Optimal pensions with endogenous labour supply

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Abstract

We show that a two-part pension system provides optimal capital accumulation without distorting labour supply, thereby achieving the *first-best*. An economy with too little retirement saving should combine a *negative* income tax with a consumption tax to replicate the first-best allocation without using any lump-sum taxes. Our results are shown in a classic Diamond overlapping generations model that is augmented with endogenous labour supply on the intensive margin.

1 Introduction

In many countries, there is a trend toward greater funding of public pensions. Current projections show that ageing populations will increase the future costs of pension provision significantly given the falling share of young workers, the limited scope for increases in labour supply of existing workers, and other demographic pressures. Despite these known risks, progress in reforming pension systems has been slow, raising several questions. Is it better to gradually phase out existing paygo systems or should policymakers rapidly implement funded pension systems? How should labour be taxed in such systems, given the importance of labour supply in making current pension promises manageable? Is there an optimal pension policy for a world where saving is too low, and if so, what does it look like?

In this paper, we consider these questions using a deliberately simplified model. We set out an overlapping generations model in the spirit of Diamond (1965) that is augmented with *endogenous* labour supply on the intensive margin. Our main result is that a two-part pension system consisting of an income tax and a consumption tax provides optimal capital accumulation without distorting labour supply, such that the first-best allocation is replicated for any initial capital inherited from past policies. For an economy with *too little* retirement saving, this policy achieves the first-best without using lump-sum taxes. The first-best is the allocation chosen by a planner who maximizes a social welfare function with positive weights on the lifetime utilities of all generations and geometric discounting.

The optimal policy depends on the savings rate in the *laissez-faire* economy. If the savings rate is too high relative to the social optimum, combining a positive income tax with

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a consumption subsidy curbs the capital overaccumulation while preventing a distortion to labour supply by making the *effective* marginal tax on labour *zero*. By contrast, if the savings rate is *too low*, the optimal policy combines a *negative* income tax and a positive consumption tax to encourage workers to make extra retirement saving, while avoiding both a labour supply distortion and the use of lump-sum taxes. Such a pension system is in the spirit of policy recommendations that aim to increase private retirement saving (see OECD, 2012), but bolder.[1] While the suggestion of a negative income tax is not new, its role here is to increase saving, not the labour supply of poor households.[2]

Several past papers add endogenous labour supply and pensions in the Diamond (1965) model, but most focus on the possibility of Pareto-improving pension reforms (Breyer and Straub, 1993; Brunner, 1996), not on policies that maximize a social welfare function. The closest papers to ours are Abio et al. (2004) and Beetsma et al. (2013). Abio et al. (2004) extend the *steady-state* result of Samuelson (1975) in the classic Diamond model to an economy with endogenous female labour and fertility: they show that *steady-state* lifetime utility is still maximized by a paygo pension in this setting, provided the benefit is linked to the number of children. Beetsma et al. (2013) focus on intergenerational risk and show that a two-tier pension system achieves the first-best in a *two-period* model with endogenous labour supply; however, their result does *not* extend to infinitely many periods.[3]

We contribute relative to these works by presenting a policy that yields a first-best *transition* in an infinite-horizon Diamond model. The key margin in our analysis – labour supply given marginal income taxes in an infinite-life economy – is absent in the above two papers. Importantly, our results suggest a way to tackle the dearth of private pension saving using widely available policy instruments: income and consumption taxes.

2 Model

Consider an overlapping generations model with discrete time $t \in \mathbb{N}$, households with two-period lives, and perfect foresight. The number of young N_t grows as $N_t = (1+n)N_{t-1}$, where $N_{-1} > 0$ is given and $n > -1$ is the fixed population growth rate. All members of a generation are identical and lifetime utility of the young born at t is

$$U_t = \ln(c_{t,y}) + \beta \ln(c_{t+1,o}) - \theta \frac{l_t^{1+\chi}}{1+\chi}, \quad \beta, \theta, \chi > 0 \tag{1}$$

where l_t is labour hours, β is a private discount factor, and χ^{-1} is the Frisch elasticity.

The economy is closed and output is devoted to consumption or investment:

$$Y_t = N_t c_{t,y} + N_{t-1} c_{t,o} + K_{t+1}, \quad \forall t \geq 0 \tag{2}$$

[1] For example, OECD (2012, 11,17) notes that pension reforms since the mid-1980s have led "to a reduction in public pension promises in many countries" and calls for "an expanded role for funded, private pensions."

[2] In the economic literature, a negative income tax was proposed by Friedman (1962); the basic idea is that government support should be withdrawn at a low marginal rate to increase incentives to work.

[3] See Section 5 of Beetsma et al. (2013) (p. 153) and their Appendix C for further details.

where we assume full depreciation of capital in a generation.

Production is Cobb-Douglas and depends on fixed total factor productivity $A > 0$:

$$Y_t = AK_t^\alpha L_t^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{3}$$

where $L_t = N_t l_t$ is aggregate labour hours, K_t is capital at date t, and $K_0 > 0$ is given.

2.1 First-best allocation

The planner maximizes a social welfare function with positive weights on the lifetime utility of each generation and geometric discounting:

$$W_0 = \sum_{t=-1}^{\infty} \omega^t U_t, \qquad 0 < \omega < 1 \tag{4}$$

where U_t is given by (1) and ω is the *social discount factor*.

The resource constraint (2) in intensive terms is

$$y_t = c_{t,y} + \frac{c_{t,o}}{1+n} + k_{t+1}, \quad \forall t \geq 0 \tag{5}$$

where $y_t := Y_t/N_t = A\tilde{k}_t^\alpha l_t^{1-\alpha}$ is output per worker and $\tilde{k}_t := k_t/(1+n)$ is capital per worker, with $k_t := K_t/N_{t-1}$ and initial capital $k_0 = K_0/N_{-1} > 0$.

The planner's maximization problem can be written as

$$\max_{\{c_{t,y}, l_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \omega^t \left(\ln(c_{t,y}) + \frac{\beta}{\omega} \ln[(1+n)(A\tilde{k}_t^\alpha l_t^{1-\alpha} - c_{t,y} - k_{t+1})] - \theta \frac{l_t^{1+\chi}}{1+\chi} \right) + t.i.p. \tag{6}$$

where $c_{t,o} = (1+n)(A\tilde{k}_t^\alpha l_t^{1-\alpha} - c_{t,y} - k_{t+1})$ is used and $t.i.p. := \omega^{-1}[\ln(c_{-1,y}) - \frac{\theta}{1+\chi}(l_{-1})^{1+\chi}]$ is a given constant that lies outside the planner's control ('terms independent of planner').

A solution to problem (6) must be Pareto efficient since each generation has a positive welfare weight $\omega^t \in (0,1)$. Thus, the planner's chosen allocation is one among many Pareto efficient allocations, namely, the one that maximizes the social welfare function (4).

The first-order optimality conditions are

$$c_{t,o} = \frac{\beta}{\omega}(1+n)c_{t,y}, \qquad \theta l_t^\chi = \frac{mpl_t}{c_{t,y}}, \qquad \frac{1}{c_{t,y}} = \beta \frac{mpk_{t+1}}{c_{t+1,o}} \tag{7}$$

where $mpl_t := (1-\alpha)y_t/l_t$ and $mpk_t := \alpha y_t/\tilde{k}_t$.

The first equation in (7) shows the optimal balance between generational consumptions; the second is optimal labour supply on the intensive margin. The third equation in (7) is the Euler equation for capital accumulation. At a steady state (i.e. as $t \to \infty$), the *modified* Golden Rule holds: $mpk = (1+n)/\omega$; for a discussion, see De La Croix and Michel (2002).

Using (5) and (7), we can solve for the first-best allocation chosen by the planner:[4]

$$k_{t+1}^* = \alpha\omega y_t^*, \qquad l_t^* = \left(\frac{(1+\frac{\beta}{\omega})(1-\alpha)}{\theta(1-\alpha\omega)}\right)^{1/(1+\chi)} \tag{8}$$

$$c_{t,y}^* = \frac{\omega(1-\alpha\omega)}{\beta+\omega} y_t^*, \qquad c_{t,o}^* = \frac{\beta(1-\alpha\omega)(1+n)}{\beta+\omega} y_t^* \tag{9}$$

where $y_t^* = A(\tilde{k}_t^*)^\alpha (l_t^*)^{1-\alpha}$.

The optimal savings rate in capital $\alpha\omega$ depends on the capital share and social discount factor; see (8). Optimal labour supply depends on the ratio of the private and social discount factors, β/ω; the ratio of the labour share $(1-\alpha)$ to the consumption share $(1-\alpha\omega)$; and the labour preference parameters θ, χ. All uninvested output is consumed, with the optimal split between young-age and old-age consumption depending on β and ω; see (9).

2.2 Decentralized economy

We now turn to the decentralized economy. Households maximize lifetime utility by choosing saving s_t^k and labour hours l_t. They face an income tax (or subsidy) at rate $\tau \in (-\infty, 1)$ and a consumption tax (or subsidy) at rate $\tau_c \in (-1, \infty)$. The proceeds from the two taxes finance a paygo-type pension $P_t \in \mathbb{R}$ to the current old.

The problem solved by a representative young born at date t is

$$\begin{aligned} &\max_{s_t^k, l_t} U_t = \ln(c_{t,y}) + \beta \ln(c_{t+1,o}) - \theta \frac{l_t^{1+\chi}}{1+\chi} \qquad \text{s.t.} \\ (1+\tau_c)c_{t,y} =& (1-\tau)w_t l_t - s_t^k, \qquad (1+\tau_c)c_{t+1,o} = r_{t+1}s_t^k + P_{t+1} \end{aligned} \tag{10}$$

where the factor prices w_t, r_t and the pension P_{t+1} are taken as given.

The first-order conditions are

$$\frac{1}{c_{t,y}} = \beta \frac{r_{t+1}}{c_{t+1,o}}, \qquad \theta l_t^\chi = \frac{(1-\tau)w_t}{(1+\tau_c)c_{t,y}}. \tag{11}$$

The Euler equation in (11) has the usual form. Intuitively, there is no marginal intertemporal distortion in this equation since consumption is taxed at the same rate τ_c at both ages. By contrast, there is a 'tax wedge' $\frac{1-\tau}{1+\tau_c}$ in the intratemporal labour supply condition in (11).

Total contributions to the pension system at date t are

$$C_t = \tau N_t w_t l_t + \tau_c (N_t c_{t,y} + N_{t-1} c_{t,o}) \tag{12}$$

and the paygo pension P_t to each old is given by a transfer rule:

$$P_t = (1+n)[\tau + \tau_c \tilde{\Phi}] w_t l_t \tag{13}$$

[4]Further details of the planner problem and its solution are provided in the Supplementary Appendix.

where $\tilde{\Phi} = \frac{1-\Phi}{1-\alpha}$ and the coefficient $\Phi \in (0,1)$ must give a balanced budget (see below).

The pension transfer rule (13) implies that a total share $\tau + \tau_c\tilde{\Phi}$ of labour income goes to pensions. Hence, for positive shares, there is a transfer of labour income from the young to the retired old, with $1 + n = N_t/N_{t-1}$ being a 'growth factor' that reflects the difference between the number of young contributors, N_t, and the number of old recipients N_{t-1}.

If $\tau_c = 0$, rule (13) has the intuitive interpretation that income tax collected from workers is remitted to pensioners, as in Belan et al. (1998) or Fedotenkov (2016). The extra term $\tau_c\tilde{\Phi}$ in square brackets arises because pensions are *also* financed by consumption taxes; as we shall see, this addition is crucial: given endogenous labour supply, a one-part pension system based on labour or consumption taxes alone cannot replicate the first-best allocation.[5]

A representative firm hires capital and labour from households in competitive factor markets at prices r_t and w_t, respectively. The firm maximizes profit each period:

$$\max_{K_t, L_t} \; AK_t^\alpha L_t^{1-\alpha} - r_t K_t - w_t L_t$$

which yields the factor prices

$$r_t = \alpha A \left(\frac{K_t}{L_t}\right)^{\alpha-1} = mpk_t, \qquad w_t = (1-\alpha)A\left(\frac{K_t}{L_t}\right)^{\alpha} = mpl_t. \tag{14}$$

Competitive equilibrium is a set of allocations and prices such that for all $t \geq 0$:

(i) s_t^k, l_t solve the maximization problem of the date t young given the taxes τ, τ_c, the pension transfer rule (13), and the profit-maximizing factor prices (14).

(ii) The pension system has a balanced budget: $P_t - C_t/N_{t-1} = 0$.

(iii) Aggregate capital equals aggregate saving, $K_{t+1} = N_t s_t^k$, and the aggregate resource constraint (2), and the per-worker resource constraint (5), hold.

Given (11)–(13) and conditions (i)–(iii), the allocations at the competitive equilibrium are:

$$k_{t+1} = \frac{\tilde{\beta}(1-\alpha)(1-\tau)}{1+\tilde{\beta}} y_t, \qquad l_t = \left(\frac{1+\tilde{\beta}}{\theta}\right)^{1/(1+\chi)} \tag{15}$$

$$c_{t,y} = \frac{(1-\alpha)(1-\tau)}{(1+\tilde{\beta})(1+\tau_c)} y_t, \qquad c_{t,o} = \left(\frac{\alpha + (1-\alpha)\tau + (1-\Phi)\tau_c}{1+\tau_c}\right)(1+n)y_t \tag{16}$$

where $\tilde{\beta} := \frac{\beta\alpha}{\alpha + (1-\alpha)\tau + (1-\Phi)\tau_c}$ and $y_t = A\tilde{k}_t^\alpha l_t^{1-\alpha}$ as above.

[5] The rule (13) contrasts with an alternative approach of paying a *fixed* pension per old by varying the contribution rate. We do not consider this approach since a time-varying income tax adds an intertemporal wedge in the Euler equation if consumption taxes are set to eliminate the labour tax wedge in (11).

Coefficient Φ is from the pension transfer rule, (13), and must be consistent with the balanced-budget condition in (ii), implying $\Phi = k_{t+1}/y_t$. In the *Supplementary Appendix* we show that Φ must solve a quadratic equation and τ, τ_c, Φ can be set to achieve the first-best.[6]

3 Optimal policy

We now turn to optimal policy. We first show that an income tax or consumption tax *alone* cannot achieve the first-best allocation. We then characterize the optimal *two-part* pension system (where 'optimal' means that the social welfare function (4) is maximized). To guarantee the existence of an optimal policy, we make the following assumption.

Assumption 1 *We assume that $\beta(1-\alpha)/\alpha > \omega(1-\omega)$.*

If Assumption 1 does not hold, the transfers required to correct the savings rate and labour supply are 'too large' (see Proposition 2 and Fn. 10 below). Since $\omega \in (0,1)$, the usual assumption $\alpha \in (0,1), \beta > 0$ does not guarantee Assumption 1 holds (as $0 < \omega(1-\omega) \leq 1/4$); instead, we require that the private discount factor β is large enough (or the capital share α small enough). Although this means our optimal policy will not apply to economies with low private discount factors and large capital shares, Assumption 1 holds for a wide range of plausible parameter values, and is comfortably satisfied in our numerical example below.[7]

3.1 Analytical results

We first clarify the conditions for which the *laissez-faire* competitive equilibrium is sub-optimal and then show that a one-part pension system cannot correct this. We then turn to a *two-part* pension system which is first-best. Proofs of all results appear in the Appendix.

Proposition 1 *Let $s := \frac{\beta}{1+\beta}$ be the savings rate out of labour income at the laissez-faire competitive equilibrium. If $s = s^* := \frac{\alpha\omega}{1-\alpha}$, then setting $\tau = \tau_c = 0$ (i.e. no intervention) achieves the first-best allocation. If $s \neq s^*$, the laissez-faire competitive equilibrium allocation deviates from the first-best allocation: there is over-saving if $s > s^*$ (under-saving if $s < s^*$). When $s \neq s^*$, no one-part pension system can achieve the first-best allocation.*

Proposition 1 shows that the *laissez-faire* competitive equilibrium (no intervention) yields the first-best *only* if a specific relationship holds between the private discount factor β, the capital income share α, and social discount factor ω. If so, then the savings rate $s = \frac{\beta}{1+\beta}$ (which is increasing in β) equals the socially-optimal value s^*, and the resulting allocation matches the planner's, since marginal distortions are avoided if taxes are absent.[8]

Otherwise, $s \neq s^*$ and government intervention may raise social welfare given by (4). What Proposition 1 clarifies, however, is that no *one-part* pension system – using τ or τ_c

[6] The quadratic is $\tau_c\Phi^2 - [\alpha(1+\beta) + (1-\alpha)\tau + \tau_c]\Phi + \alpha\beta(1-\alpha)(1-\tau) = 0$ by (15) plus $\Phi = k_{t+1}/y_t$.

[7] Note: $\omega(1-\omega) \leq 1/4$ (with equality for $\omega = 1/2$), so Assumption 1 holds for all $\alpha \in (0, 1/2]$, $\beta > 1/4$. Clearly, if the social discount factor is some $\omega \neq 1/2$, then these (sufficient) conditions can be weakened.

[8] Note that $s = s^*$ can also be expressed as $\alpha = \alpha^* := \frac{\beta}{\beta(1+\omega)+\omega}$ or $\beta = \beta^* := \frac{\alpha\omega}{1-\alpha(1+\omega)}$ (for $\alpha(1+\omega) \neq 1$).

alone – can achieve the first-best. The intuition is quite simple. If $s \neq s^*$, then saving in equilibrium is suboptimal, being too high if $s > s^*$ and too low if $s < s^*$. An income tax τ can restore the optimal saving rate, but will introduce a marginal distortion to labour supply – see (11) – that is absent in (7). A similar argument applies to the consumption tax τ_c.

Hence, the 'friction' of a *marginal tax* on labour supply prevents a one-part pension system achieving the first-best. If labour supply were instead *exogenous* as in the Diamond (1965) model, this difficulty is avoided and replicating the first-best is simple: the optimal allocation follows when $\tau_c = 0$ and the income tax τ is chosen to give optimal saving in capital (see (8)), the only choice variable.[9] With endogenous labour supply, however, getting the 'right' savings rate in capital is not enough to achieve an optimal allocation because setting $\tau \neq 0$ adds a marginal distortion to labour supply via the tax wedge $\frac{1-\tau}{1+\tau_c}$ in (11), and this wedge cannot be eliminated unless consumption taxes are used (i.e. $\tau_c \neq 0$).

We now show that a well-designed two part pension system can achieve the first-best allocation despite the friction of a marginal income tax.

Proposition 2 *The first-best allocation is achieved by a pension system with* $\Phi = \alpha\omega$ *and*

$$\tau = \frac{\beta(1-\alpha) - \alpha\omega(1+\beta)}{\beta(1-\alpha) - \alpha\omega(1-\omega)}, \qquad \tau_c = -\tau, \qquad P_t = -\tau\alpha(1+n)(1-\omega)y_t^* \tag{17}$$

where $P_t > 0$ *if and only if* $s < s^*$ *(i.e.* $\frac{\beta}{1+\beta} < \frac{\alpha\omega}{1-\alpha}$*).*

Proposition 2 gives the optimal *two-part* pension system. The consumption tax has equal magnitude to the income tax but *opposite sign* (i.e. $\tau_c = -\tau$) since this makes the effective marginal tax on labour *zero* – see Prescott (2004, p. 8) and (11) – thus preventing a marginal distortion to labour supply. When this condition is met, the first-order condition for labour supply matches the planner's in (7), given that $mpl_t = w_t$ and $\frac{1-\tau}{1+\tau_c} = 1$. Under the optimal policy, the old receive a positive share of output,[10] and $\Phi = \alpha\omega$ (= optimal savings rate out of y_t), such that the optimal coefficient in the pension transfer rule (13) is $\tilde{\Phi}^* = \frac{1-\alpha\omega}{1-\alpha}$, thereby matching this ratio in the planner's optimal labour supply equation, (8). Intuitively, this says that to 'hit' the optimal labour supply, the policymaker must correct for the income effect arising from the fact that a correction to the saving rate affects labour income.

In the case of *over*-saving, a positive income tax $\tau > 0$ is combined with a consumption subsidy $\tau_c < 0$. Intuitively, a positive income tax corrects the over-saving by curbing capital accumulation, while setting $\tau_c = -\tau$ eliminates the 'tax wedge' in the first-order condition for labour supply, (11). Interestingly, the consumption subsidy exceeds the amount raised by the income tax – see (17) – so this case results in a *negative* pension (lump-sum tax).

[9] Consumptions are implied by the capital choice. With *exogenous* labour supply, capital and consumptions follow (15)–(16) in the market economy and the optimal allocations are (8)–(9), but with exogenous labour of 1 in both cases. It is easy to show that $\tau = \frac{\beta(1-\alpha)-\alpha\omega(1+\beta)}{(1-\alpha)(\beta+\omega)}$, $\tau_c = 0$ achieves the optimal allocation.

[10] The output share of old-age consumption (see (16)–(17)) is a constant > 0 times $\alpha(1-(1-\omega)\tau) = \frac{\alpha\beta\omega(1-\alpha\omega)}{\beta(1-\alpha)-\alpha\omega(1-\omega)}$, which is > 0 if and only if Assumption 1 holds (i.e. $\beta(1-\alpha)/\alpha > \omega(1-\omega)$). Hence, Assumption 1 ensures the output share is *not* negative (or undefined), which cannot be an optimal allocation.

For *under*-saving – i.e. 'strained' pension systems – a *negative* income tax $\tau < 0$ is used alongside a consumption tax $\tau_c > 0$. This raises the savings rate to the socially-optimal level while keeping labour supply undistorted (given that $\tau_c = -\tau$). In this case, the pension to the old is *positive*: consumption tax revenue more than pays for the income subsidy; see (17). Thus, while typical policy recommendations call for governments to reduce reliance on paygo, our results suggest that a bold policy of *negative* income taxes alongside a standard consumption tax will raise retirement saving sufficiently, while avoiding lump-sum taxes.

3.2 Discussion

We have presented an optimal pension policy where the two parts – the income tax and consumption tax – are set to be equal and opposite in sign to avoid distorting labour supply. The sign of the income tax is determined by whether the economy's 'natural' savings rate at the *laissez-faire* equilibrium is too high or too low, relative to the social optimum.

In the case of *under*-saving, the optimal policy combines a *negative* income tax and a positive consumption tax, with the old receiving an unfunded pension that is strictly positive. In this important case – which seems of most relevance given demographic trends – there are *no* lump-sum taxes and thus our proposed policy looks highly attractive. In the case of *over*-saving, the pension paid to the old is *negative* (Proposition 2), implying a lump-sum tax on retirees (whose choices are past). Here the policy looks less attractive but – as argued in the Introduction – retirement saving appears to be *too low* rather than too high.

Our policy implications are quite different to existing works. In Abio et al. (2004) there is no consumption tax in the optimal policy because pensions are financed by an income taxes on males, whose labour supply is taken as *inelastic*. As a result, tax distortions to labour supply are absent, and a consumption tax has no role. In Beetsma et al. (2013), the optimal policy in their two-period economy requires a *lump-sum* tax and a distortionary tax to labour income of the young is absent; thus, again, there is no need for a consumption tax. Importantly, while intergenerational risk is a major focus in Beetsma et al. (2013), we instead emphasise a marginal income tax on the young workers, and recall that their optimality result does *not* hold in an *infinite horizon* overlapping generations economy, while ours does.

In short, our two main policy prescriptions – (i) *equal and opposite* consumption and income taxes to prevent labour supply distortion; and (ii) a *negative* income tax plus a positive pension – appear novel, and our optimal policy fills a gap in the existing literature.

3.3 Numerical example

We set the capital share at $\alpha = 0.30$ and the social discount factor at $\omega = 0.995$. The labour preference parameters are set at $\theta = 4$ and $\chi = 2$; the latter implies a Frisch elasticity of 0.5. We fix productivity at $A = 1$ and population growth at $n = 0.05$. We set the private discount factor at either $\beta = 0.85$ (over-saving) or $\beta = 0.65$ (under-saving), and initial capital per worker k_0 is set at the steady-state value in the *laissez-faire* economy for each case.

The optimal policy is implemented from date 0 onwards and we simulate the resulting transition dynamics under the assumption that the reform (i.e. the change in policy) is unanticipated. Generational welfare effects are consumption-equivalent gains (or losses) relative to the no-reform counterfactual. The results are shown in Figure 1.

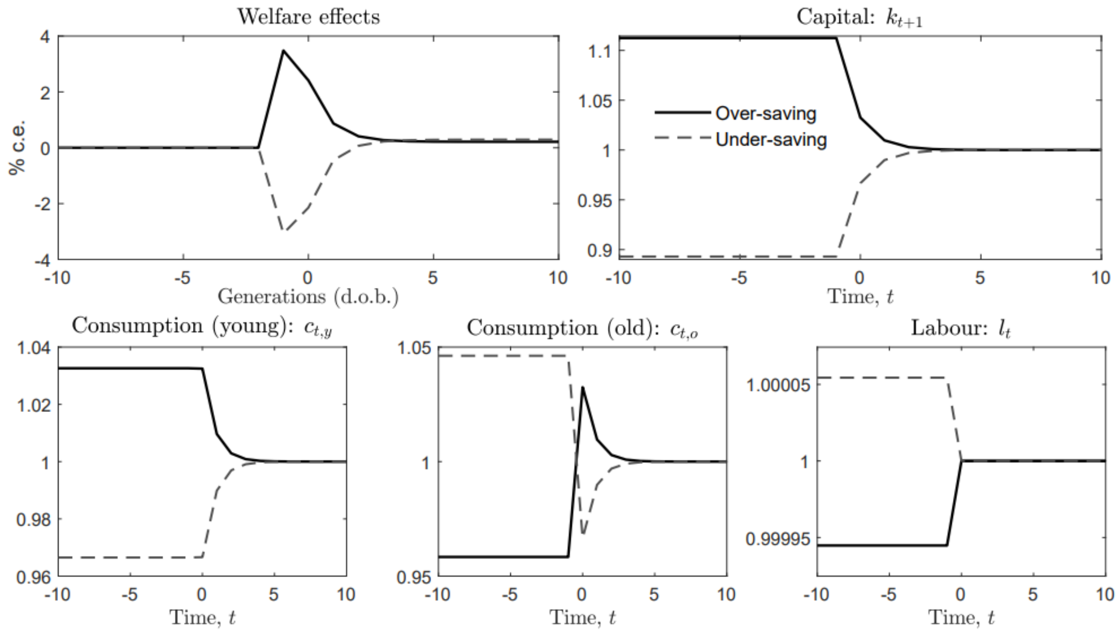

Figure 1: Transition dynamics under the optimal policy for $s^* \approx 0.43$. Optimal policy is implemented at date 0. Solid line: case of over-saving: $\beta = 0.85$ ($s = \frac{\beta}{1+\beta} > s^*$), initial capital $k_0 \approx 0.150$, optimal taxes are $\tau = 0.072$, $\tau_c = -\tau$. Dashed line: case of under-saving: $\beta = 0.65$ ($s < s^*$), initial capital $k_0 \approx 0.116$, optimal taxes are $\tau = -0.083$, $\tau_c = -\tau$. Variables in plots 2–5 are ratios to the terminal (i.e. steady-state) values as $t \to \infty$. Note: % c.e. is the consumption-equivalent increase in lifetime utility, and 'd.o.b.' is date of birth.

For the case of over-saving (solid line, $\beta = 0.85$), the optimal policy has capital falling and converging to a lower long run value (top right). This is achieved by an income tax, and a consumption subsidy, of 7.2%. Consumption when young falls only marginally at date 0 because the wage is unchanged and the effective labour tax is *zero* (only $\tilde{\beta}$ changes); by contrast, consumption when old increases sharply due to the consumption subsidy, with the subsequent reduction in private saving (capital) reducing the consumption increase for later generations (bottom panel).[11] Labour supply 'jumps' to its new value, which is slightly higher. Looking at the generational welfare effects (top left), we see that there is a Pareto improvement – all generations are better off – given initial capital overaccumulation.[12]

Now consider the case of *under*-saving (dashed lines, $\beta = 0.65$), that is, 'low' initial capital. An increase in physical capital is accomplished through a *negative* income tax and a positive consumption tax of about 8.3%. The transition paths are essentially 'mirror

[11]Since $\omega \approx 1$, the optimal pension is near zero (see Proposition 2) and $c_{t=0,o}$ increases around 8% ($\frac{1}{1-0.072} = 1.078$); for reference, see the expression for old-age consumption in (16).

[12]Recall that an optimal (first-best) allocation is Pareto efficient as generational weights are positive.

images' of the over-saving case.[13] One important difference, however, is that there is *no* Pareto improvement in this case. This is intuitive given that there is initially under-saving in capital, and we see that both the initial old and the first young generation are hit hard by the optimal policy. It takes several generations until capital has increased sufficiently that subsequent generations are better-off. In short, given the relatively high social discount factor $\omega = 0.995$, the optimal policy trades off the welfare losses of the initial generations against the gains of the subsequent generations in order to maximize the social welfare function.

4 Conclusion

We have presented a simple two-part pension system that achieves the first-best allocation with *endogenous* labour supply for any initial capital. In the case of under-saving, an unfunded pension is paid to the old which is financed by consumption tax revenue net of a labour income subsidy. This policy raises the private savings rate to a socially optimal level, avoids the use of lump-sum taxes, and leaves labour supply undistorted because the consumption tax and the labour income tax are *equal in magnitude but opposite in sign*. Such a policy seems worthy of further investigation given the urgency and apparent difficulty of solving the dearth of private pension saving using widely available policy instruments.

References

Abio, G., Mahieu, G., and Patxot, C. (2004). On the optimality of payg pension systems in an endogenous fertility setting. *Journal of Pension Economics & Finance*, 3(1):35–62.

Beetsma, R. M., Romp, W. E., and Vos, S. J. (2013). Intergenerational risk sharing, pensions, and endogenous labour supply in general equilibrium. *The Scandinavian Journal of Economics*, 115(1):141–154.

Belan, P., Michel, P., and Pestieau, P. (1998). Pareto-improving social security reform. *The Geneva Papers on Risk and Insurance Theory*, 23:119–125.

Breyer, F. and Straub, M. (1993). Welfare effects of unfunded pension systems when labor supply is endogenous. *Journal of Public Economics*, 50(1):77–91.

Brunner, J. K. (1996). Transition from a pay-as-you-go to a fully funded pension system: The case of differing individuals and intragenerational fairness. *Journal of Public Economics*, 60(1):131–146.

De La Croix, D. and Michel, P. (2002). *A theory of economic growth: dynamics and policy in overlapping generations*. Cambridge University Press.

Diamond, P. A. (1965). National debt in a neoclassical growth model. *American Economic Review*, 55(5):1126–1150.

[13] The near 'mirror images' in plots 2–5 arise from dividing each variable by the terminal (i.e. steady-state) value as $t \to \infty$. Note that these (optimal) steady state values differ in the two cases studied (since β differs).

Fedotenkov, I. (2016). Ignorance is bliss: Should a pension reform be announced? *Economics Letters*, 147:135–137.

Friedman, M. (1962). *Capitalism and Freedom*. Chicago: University of Chicago Press.

OECD (2012). *OECD Pensions Outlook*. OECD Publishing.

Prescott, E. C. (2004). *Why do Americans work so much more than Europeans?* National Bureau of Economic Research Cambridge, Mass., USA.

Samuelson, P. A. (1975). Optimum social security in a life-cycle growth model. *International economic review*, pages 539–544.

Appendix

Proof of Proposition 1

Setting $\tau = \tau_c = 0$ in (15)–(16) yields $k_{t+1} = \frac{\beta(1-\alpha)}{1+\beta} y_t$, $l_t = \left(\frac{1+\beta}{\theta}\right)^{\frac{1}{1+\chi}}$, $c_{t,y} = (\frac{1-\alpha}{1+\beta}) y_t$, and $c_{t,o} = \alpha(1+n)y_t$. Comparing with (8)–(9), the first-best allocation is achieved if and only if $\beta(1-\alpha) = \alpha\omega(1+\beta)$, which requires $\frac{\beta}{1+\beta} = \frac{\alpha\omega}{1-\alpha}$, i.e. $s = s^*$. If $s > s^*$, then $\frac{\beta(1-\alpha)}{1+\beta} > \alpha\omega$, so there is over-saving relative to the socially optimal rate; on the other hand, if $s < s^*$, then the above inequality is reversed and, by an analogous argument, there is under-saving.

We now show that if $s \neq s^*$ and one tax is zero, the first-best cannot be replicated. If $\tau_c = 0$, $\tau \in (-\infty, 1)$, then by (15)–(16), the ratios k_{t+1}/y_t, $c_{t,y}/y_t$, $c_{t,o}/y_t$ equal the planner's in (8)–(9) iff $\tau = \tau' := \frac{\beta(1-\alpha)-\alpha\omega(1+\beta)}{(1-\alpha)(\beta+\omega)} \neq 0$. But if $\tau = \tau'$, first-order condition in (11) gives (at equilibrium) $l_t^{1+\chi} = \frac{(1-\alpha)(1-\tau)}{\theta c_{t,y}/y_t} \neq (l_t^*)^{1+\chi}$; see (7). If $\tau = 0$, $\tau_c \in (-1, \infty)$, then by (15)–(16), the ratios k_{t+1}/y_t, $c_{t,y}/y_t$, $c_{t,o}/y_t$ equal the planner's in (8)–(9) iff $\tau_c = \frac{\beta(1-\alpha)-\alpha\omega(1+\beta)}{\omega(1-\alpha\omega)} \neq 0$, but then the first-order condition in (11) gives $l_t^{1+\chi} = \frac{(1-\alpha)}{\theta(1+\tau_c)c_{t,y}/y_t} \neq (l_t^*)^{1+\chi}$ since $\tau_c \neq 0$. ■

Proof of Proposition 2

Set $\tau_c = -\tau$, $\Phi = \alpha\omega$. Then k_{t+1}/y_t, $c_{t,y}/y_t$, $c_{t,o}/y_t$ in (15)–(16) equal the planner's (8)–(9) iff $\tau = \frac{\beta(1-\alpha)-\alpha\omega(1+\beta)}{\beta(1-\alpha)-\alpha\omega(1-\omega)}$. Since $\tau = \Psi\tau'$ for $\tau' = \frac{\beta(1-\alpha)-\alpha\omega(1+\beta)}{(1-\alpha)(\beta+\omega)}$, $\Psi = \frac{(1-\alpha)(\beta+\omega)}{\beta(1-\alpha)-\alpha\omega(1-\omega)} > 0$ (see Assumption 1), $sgn(\tau) = sgn(\tau')$. Labour supply equals the planner's, since $c_{t,y}/y_t = c_{t,y}^*/y_t^*$, so (11) gives $l_t^{1+\chi} = \frac{(1-\alpha)}{\theta c_{t,y}/y_t} = (l_t^*)^{1+\chi}$; see (7). Given equal ratios and $l_t = l_t^*$ for all t, $y_t = y_t^*$, $k_{t+1} = k_{t+1}^*$, $c_{t,y} = c_{t,y}^*$, $c_{t,o} = c_{t,o}^*$ for all t, so the first-best allocation is replicated. By (ii), (5), and (12)–(14), $P_t = (1+n)[\tau(1-\alpha)y_t^* + \tau_c(y_t^* - k_{t+1}^*)] = (1+n)[\tau(1-\alpha) + \tau_c(1-\alpha\omega)]y_t^*$, since $k_{t+1}^* = \alpha\omega y_t^*$. Finally, given $\tau_c = -\tau$, it follows that $P_t = -\tau\alpha(1+n)(1-\omega)y_t^*$. Since $\alpha, \omega \in (0,1)$ and $n > -1$, $P_t > 0$ iff $\tau < 0$. But $sgn(\tau) = sgn(\tau')$, so $\tau < 0$ iff $s < s^*$. ■