Generalised Performance of Neural Network Controllers for Feedforward Active Noise Control of Nonlinear Systems

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¹ Abstract

Advances in digital technologies have allowed for the 2 development of complex active noise and vibration 3 control solutions that have been utilised in a wide 4 range of applications. Such control systems are com-5 monly designed using linear filters, which cannot fully 6 capture the dynamics of nonlinear systems. To over-7 come such issues, it has been shown that replacing 8 linear controllers with Neural Networks (NNs) can 9 improve control performance in the presence of non-10 linearities. Many real systems are subject to non-11 stationary disturbances where the magnitude of the 12 system excitation time dependent. However, within 13 the literature, the performance of single NN con-14 trollers across different excitation levels has not been 15 thoroughly explored. In this paper, a method of train-16 ing Multilaver Perceptrons (MLPs) for single-input-17 single-output (SISO) feedforward acoustic noise con-18 trol is presented. In a simple time-discrete simulation, 19 the performance of the trained NNs is investigated for 20 different excitation levels. The effects of the proper-21 ties of the training data and NN controller on gener-22 alised performance are explored. It is demonstrated 23 that the generalised control performance of the MLP 24 controllers falls as the range of magnitudes included 25 in the training data is increased, and that this perfor-26 27 mance can be recovered by increasing the number of hidden nodes within the controller. 28

²⁹ 1 Introduction

Unwanted noise and vibration can be problematic in 30 both engineering systems and in public and private 31 spaces. Passive control solutions are capable of effec-32 tively reducing high frequency components of noise 33 and vibration but are typically large and/or heavy 34 for low frequency control, possibly exceeding the de-35 sign constraints of a given application. Active control 36 solutions, by contrast, are capable of effective control 37 at low frequencies, and are typically lightweight and 38 compact. Historically, feedforward active noise and 39 vibration control systems have been implemented us-40 ing linear control filters and linear plant models, com-41 monly using the FxLMS algorithm. However, it is well 42

understood that nonlinearities present in either the 43 plant or primary path of the control system can have 44 a significant impact on control performance [1], [2], 45 [3], [4]. Many approaches have been proposed to over-46 come this limitation, including polynomial, cross-term 47 or trigonometric expansion of the reference signal [5], 48 [6], genetic algorithms [7] and fuzzy logic-based meth-49 ods [8]. A further common approach, which has been 50 applied to active control over the past few decades, 51 is the application of machine learning methods. NNs 52 in particular are known to possess the property of 53 being 'universal approximators' [9] and are therefore 54 an attractive black-box method for the modelling and 55 control of unknown or uncertain nonlinear systems. 56 The similarities in structure between NNs and lin-57 ear filters provides good motivation for their use in 58 both system modelling and feedforward controller de-59 sign. Many different uses of NNs have been studied, 60 including system modelling [4], [10], [11], [12], [13], 61 feedforward controller design [4], [10], [14], inverse 62 modelling [15], signal prediction and feedback control 63 [16], [17], [18], [19], [20], linear filter selection [21], 64 adaptive parameter estimation for linear controllers 65 [20], [22], frequency-domain control [23], multichan-66 nel controller design [24], and signal classification [25]. 67 In previous work utilising NNs as feedforward con-68 trollers, however, the ability for the controller or plant 69 model NNs to generalise across a range of excitation 70 levels of the studied system has not been thoroughly 71 explored. This is clearly a desirable quality in any 72 real implementations of such a control system where 73 the properties of the excitation, and therefore the ef-74 fect of the system nonlinearity, may change over time. 75 In this paper, a simulation of a simple noise control 76 system implementing a time-domain MLP controller 77 is studied. Section 2 defines the simulated system, 78 system parameters and simulation method. Section 3 79 explains the controller training methodology. Section 80 4 presents simulated results in the time and frequency 81 domains. Finally, Section 5 discuss conclusions and 82 presents possible future research directions. 83

$_{*4}$ 2 Problem definition

⁸⁵ 2.1 Simulated System

The considered system consists of two acoustic 86 sources. The primary source, which generates the 87 acoustic disturbance, is modelled as a damped Duffing 88 oscillator, assumed to radiate as a monopole acoustic 89 source. The secondary acoustic source, which gener-90 ates the cancelling acoustic signal, is modelled as a 91 simple harmonic oscillator, also assumed to radiate 92 as a monopole source. Figure 1 presents a diagram of 93 the simulated system. 94

The displacement of the Duffing oscillator, $y_a(t)$, is caused by the motion of the floor to which it is coupled. The displacement, x(t), of this floor is also taken to be the reference signal passed to the feedforward controller. The displacement, $y_b(t)$, of the mass, m_b , is caused by the control force, $F_c(t)$, produced by the controller.

¹⁰² The equations of motion for the total system are

$$m_a \ddot{y}_a(t) + k_a p(t) + k_a^{NL} p^3(t) + c_a \dot{p}(t) = 0 \qquad (1)$$

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$$m_b \ddot{y}_b(t) + k_b y_b(t) + c_b \dot{y}_b(t) + F_c(t) = 0 \qquad (2)$$

where $p(t) = y_a(t) - x(t)$ and the remaining variables 104 are defined in Figure 1 and their values are provided 105 in Table 1. These parameter values were selected 106 such that the two oscillators have unity mass, but 107 distinct resonance frequencies of 60 Hz and 80 Hz. 108 The damping coefficients c_a and c_b were selected such 109 that each oscillator is subject to 20% of critical damp-110 ing, and the oscillators are therefore not significantly 111 underdamped or overdamped. Assuming for simplic-112 ity that the error sensor is equidistant from the two 113 point monopole sources and that the constant ampli-114 tude scaling factors are equal both cases such that 115 they can be neglected, the error signal is defined as 116

$$e(t) = y_a(t - \delta_a) + y_b(t - \delta_b) \tag{3}$$

where δ_a and δ_b are the acoustic delays, in time, between the primary and secondary sources and the error microphone, respectively. In all cases, the signal x[n] is Gaussian white noise band-limited to the frequency range [0, 250] Hz. The motion of the sources is simulated in the time domain using a 4th order Runge-Kutta method at a sample rate of $f_s = 2$ kHz. 123



Figure 1: Diagram of the simulated system, consisting of a nonlinear primary acoustic source, and a linear control source. System parameter values are given in Table 1.

Parameter	Symbol	Value
Primary oscillator mass	m_a	1 kg
Secondary oscillator mass	m_b	1 kg
Primary oscillator linear stiffness	k_a	$1.42 \times 10^5 \ {\rm Nm^{-1}}$
Primary oscillator cubic stiffness	k_a^{NL}	$1.42 \times 10^{14} \ \mathrm{Nm^{-3}}$
Secondary oscillator stiffness	k_b	$2.53 \times 10^5 \ {\rm Nm^{-1}}$
Primary oscillator damping	c_a	$151 \; {\rm N sm^{-1}}$
Secondary oscillator damping	c_b	201 Nsm^{-1}

Table 1: Simulated system parameter values.

Controller 3 design, training 124 and testing 125

Controller architecture 3.1126

A diagram of the NN architecture used to train the 127 controller is presented in Figure 2. Similarly to the 128 case of a linear controller, the NN controller and plant 129 model each take as input a tapped delay line of the 130 sampled reference signal, x[n], and sampled control 131 signal, u[n], respectively. The plant model output, 132 $\hat{y}[n]$, is linearly summed with the disturbance sig-133 nal, d[n], to generate the error signal estimate, $\hat{e}[n]$, 134 which is used to update the weights and biases of the 135 controller NN via backpropagation through the full 136 network consisting of both the controller and plant 137 model. Given a tapped delay line of length L of the 138 reference signal, x[n], given by 139

$$\mathbf{x}[n] = \begin{bmatrix} x[n] \\ x[n-1] \\ \vdots \\ x[n-L+1] \end{bmatrix}$$
(4)

the control signal, u[n], can be generated by passing 140 $\mathbf{x}[n]$ through the controller NN. If the NN architecture 141 is that of an MLP with a single hidden layer, then the 142 output of the NN is given by 143

$$u[n] = \sum_{i} w_i^o h_i + b^o \tag{5}$$

where w_i^o are the output weights of the NN, b^o is the 144 NN output bias, and h_i are the NN hidden layer node 145 values, given by 146

$$h_i = \sigma([\mathbf{W}\mathbf{x}[n]]_i + b_i^h) \tag{6}$$

where \mathbf{W} is a matrix of weights between the input 147 layer and hidden layer, $[\mathbf{Wx}[n]]_i$ is the i^{th} element 148 of the vector $\mathbf{Wx}[n]$, $\sigma(\cdot)$ is the nonlinear activation 149 function applied to the controller hidden layer, and b_i^h 150 is the bias of the i^{th} hidden layer node. In total, 151

$$u[n] = \sum_{i} w_i^o \sigma([\mathbf{W}\mathbf{x}[n]]_i + b_i^h) + b^o$$
(7)

However, a full tapped delay line $\mathbf{u}[n]$ is required to 152 infer the output of the plant model, $\hat{y}[n]$, and there-153 fore the error estimate, $\hat{e}[n]$. It is therefore necessary 154 to generate control signal values $u[n-1], u[n-2], \ldots,$ 155 u[n-I+1] for a tapped delay line of length I. A pre-156 viously presented solution to this problem [26] is to 157 train the model using the training data sequentially, 158 storing the values of the control signal in memory, and 159 calling upon them when evaluating the output of the 160 plant model at each timestep during the updating of 161 the controller weights and biases. However, a result 162 of this approach is that the error estimate, $\hat{e}[n]$, will 163

not accurately reflect the control performance of the 164 current iteration of the controller. The control signal 165 tapped delay line is calculated from the outputs of the 166 current and previous L-1 iterations of the controller, 167 and therefore so is the error estimate. This could 168 plausibly lead to stability and performance issues in 169 the training of the controller NN. Perhaps more im-170 portantly, this sequential approach will face the issue 171 of catastrophic interference [27], meaning it is unsuit-172 able for training networks without compromising gen-173 eralised control performance. As illustrated in Figure 174 2, an alternative approach is proposed here where all 175 required previous controller outputs, u[n-k], are gen-176 erated using the current iteration of the controller. In 177 general. 178

$$u[n-k] = \sum_{i} w_i^o \sigma([\mathbf{W}\mathbf{x}[n-k]]_i + b_i^h) + b^o \qquad (8)$$

where all weights and biases in equation 8 are those 179 of the current iteration of the controller during train-180 ing. Irrespective of whether the values of u[n-k]181 are called from memory or generated from the cur-182 rent iteration of the controller, standard backpropa-183 gation techniques can be used to update the weights 184 and biases of the controller to minimise a given cost 185 function of $\hat{e}[n]$. The approach presented in Figure 2 186 is clearly more computationally intensive than simply 187 storing u[n] in memory. It should be noted, however, 188 that computing u[n] is only required during the con-189 troller training. The controller NN is assumed here to 190 be fixed during operation and so, for a NN controller 191 with the same number of hidden nodes, the computa-192 tional cost to produce u[n] from x[n] in operation is 193 independent of the training method. 194

3.2Controller training

The controllers were trained to minimise the Mean Squared Error (MSE) signal, defined as 197

$$J = \overline{\hat{e}^2[n]} \tag{9}$$

where the average is calculated over the samples in 198 the training batch. The backpropagation used the 199 Adam algorithm [28] with parameters $\alpha = 3 \times 10^{-5}$, 200 $\beta_1 = 0.9, \beta_2 = 0.99, \text{ and } \epsilon = 10^{-7}$. These parameters 201 were selected through trial and error with a view to 202 reaching a trade-off between controller performance 203 and training speed. 204

In all cases, the plant model used for controller training was an FIR filter (equivalent to an MLP with no hidden layer) with 140 taps, which was capable of achieving high levels of modelling accuracy due to the linear nature of the simulated plant response.

3.3Training data

For each instance of network training, two sets of 900 $\,\mathrm{s}$ 211 of simulated data are generated. The first set is used 212

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Figure 2: Block diagram of the controller training method, assuming an FIR plant model of order 2.

for training, and the second set is used as a validation 213 set to assess model overfitting. Each dataset con-214 sists of the reference signal, x[n], and the resultant 215 displacement of the primary source, $y_a[n]$. In each 216 900 s simulation, the magnitude of the reference sig-217 nal, x[n], is increased linearly in time from the lower 218 to the upper bound of the the training range. The 219 training data is split into mini-batches of size 128, 220 selected randomly from the data. In each iteration 221 of the training, 1000 such mini-batches are employed. 222 The full set of training data are therefore not used in 223 each iteration. This is due to a training bias which is 224 implemented to optimise network training for gener-225 alised performance, explained below. All networks are 226 trained over 500 iterations. All signals used to gener-227 ate training and validation data are first normalized 228 to have a standard deviation of 1 to facilitate quick 229 and stable training of the NNs. As expected, using 230 small amounts of training data was observed to re-231 sult in model overfitting, reducing the performance of 232 the controllers and obscuring the underlying limits on 233 controller performance. The relatively large amount 234 of training data used was therefore chosen to mini-235 mize the effect of overfitting during network training, 236 and therefore no regularization was applied to the net-237 works. 238

²³⁹ During testing, it was initially observed that MLP

controllers trained using data with equal weighting 240 across all magnitudes of the reference signal produced 241 MSE attenuation that was approximately equal across 242 much of the training range. As shall be noted in Sec-243 tion 4, the maximum performance of the MLP con-244 trollers at each magnitude of the reference signal is 245 not equal. As a result, training an MLP as a con-246 troller across a range of reference signal magnitudes 247 results in control performance close to the maximum 248 for the largest training magnitudes, but underperfor-249 mance relative to the maximum at the lowest training 250 magnitudes. To counteract this, the random selection 251 of the training samples was weighted based on the 252 magnitude of the reference signal. That is, for a set of 253 N training examples with reference signal magnitudes 254 x_{mag} in the range $a < x_{mag} < b$, the probability of 255 training example q being included in a training batch 256 is given (up to a normalizing factor) by 257

$$P(q) \propto 10^{-\gamma(x_{mag}-a)} \tag{10}$$

where γ is a factor controlling the training bias. The inclusion of this training bias affects the resultant performance of the MLP controllers across the training range. Appropriate selection of γ for a given training range results in generalised control performance that approaches the maximum MLP controller performance across the training range. This effect is il²⁶⁵ lustrated in Section 4.2.

266 3.4 Testing

During the controller testing phase, the controller net-267 work is extracted and used to generate the control 268 signal u[n] in a new simulation. At each timestep, 269 the controller is input with a tapped delay line of 270 the reference signal, $\mathbf{x}[n] = [x[n], ..., x[n - L + 1]],$ 271 generating a control signal sample, u[n]. The testing 272 is undertaken across a range of magnitudes of x[n]. 273 However, this magnitude is kept constant within each 274 testing simulation. Each testing simulation is under-275 taken over 60 s, and control performance is defined 276 as MSE attenuation, measured in dB relative to the 277 sampled disturbance signal, d[n]. 278

279 4 Results

²⁸⁰ 4.1 Maximum control performance

To first establish an upper limit on the generalised 281 control performance of the MLP controllers across a 282 range of magnitudes of x[n], a set of 90 networks with 283 100 hidden nodes were trained at 30 equally spaced 284 magnitudes of x[n] from 10^{-9} to 10^{-5} m with three 285 controllers trained at each level to obtain an average 286 of the control performance. Controller weights and 287 biases were initialized with a random Gaussian dis-288 tribution with zero mean and a standard deviation 289 of 0.05. The performance of these controllers is pre-290 sented in Figure 3. Also presented is the performance 291 of FIR filter-based controllers trained using the same 292 Adam algorithm, and the control performance of FIR 293

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controllers trained in-simulation using the FxLMS algorithm.

At very low magnitudes of the reference signal, both 296 the FIR and MLP controllers achieve a similar level of 297 performance of between approximately 55 and 60 dB. 298 At the lowest magnitudes, the FIR controller outper-299 forms the MLP controller slightly. However, as the 300 magnitude of x[n] increases and the degree of nonlin-301 earity stimulated in the primary path increases, the 302 MLP controller demonstrates a clear control advan-303 tage compared to the linear controller of up to 20 304 dB. At the highest levels of nonlinearity, the perfor-305 mance of both controller architectures falls. However, 306 the MLP controller still achieves some control advan-307 tage of approximately 5 dB. Interestingly, the perfor-308 mance of the linear controller shows a standard devi-309 ation of 2.6 dB at the lowest magnitude of x[n] tested. 310 This is unexpected, and is absent in the results of the 311 FxLMS-trained controller. It is reasonable to assume, 312 therefore, that this variance is a result of the Adam 313 algorithm used to train the FIR controllers. 314

Figure 4 presents Power Spectral Density (PSD) es-315 timates of the disturbance and error signals at a range 316 of values of x_{mag} for the FIR and MLP controllers. 317 As observed in Figure 3, at the lowest value of x_{mag} , 318 the FIR controller slightly outperforms the MLP con-319 troller. However, as x_{mag} increases, the MLP con-320 troller outperforms the FIR controller. Notably, the 321 MLP controller outperforms the FIR controller across 322 all frequencies present in the reference signal. 323

4.2 Effect of training bias γ

As explained in Section 3.3, the training of the MLP $_{325}$ controllers for control across a range of x_{mag} is influ- $_{326}$



Figure 3: Average control performance of the MLP controller, and an FIR controller trained using the Adam algorithm and FxLMS. Averaging is undertaken over 5 instances of a trained controller. Solid lines represent the mean control performance. Shaded regions present 2 standard deviations of control performance around the mean.



Figure 4: Frequency-domain errors corresponding to the MLP and FIR filter-based controllers over a range of reference signal magnitudes x_{mag} , averaged over 5 instances of controller training. Black: Disturbance; Blue: FIR controller error; Red: MLP controller error. All signals used to calculate PSD estimates are normalised with respect to the RMS of the disturbance signal.

enced by a training bias which is fully defined by a 327 parameter γ . In effect, this parameter controls the 328 slope of the generalised performance curve within the 329 training region for a given controller. As the gener-330 alised performance of the controllers is limited to a 331 maximum at each magnitude of x[n], the tuning of γ 332 allows for generalised performance to be maximised 333 across the training region. Figure 5 presents the con-334 trol performance of 3 MLP controllers trained with 335 different values of the parameter γ . In each case, the 336 controller contains a hidden layer of 100 nodes, and 337 the training range is shown by the red shaded region. 338 As observed in Section 4.1, the control performance 339 of MLP networks is subject to variance. Therefore, to 340 produce smooth generalised performance curves that 341 illustrate the effect of γ more clearly, the training data 342 used for each network was identical, and the refer-343 ence signal used in each testing simulation was iden-344 tical across tests, but scaled to the appropriate mag-345 nitude. As expected, the effect of increasing γ is to 346 change the slope of the generalised performance curve 347 within the training region. We can observe that, for 348 $\gamma = 1.2 \times 10^6$, the generalised network performance 349 is close to the maximum at the bottom of the train-350

ing range, but falls away from the maximum near the top. We see the opposite effect for the network trained with $\gamma = 0$. For $\gamma = 6 \times 10^5$, the generalised performance of the controller relative to the maximum is approximately constant within the training range. 352

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4.3 Effect of training range

For a fixed number of hidden nodes in the MLP con-357 troller, the effect of increasing the range of x_{mag} 358 over which the controller is trained is investigated. 359 Figure 6 presents the control performance of 4 net-360 works with increasing training widths. The first net-361 work, trained at $x_{mag} = 5 \times 10^{-5}$ m, achieves con-362 trol performance equal to the defined maximum at 363 the trained magnitude, as expected. It also demon-364 strates some generalised performance capacity, with 365 performance relative to the maximum curve drop-366 ping by approximately 3 dB within the range of 367 $x_{mag} \in [4 \times 10^{-6}, 6 \times 10^{-6}]$. Increasing the range of 368 the training to $[4 \times 10^{-6}, 6 \times 10^{-6}]$ slightly improves 369 the control performance at the edges of this range at 370 very little cost to performance at the centre of the 371 range. Further increasing the training range contin-372



Figure 5: Performance of 3 MLP controllers trained with varying values of the training bias parameter γ . The red highlighted region represents the range of magnitudes of x[n] over which the controllers were trained.



Figure 6: MSE attenuation of 4 MLP controllers trained with reference signal magnitude within the highlighted range. Each plot presents the averaged performance of 3 controller instances trained within the presented range.

ues to reduce the performance of the controller at
the centre of the range whilst improving generalisability. This is expected, as increasing the range of the
magnitude of the training data increases the range of

³⁷⁷ nonlinear behaviour exhibited by the training data.

We should therefore expect that keeping the number of hidden nodes in the networks fixed, but increasing the training range, should reduce the peak performance of the controller relative to the maximum curve as the controller network becomes increasingly 382 under-powered to perform the required generalisationtask.

385 4.4 Effect of network size

The ability of NNs to generate nonlinear mappings 386 from input to output is derived from the nonlinear 387 activation functions applied to the nodes of the hid-388 den layer(s). It is natural, therefore, to assume that 389 increasing the number of nodes in the hidden layer of 390 the MLP controller will increase its generalised con-391 trol performance across a range of reference signal 392 magnitudes. Figure 7 shows the generalised control 393 performance of 5 MLP controllers with a range of 394 hidden nodes from 12 to 200. At each magnitude 395 of x[n] and number of hidden nodes, 3 controllers 396 were trained and the performance averaged over the 3 397 controllers. Across the training range, the controller 398 with 12 hidden nodes achieves a performance approx-399 imately 7 dB below the defined maximum MLP per-400 formance. Increasing the number of hidden nodes to 401 25 increases the performance of the controller relative 402 to the maximum by approximately 2 dB. However, 403 this approximately doubles the number of parameters 404 to be learned within the network, and approximately 405 doubles the number of operations required to infer 406 the output of the network. Further doubling of the 407 number of hidden nodes in the MLP controllers fur-408 ther increases the generalised control performance of 409 the controllers. However, this increase becomes in-410 creasingly small as the performance of the networks 411 approach the maximum. Noting that the controller 412 containing 12 hidden nodes generates a control per-413

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formance advantage over the linear controllers of up 414 to 10 dB, the computational cost of increasing the 415 number of hidden nodes from 12 to 200 to gain an 416 additional 8 dB in control performance is consider-417 able. In practical applications, the additional compu-418 tational cost of both training and inferring the out-419 puts of these networks may be prohibitive to their 420 implementation for excessively large networks. How-421 ever, it should be noted that increasing the number of 422 hidden nodes in the MLP controller does recover the 423 control performance lost by increasing the controller 424 training width. 425

5 Conclusions

In this paper, a method of training MLP NNs for 427 use as time-domain controllers for feedforward ac-428 tive noise control has been presented. For the sim-429 ple case presented, the maximum achievable perfor-430 mance of linear and MLP controllers has been esti-431 mated, demonstrating that the control performance 432 of both the linear and MLP controllers falls as the 433 degree of nonlinearity in the system primary path in-434 creases. The ability of MLP controllers to achieve gen-435 eralised performance across a range of system stimu-436 lation magnitudes has been investigated, and a train-437 ing bias parameter γ has been introduced to balance 438 the control performance across the training range. 439 The effect of varying this parameter has been pre-440 sented, demonstrating that, for a fixed number of hid-441 den nodes in the controller, the parameter can con-442 trol the relation between generalised controller per-443



Figure 7: Generalised control performance of MLP controllers trained with varying numbers of nodes in the hidden layer, trained within the highlighted range. Each curve represents the average performance of 3 trained controller instances.

- ⁴⁴⁴ formance and reference signal magnitude. Increasing
- the controller training range has been shown to reduce
- 446 controller performance over the same range. However,
- 447 it has also been demonstrated that this performance
- may be recovered by increasing the number of hiddennodes in the MLP controller.
- Additional contributions which would extend this 450 work in the future could include the study of an 451 extended set of NN architectures, a wider study of 452 the NN hyperparameters, the study of a wider range 453 of types of nonlinear behaviour, comparison of con-454 troller performance to other common nonlinear con-455 trol strategies, as well as experimental validation of 456 the results under similar nonlinear conditions. 457

458 Data Availability Statement

The data are available from the corresponding authoron request.

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