frame*, i.e., with "observable information that is irrelevant in the rational assessment of the alternatives, but nonetheless affects choice" (p. 1287; see also Bernheim and Rangel 2007, 2009). While one could consider the chronological order of menus to be a "frame", we believe that such a perspective is ill-suited. Indeed, as we shall illustrate in this paper, information about the chronological order of menus can be relevant in the rational assessment of the alternatives. Therefore, the chronological order of menus is not a frame in Salant and Rubinstein's (2008) sense. Our framework is more closely related to that of Maltz (2016), who expands the traditional choice domain with the decision-maker's past experience, such that a choice problem is the combination of the menu and the list of previously experienced alternatives presented in the order in which they were experienced. Our chronological choice framework extends Maltz's (2016) framework as it (1) not only records information about the chosen elements *but also* about the menus available to the decision-maker, and it (2) allows an observer to model forward-looking agents. Finally, our framework is also related to that of Cerigioni (2021), who proposes a choice theoretic framework that combines a chronology of choices with information about *environments* such as menus or frames *à la* Salant and Rubinstein (2008). He characterises a "dual-self" theory of choice in such a framework. As compared to his, our analysis is closer to the classical choice theory (and easier to use in empirical applications) since, except for the chronological order of menus, we do not consider any other additional element of the choice environment.

A second contribution of our analysis concerns the growing literature on preference discovery (e.g., Plott 1996; Binmore 1999; van de Kuilen 2009; Birnbaum and Schmidt 2015; Piermont et al. 2016; Cooke 2017; Wilson 2018; Delaney et al. 2020) and the large related literature on *experience goods* (e.g., Nelson 1970, 1974; Huang et al. 2009; Henze et al. 2015; Feldman et al. 2019). Experience goods are goods for which it is difficult to obtain product quality information before purchase. Because of this, consumers must learn about the product through consumption experiences. Therefore, as noted by Cooke (2017, p. 1323), experience goods such as antidepressants are notable examples of goods for which people face preference discovery processes in the "real world". We contribute to this literature by identifying novel testable revealed preference implications for various processes of preference discovery. Recent contributions to the theoretical literature examine related models, but their emphasis is on preference discovery through experimentation involving uncertain prospects (Piermont et al. 2016; Cooke 2017). Wilson (2018) proposes a utility maximisation model incorporating preference discovery costs. However, his analysis significantly differs from ours and it does not focus on revealed preference implications. As part of our analysis, we are also among the first–to the best of our knowledge–to formally distinguish between myopic and forward-looking preference discovery. The behavioural implications of these models can be empirically tested, for example, in experimental settings where decision-makers are informed in advance about future menus or not. In the former cases decision-makers can exhibit either myopic or forward-looking behaviour, while in the latter they are compelled to be myopic given their lack of information.

The rest of the paper proceeds as follows. In Sect. 2, we introduce some basic notation. In Sect. 3, we put forward our framework to study chronological choice and relate it to the classical model of a choice correspondence and rational choice. We then provide revealed preference characterisations of models of myopic preference discovery (Sect. 4.1) and forward-looking preference discovery (Sect. 4.2). Section 5 concludes.

2 General notation

A binary relation R on a set Ω is a subset of $\Omega \times \Omega$. Following common usage, we write $x\ R\ y$ instead of $(x, y) \in R$. Given a binary relation R, we define its symmetric factor I by $x\ I\ y \Longleftrightarrow x\ R\ y$ and $y\ R\ x$ and its asymmetric factor P by $x\ P\ y \Longleftrightarrow x\ R\ y$ and not $(y\ R\ x)$. A binary relation R on Ω is: (i) reflexive if the statement $x\ R\ x$ holds for every x in Ω, (ii) transitive if $x\ R\ z$ follows $x\ R\ y$ and $y\ R\ z$ for any $x, y, z \in \Omega$, (iii) complete if $x\ R\ y$ or $y\ R\ x$ holds for every distinct x and y in Ω, and (iv) antisymmetric if $x\ I\ y \Rightarrow x = y$. An ordering is a reflexive, transitive, and complete binary relation, and a linear ordering is an antisymmetric ordering.

Given two binary relations R_1 and R_2, we say that R_2 is an extension of R_1 (or is compatible with R_1) if it is the case that, for any x and y in Ω such that $x\ R_1\ y$ one has also $x\ R_2\ y$. Given a binary relation R on a set Ω, we define its transitive closure $\widehat{R}$ by $x\ \widehat{R}\ y \Longleftrightarrow \exists \{x_j\}_{j=0}^{l}$ for some integer $l \geq 1$ satisfying $x_j \in \Omega$ for all $j = 0, \ldots, l$ for which one has $x_0 = x$, $x_l = y$ and $x_j\ R\ x_{j+1}$ for all $j = 0, \ldots, l-1$. It is well-known that the transitive closure of a binary relation R is the smallest (with respect to set inclusion) transitive binary relation compatible with R.

Finally, we denote by $\mathbb{N}$ and $\mathbb{N}_+$ the set of non-negative and strictly positive integers (respectively), and by $\#A$ the cardinality of any finite set A.

3 Chronological choice framework

Let X be a universe of *alternatives* of interest for the decisionmaker, $\mathcal{P}(X)$ be the set of all finite and non-empty subsets of X, and $\mathcal{F}$ be a collection of non-empty finite subsets of X, each of which is interpreted as a possible *menu* offered to a decision-maker, whose choice behaviour is described by a *choice correspondence* $C : \mathcal{F} \rightarrow \mathcal{P}(X)$ satisfying $C(A) \subset A$ for every A in $\mathcal{F}$. If C satisfies the additional requirement that $\#C(A) = 1$ for every menu A in $\mathcal{F}$ (the decision-maker chooses only one alternative in every menu that she may face), then C is called a *choice function*. The choice-theoretic literature that has emerged in the last seventy years or so has made various assumptions on the domain $\mathcal{F}$ that depend, sometimes, upon the nature of the alternatives in X that are considered. For example, the classical theory introduced by Arrow (1959) has taken X to be an abstract set, and $\mathcal{F}$ to include all non-empty subsets of X. Such a setting is adequate to formulate representation theorems that allow us to have a deeper understanding of different behavioural phenomena. At the same time, it is clearly demanding from an observational viewpoint, since it is difficult in practice to observe all choices that an agent could make in every logically conceivable finite menu. In response, in subsequent years several authors (including Richter 1966, Hansson 1968, and Suzumura 1976, 1977 and 1983) have worked with "general" domains that do not impose any restriction on the class of menus that may be available.

The description of choice provided by the correspondence C gives rise to various "revealed preference" notions that we will appeal to below. These are the following.

Definition 1 (*Weak direct revealed preference*) For any two alternatives x and $y \in X$, we say that $C : \mathcal{F} \to \mathcal{P}(X)$ reveals a weak direct preference for x over y, denoted $x\, W_D^C\, y$, if there exists a menu $A \in \mathcal{F}$ such that $x \in C(A)$ and $y \in A$.

Definition 2 (*Strict direct revealed preference*) For any two alternatives x and $y \in X$, we say that $C : \mathcal{F} \to \mathcal{P}(X)$ reveals a strict direct preference for x over y, denoted $x\, S_D^C\, y$, if there exists a menu $A \in \mathcal{F}$ such that $x \in C(A)$ and $y \in A \backslash C(A)$.

Definition 3 (*Weak indirect revealed preference*) For any two alternatives x and $y \in X$, we say that $C : \mathcal{F} \to \mathcal{P}(X)$ reveals an indirect preference for x over y, denoted $x\, W_I^C\, y$, if there exists a sequence of $\bar{j}$ alternatives $\{z_j\}_{j=1}^{\bar{j}}$ (for some $\bar{j} \in \mathbb{N}_+$) in X such that:

(i) $z_1 = x$
(ii) $z_j\, W_D^C\, z_{j+1}$ for all $j = 1, \ldots, \bar{j} - 1$ and,
(iii) $z_{\bar{j}} = y$

Definitions 1 and 3 coincide, respectively, with Richter's (1966) definitions of direct and indirect revealed preference "in the wide sense", while Definition 2 corresponds to Arrow's (1959) definition of direct revealed preference. As shown below, we can identify the empirical implications of the theories of choice studied herein by imposing Richter's (1966) congruence axiom on suitably defined choice correspondence and domains $\mathcal{F}$. This approach has the main advantage of providing a unified methodology that is parsimonious in terms of its formal description. The original statement of Richter's (1966) congruence axiom is as follows.

Axiom 1 (Richter's congruence) The choice correspondence $C : \mathcal{F} \to \mathcal{P}(X)$ satisfies Richter's congruence if for any two alternatives x and $y \in X$ for which $x\, W_I^C\, y$, $y \in C(A)$ and $x \in A$ imply $x \in C(A)$ for any $A \in \mathcal{F}$.

We also observe that when the correspondence is a function, there is an alternative formulation of Richter's congruence axiom. The proof of this remark–and of all formal statements of this paper–is in the Appendix.

Remark 1 Let the choice correspondence $C : \mathcal{F} \to \mathcal{P}(X)$ be such that $\#C(A) = 1$ for every $A \in \mathcal{F}$. Then C satisfies Richter's congruence if and only if for every two distinct alternatives x and y in X such that $x\, W_I^C\, y$, one can not have $y\, S_D^C\, x$.

This remark shows that, when applied to a choice function, Richter's congruence is equivalent to Houthakker's (1950) Strong Axiom of Revealed Preference (SARP). We also note that these two axioms imply Samuelson's (1938) Weak Axiom of Revealed Preference (WARP).[2] However, Richter's congruence and WARP are not equivalent when the domain of the choice function (i.e., the collection of menus $\mathcal{F}$) is a strict subset of the set $\mathcal{P}(X)$ of all conceivable menus.

In this paper, we supplement this general setting with a discrete-time horizon that enables the definition of a ***chronology of choices*** as a list of pairs $\{(A^t, a^t)\}_{t=1}^T$ for some (strictly positive integer) T where, for every period $t \in \{1, \dots, T\}$, $A^t \in \mathcal{P}(X)$ and $a^t \in A^t$. The interpretation is that, at every period t, a particular menu A^t of alternatives is presented to the decision-maker who chooses the alternative a^t in that menu. In this setting, it is possible that for two different periods t and t', $a^t \neq a^{t'}$ even though $A^t = A^{t'}$. That is, a decision-maker who faces the same menu at two different periods *may* make different choices from this menu. Note that, in line with the literature working on "general" domains, we do not assume the possibility of observing choices in any conceivable combination of time period and menu at that period. We only consider, somewhat realistically, that we observe a particular chronology of choices.

We also observe that the specification of a chronology of choices is silent about the origin of the sequence of menus. This sequence can thus be either exogenously imposed on the decision-maker or endogenously chosen by her as a result of her sequential choices (as would be the case for a decision-maker whose savings decision at time t determines the budget set she will face at time $t+1$). However, a chronology of choices does not offer any information that would enable the testing of a particular theory of *choice of menus* (see Strotz 1955, Koopmans 1964, Peleg and Yaari 1973, Hammond 1976, and Kreps 1979 for seminal references).

3.1 Chronological choice and the classical rational choice model

We now briefly relate our framework to the classical model of rational choice discussed, for instance, by Richter (1966). In a traditional version of this model, the decision-maker is assumed to have a time-invariant linear preference ordering over all alternatives in X and to choose from every menu her favourite option for that preference. It is easy to see that information about the chronological order in which choices take place is useless for identifying the observable implications of this theory of choice. Indeed, one can define, for any chronology of choices $\{(A^t, a^t)\}_{t=1}^T$, an induced *timeless family of menus* $\mathcal{F}(\{(A^t, a^t)\}_{t=1}^T)$ by $\mathcal{F}(\{(A^t, a^t)\}_{t=1}^T) = \{A \in \mathcal{P}(X) : \exists s \in \{1, \dots, T\}$ s.t. $A = A^s\}$. The family $\mathcal{F}(\{(A^t, a^t)\}_{t=1}^T)$ is thus the set of all menus faced by the decision-maker in the chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ when abstracting from the period at which those menus appear. Given this timeless family $\mathcal{F}(\{(A^t, a^t)\}_{t=1}^T)$, we can define an induced *timeless choice correspondence* $C : \mathcal{F}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X)$ by $C(A) = \{x \mid x = a^t$ for some $t \in \{1, \dots, T\}$ s.t. $A^t = A\}$ for every $A \in \mathcal{F}(\{(A^t, a^t)\}_{t=1}^T)$.

[2] WARP requires a choice function to never reveal a weak direct preference–in the sense of Definition 1 –for x over y and an opposite strict direct preference–as per Definition 2 –for y over x.

Let us now define formally the (standard) property of rationalisability by a single time-invariant linear ordering R for a chronology $\{(A^t, a^t)\}_{t=1}^T$.

Definition 4 A chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the maximisation of a time-invariant preference if and only if there is a linear ordering R on X such that for every $t \in \{1, \dots, T\}$, $a^t \; R \; a'$ for all $a' \in A^t$.

It is then easy to prove the following proposition:

Proposition 1 *A chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the maximisation of a time-invariant preference if and only if its associated timeless choice correspondence $C : \mathcal{F}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X)$ is a function that satisfies Richter's congruence.*

Hence, the information about the chronological order of choices does not provide any help in identifying the empirical implications of the classical definition of rational choice. The testable (observable) implications of this model are all captured in the requirement that the induced timeless choice correspondence be a function that satisfies Richter's congruence axiom. We now turn to theories of choices for which the chronological order in which choices are made matters crucially for the identification of their empirical implications.

4 Revealing preference discovery

In this section, we provide testable implications for five models in which the decision-maker chooses as if she discovers her "definite" preference over two alternatives only after having chosen (tried) them.

To study these models, we denote, given a chronology of choices $\{(A^t, a^t)\}_{t=1}^T$, by $t(x) = \min\{t \in \{1, \dots, T, T+1\} : a^t = x\}$ the period in the chronology where $x \in X$ is chosen for the first time, with the convention that this period is $T + 1$ if x is never chosen. We also denote, for every $t \in \{1, \dots, T\}$ and alternative x, by $s^t(x)$ the *experienced status* of x at period t, defined by:

$$s^t(x) = e \text{ if } 1 \leq t(x) < t \text{ and,}$$
$$s^t(x) = u \text{ if } t \leq t(x) \leq T + 1$$

where the letters e and u stand for "experienced" and "unexperienced" respectively. In other words, $s^t(x) = e$ whenever x has been chosen before t and $s^t(x) = u$ otherwise. We observe that at any period t, s^t may be viewed as a function that maps X into the finite set $\{e, u\}$. This observation enables us to define, for every period $t \in \{1, \dots, T\}$, the set $\widehat{A}^t \subset X \times \{e, u\}$ by $\widehat{A}^t = \{(a, s^t(a)) : a \in A^t\}$. Hence, $\widehat{A}^t$ is the set of *ordered pairs* formed by the actual alternatives in the menu A^t and their *experienced status* at period t. We refer to $\widehat{A}^t$ as the *experienced-status* version of A^t. Similarly, we denote by $\widehat{a}^t \in \widehat{A}^t$ the pair $(a^t, s^t(a^t))$ made of the chosen alternative in the menu A^t and its experienced status at t. We observe that the experienced-status versions of A^t and a^t depend crucially upon the timing at which the choices are made, since this may modify the experienced-status of the chosen alternatives. As we did in Section 2, we can define for any chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ an induced *timeless experienced-status family of menus* of $X \times \{e, u\}$ by $\widehat{\mathcal{F}}(\{(A^t, a^t)\}_{t=1}^T) = \{\widehat{A} \subset X \times \{e, u\} : \exists t \in \{1, \dots, T\}$ s.t. $\widehat{A} = \widehat{A}^t\}$. Then, we can define a *timeless experienced-status choice correspondence* $\widehat{C} : \widehat{\mathcal{F}}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u\})$–where $\mathcal{P}(X \times \{e, u\})$ is the set of all non-empty subsets of $X \times \{e, u\}$–by $\widehat{C}(\widehat{A}) = \{(a, s) \in X \times \{e, u\} : \exists t$ s.t. $(a, s) = \widehat{a}^t = (a^t, s^t(a^t))$ and $\widehat{A}^t = \widehat{A}\}$ for every $\widehat{A} \in \widehat{\mathcal{F}}(\{(A^t, a^t)\}_{t=1}^T)$.

As indicated above, the identification of the empirical implications of our preference discovery models will take the form of Richter's congruence axiom imposed on suitably defined (timeless) choice correspondences. The chronological information about choices, which affects the experienced-status of the alternatives as time goes by, is used to construct these choice correspondences and their associated experienced-status-augmented chronology of choices.

4.1 Myopic preference discovery

In this subsection, we present three models of preference discovery that differ in terms of the priors that the decision-maker holds (or not) about alternatives before experiencing them. As we show below, the three models are nested: The third model implies the second and the second model implies the first. In other words, the observable implications of the models are increasingly restrictive from the first to the third model. These three first models all share the assumption that the decision-maker is "myopic" in the sense that she does not take future menus into consideration when making present choices.

4.1.1 Myopic preference discovery without a prior

We first examine a preference discovery model in which the decision-maker behaves *as if* she has no initial prior about the alternatives and "discovers" her preference via "trial-and-error". In this model, at each period, the agent either chooses the "best" alternative among those she has already "tried" (whose experienced status is e) *or* tries out an alternative that she has never chosen before (whose experienced status is u). Since the decision-maker has no prior assessment of the relative value of the untried alternatives (or, more precisely, since we make no *a priori* assumption about the decision-maker's priors in this model), this choice model imposes no restrictions whatsoever on the trial-and-error process by which knowledge of the alternatives is acquired. Let us illustrate this model with two examples.

Example 1 Consider the following chronology of choices involving choices between Indian delicacies from a traveller who has never been exposed to Indian cuisine before:

$$(A^1, a^1) = (\{dahl, paneer, dosa\}, paneer)$$
$$(A^2, a^2) = (\{dahl, paneer\}, dahl)$$
$$(A^3, a^3) = (\{paneer, dosa\}, paneer)$$
$$(A^4, a^4) = (\{dahl, dosa\}, dahl)$$

This behaviour is consistent with a model of myopic preference discovery without a prior. Albeit one observes a violation of Richter's congruence applied to the timeless choice correspondence (actually a function) induced by this chronology (paneer is revealed preferred to dahl by the first choice while the second reveals an opposite preference), this violation may be interpreted as the result of a preference discovery process (or trial and error) over these Indian meals. After having tried out paneer and dahl in periods 1 and 2 and having discovered–via these trials–her "definite" preference over these two meals, the decision-maker behaves consistently with her definite preference over these two experienced dishes from period 3 onwards.

Example 2 Consider now the following chronology of choices over the same meals:

$$(A^1, a^1) = (\{paneer, dosa\}, paneer)$$
$$(A^2, a^2) = (\{dahl, dosa\}, dahl)$$
$$(A^3, a^3) = (\{dahl, paneer, dosa\}, paneer)$$
$$(A^4, a^4) = (\{dahl, paneer\}, dahl)$$

This choice behaviour is not consistent with a model of myopic preference discovery without a prior. According to this model, in periods 1 and 2, the decision-maker tries out paneer and then dahl and discovers her preference over these meals. In the third period, given that she knows her preference over paneer and dahl, we can interpret her choice as revealing a "definite" preference for paneer over dahl. However, her choice of dahl over paneer in the fourth period is inconsistent with this "definite" preference.

We emphasize the crucial importance of introducing a chronology for characterising this behaviour. Indeed, the only difference between the two examples is the chronological order in which the menus–identical in both examples–appear. The choice behaviours of these two examples induce exactly the same timeless choice function C defined on the same family $\mathcal{F}(\{(A^t, a^t)\}_{t=1}^4)$ and induce the same violation of Richter's congruence applied to that timeless choice function. However, choices in Example 1 are consistent with a model of myopic preference discovery without a prior while choices in Example 2 are not.

The specific theory of choice examined in this subsection is defined as follows.

Definition 5 A chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the myopic maximisation of a discovered preference without a prior if and only if there exists a linear ordering R on X such that, for all $t \in \{1, \dots, T\}$, either $a^t \, R \, a$ for all $a \in A^t$ such that $s^t(a) = e$ *or* $s^t(a^t) = u$.

A chronological choice behaviour thus results from the maximisation of a discovered preference without a prior if there exists a linear ordering such that the choice made by the decision-maker at every period is either the "best" option for that ordering among all alternatives that have been previously tried once or, if this is not the case, it can only be because the chosen option has never been tried before.

To identify the observable implications of this model on the chronology of choices $\{(A^t, a^t)\}_{t=1}^T$, we can limit attention, for every period t, to the (possibly empty) *tried* version of $\widehat{A}^t$, denoted $\widehat{A}^t_e$, and defined by $\widehat{A}^t_e = \widehat{A}^t \cap X \times \{e\}$ if $s(a^t) = e$ and $\widehat{A}^t_e = \emptyset$ otherwise. Hence, $\widehat{A}^t_e$ is a subset of the *experienced-status* version of A^t that contains only the alternatives whose experienced status is e when the chosen alternative in A^t has been tried at least once before and that is empty if the chosen alternative in A^t has never been tried before. Let $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$ be the timeless family of all those non-empty tried versions of A^t defined by $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T) = \{\widehat{A}_e \subset X \times \{e\} : \exists t \in \{1, \dots, T\} \text{ s.t. } \widehat{A}_e = \widehat{A}^t_e\}$. The possibility that $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$ be empty (e.g., the decision-maker never chooses an already tried alternative) is, of course, not ruled out. For a non-empty $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$, one can define the associated *timeless tried choice correspondence* $\widehat{C}_e : \widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e\})$ by $\widehat{C}_e(\widehat{A}_e) = \{(a, e) \in X \times \{e\} : \exists t \text{ s.t. } a = a^t, s^t(a^t) = e \text{ and } \widehat{A}^t_e = \widehat{A}_e\}$ for every non-empty $\widehat{A}_e \in \widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$.

This setting enables us to identify the empirical implications of this model.

Theorem 1 *A chronology of choices* $\{(A^t, a^t)\}_{t=1}^T$ *results from the myopic maximisation of a discovered preference without a prior if and only if its timeless tried choice correspondence* $\widehat{C}_e : \widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e\})$ *is a function that satisfies Richter's congruence.*

This theorem shows that the behavioural implications of the myopic maximisation of a discovered preference without a prior are easily testable. In particular, one can restrict attention to the revealed preferences between tried alternatives (whose experience status is e) and check (i) if the choice correspondence is a function (i.e, only one alternative is chosen per menu, even if these menus are repeated in the chronology) *and* (ii) if this function satisfies Richter's congruence.

4.1.2 Myopic preference discovery with a prior ranking

We now examine a more structured myopic process of preference discovery in which the decision-maker starts with a (strict) ranking of the alternatives before trying them and updates the position of any alternative in this ranking every time she tries the alternative for the first time. This model is therefore adapted to study situations in which the decision-maker has some expectation about the ranking of the alternatives before choosing between them, but in which she may be–positively or negatively–surprised the first time she consumes the alternatives. One way to interpret this model, in the spirit of Cooke (2017), is to posit a decision-maker who is uncertain about the utility that each alternative will provide once tried. Hence, before trying the alternatives, the decision-maker assigns to each of them a specific probability distribution over all possible cardinal utility levels that can be associated with the alternatives and orders the alternatives using their expected utilities (assuming that no two distinct alternatives are initially assigned the same expected utility). Then, when facing the first menu, the decision-maker chooses from it the alternative with the largest expected utility and discovers the "true" (subjective) utility of the alternative. This leads her, in the second period, to update the position of this alternative in the ranking. This update can increase the relative position of the alternative *vis-à-vis* another or reduce it, depending upon how positive or negative the trying experience has been. In the second period, the decision-maker uses the updated ranking to make the choice and so on.

The following example illustrates the model and shows how it generates different empirical implications than the previous one.

Example 3 Consider the following chronology of choices among Indian delicacies from a traveller who has read a bit about Indian cuisine before travelling to the country:

$$(A^1, a^1) = (\{dahl, paneer\}, paneer)$$
$$(A^2, a^2) = (\{dahl, paneer\}, paneer)$$
$$(A^3, a^3) = (\{dahl, dosa\}, dahl)$$
$$(A^4, a^4) = (\{paneer, dosa\}, dosa)$$

Observe that the revealed preferences over tried alternatives do not exhibit reversals, be it for the simple reason that the decision-maker is never asked to choose between two of those. Hence, this chronology of choices is compatible with a myopic model of preference discovery without a prior. However, the choice pattern exhibited in this example is inconsistent with a model in which the decision-maker has a prior ranking over the alternatives that she updates over time. Indeed, the choice of the experienced paneer over the unexperienced dahl in the second period, followed by the choice of the unexperienced dahl over the unexperienced dosa in the third period, reveals an indirect preference for the experienced paneer over the unexperienced dosa. Yet, this revealed preference is inconsistent with the fourth period's choice of the unexperienced dosa over the experienced paneer.

From a purely ordinal perspective–the one that can be extracted from data on choices–the model we consider in this subsection, violated by the chronology of choices of Example 3, is the following.

Definition 6 A chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the myopic maximisation of a discovered preference with a prior ranking if and only if there exists a sequence of linear orderings $\{R^t\}_{t=1}^T$ such that:

(i) $a^t R^t a$ for all $t \in \{1, \dots, T\}$ and all $a \in A^t$ and,
(ii) For all distinct x and $y \in X$ and all $t = 2, \dots, T$, $x \; R^{t-1} \; y$ and $y \; R^t \; x$ implies either that $s^{t-1}(x) \neq s^t(x)$ or $s^{t-1}(y) \neq s^t(y)$.

In plain English, a chronology of choices results from the myopic maximisation of a discovered preference with a prior ranking if at every period t, the decision-maker chooses the most preferred alternative for some linear preference that is the same as the one used to choose in the previous period except, possibly, for the relative ranking of the alternative whose experienced status has changed between the previous period and the current one.

It should be clear that a chronology of choices resulting from this model satisfies the conditions set in Theorem 1 (i.e., it has an associated timeless tried choice correspondence that is a function and that satisfies Richter's congruence axiom). Yet, as illustrated in Example 3, a chronology of choices resulting from the myopic maximisation of a discovered preference with a prior ranking satisfies the stronger condition that its whole associated timeless experienced-status choice correspondence–and not only its tried one–be a function satisfying Richter's congruence. As it turns out, this latter condition contains all the empirical implications of the assumption that a chronology of choices results from the myopic maximisation of a discovered preference with a prior ranking.

Theorem 2 *A chronology of choices* $\{(A^t, a^t)\}_{t=1}^T$ *results from the myopic maximisation of a discovered preference with a prior ranking if and only if its timeless experienced-status choice correspondence* $\widehat{C} : \widehat{\mathcal{F}}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u\})$ *is a function that satisfies Richter's congruence.*

In practice, this implies that for choice behaviour to result from this preference discovery process one should never observe a cycle of revealed preferences connecting tried and (possibly) untried alternatives. Finally, note that even though this testable implication is more restrictive than that of the model where the decision-maker behaves as if she has no prior, there are situations where the two theories are indistinguishable. This would occur, for example, in a chronology of choices where no alternative is chosen more than once.

4.1.3 Myopic preference discovery with a prior reference point

In this subsection, we consider a model in which the decision-maker's prior is not a strict preference over the alternatives but is, instead, what we call a "prior reference point". By this expression we mean that the decision-maker attaches the same value (or utility) to all untried alternatives and uses this reference point to make comparisons with tried alternatives. Hence, the decision-maker considers all untried alternatives to be equivalent–and has no reason to choose one of them over another–but once she tries out an alternative, she may be positively or negatively surprised relative to the reference point of untried alternatives. There is a vast literature in economics explaining choice behaviour–such as behaviour linked to loss aversion and *status-quo* bias–with exogenously given reference points (e.g., Tversky and Kahneman 1991; Bossert and Sprumont 2003; Masatlioglu and Ok 2005, 2013). Here, the reference point can be a relevant element to explain the preference discovery process of decision-makers who have very limited prior information about alternatives. For example, this model could be applied to the analysis of what is known in psychology and decision theory as "decisions from experience", i.e., settings in which decision-makers do not know *a priori* the incentive structure and learn it via trial and error (see Hertwig et al. 2004 and Erev and Haruvy 2013).

The following example illustrates what behaviour is ruled out by such a model.

Example 4 Consider the following chronology of choices among Indian delicacies from a traveller who has very limited information about Indian cuisine:

$$(A^1, a^1) = (\{dahl, paneer\}, paneer)$$
$$(A^2, a^2) = (\{dosa, paneer\}, dosa)$$
$$(A^3, a^3) = (\{dahl, dosa\}, dosa)$$
$$(A^4, a^4) = (\{dahl, paneer\}, paneer)$$

This chronology of choices is in fact consistent with both our previous models. However, this chronology cannot result from the myopic maximisation of a discovered preference with a prior reference point. Indeed, if the decision-maker considers all untried alternatives to be equivalent and uses this as a reference point, then the choice in the second period of untried dosa over tried paneer suggests that the experience of paneer in period 1 has been a negative surprise relative to the reference point. However, this revealed negative surprise is inconsistent with the choice of tried paneer over untried dahl in the fourth period.

The formal definition of this theory of choice can be stated as follows.

Definition 7 A chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the myopic maximisation of a discovered preference with a prior reference point if and only if there is a (utility) function $U : X \times \{e, u\} \to \mathbb{R}$ satisfying $U(x, u) = U(y, u)$ for all distinct alternatives x and $y \in X$ such that:

(i) $U(a^t, s^t(a^t)) \geq U(a, s^t(a))$ for all $t \in \{1, \ldots, T\}$ and all $a \in A^t$, and

(ii) $U(a^t, s^t(a^t)) > U(a, s^t(a))$ for all $t \in \{2, \ldots, T\}$ and all $a \in A^t$ distinct from a^t such that either $s^t(a^t) = e$ or $s^t(a) = e$.

Hence, the decision-maker myopically maximises a discovered preference with a prior reference point if she initially assigns the same utility level–the "reference point–to all untried alternatives and then progressively discovers her definite preference for the alternatives after trying them and comparing them with the prior reference point.

The testable implication of this model will, once again, take the form of the Richter's congruence axiom applied to a timeless experienced-status choice correspondence suitably constructed from the chronology of choices. We construct such a correspondence in the following way. Every time the decision-maker chooses an untried alternative, we treat the (experienced-status-augmented) menu in which the choice is made as if it contains all untried alternatives of the universe and the decision-maker is choosing all of them. When the decision-maker chooses an already tried alternative, however, we treat this choice as the uniquely chosen alternative–along with its status–in the status-enlarged version of the menu in which the choice is made. Formally, we thus consider the timeless family $\widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ of subsets of $X \times \{e, u\}$ defined by:

$$\widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T) = \{\widehat{A} \subset X \times \{e, u\} \,:\, \exists t \in \{1, \ldots, T\} \text{ s.t. } \widehat{A} = \widehat{A}^t \text{ and } s(a^t) = e \\ \text{or } \widehat{A} = \widehat{A}^t \cup X \times \{u\} \text{ and } s(a^t) = u\}$$

On this family of subsets of $X \times \{e, u\}$, we define the following *timeless reference point choice correspondence* $\widehat{C}^{RP} : \widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u\})$ by:

$$\widehat{C}^{RP}(\widehat{A}) = X \times \{u\} \text{ if } \exists t \in \{1, \ldots, T\} \text{ s.t. } \widehat{A} = \widehat{A}^t \cup X \times \{u\} \text{ and } s^t(a^t) = u \text{ and,}$$
$$\widehat{C}^{RP}(\widehat{A}) = \{(a^t, e)\} \text{ if } \exists t \in \{1, \ldots, T\} \text{ s.t. } \widehat{A} = \widehat{A}^t \text{ and } s^t(a^t) = e$$

The observable implication of this model is that this correspondence satisfies Richter's congruence.

Theorem 3 *A chronology of choices* $\{(A^t, a^t)\}_{t=1}^T$ *results from the myopic maximisation of a discovered preference with a prior reference point if and only if the timeless reference point choice correspondence* $\widehat{C}^{RP} : \widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u\})$ *satisfies Richter's congruence.*

The observable implications of the myopic maximisation of a discovered preference with a prior reference point are thus easy to check. Besides the common implication of all models considered so far about the consistency over tried alternatives, myopic preference discovery with a reference point imposes the following two conditions on the chronology of choices: (i) if an already tried alternative x is indirectly revealed preferred to a yet untried alternative, we should never observe an untried alternative being chosen over x or any tried alternative indirectly revealed preferred to x, and (ii) if a tried alternative x is available at some period where an untried alternative is chosen, then x or any tried alternative that has been revealed indirectly worse than x cannot be chosen over any untried alternative.

Finally, it is worth highlighting once again that the empirical implications of this model are restrictions of (or are encompassed by) the empirical implications of the myopic maximisation of a discovered preference with a prior ranking. In other words, one can find choice behaviour that is consistent with the model of myopic preference discovery with a prior ranking *and* that is *not* consistent with the model of myopic preference discovery with a prior reference point (as illustrated in Example 4), but the reverse scenario is not possible. Since the myopic maximisation of a discovered preference with a prior ranking implies the myopic maximisation of a discovered preference without a prior, the observable implications of the models presented so far are increasingly restrictive from the first to the third model.

4.2 Forward-looking preference discovery

In this subsection, we modify the last two theories of preference discovery with a prior ranking and a reference point to allow the agent to be "forward-looking" and use her knowledge about future menus to inform her present choices. As in the previous subsection, the two models are nested: the model with a reference point implies (is more restrictive than) the model with a prior ranking.

There are at least two reasons for which one could expect a decision-maker who knows in advance the menus she will face in the future to react to this information. One is a *fear of compulsion* as in the case of addition: If a "tempting" untried alternative known to be addictive (e.g., vaping) is currently available for choice and will be again available in the future, the decision-maker might refrain from trying it due to the concern that she may be unable to resist to it in the future. If, on the other hand, this untried alternative is only available now and will never be again available in the future (e.g., due to an impending ban), the decision-maker may be tempted to "try it out" without the fear of not being able to resist to it in the future. Hence, a fear of compulsion implies that a given untried alternative is less attractive when it will be again available in the future than when it will never be again available in the future. This fear of compulsion, which can be revealed in a chronological choice setting, is related to attempts to model addiction in the literature on intertemporal decision making (see Section VI in Bernheim and Rangel 2004 for a review). One important difference between our approach and those developed in this literature is that–as mentioned in Sect. 3–we abstract from the choice of future menus, which in the case of addiction can be used by forward-looking agents to pre-commit not to have a tempting option present in the future. We also note that in our setting a decision-maker may exhibit fear of compulsion even if an alternative is available only once in the future. A more complex theory could model the fear of compulsion as having different degrees of strength depending on how many times the tempting alternative is available in the future.

A different reason for a decision-maker to use information about the future availability of alternatives, which runs in the opposite direction, is the potential benefits associated with learning the value (e.g., the taste) of untried alternatives that will be again available in the future. Indeed, it seems plausible to assume that the benefit of resolving uncertainty about the value of an untried alternative is most important when the alternative will appear again in the future. As argued by Cooke (2017), this can lead to *experimentation*: "purposeful action with the goal of resolving uncertainty” (p. 1308). From an instrumental perspective (the one considered here), experimentation is not beneficial if an alternative will not be available again in the future. In line with this perspective, Delaney et al. (2020) show in an experimental setting that incomplete learning is more prevalent for rare goods.

Having forward-looking agents implies that the future availability of alternatives becomes one of their important characteristics. Therefore, we enrich our notation and consider the set $\{e, u^a, u^n\}$ of possible “experienced status” of the alternatives at every period, with e standing, as before, for “experienced”, u^a standing for untried and available in the future, and u^n standing for untried and unavailable in the future. For any chronology $\{(A^t, a^t)\}_{t=1}^T$ and period $t \in \{1, \dots, T\}$, we define the *forward-looking experienced status* function $\hat{s}^t : A^t \to \{e, u^a, u^n\}$ by:

$$\begin{aligned}\hat{s}^t(x) &= e \text{ if } 1 \leq t(x) < t,\\ &= u^a \text{ if } t \leq t(x) \text{ and } \exists t' \in \{t+1, \dots, T\} \text{ such that } x \in A^{t'},\\ &= u^n \text{ otherwise.}\end{aligned}$$

for any $x \in A^t$.

4.2.1 Forward-looking preference discovery with a prior ranking

We first analyse a preference discovery model with a prior ranking, as defined above (Sect. 4.1.2), with forward-looking agents instead of myopic ones. The following example provides a bit more intuition about the theory of choice we are considering.

Example 5 Consider the following chronology of choices among Indian delicacies:

$$\begin{aligned}(A^1, a^1) &= (\{dahl, paneer, dosa, tandoori\ chicken\}, paneer)\\ (A^2, a^2) &= (\{dahl, dosa\}, dahl)\\ (A^3, a^3) &= (\{paneer, dosa\}, paneer)\\ (A^4, a^4) &= (\{dosa, tandoori\ chicken\}, dosa)\\ (A^5, a^5) &= (\{paneer, dahl, dosa\}, dahl)\\ (A^6, a^6) &= (\{dahl, tandoori\ chicken\}, tandoori\ chicken)\end{aligned}$$

This chronology of choices could not result from the myopic maximisation of a discovered preference with a prior ranking or with a prior reference point. Indeed, from a myopic perspective, the choice made in the fourth period reveals a (direct) preference for untried dosa over untried tandoori chicken that contradicts the indirect preference for untried tandoori chicken over untried dosa revealed by the sequence of choices made in periods 3, 5 and 6. However, it is possible to explain this chronology of choices by a forward-looking decision-maker who fears not being able to resist tempting alternatives that will show up again in the future. In this example, the direct preference revealed by the fourth period's choice is for untried dosa available in the future over untried tandoori chicken available in the future while the indirect preference revealed by the sequence of choices of periods 3, 5 and 6 is for untried tandoori chicken unavailable in the future over untried dosa available in the future. These two preferences, which show that untried tandoori chicken unavailable in the future is preferable to untried tandoori chicken available in the future, are consistent with the fear of compulsion vis-à-vis untried tandoori chicken when this alternative is available in the future. Note that if there were periods after period 6 where tandoori chicken was present, then the choice behaviour of this example could not be rationalised by the forward-looking maximisation of a discovered preference with a prior ranking.

We can define the theory of choice examined in this subsection as follows.

Definition 8 A chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the forward-looking maximisation of a discovered preference with a prior ranking if and only if there exists a sequence of linear orderings $\{R^t\}_{t=1}^T$ such that:

(i) $a^t\ R^t\ a$ for all $t \in \{1, \dots, T\}$ and all $a \in A^t$ and,
(ii) For all distinct x and $y \in X$ and all $t = 2, \dots, T$, $x\ R^{t-1}\ y$ and $y\ R^t\ x$ implies either $\widehat{s}^{t-1}(x) \neq \widehat{s}^t(x)$ or $\widehat{s}^{t-1}(y) \neq \widehat{s}^t(y)$.

In plain English, a chronology of choices results from the forward-looking maximisation of a discovered preference with a prior ranking if, at every period, the decision-maker chooses the most preferred alternative for some linear preference that is the same as the one used to choose in the previous period except, possibly, for the relative ranking of the alternative whose experienced status has changed between the previous and the current period. The difference between this theory of choice and the one resulting from the myopic maximisation of a discovered preference with a prior ranking is that there are now more sources of change of the relative position of an alternative *vis-à-vis* another that are allowed by clause (ii) of the definition. Indeed, the ranking of two alternatives can change between two periods because–as before–one of the two alternatives is tried for the first time in the first of the two periods, and now it can also change because one of the two alternatives will no longer be available in the future after the two periods.

We again formulate the empirical implication of this theory by imposing Richter's congruence on a suitably defined *timeless forward-looking experienced-status choice correspondence*, required to be a function. For this sake, we take any chronology $\{(A^t, a^t)\}_{t=1}^T$ and we define for every period $t \in \{1, \dots, T\}$ the set $\widehat{A}_{FW}^t \subset X \times \{e, u^a, u^n\}$ by $\widehat{A}_{FW}^t = \{(a, \widehat{s}^t(a)) \in X \times \{e, u^a, u^n\} : a \in A^t\}$. Hence, $\widehat{A}_{FW}^t$ is the set of ordered pairs formed by the actual alternatives in the menu A^t and their *forward-looking-experienced status* at period t. We refer to $\widehat{A}_{FW}^t$ as the *forward-looking-experienced-status* version of A^t. Similarly, we denote by $\widehat{a}_{FW}^t \in \widehat{A}_{FW}^t$ the pair $(a^t, \widehat{s}^t(a^t))$ made of the chosen alternative in A^t and its forward-looking-experienced status at

t. We then associate, to any chronology $\{(A^t, a^t)\}_{t=1}^T$, its induced *timeless forward-looking experienced-status family of menus* of $X \times \{e, u^a, u^n\}$, $\widehat{\mathcal{F}}^{FW}(\{(A^t, a^t)\}_{t=1}^T)$, defined by $\widehat{\mathcal{F}}^{FW}(\{(A^t, a^t)\}_{t=1}^T) = \{\widehat{A}_{FW} \subset X \times \{e, u^a, u^n\} : \exists t \in \{1, \dots, T\}$ s.t. $\widehat{A}_{FW} = \widehat{A}^t_{FW}\}$. Then, a *timeless forward-looking experienced-status choice correspondence* $\widehat{C}^{FW} : \widehat{\mathcal{F}}^{FW}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u^a, u^n\})$ is defined by $\widehat{C}^{FW}(\widehat{A}_{FW}) = \{(a, s) \in X \times \{e, u^a, u^n\} : \exists t$ s.t. $a = a^t$, $\widehat{s}^t(a^t) = s$ and $\widehat{A}^t_{FW} = \widehat{A}_{FW}\}$ for every $\widehat{A}_{FW} \in \widehat{\mathcal{F}}^{FW}(\{(A^t, a^t)\}_{t=1}^T)$. We can then show the following.

Theorem 4 *A chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the forward-looking maximisation of a discovered preference with a prior ranking if and only if the timeless forward-looking experienced-status choice correspondence $\widehat{C}^{FW} : \widehat{\mathcal{F}}^{FW}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u^a, u^n\})$ is a function that satisfies Richter's congruence.*

As shown in Example 5, this model can accommodate fear of compulsion in the evaluation of untried alternatives leading to a reduction of the appeal of an untried alternative when it is available in the future as compared to when it is not available in the future. It turns out that a chronology of choices from a decision-maker who follows forward-looking preference discovery with a prior ranking *cannot* reveal a preference for experimentation (i.e., a positive value of trying out an alternative to resolve uncertainty). Indeed, for this to be revealed we would need to get two (possibly identical) untried alternatives–x and y say–such that x untried and available in the future is (indirectly or directly) revealed preferred to y untried and unavailable in the future, while the ranking of the two untried alternatives should be the opposite when their future availability status is the same. For x untried and available in the future to be indirectly revealed preferred to y untried and unavailable in the future, there must be some period t where y is present, untried, unavailable in the future and not chosen (because it is dominated through a sequence of choices by x untried and available in the future). On the other hand, y untried revealed preferred to x untried (for a given availability in the future of the two alternatives) requires a period where untried y is chosen. When could this be? It cannot be before t, because otherwise, y would not be untried at t. But it cannot be after t either, because otherwise, y would be available in the future at t. It simply cannot be.

4.2.2 Forward-looking preference discovery with a prior reference point

We end our analysis by studying a theory of forward-looking preference discovery where the prior is a reference point. Contrary to the myopic setting, in this theory the decision-maker is not indifferent between all untried alternatives. Instead, the decision-maker is indifferent between untried alternatives *whose future availability is the same*. Specifically, we consider the following theory.

Definition 9 A chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the forward-looking maximisation of a discovered preference with a prior reference point if and only if there is a (utility) function $U : X \times \{e, u^a, u^n\} \to \mathbb{R}$ satisfying $U(x, u^a) = U(y, u^a)$ and $U(x, u^n) = U(y, u^n)$ for all distinct alternatives x and $y \in X$ such that:

(i) $U(a^t, \widehat{s}^t(a^t)) \geq U(a, \widehat{s}^t(a))$ for all $t \in \{1, \ldots, T\}$ and all $a \in A^t$, and
(ii) $U(a^t, \widehat{s}^t(a^t)) > U(a, \widehat{s}^t(a))$ for all $t \in \{2, \ldots, T\}$ and all $a \in A^t$ such that either $\widehat{s}^t(a^t) = e$ or $\widehat{s}^t(a) = e$.

Note that the utility function in Definition 9 assigns the same level of utility to any two untried alternatives with the same availability in the future, while it may or may not assign different utility levels to two untried alternatives with different future availability. Note as well that if the decision-maker assigns the same utility level to two untried alternatives with different future availability statuses, then the model considered in this subsection is equivalent to the myopic preference discovery model with a prior reference point characterised in Sect. 4.1.3.

As before, the observable implication of this model takes the form of a Richter's congruence requirement imposed on a specific *timeless forward-looking experienced-status choice correspondence*. Following the logic of the myopic preference discovery with a prior reference point, we construct such a correspondence as follows. First, for any period where the decision-maker chooses a yet untried alternative of a given future availability status, we treat this choice as if it consists of choosing all untried alternatives of that status. This requires that the forward-looking experienced-status version of the menu faced by the decision-maker at that period be artificially modified to include, along with all the available forward-looking experienced-status extended alternatives in that menu, all the conceivable untried alternatives with the same future availability status as the chosen alternative. Second, for any period in which the decision-maker chooses an alternative that has been previously chosen, we treat her status-augmented choice and the timeless forward-looking experienced-status version of the menu from which the choice is made as it is. Formally, we therefore consider the following timeless family $\widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ of forward-looking experienced-status subsets of $X \times \{e, u^a, u^n\}$:

$$\begin{aligned}\widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T) = \{\widehat{A}_{FW} \subset X \times \{e, u^a, u^n\} \ : \ \exists t \in \{1, \ldots, T\} \ \text{ s.t.} \\ \widehat{A}_{FW} = \widehat{A}_{FW}^t \cup X \times \{u^a\} \text{ and } s^t(a^t) = u^a \text{ or} \\ \widehat{A}_{FW} = \widehat{A}_{FW}^t \cup X \times \{u^n\} \text{ and } s^t(a^t) = u^n \text{ or } \widehat{A}_{FW} = \ \widehat{A}_{FW}^t \text{ and } s^t(a^t) = e\}\end{aligned}$$

On this timeless family of subsets of $X \times \{e, u^a, u^n\}$, we define the following *timeless reference point forward-looking choice correspondence* $\widehat{C}_{FW}^{RP} : \widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u^a, u^e\})$ by:

$$\widehat{C}^{RP}_{FW}(\widehat{A}_{FW}) = X \times \{u^a\} \text{ if } \exists t \in \{1, \dots, T\} \text{ s.t. } \widehat{A}_{FW} = \widehat{A}^t_{FW} \cup X \times \{u^a\} \text{ and } s^t(a^t) = u^a,$$
$$\widehat{C}^{RP}_{FW}(\widehat{A}_{FW}) = X \times \{u^n\} \text{ if } \exists t \in \{1, \dots, T\} \text{ s.t. } \widehat{A}_{FW} = \widehat{A}^t_{FW} \cup X \times \{u^n\} \text{ and } s^t(a^t) = u^n,$$
$$\widehat{C}^{RP}_{FW}(\widehat{A}_{FW}) = \{(a^t, e)\} \text{ if } \exists t \in \{2, \dots, T\} \text{ s.t. } \widehat{A}_{FW} = \widehat{A}^t_{FW} \text{ and } \widehat{s}^t(a^t) = e$$

We can then show the following.

Theorem 5 *A chronology of choices* $\{(A^t, a^t)\}_{t=1}^T$ *results from the forward-looking maximisation of a discovered preference with a prior reference point if and only if the timeless reference point forward-looking choice correspondence* $\widehat{C}^{RP}_{FW} : \widehat{\mathcal{F}}^{RP}_{FW}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u^a, u^e\})$ *satisfies Richter's congruence.*

To illustrate how an observer can test the empirical implication set out in Theorem 5, consider the following chronology of choices of Indian dishes.

$$(A^1, a^1) = (\{dahl, paneer, dosa, tandoori\ chicken\}, paneer)$$
$$(A^2, a^2) = (\{dahl, dosa\}, dahl)$$
$$(A^3, a^3) = (\{dahl, dosa\}, dahl)$$
$$(A^4, a^4) = (\{dahl, tandoori\ chicken\}, tandoori\ chicken)$$
$$(A^5, a^5) = (\{dahl, tandoori\ chicken, dosa\}, dosa)$$

This chronology can be rationalised by a forward-looking maximisation of a discovered preference with a prior ranking. Indeed, the status-dependent preference that rationalises these choices can be numerically represented by a utility function U such that $U(paneer, u^n) > U(dahl, u^a) > U(dosa, u^a)$, $U(dahl, e) > U(dosa, u^a)$, $U(tandoori\ chicken, u^a) > U(dahl, e)$ and $U(dosa, u^n) > U(dahl, e)$. Let us show, however, that this chronology cannot result from the forward-looking maximisation of a discovered preference with a prior reference point. For this sake, let us first construct the family $\widehat{\mathcal{F}}^{RP}_{FW}(\{(A^t, a^t)\}_{t=1}^T)$ of forward-looking experienced-status subsets of $X \times \{e, u^a, u^n\}$ associated to the chronology as follows:

$$\begin{aligned}\widehat{\mathcal{F}}^{RP}_{FW}(\{(A^t, a^t)\}_{t=1}^T) = \{&\{(dahl, u^a), (dosa, u^a), (tandoori\ chicken, u^a)\} \cup X \times \{u^n\},\\ &X \times \{u^a\},\\ &\{(dahl, e), (dosa, u^a)\},\\ &\{(dahl, e)\} \cup X \times \{u^a\},\\ &\{(dahl, e), (tandoori\ chicken, e)\} \cup X \times \{u^n\}\}\end{aligned}$$

The associated timeless reference point forward-looking choice correspondence is defined by:

$$\begin{aligned} \widehat{C}^{RP}_{FW}(\{(dahl, u^a), (dosa, u^a), (tandoori\ chicken, u^a)\} \cup X \times \{u^n\}) &= X \times \{u^n\}, \\ \widehat{C}^{RP}_{FW}(X \times \{u^a\}) &= X \times \{u^a\}, \\ \widehat{C}^{RP}_{FW}(\{(dahl, e), (dosa, u^a)\}) &= \{(dahl, e)\}, \\ \widehat{C}^{RP}_{FW}(\{(dahl, e)\} \cup X \times \{u^a\}) &= X \times \{u^a\}, \text{ and} \\ \widehat{C}^{RP}_{FW}(\{(dahl, e), (tandoori\ chicken, e)\} \cup X \times \{u^n\}) &= X \times \{u^n\} \end{aligned}$$

This correspondence violates Richter's congruence because (*dahl*, *e*) is chosen over $(dosa, u^a)$ in period 3, while the whole set $X \times \{u^a\}$ [including $(tandoori\ chicken, u^a)$ and $(dosa, u^a)$] is chosen over (*dahl*, *e*) in the menu $\{(dahl, e)\} \cup X \times \{u^a\}$ of period 4. Hence, the choice behaviour of this example cannot be rationalised by a decision-maker who is indifferent between all untried alternatives that are available in the future such as tandoori chicken and dosa in periods 3 and 4.

We conclude this subsection with two additional remarks. First, we observe that contrary to what was the case for forward-looking preference discovery with a prior ranking, with forward-looking preference discovery with a prior reference point it is possible to reveal both a fear of compulsion and a preference for experimentation. Which of the two phenomena prevails is revealed in the first period where an untried alternative available in the future and an untried alternative not available in the future are both present and one of the two is chosen (if such period exists). If the chosen alternative is not available in the future, then fear of compulsion prevails in the chronology. If the opposite preference is revealed, then a preference for experimentation prevails. Second, we note that all five models of this paper have been introduced by decreasing order of generality (i.e., with increasingly restrictive observable implications). To see this, note that (i) the myopic models are increasingly restrictive from the first to the third model ("models 1 to 3") and the forward-looking models are increasingly restrictive from the first to the second model ("models 4 to 5"), and that (ii) model 4 is more restrictive than model 3 (i.e., consistency with the model of forward-looking preference discovery with a prior ranking is implied by consistency with the model of myopic preference discovery with a prior reference point).

5 Concluding remarks

In this paper, we have argued in favour of explicitly introducing time in the description of choice behaviour provided by a choice function. We have related our framework to the traditional choice model and compared it to other existing frameworks that extend the traditional setting. We have then used this framework to provide testable implications for several preference discovery models that could not be analysed without an explicit introduction of a chronological order of choice situations. These testable implications took the form of simple revealed preference tests for both myopic and forward-looking agents.

Several extensions would be possible. For example, one could extend our analysis to the case where some exogenous number $k > 1$ of trials is needed for the experienced status e to be conferred to an alternative. On the one hand, one trial ($k = 1$), as studied here, is the most parsimonious and salient case, which aligns with previous theoretical literature (see, e.g., Piermont et al. 2016 and Cooke 2017). On the other hand, an extension to $k > 1$ could be used to test the hypothesis that a decision-maker needs "sufficient" opportunities of trial and error to discover her preferences over alternatives (see Binmore 1999, p. 17). Given a chronology of choices, one could then identify the minimum number of "trials" required to rationalise it as resulting from the maximisation of a discovered preference. Obviously, any chronology of choices can be rationalised as resulting from the maximisation of a discovered preference for some number of trials. Hence, in order to provide a refutable theory of chronological choice, the number of required trials would have to be relatively small.

While this paper focuses on preference discovery, we believe that our framework can be used to study other theories of choice. As shown above, our framework can accommodate agents who use or not their knowledge about future menus to inform their present choices, and it is not demanding in terms of the data needed to empirically test the choice models that are characterised in it. Examples of other theories include myopic and forward-looking models of exogenous and endogenous *status-quo* bias, myopic and forward-looking models of decision inertia, and myopic and forward-looking models of changing tastes.[3]

Future research could also use our revealed preference tests–and possibly others that exploit the chronological pattern of choices–to test competing explanations for people's preference discovery processes. Experimental settings may be particularly well-suited for this purpose. One of their advantages in this context is the possibility of controlling for past and future experience with goods (e.g., by guaranteeing that a decision-maker has not experienced the goods *prior* to the chronology of choices being analysed in the experiment). To the best of our knowledge, there is very limited experimental research that compares different models of preference discovery.

Appendix

Proof of Remark 1

Let $\mathcal{F}$ be a collection of subsets of X and assume, first, that $C : \mathcal{F} \to \mathcal{P}(X)$ is a choice correspondence that satisfies Richter's congruence (the requirement that $\#C(A) = 1$ for every $A \in \mathcal{F}$ is not required for this first implication). Consider two distinct alternatives x and y in X such that $x\ W_I^C\ y$ and assume by contradiction that $y\ S_D^C\ x$ holds. Hence, there is a set $A \in \mathcal{F}$ such that $y \in C(A)$ and $x \in A/C(A)$. But this contradicts Richter's congruence. To show the reverse implication, consider again

[3] For papers related to these phenomena, see, for instance, Kahneman et al. (1991) and Masatlioglu and Ok (2005) (*status-quo* bias), Charness and Levin (2005) and Alós-Ferrer et al. (2016) (decision inertia), and Gul and Pesendorfer (2005) and Mihm and Ozbek (2019) (changing tastes).

a collection $\mathcal{F}$ of subsets of X and assume that $C : \mathcal{F} \to \mathcal{P}(X)$ is a choice correspondence such that $\#C(A) = 1$ for every $A \in \mathcal{F}$ that satisfies the condition that $y \; S_D^C \; x$ does not hold for every two distinct alternatives x and y in X such that $x \; W_I^C \; y$. Consider then any two distinct alternatives x and y in X such that $x \; W_I^C \; y$ and assume that $y \in C(A)$ for some $A \in \mathcal{F}$ such that $x \in A$. Since $\#C(A) = 1$, this means that $x \notin C(A)$. But since $y \; S_D^C \; x$ does not hold, this means that there can not be any set $A \in \mathcal{F}$ such that $y \in C(A)$ and $x \in A$. Hence, Richter's congruence is (trivially) satisfied. □

Proof of Proposition 1

Suppose that the chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the maximisation of an ordering as per Definition 4. Then, there is a linear ordering R on X such that, for every $t \in \{1, \ldots, T\}$, $a^t \; R \; a'$ for all $a' \in A^t$. We observe that if there are two distinct periods t and $t' \in \{1, \ldots, T\}$ for which $A^t = A^{t'}$, then $a^t = a^{t'}$. Indeed, assuming otherwise would imply that $a^t \; R \; a^{t'}$ and $a^{t'} \; R \; a^t$ for distinct a^t and $a^{t'}$, which is incompatible with the requirement of R being antisymmetric. Hence, the induced timeless choice correspondence $C : \mathcal{F}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X)$ is a function that is rationalised by the ordering R in the sense of Richter (1966). By virtue of Theorem 1 in Richter (1966), it therefore satisfies Richter's congruence. In the other direction, suppose that the induced timeless choice correspondence $C : \mathcal{F}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X)$ is a function that satisfies Richter's congruence. By Theorem 1 in Richter (1966), there exists an ordering R that rationalises this choice function. Adapting the argument of Richter (1966) in the proof of his Theorem 1 – in particular, his Lemmas 1 and 2 and the appeal to Szpilrajn extension lemma–one concludes that in the case of a choice function, the rationalising ordering R is antisymmetric and therefore linear. This completes the proof because for any set A in the family $\mathcal{F}(\{(A^t, a^t)\}_{t=1}^T)$ where $C(A) = \{a\}$ for some $a \in A$ such that $a \; R \; a'$ for all $a' \in A$, there is a period $t \in \{1, \ldots, T\}$ such that $a = a^t$ and $A = A^t$. □

Proof of Theorem 1

For the "if" part of the theorem, assume that a chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the myopic maximisation of a discovered preference without a prior as per Definition 5. Consider the timeless tried family $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$ of subsets of $X \times \{e\}$ associated with this chronology and its associated timeless tried choice correspondence $\widehat{C}_e : \widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e\})$. Let us show that this correspondence is a function satisfying Richter's congruence. The result being trivially true if $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$ is empty, we assume that $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$ is not empty. Assume first by contradiction that $\widehat{C}_e$ is not a function. Hence, for some non-empty set $\widehat{A}_e \in \widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$, there are two distinct periods t and t' such that $a^{t'} \neq a^t$, $s^t(a^t) = s^{t'}(a^{t'}) = e$ and $\widehat{A}_e^t = \widehat{A}_e = \widehat{A}_e^{t'}$. But this implies that $\{(a^{t'}, e), (a^t, e\}\} \subset \widehat{A}_e^t = \widehat{A}_e^{t'}$ and, since $\{(A^t, a^t)\}_{t=1}^T$ results from the myopic maximisation of a discovered preference without a prior as per Definition 5, there exists a linear ordering R

such that $a^t \; R \; a^{t'}$ and $a^{t'} \; R \; a^t$. But this is a contradiction of the antisymmetry of R. Hence, $\widehat{C}_e$ must be a function. Suppose now that $\widehat{C}_e$ is a function that violates Richter's congruence axiom. Using Remark 1, there are two distinct alternatives x and y in $X \times \{e\}$ such that $x \; W_I^C \; y$ and $y \; S_D^C \; x$. By definition of W_I^C, there is a sequence of $\bar{j}$ sets $\{\widehat{A}_e^j\}_{j=1}^{\bar{j}}$ in $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$ (for some $\bar{j} \in \mathbb{N}_+$) such that $\widehat{C}_e(\widehat{A}_e^{j+1}) \subset \widehat{A}_e^j$ for $j = 1, \dots, \bar{j} - 1$, $\widehat{C}_e(\widehat{A}_e^1) = x$ and $y \in \widehat{A}_e^{\bar{j}}$. Since $\widehat{C}_e$ is a function ($\#\widehat{C}_e(\widehat{A}_e^j) = 1$ for all j) one must have, by definition of $\widehat{C}_e$ and $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$, a sequence of periods (not necessarily indexed by time) $t^1, \dots, t^{\bar{j}}$ such that $\widehat{A}_e^{t^j} = \widehat{A}_e^j$ and $\widehat{C}_e(\widehat{A}_e^j) = \{(a^{t^j}, e)\}$ for all $j = 1, \dots, \bar{j}$. Since $\{(A^t, a^t)\}_{t=1}^T$ results from the myopic maximisation of a discovered preference without a prior as per Definition 5, there is a linear ordering R on $X \times \{e\}$ (or equivalently on X) such that $x = (a^{t^1}, e) \; R \; (a^{t^2}, e) \; R \dots R \; (a^{t^{\bar{j}}}, e) \; R \; y = (z, e)$ for some $z \in A^{t^{\bar{j}}}$. By transitivity of R , one must have $x \; R \; y$. On the other hand,$y \; S_D^C \; x$ means that there is a set $\widehat{A}_e$ in $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$ such that $\{y\} = \widehat{C}_e$ ($\widehat{C}_e$ is a choice function) and $x \in \widehat{A}_e$. By definition of the family $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$ and of $\widehat{C}_e$, this implies the existence of a period $t \in \{1, \dots, T\}$ such that $y = (a^t, e)$ and $x \in \widehat{A}_e$ and, since $\{(A^t, a^t)\}_{t=1}^T$ results from the myopic maximisation of a discovered preference without a prior as per Definition 5, that $y \; R \; x$. But since x and y are distinct, this contradicts the antisymmetry of R. To prove the other implication, consider a chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ with an associated timeless tried family $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$ of subsets of $X \times \{e\}$ whose timeless tried choice correspondence $\widehat{C}_e : \widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e\})$ is a function that satisfies Richter's congruence. Following the same argument as for Proposition 1, there exists a linear ordering R that rationalises $\widehat{C}_e$. By definition of this choice function and of the family $\widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$, for every $\widehat{A}_e \in \widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$, there is some $t \in \{1, \dots, T\}$ such that $\widehat{A}_e = \widehat{A}_e^t$ and $\{(a^t, e)\} = \widehat{C}_e(\widehat{A}_e)$ and, conversely, for any t such that $s^t(a^t) = e$, there exists a set $\widehat{A}_e \in \widehat{\mathcal{F}}_e(\{(A^t, a^t)\}_{t=1}^T)$ such that $\widehat{A}_e = \widehat{A}_e^t$ and $\{(a^t, e)\} = \widehat{C}_e(\widehat{A}_e)$. This means that $(a^t, e) \; R \; (a, \; e)$ holds for all t and all $(a, e) \in \widehat{A}_e^t$ or, equivalently (modulo a redefinition of R on X instead of $X \times \{e\}$), that $a^t \; R \; a$ for all $a \in A^t$ such that $s^t(a) = e$ for all t such that $s^t(a^t) = e$. This completes the proof that the chronology $\{(A^t, a^t)\}_{t=1}^T$ results from the myopic maximisation of a discovered preference without a prior as per Definition 5. □

Proof of Theorem 2

We first show that the requirement for a chronology of choices to admit a timeless experienced-status choice correspondence that is actually a function is necessary for such a chronology of choices to result from the myopic maximisation of discovered preference with a prior ranking. By contraposition, assume that the timeless experienced-status family $\widehat{\mathcal{F}}(\{(A^t, a^t)\}_{t=1}^T)$ of subsets of $X \times \{e, u\}$ generates a choice correspondence $\widehat{C} : \widehat{\mathcal{F}}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u\})$, defined by $C(\widehat{A}) = \{(a, s) \in X \times \{e, u\} : \exists t$ such that $a = a^t$, $s^t(a^t) = s$ and $\widehat{A}^t = \widehat{A}\}$

for every $\widehat{A} \in \widehat{\mathcal{F}}(\{(A^t, a^t)\}_{t=1}^T)$, that is not a function. This means that there are two periods s and t in the set $\{1, \dots, T\}$ satisfying $s < t$ such that $\widehat{A}^s = \widehat{A}^t$ and $\widehat{a}^s = (a^s, s^s(a^s)) \neq (a^t, s^t(a^t)) = \widehat{a}^t$. Since $\widehat{A}^s = \widehat{A}^t$, $(a^s, s^s(a^s)) \in \widehat{A}^t$ and $(a^t, s^t(a^t)) \in \widehat{A}^s$ .Hence, $(a^s, s^s(a^s)) \neq (a^t, s^t(a^t))$ implies that $a^s \neq a^t$. Indeed, assuming $a^s = a^t = a$ for some $a \in X$ would imply, since $(a, s^s(a^s)) \neq (a, s^t(a^t))$, that $s^s(a) \neq s^t(a)$. But since $s < t$, $t(a) \leq s$ so that $s^t(a) = e$. Hence, assuming $s^s(a) \neq s^t(a)$ means that $s^s(a) = u$. But since $\widehat{A}^s = \widehat{A}^t$, one must have $(a, e) \in \widehat{A}^s$, which is incompatible with having also $(a, u) \in \widehat{A}^s$. Hence $a^s \neq a^t$. Observe also that $\widehat{A}^s = \widehat{A}^t$ implies, by the very definition of the experienced-status function, that $s^\tau(a) = s^s(a) = s^t(a)$ for every period $\tau \in \{s, s+1, \dots, t-1, t\}$ and every alternative $a \in A^s = A^t$. If there were linear orderings R^s and R^t that rationalise the choices of a^s and a^t at periods s and t respectively as required by the myopic maximisation of a discovered preference with a prior ranking, these two orderings would conclude that $a^s\ R^s\ a^t$ and $a^t\ R^t\ a^s$, thus contradicting–since $a^s \neq a^t$–the requirement that the orderings be time-invariant when applied to alternatives whose experienced-status has not changed. Hence, if the correspondence is not a function, it can not result from the myopic maximisation of a discovered preference with a prior ranking. Suppose now that the choice correspondence $\widehat{C} : \widehat{\mathcal{F}}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u\})$ is a function. Let us now show that it satisfies Richter's congruence if and only if the chronology of choices results from the myopic maximisation of a discovered preference with a prior ranking. We observe first that the chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the myopic maximisation of a discovered preference with a prior ranking if and only if there is a real-valued utility function U defined on the set $X \times \{e, u\}$ such that, for every $t = 1, \dots, T$, $xR^ty \iff U(x, s^t(x)) \geq U(y, s^t(y))$. The existence of utility functions U^t that numerically represent the linear orderings R^t (for $t = 1, \dots T$) used by the decision-maker who myopically maximises a discovered preference with a prior ranking is indeed guaranteed by the fact that X is finite. The fact that the different utility functions U^t for $t = 1, \dots, T$ can be written as $U^t(x) = U(x, s^t(x))$ is an immediate consequence of the requirement, imposed by the myopic preference discovery model with a prior ranking, that the ranking of any two alternatives does not varybetween two periods if their experienced-status does not change between the same two periods. Now, from Richter's (1966) theorem, we obtain that the choice correspondence $\widehat{C} : \widehat{\mathcal{F}}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u\})$ satisfies Richter's congruence if and only if there exists an ordering $\widehat{R}$ on $X \times \{e, u\}$ such that, for every $t = 1, \dots, T$, $(a^t, s^t(a^t))\ \widehat{R}$ $(a, s^t(a^t))$ for every $a \in A^t$. Since, given the finiteness of X, $\widehat{R}$ can be represented by a utility function U defined on $X \times \{e, u\}$, this completes the proof. □

Proof of Theorem 3

Assume that a chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the myopic maximisation of a discovered preference with a prior reference point as per Definition 7. Then, there exists a function $U : X \times \{e, u\} \to \mathbb{R}$ satisfying $U(x, e) \neq U(y, e)$ and $U(x, u) = U(y, u)$ for all distinct alternatives x and y in X such that $U(a^t, s^t(a^t)) > U(a, s^t(a))$ for all $t \in \{2, \dots, T\}$ and $a \in A^t$ distinct from a^t such that $s^t(a^t) = e$ or $s^t(a) = e$ and $U(a^t, s^t(a^t)) \geq U(a, s^t(a))$ for all $t \in \{1, \ldots, T\}$ and $a \in A^t$ such that $s^t(a^t) = u$. Consider then the timeless family $\widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ of subsets of $X \times \{e, u\}$ associated to $\{(A^t, a^t)\}_{t=1}^T$. We need to show that for any $\widehat{A} \in \widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$, $\widehat{C}^{RP}(\widehat{A}) = \{\widehat{a} \in \widehat{A} : U(\widehat{a}) \geq \widehat{a}'$ for all $\widehat{a}' \in \widehat{A}\}$. This is clearly true if $\widehat{A} = \widehat{A}^t \cup X \times \{u\}$ for some $t \in \{1, \ldots, T\}$ such that $s(a^t) = u$ since $\widehat{C}^{RP}(\widehat{A}^t \cup X \times \{u\}) = X \times \{u\} = \{(x, u) \in \widehat{A}^t \cup X \times \{u\} : U(x, u) = U(x', u)$ for all $(x', u) \in \widehat{A}^t \cup X \times \{u\}$ and $U(x, u) > U(a, e)$ for all $(a, e) \in \widehat{A}^t \cup X \times \{u\}\}$. If one considers now an experienced-status-augmented menu $\widehat{A} \in \widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ for which there is a period $t \in \{2, \ldots, T\}$ such that $\widehat{A} = \widehat{A}^t$ and $s(a^t) = e$, then $\widehat{C}^{RP}(\widehat{A}) = \{(a^t, e)\} = \{(a, e) \in \widehat{A}^t : U(a, e) = U(a^t, s^t(a^t)) > U(a, s^t(a))$ for all $a \in A^t\}$. Hence, the correspondence $\widehat{C}^{RP}$ chooses in any menu of its domain all alternatives that maximise the value of U which, like any real-valued function, produces an ordering of $X \times \{e, u\}$. Hence, by Theorem 1 in Richter (1966), this correspondence satisfies Richter's congruence.

In the other direction, assume that the correspondence $\widehat{C}^{RP}$ defined on the timeless family $\widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ of subsets of $X \times \{e, u\}$ associated to $\{(A^t, a^t)\}_{t=1}^T$ satisfies Richter's congruence on its domain. By the very same Theorem 1 of Richter (1966), there exists an ordering R on $X \times \{e, u\}$ such that $\widehat{C}^{RP}(\widehat{A}) = \{\widehat{a} \in \widehat{A} : \widehat{a}\ R\ \widehat{a}'$ for all $\widehat{a}' \in \widehat{A}\}$ for all $\widehat{A} \in \widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$. Since $X \times \{e, u\}$ is finite, there is a function $U : X \times \{e, u\} \to \mathbb{R}$ that numerically represents this ordering in the sense that $U(x, s) \geq U(x', s') \iff (x, s)\ R\ (x', s')$ for every $x, x' \in X$ and $s, s' \in \{e, u\}$. It is clear that U is such that $U(x, u) = U(y, u)$ for all distinct x and $y \in X$. Indeed, if there were alternatives x and y in X such that $U(x, u) > U(y, u)$, one would have $(y, u) \notin \widehat{C}^{RP}(\widehat{A}^1 \cup X \times \{u\})$ in contradiction with $\widehat{C}^{RP}(\widehat{A}^1 \cup X \times \{u\}) = X \times \{u\}$. Consider now any t. If $s^t(a^t) = u$, then $\widehat{A}^t \cup X \times \{u\} \in \widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ and $(a^t, s^t(a^t)) = (a^t, u) \in \widehat{C}^{RP}(\widehat{A}^t \cup X \times \{u\}) = X \times \{u\}$ so that $U(a^t, s^t(a^t)) \geq U(a, s^t(a))$ for all $a \in A^t$ by virtue of the fact that $\widehat{C}^{RP}(\widehat{A}) = \{\widehat{a} \in \widehat{A} : \widehat{a}\ R\ \widehat{a}'$ for all $\widehat{a}' \in \widehat{A}\}$ for all $\widehat{A} \in \widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ and that U numerically represents R. If on the other hand $s^t(a^t) = e$, then $\widehat{A}^t \in \widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ and $(a^t, s^t(a^t)) = (a^t, e) \in \widehat{C}^{RP}(\widehat{A}^t) = \{(a^t, e)\}$ so that $U(a^t, s^t(a^t)) \geq U(a, s^t(a))$ for all $a \in A^t$ (with actually a strict inequality) follows again from the fact that $\widehat{C}^{RP}(\widehat{A}) = \{\widehat{a} \in \widehat{A} : \widehat{a}\ R\ \widehat{a}'$ for all $\widehat{a}' \in \widehat{A}\}$ for all $\widehat{A} \in \widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ and that U numerically represents R . Hence, U is such that $U(a^t, s^t(a^t)) \geq U(a, s^t(a))$ for all $a \in A^t$ and all $t \in \{1, \ldots, T\}$. The last thing that remains to be shown is that $U(a^t, s^t(a^t)) > U(a, s^t(a))$ for all $t \in \{2, \ldots, T\}$ and $a \in A^t$ distinct from a^t such that either $s^t(a^t) = e$ or $s^t(a) = e$. As just indicated, the strict inequality holds for any t such that $s^t(a^t) = e$. If now $s^t(a^t) = u$, then $\widehat{A}^t \cup X \times \{u\} \in \widehat{\mathcal{F}}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ and $(a^t, s^t(a^t)) = (a^t, u) \in \widehat{C}^{RP}(\widehat{A}^t \cup X \times \{u\}) = X \times \{u\}$ while $(a, s^t(a)) \notin \widehat{C}^{RP}(\widehat{A}^t \cup X \times \{u\})$ for any $a \in A^t$ such that $s^t(a) = e$. Since U numerically represents R, we must have $U(a^t, s^t(a^t)) = U(a^t, u) > U(a, s^t(a))$ for every

$a \in A^t$ such that $s^t(a) = e$ and this completes the proof that U has all the properties mentioned in Definition 7 . □

Proof of Theorem 4

We first show that the requirement for a chronology of choices to admit a timeless forward-looking experienced-status choice correspondence that is actually a function is necessary for such a chronology of choices to result from the forward-looking maximisation of a discovered preference with a prior ranking. By contraposition, assume that the timeless forward-looking experienced-status family $\widehat{\mathcal{F}}^{FW}(\{(A^t, a^t)\}_{t=1}^T)$ of subsets of $X \times \{e, u^a, u^n\}$ induced by the chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ generates a choice correspondence $\widehat{C}^{FW} : \widehat{\mathcal{F}}^{FW}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u^a, u^n\})$, defined by $\widehat{C}^{FW}(\widehat{A}_{FW}) = \{(a, s) \in X \times \{e, u^a, u^n\} : \exists t$ such that $a = a^t$, $\widehat{s}^t(a^t) = s$ and $\widehat{A}^t_{FW} = \widehat{A}_{FW}\}$ for every $\widehat{A}_{FW} \in \widehat{\mathcal{F}}^{FW}(\{(A^t, a^t)\}_{t=1}^T)$, that is not a function. This means that there are two periods s and t in the set $\{1, \dots, T\}$ satisfying $s < t$ such that $\widehat{A}^s_{FW} = \widehat{A}^t_{FW}$ and $\widehat{a}^s_{FW} = (a^s, \widehat{s}^s(a^s)) \neq (a^t, \widehat{s}^t(a^t)) = \widehat{a}^t_{FW}$. Since $\widehat{A}^s_{FW} = \widehat{A}^t_{FW}$, $(a^s, \widehat{s}^s(a^s)) \in \widehat{A}^t_{FW}$ and $(a^t, \widehat{s}^t(a^t)) \in \widehat{A}^s_{FW}$.We observe, using a similar argument as the one used in the proof of Theorem 2, that $(a^s, \widehat{s}^s(a^s)) \neq (a^t, \widehat{s}^t(a^t))$ implies that $a^s \neq a^t$. Observe also that $\widehat{A}^s_{FW} = \widehat{A}^t_{FW}$ implies, by the very definition of the forward-looking experienced-status function, that $\widehat{s}^\tau(a) = \widehat{s}^s(a) = \widehat{s}^t(a)$ for every period $\tau \in \{s, s+1, \dots, t-1, t\}$ and every alternative $a \in A^s = A^t$. If there were two linear orderings R^s and R^t that rationalise the choices of a^s and a^t at periods s and t respectively as required by the forward-looking maximisation of a discovered preference with a prior ranking, these two orderings would conclude that a^s R^s a^t and a^t R^t a^s, thus contradicting–since $a^s \neq a^t$–the requirement on the orderings to be time-invariant when applied to alternatives whose forward-looking experienced-status has not changed. Hence, if the correspondence is not a function, it can not result from the forward-looking maximisation of a discovered preference with a prior ranking. Suppose now that the forward-looking choice correspondence $\widehat{C}^{FW} : \widehat{\mathcal{F}}^{FW}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u^a, u^n\})$ is a function. Let us now show that it satisfies Richter's congruence if and only if the chronology of choices results from the forward-looking maximisation of a discovered preference with a prior ranking. We observe first that the chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the forward-looking maximisation of a discovered preference with a prior ranking if and only if there is a real-valued utility function U defined on the set $X \times \{e, u^a, u^n\}$ such that, for every $t = 1, \dots, T$, $x\ R^t\ y \iff U(x, \widehat{s}^t(x)) \geq U(y, \widehat{s}^t(y))$. The existence of utility functions U^t that numerically represent the linear orderings R^t (for $t = 1, \dots T$) used by the decision-maker who maximises a discovered forward-looking preference with a prior ranking is indeed guaranteed by the fact that X is finite. The fact that the utility functions U^t for $t = 1, \dots, T$ can be written as $U^t(x) = U(x, \widehat{s}^t(x))$ is an immediate consequence of the requirement, imposed by the model of forward-looking preference discovery with a prior ranking, that the ranking of any two alternatives does not vary between two periods if the forward-looking experienced-status of those alternatives has not changed between these two periods. Now, from Richter's (1966) theorem, we obtain that the choice function $\widehat{C}^{FW} : \widehat{\mathcal{F}}^{FW}(\{(A^t, a^t)\}_{t=1}^T) \to \mathcal{P}(X \times \{e, u^a, u^n\})$ satisfies Richter's congruence if and only if there exists an ordering $\widehat{R}$ on $X \times \{e, u\}$ such that, for every $t = 1, \ldots, T$, $(a^t, \widehat{s}^t(a^t))\, \widehat{R}\, (a, \widehat{s}^t(a^t))$ for every $a \in A^t$. Since, given the finiteness of X, $\widehat{R}$ can be represented by a utility function U defined on $X \times \{e, u\}$, this completes the proof. □

Proof of Theorem 5

Assume that a chronology of choices $\{(A^t, a^t)\}_{t=1}^T$ results from the forward-looking maximisation of a discovered preference with a prior reference point as per Definition 9. Hence, there is a utility function $U : X \times \{e, u^a, u^n\} \to \mathbb{R}$ satisfying $U(x, u^a) = U(y, u^a)$ and $U(x, u^n) = U(y, u^n)$ for all distinct alternatives x and $y \in X$ such that $U(a^t, \widehat{s}^t(a^t)) > U(a, \widehat{s}^t(a))$ for all $t \in \{1, \ldots, T\}$ and $a \in A^t$ such that either $\widehat{s}^t(a^t) = e$, $\widehat{s}^t(a) = e$ and $U(a^t, \widehat{s}^t(a^t)) \geq U(a, \widehat{s}^t(a))$ for all $t \in \{1, \ldots, T\}$ and all $a \in A^t$. Consider then the timeless family $\widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ of subsets of $X \times \{e, u^a, u^n\}$ associated to $\{(A^t, a^t)\}_{t=1}^T$. Let us show that for any $\widehat{A}_{FW} \in \widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$, $\widehat{C}_{FW}^{RP}(\widehat{A}_{FW}) = \{\widehat{a}_{FW} \in \widehat{A}_{FW} : U(\widehat{a}_{FW}) \geq U(\widehat{a}'_{FW})$ for all $\widehat{a}'_{FW} \in \widehat{A}_{FW}\}$. Consider first $\widehat{A}_{FW}$ for which there is a $t \in \{1, \ldots, T\}$ such that $\widehat{A}_{FW} = \widehat{A}_{FW}^t \cup X \times \{u^a\}$ and $s^t(a^t) = u^a$. Then the requirement that $U(a^t, u^a) = U(a^t, \widehat{s}^t(a^t)) \geq U(a, \widehat{s}^t(a))$ for all $a \in A^t$, along with the requirement that $U(x, u^a) = U(y, u^a)$ for all distinct alternatives x and $y \in X$, implies that $U(x, u^a) \geq U(a, s^t(a))$ for every $x \in X$ and $a \in A^t$. Hence, $\widehat{C}_{FW}^{RP}(\widehat{A}_{FW}^t \cup X \times \{u^a\}) = X \times \{u^a\} = \{(x, u^a) \in \widehat{A}_{FW}^t \cup X \times \{u^a\} : U(x, u^a) \geq U(y, s)$ for all $(y, s) \in \widehat{A}_{FW}^t \cup X \times \{u^a\}\}$ in this case. Similarly, if $\widehat{A}_{FW}$ is such that there is a $t \in \{1, \ldots, T\}$ such that $\widehat{A}_{FW} = \widehat{A}_{FW}^t \cup X \times \{u^n\}$ and $s^t(a^t) = u^n$, then one obtains from the requirements that $U(a^t, u^n) = U(a^t, \widehat{s}^t(a^t)) \geq U(a, \widehat{s}^t(a))$ for all $a \in A^t$ and $U(x, u^n) = U(y, u^n)$ for all distinct alternatives x and $y \in X$, the conclusion that $U(x, u^n) \geq U(a, s^t(a))$ for every $x \in X$ and $a \in A^t$ and, as a result, that $\widehat{C}_{FW}^{RP}(\widehat{A}_{FW}^t \cup X \times \{u^n\}) = X \times \{u^n\} = \{(x, u^n) \in \widehat{A}_{FW}^t \cup X \times \{u^a\} : U(x, u^n) \geq U(y, s)$ for all $(y, s) \in \widehat{A}_{FW}^t \cup X \times \{u^n\}\}$. Consider finally a forward-looking experienced-status menu $\widehat{A}_{FW} \in \widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ for which there is a period $t \in \{2, \ldots, T\}$ such that $\widehat{A}_{FW} = \widehat{A}_{FW}^t$ and $\widehat{s}(a^t) = e$. We know that $U(a^t, \widehat{s}^t(a^t)) > U(a, \widehat{s}^t(a))$ for all $a \in A^t$ distinct from a^t. Hence, $\widehat{C}_{FW}^{RP}(\widehat{A}_{FW}^t) = \{(a^t, \widehat{s}(a^t)\} = \{(a, s) \in \widehat{A}_{FW}^t : U(a, s) \geq U(a', s')$ for all $(a', s') \in \widehat{A}_{FW}^t\}$. Hence, the timeless correspondence $\widehat{C}_{FW}^{RP}$ chooses in any menu of its domain all alternatives that maximise the value of U which, like any real-valued function, produces an ordering of $X \times \{e, u^a, u^n\}$. It follows from Theorem 1 in Richter (1966) that this correspondence satisfies Richter's congruence.

In the other direction, assume that the correspondence $\widehat{C}_{FW}^{RP}$ defined on the timeless family $\widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ of subsets of $X \times \{e, u^a, u^n\}$ associated to $\{(A^t, a^t)\}_{t=1}^T$ satisfies Richter's congruence on its domain. By Theorem 1 of Richter (1966), there exists an ordering R on $X \times \{e, u^a, u^n\}$ such that $\widehat{C}_{FW}^{RP}(\widehat{A}_{FW}) = \{\widehat{a} \in \widehat{A} : \widehat{a} \ R \ \widehat{a}' \text{ for all } \widehat{a}' \in \widehat{A}_{FW}\}$. Since $X \times \{e, u^a, u^n\}$ is finite, there is a function $U : X \times \{e, u^a, u^n\} \to \mathbb{R}$ that numerically represents this ordering in the sense that $U(x, s) \geq U(x', s') \iff (x, s) \ R \ (x', s')$ for every x, $x' \in X$ and $s, s' \in \{e, u^a, u^n\}$. Let us show that U is such that $U(x, u^a) = U(y, u^a)$ and $U(x, u^n) = U(y, u^n)$ for all distinct alternatives x and $y \in X$ and that it verifies $U(a^t, \widehat{s}^t(a^t)) \geq U(a, \widehat{s}^t(a))$ for all $t \in \{1, \dots, T\}$ and all $a \in A^t$ and $U(a^t, \widehat{s}^t(a^t)) > U(a, \widehat{s}^t(a))$ for all $t \in \{2, \dots, T\}$ and $a \in A^t$ such that either $\widehat{s}^t(a^t) = e$ or $\widehat{s}^t(a) = e$. Since all alternatives are untried at the first period, either $\widehat{s}^1(a^1) = u^a$ or $\widehat{s}^1(a^1) = u^n$. In the first case, $\widehat{A}_{FW}^1 \cup X \times \{u^a\} \in \widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ and $\widehat{C}_{FW}^{RP}(\widehat{A}_{FW}^1 \cup X \times \{u^a\}) = X \times \{u^a\} = \{(x, s) \in \widehat{A}_{FW}^1 \cup X \times \{u^a\} : U(x, s) \geq U(y, s')$ for all $(y, s') \in \widehat{A}_{FW}^1 \cup X \times \{u^a\}\}$ so that $U(x, u^a) = U(y, u^a)$ for all distinct x and $y \in X$ holds in that case. If there is a period t such that $\widehat{A}_{FW} = \widehat{A}_{FW}^t \cup X \times \{u^n\}$ and $s^t(a^t) = u^n$ for some $\widehat{A}_{FW} \in \widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$, then $\widehat{C}_{FW}^{RP}(\widehat{A}_{FW}^t \cup X \times \{u^n\}) = X \times \{u^n\} = \{(x, s) \in \widehat{A}_{FW}^t \cup X \times \{u^n\} : U(x, s) \geq U(y, s')$ for all $(y, s') \in \widehat{A}_{FW}^t \cup X \times \{u^n\}\}$ so that $U(x, u^n) = U(y, u^n)$ will be observed. If there are no period t for which $\widehat{A}_{FW} = \widehat{A}_{FW}^t \cup X \times \{u^n\}$ and $s^t(a^t) = u^n$ for some $\widehat{A}_{FW} \in \widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$, then it is never the case that an alternative untried and unavailable in the future is chosen. Hence, there are no chains of indirect revealed preferences (in the sense of Definition 3) that involve an alternative untried and unavailable in the future. In this case, all such alternatives (x, u^n) and (y, u^n) for any distinct x and y can be put at the bottom of the rationalising ordering R and be assigned the same utility value $U(x, u^n) = U(y, u^n)$. A similar reasoning can be performed (permuting the status u^a and u^n in the argument) if one assumes instead $\widehat{s}^1(a^1) = u^n$. Hence, in all cases, one has $U(x, u^a) = U(y, u^a)$ and $U(x, u^n) = U(y, u^n)$ for all distinct alternatives x and $y \in X$. To show that $U(a^t, \widehat{s}^t(a^t)) \geq U(a, \widehat{s}^t(a))$ for all $t \in \{1, \dots, T\}$ and all $a \in A^t$, consider any $t \in \{1, \dots, T\}$. If $\widehat{s}^t(a^t) = e$, then $\widehat{A}_{FW}^t \in \widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ and $\widehat{C}_{FW}^{RP}(\widehat{A}_{FW}^t) = \{(a^t, \widehat{s}^t(a^t)\}$ so that the conclusion $U(a^t, \widehat{s}^t(a^t)) \geq U(a, \widehat{s}^t(a))$ for all $a \in A^t$ follows from the definition of $\widehat{C}_{FW}^{RP}$ being rationalised by U. If $\widehat{s}^t(a^t) = u^a$, then $\widehat{A}_{FW}^t \cup X \times \{u^a\} \in \widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ and $\widehat{C}_{FW}^{RP}(\widehat{A}_{FW}^t \cup X \times \{u^a\}) = X \times \{u^a\}$ so that the conclusion $U(a^t, \widehat{s}^t(a^t)) = U(x, u^a) \geq U(a, \widehat{s}^t(a))$ for all $(a, \widehat{s}^t(a)) \in \widehat{A}_{FW}^t$ and all $x \in X$ follows again from the definition of $\widehat{C}_{FW}^{RP}$ being rationalised by U. If finally $\widehat{s}^t(a^t) = u^n$, then $\widehat{A}_{FW}^t \cup X \times \{u^n\} \in \widehat{\mathcal{F}}_{FW}^{RP}(\{(A^t, a^t)\}_{t=1}^T)$ and $\widehat{C}_{FW}^{RP}(\widehat{A}_{FW}^t \cup X \times \{u^n\}) = X \times \{u^n\}$ and the conclusion $U(a^t, \widehat{s}^t(a^t)) = U(x, u^n) \geq U(a, \widehat{s}^t(a))$ for all $(a, \widehat{s}^t(a)) \in \widehat{A}_{FW}^t$ and all $x \in X$ is again obtained from the definition of $\widehat{C}_{FW}^{RP}$ being rationalised by U. In order to show, finally, that $U(a^t, \widehat{s}^t(a^t)) > U(a, \widehat{s}^t(a))$ for all $t \in \{2, \dots, T\}$ and all $a \in A^t$ such that either $\widehat{s}^t(a^t) = e$ or $\widehat{s}^t(a) = e$, consider any $t \in \{2, \dots, T\}$. If

$\widehat{s}^t(a^t) = e$, then $\widehat{A}^t_{FW} \in \widehat{\mathcal{F}}^{RP}_{FW}(\{(A^t, a^t)\}_{t=1}^T)$ and $\widehat{C}^{RP}_{FW}(\widehat{A}^t_{FW}) = \{(a^t, \widehat{s}^t(a^t)\}$. Hence, the conclusion that $U(a^t, \widehat{s}^t(a^t)) = U(a^t, e) > U(a, \widehat{s}^t(a))$ for all $a \in A^t$ distinct from a^t follows from the definition of $\widehat{C}^{RP}_{FW}$ being rationalised by U. If $\widehat{s}^t(a^t) = u^a$, then $\widehat{A}^t_{FW} \cup X \times \{u^a\} \in \widehat{\mathcal{F}}^{RP}_{FW}(\{(A^t, a^t)\}_{t=1}^T)$ and $\widehat{C}^{RP}_{FW}(\widehat{A}^t_{FW} \cup X \times \{u^a\}) = X \times \{u^a\}$. Hence, if there is any $a \in A^t$ such that $\widehat{s}^t(a) = e$, then the conclusion that $U(a^t, \widehat{s}^t(a^t)) = U(x, u^a) > U(a, e)$ will follow from the definition of $\widehat{C}^{RP}_{FW}$ being rationalised by U. The argument for the case where $\widehat{s}^t(a^t) = u^n$ is similar, and this completes the proof. □

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