Nonreciprocity in Toroidal Electrodynamics

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According to Newton's 3rd law (action-reaction principle), the forces between objects in a closed system possess equal magnitude and are oppositely directed. Within the domain of electromagnetism, this law is usually articulated through the Lorentz and Feld-Tai reciprocity lemmas. The validity of electromagnetic reciprocity theorems was challenged in 2001 by G. N. Afanasiev [1], who predicted nonreciprocal interactions between charge-current configurations of toroidal topology. In fact, Afanasiev introduced an infinite hierarchy of such electromagnetic sources, termed supertoroidal currents, constructed by successive fractal transformation of a ring current. Remarkably, the nonreciprocal character of interacting supertoroidal currents remains largely unexplored. Here, we demonstrate that a system comprising supertoroidal currents of different order violates the action-reaction principle without breaking the Lorentz/Feld-Tai lemmas.

We consider metamaterial arrays comprising toroidal resonators supporting poloidal resonant currents (supertoroidal currents of 1st order) and split-ring resonators (SRRs) supporting ring currents (supertoroidal currents of 0th order) driven by an incident linearly polarized plane wave. We show that optical forces in such a metamaterial are non-reciprocal and the force acting on the toroidal resonators is substantially stronger than the force experienced by the SRRs. This behaviour is reminiscent of the predator-prey dynamics in living systems and chemical kinetics. Moreover, the character of the optical force depends crucially on the direction of propagation of the driving electromagnetic wave. Reversing its propagation direction, a process usually equivalent to time reversal, restores the action-reaction principle with the optical forces being repulsive and of equal magnitude.

In conclusion, we report on action-reaction principle violations in a system of toroidal charge-current configurations, similar to the interplay seen in predator-prey relationships in active matter. Our findings question the validity of the conventional reciprocity theorems and provide insights into the dynamics of many-body light-driven systems, such as photonic time crystals.

1. G. N. Afanasiev, "Simplest sources of electromagnetic fields as a tool for testing the reciprocity-like theorems," *Journal of Physics D: Applied Physics* 34, 539 (2001).