A multi-level data-driven Bayesian approach to identify probabilistic stability of aeroelastic limit cycle oscillations

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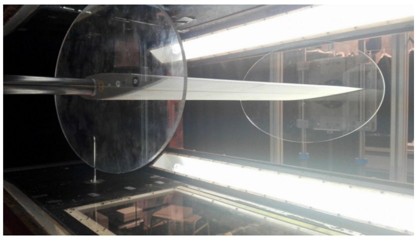
Summary. This study introduces a probabilistic approach to assess the stability of aeroelastic limit cycle oscillations. This study introduces a multi-level data driven approach to identify the probablistic stability of aeroelastic limit cycle oscillations through limit experimental data. Utilising the Hill/Koopman method, data-driven models are trained to capture eigenvalue behaviour. The stability likelihood of the limit cycle oscillations is evaluated by analysing the percentage of stable responses in Monte Carlo experiments based on parameter estimates obtained through the multilevel data-driven approach. The effectiveness of the method is demonstrated using a nonlinear aerofoil test case, revealing that it provides accurate stability information compared to experimental data.

Introduction

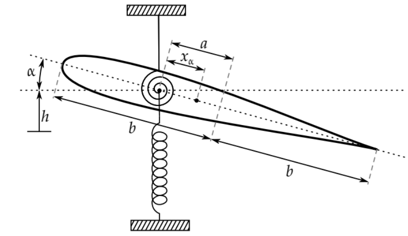
Research in nonlinear aeroelasticity is crucial for ensuring the safe operation of aerospace systems, especially with the increasing use of flexible lightweight structures and multifunctional materials. Nonlinear behaviours in aeroelastic systems, including Limit Cycle Oscillations (LCO), are commonly observed below the linear flutter threshold and can lead to stable or unstable self-sustaining oscillations. Experimental techniques like control-based-continuation (CBC) have proven successful in detecting both stable and unstable LCO occurrences [1]. However, computational analyses often overlook these nonlinear effects due to their complexity and high computational costs.
Model updating techniques, such as Bayesian model updating, offer promising avenues for capturing nonlinear behaviour in aeroelastic systems [2, 3]. Recent studies have demonstrated the effectiveness of data-driven approaches in probabilistically characterising LCO behaviour, albeit without considering stability. This study aims to introduce and validate a probabilistic data-driven method to identify LCO stability. Leveraging the Hill/Koopman method, which reduces computational costs in stability analysis, this approach aims to provide accurate and efficient estimates of aeroelastic LCO stability in the frequency domain [4]. This innovative method aims to improve computational efficiency while maintaining accuracy comparable to traditional time domain methods.

Methodology

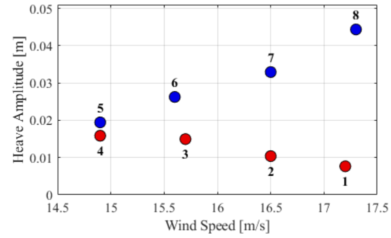
The presented methodology is a direct extension of the multilevel process outlined in previous work [3]. This process aims to probabilistically estimate nonlinear parameters based on experimental data obtained through CBC experiments, followed by a probabilistic estimation of dynamic behaviour. Initially, training data describing LCO behaviour is collected from a nonlinear aeroelastic system. The Harmonic Balance method continuation framework is employed to minimise computational costs. Within this framework, the amplitude and natural frequency of LCO are iteratively tracked and stored as training data. Kriging surrogate models are generated to describe LCO amplitude until a user-defined convergence criterion is met. Subsequently, a sufficiently accurate data-driven model is developed, and an estimate of nonlinear parameters is obtained through BMU, employing Transition Markov-Chain Monte Carlo sampling. The updated probabilistic parameter estimates serve as the design space for drawing training data in the subsequent level of the process. New data-driven models are constructed at each level to further refine the parameter estimates, thereby achieving a more precise and accurate estimation of the system. Following the final level, the converged parameter estimates are utilized to generate a probabilistic bifurcation diagram within 95% confidence bands via standard Monte Carlo sampling, thereby describing the nonlinear dynamic behaviour of the system.
This work proposes a novel step to provide a probabilistic estimate of LCO stability in the final level. Using the same dataset from the final level, Hill's matrix is constructed from each set of training data and subsequently reduced to the monodromy matrix through the Koopman operator. Typically, obtaining the monodromy matrix necessitates conversion to the time domain, which is computationally costly. Stability information can be obtained directly from the Hill's matrix but it has been shown in previous studies that reducing to the monodromy matrix lessens the number of eigenvalues to be computed, resulting in computational savings [4]. The eigenvalues of the monodromy matrix are then stored as training data, and through Kriging, data-driven models are established to relate LCO amplitude and nonlinear parameters to the real parts of the eigenvalues (Floquet multipliers). The probabilistic estimate of LCO amplitude with respect to nonlinear parameters is fed through the data-driven models describing Floquet multipliers, and stability is assessed based on whether any of the multipliers are positive, indicating instability. A percentage chance of stability is determined by considering the proportion of stable and unstable samples at each point in LCO amplitude. This approach, rather than providing a binary decision on the stability of LCO, offers a probability of stability.



(a) Experimental configuration

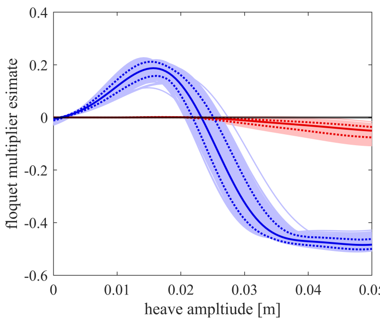


(b) 2 DOF Freebody diagram

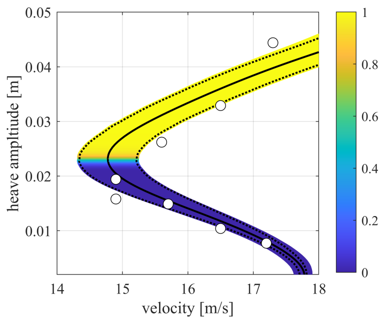


(c) Labelled experimental data (● stable LCO) (● unstable LCO)

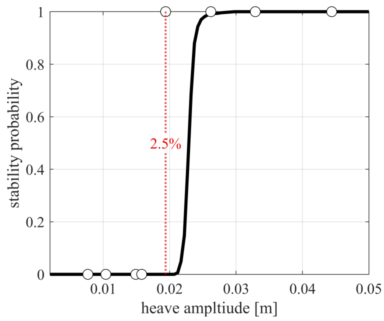
Figure 1: Case study



(a) Probabilistic eigenvalue response (■ Floquet multiplier 1), (■ Floquet multiplier 2)



(b) Probabilistic response



(c) Stability likelihood

Figure 2: Probabilistic stability response

Case Study

The implemented test case aimed at validating the proposed framework involves an aerofoil that demonstrated LCO during wind tunnel testing, as shown in Fig 1. Mathematically, the system is described by a two-degree-of-freedom model. The degrees of freedom in this study are heave (h) and pitch (α).

Nonlinearity is introduced in the pitch degree of freedom in the form of $K_{\alpha 2}\alpha^2 + K_{\alpha 3}\alpha^3$, with parameters $K_{\alpha 2}$ and $K_{\alpha 3}$ being estimated through data-driven model updating. Experimental data obtained via CBC in Fig 1c, has been previously employed to generate results in previous research [3]. The results include a bifurcation diagram that provides a probabilistic estimate of LCO amplitude. However, the existing information does not cover the stability of the LCO.

Using training data extracted from the final level of the preceding study, data-driven models were developed to characterise the key Floquet multipliers. Analysis revealed that only Multipliers 1 and 2 consistently indicated stability transitions and were thus identified as the primary Floquet multipliers. These models were trained with the same 100 inputs from the final level, which were used to train the LCO amplitude data-driven model. Notably, the accuracy of the Floquet Multiplier models was found to be 98.2% and 95.4% for Multipliers 1 and 2, respectively.

To assess stability, 1000 Monte Carlo samples of nonlinear parameters were inputted into the data-driven models across a range of LCO amplitudes. Fig 2a illustrates that at low amplitudes, both Multipliers are positive, transitioning to negativity at higher amplitudes, indicating a smooth shift from unstable to stable LCO at the bifurcation diagram's turning point. Although stability prediction for seven out of eight CBC points was accurate, the bifurcation diagram's confidence band did not fully encompass all points. Notably, the lowest amplitude stable LCO (point 5) was misclassified as unstable, likely due to inaccuracies in amplitude prediction rather than stability estimation.

This study demonstrates the feasibility of probabilistically estimating LCO stability without additional training data, as outlined in [3]. Future research should focus on refining the process to better capture experimental data behavior regarding LCO amplitude.

References

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