Predator-Prey Nonreciprocal Interactions of Toroidal Charge-Current Configurations

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Abstract – In 2001, G. N. Afanasiev predicted that electromagnetic interactions between toroidal charge-current configurations can be nonreciprocal. Here, we show that interacting toroidal coils of different orders break the Newtonian action-reaction equality without breaking Lorentz reciprocity. Such interactions between toroidal charge-current configurations resemble the predator-prey behavior in living systems and chemical kinetics and can underpin the recently identified mechanism of the mobilization phase transition to the new state of matter, the time crystal.

Action-reaction principle dictates that forces between objects in a closed system have equal magnitude and are directed in opposite directions. In electromagnetism, the action-reaction equality is typically expressed in the form of the Lorentz or Feld-Tai reciprocity lemmas. These formulations were challenged in 2001 by G. N. Afanasiev who predicted a hierarchy of charge-current configurations of toroidal symmetry violating both the action-reaction principle and the Lorentz/Feld-Tai reciprocity lemmas [1]. To date, these predictions remain largely unnoticed. Here, we demonstrate action-reaction equality violation in a system of interacting toroidal charge-current configurations driven by an incident electromagnetic wave. We show that such systems exhibit intriguing "predator-prey" dynamics, which are usually encountered in active matter.

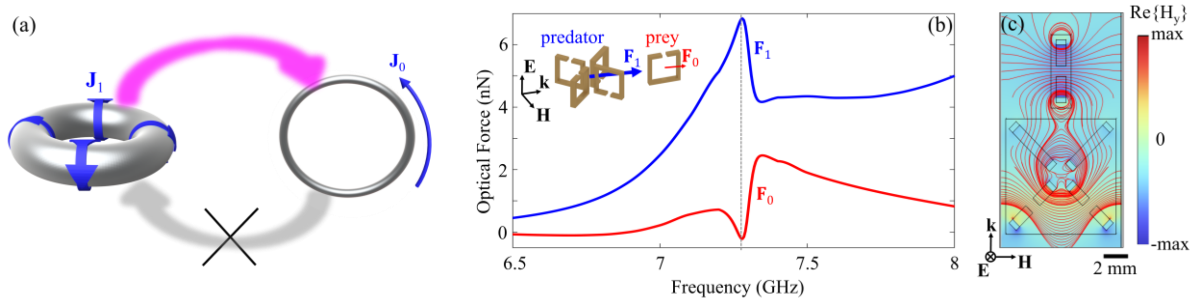

Figure 1: a) Nonreciprocal interactions of toroidal charge-current configurations of 1st order (poloidal current, left) and 0th order (ring current, right). (b) Optical forces in a metamaterial array of toroidal metamolecules (1st order toroidal current) and split-ring resonators (0th order toroidal current) under plane wave illumination with linearly polarized light. Blue and red lines correspond to the optical force component along the propagation direction acting on the 1st and 0th order toroidal currents, respectively. Inset: schematic of the metamaterial unit cell; the metamaterial extends to infinity in the plane normal to the propagation direction (**k**). The vertical dashed line marks the frequency at which the optical force on the split-ring resonators vanish. (c) Magnetic field map at a frequency of 7.28 GHz (see dashed line in (b)). Red lines represent magnetic field stream lines.

We consider the interactions in a metamaterial consisting of toroidal metamolecules [2] and split-ring resonators (see inset to Fig. 1b) corresponding to toroidal charge-current configurations of 1st and 0th order, respectively, driven by an incident linearly polarized plane wave. We model the response of the metamaterial by a finite element solver and retrieve the forces between charge-current configurations of different order by integrating Maxwell's stress tensor. Figure 1b presents frequency spectra for optical forces along the propagation direction, F_1 and F_0, acting on the toroidal metamolecules and split-ring resonators, respectively. The force acting on the former (latter) is substantially stronger (weaker) throughout the frequency spectrum indicating "predator"-like ("prey"-like) behaviour. Importantly, at ~7.28 GHz the force acting on the 1st order toroidal configuration is at a maximum, while the force on the 0th order configuration vanishes (see vertical dashed line, Fig. 1b), resembling a "captive" state.

In summary, we have demonstrated nonreciprocal behavior in charge-current configurations of toroidal topology, reminiscent of the predator-prey dynamics of living systems. Our results provide insights in the validity of "common knowledge" reciprocity lemmas in electrodynamics and will be of interest in the study of time crystals and their phase transitions [3].

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[3] T. Liu, J. Y. Ou, K. F. MacDonald, N. I. Zheludev, "Photonic metamaterial analogue of a continuous time crystal," Nature Physics 19, 986 (2023).