

Review Article

Railway Track Deterioration Models: A Review of the State of the Art

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ABSTRACT

There has been a railway renaissance in Britain since the 1990s, with passenger kilometres approximately doubling between 1990 and 2019. Despite changing habits caused by the COVID-19 pandemic, the latest data show that passenger journeys are almost back to their 2019 levels. Without building new lines (HS2 being not yet open and recently downgraded in scope), increased use has led to increased rates of infrastructure deterioration and a need for more maintenance and renewal to create the capacity on the aged existing railway network to meet this demand. Against this background, there have been on-going efforts in the field of railway track deterioration modelling to limit component failures and prolong the remaining useful life of the infrastructure. Analysis and modelling techniques have become increasingly detailed owing to advances in real-time data-acquisition and computational methods and the emergence of 'big data' approaches to interpretation. However, previous studies have generally merely confirmed the complexity of modelling track deterioration. There are few if any systematic reviews of deterioration models aimed at informing infrastructure managers (IM) from a whole-life asset management perspective. This paper addresses this knowledge gap by building on previous research to present a systematic taxonomy of track deterioration models, and proposing a hierarchical classification based on level of detail and functionality.

Introduction

Great Britain has seen a railway renaissance since the 1990s, with passenger kilometres approximately doubling between 1990 and 2019. Similar passenger and freight trends have been observed elsewhere in Europe (EU-27), with traffic rising on average by 3% per year between 2015 and 2020 [1]. The global COVID-19 pandemic caused a marked decrease in rail usage [1,2], although numbers have since recovered and the latest data show passenger journeys almost back to pre-pandemic levels [3–5]. These trends have exposed the fragility of the network, revealing the long-term deterioration of the infrastructure and a substantial backlog of maintenance and renewal (M&R) that needs to be undertaken.

To reduce track-related costs and limit component failures, scientific techniques such as mathematical optimization have attracted increasing attention in the recent years. These methods are often used to support the long-term assessment of decisions for systems with varying degrees of granularity. In the context of railway infrastructure, models can be roughly divided into those focused on either deterioration or recovery

(restoration). Models in the first group are utilized to approximate and predict the actual ageing process in terms of condition or reliability. Models in the second group aim to determine the optimal times of inspection and maintenance (or replacement), based on maintenance management policies. More recently, advances in real-time data-acquisition and computational methods have generated a growing interest in the development of models to efficiently support asset management. Additionally, a large number of studies have confirmed the complexity of modelling track deterioration. Key difficulties include [6]:

- 1) The distributed nature of the system (termed section-to-section variability), characterized by the existence of several covariates, which vary along the section lengths;
- 2) Lack of a complete understanding of the multiple interactions between different track components;
- 3) How to model the effect of a renewal on track quality, it being commonly assumed that the track returns to an 'as-built' condition per renewal cycle regardless of any changes in train use and/or the comprehensiveness of the renewal;

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Nomenclature

Acronym Description

ANFIS	Adaptive Neural network-based Fuzzy Inference System
ANN	Artificial Neural Networks
AR	Autoregressive
ARMA	Autoregressive Moving Average model
BBI	Ballast Breakage Index
CAR	Conditional Autoregressive
CART	Classification and Regression Tree method
DEM	Discrete Element Method
DSS	Decision Support System
EMGT	Equivalent Million Gross Tons
FCM	Fuzzy C-Means
FE	Finite Element
FIS	Fuzzy Inference System
GM(s)	Grey box Model(s)
IG	Inverse Gaussian
IM	Infrastructure Manager
KPI	Key Performance Indicator
LCC	Life Cycle Costs
M&R	Maintenance and Renewal
MCMC	Markov Chain Monte Carlo
MCS	Monte Carlo Simulation
MLP(s)	Multilayer perceptron(s)
MPE	Mean Percentage Error
MSE	Mean Squared Error
MVA	Multivariate Analysis
PN	Petri Nets
PSD	Power Spectral Density

SD	Standard Deviation
SFE	Stochastic Finite Element method
SVM	Support Vector Machines
TAN-TQI	Bayes-track Quality Index
TGC	Track Geometry Car
TQI	Track Quality Index
USP(s)	Under Sleeper Pad(s)
VTI	Vehicle-track Interaction
Symbol	Definition
D_f	Deterioration factor
E_s	Young's modulus of the subgrade
F_y	Yield force
S_1	Settlement at the end of the first load application
S_N	Accumulated settlement
k_s	Reaction modulus
p'	Mean effective stress
q_f	Deviatoric stress at failure
q_{max}	Maximum deviatoric stress
s_N	Settlement rate
s_u	Ultimate stress
s_y	Yield stress
ε_1	Strain at the end of the first load application
ε_N	Strain
σ_u	Ultimate (failure) vertical stress
σ_y	Yield stress
H	Hardening stiffness
N	Number of load cycles/Load cycle
T	Variation track stiffness
q	Deviatoric stress

- 4) How to express actual decision-making (in maintenance modelling), which can vary between infrastructure manager (IM) organizations depending on (i) their organizational structure, (ii) budget constraints, (iii) network constraints (track time, availability of maintenance resources, crew scheduling), (iv) organizational cultures (attitudes, beliefs, and sentiments), (v) other technical and organizational factors (design standards, in-house or outsourcing maintenance contracts). The introduction of rules based on expert knowledge or crisp values may partially resolve this (considering both dimensions of time and space);
- 5) Lack of a complete understanding of the cause and effect relationships between vehicle dynamics and track quality (displaying varying relationships even on sites of 'identical' track quality);
- 6) How to model different failure modes jointly (such as shock and deterioration failures). It is important to consider these processes together, as they tend to depend on each other;
- 7) How to jointly consider different track component M&R models, due to their different deterioration patterns, therefore losing important benefits of integrated planning through compromised M&R decisions.

There have been several review papers published related to techniques targeting different facets of the modelling process, for example: empirical track settlement equations [7–9], data collection [10], deterioration prediction modelling [11,12], maintenance planning/scheduling [13–20], and expert/decision support system (DSS) frameworks [16,21], as well as reviews considering a mixture of the above [22–24]. The main purpose of this review is to build on previous relevant studies to provide a taxonomy of the existing deterioration modelling literature, which can then be enhanced by a summary of the latest research progress. Comments on the merits, limitations and applications of existing modelling approaches are also provided.

Section 2 of this paper presents an overview of selected track deterioration models in the academic literature, and proposes a hierarchical classification based on their level of detail, functionality, and modelling technique used. This section also provides a description of these models, emphasizing recent developments and setting the scene for further research. **Section 3** presents a discussion of future research challenges.

Railway Track Deterioration Models

Some authors divide track geometry deterioration into three phases, bed-in, useful life, and wear-out [23]. The first phase starts immediately after tamping as the track beds in and the ballast grains pack closer together to form a structure able to stably support the track superstructure on a long-term basis [7,8,25,26]. This phase is relatively fast and the deterioration in geometry can be highly variable along the track, which makes it hard to model. The second phase commences after some time at a slower rate, with the deterioration path following an approximately linear relationship with time (or cumulative load) [7,25,26]. This is governed by numerous factors, such as vibration, deterioration, deviatoric stresses, and subgrade stiffness [7,27–29]. In the third and final phase, the interval required for maintenance becomes too short to be economically viable as a function of time; thus, entering this phase should be avoided [30].

In terms of fundamental mechanisms in operation within the ballast, it is suggested by Grossoni et al. [9] that the initial bedding-in stage is associated primarily with ballast densification (reduction in void ratio). Settlement magnitude and variability could both be reduced substantially by the adoption of a better (more controlled and precise) method of ballast bed preparation than tamping. The second stage is largely associated with lateral spread. Settlement could be minimized by lateral containment of the ballast or reducing the shoulder slope. The final stage may be associated with ballast grain degradation, possibly as a result of

mechanical damage caused by tamping.

Track settlement may also be attributed to settlement of the subgrade or interpenetration between the ballast and the subgrade. However, at least on a well-performing track, most plastic deformation occurs in the ballast itself [31,32].

By understanding and analysing track deterioration processes, an IM can make appropriate decisions on scheduling optimization of inspection intervals, estimate residual asset life, calculate life cycle costs (LCC), and determine suitable times and intervals for renewal interventions [33].

Broadly, approaches to deterioration modelling can be classified into mechanistic, empirical, and hybrid models (Fig. 1).

Mechanistic Models

Mechanistic or physical models are based on *a priori* physical information. This involves establishing, by theory or testing, the underlying mechanical behaviour and properties of the elements constituting the railway track and railroad vehicles [34]. Most studies focus on the trackbed as differential settlement causes most of the vertical geometry deterioration.

Despite extensive research, the mechanical behaviour and deterioration mechanisms of track geomaterials (ballast and subgrade) are difficult to capture in a generic model because of the multi-faceted nature of potential mechanisms, locally differing circumstances and a lack of comprehensive data. Ballast deterioration is generally defined as the

combination of accumulated permanent deformation and grain deterioration. It is the product of complex interdependent mechanisms at the grain scale. At a macro-scale, four processes govern plastic deformation: densification, spreading, grain deterioration, and subgrade issues including ballast-foundation interpenetration [7,9,35–37].

Permanent settlement depends on multiple factors, which may be grouped into three major categories: (1) characteristics of the constituent grains (size, shape, mineralogy and mechanical properties), (2) bulk properties of the granular assembly (grain size distribution, fines content, void ratio and density, and possibly degree of saturation), and (3) stress state (past, current and future magnitudes of stresses and vibrations).

Owing to the relatively large size of ballast stones, standard geotechnical laboratory tests and apparatus (for example, triaxial and direct shear tests) are unsuitable. In response, researchers often build bespoke or individual large laboratory test facilities, which makes comparison between results problematic. Moreover, typical laboratory tests are not easily able to replicate the operating stress conditions of ballast (for example, rotation of principal stresses during loading, attenuation of in situ stresses and loads with depth, rapid or dynamic loading).

Mechanistic models suffer from an inability to cope with the uncertainty of the deterioration path to different heterogeneous factors [38]. However, they may highlight the leading variables affecting ballast settlement (for example, ballast compaction state, rail seat load, fouling content, foundation stiffness). Recent research has attempted to

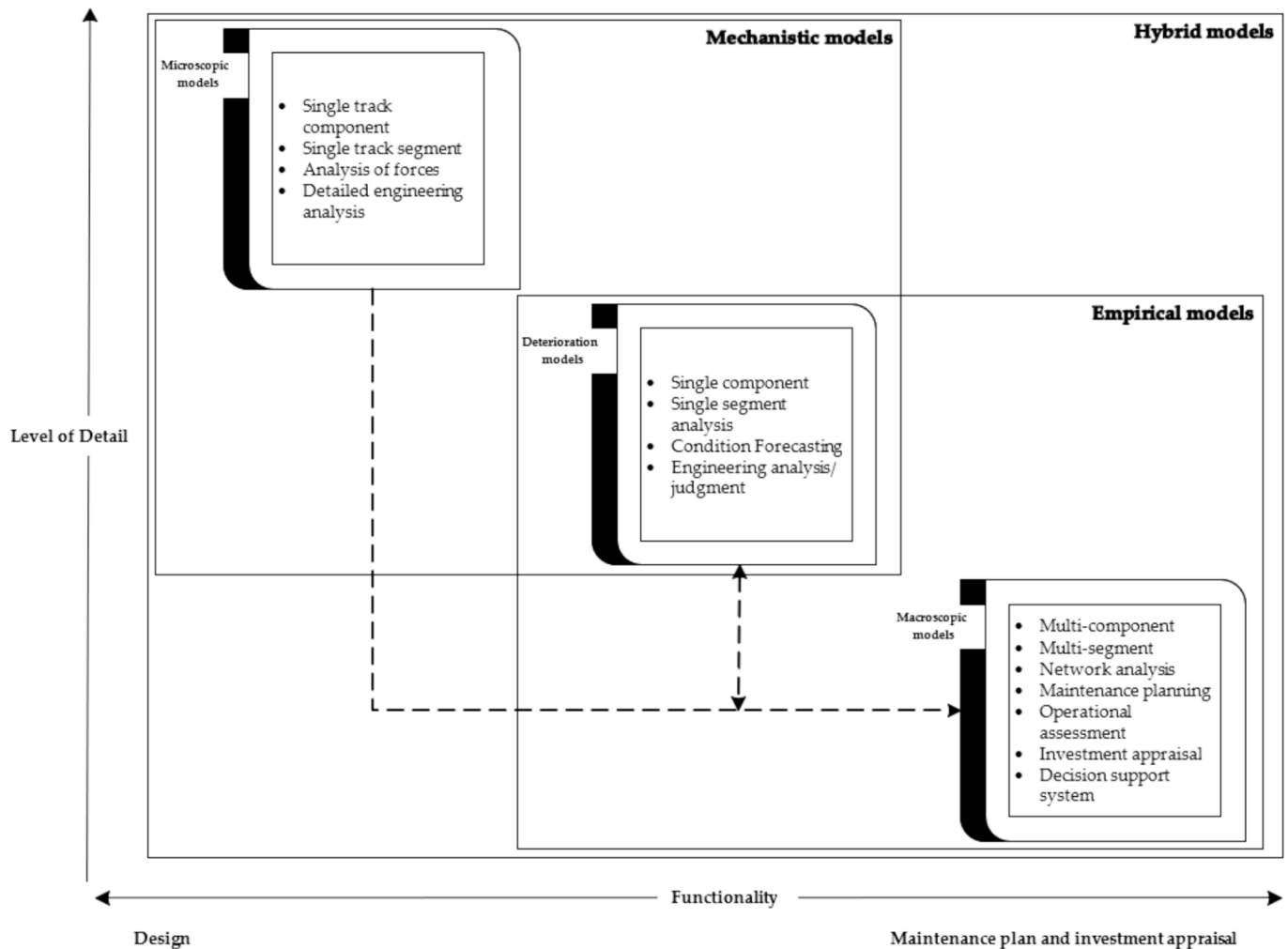


Fig. 1. Hierarchical classification of railway track deterioration prediction models based on their level of detail, functionality, and modelling technique (based on an elaboration of Ferreira and Murray [17]).

investigate the development of differential settlement using mechanistic models to simulate dynamic vehicle-track interaction (VTI). VTI models provide estimates of the response of the track (dynamic sleeper deflection, sleeper-ballast force, stresses propagated into the trackbed) for use as iterative input to mechanistic models either implicitly (for example, through a constitutive model [39]) or explicitly (for example, through a settlement equation [40–44]) to calculate the accumulation of differential permanent settlement. The VTI and mechanistic track settlement models are run iteratively to update and output the track level with number of load cycles [40,43]. Mechanistic models may be distinguished into discrete element, continuum mechanics, empirical, and semi-empirical (Fig. 2).

Discrete Element

The discrete element method (DEM) is perhaps the most fundamental approach for simulating the mechanical behaviour and deterioration of a granular material. The granular layer is represented as an assembly of two- or three-dimensional elements interacting according to a specified contact model. Each grain position, the potential contact points, and the force exchange with the surrounding elements are calculated at each calculation step [45]. DEM allows the quantification of ballast behaviour at the grain scale, which cannot easily be directly measured experimentally. For example, it permits the visualisation of the distribution of local stresses through force chains to better understand the effect of gradation, grain characteristics (size, shape and texture), fouling content and intergrain friction in the evolution of permanent settlement. It has also been used to investigate the influence of ballast gradation [46,47], to better understand the effect of USPs [48,49] and sleeper material [50] on ballast settlement.

However, there are some limitations to its use: (i) the computationally intensive nature of the analysis, (ii) the lack of an established methodology to determine the grain properties, (iii) the limited knowledge surrounding the grain-to-grain interaction mechanisms, (iv) the difficulty in capturing the irregular and variable shape of ballast stones, and (v) the limited understanding of granular material behaviour under dynamic loadings.

The limits and ranges of acceptable simplifications to adopt in the representation of stone geometry, physical properties and interactions are uncertain. At the same time, simplifications are needed to minimise both the calibration and computational time. Capturing ballast stone geometry mathematically is challenging owing to their irregular and variable size, shape and texture. Early applications represented grains as spheres [37,45]. Later applications, aided by imaging technologies such as X-ray tomography, proposed more advanced mathematical formulations and empirical indexes to quantify their geometrical characteristics.

Ballast grains have been represented using circle/sphere pairs [51,52] and clusters [53,54], superquadrics [55,56], ellipses/ellipsoids [57–59], polygons/polyhedrons [60–64] and potential particles [65–68]. Lumping together spheres to form clusters is the easiest approach but also the least efficient as it requires many spheres to obtain a realistic shape [69]. However, Ni et al. [51] and Powrie et al. [52] show that pairs of differently sized spheres bonded together can give reasonably realistic soil-like behaviour. Each cluster moves rigidly with the sphere-to-sphere bond strength within each cluster set effectively to infinity. However, setting a threshold value of bond strength, after which spheres separate, offers a way of simulating internal grain breakage [48,70].

More advanced mathematical representations may better represent angularities, but this may be at the expense of greater computational time, particularly for detecting and calculating grain contacts [71]. Nevertheless, in simulations with detailed shape modelling a certain degree of simplification is unavoidable. For example, effects resulting from the macroscopic grain texture (surface roughness) cannot be considered in the grain shape and are thus accounted for in modelling grain contacts (for example, by adapting the inter-grain coefficient of friction).

Intergrain contact forces comprise normal, tangential, and rolling components [72]. The choice of model depends on the chosen element shape. Commonly used models include the linear elastic (L) and nonlinear elastic Hertz-Mindlin (NL). However, research has shown that these cannot reproduce the suppression of dilation and reduction in peak strength that occur in triaxial tests on specimens of a given void ratio as the cell pressure is increased. However, this can be achieved through the introduction of damage models [69] or breakage criteria [73,74]. Contact parameters are calibrated based on the macroscopic behaviour of ballast (shear strength), owing to the challenges in reproducing and monitoring (experimentally) the friction forces between individual stones.

Owing to its high computational time demand, a DEM analysis is applicable for studying small track sections (usually up to three sleepers) and/or for a few loading cycles (usually between 1,000 and 5,000). A typical application of DEM is to mimic the ballast behaviour observed during triaxial tests under monotonic [68,75] or cyclic [54,76] loading, box [77,78] or full-scale cyclic tests [79]. Some studies have extended DEM to dynamic analysis or combined them with FE (finite element) models (Section 2.1.2) to consider longer track sections [80–84] where the ballast is modelled using DEM and the rest of the track using FE.

Continuum Mechanics

In continuum mechanics models, the material representing the

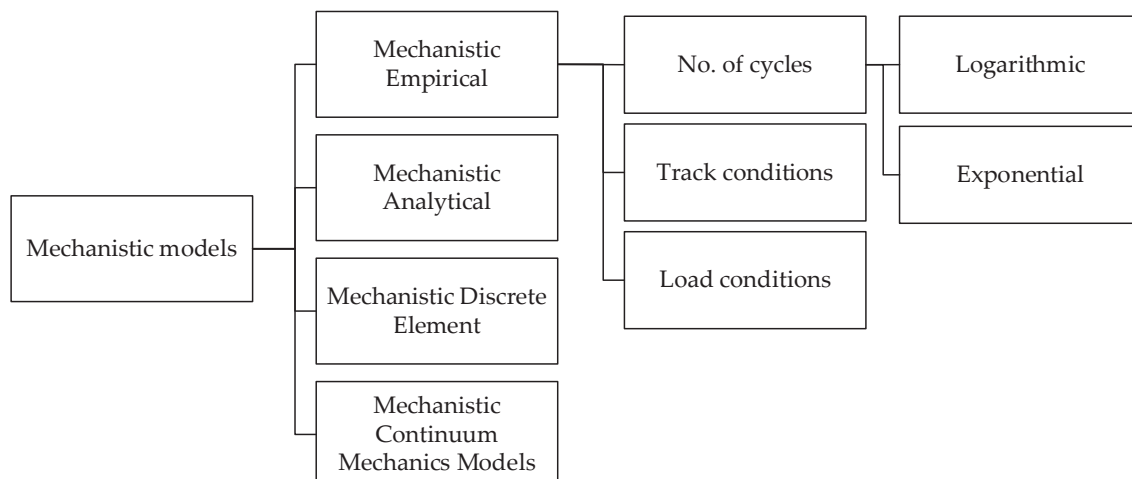


Fig. 2. Mechanistic models.

ballast layer is assumed to be continuously distributed. Mechanical properties are averaged and defined within its body at every point (ignoring intergrain interactions). The suitability of this simplification is debatable given that a single ballast grain (typically between 31 to 63 mm in size) occupies a significant proportion of the depth (usually around 300 mm) of the ballast bed. Ballast deformation is computed through constitutive models that generally relate stress to strain.

Usually, plastic settlement per cycle of load varies from millimetres on a newly tamped or poorly performing track to nanometres on a well performing bedded-in track [8,85]. Classical soil mechanics theories (elasticity, Cam clay and Mohr-Coulomb and Drucker-Prager plasticity) are unsuitable for modelling such small settlements, their variations and gradual accumulation over potentially millions of cycles.

To be suitable for cyclic loading analysis, models need to account for long-term mechanisms such as ratcheting (the accumulation of plastic deformation with number of loading cycles at a constant or increasing rate) and shakedown (the accumulation of plastic deformation with number of loading cycles at a decreasing rate). Experimental evidence suggests different long-term mechanisms depending on the stress, which may be captured by the shakedown theory [86–88]. Some researchers have proposed constitutive equations to reproduce the cyclic performance of ballast, usually using empirical or semi-empirical equations to account for shakedown and ratcheting [89–95].

Suiker and de Borst [89] proposed a mechanistic model, resembling the Perzyna [96] viscoplastic model, based on a shakedown concept. Plastic deformation is assumed to be generated by frictional sliding and volumetric compaction mechanisms, and their evolution with loading is assumed to follow a power law. Drucker-Prager cones delimit failure planes (frictional and tensile). The model was calibrated using cyclic triaxial test data and integrated into a 2D FE plane-strain model, and later extended into a 3D framework by Elsworth and Yasuhara [97], and applied in a dynamic VTI analysis by Varandas et al. [39].

Salim and Indraratna [90] proposed an elasto-plastic model based on critical state concepts, which incorporates the effect of grain breakage on ballast settlement. They proposed a semi-empirical relation between the deviatoric and mean stresses, based on an energy balance and assuming that a portion of energy dissipates through breakage [98]. Niemunis et al. [93] and Nguyen et al. [94] used a hypoplastic model [99] with intergranular strain [100] and the Matsuoka and Nakai [101] failure criterion. Hypoplastic models are superficially advantageous (but arguably less scientifically rigorous) because they do not separate deformation into elastic and plastic components or define a yield surface. In Niemunis et al. [93] the accumulated plastic deformation is defined empirically as the product of six functions accounting for the peak strain amplitude, number of load cycles, average mean pressure and stress ratio, void ratio and change of the orientation of the strain loop. However, using the strain amplitude as a principal variable might lead to computational problems and errors in dynamic analysis, where strain oscillates and is formed by small and large amplitudes [95].

François et al. [95] assumed the same plastic deformation mechanisms as Suiker and de Borst [89]. Frictional sliding was again derived using the Drucker-Prager criterion, and empirical equations were used for the volumetric compaction, with the deviatoric and volumetric plastic strain rates decreasing exponentially with accumulated plastic deformation. The model was calibrated with triaxial test data and employed in a 3D dynamic FE soil-structure problem.

Pasten et al. [102] calculated the volumetric and shear plastic strain in the first cycle using modified Cam clay [103] and used them as an input to a set of empirical equations for estimating the evolution of plastic strain with cumulative loading. Sun et al. [104] proposed a 15-parameter model with a nonlinear function relating a ballast breakage index (BBI) to the accumulated plastic deviatoric strain and initial effective mean stress. Sun et al. [105] combined bounding surface plasticity theory (Cam-clay) with fractional calculus to reduce computational time.

Continuum models are advantageous as they can be used alongside

FE. This enables the practitioner to test stress–strain relationships and classical geotechnical concepts (for example, critical state theory); also to investigate the effect of state parameters (such as void ratio) on ballast settlement. Nonetheless, they have several disadvantages. First, they require a large number of parameters in their definition, which must often be obtained using nonstandard geotechnical laboratory tests (for example, hollow cyclic cylinder and large cyclic triaxial tests) and in some cases are not directly independently measurable. Secondly, the calibration process is time-consuming and not straightforward as there may be more than a single set of parameter values that fit the underlying laboratory data set.

Empirical equations are often integrated with constitutive models to account for long-term mechanisms such as ratcheting and shakedown. In conclusion, although continuum models are generally less computationally expensive than DEM, their application is still limited to short to medium term analysis (100 to 100,000 cycles). Also, further research is needed to test these models in real 3D dynamic conditions.

Mechanistic-Empirical

Mechanistic-empirical models are usually derived by fitting a settlement curve from the experimental data expressed in terms of one or more measuring variables. A common feature is that the permanent settlement of the ballast increases with the number of load cycles N ; and in most (but not all) cases at a decreasing rate. Otherwise, most of the equations fall into one of four categories, which assume that the rate of development of permanent settlement depends on one of (i) track and traffic characteristics [25,106], (ii) the load at the superstructure/substructure interface [7,25,40], (iii) the peak elastic sleeper/rail deflections during a (possibly the initial) load cycle [107,108], and (iv) stress state (deviatoric stress q and mean effective stress p') at a representative depth in the track support system (ballast and/or subgrade) [109–112]. Settlement equations have been formulated in terms of accumulated settlement, S_N , or strain, ϵ_N , at load cycle, N (but could also be expressed in terms of cumulative loading). Other expressions use the concept of settlement rate, s_N , at a load cycle, N , defined as the ratio of the increment of settlement to the increment of number of cycles.

The settlement, S_N , or strain, ϵ_N , at load cycle N is usually defined to depend on the permanent deformation (or strain) at the end of the first load application, S_1 (or ϵ_1). Both the initial settlement, S_1 , and settlement rate, s_N , are strongly influenced by the initial level of ballast compaction and the stress conditions (cyclic vertical load and confining stress). Some authors suggested a range of values for S_1 (or ϵ_1) depending on the state of compaction and load applied, obtained from triaxial or full-scale tests [113–115]. Others attempt a more general interpretation, by defining S_1 as a function of the (constant) load applied [116,117], or give different factors related to the track type and condition [118].

Some settlement equations are defined using two or more terms based on the assumption that the evolution of settlement with number of load cycles can be distinguished into two [7,8,26,108,119–121] or possibly three stages [91]. Mechanistic-empirical equations expressed as a function of the number of cycles are usually of logarithmic, exponential, hyperbolic, or power form.

The logarithmic equations fit the short-term but not the long-term settlement, as highlighted, for example, by Chrismer and Selig [122] and Abadi et al. [22]. An example of a settlement equation that can successfully reproduce the same curve shape is given by Saussine et al. [123]. However, this does not replicate long-term settlement; this is better represented by the exponential equation of Sato [25], which includes a linear term, γN .

Settlement equations based only on the number of load cycles are not suited to predicting track geometry deterioration because they lead to a uniform settlement along the length of the track.

Mechanical empirical equations for settlement expressed as a function of track and traffic characteristic were proposed by Sato [25] and Hecke [106]. These were obtained by analysing track data of vertical rail

level acquired by monitoring one or more track sections over a certain time period and establishing correlations between inputs (traffic and track conditions) and output (track settlement). While such an approach considers actual data, it does not give insights into the underlying mechanisms involved in track settlement. Other issues include the difficulty of defining (adequately) the track characteristics and conditions (sleeper dimensions, ballast depth, soil types). Additionally, the scarcity of available field data limits the extent to which different factors affecting track settlement and its variation along the track length are accounted for.

Formulae based on the load at the superstructure/substructure interface [7,124] or peak elastic sleeper deflection [108] have been applied in iterative VTI simulations to estimate the development of differential settlement along the track, and hence maintenance requirements. Examples of such approaches are given by [42–44]. The state parameters used are the maximum stress (or force) under the sleeper, and maximum elastic sleeper deflection, respectively.

To allow explicitly for a settlement rate that reduces with the number of load cycles, Varandas et al. (2014) proposed a function, $h(F, N)$ accounting for the loading history and the amplitude of the rail seat load to control the settlement rate. This was used in a 2D VTI dynamic model and demonstrated with reference to 7 months of field data from track crossing a culvert [125].

Formulae based on these two alternatives (load at the superstructure/substructure interface and peak elastic sleeper deflection) give different trends in the calculated rate of development of permanent settlement with number of loading cycles as the track support system stiffness is increased. This arises because a stiffer track support system results in increased sleeper-ballast pressure (owing to reduced load spreading along the track), but reduced sleeper dynamic displacements. Hence an increased rate of development of permanent settlement with increasing track support system stiffness is calculated by empirical equations assuming that settlement increases with load, but a reduced rate by formulations assuming that settlement increases with elastic deflection. The second – a trend of reducing permanent settlement with increasing support stiffness – is more consistent with general empirical evidence. The issue is discussed with reference to a threshold stress approach to track settlement calculation by Grossoni et al. [9].

Some settlement equations [107,126,127] use the subgrade stiffness (elastic modulus) as the controlling parameter. The variation of track stiffness, T (or reaction modulus k_s or Young's modulus of the subgrade E_s) from a hypothetical reference value T_{ref} (or $k_{s,ref}$ or $E_{s,ref}$) is used to control the sensitivity to settlement at a number of cycles, S_N . Kennedy et al. [128] and Woodward et al. [107] calibrated their equations using data from full scale cyclic loading tests, reproducing the settlement evolution of a single sleeper.

Empirical formulations that consider the stress state in the trackbed layers provide a clear linkage to established soil mechanics behaviour. For a given soil or granular material, the plastic strain generally increases with the ratio q/p' above a threshold value [129–131]. The stress parameters p' and q also relate more directly to the bulk and shear deformations of the material, respectively. ORE [109] proposed an equation in which the permanent strain after the first cycle is proportional to the porosity of the ballast (n) and the magnitude of the deviatoric stress (q_{max}).

The equations given by Sayeed and Shahin [112], based on Li and Selig [132], relate the maximum deviatoric stress q_{max} to the deviatoric stress at failure from static compressive triaxial tests, q_f , such that a higher stress ratio q_{max}/q_f results in greater permanent strains. Another approach assumes that the plastic strain is proportional to the distance between the maximum deviatoric stress applied (q_{max}) and the failure line; for example, Ramos et al. [111], based on Chen et al. [133]. The problem with these definitions is that the failure line may change with cumulative loading due to ballast grain degradation and breakage [134].

Permanent settlement originates mainly from the ballast on well performing track built to modern standard. However, on the most

problematic sites, there is usually a more significant contribution from the subgrade. Problematic sites include locations where dynamic stresses are high (for example, bridge transitions) or where the subgrade is highly compressible (for example, peat) or otherwise volumetrically unstable (such as a high plasticity clay subject to seasonal cyclic variations in soil water content). Few iterative VTI studies incorporating the evolution of track settlement as the sum consider the ballast and subgrade explicitly. Varandas et al. [40] and Wang and Markine [43] approximated the contribution of the subgrade settlement as a linear function of time (or load cycles). Shan et al. [135] and Punetha et al. [136] evaluated subgrade settlement using empirical equations based on the stress state in the subgrade calculated with a VTI model. A useful review of mechanistic-empirical settlement models for subgrade materials is given by Ramos et al. [137].

Semi-Empirical

Recently, a new type of modelling approach has been proposed, based on a one-dimensional constitutive equation relating the vertical sleeper-ballast force (or stress) to the elastic and plastic settlement at sleeper level. This type of 'macro-model', used in geotechnical engineering to simulate soil-structure interaction (e.g., Houlsby et al. [138]), captures the entire response of the sleeper/trackbed support in a single formulation. The main advantage over traditional empirical settlement equations is that they are time-incremental (that is, they can be integrated into a time step analysis by means of a numerical integration algorithm) rather than cycle-incremental (only able to be used at the end of a time step analysis at the end of a given cycle such as the passage of a train/bogie/wheel). Thus, they can be implicitly integrated into VTI models. In this case the settlement (or strain) rate is a function of loading and plastic parameters.

Examples of such approaches given by Grossoni et al. [9] and Ognibene et al. [139] use the vertical stress in the ballast and the sleeper-ballast force as driving parameters, respectively. Plastic parameters are yield (s_y) and ultimate (s_u) stresses in Grossoni et al. [9]; and a yield force (F_y) and hardening stiffness (H) in Ognibene et al. [139]. These change with cumulative plastic settlement (or strain) to account for ratcheting. Specification of a single threshold value (whether force or stress), which delimits an elastic from a plastic regime, may be not realistic as irreversible strains in granular materials develop even at small strains. More laboratory tests are needed to assess the cumulative ballast settlement as a function of varying axle load and loading pattern.

In Grossoni et al. [9] a deterioration factor, D_f , is integrated into the definition of σ_y and σ_u , so that a lower soil stiffness leads to a higher track settlement rate, in accordance with general experiential evidence. Including a deterioration factor, D_f , in the definition of the yield stress seems reasonable. However, owing to the lack of laboratory and field data, relationships between these factors have not yet been validated. A simple linear evolution of D_f with the subgrade Young's modulus, E_s , may not be realistic.

Empirical Models

Track geometry behaviour has uncertainty as one of its prime characteristics, which by their nature the approaches discussed so far are unable to capture. It is important to incorporate uncertainty into deterioration modelling to facilitate more comprehensive decision-making. This requires the use of concepts from statistics, such as probability theory and stochastic modelling processes. A drawback with these approaches is their heavy reliance on large data sets. Recent developments in condition-monitoring techniques [140] have enabled access to so-called 'big data' [141] that can deliver valuable information and reveal the underlying condition of the track geometry. Real-time data-acquisition complements and enhances recent developments in computational methods. Against this background, there is an ever-growing trend towards the application of statistical techniques for

modelling the deterioration of track geometry.

Statistical models (Fig. 3) are structured from a given set of inputs and outputs, formulating relationships between them by utilising a large amount of data [34,142]. They constitute a powerful class of models able to account for many descriptors [23].

The main advantage of statistical models is that, being rooted in real data, an accurate estimation of the track deterioration profile can be obtained [142,143]. However, the absence of a mechanical background of any sort in relation to track components and their interactions with influencing factors (that is, a lack of insight into the underlying physics) can lead to invalid results [38,143]. Other pitfalls include spurious regressions and ecological fallacies. Yousefikia et al. [142] posit that statistical models should be preferred over the mechanistic ones when sufficient data are available. According to Jovanović et al. [144], to develop a robust deterioration model the following data requirements should be addressed: (1) layout and operating data, (2) superstructure and infrastructure inventory/register, (3) condition measurements, and (4) work history. They posit that a (reasonably) accurate long-term track quality prediction is feasible through the identification of condition parameters, suitable curves for deterioration behaviour, essential and temporal activities, and a rectification model.

Stochastic Process Methods

A significant class of statistical approaches for modelling failure progression are the stochastic approaches, including the Wiener, Gamma, and the Inverse Gaussian (IG) processes. Of these, the most commonly used model is the Gamma process [145]. An important advantage of Gamma processes is their ability to model the temporal variability associated with the evolution of deterioration [145].

However, they are unsuitable for modelling damage from sporadic shocks [145,146], or for making long-term predictions [147]. Thus, this approach is more suitable for modelling component life between individual maintenance cycles.

Fundamentally, the Gamma process is stochastic with independent, non-negative increments; hence, like the compound Poisson process, it is a jump process. The primary distinction between the two is that the Poisson process has a finite number of jumps in finite time intervals, whereas the Gamma process has an infinite one (number of jumps).

Meier-Hirmer et al. [148] adopted the Gamma process for modelling the changes in standard deviation (SD) of the longitudinal level. They postulated a dependency between environmental variables, such as ascending and descending gradients, tonnage, ballast type, and curves with the deterioration rates mean and variance. They applied the classification and regression tree method (CART) to predict the geometry deterioration of individual track sections, and subsequently classify them depending on their deterioration behaviour. The Gamma process evolves monotonically, which is useful when the deterioration behaviour is in the form of cumulative damage and wear accumulating in a sequence of tiny increments over time [145,149–151]. However, if the path of deterioration follows both positive and negative increments, it may lead to inaccuracies [150,151]. In such circumstances, the Wiener process may be employed. When the objective is to model the evolution of a deterioration path characterised by a linear increase over time with random noise, a stationary Wiener process is particularly useful [152]. However, this approach is unsuitable in modelling deterioration that proceeds in a strictly monotonic fashion or involves jump behaviour. Moreover, its underlying property of time homogeneity, makes it invalid for modelling deterioration processes that do not possess this property [150]. Another important issue is that modelling deterioration through either the Wiener or the Gamma process implies a memoryless behaviour, so that the arrival to any future states of the model depends entirely on its current state, with the associated evolution being independent of its past behaviour [150]. This property restricts how the degraded state of geometry can account for the maintenance history. In essence, Wiener processes are mostly suitable for modelling non-monotonic deterioration resulting from reduced intensity of use, minor repair, or self-healing [151].

Zhu et al. [153] modelled irregularities in the alignment and vertical profile as Gaussian random processes. They showed that methods such as power spectral density (PSD) analysis, and cross-level statistics about irregularities of track geometry could be used to enhance current approaches to track deterioration modelling.

An important issue for deterioration modelling is the simultaneous consideration of different measures of track condition. Mercier et al. [154] used a bivariate Gamma process constructed by trivariate reduction to model the development of two interdependent deterioration indicators, longitudinal and transversal level.

Vale and Lurdes [155] were the first to use the Dagum distribution in railway track deterioration modelling. They demonstrated that it fits well the track geometry deterioration process in terms of longitudinal level. They classified the changes in longitudinal level into three speed classes and inspection intervals.

Sedghi et al. [156] developed a DSS framework integrating track condition prediction at the tactical level based on a stochastic Wiener process and adopted the IG distribution to calculate the probabilities of surpassing certain predefined deterioration thresholds. This choice was made based on the argument that the stochastic process model will not provide repeatable deterioration point estimates, thus, the use of the IG distribution for calculating these thresholds could reduce the variability of modelling estimates.

While stochastic processes can capture the time variability of deterioration and may lead to a more robust maintenance plan, they also have some drawbacks [23]. As highlighted by Soleimanmeigouni et al. [23], inaccuracies may arise from their use in cases where the model mean is significantly lower than its variance.

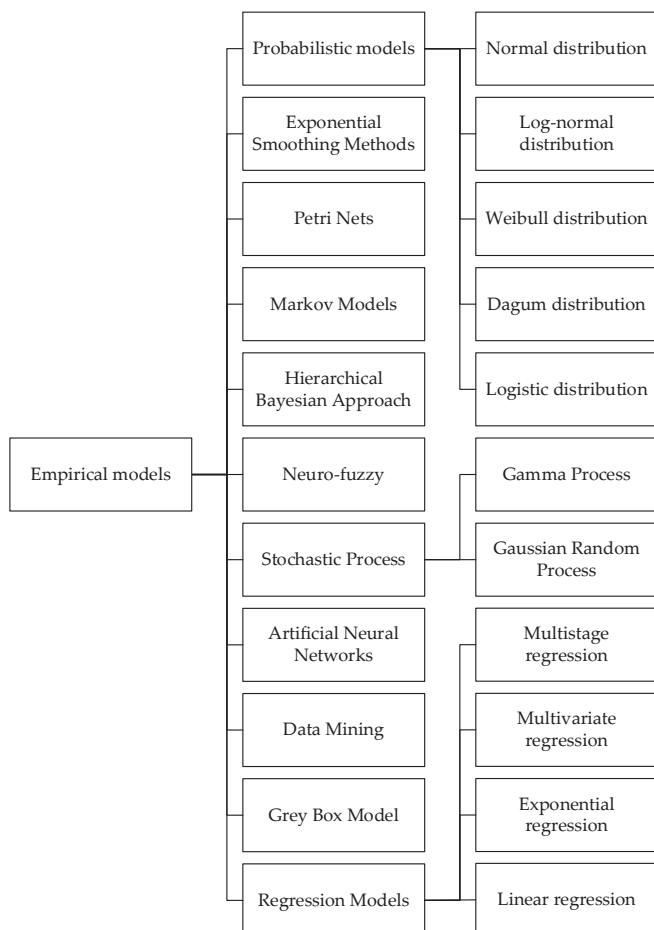


Fig. 3. Empirical models.

Path Analysis Methods

A different statistical tool that has been extensively used in deterioration modelling, and more specifically to identify the effect of different influencing factors on it, is Path Analysis. Such techniques can improve our understanding of the selection of suitable parameters for modelling track geometry deterioration.

A similar tool is multivariate analysis (MVA), which is often used to identify relationships among two or more variables of interest. Specifically, in geometry deterioration studies, this technique is employed for examining the influence of heterogeneous parameters along the track on geometry deterioration [23]. For example, Lyngby [157] proposed a method of evaluating track deterioration for geometry irregularities. He adopted an MVA regression model to determine the underlying relationships between different influencing parameters and the track deterioration variable. He modelled both the first and second phases of ballast settlement using exponential functions. Since different track sections are not identical, he segmented the track into homogeneous sections, clustered depending on their influencing parameters. Based on his analysis, he concluded that (1) axle load influences deterioration rate with a nonlinear relationship being identified, (2) deterioration after tamping is related to the tamping history since the last renewal, (3) light rail tracks settle at a higher rate than heavy ones, (4) the rate of deterioration is more rapid following heavy rainfall, and (5) clayey soils settle faster than others.

Sections of track may exhibit different deterioration behaviour even though they may have similar influencing variables. The most commonly adopted track segmentation approach is the division along its length into short sections of fixed length. However, since different sections may display different behaviour, the deterioration of each should be modelled separately. Alternatively, track segmentation can be based on similar track, traffic, maintenance history, and environmental conditions. Guler et al. [158] proposed a deterioration prediction model for different track geometry parameters utilising MVA. They first segmented the track into homogeneous sections based on their track structure characteristics, such as age, cant, curvature, rail type, and length, gradient, and speed. They then modelled the deterioration rate in terms of different independent variables by adopting a linear multiple regression model.

Westgeest et al. [159] modelled track deterioration and rectification processes using a regression method. They formulated a key performance indicator (KPI) by considering a combination of track geometry parameters. The KPI was considered as the response variable, with the examined variables being two forms of tamping (mechanical and manual), and different types of subsoil, engineering structures, tonnage, and sleeper. Their analysis demonstrated the ability of the model to reproduce changes in KPI over time, but revealed its inefficiency in predicting track behaviour. They then argued that the different deterioration rates on different segments of track depended on the presence of several heterogeneous factors. This work further emphasises the importance of, and the need to consider, section-to-section variation in track deterioration modelling.

Similarly, He et al. [160] modelled the relationship between various influencing parameters and the track deterioration rate by means of a statistical exponential model for each geometry defect. Their results showed different deterioration rates for different types of geometry defects, with most, but not all, of them exhibiting higher sensitivity to traffic volume.

Data-driven Methods

A wide range of data-driven approaches has been adopted in deterioration modelling, including data-driven statistical methods, filtering, fuzzy, and machine learning.

Machine Learning Methods. Machine learning methods can be subdivided into support vector machines (SVM), Bayesian networks, and

artificial neural networks (ANNs) [149].

Artificial Neural Networks. ANNs are biologically inspired computational models of the human brain. By using a large number of input parameters, these models can make predictions of the approximate behaviour of the track over time. They comprise a large number of simple processing elements termed neurons, coupled to each other by numerous direct links known as connections. These are associated with a synaptic weight incorporating information utilised by the networks to solve the given problem at hand. In turn, by using an activation function (also known as transfer or threshold function), each neuron generates an output. The activation function expresses the underlying relationship existing between input and output parameters of a node and a network [161].

Since their introduction, the most extensively and successfully applied approach is the multilayered neural network, which is trained in a supervised manner. This method was adopted by Guler [162] to model the deterioration of different track geometry parameters. He divided the examined railway line into homogeneous analytical segments of uniform properties and used linear regression to evaluate the deterioration rates between pairs of successive maintenance actions. His study demonstrated that ANNs are capable of modelling and predicting the deterioration of track geometry (as demonstrated by the R^2 values produced).

Sadeghi and Askarinejad [163] developed an ANN model to establish correlations between geometry irregularities (sourced from automated inspection data) and structural defects, with a view to overcoming the collection cost and time-consuming visual inspection of data. In their study, multilayer perceptrons (MLPs) with one hidden layer were used. These were trained using the error back-propagation algorithm, which is a supervised learning rule [163]. The model inputs were chosen as the SD of different track geometry indicators (i.e. gauge, profile, alignment, and twist), and the output was the density of structural defects. Instead of a single neural network, four different networks were developed, each of which was purposed for the prediction of defects of a different structural component, to obtain better accuracy.

More recently, Khajehei et al. [164] developed an ANN model to predict the rate of track geometry deterioration of the longitudinal level. They evaluated the performance of their model by conducting an extensive case study using data collected from Swedish railway network. Based on their results and performance (R^2 and mean squared error (MSE)) of their model, they concluded that ANNs can successfully capture section-to-section variability in track geometry deterioration rates. Furthermore, their application of the Garson's algorithm was proven useful on capturing the relative importance of different variables on track geometry deterioration. Their analysis demonstrated that parameters such as maintenance history, post-tamping deterioration level and train frequency passing along the track had the strongest contribution among the considered set of modelling parameters.

The main advantage of ANNs is their ability to handle well large amounts of linear and non-linear data, and potentially extract more information from them [165]. Adding to this, they can be used to explore the relative importance of different input variables on the output parameters and discover patterns in noisy and complex data sets, which makes them useful on modelling track geometry deterioration and capturing track section-inhomogeneity. However, due to their inability to extrapolate beyond the range of the defined parameters in the training set, these models have to be retrained on a case study basis. It has been also stressed by practitioners that data curation is of great importance in order for these models to be accurate, as for example, the effect of shock events has been found to be of importance with researchers [164] suggesting that such effects should be identified in the data sets and incorporated in the modelling process.

Neuro-Fuzzy Methods. Neuro-fuzzy models are hybrid-like systems combining the connectionist structure and learning abilities of ANNs with the fuzzy system's power of human-like reasoning [166]. These models seem on the verge of becoming popular. They have as yet seen

limited application in the field, but are a promising tool for deterioration modelling. This is mainly due to their ability to circumvent the rigidity of the decision-making mechanism in other, more traditional classes of model, which are essentially based on binary logic. For example, they tend to incorporate rigid rules and translate deterioration thresholds to maintenance interventions, but thereafter do not consider the amount by which the threshold is exceeded or its importance. Effective decision-making by flexible planning under a constrained budget environment is therefore almost impossible to model.

Dell'Orco et al. [167] constructed a DSS framework for planning tamping interventions by considering an adaptive neural network-based fuzzy inference system (ANFIS). The threshold limits for track interventions were chosen as linguistic variables in the fuzzy logic, while the deterioration process was modelled based on an adaptive neural network. Instead of using a length-based segmentation, they adopted a so-called 'natural' segmentation, dividing the track into homogeneous sections based on their maintenance requirements. For each of the resulting segments, the neural network was trained to calibrate the membership functions of the FIS (fuzzy inference system). Thereafter, opportunistic maintenance was considered using the fuzzy C-means (FCM) clustering method [168], to group maintenance interventions in both time and space.

Data Mining Methods. Many studies have attempted to make short-term predictions of track geometry deterioration. Xu et al. [169] suggested an approach grounded on variation in historical track irregularity to make short-term forecasts for track quality indices (TQIs) of different unit sections along the track. They then made estimates of the nonlinear behaviour of track irregularity throughout a maintenance cycle using short-range linear regression models. With continuous inspection cycles, a family of linear regression models can be obtained. Based on integral theory, a nonlinear model can be approximated using this family, and subsequently, used to make predictions of track irregularities two months beforehand. Xu et al. [170] used a track measures data mining model to make short-term forecasts of rail track deterioration. They considered alignment and twist, and subsequently observed through a validity test that the prediction errors for both follow a Gaussian distribution with a SD of less than one and a mean close to zero. Their model demonstrated the ability to provide reliable forecasts of track condition for the next two or three months.

Similarly, Liu et al. [171] suggested a short-range prediction model (SRPM) to forecast irregularities over small lengths (25 m) on a single day basis within a future short period, using track geometry car (TGC) waveform data. The proposed approach uses a linear regression model and makes daily predictions of the track state within an adjacent inspection cycle of the future. In each inspection cycle, the modelling states are sequentially updated using the latest inspection state, and the cycle is optimised on a rolling basis.

Kawaguchi et al. [172] proposed two models to predict alignment irregularities based on historical track quality data. The first evaluated different maintenance plans by estimating the mean time to maintenance of track alignment through analysis of track lateral deformation. The second made year-in-advance predictions of alignment irregularities using the exponential smoothing method. They suggested that the first model is suitable for undertaking economic analysis of different maintenance strategies, while the second can form the basis for constructing a yearly maintenance-scheduling plan.

Bai et al. [173] developed an approach termed the tree-augmented naïve Bayes-track quality index (TAN-TQI) to determine probable underlying patterns or rules for making track irregularity predictions based on the characteristic deterioration for a short-term horizon. A core component of the model is the difference in track irregularities between two successive points in time, which signifies the quantifiable manifestations of the cumulative effects of different factors during this period [173]. They demonstrated the applicability of the proposed framework to forecasting the condition for the next inspection by using irregularity data from the four previous cycles. They concluded that their framework

provided better predictability than the SRPM model.

Xu et al. [174] developed a machine learning model using a multi-stage linear fitting framework integrated with a linear regression model to describe the evolution of track irregularity over time using waveform data. Making use of control thresholds of irregularities for different tiers, track irregularities over these (tiers) were estimated by different linear regression models, each for different tiers [174].

Berggren [175] outlined techniques from the field of pattern recognition, and proposed a method for eliciting new information from existing condition data to catalogue the root causes of track issues. He demonstrated that by building a root-cause classifier, certain track-related issues could be resolved, with the main output being the classification of the features based on their effect.

Time Series Models. Track geometry deterioration models can be constructed using time series models. Sequential predictions can then be forecast using recent track geometry data through a stepwise autoregressive (AR) model. Correlation analysis can then be used to enable the AR model to compute the number of terms required to make accurate forecasts. An alternative class of time series models is the autoregressive moving average model (ARMA).

Chaolong et al. [176] studied time series track irregularity SD data and applied clustering analysis to investigate the characteristic and trend changes of track state. They determined different patterns and specifications of track irregularity behaviour. They employed the linear-ARMA and the linear recursive model to identify and forecast the variations in time series trends of track irregularity SD for a short segment of track.

Grey Box Models. Grey Box models (GMs) have their roots in Grey system theory [177]. In the current context, one of their main characteristics is that they do not follow a global trend, but attempt to follow the original deterioration pattern. Perhaps more significantly, they are advantageous in not being data-intensive, and also in having the capability of dealing with systems having partially unknown parameters [178]. Famurewa et al. [179] present an approach for assessing track geometry data and compare it with two other quality prediction models (linear, exponential, and the recommended grey model GM(1,1)¹). Their proposed model showed a lower MPE (mean percentage error) than the linear model, and approximately the same error as that of the exponential model. As the GM updates its parameters to new conditions as new data become available, this might give it an advantage over exponential or simple linear regression models [23]. Furthermore, GMs can make predictions based on only four measurements with a reasonable goodness-of-fit. However, as highlighted by Soleimanmeigouni et al. [23], the estimate may not be reliable, if a lack of sufficient measurement data hinders delineation of the deterioration path.

Liu et al. [180] found that a combination of a centre approximation GM(1,1) and a Markov chain model (termed the Markov-Grey GM(1,1) model) gave better predictability than the traditional centre approach Grey GM(1,1) models. They suggested that the incorporation of the Markov chain was able to overcome the shortcomings of the traditional GM(1,1), which are primarily employed for making predictions based on smoother data whose variation is exponential, and are thus less suited to forecasting data involving step-changes and randomness [180].

Chaolong et al. [181] presented a modified GM based on features of track cross-level data to make predictions of track irregularity at a fixed measuring point. Following validation, they concluded that the proposed approach could make accurate long-term forecasts of the changing trend in track irregularity. They also compared the use of ANN, Kalman filtering and random linear AR models to forecast short-term

¹ In Grey systems theory, GM(n, m) represent a GM. n : the order of the differential equation, and m : the number of variables. GM(1,1) is the most extensively applied GM, termed as 'Grey Model First Order One Variable'.

state changes, concluding that the ANN model had slightly better accuracy than the other two.

Multistage Linear Models. To account for potential nonlinear or periodic deterioration profiles between two successive intervention cycles, some authors have used multistage linear regression models [23]. For example, Chang et al. [182] used multistage, exponential, and cyclic profiles to model the deterioration in track geometry between two successive intervention actions. Based on the first two, they used a set of linear models to model the different stages of TQI deviation, and attempted to validate the proposed model through a case study. They concluded that their approach yielded more accurate results than simple linear models. Guo and Han [183] adopted a multistage linear model to represent the different stages of track deterioration between two successive maintenance actions, as well as the exponential growth of track irregularity.

Multistage linear models are generally held to increase the accuracy of deterioration modelling predictions. Two issues restrict their applicability: (1) determining the number of deterioration phases is data-intensive, and (2) model parameter estimation is computationally complex. While in principle, three deterioration phases should be modelled, in practice, the bedding-in and wearing-out phases are relatively short, or in some cases nonexistent, and much of the deterioration path does typically evolve linearly within an intervention cycle [38,184]. Furthermore, the common assumption of cyclic behaviour means that track deterioration behaviour between different maintenance cycles is approximately the same. However, it is commonly accepted that a maintenance intervention will affect the modelling parameters for the track geometry deterioration profile.

Random Coefficient Models. Consideration of section-level inhomogeneity in deterioration rates enables the variations in deterioration processes between segments of the same railway line to be better represented [185]. This variability can be described through the incorporation of random coefficients into the deterioration models [185–187]. Andrade and Teixeira [185] considered four track section groups: (1) bridges, (2) switches, (3) plain track, and (4) stations; and subsequently adopted a linear model for forecasting the development of the SD of longitudinal defects. Using statistical correlation analysis, they fitted a lognormal distribution to the deterioration parameters.

Andrade and Teixeira [186] adopted a Bayesian approach to estimate track geometry deterioration and deal with the uncertainty in its modelling parameters. As in their previous study [185], they assumed that the evolution of SD of the longitudinal defects (per 200 m section) followed a linear relationship with accumulated tonnage. Their approach allowed the assessment of the evolution of uncertainty linked with deterioration parameters throughout the railway track life cycle. They fitted prior probability distributions to track geometry deterioration; with the lognormal distribution found to be the most suited to the model deterioration parameters. Subsequently, they segmented the track into four section groups based on their infrastructure features, and elicited posteriors of the parameters for different stages to evaluate uncertainty reductions each time new inspection data became available (at design, after the first inspection, between the first inspection and the first tamping intervention, and between the second inspection and the remaining maintenance cycles). An important benefit of this approach is the consideration of uncertainty in the modelling parameters in simulating the deterioration process.

Andrade and Teixeira [187] proposed a statistical approach for modelling geometry deterioration based on Hierarchical Bayesian models. For railway track deterioration, conditional autoregressive (CAR) terms were introduced to account for spatial dependencies between model parameters (successive track segments in railway lines). They assumed a Gaussian prior for the SD of the longitudinal level, with its mean for different segments of track being dependent on their initial

track quality and deterioration rate before/after renewal, as well as the disturbance effect after each tamping intervention. They expressed the disturbance effect parameter through a Gaussian distribution, and each variance component in each hierarchical structure, by assigning inverse gamma distributions to each component. Finally, inference was conducted based on Markov Chain Monte Carlo (MCMC) simulation. The advantages of the proposed model lie primarily in its ability to (1) reduce the uncertainty in the geometry deterioration parameters over the length of the track by postulating a spatial dependency between successive sections, and (2) include tamping/renewal effects in the values of the deterioration parameters.

In summary, random coefficient models have the advantage of being able to capture the variability in deterioration parameters over the track length. Through the incorporation of more influencing factors, distributions with better goodness-of-fit and lower variance will be obtained in modelling [23]. However, it is also important to maintain a parsimonious structure to avoid excessive computational complexity. Finally, path analysis can aid in the identification and shortlisting of candidate factors for designing these models [23].

Markov Chain Models. Markov models represent an important class of statistical methods for modelling track geometry deterioration. The primary task in developing such models is to compute the transition probability from the sampling data by means of specific calibration techniques (two categories: state or time-based). These are used to calibrate the data and derive the transition probability matrix. In a Markov model, the probability of an event depends on the state reached in the previous event, making such models ‘memoryless’ [22,38,188,189]. So far, the Markov approach has been successfully implemented for small-scale track models.

Shafahi and Hakhamaneshi [189] proposed a cumulative damage model, based on a Markov process, which was used to model track geometry deterioration. The model has been constructed to reflect the track condition as a TQI with a range of 100, based on alignment, gauge, unevenness, and twist data, which was subsequently mapped into five-states.

Lyngby et al. [33] proposed a fifty-state Markov model to replicate the changes in twist over time in the range of 1 to 50 mm. They assigned alternative deterioration rates to the model depending on the type of the track section (straight, curved, or transition).

A later study [190] adopted a Markov stochastic process approach to examine the performance of a section unit in terms of its deterioration. The model was developed to represent track condition as a TQI, which was subsequently mapped into four ranks of the irregularity state to evaluate its condition. The authors formulated a hazard model using the heterogeneity of the section units and subsequently constructed a transition probability matrix. They took the interval of inspection between two successive interventions as Markov model time steps. Using historical data on the section units, including data on heterogeneous factors, they constructed a maximum log-likelihood function to derive the necessary transition probabilities. They concluded that the presence of these heterogeneous factors plays an essential role in the deterioration rates; with sections of the same mileage displaying different deterioration rates due to the variability of these factors.

Prescott and Andrews [191] described an eighty-state Markov model of a one-eighth mile section that was used to investigate the effects of different asset management strategies. The model maps the changes in track condition with time, as indicated by a measurement of the vertical alignment for a given strategy, which is defined through the specification of a set of model parameters. It incorporates parameters such as the deterioration of vertical alignment, restoration action by tamping, and track inspection by a track measurement train, as well as the dependence of the track deterioration rate on the maintenance history. The proposed approach was evaluated numerically through a fourth-order Runge-Kutta algorithm. It was subsequently used to study the effects of

modifying different parameters, these are: (1) mean time to perform routine maintenance, (2) inspection interval time, (3) renewal period, and (4) threshold that triggers maintenance intervention. The authors demonstrated the applicability of the model for investigating the tradeoffs for different asset management strategies in terms of cost and risk associated with different renewal, maintenance, and inspection strategies, as well as accidents and delays, and the follow-on financial penalties imposed. As highlighted by the authors, other aspects of track deterioration could be accounted for in the model (such extensions might include lateral misalignment or rail wear along with related interventions such as rail grinding). Nevertheless, such an expansion might lead to a prohibitively large model.

However, this class of models has certain limitations that restrict their applicability for representing track deterioration and maintenance [23,33,38,166,188,190]:

- The evolution of track geometry is a continuous process, while the Markov chain is a discrete model (time/state). Thus, the geometry measures should be first discretised, which could lead to inaccuracies of estimating track geometry deterioration;
- The transition between asset states must occur at a constant rate and the sojourn times are therefore governed by an exponential distribution;
- The 'memoryless' nature of the process is highly restrictive on how the way in which the degraded state of geometry was reached can account for the maintenance history;
- The model size grows exponentially relative to the number of additional components. This makes it impossible to model multiple one-eighth mile sections, and construct a model at a line level;
- Due to the section-inhomogeneity of track deterioration, individual transition matrices must be derived for different maintenance units, which include different segments with similar characteristics. Nevertheless, identifying track maintenance units with similar transition matrices can be challenging;
- The method for calculating the transition probabilities for such models is data-intensive.

Petri Nets. A Petri Net (PN) is a graphical tool that comprises three basic elements: nodes, arcs, and transitions. It combines mathematical background and graphics to represent complex dynamic systems behaviour. Two types of nodes are represented in a PN: places (circles) and transitions (rectangular boxes), linked to each other by directed edges (commonly referred to as arcs or arrows). Places reflect either an activity being modelled or a specific state of the system, while transitions incorporate the system's dynamic behaviour by enabling it to move between states. In the context of track maintenance, places could indicate: (1) the present state of an asset, (2) whether it is operating, and (3) whether any maintenance intervention is currently in process [192]. Tokens mark a system's present state at any time by residing in different places, with transitions governing their movement between these places. For example, transitions may reflect the time delay between successive inspections within a cycle, and places indicate the inspection states [192].

The MCS (Monte Carlo Simulation) method can be used to solve the PN model of a given system, by randomly sampling times for events from the appropriate statistical distribution, with the simulation commencing (starting to move tokens around) once its lifetime is selected.

Andrews [38] modelled the deterioration, maintenance, and inspection of single one-eighth mile sections. The transition times of assets degrading to different states were modelled for homogeneous segments by adopting a two-parameter Weibull distribution, for a different track (region, rail, and sleeper type, speed classes, cumulative tonnage per annum) and life phase/state features (number and sequence of interventions implemented). This family of distributions is regularly adopted in such models (failure/ deterioration) owing to their flexibility

in representing many different distribution shapes [193]. Moreover, they can provide failure analysis and prediction with a reasonable level of accuracy [194], while also dealing with small data samples [184]. This is particularly the case for the modelling of mechanical components such as rails, whose defects have been shown to evolve following a Weibull law [195]. Andrews [38] adopted these distributions to model the transition times, with the action thresholds for interventions and intervals for inspection being set as the decision variables. This methodology allowed the distribution of times to deterioration events (states defined by the SD of a number of maintenance characteristics) for a given type of track and maintenance history to be attained by monitoring the condition of respective one-eighth mile sections.

Andrews et al. [188] extended the previous work by applying a PN architecture to predict track deterioration behaviour considering the effect of different asset management strategies, through the variation of different parameters (i.e., intervention threshold, inspection, renewal, routine repair time). Their analysis revealed that the intervention intervals influence the deterioration rate, which changes accordingly from phase-to-phase. Considering the renewal times, Andrews et al [188] found that their extension had no meaningful impact on the time that the track resides in a state of good condition. They demonstrated that by including the costs of performing different maintenance actions, as well as the penalty costs (associated with potential line closures or speed restrictions), the LCC of each maintenance strategy could be estimated. While the proposed model can successfully forecast track segment condition over long timescales, it is unable to make predictions at a track line level. Thus, a potential refinement of the model by considering a series of one-eighth mile segments will be beneficial in allowing the integration of the conflicting requirements of tamping machines, as well as the ability to perform opportunistic maintenance.

Prescott and Andrews [196] developed a model based on a PN methodology permitting the analysis of a region of the railway network. The authors defined a 'region' as meaning a part of a network containing a number of one-eighth mile segments. A later study by Prescott and Andrews [197] constructed a PN model in a modular fashion that allowed a number of regions comprising a railway network to be assessed in terms of track deterioration, inspection, and maintenance. They considered the important issue of performing concurrent interventions of tamping and stoneblowing, by recognising the practical limitation of performing these activities (that is, a limited number of available machines to be allocated to different segments of track). Again, Prescott and Andrews [197] adopted a two-parameter Weibull distribution as the deterioration time distribution, considering four action limits: (i) opportunistic maintenance permitted, (ii) maintenance needed, (iii) speed reduction needed, and (iv) line closure needed thresholds. They established four corresponding states of deterioration, with the transitions between them occurring by sampling from the specified distributions. The primary innovation of this study was the integration of a maintenance DSS module into the PN architecture. The disadvantage of the model by Prescott and Andrews [196] was thereby eliminated, enabling consideration of opportunistic maintenance and leading to the extension of a single section towards a network-wide model. The decision for grouping the maintenance actions was based on: (i) states of track deterioration, (ii) machine availability, and (iii) section locations. Overall, the consideration of criteria such as the total maintenance costs and line availability for grouping major works according to opportunistic decision-making principles will enhance the model flexibility.

Rama and Andrews [198] developed a framework involving an infrastructure performance model embedded in a LCC model to carry out a whole-life costing analysis. They structured their model through a PN (including three core sub-nets: for deterioration, inspection, and maintenance), based on their previous work [199]. For the proposed infrastructure-state model, they adopted a hierarchical modular architecture, allowing a multi-asset configuration of the infrastructure (with varying degrees of complexity/detail) within a hierarchical topology of

the network (a six-level architecture) to be portrayed. This enabled the model to be used in performance prediction at both asset (single maintainable item) and system-wide (whole network) levels. This approach permitted the interdependencies among different intervention activities (i.e., opportunistic, concurrent maintenance, etc.) to be accounted for, and their subsequent effect on costs and performance to be evaluated.

Zhang et al. [192] proposed a PN-based rail maintenance model underpinned by a MCS, comprising several individual sub-nets for representing deterioration and defect/failure, inspection, maintenance, lubrication, and rail grinding. The resulting PN architecture feeds into a wider LCC framework, allowing the systematic investigation of different performance parameters including the number of interventions, maintenance costs, and deterioration profile of rails over their lifecycle. Zhang et al. [192] demonstrated the ability of the model to simulate the deterioration profile of rails and evaluate their LCC over 35 years through a case study.

Hybrid Models

Hybrid models have been developed to combine the best elements of mechanistic and empirical approaches. Fundamentally, this class of model is based on an understanding of the behaviour of system components, coupled with measurements, direct observations, and extensive data records [166]. Prior to constructing these models, track segmentation centred on building segments with homogeneous properties (e.g., influencing factors, maintenance history, etc.) is necessary [200]. Existing engineering knowledge of different covariates affecting the deterioration profile is used to explain empirical track measurement data. Often, statistical regression is employed over average values of different parameters to construct appropriate predictive relationships, for each of the partitioned section groups. For example, Sadeghi and Askarinejad [143] proposed a hybrid model combining statistical and mechanistic approaches based on regression analysis that incorporated geometry and structural condition track data recorded over two years. By adopting a deterioration coefficient, they evaluated the effect of structural condition, initial geometry, average running speed, and total equivalent million gross tons (EMGT). Based on the method of least square errors, the authors suggested that an exponential form was the most suitable approximation between the adopted coefficient and the examined set of influencing parameters [143]. Among them, the initial TQI had the greatest influence on the track deterioration rate, followed by the total passing tonnage along the track [143].

Rhayma et al. [201] suggested an adaptable stochastic approach to various mechanistic models to represent track deterioration behaviour and deal with the inherent variability of the mechanical and geometrical parameters of the railway track. They adopted a reliability-centred approach grounded on a non-intrusive stochastic finite element method (SFE), and aimed to assess the influence of different maintenance actions on the rail track probabilistically. Considering this, they used distributions instead of fixed values for the influencing factors to model the innate uncertainty of such factors.

Soleimanmeigouni et al. [202] proposed a hybrid-like approach so as to account for the spatial variations in track deterioration parameters and their potential dependencies in consecutive track segments. They developed a two-level piecewise linear model with the deterioration characteristics being modelled by a piecewise linear model with known break points at the tamping interventions and a multivariate linear regression model to link different variables on the maintenance interventions and the subsequent track section conditions with the post-intervention responses (e.g. increased deterioration rate after a tamping cycle, etc.). Finally, they used a series of ARMA models to capture the spatial correlations between different modelling parameters. They then performed an illustrative case study using data from the Main Western Line in Sweden to demonstrate the applicability of their model. Their proposed model demonstrated important benefits of accounting for both spatial and temporal variations and dependencies between

adjacent sections, which makes it more realistic when compared with other models in the literature.

More recently, Soleimanmeigouni et al. [203] proposed an integrated approach for investigating the effect of various inspection intervals on railway track geometry using the SD of the longitudinal level as the main indicator for assessing the need for maintenance actions. Similarly to their previous study, they adopted a random coefficient piecewise exponential approach to model the deterioration and restoration processes of railway track geometry. They also integrated the effect of isolated defects using an ordinal logistic regression to estimate their probability of occurrence. Their finalised model has the capability to estimate the percentage of time spent in different track geometry states, as well as the number of different maintenance interventions in a given time span. Therefore, this approach could be of use to IMs as it could help them on optimising their inspection intervals based on different direct and indirect cost parameters, which could be estimated using this model (e.g. corrective/preventive tamping interventions, risk of derailment, capacity losses from speed restrictions, etc.).

Conclusions

This paper has critically reviewed the merits and limitations of existing railway track deterioration modelling approaches and provided a systematic taxonomy of these models. From this, a hierarchical classification was proposed based on the model level of detail and functionality.

Although considerable progress has been made in extending the capabilities of track-related deterioration prediction models, there are still several challenges needing further consideration. These issues have both practical and theoretical dimensions and have not been yet adequately resolved:

- consideration of spatial and temporal variations and dependencies;
- joint consideration of competing failure modes (random shocks, deterioration-related failures);
- identification and consideration of different covariates present along the length of a track.

To address the distributed nature of the system, natural segmentation should be adopted based on each section's operational, environmental, and structural features. The use of aggregated TQI's should be carefully considered in the modelling process as they may mask localised deterioration of assets which are often prone to higher deterioration rates, such as for example, transition zones between an embankment and a bridge, level crossings, etc.

Considering spatial variations and dependencies where different track sections may exhibit different deterioration profiles, path analysis has been proposed in the literature as a means of identifying the different covariates that influence track deterioration behaviour. Random coefficient models can also be structured to address the track section-variability with random effects reflecting its characteristics.

Concerning temporal variations and dependencies of the deterioration profile, the literature makes extensive use of stochastic processes (e.g., Wiener, Gamma process, etc.) to model the deterioration of the infrastructure over time due to their ability to capture such uncertainties.

It is important to note that although these empirical data-driven methodologies have some clear advantages over the mechanistic approaches when it comes to their macroscopic predictive capabilities (see Fig. 1), their accuracy is heavily reliant on the quantity and quality of the input data sets, which makes them prone to errors and invalid results.

Physics-based (mechanistic) models have a strong theoretical basis and can be used to test assumptions regarding micro- and/or macro-mechanisms governing track deterioration components (see Fig. 1). Discrete element and continuum mechanics models are perhaps the most

suitable for analysing ballast mechanical behaviour and deterioration mechanisms at the grain and macro-scale, respectively. However, owing to their high computational demand, they can only be applied for studying relatively short track sections and/or for a small number of cycles.

Semi-analytical models are also promising as they can be integrated into dynamic VTI models to consider track and traffic characteristics and potentially simulate the accumulation of settlement over millions of cycles. However, owing to their structure (based on deterministic input–output relationships), they do not easily account for uncertainty in their calculations. It is important for future research to prioritize examining any potential interactions between the different methodologies presented in this review. For example, semi-analytical models could be complemented by machine-learning, which will help accounting for a larger set of unknown input parameters, and allow for a better representation of the ballasts' (including substructure) propensity to settle. This will enable the formulation of hybrid computational models with the capability to adequately simulate track deterioration (both micro- and macroscopically) and subsequently, enable railway researchers and practitioners to investigate the impact of improvements to the underlying system to improve track quality and drive down costs.

CRedit authorship contribution statement

Georgios Rempelos: Writing – original draft, Investigation, Conceptualization. **Giacomo Ognibene:** Writing – original draft, Investigation. **Louis Le Pen:** Writing – review & editing, Supervision. **Simon Blainey:** Writing – review & editing, Supervision, Project administration. **John Preston:** Writing – review & editing, Supervision, Project administration, Funding acquisition. **William Powrie:** Writing – review & editing, Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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References

- [1] European Commission. Eighth monitoring report on the development of the rail market under Article 15(4) of Directive 2012/34/EU of the European Parliament and of the Council. Brussels: 2023.
- [2] ORR. Passenger rail usage. Office of Rail and Road 2019. <https://dataportal.orr.gov.uk/> (accessed July 1, 2019).
- [3] ORR. Passenger rail usage October to December 2023 2024. <https://dataportal.orr.gov.uk/media/v1ilmjnt/passenger-rail-usage-oct-dec-2023.pdf> (accessed April 25, 2024).
- [4] Eurostat. Rail transport of passengers 2024. 10.2908/TTR00015.
- [5] Eurostat. Goods transport by rail 2024. 10.2908/TTR00006.
- [6] Rempelos G. A whole life carbon model for railway track system interventions. Thesis of Doctor of Philosophy. Southampton, United Kingdom: University of Southampton; 2023.
- [7] Dahlberg T. Some railroad settlement models - A critical review. Proc Inst Mech Eng F J Rail Rapid Transit 2001;215:289–300. <https://doi.org/10.1243/0954409011531585>.
- [8] Abadi T, Le Pen L, Zervos A, Powrie W. A Review and Evaluation of Ballast Settlement Models using Results from the Southampton Railway Testing Facility (SRTF). Procedia Eng 2016;143:999–1006. <https://doi.org/10.1016/j.proeng.2016.06.089>.
- [9] Grossoni I, Powrie W, Zervos A, Bezin Y, Le Pen L. Modelling railway ballasted track settlement in vehicle-track interaction analysis. Transp Geotech 2021;26: 1–14. <https://doi.org/10.1016/j.trgeo.2020.100433>.
- [10] Weston P, Roberts C, Yeo G, Stewart E. Perspectives on railway track geometry condition monitoring from in-service railway vehicles. Veh Syst Dyn 2015;53: 1063–91. <https://doi.org/10.1080/00423114.2015.1034730>.
- [11] Falamarzi A, Moridpour S, Nazem M. Development of a tram track degradation prediction model based on the acceleration data. Struct Infrastruct Eng 2019;15: 1308–18. <https://doi.org/10.1080/15732479.2019.1615963>.
- [12] de Melo ALO, Kaewunruen S, Papaelias M, Bernucci LLB, Motta R. Methods to Monitor and Evaluate the Deterioration of Track and Its Components in a Railway In-Service: A Systemic Review. Front Built Environ 2020;6:1–15. <https://doi.org/10.3389/fbuil.2020.00118>.
- [13] Lidén T. Railway infrastructure maintenance - A survey of planning problems and conducted research. Transp Res Procedia 2015;10:574–83. <https://doi.org/10.1016/j.trpro.2015.09.011>.
- [14] Lidén T. Survey of railway maintenance activities from a planning perspective and literature review concerning the use of mathematical algorithms for solving such planning and scheduling problems. Norrköping, Sweden: Linköping University, Faculty of Science & Engineering; 2014.
- [15] Lidén T. Towards concurrent planning of railway maintenance and train services. Thesis of Doctor of Philosophy. Norrköping, Sweden: Linköping University, Faculty of Science & Engineering; 2016. 10.3384/lic.diva-128780.
- [16] Turner C, Tiwari A, Starr A, Blacktop K. A review of key planning and scheduling in the rail industry in Europe and UK. Proc Inst Mech Eng F J Rail Rapid Transit 2016;230:984–98. <https://doi.org/10.1177/0954409714565654>.
- [17] Ferreira L, Murray MH. Modelling rail track deterioration and maintenance: Current practices and future needs. Transp Rev 1997;17:207–21. <https://doi.org/10.1080/01441649708716982>.
- [18] Budai-Balke G. Operations Research Models for Scheduling Railway Infrastructure Maintenance. Thesis of Doctor of Philosophy. Rotterdam, the Netherlands: Erasmus University Rotterdam; 2009.
- [19] Li R. Phase-based Planning for Railway Infrastructure Projects. Thesis of Doctor of Philosophy. Lyngby, Denmark: Technical University of Denmark; 2017.
- [20] Sedghi M, Kauppila O, Bergquist B, Vanhatalo E, Kulachi M. A taxonomy of railway track maintenance planning and scheduling: A review and research trends. Reliab Eng Syst Saf 2021;215:107827. <https://doi.org/10.1016/j.res.2021.107827>.
- [21] Guler H. Decision Support System for Railway Track Maintenance and Renewal Management. J Comput Civ Eng 2012;27:292–306. [https://doi.org/10.1061/\(asce\)cp.1943-5487.0000221](https://doi.org/10.1061/(asce)cp.1943-5487.0000221).
- [22] Higgins C, Liu X. Modeling of track geometry degradation and decisions on safety and maintenance: A literature review and possible future research directions. Proc Inst Mech Eng F J Rail Rapid Transit 2018;232:1385–97. <https://doi.org/10.1177/0954409717721870>.
- [23] Soleimanmeigouni I, Ahmadi A, Kumar U. Track geometry degradation and maintenance modelling: A review. Proc Inst Mech Eng F J Rail Rapid Transit 2018;232:73–102. <https://doi.org/10.1177/0954409716657849>.
- [24] Shang Y, Nogal M, Wang H, Wolfert ARM, (Rogier). Systems thinking approach for improving maintenance management of discrete rail assets: a review and future perspectives. Struct Infrastruct Eng 2023;19:197–215. <https://doi.org/10.1080/15732479.2021.1936569>.
- [25] Sato Y. Japanese Studies on Deterioration of Ballasted Track. Veh Syst Dyn 1995; 24:197–208. <https://doi.org/10.1080/00423119508969625>.
- [26] Shenton MJ. Ballast deformation and track deterioration. Track Technology, Derby, United Kingdom: Research & Development Division, British Railways Board; 1985, p. 253–65. 10.1680/tt.02289.0026.
- [27] Dahlberg T. Track Issues. In: Iwnicki S, editor. Handbook of Railway Vehicle Dynamics. 1st ed., Boca Raton, FL: CRC Press Taylor & Francis Group, LLC; 2006, p. 38. 10.1201/9781420004892.
- [28] Tzanakakis K. The Railway Track and Its Long Term Behaviour: A Handbook for a Railway Track of High Quality. In: Roess RP, editor. Springer Tracts on Transportation and Traffic (STTT, volume 2). 1st ed., Springer; 2013, p. 448. 10.1007/978-3-642-36051-0.
- [29] Holtzendorff K, Gerstberger U. Predicting Settlements of Ballasted Tracks due to Voided Sleepers. Proceedings of the WCRR 2001, 2001.
- [30] Jovanovic S. Railway track quality assessment and related decision making. Conf Proc IEEE Int Conf Syst Man Cybern 2004;6:5038–43. <https://doi.org/10.1109/ICSMC.2004.1400992>.
- [31] Yoo T-S, Selig ET. Field Observations of Ballast and Subgrade Deformations in Track. Transp Res Rec 1979;733:6–12.
- [32] Zhang X, Zhao C, Zhai W, Shi C, Feng Y. Investigation of track settlement and ballast degradation in the high-speed railway using a full-scale laboratory test. Proc Inst Mech Eng F J Rail Rapid Transit 2019;233:869–81. <https://doi.org/10.1177/0954409718812231>.
- [33] Lyngby N, Hokstad P, Vatn J. RAMS Management of Railway Tracks. In: Misra KB, editor. Handbook of Performability Engineering. First, London, United Kingdom: Springer-Verlag London Ltd.; 2008, p. 1123–45. 10.1007/978-1-84800-131-2_68.
- [34] Zhang Y-J, Murray M, Ferreira L. Modelling rail track performance: an integrated approach. Proceedings of the Institution of Civil Engineers - Transport 2000;141: 187–94. 10.1680/tran.2000.141.4.187.
- [35] Bathurst RJ, Raymond GP. Geogrid reinforcement of ballasted track. Transp Res Rec 1987;8–14.

- [36] Suiker ASJ, Selig ET, Frenkel R. Static and Cyclic Triaxial Testing of Ballast and Subballast. *J Geotech Geoenviron Eng* 2005;131:771–82. [https://doi.org/10.1061/\(asce\)1090-0241\(2005\)131:6\(771\)](https://doi.org/10.1061/(asce)1090-0241(2005)131:6(771)).
- [37] Lobo-Guerrero S, Vallejo LE. Discrete element method analysis of railtrack ballast degradation during cyclic loading. *Granul Matter* 2006;8:195–204. <https://doi.org/10.1007/s10035-006-0006-2>.
- [38] Andrews J. A modelling approach to railway track asset management. *Proc Inst Mech Eng F J Rail Rapid Transit* 2013;227:56–73. <https://doi.org/10.1177/0954409712452235>.
- [39] Varandas JN, Paixão A, Fortunato E, Zuada Coelho B, Hölscher P. Long-term deformation of railway tracks considering train-track interaction and non-linear resilient behaviour of aggregates – a 3D FEM implementation. *Comput Geotech* 2020;126:1–19. <https://doi.org/10.1016/j.compgeo.2020.103712>.
- [40] Varandas JN, Hölscher P, Silva MAG. Settlement of ballasted track under traffic loading: Application to transition zones. *Proc Inst Mech Eng F J Rail Rapid Transit* 2014;228:242–59. <https://doi.org/10.1177/0954409712471610>.
- [41] Wilk ST, Stark TD. Modeling Progressive Settlement of a Railway Bridge Transition. ASME/IEEE Joint Rail Conference, Columbia, South Carolina: 2016, p. 10. 10.1115/JRC2016-5715.
- [42] Nielsen JCO, Li X. Railway track geometry degradation due to differential settlement of ballast/subgrade – Numerical prediction by an iterative procedure. *J Sound Vib* 2018;412:441–56. <https://doi.org/10.1016/j.jsv.2017.10.005>.
- [43] Wang H, Markine V. Modelling of the long-term behaviour of transition zones: Prediction of track settlement. *Eng Struct* 2018;156:294–304. <https://doi.org/10.1016/j.engstruct.2017.11.038>.
- [44] Grossoni I, Andrade AR, Bezin Y, Neves S. The role of track stiffness and its spatial variability on long-term track quality deterioration. *Proc Inst Mech Eng F J Rail Rapid Transit* 2019;233:16–32. <https://doi.org/10.1177/0954409718777372>.
- [45] Cundall PA, Strack ODL. A discrete numerical model for granular assemblies. *Geotechnique* 1979;29:47–65. <https://doi.org/10.1680/geot.1979.29.1.47>.
- [46] Qian Y, Tutumluer E, Hashash YMA, Ghaboussi J, Davis DD. Ballast settlement ramp to mitigate differential settlement in a bridge transition zone. *Transp Res Rec* 2015;2476:45–52. <https://doi.org/10.3141/2476-07>.
- [47] Bian X, Huang H, Tutumluer E, Gao Y. “Critical particle size” and ballast gradation studied by Discrete Element Modeling. *Transp Geotech* 2016;6:38–44. <https://doi.org/10.1016/j.trgeo.2016.01.002>.
- [48] Li H, McDowell GR. Discrete element modelling of under sleeper pads using a box test. *Granul Matter* 2018;20:1–12. <https://doi.org/10.1007/s10035-018-0795-0>.
- [49] Kumar N, Suhr B, Marschnig S, Dietmaier P, Marte C, Six K. Micro-mechanical investigation of railway ballast behaviour under cyclic loading in a box test using DEM: effects of elastic layers and ballast types. *Granul Matter* 2019;21:1–17. <https://doi.org/10.1007/s10035-019-0956-9>.
- [50] Ferro E, Harkness J, Le Pen L. The influence of sleeper material characteristics on railway track behaviour: concrete vs composite sleeper. *Transp Geotech* 2020;23:1–12. <https://doi.org/10.1016/j.trgeo.2020.100348>.
- [51] Ni Q, Powrie W, Zhang X, Harkness R. Effect of Particle Properties on Soil Behavior: 3-D Numerical Modeling of Shearbox Tests. In: *Numerical Methods in Geotechnical Engineering*. Reston, VA: American Society of Civil Engineers; 2000. p. 58–70. [https://doi.org/10.1061/40502\(284\)5](https://doi.org/10.1061/40502(284)5).
- [52] Powrie W, Ni Q, Harkness RM, Zhang X. Numerical modelling of plane strain tests on sands using a particulate approach. *Geotechnique* 2005;55:297–306. <https://doi.org/10.1680/geot.2005.55.4.297>.
- [53] Indraratna B, Nimbalkar S, Christie D, Rujikiatkamjorn C, Vinod J. Field Assessment of the Performance of a Ballasted Rail Track with and without Geosynthetics. *J Geotech Geoenviron Eng* 2010;136:907–17. [https://doi.org/10.1061/\(asce\)gt.1943-5606.0000312](https://doi.org/10.1061/(asce)gt.1943-5606.0000312).
- [54] Indraratna B, Ngo NT, Rujikiatkamjorn C, Vinod JS. Behavior of Fresh and Fouled Railway Ballast Subjected to Direct Shear Testing: Discrete Element Simulation. *Int J Geomech* 2014;14:34–44. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0000264](https://doi.org/10.1061/(asce)gm.1943-5622.0000264).
- [55] Chakraborty N, Peng J, Akella S, Mitchell JE. Proximity queries between convex objects: An interior point approach for implicit surfaces. *IEEE Trans Rob* 2008;24:211–20. <https://doi.org/10.1109/TRO.2007.914851>.
- [56] Lopes DS, Silva MT, Ambrósio JA, Flores P. A mathematical framework for rigid contact detection between quadric and superquadric surfaces. *Multibody Syst Dyn* 2010;24:255–80. <https://doi.org/10.1007/s11044-010-9220-0>.
- [57] Rothenburg L, Bathurst RJ. Numerical simulation of idealized granular assemblies with plane elliptical particles. *Comput Geotech* 1991;11:315–29. [https://doi.org/10.1016/0266-352X\(91\)90015-8](https://doi.org/10.1016/0266-352X(91)90015-8).
- [58] Ting JM. A robust algorithm for ellipse-based discrete element modelling of granular materials. *Comput Geotech* 1992;13:175–86. [https://doi.org/10.1016/0266-352X\(92\)90003-C](https://doi.org/10.1016/0266-352X(92)90003-C).
- [59] Lin X, Ng TT. A three-dimensional discrete element model using arrays of ellipsoids. *Geotechnique* 1997;47:319–29. <https://doi.org/10.1680/geot.1997.47.2.319>.
- [60] Cundall PA. Formulation of a three-dimensional distinct element model-Part I. A scheme to detect and represent contacts in a system composed of many polyhedral blocks. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts* 1988;25:107–16. [https://doi.org/10.1016/0148-9062\(88\)92293-0](https://doi.org/10.1016/0148-9062(88)92293-0).
- [61] Saussine G, Cholet C, Gautier PE, Dubois F, Bohatier C, Moreau JJ. Modelling ballast behaviour under dynamic loading. Part 1: A 2D polygonal discrete element method approach. *Comput Methods Appl Mech Eng* 2006;195:2841–59. <https://doi.org/10.1016/j.cma.2005.07.006>.
- [62] Zhao D, Nezami EG, Hashash YMA, Ghaboussi J. Three-dimensional discrete element simulation for granular materials. *Eng Comput (Swansea)* 2006;23:749–70. <https://doi.org/10.1108/02644400610689884>.
- [63] Nezami EG, Hashash YMA, Zhao D, Ghaboussi J. Simulation of front end loader bucket-soil interaction using discrete element method. *Int J Numer Anal Methods Geomech* 2007;31:1147–62. <https://doi.org/10.1002/nag.594>.
- [64] Azéma E, Radjai F, Dubois F. Packings of irregular polyhedral particles: Strength, structure, and effects of angularity. *Phys Rev E* 2013;87:1–14. <https://doi.org/10.1103/PhysRevE.87.062203>.
- [65] Harkness J. Potential particles for the modelling of interlocking media in three dimensions. *Int J Numer Methods Eng* 2009;80:1573–94. <https://doi.org/10.1002/nme.2669>.
- [66] Houlsby GT. Potential particles: a method for modelling non-circular particles in DEM. *Comput Geotech* 2009;36:953–9. <https://doi.org/10.1016/j.compgeo.2009.03.001>.
- [67] Boon CW, Houlsby GT, Utili S. A new contact detection algorithm for three-dimensional non-spherical particles. *Powder Technol* 2013;248:94–102. <https://doi.org/10.1016/j.powtec.2012.12.040>.
- [68] Ahmed S, Harkness J, Le Pen L, Powrie W, Zervos A. Numerical modelling of railway ballast at the particle scale. *Int J Numer Anal Methods Geomech* 2016;40:713–37. <https://doi.org/10.1002/nag.2424>.
- [69] Harkness J, Zervos A, Le Pen L, Aingaran S, Powrie W. Discrete element simulation of railway ballast: modelling cell pressure effects in triaxial tests. *Granul Matter* 2016;18:1–13. <https://doi.org/10.1007/s10035-016-0660-y>.
- [70] Thakur PK, Vinod JS, Indraratna B. Effect of particle breakage on cyclic densification of ballast: A DEM approach. In: *IOP Conf Ser Mater Sci Eng*, vol. 10. Sydney: IOP Publishing; 2014. p. 1–7. <https://doi.org/10.1088/1757-899X/10/1/012229>.
- [71] Descantes Y, Tricoire F, Richard P. Classical contact detection algorithms for 3D DEM simulations: Drawbacks and solutions. *Comput Geotech* 2019;114:1–12. <https://doi.org/10.1016/j.compgeo.2019.103134>.
- [72] Jiang MJ, Yu HS, Harris D. Bond rolling resistance and its effect on yielding of bonded granulates by DEM analyses. *Int J Numer Anal Methods Geomech* 2006;30:723–61. <https://doi.org/10.1002/nag.498>.
- [73] Hossain Z, Indraratna B, Darve F, Thakur PK. DEM analysis of angular ballast breakage under cyclic loading. *Geomech Geoenviron Eng* 2007;2:175–81. <https://doi.org/10.1080/17486020701474962>.
- [74] Zhang T, Zhang C. Numerical Simulation of Particle Breakage of Granular Assemblies in Discrete Element Analyses. *Advances in Civil Engineering* 2019;2019:1–17. <https://doi.org/10.1155/2019/2048958>.
- [75] Ngo NT, Indraratna B, Rujikiatkamjorn C. Micromechanics-Based Investigation of Fouled Ballast Using Large-Scale Triaxial Tests and Discrete Element Modeling. *J Geotech Geoenviron Eng* 2017;143:04016089. [https://doi.org/10.1061/\(asce\)gt.1943-5606.0001587](https://doi.org/10.1061/(asce)gt.1943-5606.0001587).
- [76] Qian Y, Lee SJ, Tutumluer E, Hashash YMA, Ghaboussi J. Role of Initial Particle Arrangement in Ballast Mechanical Behavior. *Int J Geomech* 2018;18:04017158. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0001074](https://doi.org/10.1061/(asce)gm.1943-5622.0001074).
- [77] Ngo NT, Indraratna B, Rujikiatkamjorn C. Modelling geogrid-reinforced railway ballast using the discrete element method. *Transp Geotech* 2016;8:86–102. <https://doi.org/10.1016/j.trgeo.2016.04.005>.
- [78] Ji S, Sun S, Yan Y. Discrete element modeling of dynamic behaviors of railway ballast under cyclic loading with dilated polyhedra. *Int J Numer Anal Methods Geomech* 2017;41:180–97. <https://doi.org/10.1002/nag.2549>.
- [79] Huang H, Chrimer S. Discrete element modeling of ballast settlement under trains moving at “Critical Speeds”. *Constr Build Mater* 2013;38:994–1000. <https://doi.org/10.1016/j.conbuildmat.2012.09.007>.
- [80] Zhang X, Zhao C, Zhai W. Dynamic Behavior Analysis of High-Speed Railway Ballast under Moving Vehicle Loads Using Discrete Element Method. *Int J Geomech* 2017;17:04016157. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0000871](https://doi.org/10.1061/(asce)gm.1943-5622.0000871).
- [81] Nishiura D, Sakai H, Aikawa A, Tsuzuki S, Sakaguchi H. Novel discrete element modeling coupled with finite element method for investigating ballasted railway track dynamics. *Comput Geotech* 2018;96:40–54. <https://doi.org/10.1016/j.compgeo.2017.10.011>.
- [82] Li L, Liu W, Ma M, Jing G, Liu W. Research on the dynamic behaviour of the railway ballast assembly subject to the low loading condition based on a tridimensional DEM-FDM coupled approach. *Constr Build Mater* 2019;218:135–49. <https://doi.org/10.1016/j.conbuildmat.2019.05.102>.
- [83] Song W, Huang B, Shu X, Stránský J, Wu H. Interaction between Railroad Ballast and Sleeper: A DEM-FEM Approach. *Int J Geomech* 2019;19:04019030. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0001388](https://doi.org/10.1061/(asce)gm.1943-5622.0001388).
- [84] Lim NH, Kim KJ, Bae HU, Kim S. DEM analysis of track ballast for track ballast-wheel interaction simulation. *Appl Sci* 2020;10:1–19. <https://doi.org/10.3390/app10082717>.
- [85] Selig ET, Waters JM. *Track geotechnology and substructure management*. London, United Kingdom: Thomas Telford Limited; 1994.
- [86] Sharp RW, Booker JR. Shakedown of pavements under moving surface loads. *J Transp Eng* 1984;110:1–14. [https://doi.org/10.1061/\(ASCE\)0733-947X\(1984\)110:1\(1\)](https://doi.org/10.1061/(ASCE)0733-947X(1984)110:1(1)).
- [87] Werkmeister S, Dawson AR, Wellner F. Permanent deformation behavior of granular materials and the shakedown concept. *Transp Res Rec* 2001;1757:75–81. <https://doi.org/10.3141/1757-09>.
- [88] Alonso-Marroquín F, Herrmann HJ. Ratcheting of Granular Materials. *Phys Rev Lett* 2004;92:054301. <https://doi.org/10.1103/PhysRevLett.92.054301>.

- [89] Suiker ASJ, de Borst R. A numerical model for the cyclic deterioration of railway tracks. *Int J Numer Methods Eng* 2003;57:441–70. <https://doi.org/10.1002/nme.683>.
- [90] Salim W, Indraratna B. A new elastoplastic constitutive model for coarse granular aggregates incorporating particle breakage. *Can Geotech J* 2004;41:657–71. <https://doi.org/10.1139/T04-025>.
- [91] Indraratna B, Nimbalkar S. Stress-Strain Degradation Response of Railway Ballast Stabilized with Geosynthetics. *J Geotech Geoenviron Eng* 2013;139:684–700. [https://doi.org/10.1061/\(asce\)gt.1943-5606.0000758](https://doi.org/10.1061/(asce)gt.1943-5606.0000758).
- [92] Indraratna B, Thakur PK, Vinod JS, Salim W. Semiempirical Cyclic Densification Model for Ballast Incorporating Particle Breakage. *Int J Geomech* 2012;12:260–71. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0000135](https://doi.org/10.1061/(asce)gm.1943-5622.0000135).
- [93] Niemunis A, Wichtmann T, Triantafyllidis T. A high-cycle accumulation model for sand. *Comput Geotech* 2005;32:245–63. <https://doi.org/10.1016/j.compgeo.2005.03.002>.
- [94] Nguyen K, Villalmanzo DI, Goicolea JM, Gabaldon F. A computational procedure for prediction of ballasted track profile degradation under railway traffic loading. *Proc Inst Mech Eng F J Rail Rapid Transit* 2016;230:1812–27. <https://doi.org/10.1177/0954409715615374>.
- [95] François S, Karg C, Haegeman W, Degrande G. A numerical model for foundation settlements due to deformation accumulation in granular soils under repeated small amplitude dynamic loading. *Int J Numer Anal Methods Geomech* 2010;34:273–96. <https://doi.org/10.1002/nag.807>.
- [96] Perzyna P. Fundamental Problems in Viscoplasticity. *Adv Appl Mech* 1966;9:243–377. [https://doi.org/10.1016/S0065-2156\(08\)70009-7](https://doi.org/10.1016/S0065-2156(08)70009-7).
- [97] Elsworth D, Yasuhara H. Mechanical and transport constitutive models for fractures subject to dissolution and precipitation. *Int J Numer Anal Methods Geomech* 2010;34:533–49.
- [98] Indraratna B, Salim W. Modelling of particle breakage of coarse aggregates incorporating strength and dilatancy. *Proceedings of the Institution of Civil Engineers: Geotechnical Engineering* 2002;155:243–52. 10.1680/jeng.2002.155.4.243.
- [99] Niemunis A, Herle I. Hypoplastic model for cohesionless soils with elastic strain range. *Mechanics of Cohesive-Frictional Materials* 1997;2:279–99. [https://doi.org/10.1002/\(SICI\)1099-1484\(199710\)2:4<279::AID-CFM29>3.0.CO;2-8](https://doi.org/10.1002/(SICI)1099-1484(199710)2:4<279::AID-CFM29>3.0.CO;2-8).
- [100] Kolymbas D. Computer-aided design of constitutive laws. *Int J Numer Anal Methods Geomech* 1991;15:593–604. <https://doi.org/10.1002/nag.1610150806>.
- [101] Matsuoka H, Nakai T. Relationship Among Tresca, Mises, Mohr-Coulomb And Matsuoka-Nakai Failure Criteria. *Soils Found* 1985;25:123–8. <https://doi.org/10.3208/sandf1972.25.4.123>.
- [102] Pasten C, Shin H, Santamarina JC. Long-Term Foundation Response to Repetitive Loading. *J Geotech Geoenviron Eng* 2014;140:04013036. [https://doi.org/10.1061/\(asce\)gt.1943-5606.0001052](https://doi.org/10.1061/(asce)gt.1943-5606.0001052).
- [103] Wood DM. *Soil Behaviour and Critical State Soil Mechanics*. 1st ed. Cambridge University Press; 1991. 10.1017/CBO9781139878272.
- [104] Sun Q, Indraratna B, Nimbalkar S. An Elasto-plastic Method for Analysing the Deformation of the Railway Ballast. *Procedia Eng* 2016;143:954–60. <https://doi.org/10.1016/j.proeng.2016.06.082>.
- [105] Sun Y, Indraratna B, Carter JP, Marchant T, Nimbalkar S. Application of fractional calculus in modelling ballast deformation under cyclic loading. *Comput Geotech* 2017;82:16–30. <https://doi.org/10.1016/j.compgeo.2016.09.010>.
- [106] Hecke A. Effects of future mixed traffic on track deterioration. Sweden: 1998.
- [107] Woodward PK, Kennedy J, Laghrouche O, Connolly DP, Medero G. Study of railway track stiffness modification by polyurethane reinforcement of the ballast. *Transp Geotech* 2014;1:214–24. <https://doi.org/10.1016/j.trge.2014.06.005>.
- [108] Guérin N, Sab K, Moucheron P. Identification expérimentale d'une loi de tassement du ballast. *Can Geotech J* 1999;36:523–32. <https://doi.org/10.1139/r99-004>.
- [109] ORE. Question D71: Stresses in the Rails, the Ballast and in the Formation Resulting from Traffic Loads. Report No. D71/RP1/E. vol. 1 and 2. Utrecht, The Netherlands: 1970.
- [110] Punetha P, Nimbalkar S, Khabbaz H. Evaluation of additional confinement for three-dimensional geoinclusions under general stress state. *Can Geotech J* 2020;57:453–61. <https://doi.org/10.1139/cgj-2018-0866>.
- [111] Ramos A, Gomes Correia A, Calçada R, Alves Costa P, Esen A, Woodward PK, et al. Influence of track foundation on the performance of ballast and concrete slab tracks under cyclic loading: Physical modelling and numerical model calibration. *Constr Build Mater* 2021;277:1–13. <https://doi.org/10.1016/j.conbuildmat.2021.122245>.
- [112] Sayeed MdA, Shahin MA. Design of ballasted railway track foundations using numerical modelling. Part I: Development. *Canadian Geotechnical Journal* 2018;55:353–68. 10.1139/cgj-2016-0633.
- [113] Indraratna B, Ionescu D, Christie D, Chowdhury R. Compression and Degradation of Railway Ballast Under One-Dimensional Loading. *Aust Geomech* 1997;32:48–61.
- [114] Indraratna B, Ionescu D, Christie HD. State-of-the-art Large Scale Testing of Ballast. *Conference on Railway Engineering*, 21–23 May 2000, Adelaide, S. Aust.: 2000, p. 208–20.
- [115] Indraratna B, Shahin MA, Salim W. Stabilisation of granular media and formation soil using geosynthetics with special reference to railway engineering. *Proceedings of the Institution of Civil Engineers - Ground Improvement* 2007;11:27–43. <https://doi.org/10.1680/grim.2007.11.1.27>.
- [116] ORE. D71 RP.10: Deformation properties of ballast (Laboratory and track tests). Utrecht, The Netherlands: 1970.
- [117] Hettler A. *Bleibende Setzungen des Schotteroberbaus Eisenbahn-Technische Rundschau (ETR)* 1984;33:847–53.
- [118] Abadi TC. *Effect of sleeper and ballast interventions on rail track performance*. Southampton, United Kingdom: University of Southampton; 2015.
- [119] Jeffs T, Marich S. *Ballast Characteristics in the Laboratory*. *Conference on Railway Engineering* (4th : 1987 : Perth, W.A.), Perth, Australia: Barton, ACT: Institution of Engineers; 1987, p. 141–7.
- [120] Indraratna B, Ionescu D, Christie HD. Shear Behavior of Railway Ballast Based on Large-Scale Triaxial Tests. *J Geotech Geoenviron Eng* 1998;124:439–49. [https://doi.org/10.1061/\(asce\)1090-0241\(1998\)124:5\(439\)](https://doi.org/10.1061/(asce)1090-0241(1998)124:5(439)).
- [121] Indraratna B, Salim W. *Mechanics of ballasted rail tracks: a geotechnical perspective*. 1st ed. London: CRC Press Taylor & Francis Group, LLC; 2005.
- [122] Chrismar SM, Selig ET. *Mechanics-based model to predict ballast-related maintenance timing and costs*. *International heavy haul railway conference: TRB, Vancouver, B.C., Canada; 1991*. p. 208–11.
- [123] Saussine G, Quezada JC, Breul P, Radjai F. Railway Ballast Settlement: A New Predictive Model. In: Pombo J, editor. *Proceedings of the Second International Conference on Railway Technology: Research, Development and Maintenance*, vol. 104, Stirlingshire, Scotland: Civil-Comp Press; 2014, p. 12. 10.4203/ccp.104.121.
- [124] Sato Y. Optimization of track maintenance work on ballasted track. *Proceedings of the World Congress on Railway Research (WCRR '97)*, Florence, Italy, 16–19 November 1997, p. 405–411.
- [125] Coelho B, Hölscher P, Priest J, Powrie W, Barends F. An assessment of transition zone performance. *Proc Inst Mech Eng F J Rail Rapid Transit* 2011;225:129–39. <https://doi.org/10.1177/09544097JRR1389>.
- [126] Fröhling RD. *Deterioration of railway track due to dynamic vehicle loading and spatially varying track stiffness*. Pretoria, South Africa: University of Pretoria; 1997.
- [127] Thom NH, Oakley J. Predicting differential settlement in a railway trackbed. *Proceedings of railway foundations conference: Railfound 06*, 11–13 September 2006, Birmingham, United Kingdom: 2006, p. 190–200.
- [128] Kennedy J, Woodward PK, Medero G, Banimahd M. Reducing railway track settlement using three-dimensional polyurethane polymer reinforcement of the ballast. *Constr Build Mater* 2013;44:615–25. <https://doi.org/10.1016/j.conbuildmat.2013.03.002>.
- [129] Lackenby J, Indraratna B, McDowell G, Christie D. Effect of confining pressure on ballast degradation and deformation under cyclic triaxial loading. *Geotechnique* 2007;57:527–36. <https://doi.org/10.1680/geot.2007.57.6.527>.
- [130] Sun Q, Indraratna B, Ngo NT. Effect of increase in load and frequency on the resilience of railway ballast. *Geotechnique* 2019;69:833–40. <https://doi.org/10.1680/jgeot.17.P.302>.
- [131] Brown SF, Hyde AFL. *Significance of Cyclic Confining Stress in Repeated-Load Triaxial Testing of Granular Material*. *Transp Res Rec*, Washington, D.C.: Transportation Research Board; 1975. p. 49–58.
- [132] Li D, Selig ET. Cumulative Plastic Deformation for Fine-Grained Subgrade. *J Geotech Eng* 1996;122:1006–13. [https://doi.org/10.1061/\(asce\)0733-9410\(1996\)122:12\(1006\)](https://doi.org/10.1061/(asce)0733-9410(1996)122:12(1006)).
- [133] Chen C, Ge L, Zhang J. Modeling Permanent Deformation of Unbound Granular Materials under Repeated Loads. *Int J Geomech* 2010;10:236–41. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0000025](https://doi.org/10.1061/(asce)gm.1943-5622.0000025).
- [134] Sun QD, Indraratna B, Nimbalkar S. Deformation and Degradation Mechanisms of Railway Ballast under High Frequency Cyclic Loading. *J Geotech Geoenviron Eng* 2016;142:4015056. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0001375](https://doi.org/10.1061/(ASCE)GT.1943-5606.0001375).
- [135] Shan Y, Zhou S, Wang B, Ho CL. Differential Settlement Prediction of Ballasted Tracks in Bridge-Embankment Transition Zones. *J Geotech Geoenviron Eng* 2020;146:04020075. [https://doi.org/10.1061/\(asce\)gt.1943-5606.0002307](https://doi.org/10.1061/(asce)gt.1943-5606.0002307).
- [136] Punetha P, Nimbalkar S, Khabbaz H. Analytical Evaluation of Ballasted Track Substructure Response under Repeated Train Loads. *Int J Geomech* 2020;20:04020093. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0001729](https://doi.org/10.1061/(asce)gm.1943-5622.0001729).
- [137] Ramos A, Gomes Correia A, Indraratna B, Ngo T, Calçada R, Costa PA. Mechanistic-empirical permanent deformation models: Laboratory testing, modelling and ranking. *Transp Geotech* 2020;23:100326. <https://doi.org/10.1016/j.trge.2020.100326>.
- [138] Houlsby GT, Abadie CN, Beuckelaers WJAP, Byrne BW. A model for nonlinear hysteretic and ratcheting behaviour. *Int J Solids Struct* 2017;120:67–80. <https://doi.org/10.1016/j.jis.2017.04.031>.
- [139] Ognibene G, Le Pen L, Harkness J, Zervos A, Powrie W. An alternative approach to track settlement prediction. *4th International Conference on Transportation Geotechnics*, 23–26 May 2021, Chicago, Illinois: 2021, p. 13.
- [140] Jovanović S, Božović D, Tomićić-Torlaković M. Railway infrastructure condition-monitoring and analysis as a basis for maintenance management. *Gradjevinar* 2014;66:347–58. 10.14256/JCE.959.2013.
- [141] Ghofrani F, He Q, Goverde RMP, Liu X. Recent applications of big data analytics in railway transportation systems: A survey. *Transp Res Part C Emerg Technol* 2018;90:226–46. <https://doi.org/10.1016/j.trc.2018.03.010>.
- [142] Yousefikhia M, Moridpour S, Setunge S, Mazloumi E. Modeling Degradation of Tracks for Maintenance Planning on a Tram Line. *Journal of Traffic and Logistics Engineering* 2014;2:86–91. 10.12720/jtle.2.2.86-91.
- [143] Sadeghi J, Askarinejad H. Development of improved railway track degradation models. *Struct Infrastruct Eng* 2010;6:675–88. <https://doi.org/10.1080/15732470801902436>.
- [144] Jovanović S, Guler H, Čoko B. Track degradation analysis in the scope of railway infrastructure maintenance management systems. *GRADEVINAR* 2015;67:247–57. 10.14256/JCE.1194.2014.
- [145] van Noordwijk JM. A survey of the application of gamma processes in maintenance. *Reliab Eng Syst Saf* 2009;94:2–21. <https://doi.org/10.1016/j.res.2007.03.019>.

- [146] van Noordwijk JM, van der Weide JAM, Kallen MJ, Pandey MD. Gamma processes and peaks-over-threshold distributions for time-dependent reliability. *Reliab Eng Syst Saf* 2007;92:1651–8. <https://doi.org/10.1016/j.res.2006.11.003>.
- [147] Gorjian N, Ma L, Mittinty M, Yarlagadda P, Sun Y. A review on degradation models in reliability analysis. *Proceedings of the 4th World Congress on Engineering Asset Management*, 28–30 September 2009, Marriott Athens Ledra Hotel, Athens: 2009, p. 16.
- [148] Meier-Hirmer C, Senéa A, Riboulet G, Sourget F, Roussignol M. A decision support system for track maintenance. *WIT Trans Built Environ* 2006;88:217–26. <https://doi.org/10.2495/CR060221>.
- [149] Yea ZS, Xie M. Stochastic modelling and analysis of degradation for highly reliable products. *Appl Stoch Models Bus Ind* 2015;31:16–32. <https://doi.org/10.1002/asmb.2063>.
- [150] Si XS, Wang W, Hu CH, Zhou DH. Remaining useful life estimation - A review on the statistical data driven approaches. *Eur J Oper Res* 2011;213:1–14. <https://doi.org/10.1016/j.ejor.2010.11.018>.
- [151] Zhang Z, Si X, Hu C, Lei Y. Degradation data analysis and remaining useful life estimation: A review on Wiener-process-based methods. *Eur J Oper Res* 2018;271:775–96. <https://doi.org/10.1016/j.ejor.2018.02.033>.
- [152] Soleimanmeigouni I, Ahmadi A, Khouy IA, Letot C. Evaluation of the effect of tamping on the track geometry condition: A case study. *Proc Inst Mech Eng F J Rail Rapid Transit* 2018;232:408–20. <https://doi.org/10.1177/0954409716671548>.
- [153] Zhu M, Cheng X, Miao L, Sun X, Wang S. Advanced stochastic modeling of railway track irregularities. *Adv Mech Eng* 2013;2013:401637. <https://doi.org/10.1155/2013/401637>.
- [154] Mercier S, Meier-Hirmer C, Roussignol M. Bivariate Gamma wear processes for track geometry modelling, with application to intervention scheduling. *Struct Infrastruct Eng* 2012;8:357–66. <https://doi.org/10.1080/15732479.2011.563090>.
- [155] Vale C, M. Lurdes S. Stochastic model for the geometrical rail track degradation process in the Portuguese railway Northern Line. *Reliab Eng Syst Saf* 2013;116:91–8. <https://doi.org/10.1016/j.res.2013.02.010>.
- [156] Sedghi M, Bergquist B, Vanhatalo E, Migdallas A. Data-driven maintenance planning and scheduling based on predicted railway track condition. *Qual Reliab Eng Int* 2022;38:3689–709. <https://doi.org/10.1002/qre.3166>.
- [157] Lyngby N. Railway track degradation: Shape and influencing factors. *International Journal of Performability Engineering* 2009;5:177–86.
- [158] Guler H, Jovanovic S, Evren G. Modelling railway track geometry deterioration. *Proceedings of the Institution of Civil Engineers - Transport* 2011;164:65–75. 10.1680/tran.2011.164.2.65.
- [159] Westgeest F, Dekker R, Fischer R. Predicting rail geometry deterioration by regression models. *Advances in Safety, Reliability and Risk Management* 2012:926–33. <https://doi.org/10.1201/b11433.130>.
- [160] He Q, Li H, Bhattacharjya D, Parikh DP, Hampapur A. Track geometry defect rectification based on track deterioration modelling and derailment risk assessment. *J Oper Res Soc* 2015;66:392–404. <https://doi.org/10.1057/jors.2014.7>.
- [161] Tsoukalas LH, Uhrig RE. *Fuzzy and neural approaches in engineering*. 1st ed. New York, NY: John Wiley & Sons, Inc.; 1997. 10.1016/S0893-6080(97)00079-8.
- [162] Guler H. Prediction of railway track geometry deterioration using artificial neural networks: A case study for Turkish state railways. *Struct Infrastruct Eng* 2014;10:614–26. <https://doi.org/10.1080/15732479.2012.757791>.
- [163] Sadeghi J, Askarinejad H. Application of neural networks in evaluation of railway track quality condition. *J Mech Sci Technol* 2012;26:113–22. <https://doi.org/10.1007/s12206-011-1016-5>.
- [164] Khajehi H, Ahmadi A, Soleimanmeigouni I, Haddadzade M, Nissen A, Jebelli MJL. Prediction of track geometry degradation using artificial neural network: a case study. *International Journal of Rail Transportation* 2022;10:24–43. <https://doi.org/10.1080/23248378.2021.1875065>.
- [165] Karlaftis MG, Vlahogianni EI. Statistical methods versus neural networks in transportation research: Differences, similarities and some insights. *Transp Res Part C Emerg Technol* 2011;19:387–99. <https://doi.org/10.1016/j.trc.2010.10.004>.
- [166] Elkhoury N, Hitihamillage L, Moridpour S, Robert D. Degradation Prediction of Rail Tracks: A Review of the Existing Literature. *The Open Transportation Journal* 2018;12:88–104. <https://doi.org/10.2174/1874447801812010088>.
- [167] Dell'Orco M, Ottomanelli M, Caggiani L, Sassanelli D. New Decision Support System for Optimization of Rail Track Maintenance Planning Based on Adaptive Neurofuzzy Inference System. *Transportation Research Record: Journal of the Transportation Research Board* 2008;2043:49–54. <https://doi.org/10.3141/2043-06>.
- [168] Bezdek JC, Keller J, Krisnapuram R, Pal NR. *Fuzzy Models and Algorithms for Pattern Recognition and Image Processing* 1999;vol. 4. <https://doi.org/10.1007/b106267>.
- [169] Xu P, Sun Q, Liu R, Wang F. A short-range prediction model for track quality index. *Proc Inst Mech Eng F J Rail Rapid Transit* 2011;225:277–85. <https://doi.org/10.1177/2041301710392477>.
- [170] Xu P, Liu R-K, Wang F, Wang F-T, Sun Q-X. Railroad Track Deterioration Characteristics Based Track Measurement Data Mining. *Math Probl Eng* 2013;2013:1–7. <https://doi.org/10.1155/2013/970573>.
- [171] Liu R, Xu P, Wang F. Research on a Short-Range Prediction Model for Track Irregularity over Small Track Lengths. *J Transp Eng* 2010;136:1085–91. [https://doi.org/10.1061/\(asce\)te.1943-5436.0000192](https://doi.org/10.1061/(asce)te.1943-5436.0000192).
- [172] Kawaguchi A, Miwa M, Terada K. Actual Data Analysis of Alignment Irregularity Growth and its Prediction Model. *Quarterly Report of RTRI* 2006;46:262–8. <https://doi.org/10.2219/rtriq.46.262>.
- [173] Bai L, Liu R, Sun Q, Wang F, Wang F. Classification-learning-based framework for predicting railway track irregularities. *Proc Inst Mech Eng F J Rail Rapid Transit* 2016;230:598–610. <https://doi.org/10.1177/0954409714552818>.
- [174] Xu P, Liu R, Wang F, Sun Q, Teng H. A Novel Description Method for Track Irregularity Evolution. *International Journal of Computational Intelligence Systems* 2011;4:1358–66. <https://doi.org/10.1080/18756891.2011.9727886>.
- [175] Berggren EG. Efficient track maintenance: Methodology for combined analysis of condition data. *Proc Inst Mech Eng F J Rail Rapid Transit* 2010;224:353–60. <https://doi.org/10.1243/09544097JRR354>.
- [176] Chaolong J, Weixiang X, Lili W, Hanning W. Study of Railway Track Irregularity Standard Deviation Time Series Based on Data Mining and Linear Model. *Math Probl Eng* 2013;2013:1–12. <https://doi.org/10.1155/2013/486738>.
- [177] Deng J. Control problems of grey systems. *Syst Control Lett* 1982;1:288–94. [https://doi.org/10.1016/S0167-6911\(82\)80025-X](https://doi.org/10.1016/S0167-6911(82)80025-X).
- [178] Deng J. *Introduction to Grey System*. *Journal of Grey System* 1989;1:1–24.
- [179] Famurewa SM, Xin T, Rantatalo M, Kumar U. Comparative study of track geometry quality prediction models. *10th International Conference on Condition Monitoring and Machinery Failure Prevention Technologies* 2013, CM 2013 and MFPT 2013, vol. 2, 2013, p. 1057–1068.
- [180] Liu F, Wei X, Liu Y, Yin X. Track quality prediction based on center approach Markov-Grey GM(1,1) model. *2014 IEEE International Conference on Information and Automation, ICIA 2014*, 2014, p. 81–6. 10.1109/ICInfA.2014.6932630.
- [181] Chaolong J, Weixiang X, Futian W, Hanning W. Track Irregularity Time Series Analysis and Trend Forecasting. *Discrete Dyn Nat Soc* 2012;2012:1–15. <https://doi.org/10.1155/2012/387857>.
- [182] Chang H, Liu R, Wang W. Multistage Linear Prediction Model of Track Quality Index. *Seventh International Conference on Traffic and Transportation Studies (ICTTS)* 2010, Kunming, China: 2010, p. 1183–92. 10.1061/41123(383)112.
- [183] Guo R, Han BM. Multi-Stage Linear Prediction Model for Railway Track Irregularity. *Appl Mech Mater* 2013;361–363:1811–6. <https://doi.org/10.4028/www.scientific.net/amm.361-363.1811>.
- [184] Audley M, Andrews JD. The effects of tamping on railway track geometry degradation. *Proc Inst Mech Eng F J Rail Rapid Transit* 2013;227:376–91. <https://doi.org/10.1177/0954409713480439>.
- [185] Andrade AR, Teixeira PF. Uncertainty in Rail-Track Geometry Degradation: Lisbon-Oporto Line Case Study. *J Transp Eng* 2011;137:193–200. [https://doi.org/10.1061/\(asce\)te.1943-5436.0000206](https://doi.org/10.1061/(asce)te.1943-5436.0000206).
- [186] Andrade AR, Teixeira PF. A Bayesian model to assess rail track geometry degradation through its life-cycle. *Res Transp Econ* 2012;36:1–8. <https://doi.org/10.1016/j.retrec.2012.03.011>.
- [187] Andrade AR, Teixeira PF. Hierarchical Bayesian modelling of rail track geometry degradation. *Proc Inst Mech Eng F J Rail Rapid Transit* 2013;227:364–75. <https://doi.org/10.1177/0954409713486619>.
- [188] Andrews J, Prescott D, De Rozières F. A stochastic model for railway track asset management. *Reliab Eng Syst Saf* 2014;130:76–84. <https://doi.org/10.1016/j.res.2014.04.021>.
- [189] Shafahi Y, Hakhmaneshi R. Application of a Maintenance Management Model for Iranian Railways Based on the Markov Chain and Probabilistic Dynamic Programming. *Transaction A: Civil Engineering* 2009;16:87–97.
- [190] Bai L, Liu R, Sun Q, Wang F, Xu P. Markov-based model for the prediction of railway track irregularities. *Proc Inst Mech Eng F J Rail Rapid Transit* 2015;229:150–9. <https://doi.org/10.1177/0954409713503460>.
- [191] Prescott D, Andrews J. Investigating railway track asset management using a Markov analysis. *Proc Inst Mech Eng F J Rail Rapid Transit* 2015;229:402–16. <https://doi.org/10.1177/0954409713511965>.
- [192] Zhang D, Hu H, Roberts C. Rail maintenance analysis using Petri nets. *Struct Infrastruct Eng* 2017;13:783–93. <https://doi.org/10.1080/15732479.2016.1190767>.
- [193] Andrews JD, Moss TR. *Reliability and Risk Assessment*. 2nd ed. London, United Kingdom: Professional Engineering Publishing Limited; 2002.
- [194] Billington R, Allan RN. *Reliability Evaluation of Engineering Systems: Concepts and Techniques*. Boston, MA: Springer-Verlag, US; 2006. 10.1007/978-1-4615-7728-7.
- [195] Zhao J, Chan AHC, Roberts C, Stirling AB. Assessing the economic life of rail using a stochastic analysis of failures. *Proc Inst Mech Eng F J Rail Rapid Transit* 2006;220:103–11. <https://doi.org/10.1243/09544097JRR30>.
- [196] Prescott D, Andrews J. A Railway Track Ballast Maintenance and Inspection Model for Multiple Track Sections. *11th International Probabilistic Safety Assessment and Management Conference and the Annual European Safety and Reliability Conference. (PSAM11 ESREL 2012)*, 25–29 June 2012, Helsinki, Finland.: IAPSAM & ESRA; 2012, p. 2012.
- [197] Prescott D, Andrews J. A track ballast maintenance and inspection model for a rail network. *Proc Inst Mech Eng O J Risk Reliab* 2013;227:251–66. <https://doi.org/10.1177/1748006X13482848>.
- [198] Rama D, Andrews JD. Railway infrastructure asset management: the whole-system life cost analysis. *IET Intel Transport Syst* 2016;10:58–64. <https://doi.org/10.1049/iet-its.2015.0030>.
- [199] Rama D, Andrews JD. A holistic approach to railway infrastructure asset management. *International Journal of Performability Engineering* 2015;11:107–20.
- [200] Ahac M, Lakušić S. Tram Track Maintenance -Planning by Gauge Degradation Modelling. *Transport* 2015;30:430–6. <https://doi.org/10.3846/16484142.2015.1116464>.

- [201] Rhayma N, Bressolette P, Breul P, Fogli M, Saussine G. Reliability analysis of maintenance operations for railway tracks. *Reliab Eng Syst Saf* 2013;114:12–25. <https://doi.org/10.1016/j.ress.2012.12.007>.
- [202] Soleimanmeigouni I, Xiao X, Ahmadi A, Xie M, Nissen A, Kumar U. Modelling the evolution of ballasted railway track geometry by a two-level piecewise model. *Struct Infrastruct Eng* 2018;14:33–45. <https://doi.org/10.1080/15732479.2017.1326946>.
- [203] Soleimanmeigouni I, Ahmadi A, Khajehei H, Nissen A. Investigation of the effect of the inspection intervals on the track geometry condition. *Struct Infrastruct Eng* 2020;16:1138–46. <https://doi.org/10.1080/15732479.2019.1687528>.