Strategic use of social media influencer marketing

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Abstract

We set out a model of social media influencer marketing in which a firm may hire influencers to inform consumers about an innovation. Influencers generate sales through purchases of their followers and followers' social networks and set prices for their endorsements. In turn, the firm decides which influencers to hire, which story to convey via the influencers, and sets the retail price of the innovation. In equilibrium, influencers price according to their marginal contribution to industry profits and increase consumers' willingness to pay with their stories. In particular, under a weak condition it is the influencers with the most *reactive followers* who are hired and obtain positive profits in equilibrium. Finally, we show that the firm may be better off if it could commit to hire fewer influencers.

JEL classification: D21, L11, L15, C72, M30.

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"People do not buy goods and services. They buy relations, stories, and magic."

Seth Godin

1 Introduction

Over the last decade, social media platforms have become increasingly important, both in people's lives and for society more broadly. While today most adults use social media, these platforms are particularly popular among young people, who often visit them several times a day.[1] This has lead to the emergence of *social media influencers*, who create certain content (images, videos, livestreaming, etc.) often focussed on a specific topic. Many people "follow" influencers they do not know personally, but enjoy to be entertained by their "stories". Influencers' increasing popularity has quickly attracted the interest of the marketing industry, which traditionally has relied on celebrities to advertise new products. Today they often do not just share bits and pieces of their private life, but also advertise or "recommend" products.[2] Many influencers are paid for these recommendations—a fast-growing market whose global market value has more than doubled since 2019, reaching 16.4 billion USD in 2022 (see Figure 1).

In this paper, we build a novel model of social media influencer marketing to investigate the industrial organization of the influencer economy. Influencers compete à la Bertrand for being hired by a firm to inform consumers about and, in particular, endorse an innovation. In turn, consumers are organized in a social network and may be persuaded by influencers' posts. We characterize how profits are shared in equilibrium between influencers and producer, and establish that the firm may benefit from committing to hire fewer influencers.

In our model there is one firm that sells an innovation with unknown quality to a large set of consumers. A subset of consumers follows social media influencers, who may inform their followers of and endorse the innovation by publishing a "story". Increasing consumers' willingness to pay via their story is possible but comes at effort (and/or reputational) costs; this may be because influencers are able to shape

[1]According to a recent survey by the Pew Research Center (2021), 72% of US adults use social media (84% for age 18-29). Furthermore, more than half of Instagram and Snapchat users ages 18 to 29 visit the platform several times a day.

[2]See, e.g. Hudders et al. (2021) for an overview.

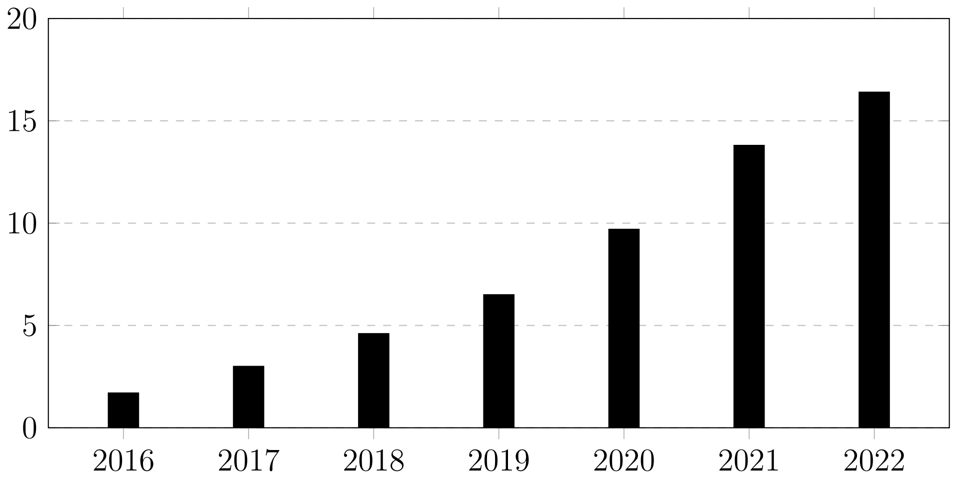

Figure 1: Global influencer marketing market size in billion USD from 2016 to 2022. Source: Statista Influencer Marketing Hub (2022).

some (e.g. naïve) consumers' beliefs about the product, but also because they create added value for at least some consumers. We abstract from differences in the rate at which followers react to postings between influencers. Instead, an influencer's number of followers will measure the number of *reactive (or "engaged") followers*.

After the influencers have announced their pricing policies, the firm decides which influencers to hire, which story to convey via the influencers, and sets the retail price of the innovation. Influencers' endorsements then generate additional demand for the innovation. Note that a key difference to traditional Bertrand competition is that the firm may hire multiple influencers.

We then show that our assumptions on the structure of demand hold in a microfoundation based on Galeotti and Goyal (2009), which features both sophisticated and naïve consumers, and influencers' endorsements generate sales both directly through purchases of their followers and indirectly through subsequent word-of-mouth communication within followers' social networks.

We allow influencers to coordinate their pricing policies, which implies that industry profits will be maximized in equilibrium.[3] In a first step, we then show that influencers generally increase consumers' willingness to pay. Moreover, they even "overprice" and sell only to consumers who are responsive to such behavior if product quality is low.

[3]Formally, we consider strong subgame-perfect equilibria, which are stable with respect to coalitional deviations.

Second, we refer to an influencer as *essential* if hiring her is necessary to maximizing industry profits. Essential influencers are those with the largest number of (reactive) followers under a weak condition on effort costs. Roughly speaking, it ensures that influencers' stand-alone profit is increasing in the number of followers. We then show that in equilibrium influencers price according to their marginal contribution to industry profits, and thus obtain a positive profit in equilibrium if and only if they are essential.

We then investigate the trade-off between macro- and micro-influencers. Marketing practitioners broadly draw the line between these two categories at 100,000 followers, and also use the term nano-influencers for those with fewer than 3,000 followers (Fainmesser and Galeotti, 2021). Note, however, that these numbers are raw numbers, disregarding followers' engagement or reaction rate. Imposing the condition on effort costs discussed above, we show that it is not macro-influencers per se (i.e. in raw numbers) but those with the most *reactive* followers who are hired and obtain positive profits in equilibrium. Those influencers reach a large audience relative to their costs, and are thus preferred by the firm. However, we also show that, if we assume that influencers are able to shape some consumers' beliefs and, additionally, followers of micro-influencers (in terms of reactive followers) react (weakly) more strongly to exaggerated posts (i.e. reputational costs per follower are larger), then micro-influencers will exaggerate less, making them better for naïve consumers.

Finally, we illustrate in the case of two influencers that the firm may benefit from committing to hire fewer influencers: The influencer who can generate the most profit benefits from the second influencer being hired as well if both are essential. In this case, hiring both influencers maximizes industry profits and therefore also yields a larger marginal contribution of the first influencer. In turn, this lowers the firm's profits, which thus would benefit from committing to hire only one influencer.

Related literature. Our paper contributes to the recent and growing literature on marketing in social networks. Galeotti and Goyal (2009) have been one of the first to investigate the effects of social networks on firm profits. Subsequently, many papers have studied monopolistic pricing in social networks (Candogan et al., 2012; Bloch and Quérou, 2013; Campbell, 2013; Chatterjee and Dutta, 2016; Fainmesser and Galeotti, 2016; Leduc et al., 2017; Ajorlou et al., 2018). Furthermore, Bimpikis et al. (2016) and Goyal et al. (2019) have studied competitive targeted advertising in a contagion model. Our approach differs from these papers in that we introduce intermediaries, the social media influencers, which the monopolist can hire in order for them to inform (and potentially persuade) consumers in a social network.

This relates our paper to contemporary work by Cong and Li (2023), who consider an influencer economy in which two influencers first choose their type (style, status, etc.), are then matched with a firm and subsequently compete for consumers to follow them, in which case they also consume the firm's product. They show that endogenous pluralism in influencers' types prevents market concentration and substitutes horizontal but not necessarily vertical product differentiation. In contrast, we abstract from differences in influencer type and build a general model of Bertrand competition between many influencers. In particular, we allow the firm to hire multiple influencers, while consumers are organized in a social network and may be persuaded by influencers' posts.

In another closely related paper, Fainmesser and Galeotti (2021) consider many marketers and many influencers. They focus on the trade-off between paid and organic endorsements and show that policies that make paid endorsements more transparent may have negative welfare effects, whereas better search technology that matches followers to influencers has positive welfare effects.

Another strand of the literature focusses on a single influencer's choice between the best advice and the most revenue (Mitchell, 2021; Pei and Mayzlin, 2022). Nistor and Selove (2023) additionally assume that purchasers can leave comments for future consumers. They derive conditions under which macro-influencers endorse all products, whereas micro-influencers endorse only high-quality products. Furthermore, Jain and Qian (2021) and Bhargava (2022) focus on the relationship between influencers and platforms, investigating how advertising revenue is shared between them.

Finally, social media influencers can be interpreted as media outlets, which relates our paper to the literature on the political economy of mass media and media bias (Prat and Strömberg, 2013; Anderson et al., 2015). Similarly to Gentzkow and Shapiro (2006) and Foerster (2023), misrepresenting information comes with reputational costs in one interpretation of our model. Different to these papers, however, we explicitly model a firm that sells a product to consumers and focus on how the firm strategically uses influencers to market the product.

The rest of the paper is organized as follows. In Section 2 we present the model and notation. We discuss the micro-foundation with respect to consumer behavior and word-of-mouth communication in Section 2.1 and 2.2, respectively. Section 3 solves the game backwards. We first determine firm behavior in Section 3.1 and then influencer pricing in Section 3.2. We discuss the trade-off between macro- and micro-influencers in Section 3.3 and illustrate the results in case of two influencers in Section 3.4. Section 4 concludes.

2 Model

We consider a monopolistic *firm* f, a finite set of *social media influencers* $\mathcal{M}$, with $|\mathcal{M}| = m \geq 2$, and a continuous set of consumers $N = [0, 1]$. The firm sells one durable good with quality $\theta \in \Theta = \mathbb{R}_{>0}$, which we call *innovation*. The per unit cost of the innovation is normalized to zero.

Influencers can contribute to sales by advertising the product and they offer that service to the firm. We assume that influencers $i \in \mathcal{M}$ are characterized by their share of *followers* $\eta_i \in (0, 1)$, which may reflect, e.g. the attractiveness of the influencers' content. Each consumer follows influencer i with probability η_i, independently from other consumers and across influencers. Thereby, a subset $M \subseteq \mathcal{M}$ of social media influencers has a *total share of followers*

$$\bar{\eta}(M) \equiv 1 - \prod_{i \in M} (1 - \eta_i). \tag{1}$$

Influencers can advertise the product by publishing a *story* $\tilde{\theta}$. A story $\tilde{\theta} > \theta$ will increase consumers' willingness to pay (explained below) but comes at an increasing and convex effort (and/or reputational) cost. We assume that for any influencer i the total cost of story $\tilde{\theta}$, $c_\theta(\tilde{\theta}, \eta_i)$, consists of a fixed cost $c_0 > 0$ and a separable function of $\tilde{\theta} - \theta$ and her share of followers η_i. We therefore write $c_\theta(\tilde{\theta}, \eta_i) = c_0 + c_1(\tilde{\theta} - \theta) c_2(\eta_i)$, where c_1 and c_2 are twice continuously differentiable with $c_1(0) = 0$, $c_1'(0) = 0$, $c_1'' > 0$, $c_2(\eta_i) > 0$ for all $\eta_i > 0$ and $c_2' \geq 0$. We may interpret a story $\tilde{\theta} > \theta$ as the influencer being able to either shape some consumers' beliefs about the product (with $\tilde{\theta} - \theta$ being the level of exaggeration), which then may result in reputational costs. Alternatively, a story $\tilde{\theta} > \theta$ may create added value $\tilde{\theta} - \theta$ for at least some consumers at an effort cost. To illustrate our model, we will sometimes use costs that are quadratic in the level of exaggeration and linear in the number of followers, $c_\theta(\tilde{\theta}, \eta_i) = c_0 + \alpha \frac{\eta_i}{2}(\tilde{\theta} - \theta)^2$ with $\alpha > 0$.

Influencers and firm engage in a two-stage game. In the first stage, both the firm and the influencers learn the quality θ and influencers then simultaneously determine their pricing policies $q_i : [\theta, +\infty) \to \mathbb{R}_+ \cup \{+\infty\}$, where $q_i(\tilde{\theta})$ is the price the firm has to pay influencer $i \in \mathcal{M}$ to use the influencer to deliver the story $\tilde{\theta} \geq \theta$. Note first that we thus impose a lower bound on the story, which may reflect that the firm has no incentives to downplay the true quality or subtract value. Second, influencers may set an infinite price for many (perhaps even all but one) stories, in which case they would not be willing to transmit these stories. Denote the set of possible pricing policies, i.e. the strategy set for each $i \in \mathcal{M}$, by $Q_i(\theta)$.

In the second stage, the firm observes the pricing policies $q_\mathcal{M} = (q_1, \ldots, q_m)$ and, then, sets a price $p \geq 0$ and chooses which influencers to use, $M \subseteq \mathcal{M}$, as well as the story $\tilde{\theta} \geq \theta$ to deliver to their followers, i.e. the firm's strategy is a function

$$q_f = (p_f, M_f, \tilde{\theta}) : Q_\mathcal{M}(\theta) \to \mathbb{R}_+ \times 2^M \times [\theta, +\infty),$$

where $Q_\mathcal{M}(\theta) = \prod_{i \in \mathcal{M}} Q_i(\theta)$ denotes the set of all possible pricing policies and 2^M denotes the power set of M. Denote the firm's strategy set by $Q_f(\theta)$. Whenever the reference is clear we will subsequently drop dependence of the strategies on θ.

Given influencers' pricing strategies $q_\mathcal{M} \in Q_\mathcal{M}$ and the firm's strategy $q_f = (p_f, M_f, \tilde{\theta}_f) \in Q_f$ the profits of each influencer $i \in \mathcal{M}$ are given by

$$\pi_i\left(q_\mathcal{M}, q_f\right) = 1_{\{i \in M_f(q_\mathcal{M})\}} \left(q_i(\tilde{\theta}_f(q_\mathcal{M})) - c_\theta(\tilde{\theta}_f(q_\mathcal{M}), \eta_i) \right). \tag{2}$$

Profits of the firm depend on generated sales. We assume that by choosing price p, the set of influencers M that reaches $\bar{\eta}(M)$ followers, and story $\tilde{\theta}$ in state θ yields the following demand:

$$G(p, M, \tilde{\theta}|\theta) = 1_{\{p \leq \theta\}} g_1(p, \bar{\eta}(M), \tilde{\theta}|\theta) + 1_{\{p > \theta\}} g_2(p, \bar{\eta}(M), \tilde{\theta}|\theta), \tag{3}$$

where, for fixed quality θ, $g_1, g_2 : \mathbb{R}_+ \times [0,1] \times [\theta, +\infty) \to [0,1]$ assign a share of buyers to each price p, share of followers $\bar{\eta}(M)$ generated by the influencers M, and story $\tilde{\theta}$, such that g_1 is the demand if the price is below or equal to the true quality θ, and g_2 is the demand if the price is strictly above θ. We further impose the following assumptions on g_1 and g_2:

Assumption 1. *Let g_1 and g_2 be twice almost everywhere differentiable for fixed θ, and twice almost everywhere on $\theta \leq \tilde{\theta}$ differentiable functions of θ.*

(i) a. $\frac{\partial}{\partial p} g_1 < 0$ *if* $p \in [0, \theta]$ *and either* $\frac{\partial^2}{\partial p^2} g_1 < -\frac{2}{p} \frac{\partial}{\partial p} g_1$ *for all* $p \in [0, \theta]$ *or there exists* $\bar{p} \in (0, \theta)$ *such that* $\frac{\partial^2}{\partial p^2} g_1 < -\frac{2}{p} \frac{\partial}{\partial p} g_1$ *if and only if* $p < \bar{p}$*,*

b. $\frac{\partial}{\partial p} g_2 < 0$ *and* $\frac{\partial^2}{\partial p^2} g_2 \leq 0$ *if* $p \in [\theta, \tilde{\theta}]$ *and* $\bar{\eta} > 0$*,*

(ii) $\frac{\partial}{\partial \tilde{\theta}} g_k > (=)\ 0$ *and* $\frac{\partial^2}{\partial \tilde{\theta}^2} g_k < (=)\ 0$ *if* $\bar{\eta} > (=)\ 0$ *for* $k = 1, 2$*,*

(iii) $\frac{\partial}{\partial \bar{\eta}} g_k > 0$ *and* $\frac{\partial^2}{\partial \bar{\eta}^2} g_k \leq 0$ *for* $k = 1, 2$*,*

(iv) $\frac{\partial}{\partial \theta} g_1 > 0$ *and* $\frac{\partial}{\partial \theta} g_2 = 0$*,*

(v) $g_1 = g_2$ *and* $\frac{\partial}{\partial p} g_1 < \frac{\partial}{\partial p} g_2$ *if* $p = \theta$*, and*

(vi) $g_2 = 0$ *if* $\bar{\eta} = 0$ *or* $p \geq \tilde{\theta}$*.*

First, demand is decreasing in the price p and not "too convex" (such that the second-order condition holds up to some point $\bar{p}$). Second, demand is (strictly) increasing and (strictly) concave in the story $\tilde{\theta}$ (conditional on the firm hiring influencers), which reflects that influencers can increase consumers' willingness to pay with their stories. Third, demand is strictly increasing and concave in the number of followers $\bar{\eta}$ that the hired influencers reach with their story, which reflects that consumers may not be aware of the product and have limited knowledge about the true quality but can be informed and persuaded by the influencers.

Fourth, demand is increasing in the quality θ if and only if $p \leq \theta$. Fifth, demand is continuous but not differentiable at $p = \theta$. Sixth, positive demand at price $p > \theta$ requires influencers. The last three points reflect that consumers' willingness to pay for the product *per se* is at most equal to the true quality θ, such that selling at $p > \theta$ requires influencers to persuade consumers' into buying the product.

We will discuss one possible micro-foundation of the assumptions on the demand function (3) based on the interpretation that influencers are able to shape consumers' beliefs in the subsequent Section 2.1.

Given the influencers' pricing strategies $q_{\mathcal{M}}$ and the firm's strategy $q_f = (p_f, M_f, \tilde{\theta}_f)$, the profits of the firm f are given by the total revenue net of prices charged by the hired influencers charge,

$$\pi_f (q_{\mathcal{M}}, q_f) = p_f(q_{\mathcal{M}}) G\left(p_f(q_{\mathcal{M}}), M_f(q_{\mathcal{M}}), \tilde{\theta}_f(q_{\mathcal{M}}) | \theta\right) - \sum_{i \in M_f(q_{\mathcal{M}})} q_i \left(\tilde{\theta}_f(q_{\mathcal{M}})\right). \quad (4)$$

To summarize, the timing of events is as follows:

1. Both the firm and influencers learn the true quality θ.

2. Influencers simultaneously choose their pricing policies $q_{\mathcal{M}} = (q_1, \ldots, q_m)$.

3. After observing $q_{\mathcal{M}}$, the firm sets a price $p \geq 0$ and chooses the set of influencers to use $M \subseteq \mathcal{M}$ and the story $\tilde{\theta} \geq \theta$ to deliver.

4. Payoffs realize.

We will study subgame perfect Nash equilibria (SPE) of the game. Due to multiplicity of equilibria, we will restrict attention to SPEs in pure strategies which are also strong Nash equilibria. We apply the following notion of strong Nash equilibrium:

Definition 1. *A profile of pricing policies and firm strategy* $(q^*_{\mathcal{M}}, q^*_f)$ *is a* strong Nash equilibrium*, if and only if for all* $J \subset M \cup \{f\}$ *and all* $q'_J \in \prod_{j \in J} Q_j \backslash q^*_J$ *there exists* $j' \in J$ *such that* $\pi_{j'}(q'_J, q^*_{-J}) \leq \pi_{j'}(q^*)$ *(where* $q^*_{-J} = (q^*_k)_{k \notin J}$*).*

In what follows we denote by a strong subgame-perfect Nash equilibrium (SSPE) a strategy profile which is both a strong Nash equilibrium and subgame perfect.

2.1 Micro-foundation 1: Consumer behavior

So far we have modelled consumer behavior only indirectly via the structure of the demand function (3) and Assumption 1. As we will see, this general approach is sufficient to derive the key results in this paper. One possible micro-foundation giving rise to such a structure of the demand function is based on the interpretation that influencers are able to shape consumers' beliefs.

Suppose that initially only a share of consumers $\mu \in (0, 1)$ is aware of the innovation, while no consumer has any information about its quality. Consumers follow social media influencers who may make their followers aware of the innovation and post a story $\tilde{\theta} \geq \theta$ that is supposed to reflect the true quality of the innovation. After becoming aware of the product, a share of consumers $\rho \in [0, 1]$ is able to gather costless and accurate information about its quality through an external source (e.g. product reviews). These consumers will thus form the correct belief $\hat{\theta} = \theta$ once becoming aware of the innovation, and are henceforth called *sophisticated consumers*. The consumers who do not have access to this source, henceforth *naïve consumers*, form the pessimistic belief $\hat{\theta}(\emptyset) = 0$ if they do not get to read a story but upon reading story $\tilde{\theta}$ take it at face value and form the belief $\hat{\theta}(\tilde{\theta}) = \tilde{\theta}$.

Each consumer $i \in N$ has valuation $v_i(\theta) = i \cdot \theta$ for a product of quality θ, has unit demand, and is myopic, i.e. she purchases the product if her expected valuation exceeds the price, $E[v_i(\theta)|\hat{\theta}] = v_i(\hat{\theta}) \geq p$. After purchase, naïve consumers learn the true quality with probability $\lambda \in [0, 1]$ and otherwise do not update their belief. All consumers use word-of-mouth communication to make others aware of the product and communicate their belief. Hence, a sophisticated consumer i buys the product if and only if she becomes aware of the product (either initially, through an influencer, or by word-of-mouth communication) and $v_i(\theta) \geq p$, while a naïve consumer i buys the product if and only if she becomes aware and reads at least one story $\tilde{\theta}$ such that $v_i(\tilde{\theta}) \geq p$ (either through an influencer, or by word-of-mouth communication).[4]

Optimal consumer behavior then gives rise to a demand function as in (3) and satisfies Assumption 1.

2.2 Micro-foundation 2: Word-of-mouth communication

In a second step, we now model word-of-mouth communication explicitly. Suppose that there are two periods and that consumer behavior is as described in Section 2.1 for $\lambda = 1$, i.e. after purchase naïve consumers learn the true quality; we provide a generalization to imperfect learning in Appendix B.

Consumers are further organized in a social network à la Galeotti and Goyal (2009): each consumer observes k other consumers with probability $P(k) \geq 0$, where $k \in \{1, 2, \ldots, \bar{k}\}$ and $\sum_{k=1}^{\bar{k}} P(k) = 1$. A consumer with degree k makes k independent and uniformly distributed draws from N.[5] Consumers' degrees are independently distributed. Hence, following a standard "abuse" of the law of large numbers, there is a fraction $P(k)$ of consumers with degree k. The firm knows the degree distribution P but not the actual network.

In the first period, a share of individuals is aware of the product either because they are followers or because they were initially aware of the product. From our

[4] Note that this assumption is for simplicity and only affects demand quantitatively. Any other rule would also work, e.g. that all messages $\tilde{\theta}$ that a naïve consumer i has received must be such that $v_i(\tilde{\theta}) \geq p$.

[5] Notice that the probability to draw the same consumer twice is zero, as there is a continuum of consumers.

independence assumption, the total share of consumers who are aware of the innovation in the first period can be calculated to be $1-(1-\bar{\eta})(1-\mu)$. Sophisticated consumers buy the product if and only if they are aware of it and $v_i(\theta) \geq p$, while naïve consumers do so if and only if they follow an influencer and $v_i(\tilde{\theta}) \geq p$. The total share of buyers in the first period is therefore

$$G_1^P(p, M, \tilde{\theta}|\theta) = (1-\rho)\bar{\eta} \max\left\{1 - \frac{p}{\tilde{\theta}}, 0\right\} + \rho\left(1-(1-\bar{\eta})(1-\mu)\right) \max\left\{1 - \frac{p}{\theta}, 0\right\}.$$

In the second period, consumers communicate with their neighbors in the network. All consumers who purchased the product in the first period make others aware and communicate the true quality of the product. Each naïve consumer who does not follow an influencer and each sophisticated consumer who was not aware of the product, i.e. the share

$$(1-\bar{\eta})(1-\rho) + (1-\bar{\eta})\rho(1-\mu) = (1-\bar{\eta})(1-\rho\mu),$$

makes k observations with probability $P(k)$ in the second period and ends up buying if she is informed by one of the k observed consumers and $v_i(\theta) \geq p$. Note that the probability that none of the k observed consumers informs her of the innovation is given by

$$\left(1 - G_1^P(p, M, \tilde{\theta}|\theta)\right)^k.$$

Thus, demand in period 2 is given by

$$G_2^P(p, M, \tilde{\theta}|\theta) = \max\left\{1 - \frac{p}{\theta}, 0\right\}(1-\bar{\eta})(1-\rho\mu)\left(1 - \sum_k P(k)\left(1 - G_1^P(p, M, \tilde{\theta}|\theta)\right)^k\right).$$

Total demand G^P then is given by (3), with

$$\begin{aligned} g_1^P(p, \bar{\eta}, \tilde{\theta}|\theta) &= \left(1 - \frac{p}{\theta}\right)\left(\rho + (1-\bar{\eta})\left(1 - \rho - (1-\mu\rho)\cdot\right.\right. \\ &\qquad \left.\left.\sum_k P(k)\left(1 - G_1^P(p, M, \tilde{\theta}|\theta)\right)^k\right)\right) + \left(1 - \frac{p}{\tilde{\theta}}\right)(1-\rho)\bar{\eta}, \\ g_2^P(p, \bar{\eta}, \tilde{\theta}|\theta) &= \max\left\{1 - \frac{p}{\tilde{\theta}}, 0\right\}(1-\rho)\bar{\eta}, \end{aligned}$$

see Figure 2 for an illustration.

Finally, we derive a sufficient condition under which g_1^P and g_2^P satisfy Assumption 1 for $p \leq \theta$ and $\theta < p \leq \tilde{\theta}$, respectively. While most properties in Assumption 1 are straightforward to verify, the assumption on the second-order condition in

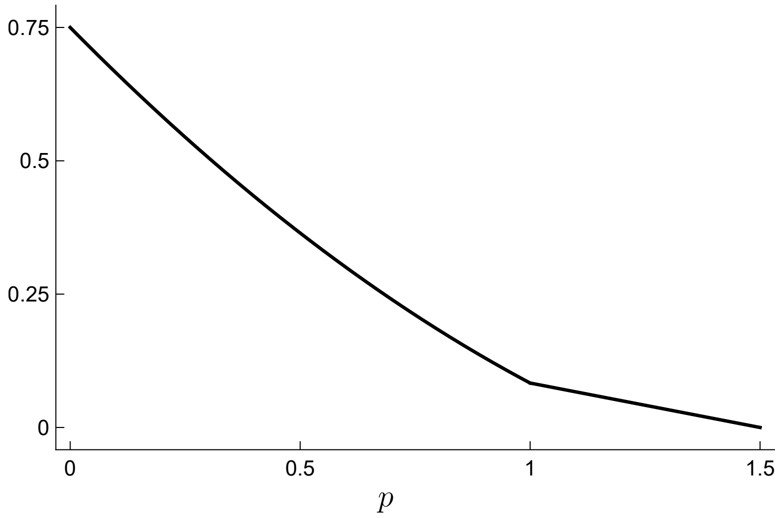

Figure 2: Demand G^P for $\mu = 0$, $\bar{\eta} = \rho = \frac{1}{2}$, $\bar{k} = 1$, and $\tilde{\theta} = 1.5 > \theta = 1$.

part (i) does not necessarily hold. We show that it holds if the upper bound on consumers' degrees is at most three:

Lemma 1. *g_1^P and g_2^P fulfill Assumption 1 if P is such that $\bar{k} \in \{1, 2, 3\}$.*

All proofs are relegated to Appendix A. We have further verified numerically that the assumption on the second-order condition in Assumption 1 (i) holds on a wide range of parameters beyond Lemma 1.[6] We did not find a single instance where the assumption does not hold.

3 Equilibrium analysis

We denote *industry profits* under strategy profile $(q_{\mathcal{M}}, q_f)$ by

$$\begin{aligned} \Pi(q_{\mathcal{M}}, q_f) &\equiv \sum_{i \in \mathcal{M} \cup \{f\}} \pi_i(q_{\mathcal{M}}, q_f) \\ &= p_f(q_{\mathcal{M}}) G(p_f(q_{\mathcal{M}}), M_f(q_{\mathcal{M}}), \tilde{\theta}_f(q_{\mathcal{M}}) | \theta) - \sum_{i \in M_f(q_{\mathcal{M}})} c_\theta(\tilde{\theta}_f(q_{\mathcal{M}}), \eta_i). \end{aligned} \tag{5}$$

[6]We have checked the second-order condition on a grid of values for θ, $\tilde{\theta}$, ρ, μ, $\bar{\eta}$ and $\bar{k}$ and for two distributions P. The grid size was such that $\theta \in (0, 3)$, $\tilde{\theta} \in (\theta, \theta + 3)$, $\rho \in (0, 1)$, $\mu \in (0, 1)$, $\bar{\eta} \in (0, 1)$ (with increments of 0.2 in the first two cases and of 0.1 else) and $\bar{k} = 4, 5, \ldots, 30$. P was such that either $P(k) = \frac{1}{k}$ for all $k = 1, 2, \ldots \bar{k}$ or $P(\bar{k}) = 1$.

Note that (5) depends only indirectly on influencers' pricing $q_{\mathcal{M}}$, through the firm's decision $q_f = (p_f, M_f, \tilde{\theta}_f)$. We will hence sometimes omit the dependence of $\Pi(q_{\mathcal{M}}, q_f)$ on $q_{\mathcal{M}}$. We first establish that any SSPE is such that industry profits are maximized.

Lemma 2. *Let $(q^*_{\mathcal{M}}, q^*_f)$ be a SSPE. Then there does not exist $(q'_{\mathcal{M}}, q'_f) \in Q_{\mathcal{M}} \times Q_f$ such that $\Pi(q'_{\mathcal{M}}, q'_f) > \Pi(q^*_{\mathcal{M}}, q^*_f)$.*

Lemma 2 implies that any SSPE is efficient in the sense that it maximizes industry profits. Note that industry profits only depend on the realized choices of the firm, while influencer pricing determines how the pie is divided among the firm and the chosen influencers. If the resulting choices of the firm were not efficient, then we could always find coalitional deviations that benefit all members of the coalition strictly. In what follows, we solve the game backwards and first determine the firm's equilibrium behavior. We then solve for the influencers' equilibrium pricing.

3.1 Firm behavior

In principle, the firm maximizes its profits by choosing the set of influencers to hire M_f, the story $\tilde{\theta}_f$, and the price p_f given the influencers' pricing policies $q_{\mathcal{M}}$. However, since industry profits (5) only depend on the firm's realized choices and are maximized in equilibrium by Lemma 2, the firm's equilibrium behavior $q_f = (p_f, M_f, \tilde{\theta}_f)$ will maximize (5).

3.1.1 The optimal price

Since demand $G(p, M, \tilde{\theta}|\theta)$ is given by (3) and the state θ is known to the firm, Assumption 1 implies that there is a unique optimal price conditional on the story and the set of influencers.

Lemma 3. *Generically, there is a unique price $p^*(\tilde{\theta}, M|\theta)$ that maximizes industry profits (5) conditional on choosing story $\tilde{\theta}$ and the set of influencers M given state θ. If moreover $\frac{\partial^2}{\partial p \partial \theta} g_1 \geq 0$ and $\frac{\partial^2}{\partial p \partial \tilde{\theta}} g_1 \geq 0$, then $\frac{\partial}{\partial \tilde{\theta}} p^*(\tilde{\theta}, M|\theta) \geq (>) 0$ (if $\bar{\eta}(M) > 0$) and $\frac{\partial}{\partial \theta} p^*(\tilde{\theta}, M|\theta) \geq (>) 0$ (if $p^*(\tilde{\theta}, M|\theta) < \theta$).*

We henceforth assume that the firm chooses the smallest optimal price in non-generic cases.

3.1.2 The optimal story

We next determine the optimal story conditional on the set of influencers. Suppose first that no influencer is being hired, $M = \emptyset$. Then demand is independent of the story $\tilde{\theta}$ by Assumption 1 (ii), such that any story is optimal. For any choice of the set of influencers $M \neq \emptyset$, we now determine the story that maximizes industry profits (5) given the optimal price $p^*(\tilde{\theta}, M|\theta)$ (Lemma 3).

Proposition 1. *There is a unique story $\tilde{\theta}^*(M|\theta) > \theta$ that maximizes industry profits conditional on choosing $M \neq \emptyset$ and $p^*(\tilde{\theta}, M|\theta)$ given state θ. If moreover $\frac{\partial^2}{\partial\theta\partial\tilde{\theta}} g_1 \geq 0$, then $\frac{\partial\tilde{\theta}^*(M|\theta)}{\partial\theta} > 0$.*

Proposition 1 shows that influencers generally increase consumers' willingness to pay in equilibrium, and that higher product quality yields a "higher" story that is associated with higher willingness to pay. Furthermore, if product quality is low, the firm even "overprices" the product, selling only to consumers who are responsive to the story.

Corollary 1. *For each $M \neq \emptyset$, there exists $\bar{\theta}(M) > 0$ such that $p^*(\tilde{\theta}^*(M|\theta), M|\theta) > \theta$ if $\theta < \bar{\theta}(M)$.*

3.1.3 The optimal choice of influencers

Finally, it remains to determine the choice of influencers which yield maximal industry profits. By Lemma 3 and Proposition 1, the optimal message $\tilde{\theta}^*(M|\theta)$ and price $p^*(\tilde{\theta}^*(M|\theta), M|\theta)$ are (generically) unique for any choice of influencers $M \subseteq \mathcal{M}$. Thus, we can henceforth write $\tilde{\theta}^*_M \equiv \tilde{\theta}^*(M|\theta)$ and $p^*_M \equiv p^*(\tilde{\theta}^*_M, M|\theta)$; note in particular that p^*_M is optimal given that the message is chosen optimally, $\tilde{\theta} = \tilde{\theta}^*_M$. By (5), maximal industry profits given the choice of influencers M then are

$$\Pi_M \equiv p^*_M G(p^*_M, M, \tilde{\theta}^*_M|\theta) - \sum_{i \in M} c_\theta(\tilde{\theta}^*_M, \eta_i).$$

For a general cost function, there is no simple relation between the optimal choice of influencers and initial conditions, i.e. number of followers of each influencer. To ease the exposition, we henceforth order influencers according to their "stand-alone" profit:

Assumption 2. $\Pi_1 \geq \Pi_2 \geq \ldots \geq \Pi_m$.

It seems natural that the ranking of the influencers according to their stand-alone profit coincides with that according to their number of followers. As we now show, this will hold in our model if influencers' effort costs do not increase too rapidly in the number of followers.

Lemma 4. *There exists $\varepsilon^* > 0$ such that if*

$$c_2'(\eta) < \varepsilon^* \text{ for all } \eta \in (0,1), \tag{C1}$$

we have $\Pi_{M\cup\{i\}} \geq \Pi_{M\cup\{j\}}$ if and only if $\eta_i \geq \eta_j$ for all $M \subset \mathcal{M}\backslash\{i,j\}$.

Clearly, influencers with a higher number of followers being able to generate higher industry profits requires that effort costs are not exploding for large influencers compared to their smaller counterparts. This seems a weak and natural assumption, and one may even argue—at least in the interpretation of our model in which influencers create added value with their stories—that the effort necessary for creating a certain story is independent of follower numbers. However, for most results we will not require Condition (C1). We immediately get the desired result for the stand-alone values by setting $M = \emptyset$ in Lemma 4:

Remark 1. *Suppose that Condition* (C1) *holds. Then $\Pi_i \geq \Pi_j$ if and only if $\eta_i \geq \eta_j$.*

We next introduce the notion of an essential influencer:

Definition 2. *An influencer $i \in \mathcal{M}$ is* essential *(to maximizing industry profit) if $M \in \operatorname{argmax}_{M'\subseteq\mathcal{M}} \Pi_{M'}$ only if $i \in M$. Let M^e denote the set of essential influencers.*

In a first step, we derive conditions under which the influencers with the highest stand-alone profit are essential. Let $M^k \equiv \{1, 2, ..., k\}$ for any $k \in \mathcal{M}$ and $M^0 \equiv \emptyset$.

Proposition 2. *$M^e = M^{k^*-l(\eta_{k^*+1})}$ if either $m = 2$ or Condition* (C1) *holds, where*

$$k^* \equiv \min \operatorname*{argmax}_{k\in\mathcal{M}\cup\{0\}} \Pi_{M^k} \text{ and } l(\eta_{k^*+1}) \equiv |\{i \in M^{k^*} : \eta_i = \eta_{k^*+1}\}|.$$

3.2 Influencer pricing

Having determined firm behavior in Section 3.1, we now consider the influencers' pricing. Given a quality θ, influencers will charge maximal prices under the two conditions that they will be hired and that they obtain non-negative payoffs. Suppose that influencers $M \subseteq \mathcal{M}$ are hired, and hence $p = p_M^*$ and $\tilde{\theta} = \tilde{\theta}_M^*$. Then each influencer $i \in M$ can charge at most her marginal contribution to industry profits, i.e.

$$q_i(\tilde{\theta}^*_M) \leq \Pi_M - \max_{M' \subseteq \mathcal{M} \setminus \{i\}} \Pi_{M'} + c_\theta(\tilde{\theta}^*_M, \eta_i).$$

It turns out that each hired influencer indeed obtains her marginal contribution in equilibrium if all influencers play weakly undominated strategies, i.e. do not price below marginal cost:

Lemma 5. *In any undominated SSPE in which the firm chooses M^*, each influencer $i \in M^*$ receives a payoff of*

$$\pi^*_i = \bar{\pi}_i(M^*) \equiv \Pi_{M^*} - \max_{M' \subseteq \mathcal{M} \setminus \{i\}} \Pi_{M'}.$$

Second, it follows from Lemma 2 and Lemma 5 that only essential influencers obtain strictly positive profits in equilibrium. This yields the following equilibrium characterization:

Theorem 1. *Any undominated SSPE is such that $M^* \in \operatorname{argmax}_{M \subseteq \mathcal{M}} \Pi_M$ and yields the same payoffs $\pi^*_i = \bar{\pi}_i(M^*) > 0$ if $i \in M^e$, $\pi^*_i = 0$ otherwise, and $\pi^*_f = \Pi_{M^*} - \sum_{i \in M^e} \bar{\pi}_i(M^*)$. Each influencer $i \in M^*$ charges price*

$$q^*_i(\tilde{\theta}_{M^*}) = \Pi_{M^*} - \max_{M' \subseteq \mathcal{M} \setminus \{i\}} \Pi_{M'} + c_\theta(\tilde{\theta}_{M^*}, \eta_i).$$

So, in equilibrium the firm chooses influencers as to maximize industry profits, while influencers price according to their marginal contribution and obtain strictly positive profits if and only if they are essential.

3.3 Macro- and micro-influencers

We next compare macro-influencers with many followers to micro-influencers with few followers each. Note that in our model these terms do not refer to the raw number of followers but rather to the number of *reactive* followers. Recall from Remark 1 that Condition (C1) implies $\eta_1 \geq \eta_2 \geq \ldots \geq \eta_m$. We can thus interpret influencers with a low index as macro-influencers and those with a high index as micro-influencers. It then follows from Proposition 2 and Theorem 1 that it is macro-influencers who obtain positive profits in equilibrium:

Corollary 2. *Suppose that Condition* (C1) *holds. Any undominated SSPE is such that $\pi^*_i > 0$ if and only if $\eta_i \geq \underline{\eta} \equiv \eta_{\max M^e}$.*

Under Condition (C1), macro-influencers are simply more cost-effective, reaching a larger audience relative to costs.

Moreover, we can ask whether macro or micro-influencers are better for consumers given that they reach the same number of followers. In our model, consumers' willingness to pay and hence demand increases in the story $\tilde{\theta}$ that influencers deliver to their followers for a given price and product quality. Suppose that this occurs because influencers are able to shape naïve consumers' beliefs about the product as described in our micro-foundation in Section 2.1. If additionally reputational costs are (weakly) concave in the number of followers, i.e. *per follower* larger for micro- than for macro-influencers, then micro-influencers will exaggerate less than macro-influencers, making them better for naïve consumers:

Proposition 3. *Suppose that Condition* (C1) *holds and that* $c_2''(\eta) \leq 0$ *for all* $\eta \in (0,1)$*, such that the firm chooses* $M^* = M^{k^*}$ *in any undominated SSPE. If* $M' \subseteq \mathcal{M} \backslash M^*$ *is such that* $\bar{\eta}(M') = \bar{\eta}(M^*)$*, then* $\tilde{\theta}^*_{M'} < \tilde{\theta}^*_{M^*}$*.*

3.4 Two influencers

To derive more concrete results we now consider two influencers, $m = 2$. Recall first from Proposition 2 that in this case the influencers with the highest stand-alone profit are essential, in particular $M^e \neq \{2\}$. Second, Assumption 1 (iii) implies that an influencer's marginal contribution is decreasing in the number of followers that is already reached:

Remark 2. $\Pi_i - \Pi_\emptyset > \Pi_{\mathcal{M}} - \Pi_j$ *for all distinct* $i, j \in \mathcal{M}$*.*

We can now establish that:

Corollary 3. *Suppose that* $m = 2$*. Any undominated SSPE yields payoffs*

(i) $\pi_1^* = \Pi_{\mathcal{M}} - \Pi_2$*,* $\pi_2^* = \Pi_{\mathcal{M}} - \Pi_1$*, and* $\pi_f^* = \Pi_1 + \Pi_2 - \Pi_{\mathcal{M}}$ *if* $\Pi_{\mathcal{M}} > \Pi_1$*,*

(ii) $\pi_1^* = \Pi_1 - \max\{\Pi_2, \Pi_\emptyset\}$*,* $\pi_2^* = 0$*, and* $\pi_f^* = \max\{\Pi_2, \Pi_\emptyset\}$ *if* $\Pi_1 - \Pi_\emptyset > 0 \geq \Pi_{\mathcal{M}} - \Pi_1$*,*

(iii) $\pi_1^* = \pi_2^* = 0$*,* $\pi_f^* = \Pi_\emptyset$ *if* $\Pi_\emptyset \geq \Pi_1$*.*

Note that we have used that by Remark 2 $\Pi_{\mathcal{M}} > \Pi_1$ implies $\Pi_2 > \Pi_\emptyset$. This further yields:

Proposition 4. *Suppose that* $\Pi_{\mathcal{M}} > \Pi_1$*, such that* $M^e = \mathcal{M}$*.*

(i) Influencer 1 benefits from influencer 2 being hired as well, as $\Pi_{\mathcal{M}} - \Pi_2 > \Pi_1 - \max\{\Pi_2, \Pi_{\emptyset}\}$*.*

(ii) The firm would benefit from committing to hire only one influencer, as $\Pi_1 + \Pi_2 - \Pi_{\mathcal{M}} < \max\{\Pi_2, \Pi_{\emptyset}\}$*.*

4 Concluding remarks

We have proposed a model of social media influencer marketing to investigate the industrial organization of the influencer economy. A firm may hire influencers to inform consumers about an innovation. After influencers have set prices for their endorsements, the firm decides which influencers to hire, which story to convey via the influencers, and sets the retail price of the innovation. In equilibrium, influencers price according to their marginal contribution to industry profits and generally increase consumers' willingness to pay. In particular, it is the influencers with the most reactive followers who are hired and obtain positive profits in equilibrium if costs do not increase too rapidly in the number of followers. Finally, we have shown that the firm may be better off if it could commit to hire fewer influencers.

We have presented two possible interpretations of the assumed structure of the cost and demand functions: The first is that influencers are able to shape (some) consumers' beliefs about the product. We have presented a micro-foundation where some individuals are sophisticated while others are naïve. Clearly, this assumption is extreme, and stronger than necessary since as long as influencers' story has a positive effect on consumers' beliefs, the main structure of the demand function will remain unchanged. The other interpretation is that influencers can exert effort to create added value to the product through their story, the degree of which can differ among consumers.

In the next step, we plan to use the framework to understand when and how the market for influencers should be regulated and to extend the framework to competition between multiple firms.

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A Appendix: Proofs

Proof of Lemma 1. The only property that requires proof is the second-order condition with respect to $H(p) \equiv pg_1^P(p, \bar{\eta}, \tilde{\theta}|\theta)$ in part (i), where $\bar{\eta} = \bar{\eta}(M)$. Let $p \leq \theta$ and

$$h(p) \equiv \rho + (1-\bar{\eta}) \left(1 - \rho - (1-\mu\rho) \sum_k P(k) \left(1 - G_1^P(p, M, \tilde{\theta}|\theta) \right)^k \right).$$

Then $H(p) = p\left(1 - \frac{p}{\theta}\right) h(p) + p\left(1 - \frac{p}{\tilde{\theta}}\right)(1-\rho)\bar{\eta}$ and $h^{(k)}(p) \leq 0$ for $k = 1, 2, 3$. We obtain

$$H''(p) < 0 \Leftrightarrow 2 \underbrace{\left(1 - \frac{2p}{\theta}\right)}_{>0 \Leftrightarrow p < \frac{\theta}{2}} \underbrace{h'(p)}_{\leq 0} + p\left(1 - \frac{p}{\theta}\right) \underbrace{h''(p)}_{\leq 0} < 2\left(\frac{1}{\theta} h(p) + \frac{1}{\tilde{\theta}}(1-\rho)\bar{\eta}\right). \quad (6)$$

Since the right-hand side of (6) is strictly positive, there exists $\bar{p} > \frac{\theta}{2}$ such that $H''(p) < 0$ for all $p < \bar{p}$. Furthermore, $H''(\bar{p}) = 0$ if $\bar{p} < \theta$.

The claim follows immediately if $\bar{p} = \theta$. Hence, suppose that $\bar{p} < \theta$. It is left to show that $H''(p) > 0$ for all $p > \bar{p}$ if P is such that $\bar{k} \in \{1, 2, 3\}$. Note first that $H''(p) < 0$ for all $p < \bar{p}$ and $H''(\bar{p}) = 0$ implies that there exists $\varepsilon_1 > 0$ such that $H'''(p) > 0$ for all $p \in (\bar{p} - \varepsilon_1, \bar{p})$. Second, we have $h^{(4)}(p) = 0$, which yields

$$H^{(4)}(p) = -\frac{10}{\theta} h''(p) + 2 \underbrace{\left(1 - \frac{2p+1}{\theta}\right)}_{<0 \text{ for } p > \frac{\theta}{2}} h'''(p).$$

Since $\bar{p} > \frac{\theta}{2}$, there exists $\varepsilon_2 > 0$ such that $H^{(4)}(p) \geq 0$ for all $p \geq \bar{p} - \varepsilon_2$. Hence, $H'''(p) > 0$ for all $p \geq \bar{p}$, which yields the claim. □

Proof of Lemma 2. Suppose to the contrary that $(q^*_{\mathcal{M}}, q^*_f)$ is a SSPE and there exists $(q'_{\mathcal{M}}, q'_f) \in Q_{\mathcal{M}} \times Q_f$ with $\Pi(q'_{\mathcal{M}}, q'_f) > \Pi(q^*_{\mathcal{M}}, q^*_f)$. Let $\Delta \equiv \Pi(q'_{\mathcal{M}}, q'_f) - \Pi(q^*_{\mathcal{M}}, q^*_f) > 0$ and let $p' = p(q'_{\mathcal{M}})$, $M' = M_f(q'_{\mathcal{M}})$, and $\tilde{\theta}' = \tilde{\theta}(q'_{\mathcal{M}})$. Denote by $q'_f = (p', M', \tilde{\theta}')$ the (constant) strategy of the firm making these choices independently of influencer pricing. Further let $q'_i(\tilde{\theta}) = q^*_i(\tilde{\theta}(q^*_{\mathcal{M}})) - c_\theta(\tilde{\theta}(q^*_{\mathcal{M}}), \eta_i) + c_\theta(\tilde{\theta}', \eta_i) + \frac{\Delta}{m+1}$ for all $\tilde{\theta} \geq \theta$ and $i \in M'$ be the constant pricing of influencers. Then clearly $\pi_i\left((q'_{M'}, q^*_{M\setminus\{M'\}}), q'_f\right) > \pi_i\left(q^*_M, q^*_f\right)$ for all $i \in M'$, as these influencers are chosen and charge higher prices net of costs. Further $\pi_f\left((q'_{M'}, q^*_{M\setminus\{M'\}}), q'_f\right) - \pi_f\left(q^*_M, q^*_f\right) = \Delta - |M'|\frac{\Delta}{m+1} > 0$ since $|M'| \leq m$. Hence the coalition $J = M' \cup \{f\}$ has a deviation that strictly benefits all of its members, contradicting that $(q^*_{\mathcal{M}}, q^*_f)$ is a SSPE. $\square$

Proof of Lemma 3. Suppose first that the optimal price p^* satisfies $p^* > \theta$, i.e. in particular p^* solves

$$\operatorname*{argmax}_{p>\theta} p g_2(p, \bar{\eta}, \tilde{\theta}|\theta),$$

where $\bar{\eta} = \bar{\eta}(M)$. Note that $p^* < \tilde{\theta}$ by Assumption 1 (vi). The solution is given by the first-order condition

$$g_2(p^*, \bar{\eta}, \tilde{\theta}|\theta) + p^* \frac{\partial}{\partial p} g_2(p^*, \bar{\eta}, \tilde{\theta}|\theta) = 0$$

and unique, since the second-order condition

$$2\frac{\partial}{\partial p} g_2(p, \bar{\eta}, \tilde{\theta}|\theta) + p\frac{\partial^2}{\partial p^2} g_2(p, \bar{\eta}, \tilde{\theta}|\theta) < 0$$

is satisfied for all $\tilde{\theta} > p > \theta$ by Assumption 1 (i). Second, suppose that $p^* \leq \theta$, i.e. in particular p^* solves

$$\operatorname*{argmax}_{p\in[0,\theta]} p g_1(p, \bar{\eta}, \tilde{\theta}|\theta).$$

Note that $p^* \neq 0$. Next, suppose that $p^* = \theta$. Then

$$g_1(p^*, \bar{\eta}, \tilde{\theta}|\theta) + p^* \frac{\partial}{\partial p} g_1(p^*, \bar{\eta}, \tilde{\theta}|\theta) \geq 0.$$

Assumption 1 (v) implies

$$g_2(p^*, \bar{\eta}, \tilde{\theta}|\theta) + p^* \frac{\partial}{\partial p} g_2(p^*, \bar{\eta}, \tilde{\theta}|\theta) > 0,$$

a contradiction. Thus, $p^* \in (0, \theta)$, such that the first-order condition

$$g_1(p^*, \bar{\eta}, \tilde{\theta}|\theta) + p^* \frac{\partial}{\partial p} g_1(p^*, \bar{\eta}, \tilde{\theta}|\theta) = 0 \tag{7}$$

and the second-order condition

$$2\frac{\partial}{\partial p} g_1(p^*, \bar{\eta}, \tilde{\theta}|\theta) + p^* \frac{\partial^2}{\partial p^2} g_1(p^*, \bar{\eta}, \tilde{\theta}|\theta) < 0 \tag{8}$$

must hold. Therefore, by Assumption 1 (i), $p^* < \bar{p}$ is unique. Altogether, the optimal price is generically unique and satisfies the respective first-order condition, which proves the first claim.

Suppose now that $\frac{\partial^2}{\partial p \partial \theta} g_1 \geq 0$ and $\frac{\partial^2}{\partial p \partial \tilde{\theta}} g_1 \geq 0$. For the second claim, consider $p^* < \theta$ ($p^* > \theta$ analogous), then there exists $\varepsilon > 0$ such that (8) still holds for $\tilde{\theta}' > \tilde{\theta}$ such that $|\tilde{\theta}' - \tilde{\theta}| < \varepsilon$. Assumption 1 (ii) and $\frac{\partial^2}{\partial p \partial \tilde{\theta}} g_1 \geq 0$ imply that the left-hand side of (7) is weakly (strictly if $\bar{\eta} > 0$) increasing in $\tilde{\theta}$. By (8), $\tilde{\theta}' > \tilde{\theta}$ yields an optimal price $p' \geq p^*$ ($> p^*$ if $\bar{\eta} > 0$).

The third claim follows analogously from Assumption 1 (iv) and $\frac{\partial^2}{\partial p \partial \theta} g_1 \geq 0$. $\square$

Proof of Proposition 1. Fix M and θ. The optimal message $\tilde{\theta}^*$ solves

$$\max_{\tilde{\theta}} \Pi(p^*(\tilde{\theta}, M|\theta), M, \tilde{\theta}) = \max_{\tilde{\theta}} p^*(\tilde{\theta}, M|\theta) G(p^*(\tilde{\theta}, M|\theta), M, \tilde{\theta}|\theta) - \sum_{i \in M} c_\theta(\tilde{\theta}, \eta_i).$$

In the following we drop the arguments of the functions to save notation. If $p^*(\tilde{\theta}^*(M|\theta), M) < \theta$ ($p^*(\tilde{\theta}^*(M|\theta), M) > \theta$ is analogous), the first-order condition is given by

$$\frac{\partial \Pi}{\partial \tilde{\theta}} = \frac{\partial p^*}{\partial \tilde{\theta}} \underbrace{\left(g_1 + p^* \frac{\partial g_1}{\partial p} \right)}_{=0} + p^* \frac{\partial g_1}{\partial \tilde{\theta}} - \sum_{i \in M} \frac{\partial c_\theta}{\partial \tilde{\theta}} = p^* \frac{\partial g_1}{\partial \tilde{\theta}} - \sum_{i \in M} \frac{\partial c_\theta}{\partial \tilde{\theta}} = 0, \tag{9}$$

where the second equality follows from the first-order condition (7). Note that Assumption 1 (ii) and $c'(0) = 0$ imply that (9) cannot hold for $\tilde{\theta}^*(M|\theta) = \theta$. Since (7) and (8) hold for all $\tilde{\theta}$, we further obtain that the second-order condition holds:

$$\begin{aligned} \frac{\partial^2 \Pi}{\partial \tilde{\theta}^2} =& \frac{\partial^2 p^*}{\partial \tilde{\theta}^2} \underbrace{\left(g_1 + p^* \frac{\partial g_1}{\partial p} \right)}_{=0} + \frac{\partial p^*}{\partial \tilde{\theta}} \underbrace{\frac{\partial}{\partial \tilde{\theta}} \left(g_1 + p^* \frac{\partial g_1}{\partial p} \right)}_{=0} \\ &+ \frac{\partial p^*}{\partial \tilde{\theta}} \frac{\partial g_1}{\partial \tilde{\theta}} + p^* \left(\frac{\partial^2 g_1}{\partial \tilde{\theta} \partial p} \frac{\partial p^*}{\partial \tilde{\theta}} + \frac{\partial^2 g_1}{\partial \tilde{\theta}^2} \right) - \sum_{i \in M} \frac{\partial^2 c_\theta}{\partial \tilde{\theta}^2} \end{aligned}$$

$$
\begin{aligned}
=&\frac{\partial p^*}{\partial \tilde{\theta}}\left(\frac{\partial g_1}{\partial \tilde{\theta}}+p^*\frac{\partial^2 g_1}{\partial \tilde{\theta}\partial p}\right)+p^*\frac{\partial^2 g_1}{\partial \tilde{\theta}^2}-\sum_{i\in M}\frac{\partial^2 c_\theta}{\partial \tilde{\theta}^2}\\
=&\frac{\partial p^*}{\partial \tilde{\theta}}\underbrace{\left(\frac{\partial g_1}{\partial \tilde{\theta}}+p^*\frac{\partial^2 g_1}{\partial \tilde{\theta}\partial p}+p^*\frac{\partial^2 g_1}{\partial p^2}\frac{\partial p^*}{\partial \tilde{\theta}}+\frac{\partial p^*}{\partial \tilde{\theta}}\frac{\partial g_1}{\partial p}\right)}_{=0 \text{ by } (7)}-\left(\frac{\partial p^*}{\partial \tilde{\theta}}\right)^2\underbrace{\left(p^*\frac{\partial^2 g_1}{\partial p^2}+\frac{\partial g_1}{\partial p}\right)}_{<-\frac{\partial g_1}{\partial p}\text{ by } (8)}\\
&+p^*\frac{\partial^2 g_1}{\partial \tilde{\theta}^2}-\sum_{i\in M}\frac{\partial^2 c_\theta}{\partial \tilde{\theta}^2}\\
=&\left(\frac{\partial p^*}{\partial \tilde{\theta}}\right)^2\underbrace{\frac{\partial g_1}{\partial p}}_{<0}+p^*\underbrace{\frac{\partial^2 g_1}{\partial \tilde{\theta}^2}}_{\leq 0}-\sum_{i\in M}\underbrace{\frac{\partial^2 c_\theta}{\partial \tilde{\theta}^2}}_{>0}<0.
\end{aligned}
$$

Note that the last equality follows because the first parenthesis is equal to $\frac{\partial}{\partial \tilde{\theta}}\left(g_1+p^*\frac{\partial g_1}{\partial p}\right)=0$ by (7), while the last inequality follows from Assumption 1 (i) and (ii) and the assumptions on the cost function.

Suppose now that $\frac{\partial^2}{\partial\theta\partial\tilde{\theta}}g_1 \geq 0$ and consider $\tilde{\theta}^*(M|\theta)$, i.e. the unique solution to the first-order condition (9) expressed as a function of θ. By Lemma 3 and Assumption 1 (ii), the left-hand side of (9) is strictly increasing in θ since $\frac{\partial^2 c_\theta}{\partial\tilde{\theta}\partial\theta}<0$. By the second-order condition, this implies that $\tilde{\theta}^*(M|\theta)$ is strictly increasing in θ. □

Proof of Corollary 1. If $p^*(\tilde{\theta}^*, M|\theta) \leq \theta$, then industry profits are

$$
p^*(\tilde{\theta}^*, M|\theta)\underbrace{g_1(p^*(\tilde{\theta}^*, M|\theta), \bar{\eta}(M), \tilde{\theta}^*|\theta)}_{\leq 1}-\sum_{i\in M}\underbrace{c_\theta(\tilde{\theta}^*, \eta_i)}_{\geq c_0}\leq \theta-|M|c_0.
$$

If $p^*(\tilde{\theta}^*, M|\theta) > \theta$, then industry profits are

$$
p^*(\tilde{\theta}^*, M|\theta)g_2(p^*(\tilde{\theta}^*, M|\theta), \bar{\eta}(M), \tilde{\theta}^*|\theta)-\sum_{i\in M}c_\theta(\tilde{\theta}^*, \eta_i). \tag{10}
$$

At $\theta = 0$, (10) is strictly larger than $-|M|c_0$, since $\tilde{\theta}^* > \theta$ by Proposition 1. The claim follows by continuity. □

Proof of Lemma 4. Define

$$
\varepsilon^* \equiv \min_{i,j\in\mathcal{M}:i<j}\min_{M\subseteq\mathcal{M}\setminus\{i,j\}}\frac{\begin{aligned}p^*_{M\cup\{j\}}\Big(&G(p^*_{M\cup\{j\}}, M\cup\{i\}, \tilde{\theta}^*_{M\cup\{j\}})\\&-G(p^*_{M\cup\{j\}}, M\cup\{j\}, \tilde{\theta}^*_{M\cup\{j\}})\Big)\end{aligned}}{c_1(\tilde{\theta}^*_{M\cup\{j\}}-\theta)(\eta_i-\eta_j)}.
$$

Note that since $\eta_i > \eta_j$, $\tilde{\theta}^*_{M\cup\{j\}} > \theta$ by Proposition 1, and by Assumption 1 (iii), $\varepsilon^* > 0$ is well defined. Suppose that $c_2'(\eta) < \varepsilon^*$ for all $\eta \in (0,1)$ and fix $i, j \in \mathcal{M}$ such that $i < j$ and $M \subseteq \mathcal{M}\backslash\{i,j\}$. By definition of $\Pi_{M\cup\{i\}}$,

$$
\begin{aligned}
&\Pi_{M\cup\{i\}} - \Pi_{M\cup\{j\}} \\
\geq & p^*_{M\cup\{j\}} G(p^*_{M\cup\{j\}}, M\cup\{i\}, \tilde{\theta}^*_{M\cup\{j\}}) - \sum_{k\in\mathcal{M}\cup\{i\}} c_\theta(\tilde{\theta}^*_{M\cup\{j\}}, \eta_k) \\
&- \left(p^*_{M\cup\{j\}} G(p^*_{M\cup\{j\}}, M\cup\{j\}, \tilde{\theta}^*_{M\cup\{j\}}) - \sum_{k\in\mathcal{M}\cup\{j\}} c_\theta(\tilde{\theta}^*_{M\cup\{j\}}, \eta_k) \right) \\
= & p^*_{M\cup\{j\}} \Big(G(p^*_{M\cup\{j\}}, M\cup\{i\}, \tilde{\theta}^*_{M\cup\{j\}}) - G(p^*_{M\cup\{j\}}, M\cup\{j\}, \tilde{\theta}^*_{M\cup\{j\}}) \Big) \\
&- c_1(\tilde{\theta}^*_{M\cup\{j\}} - \theta) \Big(c_2(\eta_i) - c_2(\eta_j) \Big) \\
> & p^*_{M\cup\{j\}} \Big(G(p^*_{M\cup\{j\}}, M\cup\{i\}, \tilde{\theta}^*_{M\cup\{j\}}) - G(p^*_{M\cup\{j\}}, M\cup\{j\}, \tilde{\theta}^*_{M\cup\{j\}}) \Big) \\
&- c_1(\tilde{\theta}^*_{M\cup\{j\}} - \theta) \varepsilon^* (\eta_i - \eta_j) \\
> & 0,
\end{aligned}
$$

where the last inequality follows from the definition of ε^*. □

Proof of Proposition 2. First, suppose that $m = 2$. Since $\Pi_1 \geq \Pi_2$, we have that $M^{\mathrm{e}} \neq \{2\}$. Therefore, there exists $k \in \mathcal{M} \cup \{0\}$ such that $M^{\mathrm{e}} = M^k$. If $k^* = 0$ or $k^* = 2$, then there is nothing to show. For $k^* = 1$, if $\Pi_1 = \Pi_2$ (i.e. $l(\eta_2) = 1$) neither 1 or 2 are essential since $M = \{2\}$ is also a maximizer, hence $M^{\mathrm{e}} = M^0$. If $\Pi_1 > \Pi_2$ (i.e. $l(\eta_2) = 0$) then only 1 is essential, hence $M^{\mathrm{e}} = M^1$.

Second, suppose that Condition (C1) holds. Then, by Remark 1 we have $\eta_i \geq \eta_j$ if $i < j$. Lemma 4 then yields $\Pi_{M^k} \geq \Pi_M$ for all M such that $|M| = k$. Thus, $M^{\mathrm{e}} \subset M^{k^*}$. Now, for any influencer $i \in M^{k^*}$ with $\eta_i = \eta_{k^*+1}$ we have $\Pi_{M^{k^*}} = \Pi_{(M^{k^*}\backslash\{i\})\cup\{k^*+1\}}$, implying $i \notin M^{\mathrm{e}}$, while we get strict inequalities for $i \in M^{k^*}$ with $\eta_i > \eta_{k^*+1}$, proving the claim. □

Proof of Lemma 5. Suppose first that in some SSPE the firm chooses M and there exists an influencer $i \in M$ with payoff $\pi_i > \bar{\pi}_i(M)$. Note that $\sum_{j\in M} \pi_j + \pi_f \leq \Pi_M$ by definition of Π_M. If $\emptyset \in \operatorname{argmax}_{M'\subseteq\mathcal{M}\backslash\{i\}} \Pi_{M'}$, then

$$\pi_f \leq \Pi_M - \sum_{j\in M} \pi_j \leq \Pi_M - \pi_i < \Pi_M - \bar{\pi}_i(M) = \Pi_\emptyset,$$

i.e. the firm has a profitable deviation to $M' = \emptyset$, a contradiction.

If $\emptyset \notin \text{argmax}_{M' \subseteq \mathcal{M} \backslash \{i\}} \Pi_{M'}$, consider any $M' \in \text{argmax}_{M' \subseteq \mathcal{M} \backslash \{i\}} \Pi_{M'}$ and a deviation to pricing strategies

$$q'_j(\tilde{\theta}) = \begin{cases} \pi_j + c_\theta(\tilde{\theta}, \eta_j) + \frac{\pi_i - \bar{\pi}_i(M)}{|M|} & \text{if } \tilde{\theta} = \tilde{\theta}^*_{M'} \\ \infty & \text{else} \end{cases} \text{ for all } j \in M'.$$

Note that all $j \in M'$ are strictly better off with this deviation. By choosing $p' = p^*_{M'}$ and $\tilde{\theta}' = \tilde{\theta}^*_{M'}$, the firm obtains

$$\begin{aligned} \pi'_f = p^*_{M'} G(\tilde{\theta}^*_{M'}, M', \tilde{\theta}^*_{M'}) - \sum_{j \in M'} q'_j(\tilde{\theta}^*_{M'}) =& \Pi_{M'} - \sum_{j \in M'} \left(\pi_j + \frac{\pi_i - \bar{\pi}_i(M)}{|M|} \right) \\ >& \Pi_{M'} - \sum_{j \in M'} \pi_j - \pi_i + \bar{\pi}_i(M) \\ =& \Pi_M - \sum_{j \in M} \pi_j, \end{aligned}$$

i.e. also the firm obtains a higher payoff, a contradiction. Thus, $\pi_i \leq \bar{\pi}_i(M)$ for all $i \in M$.

Second, suppose that in some SSPE the firm chooses M and there exists an influencer $i \in M$ with payoff $0 \leq \pi_i < \bar{\pi}_i(M)$. Consider a deviation by i to pricing strategy

$$q'_i(\tilde{\theta}) = \begin{cases} q_i(\tilde{\theta}) + (\bar{\pi}_i(M) - \pi_i)/2 & \text{if } \tilde{\theta} = \tilde{\theta}^*_M \\ \infty & \text{else} \end{cases}.$$

By definition of $q'_i(\tilde{\theta})$ it is still optimal for the firm to hire influencer 1 if all influencers $j \neq i$ play an undominated strategy $q_j \geq c_\theta$, since in this case $\pi'_i = \pi_i + (\bar{\pi}_i(M) - \pi_i)/2 = (\pi_i + \bar{\pi}_i(M))/2 < \bar{\pi}_i(M)$, i.e. $1 \in M$. Since further also $\pi_j \leq \bar{\pi}_j(M)$ for all $j \in M \backslash \{i\}$ by the first part, the firm will still choose M, such that i obtains payoff $\pi'_i = \pi_i + (\bar{\pi}_i(M) - \pi_i)/2 > \pi_i$, a contradiction. □

Proof of Proposition 3. Consider $M' \subseteq \mathcal{M} \backslash M^{k^*} = \{k^*+1, k^*+2, \ldots, m\}$ such that $\bar{\eta}(M') = \bar{\eta}(M^{k^*})$.

We first prove that $\sum_{i \in M^{k^*}} c_2(\eta_i) < \sum_{i \in M'} c_2(\eta_i)$. Suppose without loss of generality that $M' = \{k^*+1, k^*+2, \ldots, k^{**}\}$ and let $\tilde{\eta}_i = \eta_{i-k^*}$ for all $i = k^*+1, k^*+2, \ldots, 2k^*$ and $\tilde{\eta}_i = 0$ otherwise. Since, $\tilde{\eta}_i > \eta_i$ for all $i = k^*+1, k^*+2, \ldots, 2k^*$,

$$\bar{\eta}(M^{k^*}) = \bar{\eta}(M') \Leftrightarrow \Pi_{i \in M^{k^*}}(1 - \eta_i) = \Pi_{i \in M'}(1 - \eta_i)$$

$$\Leftrightarrow \Pi_{i\in M'}(1-\tilde{\eta}_i) = \Pi_{i\in M'}(1-\eta_i)$$

implies $\sum_{i\in M^{k^*}} \eta_i = \sum_{i\in M'} \tilde{\eta}_i < \sum_{i\in M'} \eta_i$. The claim follows by concavity of c_2.

Next, consider the choice of influencers M' and suppose without loss of generality that $p^* = p^*(\tilde{\theta}^*_{M'}, M'|\theta) < \theta$. We know from (9) that $\tilde{\theta}^*_{M'}$ solves

$$\begin{aligned} 0 =& p^* \frac{\partial g_1}{\partial \tilde{\theta}}(p^*, \bar{\eta}(M'), \tilde{\theta}^*_{M'}|\theta) - c_1'(\tilde{\theta}^*_{M'} - \theta) \sum_{i\in M'} c_2(\eta_i) \\ <& p^* \frac{\partial g_1}{\partial \tilde{\theta}}(p^*, \bar{\eta}(M^{k^*}), \tilde{\theta}^*_{M'}|\theta) - c_1'(\tilde{\theta}^*_{M'} - \theta) \sum_{i\in M^{k^*}} c_2(\eta_i), \end{aligned}$$

where the inequality follows from the claim established first. Thus, the optimal message conditional on choosing influencer 1 satisfies $\tilde{\theta}^*_1 > \tilde{\theta}^*_{M'}$. □

Proof of Corollary 3. First, note that by Proposition 2, we have that $M^{\mathrm{e}} = M^{k^*-l(\eta_{k^*+1})}$, i.e. either $M^{\mathrm{e}} = M$, $M^{\mathrm{e}} = \{1\}$ or $M^{\mathrm{e}} = \emptyset$. We proceed by case distinction:

(i) $\Pi_{\mathcal{M}} > \Pi_1$. In this case Assumption 2 and Remark 2 yield $\Pi_1 > \Pi_2 > \Pi_\emptyset$, i.e. $M^{\mathrm{e}} = \mathcal{M}$. Thus, by Proposition 1, each $i \in M^{\mathrm{e}}$ receives $\pi_i = \bar{\pi}_i(M^{\mathrm{e}}) = \Pi_{\mathcal{M}} - \Pi_j$, where $j \in \mathcal{M}\backslash\{i\}$, and $\pi_f = \Pi_{M^{\mathrm{e}}} - \sum_{i\in M^{\mathrm{e}}} \bar{\pi}_i(M^{\mathrm{e}}) = \Pi_1 + \Pi_2 - \Pi_{\mathcal{M}}$.

(ii) $\Pi_1 - \Pi_\emptyset > 0 \geq \Pi_{\mathcal{M}} - \Pi_1$. Then $M^{\mathrm{e}} = \{1\}$, and hence by Proposition 1 $\pi_1 = \bar{\pi}_1(M^{\mathrm{e}}) = \Pi_1 - \max\{\Pi_2, \Pi_\emptyset\}$, $\pi_2 = 0$, and $\pi_f = \max\{\Pi_2, \Pi_\emptyset\}$.

(iii) $\Pi_\emptyset \geq \Pi_1$. In this case Assumption 2 and Remark 2 yield $\Pi_1 > \Pi_2 > \Pi_{\mathcal{M}}$, i.e. $M^{\mathrm{e}} = \emptyset$. Thus, $\pi_1 = \pi_2 = 0$, and $\pi_f = \Pi_\emptyset$.

□

B Appendix: Word-of-mouth communication with imperfect learning

We generalize the micro-foundation on word-of-mouth communication presented in Section 2.2 to imperfect learning. To this end, suppose that after purchase naïve consumers learn the true quality with probability $\lambda \in [0, 1]$ and otherwise do not update their belief. Recall from Section 2.2 that the total share of buyers in the first period is given by

$$G_1^P(p, M, \tilde{\theta}|\theta) = (1-\rho)\bar{\eta}\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\} + \rho\left(1-(1-\bar{\eta})(1-\mu)\right)\max\left\{1-\frac{p}{\theta}, 0\right\}.$$

In the second period, consumers communicate with their neighbors in the network. The share of individuals communicating belief $\hat{\theta} = \tilde{\theta} \geq \theta$ is given by $(1-\lambda)(1-\rho)\bar{\eta}\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\}$, while that communicating the true quality $\hat{\theta} = \theta$ is given by

$$\lambda(1-\rho)\bar{\eta}\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\} + \rho\left(1-(1-\bar{\eta})(1-\mu)\right)\max\left\{1-\frac{p}{\theta}, 0\right\}.$$

Out of the share of individuals who are naïve and were not informed by an influencer in the first period, $(1-\rho)(1-\bar{\eta})$, each consumer makes k observations with probability $P(k)$ in the second period. A consumer ends up buying if either at least one of the k observed consumers reports $\hat{\theta} = \tilde{\theta}$ and $v_i(\hat{\theta}) \geq p$, or at least one of the k observed consumers reports $\hat{\theta} = \theta$ and none of the observed consumers reports $\hat{\theta} = \tilde{\theta}$ and $v_i(\theta) \geq p$. Note that the probability that none of the k observed consumers reports $\hat{\theta} = \tilde{\theta}$ and the probability that none reports $\hat{\theta} = \tilde{\theta}$ but at least one reports $\hat{\theta} = \theta$ is given by

$$\left(1-(1-\lambda)(1-\rho)\bar{\eta}\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\}\right)^k$$

and

$$\begin{aligned}
&\left(1-(1-\lambda)(1-\rho)\bar{\eta}\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\}\right)^k \\
&\quad - \left(1-\lambda(1-\rho)\bar{\eta}\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\} - \rho\left(1-(1-\bar{\eta})(1-\mu)\right)\max\left\{1-\frac{p}{\theta}, 0\right\}\right)^k \\
&= \left(1-(1-\lambda)(1-\rho)\bar{\eta}\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\}\right)^k \\
&\quad - \left(1-G_1(p, M_f, \tilde{\theta}|\theta) + (1-\lambda)(1-\rho)\bar{\eta}\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\}\right)^k,
\end{aligned}$$

respectively. Therefore, total sales to naïve consumers are

$$\begin{aligned}
&\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\}(1-\rho)\bar{\eta} \\
&+\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\}(1-\rho)(1-\bar{\eta})\sum_{k=1}^{\bar{k}} P(k)\left(\left(1-\left(1-(1-\rho)\bar{\eta}(1-\lambda)\max\left\{1-\frac{p}{\tilde{\theta}}, 0\right\}\right)^k\right)\right.
\end{aligned}$$

$$
\begin{aligned}
&+\max\left\{1-\frac{p}{\theta},0\right\}(1-\rho)(1-\bar{\eta})\sum_{k=1}^{\bar{k}}P(k)\Bigg(\left(1-(1-\rho)\bar{\eta}(1-\lambda)\max\left\{1-\frac{p}{\tilde{\bar{\theta}}},0\right\}\right)^k \\
&\quad-\left(1-G_1(p,M_f,\tilde{\theta}|\theta)+(1-\rho)\bar{\eta}(1-\lambda)\max\left\{1-\frac{p}{\tilde{\bar{\theta}}},0\right\}\right)^k\Bigg) \\
&=\max\left\{1-\frac{p}{\tilde{\bar{\theta}}},0\right\}(1-\rho)\left(1-(1-\bar{\eta})\sum_{k=1}^{\bar{k}}P(k)\Bigg(\left(1-(1-\rho)\bar{\eta}(1-\lambda)\max\left\{1-\frac{p}{\tilde{\bar{\theta}}},0\right\}\right)^k\right) \\
&\quad+\max\left\{1-\frac{p}{\theta},0\right\}(1-\rho)(1-\bar{\eta})\sum_{k=1}^{\bar{k}}P(k)\Bigg(\left(1-(1-\rho)\bar{\eta}(1-\lambda)\max\left\{1-\frac{p}{\tilde{\bar{\theta}}},0\right\}\right)^k \\
&\quad-\left(1-G_1(p,M_f,\tilde{\theta}|\theta)+(1-\rho)\bar{\eta}(1-\lambda)\max\left\{1-\frac{p}{\tilde{\bar{\theta}}},0\right\}\right)^k\Bigg).
\end{aligned}
$$

Similarly, out of the share of individuals who are sophisticated and were not aware of the innovation in the first period, $\rho(1-\bar{\eta})(1-\mu)$, each consumer makes k observations with probability $P(k)$ in the second period and ends up buying if she is informed by one of the k observed consumers and $v_i(\theta) \geq p$. Note that the probability that none of the k observed consumers informs her of the innovation is given by

$$\left(1-G_1(p,M_f,\tilde{\theta}|\theta)\right)^k,$$

such that total sales to sophisticated consumers are

$$
\begin{aligned}
&\rho\left(1-(1-\bar{\eta})(1-\mu)\right)\max\left\{1-\frac{p}{\theta},0\right\} \\
&+\rho(1-\bar{\eta})(1-\mu)\max\left\{1-\frac{p}{\theta},0\right\}\left(1-\sum_{k=1}^{\bar{k}}P(k)\left(1-G_1(p,M_f,\tilde{\theta}|\theta)\right)^k\right) \\
&=\max\left\{1-\frac{p}{\theta},0\right\}\rho\left(1-(1-\bar{\eta})(1-\mu)\sum_{k=1}^{\bar{k}}P(k)\left(1-G_1(p,M_f,\tilde{\theta}|\theta)\right)^k\right).
\end{aligned}
$$

Total demand is the sum of both.