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Supertoroidal Non-Radiating Configurations

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Abstract – **We report on a new type of non-radiating charge-current configurations, termed supertoroidal anapoles, excited in dielectric particles under illumination with toroidal light pulses. We show that such non-radiating excitations are linked to supertoroidal currents induced in the particle leading to suppression of scattering by over 70%.**

The emergence of toroidal electrodynamics has highlighted the central role of toroidal multipoles in the description of electromagnetic excitations in matter. In particular, the toroidal dipole, represented by poloidal currents on the surface of a torus, is known to interfere with the electric dipole to form non-radiating configurations with null electromagnetic fields outside the source. Moreover, toroidal light pulses, also known as Flying Doughnuts, have been shown to efficiently excite such anapole modes in dielectric nanospheres. Alongside toroidal multipoles, the full multipole expansion includes their mean square radii, related to the finite size of the electromagnetic excitation. Such terms can be represented by supertoroidal currents (or equivalently by nested poloidal currents, see Fig. 1a), fractal iterations of poloidal currents, which are considered as higher order corrections and thus are typically omitted. Here, we show that torus-shaped dielectric particles under illumination with toroidal light pulses support resonant modes, where the toroidal dipole mean square radius not only provides a major contribution to the scattering response of the particle but in fact interferes with the electric and toroidal dipoles to substantially suppress scattering.

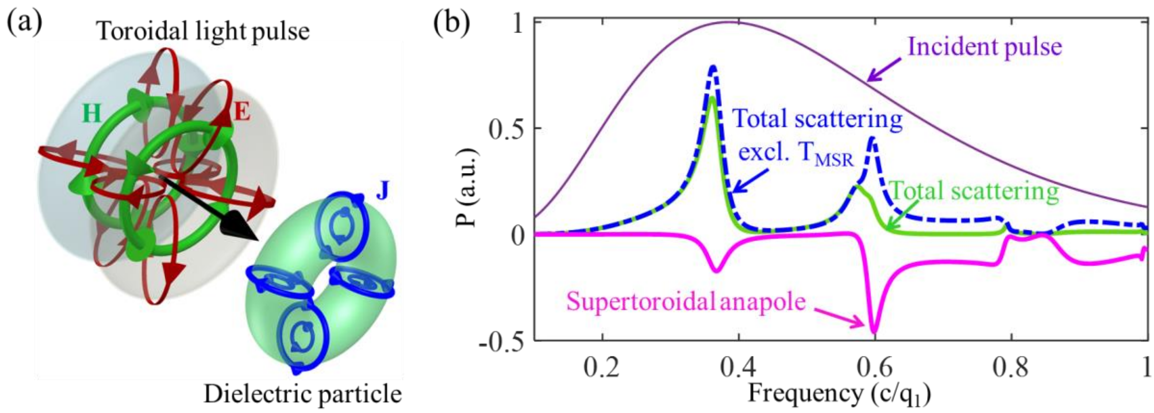

Fig. 1. (a) A radially polarized toroidal light pulse with effective wavelength q_1 and Rayleigh range $q_2=5q_1$ interacts with a torus-shaped dielectric particle with major radius $R=q_1/2$, minor radius $r=q_1/4$, and refractive index n=4. Red, green, and black arrows represent the electric field (**E**), magnetic field (**H**), and the propagation direction of the toroidal light pulse, respectively, while blue arrows indicate the induced displacement current in the dielectric particle. (b) Power as a function of frequency. Green and dashed blue lines show the total scattered power calculated by considering all multipoles up to electric octupole order including (solid green line) or excluding (dashed blue line) the toroidal dipole mean square radius, T_{MSR}. The contribution of the anapole term resulting from the interference of the toroidal dipole mean square radius (T_{MSR}) with the electric and toroidal dipoles is shown by the solid magenta line, where negative values indicate destructive interference and thus suppression of scattering. The power spectrum of the incident pulse is represented by the solid purple line.

We studied numerically the response of a dielectric toroidal particle under illumination with a toroidal light pulse by finite element calculations (see Fig. 1a). The particle was considered as lossless and dispersionless with refractive index n=4 and with major and minor radii $q_1/2$ and $q_1/4$, respectively. It was placed at the focus of a radially polarized (transverse magnetic) toroidal light pulse with effective wavelength q_1 and Rayleigh range $q_2/2=2.5q_1$. The axis of the toroidal particle coincided with the axis of the toroidal pulse (see Fig. 1a). The response of the particle was analysed in terms of multipole moments calculated by the displacement currents induced by the toroidal light pulse. We considered multipole terms up to electric octupole including toroidal dipole and its mean square radius. The frequency response of the particle is presented in terms of scattered power calculated by summing the contributions of all multipoles (see green solid line in Fig. 1b). Of particular interest here is the toroidal dipole mean square radius, T_{MSR}, which can interfere with the electric and toroidal dipole moments, resulting in a new type of non-radiating anapole configuration. Indeed, Fig. 1(b) highlights the importance of T_{MSR}: comparing the scattered power of the particle including (green line) and excluding (blue dashed line) the contribution of T_{MSR}, we observed that in the former case scattering is suppressed by 70% at the frequency of the supertoroidal anapole peak (ν~$0.6c/q_1$) and by 36% when scattering is integrated over the full bandwidth of the incident pulse.

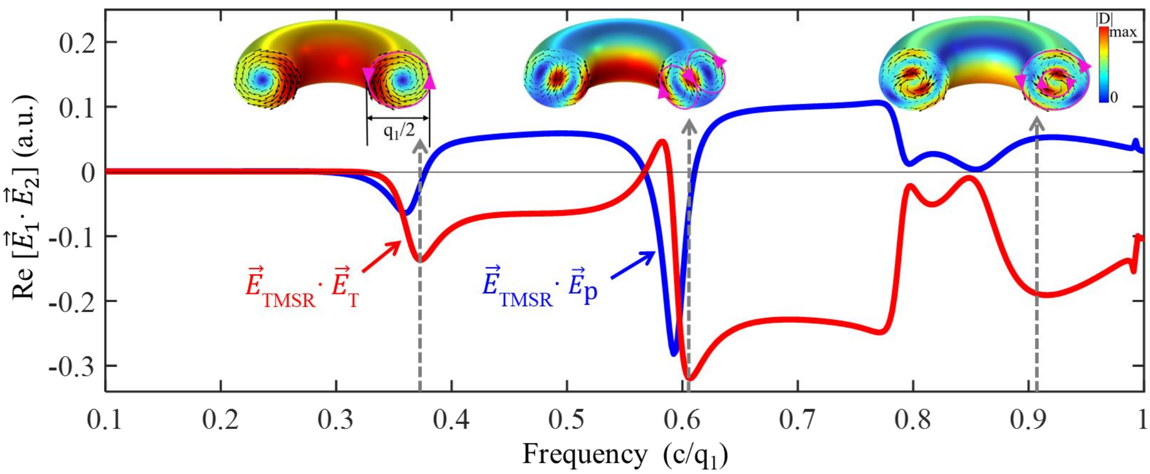

Fig. 2. Interference spectra of fields radiated by the toroidal mean square radius and electric and toroidal dipoles. $\vec{E}_{TMSR} \cdot \vec{E}_T$ (red solid line) represents the interference of toroidal dipole mean square radius, T_{MSR}, with the toroidal dipole, T; $\vec{E}_{TMSR} \cdot \vec{E}_P$ (blue solid line) represents the interference T_{MSR} with the electric dipole, p. Field maps of the displacement field modulus, |D|, corresponding to the three dips (grey vertical dashed lines) of $\vec{E}_{TMSR} \cdot \vec{E}_T$ curve are presented as insets, where black arrows show the direction of the instantaneous displacement field and magenta arrows serve as guides to the eye.

As shown in Fig. 2, fields radiated by the toroidal dipole mean square radius and the electric dipole (blue line) undergo mainly constructive interference except for two dips at $0.36c/q_1$ and $0.59c/q_1$. On the other hand, interference of T_{MSR} with toroidal dipole, T (red line) is predominantly destructive with three resonant dips at $0.38c/q_1$, $0.6c/q_1$ and $0.92c/q_1$ (grey vertical dashed lines). The corresponding field configurations are presented in the inset to Fig. 2. The first field map (from the left) shows strong poloidal currents that contribute to a strong toroidal dipole, while a weak azimuthal asymmetry results in a significant electric dipole and toroidal dipole mean square radius. The second field map shows two side-by-side counter-circulating poloidal current configurations. The difference in strength between these two current configurations leads to the excitation of strong T and T_{MSR}. The third field map shows two nested poloidal currents circulating in opposite directions. Such a configuration is equivalent to a supertoroidal current, a fractal iteration of the poloidal current associated with the toroidal dipole and indicates the presence of a strong toroidal mean square radius.

In summary, we show that dielectric particles under illumination with toroidal light pulses support a new type of non-radiating configuration, termed supertoroidal anapole, related to the excitation of supertoroidal currents. We show that engaging such current configurations can substantially suppress scattering by 70%. Our results will be of interest for applications in light emission, spectroscopy, sensing, and absorption engineering applications.