Drag Characteristics of a Volumetric Tree Model in Computational Fluid Dynamic Simulations

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1 INTRODUCTION

Trees play a crucial role in urban environments, offering potential solutions to significant challenges like urban air quality and the urban heat island effect by modifying the local exchanges of momentum, radiation, moisture, heat and pollution. Given the multi-fold impact of trees on urban ecosystem, they must be included in the microscale models of urban environment to obtain an accurate prediction of the urban wind flow [1]. The assessment of trees' impact on urban areas is often investigated through urban microclimate models relying on either wind tunnel experiments with small-scale tree models [2, 3] or volumetric source models in computational fluid dynamics (CFD) simulations [4]. However, it remains unclear how to map the aerodynamic characteristic of model trees in experiments or of large urban trees in CFD simulations accurately.

Often in CFD based micorscale urban models, trees are modelled following a 'grey scale' approach [4], *i.e.*, neither the tree effects are entirely parameterized as done in mesoscale models, nor is the tree behaviour resolved going in the level of tree branches and leaves. Instead, trees are modelled as porous volumetric blocks using a volumetric source/sink term (S_{u_i}) in the momentum equation. This source/sink term accounts for the combined inertial and viscous losses introduced by the presence of the tree, and has a quadratic dependence on the local wind velocity as given by [5],

$$S_{u_i} = -aC_d^V |u_i| u_i \tag{1}$$

where u_i indicates wind velocity, a is the leaf-area density defining the one-sided surface area of leaves over the volume of air that they are present in [6], and C_d^V is the volumetric drag coefficient. The flow turbulence due to the tree-air interactions is resolved using appropriate turbulence models. Notably, a and C_d^V are the two main parameters required as inputs to perform a CFD simulation with a tree canopy.

In contrast to CFD simulations, wind tunnel experiments with model trees compute the bulk drag coefficient C_d based on the bulk drag force measured using the force measuring instruments. The bulk drag coefficient is defined as

$$C_d = \frac{2F_D}{\rho U_\infty^2 A_F} \tag{2}$$

where F_D indicates the bulk drag force, ρ is wind density, U_∞ is the streamwise free stream velocity, and A_F is the projected frontal area. Another important quantity often measured in wind tunnel experiments, that in a way represents the aerodynamic resistance of a tree, is the aerodynamic porosity (α) defined as the ratio of volume flux going through the tree to the volume flux entering at the windward plane of the tree canopy [7],

$$\alpha(x) = \frac{\iint_{A_F} \bar{u}(x,y,z)\, dy\, dz}{\iint_{A_F} \bar{u}(0,y,z)\, dy\, dz} \tag{3}$$

where $\bar{u}$ is the mean streamwise wind speed, and x, y, z indicate the coordinate directions.

As listed above, the CFD simulation with a tree canopy requires a and C_d^V as inputs whereas the wind tunnel experiments with model trees measure the quantities C_d and α. However, a direct correlation between these is yet to be achieved. Note that C_d being defined based on a two-dimension projected area, is different from the volumetric coefficient C_d^V. Thus it is not clear what values for a and C_d^V to use as inputs to replicate the aerodynamic characteristics of an experimental model tree in a corresponding CFD simulation accurately. The present study aims to establish a connection between the two set of parameters

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in an appropriate manner such that both the CFD and wind tunnel model trees yield comparable aerodynamic traits. To this regard, a simplified canopy model for the flow inside a vegetation canopy is proposed in Section 2 and the supporting results are presented in Section 3. Finally, the main findings are concluded in Section 4.

2 ANALYTICAL MODEL FOR THE FLOW INSIDE A VEGETATION CANOPY

A simplified steady-state Reynolds-averaged horizontal momentum equation considering only the horizontal advection, the pressure gradient, and the source/sink term modelling the tree drag can be written as

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{p}}{\partial x} = -aC_d^V \bar{u}^2 \tag{4}$$

where $\bar{p}$ indicates mean pressure upon density. Here, we consider a homogeneous vegetation canopy of length L, width W and canopy height h, with the base of the canopy being at a height of h_0 from the ground; see in Fig. 1. Averaging Eq. 1 over the canopy frontal area gives,

$$\frac{dU^2}{dx} + \frac{2}{c}\frac{dP}{dx} = -2aC_d^V U^2 \tag{5}$$

where $U = (Wh)^{-1}\iint_{A_F} \bar{u}\, dA$, $P = (Wh)^{-1}\iint_{A_F} \bar{p}\, dA$, and c is a shape factor defined such that $cU^2 = (Wh)^{-1}\iint_{A_F} \bar{u}^2\, dA$. In order to be able to obtain a closed form solution for Eq. 5, a key assumption is made at this stage that the pressure gradient inside the tree canopy evolves in the same way as the inertial term, *i.e.*,

$$\frac{2}{c}\frac{dP}{dx} = \beta\frac{dU^2}{dx} \tag{6}$$

where β is a coefficient assumed to be independent of x and accounts for all the complex three dimensional flow phenomena not considered in the flow governing equations above (Eqs. 4 and 5). The results will show that this assumption works reasonably well; although other parameterisations are conceivable, this one stands out for its simplicity. Additionally, the leaf area density and the volumetric drag coefficients are combined to a single quantity which we define as the drag length $\ell_d = (aC_d^V)^{-1}$. Substituting these assumption into Eq. 5, one can obtain the equation for the bulk flow inside a tree canopy as

$$(1+\beta)\frac{dU^2}{dx} = -\frac{2}{\ell_d}U^2 \tag{7}$$

which has a closed form solution given by,

$$U^2 = U_0^2\, e^{-\frac{2}{1+\beta}\cdot\frac{x}{\ell_d}} \tag{8}$$

where U_0 is the average streamwise wind speed at the windward plane of the canopy, *i.e.*, $U_0 = U|_{x=0}$ assuming the vegetation canopy starts at $x = 0$. From Eq. 7, the aerodynamic porosity can be derived following Eq. 3 and is given by,

$$\alpha^2 = \frac{U^2}{U_0^2} = e^{-\frac{2}{1+\beta}\cdot\frac{x}{\ell_d}} \tag{8}$$

This Eq. 9 is the key equation that describes the evolution of bulk flow inside the tree canopy. This simplified analytical model finds an exponential decay of the bulk flow inside the canopy along its length.

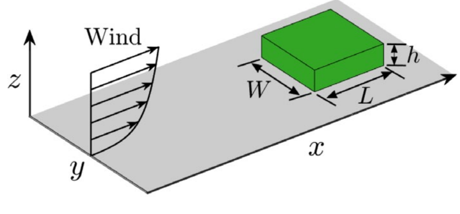

Figure 1, schematic diagram showing a simplified tree canopy model

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Note in Eq. 9, the aerodynamic porosity α in the left hand side can be directly measured in a wind tunnel experiment by measuring the velocity field at the windward and leeward plane of the tree canopy. On the other hand, the drag length ℓ_d in the right hand side of Eq. 9 is directly related to the input parameters a and C_d^V required in the CFD simulations. Thus the Eq. 9 establishes the direct interconnection between the wind tunnel experiments and CFD simulations. We propose the drag length ℓ_d to be the main parameter that dictates the bulk aerodynamic signature of a vegetation canopy. For a tree canopy in a wind tunnel experiment and in a CFD simulation (of same length scale) to be of comparable aerodynamic traits, the drag length must remain the same in both these situations.

3 VALIDATION OF THE PROPOSED ANALYTICAL MODEL

The credibility of the proposed analytical model for the bulk flow inside a vegetation canopy (Eq. 9) is tested through a series of large eddy simulations. The simulations are performed using uDALES [8] which is a multi-physics micro-climate modelling framework for urban environment. It performs the large eddy simulation of the incompressible Navier-Stokes equations within the Boussinesq approximation. Grylls and van Reeuwijk [4] included trees in the uDALES framework, modelling the drag, shading, evaporation and deposition. Trees are modelled in uDALES as rectangular blocks, and the source/sink term defined in Eq. 1 is applied to the grid points that fall within the volumetric tree blocks. In the present simulations, a rectangular tree block is considered with $L \times W \times h = 4.5 \times 4.5 \times 6.5$ m^3, being inspired by the dimension of the model tree used in the wind tunnel experiments of Fellini et al. [3]. The base of the tree block is at $h_0 = 2$ m height from the ground, hence the crown of the tree is at a height of $h_{ref} = h_0 + h = 8.5$ m from the ground. A comprehensive parametric study is performed covering wide ranges of C_d^V and a values, summarized systematically in Table 1 along with the resulting C_d and α_L $(= \alpha|_{x=L})$ computed from the simulation outputs. Note in Table 1, the individual values of the input parameters C_d^V and a are not important and are interchangeable; the output aerodynamic properties are primarily governed by the combined drag length ℓ_d. All the cases C11 - C15 results in the same C_d and α_L as dictated by $\ell_d = 5$ m in these cases, though a and C_d^V took values of various combinations. This finding corroborates the theory proposed in Section 2. Moreover, it is observed that a decrease in drag length increases C_d and decreases α_L.

Next, the contour plots of mean streamwise wind velocity and the turbulent kinetic energy at the mid-span vertical plane are presented in Figs. 2a and 2b respectively, for a typically chosen case form Table 1. Presence of tree canopy slows down the incoming flow as it enters into the canopy. Both flow velocity and TKE decay towards the leeward side of the canopy. Similar trend is seen for all remaining cases as well, and therefore not repeated here only for the sake of brevity. For a quantitative representation, the variation of aerodynamic porosity along the length of the tree canopy is shown in Fig. 2c. Note that the evolution of α obtained from the simulation results indeed reflects an exponential decay agreeing well to

Table 1, parameter space: cases C1-C5 vary C_d^V, C6-C10 vary a, C11-C15 vary both C_d^V and a such that ℓ_d remains constant, and C16-C20 are representative limiting ℓ_d cases

Type	C_d^V	a [m^2/m^3]	ℓ_d [m]	C_d	α_L	Type	C_d^V	a [m^2/m^3]	ℓ_d [m]	C_d	α_L
C1	0.1	1.0	10	0.657	0.61	C11	0.1	2.0	5	0.893	0.43
C2	0.15	1.0	6.67	0.804	0.48	C12	0.15	1.33	5	0.893	0.43
C3	0.2	1.0	5	0.893	0.43	C13	0.2	1.0	5	0.893	0.43
C4	0.25	1.0	4	0.954	0.37	C14	0.25	0.8	5	0.893	0.43
C5	0.3	1.0	3.33	0.998	0.32	C15	0.3	0.67	5	0.893	0.43
C6	0.2	0.5	10	0.657	0.61	C16	0.0	0.0	∞	0.0	1.0
C7	0.2	0.75	6.67	0.804	0.48	C17	0.2	0.1	50	0.189	0.91
C8	0.2	1.0	5	0.893	0.43	C18	0.15	0.5	13.33	0.55	0.69
C9	0.2	1.25	4	0.954	0.37	C19	0.3	2.0	1.67	1.1	0.15
C10	0.2	1.5	3.33	0.998	0.32	C20	0.3	3.0	1.11	1.127	0.07

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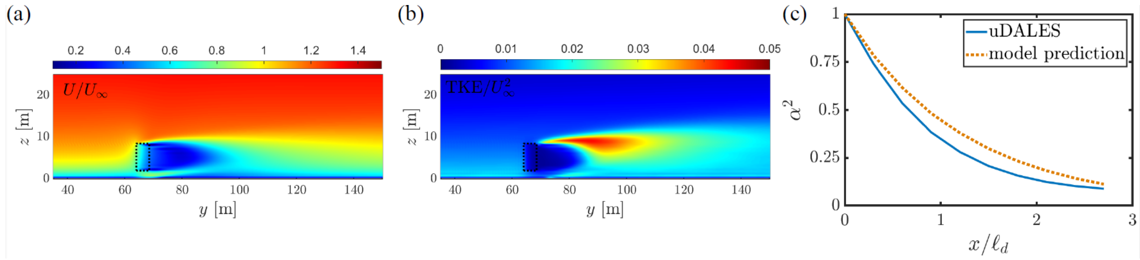

Figure 2, contour of mean (a) streamwise velocity and (b) turbulent kinetic energy at the mid-span vertical plane for the case C3, and (c) the corresponding evolution of aerodynamic porosity inside the canopy

Eq. 9 derived in Section 2, thus establishing the credibility of the proposed theoretical model. An appropriate value of the coefficient β in Eq. 9 is estimated by extracting $\alpha^2(x)$ inside the canopy such that the simulation output result best fits Eq. 9.

4 CONCLUSIONS

The current study contributes towards better understanding of the interplay between volumetric tree modelling parameters and the drag behaviour of wind tunnel tree models. It reveals that the aerodynamic characteristics of the tree model does not depend on the individual modelling parameters C_d^V and α, but the combined drag length ℓ_d defined in this work plays the pivotal role in determining C_d and α. The present work proposes a simplified analytical model for the evolution of bulk flow inside a tree canopy, and derives an equation for aerodynamic porosity as it evolves along the length of the canopy. This theoretical model provides a direct interlinking between aerodynamic porosity, which is a measurable quantity in wind tunnel experiments, and the volumetric tree modelling input parameters required in CFD simulations. Drag length ℓ_d is the novel parameter defined here to act as the bridge between experimental and simulation approaches, leading to potential equivalences in aerodynamic characteristics of the tree models used in these approaches. This establishes a practical mean for emulating an experimental tree model/natural tree to reproduce the corresponding drag behaviour in CFD simulations accurately.

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