Superoscillatory Space-Time Nonseparable Optical Pulses

***Yijie Shen*[1], *Nikitas Papasimakis*[1], *Mark R. Dennis*[2], *Nikolay I. Zheludev*[1,3]**

1. Optoelectronics Research Centre & Centre for Photonic Metamaterials, University of Southampton, Southampton SO17 1BJ, UK
2. School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, UK
3. Centre for Disruptive Photonic Technologies & TPI, SPMS, Nanyang Technological University, Singapore 637371, Singapore

Abstract: We show that space-time non-separable band-limited light fields can exhibit superoscillations simultaneously in the spatial and temporal domains, i.e. can oscillate faster that the highest harmonics of their spectra. We demonstrate that such behavior is exhibited by the supertoroidal light pulses and discuss possible applications of the effect.

Superoscillation (SO) is a phenomenon in which *a band-limited function contains local segments oscillating faster than its fastest Fourier component* [1], as demonstrated in Fig.1(a-b). Superoscillatory behavior has been demonstrated in a variety of systems, including acoustics, quantum mechanics, electromagnetism, to name a few. In particular, in electrodynamics, superoscillations are typically considered either in the temporal or spatial domain, leading to applications in superresolution imaging and nanoscale metrology. Here, we extend the concept of superoscillations to the spatiotemporal domain introducing Space-Time Superoscillations (STSOs), whereby *a band-limited spatiotemporal signal, at a certain time, locally oscillates faster than the fastest spatial frequency component of its entire spectrum, and, at a certain position, temporally oscillates faster than the fastest temporal frequency component of its entire spectrum*. We show that such fine-scale features are present in space-time nonseparable light fields, i.e. fields whose spatial and temporal dependence cannot be factorized, such as the supertoroidal light pulse [2], leading to arbitrarily small spot sizes at focus.

A space-time superoscillatory field, $F(x,t)$, should exhibit five main features: (1) Band-limited nature. Both the spatial and temporal frequency spectrum of $F(x,t)$ should be band-limited. (2) Fast local oscillations. The field should contain local spatial segments oscillating faster than its fastest spatial Fourier components, and a local temporal segment oscillating faster than its fastest temporal Fourier component. (3) Rapid phase variations. Superoscillatory regions should be accompanied by rapid phase changes in space and in time, where the phase variation is faster than the phase variation of the highest harmonic in the spectrum. (4) Broad local spectrum. The spatial and temporal frequency spectrum of local segments of a space-time superoscillatory field can be much broader than that of the entire field. Figure 1 contrasts a common SO function to a band-limited STSO pulse, which is obtained from a supertoroidal pulse [2] with cutting-off frequency at bandlimit.

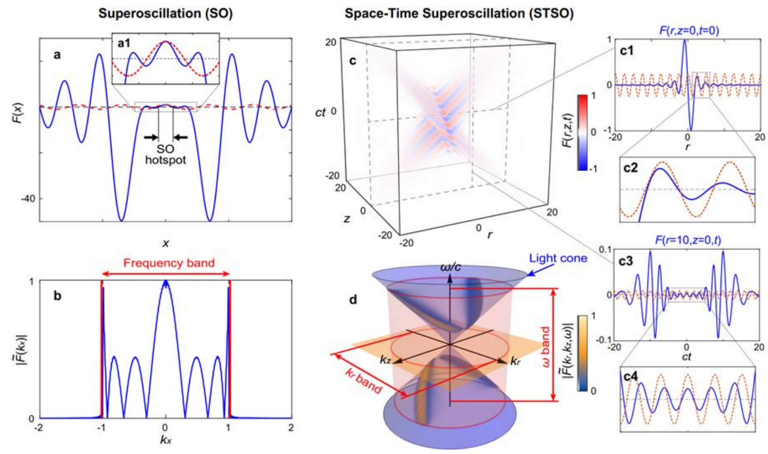

Fig. 1 Conventional SO vs STSO. **a, b,** An example of a band-limited function, $F(x)$, exhibiting SOs in the spatial domain, and spatial frequency spectrum bounded by $|k_x|<1$. **c, d,** A supertoroidal light pulse [2], $F(r,z,t)$, with truncated spatial and temporal frequency spectra exhibits space-time superoscillations: (**c**) pulse profile in space-time (r,z,t), insets (c1-c4) show the spatial and temporal traces of the pulse (blue) and its highest spatial frequency component (red); (**d**) pulse spectrum in the spatial and temporal frequency domain (k_r,k_z,ω) with pulse components being confined on the light cone. Red rings indicate the spatial and temporal frequency bandwidth.

Our work extends the counterintuitive superoscillatory effects to the space-time domain, where an extremely localized oscillation can emerge at a specific location and at a specific moment in time and may diffuse as time passes by. We demonstrate that STSOs are present in the recently introduced space-time nonseparable supertoroidal pulses [2] with truncated spatial and temporal frequency spectra. STSO light fields will lead to novel approaches in imaging, sensing, metrology and spectroscopy.

References

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