Parameter Optimization and Energy Flow Analysis of the Digital Piezoelectric Vibration Absorber with Nonlinear Synthetic Inductance

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Abstract. A digital nonlinear synthetic inductance is incorporated into the resonant shunt circuit in order to effectively mitigate nonlinear vibrations. The tuning of the nonlinear inductance is determined based on the principle of similarity. The dynamic response is obtained using the harmonic balanced method (HBM). The vibration power flow characteristics of the nonlinear system with the nonlinear piezoelectric tuned vibration absorber (NPTVA) are analysed to understand the mechanism behind the vibration suppression provided by the nonlinear inductance shunt circuit. A comparison with the linear piezoelectric tuned vibration absorber (LPTVA) reveals that the proposed NPTVA exhibits adaptability to the hardening phenomenon associated with nonlinear vibrations. Based on the power flow analysis, the surrogate model-based optimization is employed to determine the circuit parameters resulting in improved vibration performance compared to the analytical approach. The optimization results reveal that as the excitation amplitude increases, the optimal values for nonlinear inductance also need to be increased.

1. Introduction

Piezoelectric tuned vibration absorbers (PTVAs) have attracted significant interest in vibration control [1,2]. Among the various shunt circuit options, the resonant circuit (RL circuit) has emerged as a widely used configuration [3]. Previous research has provided tuning rules for the RL shunt circuit based on pole placement techniques and receptance transfer function methods [1]. However, linear resonant shunt circuits are restricted by the need for precise tuning of the electrical resonance frequency, which can be challenging due to structural nonlinearities. Consequently, this technology is often criticized for its lack of robustness [4].

The concept of a nonlinear piezoelectric tuned vibration absorber (NPTVA) was introduced by Soltani and Kerschen [5]. Unlike its linear counterpart, the NPTVA offers a broader range of vibration suppression capabilities for nonlinear vibrations. However, implementing complex nonlinear laws using analogue circuits poses significant challenges. To address this limitation, Alfahmi et al. [6] recently proposed a new nonlinear synthetic inductance. This research paves a new way for the design of NPTVA.

In this research, the integration of a digital nonlinear synthetic inductance into the PTVA is proposed to suppress nonlinear vibrations. The tuning rule is established based on the principle of similarity firstly. Using the harmonic balanced method (HBM), a comparative analysis of vibration suppression performance is conducted between the LPTVA and the NPTVA subjected to varying excitation levels. Furthermore, a comprehensive analysis of power flow is conducted. Based on the power flow analysis, the surrogate model-based optimization technique is employed to accurately obtain the circuit parameters with enhanced vibration performance compared to the analytical method. The optimization results demonstrate a clear relationship, indicating that as the excitation amplitude progressively increases, there is a corresponding need for the optimal nonlinear inductance values to be incremented. The investigation highlights the adaptability of the NPTVA in handling nonlinear vibrations.

2. System model and derivation validation

In this section, the governing electro-mechanical model for a NPTVA is derived. The PTVA's nonlinearity is introduced through a programmable cubic inductive shunt.

The system is governed by the following equations:

$$\begin{gathered} m\ddot{x} + c\dot{x} + k_{oc}x + k_{nl}x^3 - k_{cp}v_p = f \\ k_{cp}\dot{x} + C_p\dot{v}_p + K_R v_p + K_L\int_0^t v_p(\tau)dt + K_{Lnl}\left(\int_0^t v_p(\tau)d\tau\right)^3 = 0 \end{gathered} \tag{1}$$

in which k_{oc} denotes the total structural stiffness, m is the mass, c is the damping, x is the displacement of the oscillator, k_{cp} is the coupling coefficient; f represents harmonic external force, C_p is the capacitance of the piezoelectric transducer at constant strain, v_p is the voltage generated by the piezoelectric transducer. K_R, K_L, K_{Lnl} denote the equivalent electrical component: resistor, inductor and cubic inductor.

Introducing auxiliary variables $\boldsymbol{r}$, the dynamic equations of the coupling system can by rewritten as:

$$\begin{cases} m\dot{v}_1 + cv_1 + ky_1 + k_{nl}y_1r_1 - k_{cp}v_2 - f = 0 \\ C_p\dot{v}_2 + K_R v_2 + K_L y_2 + K_{Lnl}y_2r_2 + k_{cp}v_1 = 0 \end{cases} \tag{2}$$

The nonlinear electro-mechanical system of Eq.(2) is solved by the harmonic balance method (HBM) combined with asymptotic numerical method (ANM) using the nonlinear dynamic analysis research software MANLAB [7].

3. Parameter study and energy dissipation of nonlinear PTVA

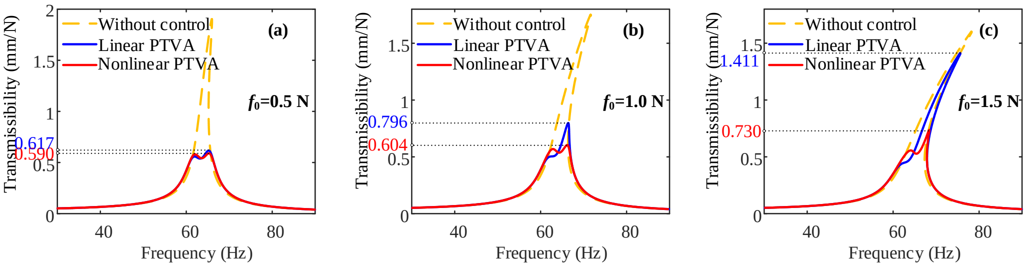

Figure 1. Frequency response of the system in Figure 1 controlled by linear and nonlinear PTVA under different excitation level: (a) f_0=0.5N, (b) f_0=1.0N, (c) f_0=1.5N.

Soltani and Kerschen [5] established an analytical tuning rule for nonlinear capacitance within piezoelectric shunt damping using the principle of similarity. For the nonlinear inductance employed in this research, the tuning rule can be written as: $K_{Lnl} = 2\left(C_p / m\right)^2 \times k_{nl}$.

The vibration suppression comparison results are shown in Figure 1. As the forcing amplitude increases, the linear absorber becomes severely detuned. The suppression effect of linear PTVA dropped from 67.5% to 11.9%. When the excitation force increases to 1.5 N, the vibration suppression performance decreases significantly because of the detuning. Conversely, the nonlinear PTVA performance remains almost unchanged as the force amplitude increase from 0.5N to 1.5 N. As shown in Figure 1 (c), with the increase of the force amplitude, the nonlinear circuit will still be out of tune. To further investigate the vibration suppression principle of the nonlinear PTVA, it is important to establish comprehensive understanding on the mechanism of the energy dissipation of the PTVA. As shown in Figure 2, the ratio of the NPTVA bends to the high-frequency range with the force level increases, while the LPTVA remain unchanged. In other word, the NPTVA has certain adaptability to the hardening phenomenon of the nonlinear vibration.

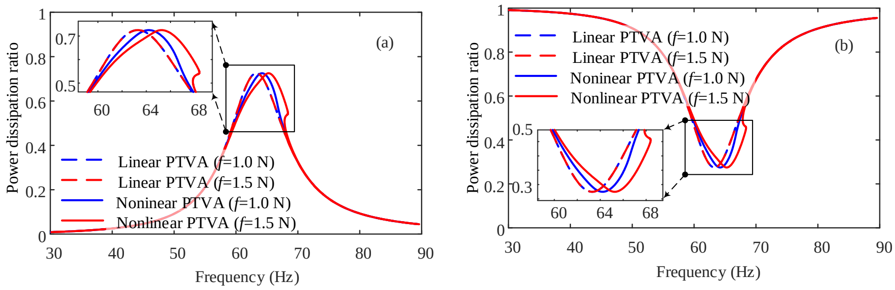

Figure 2. Comparison of power dissipation ratios of the system under the control of linear and nonlinear PTVA with different level of excitation: (a) R_{dR} (b) R_{dc}.

4. Surrogate model-based optimization

To further optimise the parameters, the surrogate model-based optimization is carried out. This algorithm considers a function $E_k(\mathbf{p})$, which represents the maximum kinetic energy generated through the circuit parameters $\mathbf{p}=(R, L, L_{nl})$. An objective function is defined as: $\max\{E_k(\mathbf{p})\}$ when $b_{li}<p_i<b_{ui}$ (i=1,2,3). Firstly, a surrogate of the objective function is constructed by evaluating it at

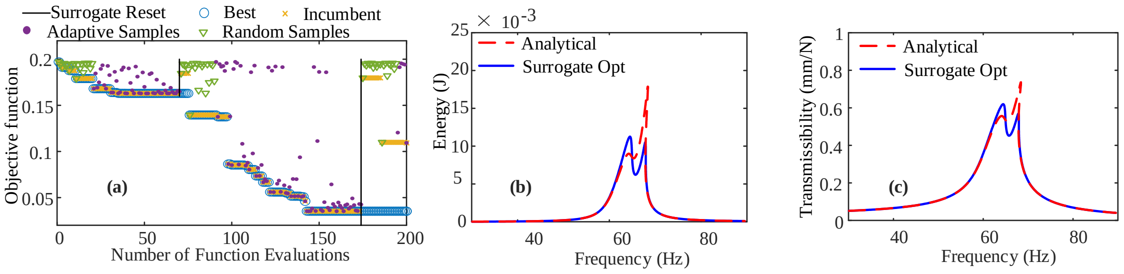

Figure 3. Surrogate model-based optimization of NPTVA: (a) monitor of the process (b) maximum kinetic energy and (c) transmissibility of the coupling system with f_0=1.5 N.

randomly generated points. The minimum value is found by evaluating a merit function based on the surrogate value [8]. Subsequently, the surrogate model is updated, and the search process is repeated until the distance is less than the tolerance.

As shown in Figure 3, when the excitation amplitude is low, the vibration suppression performance achieved with the parameters obtained through the surrogate optimization algorithm is comparable to that achieved with the analytical tuning rule. However, as the excitation amplitude increases, the former outperforms the latter in terms of performance. The optimal parameters obtained from both methods indicate that the resistance and linear inductance values have minimal deviations from the theoretical calculations. The significant variation observed is in the nonlinear inductance values optimized by the surrogate model: as the excitation amplitude increases, the nonlinear inductance values also increase.

5. Conclusion

This study proposed a digital NPTVA composed of the digital nonlinear synthetic inductance. The tuning rule of the nonlinear inductance is given based on the principle of similarity in which the ratio inherent capacitance of the piezoelectric patch and the mass of the primary structure is taken as the mass ratio. The HBM is used to obtain the dynamic response of the electro-mechanical coupling system. The nonlinear vibration suppression performance comparison between the LPTVA and the NPTVA is conducted under different excitation level. The power dissipation ratios comparisons reveal that the NPTVA has certain adaptability to the hardening phenomenon of the nonlinear vibration. This characteristic of NPTVA allows it to better control nonlinear vibration. Drawing upon a comprehensive power flow analysis, the utilization of surrogate model-based optimization emerges as an effective strategy to determine the most appropriate circuit parameters The optimization outcomes substantiate a positive correlation, indicating that with an increase in the excitation amplitude, there exists a corresponding requirement for an upward adjustment in the optimal values of the nonlinear inductance.

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