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Picometrology with Topologically Structured Light: The Information Aspect

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***Abstract* – Optical localization metrology of nanoscale objects achieving sub-atomic levels of precision with topologically structured light has recently been demonstrated (Nat. Mater. 22, 844, 2023). Here we report a Fisher Information analysis of this metrology, which provides understanding of how such performance – reaching orders of magnitude beyond the conventional diffraction limit – is possible. We show that light scattering by sharp features of the object located near zones of high phase and intensity gradient in the optical field carries enhanced information content on the object. The Fisher Information methodology can be used to optimize metrological resolution by enabling the topology of light to be tailored to the class of object being measured.**

Recently, dimensional and positional measurements with deeply subwavelength resolution have been demonstrated via deep learning-enabled analysis of diffraction patterns of topologically structured light scattered by objects. With such metrology for example, one can optically localize the mean position of a nanowire beyond its thermal fluctuations, with precision down to ~100 pm using visible light, thereby beating the diffraction limit of conventional optical instruments thousands of times over. Here, we show that the scattering of topologically structured light by an object located near a singularity in the light field has a higher information content as compared to the scattering of a plane wave, enabling greater precision in metrology upon its analysis.

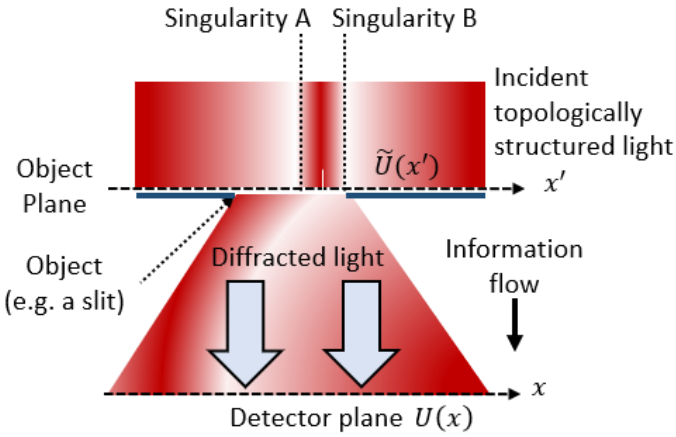

Fig. 1. Metrology with topologically structured light. When a phase singularity in the incident light field encounters a sharp feature of an object of the target object (e.g. the edges of a slit in a screen, in this example) the Fisher information content of the scattered light increase multi-fold.

Considering the archetypal case of a narrow slit in an opaque screen, we show that when illuminated with a complex light field, $\tilde{\boldsymbol{U}}(\boldsymbol{x}') = \boldsymbol{A}(\boldsymbol{x}')\boldsymbol{e}^{\boldsymbol{i\phi}(\boldsymbol{x}')}$ that the scattered field $\boldsymbol{U}(\boldsymbol{x})$ at the detector plane can be written as a sum of three contributions: The first component is always present, mapping the field intensities from the input to the output plane, as if they were planar, i.e having a flat, and constant phase front. The other two terms depend on the field and phase gradients respectively in the object plane. They are zero for a plane wave incident light field but can become large when the field is highly structured. For example, the anomalously high local wavevectors found in the vicinity of phase singularities in an incident field can cause the phase gradient-dependent term to become large. Both of these terms can be highly sensitive to changes in the object plane (e.g in the slit position) when there are deeply sub-wavelength intensity and phase features in a topologically structured incident light field.

To illustrate the advantage of using light with phase singularities in metrology, we consider a nano-slit probed with a super-oscillatory light field (intensity and phase profiles of which are shown in Fig. 2a), with the scattered field observed in a 'detector plane' at a distance 4λ from the object plane. We employ the concept of Fisher information to evaluate the amount of information $I(a_{fix})$ contained within the scattered field's intensity profile (normalised with respect to the intensity $i(x;a)$ incident upon the slit). As such, $i(x;a)$, becomes a probability distribution function describing the likelihood of a given photon being detected at a position x on the screen, as a function of slit position a. Fisher information is then given by the expression:

$$I(a_{\text{fix}}) = -\int_{-\infty}^{\infty} \frac{\partial^2 \ln i(x; a_{\text{fix}})}{\partial a^2} i(x; a_{\text{fix}}) dx,$$

The faster the probability distribution function changes as a function of our fixed parameter, the slit position a_{fix}, the greater the information contained within the scattered field. Figure 2b shows that information is maximised at the singularity, where the *information per photon is enhanced by around 5 orders of magnitude* relative to the same measurement performed with a plane wave having the same intensity as the central bright spot of the superoscillatory field.

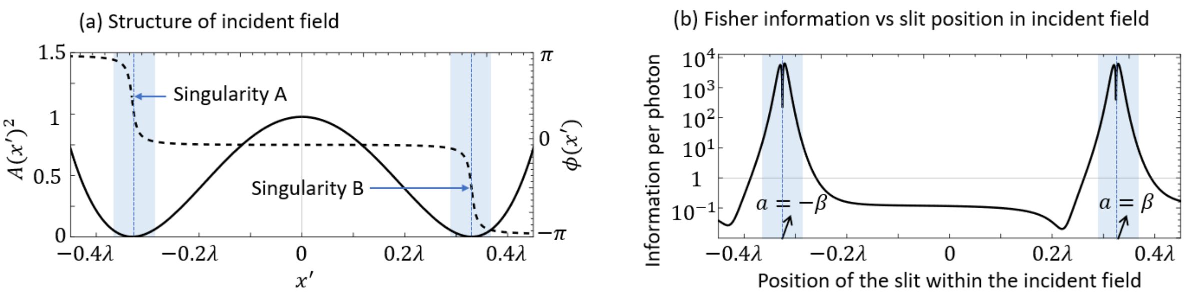

Fig. 2. Information content of nano-slit optical metrology using topologically structured light. (a) Intensity $A(x')^2$ [solid line] and phase $\phi(x')$ [dashed line] of the incident superoscillatory field. Phase singularities are located at $x' = \pm\beta$; (b) Fisher information per photon as a function of the position of the centre of the slit in the object plane. Information per photon is enhanced by approximately 5 orders of magnitude when the slit is located in close proximity to a singularity [the blue shaded bands], as compared to when it is positioned at the centre of the superoscillatory intensity hotspot (where there is an approximately flat phase and intensity profile).

Our work provides a fundamental explanation, and justification, for singularity-based optical metrology. We demonstrate that despite the exponentially low field intensities around phase singularities within topologically structured fields, the information content per measurement can be superior.