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Beginning mathematics teachers' values and beliefs about pedagogy during a time of policy flux

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Abstract

This study interrogates how beginning secondary mathematics teachers align their beliefs with their practice when they justify their pedagogical choices in the context of recent English mathematics education policy which is strongly influenced by approaches to mathematics teaching in Shanghai and Singapore currently referred to as 'mastery' approaches. It seeks to understand beginning teachers' perceptions and understandings of these approaches and the extent to which they recognise aspects of 'mastery' in practice. In setting the context, pre-service teachers' beliefs were surveyed and found to be congruent with constructivist approaches to learning. We then draw on qualitative data from semi-structured interviews secondary mathematics teachers in their first year post-qualification. The interviews were designed to interrogate and capture understanding of the features of mastery within their own classrooms. By using vignettes to capture participants' beliefs, our aim was to present a 'more nuanced understanding of the phenomena' (Skilling and Stylianides in Int J Res Method Educ 43(5):541–556, 2019, 10.1080/1743727x.2019.1704243). The analytical framework developed draws on Guskey's (In: Wright J (ed) International encyclopedia of the social & behavioural sciences, 2015, vol 14, 2nd edn, Elsevier, pp 752–759) interpretation of Bloom's theory of mastery learning together with features of mastery learning in mathematics articulated by Drury (How to teach mathematics for mastery, 2018, Oxford University Press) and Boylan et al. (Edu Sci 8(4):202, 2018, 10.3390/educsci8040202). This posed a research design challenge given the variation in interpretation of mastery learning as it is understood in practice. The data exposes differences in the interpretation of mastery approaches in the settings where they learn to teach, as well as the tensions that arise between beginning teachers' beliefs, practice, professional knowledge and agency in their developing classroom roles.

Keywords Mastery · Vignettes · Beliefs · Pedagogy

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Introduction

As teacher educators we are interested in the discourses of policy, practice and research (Biesta, 2019), and the intersections between them. In our teacher education context, we are immersed in what we see as the contradictions between how national policy regarding the teaching of secondary school mathematics interacts with our work in Initial Teacher Education (ITE). As teacher educators, these contradictions impact on how we work with beginning teachers in order to effectively support their development of personal pedagogies and critical practices that may allow them to become informed decision makers as they navigate the early stages of becoming a secondary mathematics teacher. We are also interested in the relationship between beginning secondary mathematics teachers' beliefs and practices and the extent to which they draw on aspects of their learning from their preparation programme in their classrooms.

The paper first conceptualises the current English government-promoted approach to teaching and learning school mathematics commonly referred to as 'mastery'. This section develops a framework for our understanding of this term through drawing on illustrative literature that refers both directly and indirectly to mastery approaches to teaching. This then provides both a foundation and framework for the empirical work. We first analyse empirical data regarding the beliefs of a larger sample of pre-service teachers of mathematics. This then sets the context for gathering the views of a smaller group of beginning teachers (near the end of their first year post-qualification) towards mastery approaches for secondary mathematics teaching. The analysis and discussion consider the ways in which this nationally promoted approach to teaching is interpreted and understood by beginning teachers with reference to their beliefs. The paper concludes with a consideration of the implications and opportunities for teacher educators working within the complex intersections between policy, practice and research.

Initial teacher education for secondary school mathematics in England

Most secondary school (ages 11–18) mathematics teachers in England qualify through a Post Graduate Certificate in Education (PGCE) programme following a Bachelor's degree. Long-standing shortages of secondary mathematics teachers in England mean that eligible pre-service teachers receive a substantial bursary over the period of their Initial Teacher Education. ITE in England is highly regulated with a set of competencies for qualification, the current version of which is called the Teachers' Standards (Department for Education, 2011). Currently pre-service teachers spend at least two-thirds of their 36 week postgraduate programme on placement in two different schools. A significant and far-reaching change announced in 2010 led to what is now a multiplicity of routes to qualification and a shift away from university-led provision to the position where, in the year 2021/22, 55% of postgraduate pre-service teachers followed routes where schools took the lead role in the organisation and provision of their ITE programme (gov.uk, 2022). In recent years, highly influential reports have criticised the cost effectiveness of ITE in England (Allen et al., 2014) and the content of programmes (Carter, 2015). More recently, the Core Content Framework (Department for Education, 2019) was introduced by the government to regulate the content of programmes through the setting of 'minimum expected standards' and the ITT Market Review (Department for Education, 2021) has led to a requirement for all providers to apply for re-accreditation to run their programmes, indicating unprecedented political intervention in the design and implementation of ITE programmes. This move towards what are called 'school-led' models and the emphasis in the Core Content Framework focus on specific practices and approaches has also led to debate around the role of the university in ITE and different understandings of the role of the teacher (see, for example, Orchard & Winch, 2015).

The school curriculum in England is centralised through the provision of a National Curriculum (Department for Education, 2013), which provides the subject content for the curriculum in schools. For mathematics, the three aims of the National Curriculum are that students should become "fluent in the fundamentals of mathematics… reason mathematically" and "solve problems" (Department for Education, 2013, p. 2). Concern regarding what was considered to be England's poor performance in international comparative tests in mathematics (Wiliam, 2021) caused the government to look to the mathematics curricula and practice in countries who performed highly on such tests. The National Centre for Excellence in the Teaching of Mathematics (NCETM) was set up in 2006 and is funded by the Department for Education to improve the teaching of mathematics and student attainment through a programme of professional development for teachers. Since 2014, a range of programmes, delivered through a network of 40 hubs, has aimed to support teachers with developing approaches to teaching mathematics that mirror more closely practice seen to be successful in East Asia, and Singapore and Shanghai in particular. These practices promoted by the NCETM and the Maths Hubs have come to be known as 'teaching for mastery' in England (Simpson & Wang, 2023).

Working with pre-service teachers to develop pedagogy and practice, and the role of beliefs

Our pedagogical approach to working with pre-service mathematics teachers is premised the view that teacher education has the potential to be transformative (Boylan et al., 2023; Darling-Hammond, 2017), whilst recognising the complexity of the role that a Higher Education Institution (HEI) and school play in influencing the beliefs and practices of beginning teachers (Burn & Mutton, 2015; Ellis & McNicholl, 2015). We conceptualise theory as "cutting edge analytical engagement with new situations" (Hodson et al., 2012, p. 181) and use Knight's (2015, p. 158) approach of creating a space for problematising "practice through exposure to challenging questions, wider perspectives and de-familiarising experiences". Our approach is also more specifically informed by the principles outlined in 'Developing mathematics-specific pedagogy in Initial Teacher Education' (The Joint Mathematical Council for the United Kingdom, 2017) with research-informed approaches focused on teaching for understanding aligned with a broadly constructivist model of teaching mathematics. Beswick (2007, p. 98) identifies the following features of classrooms as being consistent with a constructivist view of learning: a "focus on the students—their needs, backgrounds, interests and particularly their existing mathematical understandings", facilitation of dialogue to develop understanding, and purposeful use of "tasks, materials, questions, or information to stimulate reflection on and possible restructuring of students' understandings". Hence, aspirations for beginning teachers are ambitious because mathematics is viewed as a field of connected ideas that can be learned through rich, reasoned experiences that allow the teacher to respond to pupils' ways of knowing mathematics, situated in a context that enables high expectations of all learners (Swan & Burkhardt, 2014).

With decades of experience of working with pre-service and beginning teachers we recognise that the beliefs of pre-service teachers are important because of the impact they have on the decisions they make in practice and the extent to which they are willing to engage with theory. We use Philipp's (2007) definition of beliefs as "psychologically held understandings, premises and propositions about the world that are thought to be true… lenses that affect one's view of some aspect of the world or as dispositions toward action" (p. 239). As Tatto et al. (2012) say, there is "widespread agreement that the beliefs held by teachers and students are an important influence on teaching and learning" and that "it is reasonable to expect that teachers holding [these] different patterns of belief will engage in different classroom practice" (p. 153). Our experience also supports the literature regarding the socialisation effects of classroom experience, recognising the situated nature of learning to teach within a particular social and cultural context, integrated with the individual teachers' beliefs and experiences that influence their emerging professional identity of the beginning teacher (Brown & McNamara, 2011; Steadman, 2023). Pre-service teachers realise that whilst learning to teach mathematics in the classrooms of their placement school, their immediate responsibility is to the students in front of them and the teachers whose classrooms they share. Therefore, seeking to emulate their mentors and supervising teachers in a manner that might appear to conflict with the practices experienced in the HEI is understood as one of the central tensions in developing partnerships in ITE programmes (Burn & Mutton, 2015; Ellis et al., 2011; Zeichner et al., 2014). Our experiences also suggest that pre-service teachers' actions might emulate their beliefs about the way that they want to learn or were taught themselves (Brown & McNamara, 2011). To better understand the process of change and development in beginning teachers, we find it helpful to work with Fujii's (2014) research identifying three levels of expertise in mathematics teaching:

Level 1 The teacher can tell students the important basic ideas of mathematics using procedures and practices.

Level 2 The teacher can explain the meanings and reasons for the important basic content and practices of mathematics in order for students to understand them.

Level 3 The teacher can provide students with opportunities to understand mathematical content and develop mathematical practices, and support students to become independent learners.

Japanese teacher educators also believe that each level needs to be mastered before moving to the next and that it takes ten years and a great deal of effort for a teacher to reach level 3 (Fujii, 2014). Therefore, working with beginning teachers to stimulate critical approaches to learning to teach, and allowing them to challenge their own assumptions about mathematics education, is important to us in supporting beginning teachers who are able to use research-informed classroom practices and become educators capable of realising their own potential as teachers and supporting high aspirations for the students that they teach.

Constructs of mastery in mathematics education

In order to study pre-service and beginning mathematics teachers' perceptions of a mastery approach to teaching, we have aligned current perceptions of mastery learning in mathematics in policy and practice with an interrogation of the context of learning to teach mathematics in English secondary schools. The nature of this alignment has an influence on beginning teachers' actions and decisions because, as we note in the previous section, the contexts in which they are situated in school are likely to influence their practice more significantly than professional learning situated elsewhere such as the university (Ellis et al., 2011; Nelson & Campbell, 2017). A mastery approach to teaching mathematics has become an aspect of practice in many English secondary schools that evokes far more than mathematical pedagogical considerations, because the plethora of uses and situated meanings of the term are influenced by political and cultural factors (Boylan, 2016; Simpson & Wang, 2023). This is further complicated by the context of learning to teach in England, whereby ITE and Early Career Teacher accreditation policy is structured within a proposed research framework that may be in tension with the nature of learning to teach (Turvey et al., 2019).

Some aspects of modern interpretations of mastery learning are consistent in many sources, such as using evidence of pupils' knowledge and experience to inform lesson design (Guskey & McTighe, 2016), whereas others are more subjective both in interpretation and enactment, illustrated by the way in which pupils might be taught to make connections between different *representations* of mathematical knowledge, if at all, or the manner in which variation theory is understood and applied to practice (Askew et al., 2015; Kullberg et al., 2017). Nonetheless, we cannot assume, nor reasonably expect, that beginning teachers have a clear understanding of what mastery in mathematics education means, even in aspects with greater synthesis between policies, nor how it is represented within the activity of teachers and learners in their schools and colleges. In order to expose and interrogate their perceptions, we identified the following features of mastery in mathematics education that enabled us to analyse the perceptions of the teachers through interviews.

The summary in Table 1 is not intended to represent a detailed analysis or synthesis of available mastery in mathematics education literature. Rather is an illustrative framework of literature that directly informs the ongoing dialogue about what mastery might be, alongside features that have an indirect connection so that we might understand how these ideas are perceived or used by beginning teachers. This framework is exemplified using illustrative quotations from research and theories that align with the supposed feature of mastery in mathematics education is derived from publications explaining a mastery approach to mathematics education. The publications identified either have a direct connection to mastery in mathematics education or are those where we consider the publication to have an indirect link, but nonetheless one that is representative of sources that inform initial teacher education in our own practice and that of other mathematics teacher educators. Whilst this is not an exhaustive account, the table is illustrative of potential influences on beginning teachers (in ITE and beyond) in England. It is offered to explain what we understand about learning to teach mathematics through the construct of mastery in mathematics education, in order that it can be used as a lens through which to analyse and understand beginning teachers' responses in the interviews.

Claims of misinterpretation and misconceptions about mastery learning are as well documented as those claims explaining what mastery might be (Jain & Hyde, 2020; Stevenson & Shearman, 2021). Guskey (2007, 2010) explains what Bloom's theory of mastery learning is *not,* alongside accounts of what he interprets it to *be*. Despite the absence of a unified meaning and cohesive practice of mastery in mathematics education, situated within current policy shifts influenced by Singapore and Shanghai (Boylan et al., 2019; Simpson & Wang, 2023) the characteristics of a meaningful and accessible contemporary mathematics education have been a unified endeavour for educators for many decades (Stevenson & Shearman, 2021). For example, Watson and Mason (2006a) have been publishing insights into variation in task design for two decades, years before the NCETM ventured into Shanghai schools in the Shanghai-England exchange (Boylan et al., 2019). Similarly,

Table 1 Features of mastery in mathematics education illustrated through direct and indirect connections to publications

Feature of mastery in mathematics education	Indicative quotations showing direct connection	Indicative quotations showing indirect connection
Teachers' actions in the classroom are informed by the belief that the *majority of learners are capable* of learning mathematics when it is *encountered in a learnable manner*	"A mastery approach […] includes a belief that all pupils are capable of understanding and doing mathematics, given sufficient time. Pupils are neither 'born with the maths gene' nor 'just no good at maths'" (Askew et al., 2015, p. 5) "Teachers have high expectations for every single learner, rooted in a belief that anyone can become an excellent mathematician" (Drury, 2018, p. 8) "Bloom believed that all students could be helped to reach a high criterion of learning if both the instructional methods and time were varied to better match students' individual learning needs" (Guskey, 2007, p. 9)	"The connectionist teachers indicated a commitment not to label pupils as being inevitably poor at mathematics. They had high levels of expectations for all pupils irrespective of ability. Intelligence was not seen as static and all pupils were regarded as having the potential to succeed" (Askew et al., 1997, p. 34) "There is the "untapped potential" view—a belief that starts from the assumption that so-called ability is a complex of skills that can be learned. […] the evidence is that ways of managing formative assessment that work with the assumptions of "untapped potential" do help all pupils to learn and can give particular help to those who have previously struggled" (Black & Wiliam, 1998, p. 146)
Teachers using formative assessment including evidence of *pupils' prior knowledge and experience* to inform lesson design and responsive teaching	"Nearly every modern instructional planning model, differentiation approach, and personalized learning system incorporates some form of pre-assessment. The designers of these systems clearly believe that it's essential to tap students' prior knowledge, experience, skill levels, and potential misconceptions to target instruction to individual students' learning needs" (Guskey & McTighe, 2016, p. 1)	"Teaching is more effective when it builds on the knowledge learners already have. This means developing formative assessment techniques and adapting our teaching to accommodate individual learning needs" (Swan & Burkhardt, 2014, p16)
Pupils acquiring *depth* and *meaning* in their understanding of mathematics through *reasoned connections, by resolving conflicts* and gaining insight into *structure, relationships and procedures*	"A mathematical concept or skill has been mastered when, through exploration, clarification, practice and application over time, a person can represent it in multiple ways, has the mathematical language to be able to communicate related ideas, and can think mathematically with the concept so that they can independently apply it to a totally new problem in an unfamiliar situation" (Drury, 2014, p. 9)	*"Mathematical 'richness' comes from how the teacher engenders curiosity, experimentation and realisations about the underlying structure,* rather than from focusing on successful completion of final answers" (Watson, 2021, p. 145)

Table 1 (continued)

Feature of mastery in mathematics education	Indicative quotations showing direct connection	Indicative quotations showing indirect connection
Pupils being taught to make connections between different *representations* of mathematical knowledge and realisable *contexts*	"In the implementation [of teaching for mastery] in England, […] a much increased and more sophisticated use of mathematical models, including visual and concrete representations [are prominent]" (Boylan et al., 2019, p. 11)	"The Concrete-Pictorial-Abstract (CPA) approach, based on Bruner's conception of the enactive, iconic and symbolic modes of representation, is a well-known instructional heuristic advocated by the Singapore Ministry of Education since early 1980's" (Leong et al., 2015, p. 2)
Language is considered as a core representation of mathematical ideas as well as a means of creating an inclusive environment for discussing, problematising and knowing mathematics	"A mathematical concept or skill has been mastered when, through exploration, clarification, practice and application over time, ….a person can represent it in multiple ways, has the mathematical language to be able to communicate related ideas" (Drury, 2014, p. 9)	"Children need to learn how to mean mathematically, how to use mathematical language to create, control and express their own mathematical meanings as well as to interpret the mathematical language of others" (Pimm, 1995, p. 179)
Teachers stimulating *interactive dialogue* that allows them to teach *responsively, recognising* and working with *difficulties*	"However, the best forms of ongoing, formative assessment arise from well-structured classroom activities involving interaction and dialogue (between teacher and pupils, and between pupils themselves)" (Askew et al., 2015, p. 5)	"When problem situations are discussed in mathematics classrooms, several levels of conversation are going on at the same time. There is the use of the symbolic mathematical language to describe mathematically the phenomena (usually) expressed in natural language. We employ both languages, the one that describes our world in natural language, and the one that attends to it in a formal language. Yet, it is important to keep in mind that these are two distinct languages" (Sfard et al., 1998, p. 42) "To learn mathematics effectively, pupils primarily need to talk about their mathematical ideas, negotiate meanings, discuss ideas and strategies and make mathematical language their own" (Lee, 2006, p. 11)

Table 1 (continued)

Feature of mastery in mathematics education	Indicative quotations showing direct connection	Indicative quotations showing indirect connection
Teachers explicitly integrating assessment-informed *intervention* into, lesson design and teaching, guided by pupils' learning	"Through this process of formative classroom assessment, combined with the systematic correction of individual learning difficulties, Bloom believed all students could be provided with a more appropriate quality of instruction than is possible under more traditional approaches to teaching. As a result, nearly all might be expected to learn well and truly master the unit concepts…" (Guskey, 2015, p. 753)	"Teaching is more effective when it builds on the knowledge learners already have. This means developing formative assessment techniques and adapting our teaching to accommodate individual learning needs" (Swan & Burkhardt, 2014, p. 9)
Teachers differentiating through *enrichment* and *support,* not through acceleration	"All pupils need access to these concepts and ideas and to the rich connections between them. There is no such thing as 'special needs mathematics' or 'gifted and talented mathematics'. Mathematics is mathematics and the key ideas and building blocks are important for everyone" (Askew et al., 2015, p. 4)	"We do not offer students different tasks related to some pre-determined notion of 'ability' ("differentiation by prejudice") but rather offer all students the same task and then, as needs emerge, extension challenges or additional support as necessary ("differentiation by outcome")" (Swan & Burkhardt, 2014, p. 18)
Teachers carefully consider lesson design that allows for the realisation of the *object of learning* through the use of *variation* in meaningful whole-class teaching	"Variation theory has several dimensions, including use of multiple representations of what a concept is, and what it is not. It is characterised by a carefully constructed small-step journey through learning. It pays attention to what is kept the same and what changes, in order that pupils might reason… Variation is applied to practice questions where attention is paid to the selection and order of the examples. Often just one aspect is changed whilst others are kept the same. The intention is to avoid mechanical repetition but instead to promote thinking to make connections. This is also known as 'intelligent practice'" (NCETM, 2022)	"[Variation] theory has been developed by Ference Marton (2005) and his co-researchers, some of whom are using it to study mathematics lessons and learning. He believes that learning only happens if there is some variation to discern and he sees learning as the discernment of variation" (Watson & Mason, 2006b, p. 3) "Marton (2015) suggests a prototype for how to sequence patterns of variation and invariance to bring about learning, as follows: starting with the undivided object of learning, usually a problem to solve aiming at getting the learners acquainted with the situation or what is to be mastered, followed by contrast, generalization and finally fusion" (Kullberg et al., 2017, p. 263)

Swan & Burkhart's (2014) work encompassing the connectionist teacher orientation demonstrated how mathematics teaching can expose hidden conflicts to be resolved through reasoned dialogue in the knowledge that a successful mathematics student wants mathematics to make sense, whether that be in a classroom applying principles of mastery learning or not. These, and many more, features of mathematics teaching and learning are an ongoing concern for teacher educators and beginning teachers and not merely because of a recent resurgence of the use of the term 'mastery' in mathematics education in England.

The purpose of this paper is not to account for, nor resolve, tensions arising from different interpretations of mastery but to provide an illustrative framework for interpreting beginning teachers' perceptions of mastery in mathematics education. Similarly, these illustrative quotations are not intended to suggest an idealised model of mathematics teaching that all mathematics educators should aspire to. However, they do illuminate the many features of learning to teach mathematics that beginning teachers may be exposed to, offering a way of understanding how these features are used or understood by beginning teachers in the early stages of their career. All the features of mastery in mathematics education identified in Table 1 correspond to the classroom features Beswick (2007) considers as consistent with a constructivist view of learning. We therefore see policy intentions, as expressed through this framework, as consistent with our own intentions as teacher educators.

Beginning teachers' beliefs and understandings

Methods

Our empirical work is a qualitative study preceded by a survey analysed using descriptive statistics to provide context. This approach is supported by Thompson (1992) who argues that researchers need to go further than solely analysing teachers' professed views in order to characterise their conceptions of the discipline. The survey sought to identify the beliefs of pre-service teachers regarding the nature of mathematics, mathematics learning and mathematics achievement at the very beginning of their teacher preparation programme. Pre-service teachers following one-year postgraduate programmes in either primary education or secondary mathematics at universities in England were surveyed and responses were received from 135 individuals from 13 different universities. The survey used an online version of the TEDS-M future teachers beliefs survey (Brese, 2008). Data were collected anonymously at the very beginning of participants' pre-service teacher education to reduce the opportunity for participants to be influenced by their programme and tutors' perspectives.

For the main interview phase with beginning secondary mathematics teachers towards the end of their first year as qualified teachers, the focus was on capturing beginning secondary mathematics teachers' perceptions of mastery approaches with the following research questions:

- What aspects of mastery approaches to teaching do beginning secondary mathematics teachers recognise in their own practice and that of others?
- How do beginning teachers align and justify their beliefs with their practice?
- To what extent do their accounts of their teaching reflect mastery approaches?

We were conscious of our position as teacher educators and recognised the difficulties associated with finding the most appropriate way to collect data regarding participants' beliefs about the complex topic of mastery approaches without them responding in socially desirable ways. As with Stecher et al.'s (2006, p. 1) work we identified observations of participants' teaching as "difficult to conduct, time consuming and expensive". Observations provide snapshots of specific examples of practice and this, we felt, was not suitable for our purpose. Stecher et al. (2006) further points to surveys and logs as being limited measures of classroom practice and we were also concerned not to increase the workload of beginning teachers. Skilling and Stylianides' (2019) paper on using vignettes in educational research provided us with a potential way forward. Vignettes have been used in a variety of fields, and in different ways, to collect data. The literature (for example, Jeffries & Maeder, 2005; Stecher et al., 2006; Skilling & Stylianides, 2019) suggests that they are an established tool for educators and researchers with a range of applications and that they are used for a variety of purposes. Veal (2002, p. 2) identifies effective use of vignettes as leading to the eliciting of discussion, development of knowledge, fostering problem solving, promoting decision making and initiating reflection. Bradbury-Jones et al. (2012) identifies them as supporting the understanding of beliefs, perceptions, values and dispositions and Skilling and Stylianides (2019, p. 514) as further "leading to a more nuanced understanding of the phenomena". Jenkins et al. (2010) further identify their usefulness for achieving "insight into the social components of the participants' interpretative framework and perceptual processes" (p. 178). Vignettes were further identified as suitable for our purpose given that we were not primarily interested in teacher behaviour or what they would do in a particular situation, but rather we wanted to capture their perceptions of mastery approaches to secondary mathematics education in order to expose their values and beliefs.

A working definition of a vignette for our purposes was as a short, realistic, contextual, and specific piece of writing that would act as a stimulus for a semi-structured interview by presenting participants with something that was plausible (Jenkins et al., 2010), of interest, relevant and realistic (Hughes & Huby, 2004) and would promote some reflection on the part of participants. Following Veal's (2002) advice, the vignettes used have a brief introduction identifying the setting (either a part of an assignment or a piece of reflective writing) and the participant (in this case a pre-service teacher). We chose to develop two vignettes, drawn from our own work as teacher educators, and adapted (with the permission of those concerned) from pieces of pre-service mathematics teacher writing. Both the pieces we developed were intended to 'feel' like real pieces of pre-service teacher writing, meeting Bradbury-Jones et al.'s (2012) criteria for capturing reality and relevance, and to allow for questioning to explore some of the complexity of the features of mastery approaches. Bradbury-Jones et al. (2012, p. 430) identify vignettes as having the "potential to cut through multifaceted issues" and as creating "distance between the researcher and participant"; both features we were looking for in our data collection tool. The vignettes developed are shown in Fig. 1.

The vignettes along with the questions for the semi-structured interview were sent to participants in advance. The interview schedule included brief instructions and a second copy of the vignettes with line numbers to ease identification of the correct places in the text when questioning. Participants were initially asked to explain why they thought the person in the vignette had written as they had before the questioning broadened out to an exploration of their own understanding and experience. The interview schedule was piloted, and minor changes were made to improve the clarity of the questions. Examining the transcripts from the pilot interviews supported consideration of internal validity in terms of ensuring that the vignette and subsequent questioning exposed the sorts of

Vignette 1

Suppose a trainee teacher has written the following in an assignment:

When pupils are taught using rules they often misapply them which results in misconceptions because "surface mastery of rules in the short term is quickly lost" (Ojose, 2015, p.31). To avoid this, teachers can use the mastery approach to enforce understanding of the mathematical concepts to the students. The mastery approach involves starting from the learning procedures and applying it to questions that progressively become more challenging where understanding of the procedure is required. This approach forces students to understand the use of formulas as "inadequate learning of formulas may result in misconceptions" (Özerem, 2012, p.7). The mastery approach is effective but can be time consuming as a long time needs to be spent on each topic for the concepts to be mastered. This then means that less time is available to teach other topics in the National Curriculum.

Vignette 2

A trainee teacher wrote the following in her weekly reflections:

Despite learning about many different ways to teach maths most lessons I have observed seem to settle to the standard lesson format of:

1. Demonstration and exposition by the teacher
2. Examples modelled by the teacher
3. Questions/Practise, for example using a worksheet, by pupils

with a big focus on procedural teaching and following rules. The differences between a 'good lesson' and a 'bad lesson' are remarkably subtle. For example, even if a lesson follows the format above, the "demonstration" or "example" section could involve a fruitful diagnostic discussion or be designed to be student-led in which case the lesson is elevated substantially.

Fig. 1 Vignettes

responses we were anticipating. Piloting also confirmed that a single vignette was sufficient for generating a 30 min interview and indicated that the interviews would be successful in our intention of drawing unprompted on participants' experience (Hughes & Huby, 2004).

We approached beginning teachers all in their first full year of teaching for volunteers who would be willing to participate. A convenience sample of six participants, two per institution, were recruited to the study. Although participants were following programmes in different institutions, the prescriptive nature of ITE in England (Sect. "Introduction") and our shared understandings of teacher preparation (Sect. "Initial teacher education for secondary school mathematics in England") mean that the programmes share common features. Table 2 provides brief biographical detail and indicates that participants have typical characteristics of pre-service teachers in England. All were interviewed approximately two-thirds of the way through their first year as a qualified secondary mathematics teacher. They were interviewed by someone from a different institution to the one they had attended to reduce the chances of participants seeking to give responses to please the interviewer. Each interview used one of the two vignettes, chosen at random, and lasted for approximately 30 min. Interviews took place online and were recorded with the participant's permission following ethical protocol and approval from the relevant institutions.

The approach to thematic analysis applied to this study is rooted in our acknowledgement that our shared beliefs and experiences as teacher educators inform our positions

Table 2 Biographical information on participants

Pseudonym	Gender	Age	Context as shared in interview
Cleona	Female	Under 25 and recent graduate	Engineering degree
Joanne	Female	Mature student	Career changer, previous career in business
Mark	Male	Mature student	Some teaching experience as an unqualified teacher prior to starting the programme. Mathematics degree
Katie	Female	Mature student	Career changer
Jonathan	Male	Under 25 and recent graduate	Mathematics degree
Komol	Female	Under 25 and recent graduate	Environmental science degree

as researchers; acknowledging that the influence of our experiences as mathematics teachers and teacher educators is an irrepressible part of our identities and therefore our beliefs and actions as practitioners and researchers. Merely acknowledging this is insufficient to account for our reflexivity within the study because we were each aware of our responsibility to immerse ourselves in the data from the interviews intuitively and iteratively to allow us to interrogate our own assumptions about beginning mathematics teachers' perceptions. We aimed to research in a manner that would not undermine the accurate representation of patterns of meaning in the teachers' perceptions (Terry et al., 2017). This dilemma stimulated the systematic and rigorous approach that we set out to use in the thematic analysis because we wanted to apply our insider view of learning to teach mathematics alongside our knowledge of research that informs mathematics education and mathematics teacher education. We acknowledged this understanding within complex and situated experiences that beginning teachers encounter as well as imposed policies that regulate the criteria for qualification in English schools (Ellis, 2023). We are each experienced educators who adopt research from cognitivist and constructivist perspectives on learning mathematics, albeit situating these positions in the different culture and practices of the teacher preparation programmes on which we each teach.

Familiarisation with the data began with each researcher transcribing and reviewing the interviews allowing us to immerse ourselves in the words of each participant. We did not interview the teachers that the individual researcher had taught so that we could limit the potential for shared assumptions that would exist between the tutor and beginning teacher to be overlooked. Review of each transcript led to initial cautious and intuitive coding from each interviewer. The sharing of each transcript and the initial codes was both tentative and expansive for each of us because we were offering an interpretation of the perceptions of teachers that one researcher knew, but the other two did not. This allowed us to deepen our analysis of the initial codes that we had generated by sharing possible meaning generated by our initial analysis and situating this in the context of each researcher's insider view of the experience of each participant (Terry et al., 2017). This was designed to add depth to our analysis because we were seeking implied meanings within the participants' words. We were able to challenge each other's assumptions about the participants' perceived understanding. Each researcher returned to the transcripts to review the patterns of meaning that had been generated, before once again reviewing the transcripts collaboratively. Extracts that illuminated the first iteration of themes were chosen because of their significance to the aims of the study and not for their frequency of occurrence nor their alignment between each participant's responses (ibid).

Table 3 Themes

Theme	Sub themes
Myths about mastery approaches	Everything in their ITE programme was about mastery Mastery is unattainable in their setting
The continuing impact of beliefs about teaching	Constructivist Own learning experience considered 'old-fashioned'
Complexities and difficulties relating to professional practice	Difficulties in articulating their understanding Focus on a single aspect of practice
Tensions between beliefs, actions and intentions	Lack of confidence Issues of equity for learners Pupils wanting to 'be told'—and this is 'easier' to do Lack of agency

Table 4 Pre-service teachers' beliefs about mathematics and mathematics learning, percent of statements endorsed

Belief	Percentage endorsement
Mathematics as a set of rules and procedures	46.9
Mathematics as a process of inquiry	74.0
Learn mathematics by following teacher direction	6.1
Learn mathematics through active involvement	50.9
Mathematics as a fixed ability	7.2

The analytic process described by Braun and Clarke (2021, p. 333) as involving "immersion in the data, reading, reflecting, questioning, imagining, wondering, writing, retreating, returning" allowed us to **represent** the participants' perceptions of their early experiences of teaching so that it is interpreted and understood through what Braun and Clarke (2021, p. 342) call "patterns of shared meaning, united by a central concept or idea". These are shown in Table 3 and explored further through the findings and discussion.

Findings and discussion

In reporting the findings from our empirical work, we turn first to the survey results in Table 3 and discuss pre-service teacher beliefs at the beginning of their teacher preparation programme.

Survey findings

As with the TEDS-M study, the number of respondents indicating agreement or strong agreement with a statement are considered to endorse that view. The statements were then grouped according to the five belief scales used in the TEDS-M report (Tatto et al., 2012) and the percentage of statements endorsed calculated and show in Table 4.

In common with the TEDS-M results, pre-service teachers in our survey strongly endorsed the view that mathematics is a process of enquiry. The view that mathematics is learnt through active involvement, which would generally be seen as consistent with this (Tatto et al., 2012), was supported by our sample, but not as strongly supported as it was in the TEDS-M study. In our data, the view that mathematics is best learnt by following teacher direction was not well-supported, nor was the view of mathematics as a fixed ability. As with the TEDS-M survey, our survey indicates stronger support for beliefs consistent with conceptual and cognitive-constructive views of mathematics teaching than for conceptual and calculational views of mathematics learning. The survey provides some context before our attention turns to probing interview data from a small number of teachers collected at a point at which they have some teaching experience in order to explore their understandings of practice, in the current policy context, and to consider the extent to which their views are congruent with broadly constructivist teaching practices.

Interviews

To help us understand the nuances of beginning teachers' understanding we conducted six semi-structured interviews with secondary mathematics teachers towards the end of their first full year as a qualified teacher. The data were analysed as described in Sect. "Methods" using the mastery framework developed in Sect. "Constructs of mastery in mathematics education" and is discussed using the themes from Table 3.

Myths about mastery approaches As explained in Sect. "Constructs of mastery in mathematics education", there are multiple meanings of the word 'mastery' in the English context regarding mathematics teaching. Joanne articulates this, saying '*I think you know, the word 'mastery' … people tend to band it about and it can be used to cover a multitude of ideas and ways of teaching'* thus providing evidence as to the degree of complexity commonly found among English teachers regarding 'mastery' and identified by Blausten et al. (2020). Evidence of a range of understandings of mastery teaching were found in the interviews, for example, mastery teaching is '*more open-ended, like, discovery task*' (Jonathan), '*mastery is mixed ability sets*' (Cleona) and, from Komol, mastery means to use '*a practical aspect of the real-life application of a topic, do some experiments or something that involves the practical use of that topic or concept'*. There was evidence from some participants of a difficulty distinguishing between teaching for mastery and problem solving:

> I like to draw out a problem and quite a lot of the time I will do that. And it's trying to get them into that way as well, so I think …. getting it student led but… and like we said doing the diagnostic, doing the questioning of the student (Katie)
> So that no matter what problem they approach, they have a. really solid understanding of how things work, so they're not confronted by things that are slightly different and that they're not presented in the way they expect and can still come to a conclusion and get an answer (Mark)

We found evidence in the interviews for aspects of the features of the mastery framework developed in Table 1 but often not in the ways that these are articulated in Table 1. Mark, for example, echoes aspects of the view that the majority of learners are capable of learning mathematics when he justifies mixed attainment classes at lower secondary level by saying this lets '*everybody believe that they are capable'*. Joanne reflects aspects of using pupils' prior knowledge and experience to inform lesson design when she says, '*if they don't get it let's expand it in a different way*'. Some of the teachers articulated aspects of teaching for mastery that were about depth of understanding, considering misconceptions, reasoning and connections: '*all connecting up instead of doing them [topics] all separately*'

(Jonathan), '*reasoning your answers'*, (Jonathan), '*taking time to teach for understanding'* (Mark) and '*bringing up some, like, misconceptions'* (Katie). Joanne demonstrated an awareness of using different representations when she mentions '*tiles, cups and counters'*. Elements of the value of language and interactive dialogue are evident in Cleona's view that mastery teaching means *'let's have a good discussion on it'* and Jonathan that it includes '*speaking mathematically, improving your answers'*. There were small mentions of assessment-informed intervention in Joanne's view that it is '*a lot about assessment for learning*' and '*assessing before you move on'*. Some understanding of differentiation by enriching was evidenced by Jonathan's comment that '*you could have the exact same question and the exact same procedures, but you have just used…numbers that have decimals or fractions or something like that'*. Katie picks up principles of variation when she says the teacher needs to think about '*the subtle changes you're making between your examples so you know that there's subtly something different each time, seeing if the children notice what's different each time'*. These partial understandings are what one would expect from teachers working at the first of Fujii's (2014) levels and our data provides evidence that the features of mastery in Table 1 are at a higher level than these beginning teachers are yet able to achieve.

In contrast to the results where only 7% of the pre-service teachers surveyed believed that mathematics was a fixed ability, some of the teachers we interviewed suggested that mastery was only for particular groups of students, for example Joanne says: '*it could work really well than a certain way with SEN group and nurture group. But then a different approach would be good for a top set or a middle set'.* Komol works with a different definition of mastery and suggests that '*maybe the top ten kids are able to master it'* and that these are the students who go on to have the opportunities to tackle problem solving. In the following quote Komol seems to imply that mastery is a lot easier to achieve with well-behaved classes, where teachers can focus on the pedagogy and don't have to worry about managing behaviour, and that higher attainers are often better behaved:

> Having good control of the behaviour management, you know, because that as a new teacher as a trainee is best…most of the time that's in your head like you need to just tackle the behaviour so it kind of slips sometimes. Your bigger aim was…. yes, but it could be possible and maybe for trainee teacher to start using it more towards their top sets which where you don't have to deal with a lot of bad behaviour and then kind of you know getting the experience of it there and then bringing it back to their other classes (Komol)

Beliefs about teaching The interviews indicate that all these beginning teachers hold beliefs that support constructivist approaches to teaching and that they have a continued commitment to teaching in this way. They are clear about the need to teach for understanding and for learners to be active in constructing their own learning. For example, Mark says that students need a '*really solid understanding of how things work, so they're not confronted by things that are slightly different and that… they're not presented in the way they expect and can still come to a conclusion and get an answer*'. Komol says *'sometimes I'm not giving the answer directly and letting them kind of struggle'.* Several of those interviewed specifically indicated that they reject more transmissive approaches to teaching, supporting the finding that 6% of the pre-service teachers surveyed held the view that mathematics was learnt by following teacher direction. For example, Cleona says: '*when I think of a bad lesson… is when I've just kind of stood at the board and lectured—I've had too much teacher talk'.* Joanne refers to this sort of approach as '*the old fashioned stand at the front, tell them'* and

Katie '*a bit of an old style of teaching that you were given all of it at the start and then you just get on with the task'*. However, it is also clear from the interviews that these beginning teachers were struggling to enact their beliefs in their practice. Joanne, for example, says that her 'top sets' '*want to be told and then they will understand it through working through it themselves*'. Mark expresses a similar view suggesting that the expectations of students, school management and parents limit what he feels able to do in the classroom. Joanne also says that '*for some lessons it just has to be easy, if you're absolutely shattered yourself, or the class is difficult*'.

> I still think you could make this style [demonstration and exposition] work, but I do think you've got to think about how you are modelling them for the students, whether you're bringing in a misconception in one of the examples you're modelling, the subtle changes you're making between your examples so you know that there's subtly something different each time, seeing if the children notice what's different each time, involving them in the examples (Cleona).

Here, Cleona begins to articulate that simplistic identification of teaching style using surface features is insufficient through trying to reason that such approaches can effectively incorporate approaches such as conceptual learning and modelling.

Beliefs about the nature of good teaching, and the beliefs held at the beginning of their pre-service teacher education, are still evident in the interview data. In their interviews, they made references to the constructivist practices they had been exposed to on their teacher preparation programme. The practices they mentioned included the use of manipulatives, diagnostic questioning, formative assessment, collaboration, practical work and problem-solving Jonathan explains how he was taught trigonometry very procedurally as a school student himself before experiencing a more conceptual approach during his teacher preparation programme: '*I got taught trigonometry … like sohcahtoa, never really understood it, never meant anything to me. My class got taught a unit circle method for it. We were just so much more confident'*. However, relatively little of what teachers said in their interviews was mathematics-specific and this may indicate that, whilst continuing to hold constructivist views, beginning teachers are still struggling with applying their constructivist beliefs specifically to secondary mathematics teaching and that they are still developing subject-specific pedagogy for teaching mathematics.

Complexities and difficulties in professional practice The transcripts indicate that the beginning teachers interviewed identified several complexities in their lived experience as teachers. In the following quote Katie is trying to explain how the 'success' or otherwise of a lesson in school might depend on the expectations of the school or department.

> hard to say whether it's a bad lesson, 'cause if they're doing what exactly they've been told to do, if that's how they've been told: to demonstrate, to do the examples and then go and do worksheets. If that's the department doing it, well, it's quite hard to say that was a bad lesson 'cause technically, if the school, somebody from that department came and observed it, well, they're doing exactly the right steps they've been told to do (Katie).

Cleona makes a very similar point in her interview. In many schools in England teaching methods are prescribed and teachers are given a set of expectations regarding the structure and activities that are expected in a lesson in their department or school (Hutchings, 2015). There are many good reasons for such approaches and, when based on shared and negotiated pedagogical understanding, these can lead to a more cohesive learning journey for students and greater collaboration between teachers. However, if the teacher does not understand the underpinning pedagogy and reasons for the shared approach then the promoted teaching practices can become distorted and less effective. Cleona and Katie both express that understanding of 'good teaching' can be highly contextualised based on the expectations and norms in the department or school the teacher works in. Professional practice is therefore exposed as complex and difficult. Mark identifies another of these complexities when he explains that he feels completing pressures from different stakeholders who all have expectations of him: '*limitations in my experience are not so much from the teaching, … it's from students and* [*school*] *management, parents, who kind of go, 'How come you're doing that? Oh, that looks really interesting but, it's all good fun, but what are you doing that for? Why aren't you just teaching them something?''* His response demonstrates a difficulty with transforming practice because this is seen as taking a risk.

The beginning teachers interviewed further articulated that, in their understanding, 'mastery' is not 'normal teaching' and justified their lack of use of some of the approaches identified in Table 1 as being unattainable in their context. When asked about what they understood mastery to mean, the teachers interviewed listed many different elements of their teacher preparation programmes. There was very little said by any of the participants as to why mastery approaches might be beneficial to learners.

In the following example, Katie is trying to explain how she would teach some arithmetic with a focus on questioning for understanding but is not using language that supports the development of that understanding and is instead reproducing the language she learnt at school. Hence she refers to 'borrowing' rather than regrouping or exchanging.

> Why am I taking a number from that side and bringing it over? Why am I borrowing? What is that from? You're not just getting them to do the simple maths. You're getting them to model with you and understand, actually yeah, I'm borrowing because I need to take, so if we're doing it with the hundreds, that I need to borrow from the hundreds column and I need to increase my 10s (Katie)

We found that beginning teachers were not yet ready to talk about mathematics pedagogy early on in their teaching career. In several cases it was quite some time through the interview, and after several attempts, before the beginning teacher was able to articulate aspects of pedagogy with a degree of clarity. However, teachers tended to focus their explanations on a single aspect of practice, for example, problem solving or assessment for learning. In these ways teachers expressed specific and limited understanding with an embryonic development of pedagogy.

> **Tensions** I find some that my top sets just want to be told it - and if you start to do too much of an investigation or too much of a scaffolded …. they almost turn off. They just want to be told it and then almost the opposite. They want to be told (Joanne).

Joanne expresses a tension between wanting to take the more constructivist approach encouraged by her teacher preparation but feeling that such approaches are resisted in her school practice, in this example by higher-attaining students. As teacher educators using transformative and reflective pedagogies, we strive to provide safe spaces in university where pre-service teachers can consider and explore learning and teaching without having immediate responsibility for responding pupils. School placements are also intended to be a safe space where pre-service teachers are supported in trying out learner-centred teaching approaches with the support of their mentor, who is a more experienced teacher. Yet the tensions identified by the teachers interviewed included the notion of classroom activity that was 'safe' or 'risky' implying that they felt a lack of agency and had a deficit in their thinking here. The beginning teachers interviewed clearly felt pressure to 'perform' and to meet what they perceived to be the expectations of others. For example, in her interview, Joanne describes a structured investigation task finding the angle sum of polygons as a 'risky' choice for a lesson that was to be observed by a more senior colleague. Having been encouraged by the colleague to use the task she later reflected that taking the 'risk' had been worthwhile. In line with Gainsburg's (2012) study, the expectation that lessons needed to be 'successful' meant that the teachers interviewed felt less willing to take what they considered more 'risky' approaches and to try things out, despite having learnt this way in their teacher preparation. Our findings support Gainsburg's (2012) that teacher preparation programme-promoted teaching practices were considered to be time-consuming. For example, referring to teaching using the mastery features identified in Table 1 promoted by our practice as teacher educators, Komol said '*Generally it could be a bit more time consuming than the normal teaching*' and Cleona said that these were *'not as easy to use'*. Komol went on to say that she learnt about mastery approaches in her teacher preparation programme but that she *'wouldn't say I am using it at the moment myself'* and Cleona said *'even though you're taught lots of different ways, most people seem to just…. forward to the…. like a standard….'*. The tension felt was considered by these teachers to be between a traditional transmissive approach supported with closed exercises from a textbook and the more constructivist approaches they explored on their ITE programmes. Equity for all learners is central to the features of mastery in Table 1, and a key part of the curriculum in our teacher preparation programmes. However, tensions between these and actual classroom practice can be seen in Joanne's view that her '*top sets' 'want to be told it'* and that her *'low ability sets'* are *'the ones that they need to get something out to visualise it…. the conceptual side'.* This lack of equity for all can also be seen in Komol's suggestion that mastery approaches are more suitable for well-behaved classes.

Conclusions, implications and opportunities for teacher education

We now return to our three research questions. Regarding the first question, our evidence indicates that beginning teachers partially recognise aspects of mastery approaches in their own practice, and the practice of others. Elements of all the features articulated in Table 1 were found in our interview data although the features were not developed in full, did not realise the breadth and full meaning of the features in the table, and did not use the language of our framework. The beginning teachers interviewed tended to reduce the features of mastery to 'teaching for understanding' which they then set in opposition to more transmissive approaches characterised as 'teaching as telling'. Whilst our mastery features framework provides a useful tool for analysing our transcripts, we acknowledge that it does not capture the emotional aspects of either teaching or learning mathematics implicit in Beswick's (2007) description of the features of classrooms consistent with a constructivist view of learning.

Turning our attention to the second research question, 'How do beginning teachers align and justify their beliefs with their practice?', our data finds that this is an area of challenge and difficulty. The survey results with pre-service teachers indicated that they started their teacher preparation programme holding beliefs consistent with constructivism, which those interviewed still held 18 months later. However, the beginning teachers we interviewed gave justifications as to why they felt that they were currently unable to enact constructivist approaches in practice. We consider this as indicative of continuing challenges regarding the role of theory and its connection to practice.

Finally, our third research question: 'To what extent do their accounts of their teaching reflect mastery approaches?' The vignettes provided a mechanism for the beginning teachers interviewed to articulate practice through responding to an external stimulus. They then naturally moved on to discuss their own practice. Our evidence indicates that the elements of the mastery framework noticed in their own practice included an intention to teach for depth and meaning in pupils' understanding of mathematics. The beginning teachers interviewed gave examples showing that identifying misconceptions, and their awareness of these, was an important part of their practice. Aspects of taking into account prior learning were also evidenced through mention of the use of diagnostic questioning approaches and formative assessment. Some also indicated use of manipulative and of structural approaches to mathematics such as bar models.

Our work illuminates some of the clear contradictions and tensions present between policy and practice for both beginning teachers and for teacher educators. What our beginning teachers learnt whilst they were on their teacher preparation programme about constructivist approaches to learning was consistent with their previously held beliefs but was inconsistent with some of their interpretations of the practice they saw in schools and that they felt they were expected to follow. For us as teacher educators the contradiction is between preparing teachers for the realities in school and preparing them to be aspirational in their expectations of themselves and their own practice. As teacher educators taking a transformative approach to our practice, we consider our findings in the light of how we can best effect change and development in the practice of our pre-service teachers. We concur with those of Gainsburg (2012) whose research found that recent graduates had not made the practices emphasised in their pre-service programme central to their teaching but mainly employed traditional teaching practices. Her findings that "many of their actions are being "tried out" and are not yet rooted in their identity or philosophy" (2012, p. 362) resonates with our own findings, despite a very different methodological approach. That the challenges we face being transformative are significant is clear from both Wilkie (2019) and Hiebert (2013) who identify structural reasons as to why it is difficult to change classroom practice. Wilkie (2019) suggests that there are characteristics of schools that constrain the effectiveness of professional development. Hiebert (2013) further asserts that mathematics needs to be better understood as inherited cultural activity where change is often resisted. Our findings support this view of mathematics teacher development given that, as indicated earlier, the embedded use of a range of mastery approaches would be situated at Fujii's (2014) level 3. In an education system such as England's where there is a great deal of teacher turnover, beginning teachers often do not stay in the profession long enough to move up Fujii's levels and often leave whilst they are still at level 1 or in transition to level 2. Further, in a content-heavy and prescriptive teacher preparation system within a performative and prescriptive school system, such as that in England, there are expectations that teachers finish their preparation programme as fully-fledged teachers, and this provides a culture in which it is difficult for beginning teachers to change and develop. This provides a context for further levels of contradiction at policy level. Schools, and groups of schools, often have high levels of performativity, accountability, and prescription, supported by policy narrative that engages with 'what works' supporting a technicist role for teachers (Orchard & Winch, 2015). Yet the same policy narrative sees the English government funding the National Centre for Excellence in the Teaching of Mathematics to develop mathematics teachers who are highly skilled with the mastery features in Table 1 and for which teachers need to be of Orchard and Winch's (2015) 'professional' disposition and have sufficient professional learning to be working a Fujii's level 3.

Our work here on mastery approaches allows us to respond to external constraints as mathematics teacher educators to embed and connect current expectations and requirements such as the Core Content Framework (Department for Education, 2019) with elements of the key research literature in the field. It allows us to further our goal in preparing mathematics teachers who are critical and reflective with skills that will help them develop a career that will be longer term and adaptive to the future winds of policy change. Teacher education in England has a long history of policy flux. A research-informed foundation should allow beginning teachers to respond to policy changes in a way that is consistent with their beliefs. If teachers are not able to do this, concerns about recruitment, retention and long-term professional learning may be a continuing reality.

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