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Asset pricing in African frontier equity markets

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A B S T R A C T

This paper undertakes a horse races style comparison of the efficacy of a range of multifactor asset pricing models in explaining the cross section of stock returns in African securities markets. Valuation factors used include size, book-to-market value, momentum, operating profit, asset growth or investment, liquidity and investor protection. Using monthly returns of 375 blue chip firms from 8 African equity markets over 23 years, we undertake a horse-race style comparison of various classes of augmented CAPM models. We show that both the Fama & French (2015) five factor and Fama & French (2018) six factor framework yield the highest explanatory power. Analysis of costs of equity and optimized portfolio opportunity set simulations reveal substantial differences arising and borne by practitioners from the contrasting application of different asset pricing models underscoring the timely importance of our study.

Usiweke mayai yako yote kwenye kikapu kimoja [Kiswahili, East Africa]

Mɛnfa wo nkosoa nyinaa nwu kɛntɛn baako mu [Ashanti Twi, Ghana & West Africa]

Do not put all your eggs in one basket [Warren Buffett]

1. Introduction

The concepts of risk versus return and risk diversification are mainstays in the finance literature, while in the related field of asset pricing, they have spawned a vast and evolving array of risk hedging factors designed to hedge against various diversification risks. Much of the susceptibility of the asset pricing literature to “data mining” (Fama & French, 2018: 237) in the quest for supplementary risk hedging valuation factors arises from the simplicity of the intertemporal capital asset pricing model (ICAPM), which augments a ubiquitous market factor with a potentially unlimited number of additional factors (Fama & French, 2018). These factors are typically based on a multitude of financial ratios derived from firms’ balance sheets. Moreover, following inclusion within an augmented CAPM, such factors often exhibit strong out-of-sample modelling performance (Fama & French, 2018) despite lacking theoretical support for their inclusion (Maio & Santa-Clara, 2012). This motivates our study to undertake a “horse races” style (e.g., Cooper, Ma, Maio, & Philip, 2021; Cooper & Maio, 2019a; Hou, Mo, Xue, & Zhang, 2019) comparison of the most established risk hedging factors, which also have strong theoretical grounding within the literature.

Our theoretical approach is based on Merton’s (1973) intuition that minority portfolio investors’ utility of wealth depends on how it can be used to generate future consumption and on the portfolio opportunities that will be available to move wealth through time for future consumption. Critically, wealth is contingent on stochastic state variables related to specific future consumption investment risks, such as the relative prices of consumption goods and the risk–return trade-offs in capital markets. Moreover, changes in wealth over time are attributable to capital gains in equity assets held within portfolios. A drawback of Merton’s (1973) approach is its complex mathematical exposition, which led to Fama (1996) and Fama and French (1996) outlining a simple, tractable intertemporal capital asset pricing model (ICAPM). This model is a multifactor efficient augmentation of the mean-variance

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efficient market factor of Sharpe's (1964) and Lintner's (1965) underlying CAPM, with additional returns-based factors mimicking state variables.

Our study empirically considers a range of factors from those of the more established market, size and book to market value introduced by Fama and French (1993) to momentum by Carhart (1997) through to illiquidity introduced by Liu (2006), then the recent operating profit and asset growth, or investment, terms in Fama and French (2015a, 2015b), Fama & French, 2018), as well as investor protection, based on concentrated ownership and national institutional quality, in Hearn, Phylaktis, and Piesse (2017). We utilize these factors both in the asset pricing format they were originally introduced as well as jointly altogether within a grand asset pricing model. Consequently, we consider the single factor CAPM of Sharpe (1964) and Lintner (1965), with this augmented by size and book to market value factors in Fama and French's (1993) three factor format (abbreviated as FF3F), then the addition of momentum on top within Carhart's (1997) four factor format (abbreviated as C4F). We then consider Fama and French (2015a, 2015b) profit and investment factors within a five-factor configuration on top of the constituents of their original three factor model (henceforth FF5F), which was subsequently followed by Fama and French's (2018) proposal of additional augmentation of these five factors by momentum in constituting a six-factor model (henceforth FF6F). Importantly, all of these six factors are formed through a 3 × 3 "double sort" procedure, with stocks sorted into three portfolios first by size and then each of these subsequently sorted by the factor in question, such as book to market value or momentum or operating profit. The size factor returns are those of the smallest portfolio minus the biggest, while those of the factor in question are the difference between the average returns on three highest and lowest portfolios in second stage. Next, we consider the simpler two factor models proposed by Liu (2006) in augmenting the market term with an illiquidity factor and Hearn et al. (2017) in similarly augmenting the market term with an investor protection factor. Notably, both the illiquidity and investor protection factors considered in this two-factor format are formed through the returns' differences between lowest and highest of ten decile portfolios sorted on the factor in question – so a "single pass" procedure. Finally, we consider all factors jointly together in a grand asset pricing format with all factors constituent to this having been formed through the distinctive 3 × 3 "double sort" procedure.[1]

Our empirical contribution to the asset pricing literature is through the application of some of the most recently proposed asset pricing methods. Our asset pricing methods include Fama and Macbeth (1973) cross sectional style regressions alongside traditional time series regression analysis accompanied with related Gibbons, Ross, & Shanken (henceforth GRS, Gibbons, Ross, & Shanken, 1989) test statistics based on the application of regressions to groups of underlying portfolio returns-based assets. These test assets are representative of a broad selection of well-known pricing anomalies in the cross section of stock returns from similar asset pricing studies undertaken worldwide and include a variety of balance sheet metrics, as well as a range of liquidity-based measures. We also apply joint time series – cross sectional methods to gauge the relative explanatory power of models in relation to one another in terms of a "constrained" cross-sectional R-squared, R_C^2, proposed in Maio and Santa-Clara (2017) and Cooper and Maio (2019b). Next, we apply spanning regressions (Barillas & Shanken, 2017, 2018; Fama & French, 2015b, 2018 and Hou, Xue, & Zhang, 2020) to ascertain the potential redundancy of individual factors constituent to a given model. Finally, we undertake a novel extension in exploring the implications of using each of the asset pricing models for firms in terms of estimating costs of equity or discount functions that would otherwise be applied to their cash flows and, as a flipside to minority portfolio investors in relation to the profiles of their investment opportunity sets.

We focus on novel emerging African universe comprising listed stocks from the major stock markets across the continent. Given the lack of appropriate region-specific benchmarks, we first gather data on all available stocks' constituent to various respective national blue-chip indices and then screen these in relation to illiquidity or activity. This is important owing to the severity of extreme illiquidity implying near static immobile returns time series, which would otherwise violate statistical Normality distributional assumptions within later asset pricing modelling techniques. This leads to a reduction of over half from the initial universe to our final sample of 375 stocks from January 2001 to October 2023. While there are a number of studies that have considered asset pricing within an African context, such as Hearn and Piesse (2010a, 2010b), Hearn, Strange, & Piesse, 2012), these are relatively limited in scope of the breadth of asset pricing models and underlying constituent factors while also typically solely focusing on illiquidity, which is a major phenomenon in smaller emerging and developing securities markets. Our study is the first to consider practitioner implications relating to risk diversification in the African context – something also reflected in the selection of indigenous Twi, and Kiswahili language phrases at the start of this introduction.

The results suggest that the traditional single factor CAPM is inadequate as a pricing model in its own right with only its augmented multifactor counterparts yielding some degree of efficacy in accounting for the hedging of risks. In particular, the C4F and FF6F models consistently outperform rival models across a battery of tests, although there is strong evidence pointing towards the redundancy of the operating profit valuation factor, which is especially visible in spanning regression tests. Moreover, the considerable differences in efficacy of each of the rival asset pricing models is visibly reflected in significant distortions of the portfolio opportunity sets, as well as cost of equity estimations. These underscore the value of our work in enabling practitioners to differentiate between the application of rival asset pricing models.

The paper proceeds as follows. Section 2 describes the sample selection and data sources. Sections 3 and 4 discuss the asset pricing methodology and then outline the techniques used to construct the factor mimicking portfolios (FMPs), accompanied by summary statistics. Section 5 reviews the results from the application of the various asset pricing tests, while Section 6 discusses the practitioner implications. The final section concludes with some policy implications for African equity markets.

2. Data

2.1. Data

One may consider a number of unique attributes whilst forming an African universe of stocks drawn from constituent national stock exchanges across the continent. The first is that none of the continent's markets are designated as "developed" within the MSCI world index,[2] while only Egypt and South Africa fall within the "emerging" category. Of the remainder, Kenya, Morocco, Tunisia, Mauritius, Nigeria and the Francophone West African regional bourse, the BRVM, in Côte d'Ivoire are designated as "frontier" markets while Botswana, Zimbabwe and

[1] We also acknowledge that a number of other recently proposed models are available in the literature such as the Hou et al. (2015) four-factor q model, the Hou et al. (2019) five-factor q5 model, the Stambaugh and Yuan (2017) four-factor model, and Barillas and Shanken (2018) six-factor model. However, these have been omitted from consideration in our study owing to the severity of data limitations on firm level data essential to the construction of one or more of the constituent valuation factors of these models. Such data limitations on firm level data are especially pertinent in smaller emerging and frontier equity markets.

[2] MSCI definitions are sourced from MSCI website https://www.msci.com/market-classification.

Table 1
Summary statistics.

Market	Incl.	Start date	N	Liquidity			Trading statistics					
				BID-ASK	ZERO	LIU	RET	MCAP	BTMV	MOM	FF	IP
			#	%	%	#	%	US$m	#	%	%	#
North Africa												
Algeria	No	1/2000	7	3.01	94.17	108,431	– –	56.14	– –	−1.73	21.22	698
Egypt Hermes	Yes	1/2000	43	12.41	33.67	3635	1.59	438.07	1.0611	29.36	40.40	1520
Morocco	Yes	1/2000	71	5.87	47.37	1535	0.90	646.73	0.5587	6.69	26.69	1213
Sudan	No	1/2010	67	0.69	99.85	6,512,155	– –	85.76	– –	5.54	– –	– –
Tunisia	Yes	1/2006	86	2.82	48.09	1325	0.65	81.19	0.4276	7.34	35.12	1721
West Africa												
BRVM Top10	Yes	2/2007	10	2.99	40.38	1351	0.62	538.72	0.5349	5.71	54.81	1955
BRVM	No	2/2007	46	8.80	61.24	3304	1.68	142.88	0.4064	14.41	40.85	1429
Ghana	No	9/2009	33	24.74	91.56	1,468,935	−0.12	97.04	0.9533	18.24	25.39	1348
Nigeria Ngx30	Yes	10/2009	30	4.70	37.04	2048	1.05	1537.11	0.9455	22.67	47.87	1295
Nigeria	No	10/2009	164	12.74	61.49	223,421	0.29	287.87	0.8226	15.61	46.34	1254
East Africa												
Kenya Top40	Yes	1/2000	41	17.28	35.39	674	0.98	316.58	1.1772	12.77	46.51	1739
Kenya	No	1/2000	60	27.22	48.19	3668	1.08	223.25	1.5907	14.03	42.19	1579
Rwanda	No	8/2011	6	– –	79.58	10,493	0.76	173.35	0.6021	9.08	– –	– –
Tanzania	No	3/2009	22	2.06	90.77	199,965	0.72	207.68	0.5317	8.44	38.99	1632
Uganda	No	5/2009	10	7.09	85.69	21,501	0.43	125.55	1.3775	7.48	– –	– –
Southern Africa												
Botswana	No	2/2009	25	12.53	89.35	42,158	0.71	191.31	0.6598	4.15	45.57	3075
Malawi	No	11/2010	14	20.47	92.69	17,145	4.07	136.39	1.0516	40.84	– –	– –
Mauritius Top7	Yes	8/2008	7	1.50	48.51	2156	0.12	617.67	1.0411	2.73	45.47	3237
Mauritius	No	8/2008	37	4.96	73.18	4423	1.03	174.38	1.4318	5.01	42.47	3024
Namibia	No	7/2011	12	4.28	91.06	2,146,826	1.49	3717.77	0.6029	8.34	69.22	4206
South Africa	Yes	1/2000	87	6.00	13.81	93	1.37	3495.87	0.6178	14.90	63.67	3700
Zambia	No	2/2011	22	14.80	92.37	78,538	3.83	962.23	1.0655	14.26	43.87	1948
Zimbabwe	No	4/2009	25	23.65	78.65	27,311	29.96	36.07	7.0701	199.73	63.29	1415
Africa overall		1/2000	867	10.68	61.41	6,812,064	1.79	722.10	1.0946	16.58	42.34	1860
Africa sample		1/2000	375	6.75	34.23	1396	0.83	1215.28	0.7191	11.78	42.53	2010

This table reports summary statistics for the sample markets. The first column in table is the name of the national equity market, namely the country within which it is situated. The second column entitled "Incl." is indicative of whether the market is included in the final sample with a "yes" indicating inclusion versus a "no" indicating otherwise. The third column provide the start date for data obtained on constituent firms within the national equity market – as provided by Thomson Refinitiv Datastream database or in exceptional circumstances such as in Sudan direct from the national stock exchange itself. The fourth column entitled "N" indicates the number of constituent stocks listed within the national market or a particular segment of it, such as those stocks constituent to a blue-chip index – such as the Ngx30 in Nigeria. Importantly, the decision upon which firms are included in our African sample and which are omitted (as indicated in the second column entitled "Incl.") is based on the liquidity/activity characteristics highlighted between columns five to seven, under the banner of "Liquidity". These are bid ask spread, "BID-ASK", (column five), proportion of daily zero returns, "ZERO", a metric gauging the static nature of prices and returns (column six) and the multi-dimensional trading speed metric of Liu (2006), "LIU", in column seven. While bid ask spreads are among the highest in the world and subject to huge variation across the sample, by far the most important discriminating criteria are the proportion of zero returns and the Liu metric. Notably, those stocks with zero returns in excess of 60 % are omitted from inclusion in the sample universe. This is because such highly static time series violate statistical Normality assumptions in later empirical modelling while undermining the mean-variance optimization methodology employed. Trading statistics are reported between columns nine to fourteen. These are monthly returns, "RET", are the average returns of each stock over a monthly interval. Market capitalization, "MCAP", is measured at 1 January for each country and is the equity market value for each firm in billions of US$. The US$ market capitalization is measured at the end of month exchange rate for each country and each month. The book to market value ratio, "BTMV", is the inverse of the Datastream price-to-book value, for each stock. Momentum, "MOM", is the time series average of the percentage cumulative return for each stock over the prior six months, omitting the most recent month, and is monthly, following Jegadeesh and Titman (1993). The percentage free float, "FF", of shares held outside block owners. The investor protection metric, "IP", is based on a stock-by-stock basis and is the product of free float proportion and country-level aggregate institutional quality, in units of 0–10,000. Datastream reports the free float proportions (%). Institutional quality is reported on a 0–1 scale, where this is the average of the rescaled six underlying World Governance indicators. Indicators from Kaufmann et al. (2009) "Governance Matters VIII: Governance Indicators for 1996-2008". World Bank Policy Research June 2009. These are downloadable from http://www.govindicators.org. Descriptive statistics for trading and liquidity measures use Datastream for the daily prices, volume, market capitalization and free float information.

Mauritius are the subject of distinct standalone MSCI "country" indices. To facilitate a comprehensive overview of the continent's capital markets, we additionally include the constituent listed firms (stocks) from all of the fledgling and micro-markets such as Algeria,[3] Sudan (Khartoum),[4] Uganda, Tanzania, Rwanda, Malawi, Zambia, Zimbabwe, Namibia. Firm level trading, ownership and balance sheet accounting data are all sourced from Thomson Refinitiv Datastream's coverage of the continent with this augmented by data sourced direct from the national stock exchanges in the case of Namibia, Sudan, Malawi and Algeria. Owing to a lack of data availability and also extreme smallness and inactivity, we omit the fledgling Eswatini (Swaziland),[5] Mozambique and Cameroon stock markets and then the newly formed markets of Somalia,[6] Lesotho, and Angola. These criteria lead to our initial consideration of 19 African equity markets, as outlined in Table 1.

Next, we apply several distinct screening steps in order to isolate our final working sample. The first is that of universal data availability for individual firms' constituent to each of market. This eliminates Algeria and Sudan given a wholesale lack of ownership and balance sheet accounting data essential in forming the basis of our anomaly variables.

[3] See Hearn (2014) for a detailed overview of liquidity and trading on the Bourse d'Alger in Algeria.

[4] See Hearn et al. (2012) for a detailed overview of the Khartoum stock exchange in Sudan.

[5] See Hearn and Piesse (2010a, 2010b) for a detailed overview of "micro-market" stock exchanges of Eswatini and Mozambique.

[6] See http://www.somalistockexchange.so/.

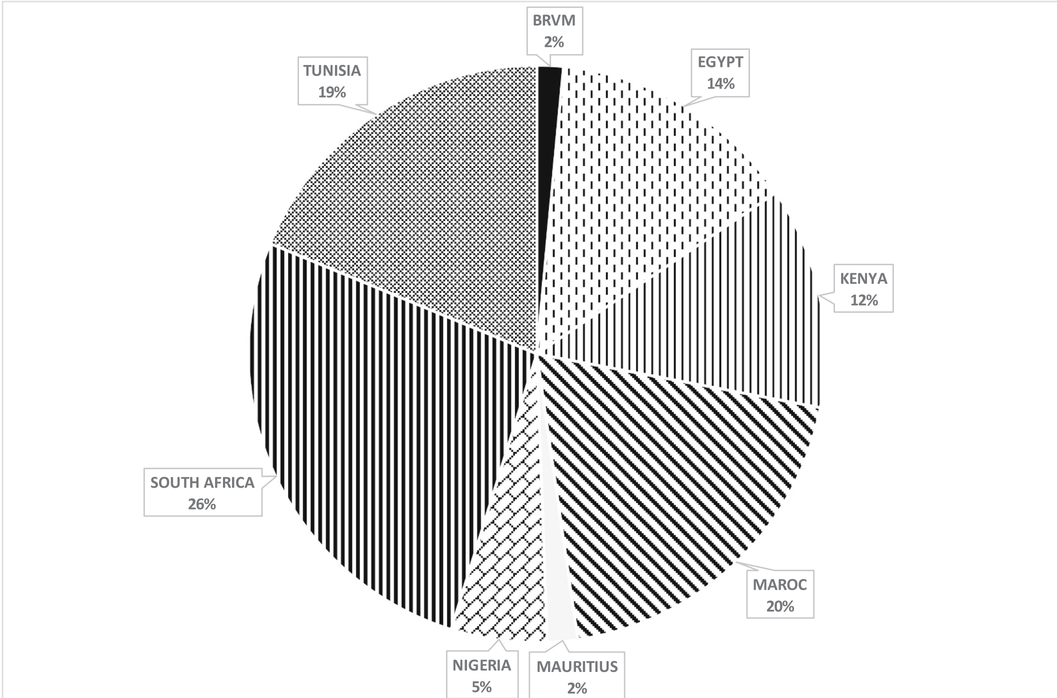

Fig. 1. Sample construction.

The second is the level of activity or liquidity of firms' stocks with this varying wildly both within and across markets. An immediate consequence of extreme inactivity, reflected in largely immobile time series of stock prices and static or non-existent returns, is that these are likely to proliferate within certain decile portfolios formed in the cross-sectional sorting and rebalancing based on anomy variables. This is reflected in violations of statistical Normality assumptions behind returns series, which are essential for their modelling. It is also reflected in zero-cost hedging portfolios that lack effective representation of the anomalies which they should capture given that underlying returns series are overwhelmingly dominated by severe inactivity. Such consideration of illiquidity leads to our omission of the entire markets of Ghana, Rwanda, Tanzania, Uganda, Botswana, Malawi, Namibia, Zambia and Zimbabwe. All have exceptionally high bid ask spreads. They also notably have monthly proportions of daily zero returns in excess of 90 % revealing the essentially static nature of constituent listed firms' returns-based time series. Furthermore, these levels of extreme inactivity are reflected in the Liu (2006) metric at excessively high values in Sudan (6,512,155), Namibia (2,146,826) and Ghana (1,468,935), which starkly contrast with the levels in larger, more liquid markets, such as South Africa (93). This justifies the omission of these less active markets from further consideration in our sample.

However, there is considerable intra-market variation among individual constituent listed firms in terms of their illiquidity within the markets of Egypt, Kenya, Nigeria, Mauritius and BRVM. An additional issue mirroring the illiquidity within Egypt is that of universality of data availability with this notably lacking in stocks that are not constituent to the blue-chip Hermes Financial index. This led to our inclusion of only those constituents of this index for Egypt. Similarly, in Nigeria, we only considered constituents of the blue-chip NGX30 index owing to huge intra-market variation in illiquidity. In the case of Kenya, we included the most active 40 listed firms which had proportions of daily zero returns less than 50 %, while this criterion was also applied to Mauritius and Francophone West African regional exchange BRVM resulting in only the top 7 and top 10 listed firms being included respectively. Our final sample comprised 8 African equity markets of Morocco, Tunisia, Egypt, Kenya, Mauritius, Nigeria, BRVM and South Africa. Next, the constituent listed firms of these equity markets were screened to ensure all were listings of ordinary (one share equals one vote) shares. Here, we have omitted funds (also known as Sicafs or Sicavs in Francophone markets), real estate investment trusts (REITS), secondary or tertiary lines of shares and non-ordinary preference shares or multiple classes of shares based on variations in voting rights conferred to each share. The imposition of these criterion led to a substantial reduction in the member firms of South Africa's blue-chip Johannesburg Stock Exchange, FTSE/JSE-All Share index from an initial 127 stocks down to 87 finally included in our sample. The final sample size was 375 listed firm's stock. Importantly, our sample yields significant reductions in illiquidity, as visible in the last two rows of Table 1. Notably, our sample yields a dramatic reduction in Liu (2006) multi-dimensional illiquidity metric, while both bid-ask spread and proportion of daily zero returns are halved vis-à-vis the entirety of African universe. These statistics further corroborate our sample selection criterion in forming our African sample group.

Our final consideration is that of data availability. This varies significantly both across the markets in our sample, as well as within them. Some of this variation at an aggregate market level is visible in the differences in start dates of data in Table 1. Data from South Africa, Kenya, Morocco and Egypt is available from 2000, while that for Tunisia starts in 2006 and then that for Mauritius and Nigeria is more recent in 2008 and 2009, respectively. Our final headline sample time frame is from January 2001 to October 2023.

2.2. Summary statistics

Summary statistics are reported in Table 1. A few observations are apparent. The first is that listed firms' constituent to North African equity markets have the highest ownership concentration and subsequent lowest levels of free float while listed firms' constituent to Southern Africa's equity markets have correspondingly the lowest concentrated ownership and highest dispersed ownership levels. This huge variation between regions is also reflected in market capitalization, with this being lowest across North Africa and biggest across the Sub-Saharan African regions of East, West and Southern Africa. Intuitively, this is

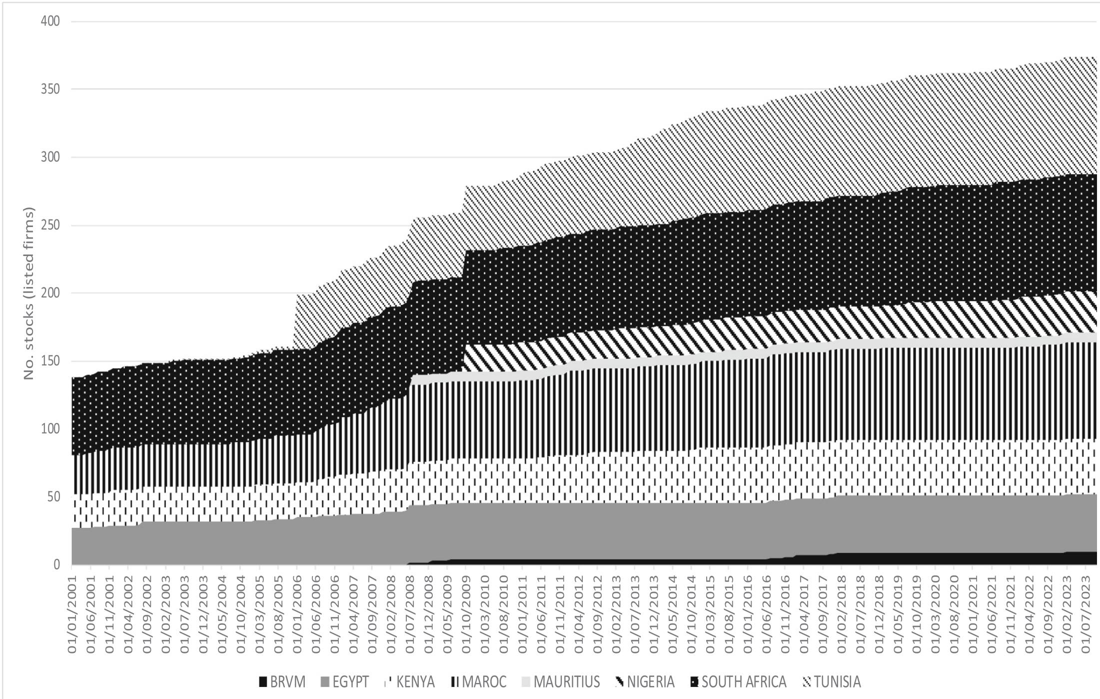

Fig. 2. Evolution of sample constituency over time.

reflective of North African firms being subject to concentrated control associated with an increased reliance on primarily relational capital provision with far less motivation to raise finance from external constituencies and investors. This evidence fits with prior literature observations of North African economies' dominance by powerful local extended families (see Hearn, 2011, 2014). The second is that momentum is similarly considerably lower in North Africa while being markedly higher across Sub-Saharan Africa. The third observation centres on the extreme variation in levels of inactivity and illiquidity across the continent and within subordinate regions. This observation fits with recent literature addressing liquidity measurement (Hearn & Piesse, 2013) and its incorporation into asset pricing models (e.g., Hearn et al., 2012; Hearn & Piesse, 2010a, 2010b) and is very much reflective of the region's capital markets' frontier status within MSCI.

A fourth observation concerns the composition of our sample, which can be visibly seen in Fig. 1. Owing to the severity of illiquidity across the majority of Sub-Saharan African equity markets, stocks constituent to these have at best a minimal presence within the sample. Instead over a quarter of the sample is comprised of South African stocks, while over a half again originate from the North African equity markets of Egypt, Morocco and Tunisia. Out of Sub-Saharan Africa, Mauritius and BRVM account for 2 % each, Nigeria 5 % and then Kenya with a slightly larger stake of 14 %. The inclusion of these Sub-Saharan equity markets is likely the result of their implementation of far-reaching market microstructural reforms over the last 15 years, which has resulted in visible enhancements of trading activity and the liquidity profiles of listed firms' stocks. A related observation to the composition of the sample is that of its evolution over time, which is visible in Fig. 2. From inception to approximately 2006 the sample comprised South Africa, Egypt, Morocco and Kenya. Thereafter, Tunisia was included and then after 2009 the remaining equity markets in our aggregate sample were gradually included through increasing numbers of stocks.

2.3. *The construction of factor mimicking portfolios (FMPs)*

To study the influence of factors, such as size, book to market value, momentum, and liquidity on the variation of African stock returns, we construct returns-based proxies using zero-investment portfolios. These portfolios go long in stocks with high values of a given characteristic and short in stocks with low values for that characteristic. We use the time-series regressions of Black, Jensen, and Scholes (1972), following Fama and French (2015a, 2015b), Fama & French, 2018) and Hearn et al. (2017), to assess the pricing implications arising from the underlying factors. In this approach, the excess returns on test portfolios are regressed on a combination of a market factor, denominated in excess returns over the risk-free rate, and the returns of FMPs. The time series slopes are interpreted as factor loadings that inform how various combinations of these FMPs explain the average returns across the portfolios. We form market portfolios based on both value and equal weighted returns of all stocks within a universe at a given time and use the yield on the 1-year US Treasury bill as our risk-free rate. The application of value-weighting of returns is essential given the acute risk of the typically overwhelming majority of microcap stocks "crowding out" simple equally weighted average returns series. Moreover, the formation of such equal weighted portfolios are typically prohibitively costly to investors given the significant transaction costs in taking multiple positions in smaller stocks which are less frequently traded and subject to often formidable informational asymmetries. Therefore, value-weighted returns while being largely determined by the biggest size stocks' returns are more reflective of realistic and viable positions an investor might take in practice.

We employ four different techniques to construct FMPs. The first is the formation of a time series of equal and value-weighted average returns from the monthly rebalancing of all individual stocks' returns within the universe at each particular monthly interval. While rebalancing is undertaken monthly across all individual stock returns, this also follows through in accounting for each stocks' various ownership and balance sheet metrics thereby providing a broader set of summary statistics (also equal and value weighted) behind the aggregate market return.

The second technique involves annual rebalancing into ten decile sorted portfolios based on each stocks' relative value of a factor followed by annual holding period for returns. Rebalancing is undertaken in December each year. FMPs are then formed from the returns-based difference between the extreme portfolios (D1 and D10). In this way, the returns-based FMP accounting for Liu (2006) liquidity is formed from the difference between least liquid (most illiquid) portfolio D1 returns and most liquid (least illiquid) portfolio D10. Similarly, the returns-based FMPs for free float and investor protection are calculated from the difference between returns on lowest free float (and investor protection) decile D1 minus those of the corresponding highest free float (and investor protection) decile D10.

The third technique is associated with momentum as calculated by the cumulative returns difference over preceding 6 and then 12-month periods. The FMP for momentum follows the Jegadeesh and Titman (1993) six-month/six-month strategy, where monthly returns are both value and equal weighted average of six individual strategies of buying the winning decile portfolio and selling the losing decile portfolio. These use 10 decile portfolios with stocks ranked on momentum across portfolios, where momentum is defined as the cumulative return over the preceding six months. Rebalancing occurs monthly.[7] The same technique is used for twelve-month/twelve-month strategy in momentum. To minimize the bid-ask bounce effect, we skip one month between ranking and holding periods when constructing the momentum FMP.

The fourth and final technique is that of a double-sort procedure initially introduced in Fama and French (1993) for an initial sort by size of stocks into five quintile portfolios, each of whom is sorted by book to market value in a further second stage sort. The size FMP is formed from the average returns on smallest portfolio minus those of the biggest portfolio and is denoted SMB. The book to market value FMP is then formed from average returns across the five highest portfolios minus those of the five lowest portfolios, denoted as HML. Fama and French (2016) extended this concept into the formation of four FMPs over and above the market term. Each is formed from same double sort procedure, namely first step by size followed by a second step sorting of each of the initial size-based portfolios by book to market value, then momentum and finally operational profit scaled by book equity. The final size FMP, SMB, is formed from the average returns across the four consecutive size FMPs obtained through the four separate double-sort procedures.

2.4. Anomaly portfolios: test assets

The previous asset pricing literature has identified hundreds of potential anomaly factors explaining the cross section of stock returns (see Fama & French, 2018; Hou et al., 2020). In this paper, we focus on 12 of the most prominent. Each factor is formed by sorting stocks into 10-decile portfolios based on each stock's relative ranking in terms of the underlying anomaly variable, from the highest decile (D10) to the lowest (D1). Rebalancing occurs annually in December. A detailed outline of the construction of each and the sourcing of the data is provided in Panel 2 of the appendix. Below, we introduce the 18 variables, which form the right-hand side test assets in subsequent asset pricing tests.

The first factor is *FF*, which is defined as the percentage of shares available to minority investors at any given time, which are not closely held (i.e., it is the opposite of concentrated ownership). Next, β (beta) is estimated using the preceding five years (rolling window) of previous monthly returns against a benchmark index formed from the value-weighted average of stock returns across our African sample universe. Our construction of such a benchmark from the African sample is important given the minimal availability of suitable benchmarks in this region and of those that there are stymied by the negligible coverage across the continent, which questions potential fit with our sample. A sizeable number of studies, from Black et al. (1972) and Fama and Macbeth (1973) to Frazzini and Pedersen (2014), find that the relation between the univariate market beta and the average stock return is flatter than predicted by Sharpe's (1964) and Lintner's (1965) CAPM. Fama and French (2016) qualify this beta anomaly as a purported violation of the CAPM. *MCAP* (size) is based on firms' market capitalizations, with stocks in the bottom and top size deciles referred to as microcaps and megacaps, respectively. The rationale for the *MCAP* factor is that smaller firms are more prone to earnings disparities during recessionary periods than their larger counterparts (Fama & French, 1993), although this has been questioned by Bebchuk, Cohen, & Ferrell (2009). *B/M* (book-to-market value) is defined as the ratio of the book value of equity to the market value of equity. Fama and French (1993) attribute persistent earnings variation to differences between value and growth stocks, as differentiated by their *B/M*, and this ratio has remained a cornerstone in asset pricing since then. However, more recent studies, such as Fama and French (2018), have questioned the importance of the book-to-market value.

Next, we consider *Vol* (volatility), defined as the variance in the daily closing price returns over the preceding 12 months. Ang, Hodrick, Xing, and Zhang (2006) find that stocks with highly volatile returns tend to have low average returns. Also included is *Asset growth*, defined as the change in total assets from five years before the preceding year. Fama and French (2015a) attribute this change in total assets to investment or disinvestment by the firm in its own asset base. We also include *Acc* (*accruals*), which is defined as the change in operating working capital per split-adjusted share from the fiscal year-end two years before that in the preceding year divided by the book equity per share in the preceding year. Our inclusion of accruals follows Sloan (1996) in attributing low returns to high accruals, while Fama and French (2018) argue that accruals differences arise because accounting decisions cause book earnings to differ from cash earnings. Next, *Op* (*operating profit*) is defined as revenue minus the cost of goods sold, minus selling, general, and administrative expenses, minus interest expense, all divided by book equity in the preceding fiscal year. Fama and French (2015b) focus on operating profit as an anomaly that explains the variance in the cross section of stock returns.

We also include four additional factors. *DY* (dividend yield) is defined as the total dividends paid out from July of the preceding year to June of the current year divided by the market equity at the end of June of the current year. Hou, Mo, & Xue (2021) argue that the dividend yield is directly related to variation in the cross section of stock returns. *P*/CF (price to cash flow) is defined as the stock price to cash flow per share. *P*/CF is measured as the market value of equity at the end of December of the previous year divided by the cash flows for the preceding fiscal year. *NSI* (net stock issues) is defined as the change in the natural logarithm of split-adjusted shares outstanding from the fiscal year two years before the fiscal year immediately before. The inclusion of this variable follows from share repurchases tending to be followed by large average returns (Ikenberry, Lakonishok, & Vermaelen, 1995), and average returns after share issues tending to be low (Loughran & Ritter, 1995). *P/E* (price-to-earnings ratio) is defined as the market equity at the end of December in the preceding year divided by earnings, which is defined as income before extraordinary items in the preceding fiscal year.

We also include *capital expenditure* (*CAPX*) *growth*, which captures variation in abnormal corporate investment by firms (Hou, Xue & Zhang, 2020). This is calculated from the change in the capital expenditure (CAPX) of the firm between the CAPX reported three years prior to one year prior and is expressed in percentage change terms. CAPX represent the funds used to acquire fixed assets other than those associated with acquisitions. We also include *sales growth* which captures growth in gross cash flows and reflects earnings variability. It is defined as the current year's net sales or revenues divided by net sales six years ago minus unity and is expressed as a percentage.

Finally, since our focus is on smaller developing and emerging stock markets, we include four anomaly variables related to illiquidity. The first is the "trading speed" *metric of* Liu (2006), which provides a scaled measure of zero trading volumes and has been lauded as beneficial

[7] That is, the momentum FMP return for January 2002 is 1/6 of the return spread between the winners and losers from July – November 2001, 1/6 of the return spread between winners and losers from June – October 2001.

owing to its ability to capture the multidimensional nature of illiquidity. The second is the "price impact" measure of *Amihud (2002)*, which measures the traded value impact from a stock's return. However, a caveat associated with this metric is that under conditions of severe illiquidity it is likely to be undefined for much of the same time frame. The third is *turnover*, namely a scaled mean of number of shared traded to the number of shares of a given firm issued and outstanding. While ubiquitous to the finance liquidity literature, this metric too has a shortcoming in being undefined under conditions of extreme illiquidity. Our final illiquidity metric is that of the mean monthly *proportion of daily zero returns*. This is simple to construct, tractable and theoretically associated with price rigidity or freezing from extreme informational asymmetry which inhibits minority investors' ability to achieve pareto optimality in trade. All four illiquidity measures are highly applicable to smaller developing stock markets such as Africa.

2.5. Asset pricing models

Our analysis is based on seven asset pricing models, all utilizing time-series ordinary least squares, OLS, regressions ubiquitous to the asset pricing literature. The first is the traditional CAPM, defined as follows:

$$R_{it} - r_{f,t} = \alpha_{i,t} + \beta_{i,M}\left(R_{M,t} - r_{f,t}\right) + \varepsilon_{i,t} \tag{1}$$

where $R_{it} - r_{f,t}(\mathrm{R_{it}} - \mathrm{r_{f,t}})$ are the returns of the portfolio or test asset i ($i = 1, \ldots, N$) in excess of the risk-free rate ($t = 1, \ldots, T$), $\alpha_{i,t}$ is the Jensen alpha or a $1 \times T$ vector of constant coefficients, and $\left(R_{M,t} - r_{f,t}\right)$ is the value-weighted market return in excess of the risk-free rate of return (one-month US Treasury rate). $\beta_{i,M}$ is a vector of factor loadings for asset i on the market excess returns, and $\varepsilon_{i,t}$ represents the idiosyncratic i.i.d. errors, which are allowed to have a limited correlation among returns.

Fama and French (1993), henceforth FF3F, extended this basic specification to additionally capture the relation between the average return and the size and the accounting book-to-market value, *B/M*, leading to a three-factor model:

$$R_{it} - r_{f,t} = \alpha_{i,t} + \beta_{i,M}\left(R_{M,t} - r_{f,t}\right) + \beta_{i,SMB}(SMB_t) + \beta_{i,HML}(HML_t) + \varepsilon_{i,t}, \tag{2}$$

where SMB_t is the difference between the returns on a diversified portfolio of small stocks minus those of an equally diversified portfolio of big stocks, and HML_t is similarly the difference between the returns on a diversified portfolio of high *B/M* stocks minus those of a diversified portfolio of low *B/M* stocks. The formation of SMB_t and HML_t uses a 3 × 3 double-sort procedure in which stocks are sorted into three-tercile portfolios based on their size, or market capitalization, each of which is further sorted into another three-tercile portfolios based on *B/M*. At any time, stocks with missing values for either characteristic are omitted, as are stocks with negative book-to-market values. FMPs related to size are created from the average returns on small portfolios minus those on large portfolios (SMB_t factor) and similarly with high book-to-market value portfolios minus low book-to-market portfolios (HML_t factor). Portfolio rebalancing takes place annually in December. The SMB_t and HML_t factors are formed from value-weighted returns. $\beta_{i,SMB}$ and $\beta_{i,HML}$ are vectors of factor loadings for asset i on the SMB_t and HML_t factors, respectively.

Carhart (1997), henceforth Carhart 4F, augmented the Fama–French three-factor model with a fourth factor related to momentum or persistence in stock price returns. This is defined as follows:

$$\begin{aligned} R_{it} - r_{f,t} = \alpha_{i,t} &+ \beta_{i,M}\left(R_{M,t} - r_{f,t}\right) + \beta_{i,SMB}(SMB_t) + \beta_{i,HML}(HML_t) \\ &+ \beta_{i,UMD}(UMD_t) + \varepsilon_{i,t} \end{aligned} \tag{3}$$

where UMD_t is the difference between the returns on a diversified portfolio of high-performing or "up" stocks minus those of an equally diversified portfolio of underperforming "down" stocks. The FMP for momentum involves a 3 × 3 double sort procedure of first sorting cross section of stock returns into three-tercile portfolios followed by each being subsequently sorted into a further three-tercile portfolios based on momentum. FMPs related to size are created from the average returns on small portfolios minus those on large portfolios (SMB_t factor) then with momentum it follows in the spirit of Jegadeesh and Titman (1993) in adopting a strategy of buying the winning, or "up," portfolio and selling the losing, or "down," portfolio leading to the $UMD_t\mathrm{HML_t}$ factor. Portfolio rebalancing takes place annually in December. The SMB_t and $UMD_t\mathrm{HML_t}$ factors are formed from value-weighted returns. $\beta_{i,SMB}$ and $\beta_{i,UMD}\beta_{\mathrm{i,HML}}$ are vectors of factor loadings for asset i on the SMB_t and $UMD_t\mathrm{HML_t}$ factors, respectively. This FMP is also formed from value-weighted returns. $\beta_{i,UMD}$ is a vector of factor loadings for asset i on the UMD_t (factor)

Fama and French (2015a, 2015b), henceforth FF5F, augmented their earlier three-factor model with additional factors relating to cross sectional differences in operating profitability (OP) and asset growth, or investment (INV). This is defined as follows:

$$\begin{aligned} R_{it} - r_{f,t} = \alpha_{i,t} &+ \beta_{i,M}\left(R_{M,t} - r_{f,t}\right) + \beta_{i,SMB}(SMB_t) + \beta_{i,HML}(HML_t) \\ &+ \beta_{i,UMD}(UMD_t) + \beta_{i,RMV}(RMW_t) + \beta_{i,CMA}(CMA_t) + \varepsilon_{i,t} \end{aligned} \tag{4}$$

where $RMW_t\mathrm{UMD_t}$ is the difference between the returns on diversified portfolios of stocks with robust and weak profitability, and $CMA_t$is the difference between the returns on diversified portfolios of the stock so flow and high investment firms, which we call conservative and aggressive. The formation of $RMW_t\mathrm{SMB_t}$ and $CMA_t\mathrm{HML_t}$ uses a 3 × 3 double-sort procedure in which stocks are sorted into five-tercile portfolios based on their size, or market capitalization, each of which is further sorted into another five-tercile portfolios based on *OP* or alternatively *INV*. At any time, stocks with missing values for either characteristic are omitted. Portfolio rebalancing takes place annually in December. The SMB_t, $RMW_t\mathrm{HML_t}$ and CMA_t factors are formed from value-weighted returns. $\beta_{i,SMB}$, $\beta_{i,RMV}$and $\beta_{i,CMA}\beta_{\mathrm{i,HML}}$ are vectors of factor loadings for asset i on the SMB_t, $RMW_t\mathrm{HML_t}$ and CMA_t factors, respectively.

Fama and French (2018), henceforth FF6F, further augment their preceding five factors in model (4) above with an additional momentum term to form a six-factor model. This is defined as follows:

$$\begin{aligned} R_{it} - r_{f,t} = \alpha_{i,t} &+ \beta_{i,M}\left(R_{M,t} - r_{f,t}\right) + \beta_{i,SMB}(SMB_t) + \beta_{i,HML}(\mathrm{HML}_t) \\ &+ \beta_{i,UMD}(UMD_t) + \beta_{i,RMV}(RMW_t) + \beta_{i,CMA}(CMA_t) \\ &+ \beta_{i,UMD}(UMD_t) + \varepsilon_{i,t} \end{aligned} \tag{5}$$

where the additional UMD_t factor is formed through a 3 × 3 double sort procedure as in the Carhart four-factor model (3) above which involves the formation of an initial SMB_t factor followed by the UMD_t factor. The SMB_t and $UMD_t\mathrm{HML_t}$ factors are formed from value-weighted returns. $\beta_{i,SMB}$ and $\beta_{i,UMD}\beta_{\mathrm{i,HML}}$ are vectors of factor loadings for asset i on the SMB_t and $UMD_t\mathrm{HML_t}$ factors, respectively. This FMP is also formed from value-weighted returns. $\beta_{i,UMD}$ is a vector of factor loadings for asset i on the UMD_t (factor)

Liu (2006), henceforth ILLIQ2F, proposed augmentation of the traditional CAPM with an additional liquidity factor. Liu's measure is based on the turnover-adjusted number of zero daily volume, which better captures the multidimensional nature of liquidity and overcomes shortcomings in numerous unidimensional measures prevalent in the literature. This two-factor model is defined as follows:

$$R_{it} - r_{f,t} = \alpha_{i,t} + \beta_{i,M}\left(R_{M,t} - r_{f,t}\right) + \beta_{i,ILLIQ}(ILLIQ_t) + \varepsilon_{i,t}, \tag{6}$$

where $ILLIQ_t$ is the difference between the returns on a diversified portfolio of highly illiquid stocks minus those of an equally diversified portfolio of low illiquidity stocks. Stocks are sorted into 10-decile portfolios, and the FMP is formed from the returns difference between high and low illiquidity decile portfolios. Then, the FMP is based on annual rebalancing each December, as in Liu (2006), and is value weighted again to minimize the effects of illiquid microcap stocks. $\beta_{i,ILLIQ}$ is a vector of factor loadings for asset *i* on the $ILLIQ_t$ (factor)

Hearn et al. (2017), henceforth IP2F, introduced a two-factor augmented-CAPM analogous to the preceding liquidity two-factor model, which augments the single market factor with an investor protection term, which is the product of firm-level free float and national institutional quality of the primary listing jurisdiction. This is defined as follows:

$$R_{it} - r_{f,t} = \alpha_{i,t} + \beta_{i,M}\left(R_{M,t} - r_{f,t}\right) + \beta_{i,IP}(IP_t) + \varepsilon_{i,t}, \tag{7}$$

where $IP_t\mathrm{FF}_t$ is the difference between the returns on a diversified portfolio of stocks with a low investor protection, enumerated as percentage free float times national institutional quality, minus those of an equally diversified portfolio of high percentage investor protection stocks. Stocks are sorted into 10-decile portfolios, and the FMP is formed from the returns difference between low and high investor protection decile portfolios. Then, the FMP is based on annual rebalancing each December and is value weighted again to minimize the effects of illiquid microcap stocks. $\beta_{i,IP}\beta_{i,FF}$ is a vector of factor loadings for asset *i* on the $IP_t\mathrm{FF}_t$ (factor)

Finally, we test a grand asset pricing model comprising the traditional CAPM plus all of the factors introduced above, namely, SMB_t, HML_t, UMD_t, RMW_t, $\mathrm{HML}_t CMA_t$, $ILLIQ_t$, and IP_t, henceforth referred to as $8\mathrm{FFF}_{\mathrm{size},t}\mathrm{ModFF}_t$:

$$\begin{aligned} R_{it} - r_{f,t} = \alpha_{i,t} &+ \beta_{i,M}\left(R_{M,t} - r_{f,t}\right) + \beta_{i,SMB}(SMB_t) + \beta_{i,HML}(HML_t) \\ &+ \beta_{i,UMD}(UMD_t) + \beta_{i,RMV}(RMW_t) + \beta_{i,CMA}(CMA_t) \\ &+ \beta_{i,UMD}(UMD_t)\beta_{i,ILLIQ}(ILLIQ_t) + \beta_{i,IP}(IP_t) + \varepsilon_{i,t} \end{aligned} \tag{8}$$

where it should be noted that all FMPs are formed through 3×3 double sorting procedure of first the SMB_t followed by the respective factor FMP. The final SMB_t is the average of all SMB_t FMPs formed through each of the respective double sorts.

We adopt value-weighted returns across all the FMPs and test portfolios in the subsequent analysis. This is intended to mitigate the effects of the proliferation of microcap stocks across all factor-sorted portfolios on which the FMP formation is based. This issue is especially important, given that our breakpoints are freely determined by an even distribution of stock numbers based on the sorting procedure of the underlying variable of interest. This procedure is especially susceptible to the spread of microcaps throughout the factor-sorted portfolios. In particular, microcaps can inflate the magnitude of anomalies, especially when combined with equal-weighted returns. Fama and French (2008) highlight that microcaps account for at most 3 % of the aggregate market capitalization of the NYSE-Amex-NASDAQ universe, from which the S&P 1500 index constituents are drawn, but account for about 60 % of the total number of stocks. However, in smaller, developing stock markets this issue of a proliferation of microcap stocks is likely to be as important if not more important than in the case of larger, development stock markets.

2.6. Time-series regressions and GRS tests

We follow the time-series approach of Black et al. (1972) and studies in this vein, such as Fama and French (1993, 1996, 2015b) and Hou, Xue, and Zhang (2015), in regressing the excess returns on test portfolios on those of the value-weighted market portfolio plus the returns of the FMPs for each of the seven asset pricing models outlined in expressions (1) to (8). The test portfolios are formed by sorting the cross section of stock returns by each firm's Liu (2006) illiquidity metric with the average returns associated with each minus the risk-free rate to produce test portfolio excess returns. Our focus on Liu's illiquidity metric resonates with the wider importance of liquidity as a major phenomenon impeding African and smaller developing stock markets worldwide. The time-series slopes are interpreted as factor loadings that inform how various combinations of these FMPs explain the average returns across portfolios. Of central importance to this approach is the expectation that the Jensen alpha is not statistically different from zero, given the relationship between an individual portfolio's or test asset's expected returns and those of the market (Markowitz, 1959).

Next, we extend this analysis by investigating the relative effectiveness of each of the eight models in fully explaining the returns on difference portfolio formed from the extremes across the cross section of stock returns sorted by *ILLIQ*. This is the investor protection FMP itself. A large and significant Jensen alpha is viewed as an *abnormal return* that cannot be attributed to any of the included FMPs or "the return in excess of what could have been achieved by passive investments in any of the factors" (Gompers et al., 2003; 122).

Finally, we utilize the GRS statistic to test the null hypothesis that if an asset pricing model captures expected returns, then the intercept is indistinguishable from zero in the time-series regression of any asset's excess return (its return in excess of the risk-free rate) on the model's factor returns. The GRS statistic tests the validity of this hypothesis for each of the seven asset pricing models applied to 10-decile portfolios sorted based on the 18 anomalies (e.g., dividend yield, net stock issues, and accruals) outlined in the preceding section. Therefore, for each of the 18 anomaly variables, we utilize 10 sorted decile portfolios as left-hand side test assets to which we apply the seven asset pricing models in succession. In addition to the GRS statistic, we report the mean absolute alpha (MAA) from the application of each asset pricing model across each set of 10-decile portfolios per each of the 12 anomalies in succession. The MAA is defined as follows:

$$MAA = \frac{1}{N}\sum_{i=1}^{N}|\widehat{\alpha}_i|, \tag{9}$$

where $\widehat{\alpha}_i$ 1, …, *N* represents the first *N* moments, i.e., the pricing errors associated with the *N* testing assets. It is notable that we estimate all regression slopes as constants; therefore, time variation in the slopes is a potential problem. Similar to most asset pricing literature, our models and tests also assume there are no market frictions, such as transaction costs and taxes.

2.7. Joint time-series and cross-sectional methods

The time-series models in expressions (1) to (8) can also be expressed in the basic general form of

$$\left(R_{it} - r_{f,t}\right) = \alpha_{i,t} + \beta_i F_t + \varepsilon_{i,t}, \tag{10}$$

where $\left(R_{it} - r_{f,t}\right)$ are the returns of portfolio or test asset *i* ($i = 1, \ldots, N$) in excess of the risk-free rate ($t = 1, \ldots, T$), $\alpha_{i,t}$ is the Jensen alpha or a 1 x *T* vector of constant coefficients, F_t is a *K*-dimensional vector of common FMPs (returns-based factors) at *t*, with *K* the number of FMPs included in the particular model, β_i is a *K*-dimensional vector of factor loadings for the excess return on asset *i*, and $\varepsilon_{i,t}$ represents the idiosyncratic i.i.d. errors, which are allowed to have a limited correlation among the returns. Next, we follow Maio and Santa-Clara (2017), Cochrane (2005), and Lewellen, Nagel, and Shanken (2010) in performing a two-step analysis. This analysis builds on an initial time-series regression step, which is outlined in expressions (1) to (8). Using example model (9), we form the "constrained" model for each of the test asset regressions across all 10-decile portfolios of each of the 18 anomalies:

$$\left(\overline{R_i - r_f}\right) = \widehat{\beta}_{i,F}\overline{F} + \widehat{\alpha}_{i,C}, \tag{11}$$

and with an example again in the form of the FF3F (from expression (2)):

$$\left(\overline{R_i - r_f}\right) = \widehat{\beta}_{i,M}\left(\overline{R_M - r_f}\right) + \widehat{\beta}_{i,SMB}\left(\overline{SMB}\right) + \widehat{\beta}_{i,HML}\left(\overline{HML}\right) + \widehat{\alpha}_{i,C}, \tag{12}$$

where $\left(\overline{R_i - r_f}\right)$ is the average of the time-series excess returns on test asset *i*; $\left(\overline{R_M - r_f}\right)$, $\left(\overline{SMB}\right)$ and $\left(\overline{HML}\right)\left(\overline{\mathrm{ModFF}}\right)$ are the averages of the time-series FMP returns; $\widehat{\beta}_{i,M}$, $\widehat{\beta}_{i,SMB}$, and $\widehat{\beta}_{i,HML}\widehat{\beta}_{\mathrm{i,ModFF}}$ are the estimated factor loadings on each FMP obtained from the time-series regression; and $\widehat{\alpha}_{i,C}$ is the pricing error.

Expressions (11) and (12) are considered constrained as they restrict the risk price estimates to the factor means (see Maio & Santa-Clara, 2017). Therefore, rather than estimating an additional cross-sectional regression based on the FMP betas to obtain risk price estimates, we conduct a sensitivity analysis based on expression (12) obtained from the application of each of the eight asset pricing models for each of the 10-decile portfolios for each of the 18 anomalies: therefore, 10 times 18 equals 180 test asset portfolios.

We report four sensitivity statistics for this configuration. The first is the mean absolute error or alpha obtained from $\widehat{\alpha}_{i,C}$ across all 180 test assets, utilizing expression (9). The second is the number of instances when the Jensen alpha is statistically significant; that is, $p \leq 0.05$ across the 120 test assets. The third is the number of instances (out of the application of each model to 180 test assets) in which a Wald test, with a chi-square distribution, examines the validity of the null hypothesis of the expectation of pricing errors; $\widehat{\alpha}_{i,C}$ is equal to zero. Assume that E(F) is the vector of FMP means, T is the number of time-series observations, N is the number of test assets, K is the number of FMPs, and $\widehat{\alpha} = (\widehat{\alpha}_{1,C}, \ldots.., \widehat{\alpha}_{N,C})$ denotes the vector of alphas estimated from the first-stage time-series regression, which represents the pricing error in the second-stage cross-sectional constrained model. A formal statistical test for the null hypothesis that the alphas are jointly equal to zero is the following Wald test:

$$T\left[1 + E(f)'\widehat{\Omega}^{-1}E(f)\right]^{-1}\widehat{\alpha}'\widehat{\Sigma}^{-1}\widehat{\alpha} \sim \chi^2(N). \tag{13}$$

In expression (2), the covariance matrices of the FMP returns $\left(f_t \equiv (f_{1t}, \ldots\ldots.., f_{Kt})'\right)$ and the residuals from the time-series regressions $\left(\widehat{\varepsilon}_t \equiv (\widehat{\varepsilon}_{1t}, \ldots\ldots.., \widehat{\varepsilon}_{Kt})'\right)$ are given by:

$$\widehat{\Omega} = \frac{1}{T}\sum\nolimits_{t=1}^{T}\left[(f_t - E(f))(f_t - E(f))'\right] \tag{14}$$

$$\widehat{\Sigma} = \frac{1}{T}\sum\nolimits_{t=1}^{T}\left[\widehat{\varepsilon}_t\widehat{\varepsilon}_t'\right]. \tag{15}$$

Following Cooper and Maio (2019b), this statistic generalizes the GRS test by relaxing the restrictive assumptions that the errors from the time-series regressions are jointly normally distributed and have a spherical variance (i.e., the errors are homoscedastic and jointly orthogonal). The statistic is valid for finite samples. Although the chi-square statistic represents a formal test of the validity of a given model for explaining a given cross section of average returns, it lacks consistent robustness. This potential weakness is due to the problematic inversion of $\widehat{\Sigma}$ when there the number of test assets is large. Thus, one might reject a model because of a large estimate of $\widehat{\Sigma}^{-1}$ even with low magnitudes of the alphas. This problem might be accentuated by the term involving $\widehat{\Omega}^{-1}$, which also might be poorly estimated with a large number of factors, such as for the eight-factor 8F model (expression (8)). Consequently, for the full estimation with the 18 anomalies, we report the number of anomalies (or portfolio groups) for which the model is not rejected (at the 0.05 level), rather than reporting the p values for the null hypothesis that the alphas for the 180 portfolios are jointly equal to zero. We also report the number of alphas that are individually statistically significant (at the 0.05 level) in each cross-sectional test.

Finally, the fourth sensitivity statistic is the "constrained" cross-sectional R-squared, R_C^2, proposed in Maio and Santa-Clara (2017) and Cooper and Maio (2019b), and is defined as follows:

$$R_C^2 = 1 - \frac{Var_N\left(\widehat{\alpha}_{i,C}\right)}{Var_N\left(\overline{R_i - r_f}\right)}, \tag{16}$$

where the $\widehat{\alpha}_{i,C}$'s are the pricing errors, and $\overline{R_i - r_f}$ is the predicted mean of excess returns of the test asset (as calculated from the estimated beta coefficients) from expression (12), and Var_N is the variance. This R_C^2 metric overcomes shortcomings in the MAA (expression (9)) relating to the inability to relate the magnitudes of the pricing errors to the magnitudes of the raw portfolio risk premiums that we seek to explain in the first place. This is exemplified by the given model, possibly producing an average pricing error that is apparently large but is actually small in comparison with the scale of the raw risk premiums that we are trying to explain. This is especially important in our case, as we have joint asset pricing tests involving many different anomalies, and thus, different magnitudes of risk premiums.

Finally, we follow Cooper and Maio (2019b) and Cooper et al. (2021) in undertaking a statistical bootstrap simulation exercise to assess the statistical significance of the R_C^2 through the computation of empirical p-values. These correspond to the proportions of artificial samples in which the pseudo statistics are higher than the corresponding sample estimates. In the simulation, we impose the condition that the factors are independent from the test portfolio returns (akin to the "useless factors" in Kan & Zhang, 1999) while preserving the correlations among factors in a given asset pricing model (models (1) to (8)). A fully detailed outline of the bootstrap simulation is presented in the online appendix.

2.8. Spanning regression tests

Following Fama and French (2015b, 2018), Hou et al. (2020), and Barillas and Shanken (2017, 2018), spanning regression tests provide a useful means of comparing the efficacy of individual factors within a given model in terms of the joint ability of all the other factors within the model to explain the variation in any given factor. Barillas and Shanken argue that for models with traded factors, the extent to which the combination of all other factors within a model is able to price the focal factor is all that matters for model comparison. Thus, if an individual factor is "spanned" by the other factors, then it is effectively redundant in asset pricing and in explaining the cross section of stock returns in comparison to the other factors within the model. We utilize spanning regression tests as an informative and concise way to compare asset pricing models.

Our starting point is estimating the maximum Sharpe ratios for each of the eight asset pricing models. Following MacKinlay (1995), we define a given model's maximum Sharpe ratio as follows:

$$Sh(F)^2 = \overline{\mu}_F \Pi^{-1} \overline{\mu}_F', \tag{17}$$

where $\overline{\mu}_F$ is the vector of returns-based FMP means, including the mean of the excess returns on the initial market factor, and Π is the variance-covariance matrix of the FMP means. Second, following Fama and French (2018), we focus on the contribution of each FMP to the maximum Sharpe ratio associated with the overall asset pricing model. Fama and French define the incremental increase in the maximum Sharpe ratio arising from FMP_i over and above that attributable to the other FMPs collectively as follows:

$$\frac{\alpha_i^2}{\sigma_i^2} = \textit{Incremental contribution of } FMP_i \textit{ to overall model} \tag{18}$$

This expression implies that an FMP's marginal contribution to a model's maximum squared Sharpe ratio is small if the FMP's expected return is explained well by other FMPs (α_i is close to zero), and its variation not explained by other factors (σ_i) is large, or both.

3. Preliminary analysis

3.1. Value effects

We conduct a horse-race style comparison among valuation ratios through an application of cross-sectional Fama and Macbeth (1973) regressions of individual monthly stock returns on the ratios. In this

Table 2
Fama–MacBeth (two-step procedure) regressions of stock returns on beta, size, and valuation ratios.

	α	β_t	MCAP	B/M	MOM	OP	Asset growth	LIU	IP	R^2
Panel 1: Overall										
(1)	0.0112	0.0033								
	[4.85]**	[0.92]	– –	– –	– –	– –	– –	– –	– –	0.0522
(2)	0.0035	0.0041	0.0001	0.0057						
	[0.38]	[1.06]	[0.26]	[3.62]**	– –	– –	– –	– –	– –	0.0849
(3)	0.0053	0.0035	−0.0001	0.0055	0.0093					
	[0.57]	[0.89]	[−0.01]	[3.56]**	[3.91]**	– –	– –	– –	– –	0.1107
(4)	0.0048	0.0039	−0.0001	0.0059		0.0045	0.0122			
	[0.52]	[0.99]	[−0.14]	[3.40]**	– –	[2.07]*	[4.98]**	– –	– –	0.1030
(5)	0.0079	0.0041	−0.0003	0.0056	0.0085	0.0038	0.0098			
	[0.85]	[1.03]	[−0.52]	[3.30]**	[3.56]**	[1.59] †	[4.16]**	– –	– –	0.1278
(6)	0.0384	0.0018						−0.0020	−0.0022	
	[2.33]*	[0.50]	– –	– –	– –	– –	– –	[−2.15]*	[−1.29] †	0.0870
(7)	0.0249	0.0038	−0.0006	0.0049	0.0071	0.0031	0.0130	−0.0009	−0.0008	
	[1.20]	[0.90]	[−1.24]	[3.02]**	[2.77]**	[1.32] †	[4.84]**	[−0.88]	[−0.44]	0.1613
Panel 1a: Smallest size (D1 & D2 & D3)										
(1a)	0.0150	0.0034								
	[5.54]**	[0.60]	– –	– –	– –	– –	– –	– –	– –	0.0423
(2a)	−0.0763	0.0073	0.0052	0.0052						
	[−2.73]**	[1.09]	[3.15]**	[2.28]*	– –	– –	– –	– –	– –	0.1156
(3a)	−0.0697	0.0059	0.0049	0.0051	0.0016					
	[−2.12]*	[0.84]	[2.45]*	[2.13]*	[0.34]	– –	– –	– –	– –	0.1612
(4a)	−0.0638	0.0104	0.0043	0.0043		0.0001	0.0135			
	[−2.22]*	[1.41] †	[2.55]*	[2.01]*	– –	[0.02]	[1.78]*	– –	– –	0.1694
(5a)	−0.0691	0.0102	0.0047	0.0053	−0.0019	0.0029	0.0180			
	[−1.98]*	[1.31] †	[2.23]*	[2.38]*	[−0.34]	[0.57]	[2.14]*	– –	– –	0.2198
(6a)	0.0328	0.0009						−0.0026	−0.0010	
	[3.52]**	[0.16]	– –	– –	– –	– –	– –	[−1.86]*	[−1.15]	0.0928
(7a)	−0.0368	0.0104	0.0031	0.0045	−0.0029	0.0038	0.0249	−0.0004	−0.001	
	[−0.94]	[1.24]	[1.31] †	[2.01]*	[−0.49]	[0.64]	[2.97]**	[−0.20]	[−1.65]*	0.2952
Panel 1b: Biggest size (D8 & D9 & D10)										
(1b)	0.0039	0.0072								
	[1.24]	[1.66] †	– –	– –	– –	– –	– –	– –	– –	0.1062
(2b)	−0.0477	0.0058	0.0023	0.0081						
	[−2.46]*	[1.32] †	[2.45]*	[2.56]**	– –	– –	– –	– –	– –	0.1640
(3b)	−0.0446	0.0052	0.0020	0.0094	0.0153					
	[−2.29] †	[1.14]	[2.12]*	[3.22]**	[3.33]**	– –	– –	– –	– –	0.2203
(4b)	−0.0432	0.0061	0.0018	0.0102		0.0031	0.0001			
	[−2.25] †	[1.38] †	[2.02]*	[2.95]**	– –	[1.27]	[3.11]**	– –	– –	0.2040
(5b)	−0.0432	0.0059	0.0017	0.0107	0.0148	0.0019	0.0001			
	[−2.24] †	[1.29] †	[1.91]*	[3.39]**	[3.15]**	[0.76]	[2.56]**	– –	– –	0.2567
(6b)	0.0027	0.0072						−0.0002	0.0003	
	[0.14]	[1.62] †	– –	– –	– –	– –	– –	[−0.15]	[0.13]	0.1639
(7b)	−0.0479	0.0081	0.0016	0.0105	0.0148	0.0010	0.0001	0.0008	0.0002	
	[−1.64] †	[1.76]*	[1.80]*	[3.35]**	[3.05]**	[0.42]	[2.70]**	[0.63]	[0.11]	0.3099
Panel 1c: Lowest illiquidity (D1 & D2 & D3)										
(1c)	0.0108	0.0033								
	[2.33]**	[0.63]	– –	– –	– –	– –	– –	– –	– –	0.0758
(2c)	−0.0069	0.0037	0.0003	0.0189						
	[−0.39]	[0.77]	[0.27]	[3.10]**	– –	– –	– –	– –	– –	0.1702
(3c)	−0.0125	0.0041	0.0003	0.0212	0.0001					
	[−0.78]	[0.87]	[0.36]	[3.23]**	[3.76]**	– –	– –	– –	– –	0.2171
(4c)	−0.0086	0.0052	−0.0001	0.0223		0.0099	0.0219			
	[−0.51]	[1.10]	[−0.06]	[3.52]**	– –	[1.98]*	[4.75]**	– –	– –	0.2087
(5c)	−0.0107	0.0056	−0.0001	0.0241	0.0001	0.0101	0.0181			
	[−0.67]	[1.20]	[−0.11]	[3.66]**	[3.36]**	[2.14]*	[4.27]**	– –	– –	0.2512
(6c)	0.0535	0.0050						0.0010	−0.0057	
	[1.48] †	[0.99]	– –	– –	– –	– –	– –	[0.18]	[−1.42] †	0.1535
(7a)	−0.0065	0.0061	−0.0003	0.0223	0.0001	0.0092	0.0159	0.0036	−0.0013	
	[−0.18]	[1.30] †	[−0.31]	[3.84]**	[3.21]**	[1.94]*	[4.03]**	[0.82]	[−0.40]	0.3016
Panel 1d: Highest illiquidity (D8 & D9 & D10)										
(1d)	0.0097	0.0072								
	[4.36]**	[1.85]*	– –	– –	– –	– –	– –	– –	– –	0.0311
(2d)	0.0115	0.0083	−0.0003	0.0029						
	[0.61]	[1.75]*	[−0.26]	[1.19]	– –	– –	– –	– –	– –	0.1305
(3d)	−0.0003	0.0078	0.0003	0.0024	0.0001					
	[−0.02]	[1.62] †	[0.39]	[0.92]	[0.66]	– –	– –	– –	– –	0.1902

(*continued on next page*)

Table 2 (*continued*)

	α	$β_t$	MCAP	B/M	MOM	OP	Asset growth	LIU	IP	R^2
(4d)	0.0188	0.0074	−0.0010	0.0043	– –	0.0079	0.0132	– –	– –	0.2101
	[0.76]	[1.40] †	[−0.71]	[1.39] †		[0.82]	[1.46] †			
(5d)	0.0058	0.0062	−0.0004	0.0055	0.0001	0.0083	0.0159	– –	– –	0.2606
	[0.26]	[1.15]	[−0.31]	[1.56] †	[0.34]	[0.83]	[1.61] †			
(6d)	0.0441	0.0075						−0.0048	−0.0006	
	[2.72]**	[1.94]*	– –	– –	– –	– –	– –	[−3.82]**	[−0.35]	0.0790
(7d)	0.2977	0.0284	−0.0092	0.0062	−0.0001	0.0772	0.0081	−0.0050	−0.0159	
	[0.63]	[0.74]	[−0.65]	[0.62]	[−0.34]	[0.64]	[0.20]	[−0.57]	[−0.58]	0.3288

The table reports the average slope coefficients from month-by-month Fama–MacBeth regressions for the African market universe in the sample period of 2001:01 to 2023:11. Individual stock returns are regressed cross-sectionally on stock characteristics as of the previous month. The columns correspond to different regression specifications, with nonempty rows indicating the included regressors. The regressors include pre-ranking CAPM $β_t$ estimated using the previous 60 months (over 5 years) of monthly returns; the log of the month-end market cap (MCAP); the book-to-market value ratio (B/M); and MOM, namely, momentum, i.e., the time-series average of the percentage cumulative return for each stock over the previous 12 months, omitting the most recent month, and is monthly, following Jegadeesh and Titman (1993). Op is operating profit of the firm scaled by its book equity while asset growth is the one-year growth in total assets of the firm between two years previous and the previous year. Liu is Liu's (2006) liquidity measure estimated over the previous one-year ranking period, and IP is the investor protection metric of Hearn et al. (2017) which is product of firm's free float capitalization proportion and national institutional quality. The last row reports the average R^2 for each specification. The aggregate sample is considered in panel 1 before being disaggregated into a combination of the three lowest extreme portfolios (D1, D2 & D3) and then the highest three extreme portfolios (D8, D9 & D10) based on two respective criteria: the first being size or market capitalization, between panels 2 and 3, the second being illiquidity – as defined by Liu metric between panels 4 and 5. †, *, and ** indicate significance at the 10 %, 5 %, and 1 % levels, respectively.

Table 3
Factor mimicking portfolio descriptive statistics.

	R_m-R_f	SMB	HML	UMD	OP	INV	ILLIQ	IP
Panel 1: Descriptive statistics								
Mean (%)	1.107	0.881	1.025	0.651	0.099	0.724	−0.259	−0.083
t-statistic	2.878**	2.510**	4.222**	2.510**	0.462	3.114**	−0.743	−0.307
Standard deviation (%)	6.367	5.809	4.018	4.293	3.565	3.849	5.762	4.458
Skewness	−0.441	0.358	0.799	0.025	−1.292	−1.983	0.103	0.835
Kurtosis	3.903	3.793	4.733	3.383	10.900	22.369	4.019	7.491
Jarque-Bera statistic [*p* value]	18.18	13.01	63.45	1.70	788.77	4462.52	12.34	262.12
	[0.00]	[0.00]	[0.00]	[0.43]	[0.00]	[0.00]	[0.00]	[0.00]
Number of months	274	274	274	274	274	274	274	274
Panel 2: Pearson correlations								
Market	1.000							
SMB	−0.609**	1.000						
HML	0.206**	0.024**	1.000					
UMD	−0.074	0.155	−0.255**	1.000				
OP	−0.028	−0.044	−0.228**	0.138*	1.000			
INV	0.040	0.006	−0.186**	0.232**	0.446**	1.000		
ILLIQ	−0.490**	0.215**	−0.200**	−0.047	0.143**	−0.092†	1.000	
IP	−0.439**	0.427**	−0.117*	−0.013	−0.006	−0.109*	0.412**	1.000

This table reports the descriptive statistics for returns-based valuation factors with all having been obtained through 3 × 3 double sort procedure in the spirit of Fama and French (2015). Consequently, a size or small minus big (SMB) factor is produced through an initial sorting procedure followed by secondary sorting of the initial three portfolios by book to market value, producing high minus low (HML), momentum derived from up minus down (UMD) and then operating profit scaled by book equity (OP), one year investment as measured by asset growth over preceding financial reporting year (INV), Liu's illiquidity metric (ILLIQ), and Hearn et al. (2017)'s investor protection metric, a product of free float and national institutional quality (IP). The market universe is the aggregate African universe and in addition to all factors is value weighted. All factors bar IP are formed through a 3 × 3 double sorting process where the first sort produces size factor based on "small minus big" portfolio returns followed by a second stage entailing "high minus low" returns difference from the average of returns on each of the 3 high second stage portfolios minus the average on the three respective low second stage portfolios. In the case of IP this second stage is reversed with the factor produced from "low minus high" returns difference from the average of returns on each of the 3 low second stage portfolios minus the average on the three respective high second stage portfolios. Descriptive statistics reported include the monthly average or mean returns in addition to the t-statistic indicating the significance from zero, standard deviation, skewness and kurtosis in distribution, Jarque-Bera statistics for non-normality, and the sample period in number of months. †, *, and ** indicate significance at the 10 %, 5 %, and 1 % levels, respectively.

exercise, we perform five groups of Fama-Macbeth regressions. The first is that of the overall aggregate sample. The next two are the extreme three largest and extreme three smallest of ten decile portfolios sorted by size, or market capitalization each December. The final two are the extreme highest and then the extreme three lowest of ten decile portfolios sorted by the Liu (2006) illiquidity metric. This reflects the profound importance of illiquidity in smaller developing stock markets such as Africa. Consideration of these extreme size portfolios facilitates the unpicking of the consistency of relationships between stock returns and the variables across the extremities of the universe.

Each of the three groups of regressions contains a pre-ranking CAPM beta (β) estimated on the rolling window of the previous five years' monthly returns on its own. We then mimic the configurations of the asset pricing models in expressions (1) to (7) in our selection of additional variables to accompany the initial pre-ranking β. The first is the pre-ranking β on its own. The second is only the pre-ranking β on its own (mimicking the one-factor CAPM), followed by the addition of the firm's market capitalization, *MCAP*, and by the book-to-market value ratio, *B/M* (mimicking the FF3F model). The third augmented these size and book to market value factors with momentum, *MOM* (mimicking the Carhart 4F model). The fourth includes the pre-ranking β along with size, book to market value and then operating profit, OP, and asset growth in mimicking the FF5F model. The fifth additionally augments these preceding factors with momentum, MOM, thereby mimicking

Table 4
Factor mimicking portfolio summary statistics.

	D1 (Low illiquidity)	D2	D3	D4	D5	D6	D7	D8	D9	D10 (High illiquidity)
Panel 1: Stock count (#)										
North Africa										
Morocco	0.61	0.65	0.95	2.12	4.42	5.44	7.57	10.02	11.75	10.37
Tunisia	0.53	1.27	3.78	5.36	6.14	6.31	6.39	4.65	5.56	9.66
Egypt	9.49	7.48	6.14	4.84	3.24	2.52	1.08	1.08	0.70	1.21
West Africa										
BRVM	0.00	0.04	0.18	0.52	0.26	0.43	0.74	0.86	0.78	0.13
Nigeria	0.00	0.04	0.57	2.51	2.46	1.79	2.31	2.31	1.35	1.22
East Africa										
Kenya	0.74	0.61	1.14	2.31	4.62	5.22	5.22	5.40	5.46	3.00
Southern Africa										
Mauritius	0.00	0.00	0.13	0.18	0.52	1.13	0.53	0.74	0.26	0.53
South Africa	19.74	16.57	13.70	8.69	4.87	3.74	2.78	1.56	0.79	0.43
Total	31.10	26.67	26.58	26.53	26.53	26.58	26.61	26.62	26.65	26.55
Panel 2: Summary statistics										
Returns – value weight (%)	1.392†	1.256	1.252	1.416	1.138	1.231	1.049	1.351	0.958	1.404
Returns – equal weight (%)	1.370	1.496	1.371	1.395	1.413	1.125	1.516	1.359	0.880	1.156
FF	74.43**	66.25	46.84	47.91	42.75	37.61	35.12	30.25	24.38	21.55
MCAP	13.792**	14.541	10.020	4.828	4.265	4.625	2.523	3.880	4.146	3.316
NSI	0.0368**	0.0180	0.0512	0.0445	0.0840	0.0374	0.0953	0.0354	0.0549	0.0881
OP	0.2450*	0.2740	0.3993	0.2355	0.3110	0.3707	0.3257	0.3968	0.3208	0.2720
P/CF	24.00	−1.50	3.51	0.33	3.35	10.12	7.66	12.43	10.40	18.25
P/E	31.70**	20.52	23.10	21.41	17.45	45.72	21.24	105.99	30.23	20.08
DY	2.64**	3.19	4.04	3.68	3.46	3.32	3.51	4.12	3.97	3.20
Asset growth	16.901**	16.648	15.348	13.484	16.684	12.835	12.141	11.463	9.232	7.986
CAPX Growth	−0.058**	0.037	−0.091	−0.332	−3.429	−0.139	−0.219	−0.084	−2.666	−0.358
Sales growth	12.116**	13.356	15.168	12.688	12.622	11.909	12.306	8.869	11.254	7.485
B/M	0.4529**	0.4688	0.4301	0.6379	0.5253	0.4546	0.4753	0.4092	0.4042	0.4098
ACC	0.0521*	0.0211	0.0178	0.0098	−0.0334	0.0527	0.0392	−0.0094	0.0189	0.0226
β	1.1205**	1.1362	1.0991	0.8922	0.6264	0.5487	0.4166	0.2925	0.2854	0.2545
VOL	2.1079**	1.9457	1.9540	1.9435	1.6203	1.6308	1.6043	1.5218	1.6321	1.5559
MOM	19.12**	17.27	14.23	15.10	13.41	17.69	11.49	16.80	10.67	13.76
VOLUM	51.490**	67.464	55.548	79.419	31.425	30.897	19.176	8.919	4.448	6.369
P	28.404**	23.035	18.601	15.950	23.887	24.839	31.490	46.363	58.435	38.875
Liu	21.47**	28.09	355.51	57.03	121.34	229.74	232.34	312.07	648.26	8988.81
Amihud	90**	128	3819	3838	11,600	9047	16,720	18,462	22,574	164,223
To	1483**	727	456	305	200	132	98	71	35	13
Zero	6.32**	6.10	7.82	13.19	18.79	23.19	29.35	30.78	39.80	53.53
Panel 3: Block ownership										
Cross-shareholder networks (%)	6.379**	10.762	34.404	26.947	25.008	21.599	25.817	23.624	25.569	24.770
Employee/Family (%)	7.474**	5.627	3.432	4.439	3.871	4.251	6.872	6.454	12.959	13.713
Foreign (%)	4.174**	5.334	20.095	9.027	11.951	15.152	15.321	22.541	20.867	17.785
State (%)	4.856*	6.180	5.451	6.726	4.785	8.447	4.032	8.676	5.774	3.823
Investment companies (%)	3.078**	2.156	1.193	1.359	1.622	2.131	1.017	0.374	0.628	0.931
Other (%)	0.615**	4.588	4.103	3.974	8.308	8.604	7.482	5.367	2.923	2.570
Pension funds (%)	0.003**	0.002	0.149	0.161	0.578	0.381	0.551	0.601	0.409	0.185

This table reports the stock counts, summary descriptive statistics, and categories of block ownership for each of the 10 value-weighted Liu (2006) illiquidity sorted decile portfolios (D1–D10). Panel 1 reports the stock counts per constituent national equity market within the African universe. Panel 2 reports the average equal and value-weighted returns followed by the average proportions of a series of distinct categories of anomalies. These are first, the average proportions of FF (free float, %), MCAP (market capitalization, US$ billions), NSI (change in net stock issues from year 2 to year 1), then OP (operating profit again scaled by book equity), P/CF (stock price to cash flow ratio), P/E (Price to Earnings ratio), DY (dividend yield), Asset growth (1-year asset growth, year 2 – year 1), CAPX growth (growth in capital expenditure from two reporting years prior to current year to the immediately preceding reporting year), Sales growth (5-year gross sales growth, year 5 – year 1), B/M (book-to-market ratio), ACC (two-year change in accruals – (year 2 – year 1) scaled by book equity). The third category of trading anomalies are β (beta), VOL (daily price volatility, %), MOM (momentum change over 12 months, %), VOLUM (daily traded volume, US$ millions) and P (stock price – an indicator of risk, US$). The fourth category include The Liu (2006) multidimensional trading speed metric of illiquidity measurement, Amihud's (2002) price impact from a currency unit's traded volume measure, To which is turnover in relation to daily traded volume scaled by number of shares issued and outstanding, and ZERO (daily zero returns, price rigidity, %). In the first column of Panel 2, a t-difference in the means' statistical significance confidence level is provided for the mean values in decile portfolio D1 in relation to the differences between these values and those for D10. †, *, and ** indicate significance at the 10 %, 5 %, and 1 % levels, respectively. Panel 3 reports average ownership per block owner categories reported by Refinitiv Datastream, namely, cross-shareholder networks, employee/family, foreign, state, investment companies, and other and pension funds.

FF6F model. The sixth includes the pre-ranking β in addition to both the Liu (2006) illiquidity metric and the Hearn et al. (2017) investor protection metric which is loosely tied to proxying the ILLIQ2F and IP2F models while the final seventh model contains all factor variables as a proxy of the 8F grand asset pricing model.

Table 2 reports the average slopes from the month-by-month Fama–MacBeth regressions applied to the overall sample (Panel 1), followed by the small (Panel 2) and large (Panel 3) the lowest illiquidity (Panel 4) and highest illiquidity (Panel 5) subsamples. Notably, the statistical significance of the pre-ranking β is generally negligible and minimal at best across all models and all five panels. The addition of *MCAP* is generally lacking in statistical significance across the main sample and all subsamples – with the sole exception of the biggest size stocks. This implies a possible size-related effect in the cross section of

Table 5
Regression results for 10 illiquidity sorted decile portfolios.

	D1 (Low illiquidity)	D2	D3	D4	D5	D6	D7	D8	D9	D10 (High illiquidity)	ILLIQ
Panel 1: CAPM											
Alpha (%)	0.002	0.001	0.001	0.004†	0.003	0.005†	0.005†	0.009**	0.006*	0.008*	0.006†
	[1.05]	[0.05]	[−0.18]	[1.39]	[1.13]	[1.66]	[1.42]	[2.42]	[1.94]	[2.21]	[1.40]
Beta: Market (excess return)	1.090**	1.124**	1.160**	0.909**	0.714**	0.624**	0.518**	0.406**	0.314**	0.343**	−0.747**
	[30.90]	[24.17]	[34.47]	[16.92]	[8.27]	[8.30]	[7.28]	[7.65]	[6.48]	[5.07]	[−9.35]
R^2	0.8366	0.8211	0.8316	0.6224	0.5309	0.4269	0.2909	0.2451	0.1654	0.1327	0.3559
Panel 2: FF3F											
Alpha (%)	0.003*	0.002	0.002	0.001	−0.002	0.001	−0.001	0.004†	0.001	0.003	−0.001
	[1.93]	[0.62]	[0.84]	[0.13]	[−0.56]	[0.09]	[−0.27]	[1.51]	[0.05]	[0.71]	[−0.22]
Beta: Market (excess return)	1.057**	1.041**	1.094**	0.939**	0.875**	0.752**	0.602**	0.641**	0.496**	0.552**	−0.505**
	[25.01]	[21.84]	[28.63]	[16.08]	[9.21]	[9.98]	[7.40]	[7.70]	[9.46]	[6.91]	[−5.60]
Beta: SMB	−0.072*	−0.134*	−0.123**	0.109†	0.310**	0.265**	0.216**	0.402**	0.352**	0.382**	0.454**
	[−1.83]	[−2.81]	[−3.27]	[1.28]	[4.66]	[4.37]	[2.64]	[5.14]	[7.06]	[5.16]	[5.09]
Beta: HML	−0.056	0.067	−0.013	0.234**	0.074†	0.138*	0.265*	−0.103	0.087	0.009	0.065
	[−1.08]	[0.94]	[−0.21]	[2.95]	[1.37]	[2.21]	[1.86]	[−1.10]	[1.25]	[0.12]	[0.80]
R^2	0.8386	0.8260	0.8354	0.6435	0.5862	0.4780	0.3529	0.3613	0.2836	0.2148	0.4243
Panel 3: Carhart 4F											
Alpha (%)	0.003*	0.001	0.002	0.001	−0.002	0.001	−0.001	0.004†	−0.001	0.003	−0.001
	[1.74]	[0.47]	[0.90]	[0.26]	[−0.59]	[0.04]	[−0.27]	[1.39]	[−0.10]	[0.80]	[−0.08]
Beta: Market (excess return)	1.054**	1.037**	1.096**	0.944**	0.873**	0.750**	0.602**	0.642**	0.492**	0.556**	−0.498**
	[24.58]	[21.47]	[28.93]	[16.42]	[9.39]	[10.31]	[7.54]	[7.41]	[9.49]	[6.73]	[−5.34]
Beta: SMB	−0.077*	−0.143**	−0.119**	0.120†	0.306**	0.261**	0.215**	0.403**	0.342**	0.391**	0.469**
	[−2.00]	[−3.03]	[−3.13]	[1.50]	[4.91]	[4.47]	[2.66]	[4.73]	[6.92]	[5.21]	[5.19]
Beta: HML	−0.047	0.081	−0.020	0.215**	0.082†	0.144**	0.267*	−0.105	0.103†	−0.007	0.040
	[−0.81]	[1.05]	[−0.34]	[2.54]	[1.37]	[2.32]	[1.91]	[−1.02]	[1.54]	[−0.08]	[0.44]
Beta: UMD	0.030	0.045	−0.024	−0.060	0.025	0.020	0.005	−0.008	0.053	−0.052	−0.082
	[0.48]	[0.78]	[−0.46]	[−0.77]	[0.44]	[0.31]	[0.06]	[−0.10]	[1.01]	[−0.58]	[−0.79]
R^2	0.8382	0.8259	0.8349	0.6433	0.5849	0.4762	0.3505	0.3589	0.2830	0.2132	0.4240
Panel 4: FF5F											
Alpha (%)	0.004**	0.001	0.002	0.001	−0.003	−0.001	−0.003	0.004†	−0.001	0.003	−0.002
	[2.44]	[0.58]	[0.83]	[0.08]	[−0.82]	[−0.07]	[−1.25]	[1.32]	[−0.18]	[0.74]	[−0.38]
Beta: Market (excess return)	1.061**	1.037**	1.098**	0.945**	0.874**	0.754**	0.587**	0.640**	0.497**	0.557**	−0.505**
	[25.08]	[21.53]	[28.03]	[16.93]	[9.43]	[10.45]	[8.09]	[7.49]	[9.33]	[6.89]	[−5.59]
Beta: SMB	−0.071*	−0.140**	−0.117**	0.118†	0.314**	0.270**	0.212**	0.403**	0.357**	0.387**	0.458**
	[−1.98]	[−2.98]	[−3.13]	[1.49]	[4.93]	[4.46]	[2.76]	[5.19]	[7.25]	[5.19]	[5.24]
Beta: HML	−0.088†	0.060	0.001	0.257**	0.117*	0.168**	0.365**	−0.084	0.122*	0.013	0.101
	[−1.60]	[0.81]	[0.01]	[3.15]	[2.01]	[2.59]	[2.44]	[−0.84]	[1.72]	[0.16]	[1.13]
Beta: OP	−0.104	−0.084	0.115*	0.177*	0.193*	0.164*	0.303**	0.081	0.176*	0.074	0.178†
	[−1.25]	[−1.10]	[1.78]	[2.27]	[1.98]	[2.13]	[2.72]	[1.11]	[1.85]	[0.67]	[1.28]
Beta: INV	−0.053	0.049	−0.049	−0.056	0.019	−0.011	0.185*	0.011	0.002	−0.051	0.002
	[−0.63]	[0.70]	[−0.72]	[−0.88]	[0.30]	[−0.17]	[2.00]	[0.13]	[0.04]	[−0.55]	[0.02]
R^2	0.8415	0.8259	0.8362	0.6467	0.5961	0.4825	0.4089	0.3599	0.2946	0.2107	0.4263
Panel 5: FF6F											
Alpha (%)	0.004*	0.001	0.002	0.001	−0.003	−0.001	−0.003	0.004	−0.001	0.003	−0.001
	[2.24]	[0.47]	[0.89]	[0.19]	[−0.80]	[−0.08]	[−1.10]	[1.26]	[−0.27]	[0.81]	[−0.24]
Beta: Market (excess return)	1.058**	1.033**	1.100**	0.950**	0.874**	0.753**	0.591**	0.642**	0.494**	0.560**	−0.498**
	[24.58]	[21.31]	[28.35]	[17.28]	[9.60]	[10.71]	[8.07]	[7.26]	[9.34]	[6.73]	[−5.34]
Beta: SMB	−0.080**	−0.148**	−0.112**	0.129*	0.313**	0.269**	0.221**	0.406**	0.350**	0.396**	0.475**
	[−2.38]	[−3.13]	[−3.06]	[1.73]	[5.18]	[4.52]	[2.80]	[4.83]	[7.12]	[5.26]	[5.45]
Beta: HML	−0.075†	0.072	−0.007	0.240**	0.118*	0.170**	0.351*	−0.089	0.133*	0.001	0.075
	[−1.31]	[0.90]	[−0.11]	[2.82]	[1.92]	[2.65]	[2.45]	[−0.82]	[1.91]	[0.01]	[0.79]
Beta: OP	−0.105†	−0.085	0.116	0.178*	0.193*	0.163*	0.304**	0.081	0.175*	0.075	0.180†
	[−1.28]	[−1.10]	[1.80]	[2.30]	[1.99]	[2.12]	[2.73]	[1.12]	[1.85]	[0.68]	[1.31]
Beta: INV	−0.063	0.041	−0.044	−0.043	0.017	−0.013	0.195†	0.014	−0.006	−0.041	0.022
	[−0.72]	[0.56]	[−0.62]	[−0.71]	[0.27]	[−0.20]	[2.12]	[0.16]	[−0.09]	[−0.43]	[0.17]
Beta: UMD	0.049	0.045	−0.026	−0.067	0.006	0.009	−0.052	−0.017	0.040	−0.051	−0.100
	[0.76]	[0.76]	[−0.47]	[−0.94]	[0.12]	[0.14]	[−0.58]	[−0.22]	[0.74]	[−0.57]	[−0.96]
R^2	0.8416	0.8258	0.8358	0.6468	0.5946	0.4806	0.4079	0.3577	0.2930	0.2089	0.4267
Panel 6: IILIQ2F											
Alpha (%)	0.003**	0.001	−0.001	0.003	0.002	0.004†	0.004	0.008*	0.004†	0.003**	– –
	[2.35]	[0.30]	[−0.12]	[1.19]	[0.69]	[1.38]	[1.13]	[2.19]	[1.54]	[2.35]	
Beta: Market (excess return)	0.909**	1.054**	1.149**	0.980**	0.889**	0.762**	0.643**	0.530**	0.525**	0.909**	– –
	[19.94]	[20.56]	[30.15]	[18.36]	[8.44]	[9.14]	[8.74]	[9.36]	[8.64]	[19.94]	
Beta: ILLIQ	−0.242**	−0.094**	−0.015	0.094**	0.235**	0.185**	0.168**	0.166**	0.283**	0.758**	– –
	[−7.09]	[−2.51]	[−0.48]	[2.55]	[3.72]	[3.50]	[3.60]	[4.72]	[5.31]	[22.21]	
R^2	0.8775	0.8262	0.8311	0.6278	0.5874	0.4629	0.3195	0.2838	0.3005	0.7990	– –

(*continued on next page*)

Table 5 (*continued*)

	D1 (Low illiquidity)	D2	D3	D4	D5	D6	D7	D8	D9	D10 (High illiquidity)	ILLIQ
Panel 7: IP2F											
Alpha (%)	0.003†	0.001	−0.001	0.004†	0.002	0.005†	0.004	0.008**	0.004*	0.007*	0.004
	[1.61]	[0.27]	[−0.10]	[1.29]	[0.83]	[1.51]	[1.18]	[2.33]	[1.87]	[2.04]	[1.11]
Beta: Market (excess	0.968**	1.030**	1.135**	0.959**	0.946**	0.777**	0.704**	0.628**	0.606**	0.647**	−0.321**
return)	[23.67]	[19.27]	[28.29]	[15.57]	[9.76]	[9.88]	[9.84]	[9.95]	[10.29]	[6.26]	[−3.46]
Beta: IP	−0.162**	−0.126**	−0.033	0.067	0.309**	0.204**	0.248**	0.296**	0.389**	0.405**	0.568**
	[−5.22]	[−2.74]	[−1.08]	[0.91]	[5.13]	[4.26]	[4.65]	[6.33]	[6.43]	[4.79]	[7.68]
R^2	0.8542	0.8306	0.8316	0.6243	0.6277	0.4699	0.3546	0.3716	0.4179	0.3166	0.5572
Panel 8: 8F											
Alpha (%)	0.004**	0.002	0.002	0.001	−0.003	−0.001	−0.004	0.002	−0.002	0.001	– –
	[2.42]	[0.80]	[0.97]	[0.10]	[−0.84]	[−0.22]	[−1.27]	[0.86]	[−0.67]	[0.14]	
Beta: Market (excess	1.034**	0.969**	1.092**	0.974**	0.887**	0.784**	0.632**	0.733**	0.577**	0.740**	– –
return)	[21.00]	[17.68]	[23.13]	[15.85]	[9.45]	[9.82]	[7.40]	[8.02]	[8.70]	[8.91]	
Beta: SMB	−0.054†	−0.154**	−0.116**	0.113†	0.287**	0.266**	0.209**	0.382**	0.299**	0.353**	– –
	[−1.53]	[−3.12]	[−2.80]	[1.43]	[5.11]	[4.05]	[2.62]	[4.02]	[6.18]	[4.96]	
Beta: HML	−0.090†	0.050	−0.009	0.253**	0.130	0.183**	0.369**	−0.049	0.177	0.077	– –
	[−1.52]	[0.66]	[−0.15]	[2.97]	[2.08]	[2.91]	[2.64]	[−0.44]	[2.56]	[0.90]	
Beta: OP	−0.096	−0.045	0.122*	0.167	0.190*	0.145*	0.281	0.030	0.134†	−0.027	– –
	[−1.22]	[−0.55]	[1.85]	[2.13]	[1.98]	[1.75]	[2.52]	[0.42]	[1.46]	[−0.27]	
Beta: INV	−0.081	0.008	−0.047	−0.027	0.030	0.004	0.220**	0.068	0.050	0.064	– –
	[−1.08]	[0.10]	[−0.67]	[−0.41]	[0.46]	[0.07]	[2.48]	[0.73]	[0.69]	[0.75]	
Beta: UMD	0.038	0.027	−0.027	−0.058	0.014	0.019	−0.039	0.014	0.073	0.008	– –
	[0.57]	[0.43]	[−0.49]	[−0.77]	[0.26]	[0.28]	[−0.42]	[0.18]	[1.26]	[0.09]	
Beta: ILLIQ	−0.012	−0.135**	−0.023	0.026	−0.010	0.058	0.066	0.149**	0.097†	0.300**	– –
	[−0.21]	[−2.67]	[−0.57]	[0.51]	[−0.26]	[0.68]	[0.91]	[3.21]	[1.50]	[4.69]	
Beta: IP	−0.111*	−0.013	0.010	0.074	0.108†	0.028	0.066	0.143*	0.240**	0.266*	– –
	[−2.27]	[−0.19]	[0.17]	[0.88]	[1.36]	[0.30]	[0.77]	[2.24]	[3.26]	[2.44]	
R^2	0.8485	0.8368	0.8395	0.6567	0.6078	0.4950	0.4265	0.4098	0.3635	0.3380	– –

This table reports the results for the time-series regression beta coefficients for valuation factors with t-statistics and explanatory power (R^2) for the eight models. The dependent variables are each of the value-weighted ten-decile Liu (2006) illiquidity metric sorted portfolio returns in addition to a final difference portfolio formed from the difference of D1 and D10 Liu-metric portfolio returns. D1 is the lowest illiquidity (most liquid), and D10 the highest illiquidity (least liquid). Panel 1 reports the capital asset pricing model (CAPM) comprising only the market factor denominated in excess returns. Panel 2 reports the Fama and French (1993) three factor model, FF3F, comprising the CAPM plus SMB (small minus big) and HML (high minus low) factors, related to the size and the book-to-market value. Panel 3 reports the Carhart (1997) four factor model, Carhart 4F, comprising the FF3F factors plus an additional momentum factor, UMD (up minus down) factor. Panel 4 reports the Fama French (2015) five factor model, FF5F, which includes a market excess returns factor plus SMB, HML and then additional OP, operating profit scaled by book equity, and INV, investment derived from one year asset growth, factors. Panel 5 reports the Fama and French (2016) six factor model, FF6F, which is the previous five factors of FF5F in addition to momentum, UMD. Panel 6 outlines Liu's (2006) two-factor model, LIQ2F, comprising the CAPM plus by an illiquidity factor (ILLIQ) formed from the Liu (2006) liquidity metric. In Panel 7, in a similar manner, the investor protection (IP) is augmented on the market factor to result in the IP2F model. Finally, panel 8 reports a grand asset pricing model comprising CAPM, SMB, HML, OP, INV, UMD, ILLIQ, and IP. The market universe is the aggregate African sample with all factors being value weighted. All factors are formed through a 3 × 3 double sort procedure comprising an initial sort based on size followed by a subsequent sort based on the factor. However, only in the two factor models of ILLIQ2F and IP2F do we utilize an alternative factor construction from the ILLIQ and IP deciles formed from the returns difference between highest (lowest) and lowest (highest) decile portfolios are a single pass sort into ten decile portfolios based on the underlying factor. Rebalancing in all cases is annual in December of each year. The one-month US deposit yield is used as the risk-free rate. Numbers in square brackets are t-statistics. †, *, and ** indicate significance at the 10 %, 5 %, and 1 % levels, respectively. Standard errors are Huber-White heteroscedasticity-robust.

stock returns, with this especially visible in the extremes of size-sorted stocks. Fama and French (1993) argue smaller stocks in particular are more prone to prolonged earnings downturns in periods of economic recession. Next, the addition of *B/M* leads to a positive coefficient across all models between main and subsamples. However, while these are generally statistically significant this is wholly lacking in both highest (Panel 5) and lowest illiquidity (Panel 4) subsamples. The addition of *MOM* results in a positive and significant coefficient, alluding to a momentum effect in the valuation of the stock returns. However, the absolute size of the *MOM* coefficient is extremely small and smaller than any other variable coefficient, thus bringing into question its economic significance. This evidence is consistent with Fama & French (1993) previous findings for the US and more recently, Liu, Stambaugh, and Yuan's (2019) findings. Moreover, *MOM* notably lacks statistical significance in the smallest size (Panel 2) and highest illiquidity (Panel 5) stocks.

Next, operating profitability, *OP*, and *asset growth* have positive coefficients across the main sample and all subsamples. However, only in the latter are coefficients consistently statistically significant across all subsamples while the former only attains statistical significance in the lowest illiquidity stocks (Panel 4) which is accompanied by at best very marginal statistical significance in the main sample (Panel 1). This evidence points to the potential redundancy of operating profit as a priced factor while contrastingly the potential importance of asset growth or investment as a priced factor. Finally, coefficients on both illiquidity, *LIU*, and investor protection, *IP*, are consistently negative across all samples and subsamples yet almost wholly lack statistical significance which undermines confidence in their potentially being priced.

3.2. Descriptive statistics of factor mimicking portfolios

The means, standard deviations, autocorrelations, and cross-correlations of the monthly returns of the FMPs are reported in Table 3. Several observations are apparent. The first in Panel 1 is that the largest average monthly returns are also those with the highest statistical significance. This is especially evident in relation to OP (0.099, t-statistic: 0.462), ILLIQ (−0.259, t-statistic: −0.743) and IP (−0.083, t-statistic: −0.307) – all of whom are small average returns and negligible statistical significance. This evidence questions the potential viability of premiums attached to operating profit, illiquidity and investor protection, in terms of ownership concentration and national institutional quality. However, the market excess returns premium, SMB, HML, UMD and INV are all statistically significant (t-statistic: 1.143; $p \leq 0.01$) indicating their being potential statistically significant premiums in rationalizing the cross section of stock returns. The analysis in Panel 2 of the correlations reveals minimal correlations among all factors with the sole exception of a high correlation between *SMB* and *market excess returns* (−0.609**, $p \leq 0.01$).

3.3. Descriptive statistics of free float portfolios

Descriptive statistics for each of the 10 value-weighted *ILLIQ (*Liu, 2006*)* sorted portfolios are reported in Table 4. A number of observations are apparent in panel 1. The first is the smallness of the sample with each of the ten decile sorted portfolios having an average of 26 to 31 constituent stocks. The second is a visible progressive trend in increasing proportions of the constituent stocks of increasingly high illiquid portfolios (so from D5 towards D10) drawn from the equity markets of Morocco, Egypt, Tunisia and Kenya. The third relates to an opposite trend in terms of an increasing concentration of South African stocks in progressively less illiquid portfolios (D5 to D1). However, this dichotomy in opposing trends between North and South Africa is also revealing of a potential mirroring polarization within any regional benchmark universe with factor mimicking portfolios lacking effective inclusivity of stocks from jurisdictions outside North and South Africa.

A comparison of the trading and descriptive statistics for the stocks sorted into the 10 *ILLIQ* value-weighted decile portfolios is provided in Panel 2 of Table 4. A number of observations are apparent. There is evidence of a statistically significant difference in the average value-weighted monthly returns ($p \leq 0.10$) between D1 and D10. This is also accompanied by a progressive rise in the average monthly returns from D10 to D1. However, the statistical significance as with the absolute size of difference is minimal. Similar trends are very visible in relation to almost all of the other anomaly variables. Notably, FF ($p \leq 0.01$), MCAP ($p \leq 0.01$), P/E ($p \leq 0.01$), Sales growth ($p \leq 0.01$), B/M ($p \leq 0.01$), ACC ($p \leq 0.01$) are all progressively increasing as illiquidity correspondingly decreases (so D10 towards D1). Notably P/CF lacks statistical significance. The opposite directional trend is visible with NSI ($p \leq 0.01$), OP ($p \leq 0.01$), DY ($p \leq 0.01$), CAPX Growth ($p \leq 0.01$) with these increasing towards higher illiquidity. Similarly, pre-ranking β ($p \leq 0.01$), VOL ($p \leq 0.01$), MOM ($p \leq 0.01$) and VOLUM ($p \leq 0.01$) all progressively increase in line with decreasing illiquidity while the opposite is true for stock price, P ($p \leq 0.01$). Finally, all illiquidity measures, namely Liu ($p \leq 0.01$), Amihud ($p \leq 0.01$), To ($p \leq 0.01$) and Zero ($p \leq 0.01$) are all markedly lower in absolute value in lower illiquidity portfolios than in the progressively higher illiquidity counterparts. Together, these findings are indicative of the wide-ranging influence of illiquidity over firm level attributes.

Finally, in Panel 3 of Table 4, we consider the dispersion of different types of block ownership over the 10 illiquidity decile portfolios. All seven categories of block ownership ($p \leq 0.01$) are highest in the lowest *ILLIQ* decile (D1) and decrease progressively to negligible values in the highest *ILLIQ* decile (D10) with the sole exception of state ownership which has the opposite trend. Notably, stocks in decile D1 are dominated by high levels of block ownership of cross-shareholder networks (6.379 %) and employee/family insiders (7.474 %). These levels of ownership dramatically increase in tandem with illiquidity to 24.770 % and 13.713 % respectively in D10.

4. Main results

The results from the application of the time-series regressions on the 11 test asset portfolios are reported in Table 5. These comprise 10 *ILLIQ* sorted decile portfolios in addition to an *ILLIQ* FMP formed from the returns difference between extreme decile-sorted portfolios. A number of observations are apparent.

The first observation is related to the differentiation between the seven asset pricing models based on their explanatory power or adjusted-R^2 when applied to the average returns of the 10 *ILLIQ* sorted portfolios. There is a visible jump in the adjusted-R^2 of between 1 % and 2 % across all 10 *ILLIQ* decile portfolios between the CAPM (Panel 1) and the FF3F model (Panel 2) although this jumps to nearer 12 % in the higher illiquidity portfolios of D8 to D10. This is followed by a negligible change in the adjusted-R^2 upon progression from the FF3F model to the Carhart 4F model (Panel 3). However, this is followed by another small jump between the Carhart 4F model (Panel 3) and both FF5F and FF6F (between Panels 4 and 5 respectively) of between 1 and 2 %, although there is negligible change between FF5F and FF6F. There is a substantial increase in the adjusted-R^2 to the ILLIQ2F model (Panel 6) compared to all preceding models (and panels) which is especially evident in the extreme decile portfolios of D1 and D10 although this is subject to some collinearity influence given the ILLIQ FMP is formed from the difference between these two extreme decile portfolios. This is followed by a reduction in the adjusted-R^2 in progression to the IP2F (Panel 7) and only a minimal increase in the adjusted-R^2 in subsequent progression towards the 8F model (Panel 8). Together, these findings support the relative strength of the Carhart 4F, FF5F and FF6F models over rival models. The second observation relates to the absolute size and statistical significance of the regression alpha associated with the seven asset pricing models in their application to the average returns of the 10 *ILLIQ* sorted decile portfolios. Notably, the size and significance are lowest for the Carhart 4F (Panel 3), FF5F model (Panel 4) as well as the FF6F (Panel 5) models compared to all other models. This reflects stronger additional support for the superiority of the Carhart 4F, FF5F and FF6F models over all alternative models.

The third observation is a distinct trend in the direction (sign) of both the *ILLIQ* beta in the ILLIQ2F model (Panel 6) and *IP* beta in the IP2F model (Panel 7). Notably, these betas are large, statistically significant, and positive in portfolios D1 and D2 and then progressively transition into equally large, statistically significant yet negative betas in D9 and D10. We argue that this is evidence regarding the minority investor welfare implications arising from both dispersed ownership and illiquidity.

Our final observation is about the value-weighted difference portfolio, or FMPs, in the last column of Table 4. They correspond to the returns generated from a strategy of a long position in high ILLIQ stocks and shorting those with low ILLIQ stocks. The estimation results reveal that the CAPM has a statistically significant alpha implying a lack of explanatory power of ILLIQ and reveal statistically significant regression alphas, i.e., *abnormal returns* that cannot be attributed to any of the FMPs. The opposite is true for Carhart 4F, FF5F, FF6F models (panels 3 to 5) with all yielding equally good explanatory power. This reveals further statistical support for the Carhart 4F, FF5F, FF6F asset pricing models. Moreover, similar evidence is visible from the analysis conducted on equal-weighted portfolios,[8] which supports our findings.

Table 6 reports the GRS test statistics and mean absolute alphas for the application of each of the eight asset pricing models to 10 test portfolios formed from the underlying stocks sorted according to one of the 18 anomalies. Our first observation is that of an overwhelming rejection of the null hypothesis ($p \leq 0.01$) that all regression intercepts are jointly equal to zero across all seven models and all 18 anomalies, that is, all 180 test portfolios. This largely uniform rejection of the GRS test statistic is in line with previous literature, with Fama and French (1993) for a universe of US stocks and with Hearn (2011) for a multi-country sample comprised of 49 major equity markets worldwide. The second observation is the relatively strong statistical support for the Carhart 4F model given that it has the lowest GRS statistic vis-à-vis the other seven rival asset pricing models across 7 of the 18 anomalies. Notably, the evidence is more mixed for equal-weighted portfolios.[9] Together, this evidence supports the superiority of the Carhart 4F model

[8] The results are available in Table 9 in the online appendix.

[9] The results are available in Table 12 in the online appendix.

Table 6
GRS statistical differentiation between models applied to anomalies in cross section of stock returns.

	FF				**β**			
	R^2	SE (model)	$\lvert a \rvert$	GRS	R^2	SE (model)	$\lvert a \rvert$	GRS
CAPM	0.6337	0.00240	0.00235	14.40	0.4198	0.002761	0.003556	14.95
FF3F	0.6514	0.00251	0.00195	9.87	0.4859	0.002795	0.001521	5.60
Carhart 4F	0.6529	0.00254	0.00189	**9.05**	0.4888	0.002828	0.001752	7.50
FF5F	0.6583	0.00254	0.00204	10.87	0.4967	0.002828	0.001634	**4.95**
FF6F	0.6599	0.00255	0.00196	10.22	0.4996	0.002847	0.001858	6.66
ILLIQ2F	0.6542	0.00237	0.00206	12.78	0.4699	0.002678	0.002615	12.93
IP2F	0.6940	0.00225	0.00210	13.91	0.4851	0.002628	0.002781	13.75
8F	0.6764	0.00252	0.00202	10.72	0.5225	0.00282	0.00186	6.19

	MCAP				B/M			
	R^2	SE (model)	$\lvert a \rvert$	GRS	R^2	SE (model)	$\lvert a \rvert$	GRS
CAPM	0.4686	0.00267	0.00892	48.06**	0.6188	0.002771	0.004410	27.70**
FF3F	0.6271	0.00241	0.00246	17.04	0.6543	0.002818	0.002375	9.72
Carhart 4F	0.6291	0.00244	0.00243	**15.08**	0.6594	0.002840	0.002233	**9.10**
FF5F	0.6402	0.00242	0.00277	20.87**	0.6588	0.002862	0.002176	10.48
FF6F	0.6414	0.00244	0.00269	19.67**	0.6646	0.002866	0.002107	9.65
ILLIQ2F	0.5013	0.00261	0.00794	45.07**	0.6289	0.002757	0.004155	25.18**
IP2F	0.4987	0.00261	0.00833	46.93**	0.6338	0.002732	0.004213	26.46**
8F	0.6553	0.00242	0.00225	17.03	0.6695	0.00288	0.00207	9.75

	Vol				Asset growth			
	R^2	SE (model)	$\lvert a \rvert$	GRS	R^2	SE (model)	$\lvert a \rvert$	GRS
CAPM	0.6150	0.00271	0.00333	19.14**	0.6633	0.002429	0.002304	14.84
FF3F	0.6262	0.00286	0.00271	11.39	0.6717	0.002555	0.002357	15.72
Carhart 4F	0.6337	0.00287	0.00206	**8.81**	0.6761	0.002578	0.002397	16.73
FF5F	0.6300	0.00291	0.00281	12.56	0.6752	0.002599	0.002577	19.73**
FF6F	0.6373	0.00291	0.00240	10.75	0.6790	0.002610	0.002576	20.22**
ILLIQ2F	0.6212	0.00271	0.00314	17.13	0.6648	0.002439	0.002281	**14.87**
IP2F	0.6289	0.00269	0.00313	17.93	0.6697	0.002418	0.002324	14.93
8F	0.6419	0.00292	0.00239	10.88	0.6822	0.00263	0.00276	24.00**

	Acc				Op			
	R^2	SE (model)	$\lvert a \rvert$	GRS	R^2	SE (model)	$\lvert a \rvert$	GRS
CAPM	0.5870	0.00290	0.00229	11.27	0.6587	0.002522	0.002406	12.47
FF3F	0.5938	0.00306	0.00228	**9.20**	0.6668	0.002657	0.001906	8.98
Carhart 4F	0.5963	0.00310	0.00230	9.26	0.6698	0.002685	0.001736	**8.34**
FF5F	0.5990	0.00311	0.00239	10.90	0.6857	0.002639	0.001945	9.14
FF6F	0.6015	0.00313	0.00249	10.68	0.6883	0.002654	0.001892	8.92
ILLIQ2F	0.5893	0.00291	0.00241	12.79	0.6613	0.002529	0.002426	11.91
IP2F	0.5944	0.00288	0.00236	11.48	0.6663	0.002502	0.002385	11.78
8F	0.6072	0.00315	0.00264	12.66	0.6958	0.00265	0.00198	8.79

	DY				P/CF			
	R^2	SE (model)	$\lvert a \rvert$	GRS	R^2	SE (model)	$\lvert a \rvert$	GRS
CAPM	0.6332	0.00267	0.00314	14.68	0.6174	0.00275	0.00229	8.61
FF3F	0.6414	0.00282	0.00224	6.93	0.6302	0.00288	0.00163	4.74
Carhart 4F	0.6430	0.00285	0.00199	5.50	0.6359	0.00290	0.00184	5.41
FF5F	0.6541	0.00283	0.00228	6.48	0.6399	0.00291	0.00153	**4.45**
FF6F	0.6556	0.00285	0.00207	5.47	0.6458	0.00291	0.00180	4.99
ILLIQ2F	0.6365	0.00268	0.00327	15.32	0.6250	0.00274	0.00215	7.81
IP2F	0.6446	0.00264	0.00308	14.36	0.6233	0.00273	0.00228	8.26
8F	0.6631	0.00285	0.00185	**4.16**	0.6517	0.00292	0.00153	4.53

	NSI				P/E			
	R^2	SE (model)	$\lvert a \rvert$	GRS	R^2	SE (model)	$\lvert a \rvert$	GRS
CAPM	0.3976	0.00328	0.00324	18.86**	0.6193	0.00269	0.00202	13.55
FF3F	0.4279	0.00341	0.00290	9.84	0.6335	0.00281	0.00151	10.33
Carhart 4F	0.4344	0.00344	0.00280	**7.68**	0.6341	0.00285	0.00150	9.83
FF5F	0.4352	0.00346	0.00342	11.12	0.6504	0.00281	0.00162	9.22

(*continued on next page*)

Table 6 (*continued*)

	NSI				P/E			
	R^2	SE (model)	|a|	GRS	R^2	SE (model)	|a|	GRS
FF6F	0.4433	0.00347	0.00285	8.70	0.6512	0.00284	0.00149	**9.02**
ILLIQ2F	0.4144	0.00326	0.00301	17.59	0.6263	0.00268	0.00196	12.28
IP2F	0.4287	0.00322	0.00298	17.64	0.6255	0.00268	0.00192	13.25
8F	0.4553	0.00347	0.00298	8.89	0.6564	0.00285	0.00160	9.07

	CAPX Growth				Sales Growth			
	R^2	SE (model)	|a|	GRS	R^2	SE (model)	|a|	GRS
CAPM	0.6199	0.00253	0.00103	**2.42**	0.6541	0.002506	0.001680	5.63
FF3F	0.6321	0.00265	0.00168	6.83	0.6597	0.002652	0.002442	8.87
Carhart 4F	0.6327	0.00269	0.00167	7.96	0.6637	0.002675	0.002111	6.60
FF5F	0.6404	0.00268	0.00192	9.44	0.6678	0.002680	0.002459	9.17
FF6F	0.6410	0.00270	0.00189	9.84	0.6713	0.002692	0.002188	7.70
ILLIQ2F	0.6262	0.00253	0.00113	2.44	0.6556	0.002518	0.001689	5.52
IP2F	0.6323	0.00250	0.00123	2.77	0.6578	0.002502	0.001578	**5.09**
8F	0.6462	0.00272	0.00167	8.74	0.6747	0.00271	0.00214	7.44

	LIU				AMIHUD			
	R^2	SE (model)	|a|	GRS	R^2	SE (model)	|a|	GRS
CAPM	0.4931	0.00262	0.00344	12.69	0.4608	0.00280	0.00611	32.27**
FF3F	0.5470	0.00266	0.00148	**4.10**	0.5234	0.00281	0.00205	12.91
Carhart 4F	0.5475	0.00270	0.00155	4.33	0.5278	0.00284	0.00207	12.51
FF5F	0.5584	0.00268	0.00197	6.54	0.5296	0.00286	0.00189	12.65
FF6F	0.5591	0.00271	0.00198	6.32	0.5345	0.00287	0.00183	12.40
ILLIQ2F	0.5956	0.00236	0.00231	7.96	0.4847	0.00276	0.00532	29.38**
IP2F	0.5738	0.00244	0.00263	12.08	0.4784	0.00276	0.00570	31.28**
8F	0.5819	0.00268	0.00213	6.06	0.5518	0.00285	0.00158	**10.41**

	TO				ZERO			
	R^2	SE (model)	|a|	GRS	R^2	SE (model)	|a|	GRS
CAPM	0.5395	0.00271	0.00229	10.39	0.4755	0.00269	0.00346	9.37
FF3F	0.5725	0.00280	0.00206	11.07	0.5211	0.00275	0.00100	4.50
Carhart 4F	0.5763	0.00283	0.00163	8.78	0.5227	0.00278	0.00099	**4.10**
FF5F	0.5832	0.00283	0.00235	12.11	0.5347	0.00277	0.00146	6.21
FF6F	0.5873	0.00284	0.00197	10.22	0.5365	0.00279	0.00144	5.63
ILLIQ2F	0.5996	0.00258	0.00173	**7.76**	0.5336	0.00257	0.00235	6.71
IP2F	0.5971	0.00259	0.00186	9.25	0.5296	0.00257	0.00272	8.39
8F	0.6089	0.00281	0.00242	9.53	0.5526	0.00277	0.00196	6.42

This table presents the Gibbons, Ros, and Shanken (GRS; 1989) statistics for each of the eight asset pricing models as applied to 10-decile portfolios formed from sorting of stocks by 18 anomaly variables. These are first, the average proportions of FF (free float), MCAP (market capitalization), NSI (net stock issues), then second OP (operating profit), P/CF (price to cash flow ratio), P/E (price to earnings ratio), DY (dividend yield), Asset growth (one year asset growth), CAPX growth (growth in capital expenditure), Sales growth, B/M (book-to-market ratio), ACC (accruals scaled by book equity). The third category of trading anomalies are β (beta), VOL (volatility), MOM (momentum change over 12 months), VOLUM (volume) and P (stock price). The fourth category include The Liu (2006) multidimensional trading speed metric of illiquidity measurement, Amihud's (2002) price impact from a currency unit's traded volume measure, To which is turnover in relation to daily traded volume scaled by number of shares issued and outstanding, and ZERO (daily zero returns, price rigidity, %). The eight asset pricing models are the CAPM, FF3F (including the additional SMB and HML), Carhart 4F (including the additional momentum factor, UMD), FF5F, based on FF3F plus additional OP ad INV, FF6F, based on FF5F plus additional UMD. Then the two-factor liquidity augmented CAPM by the Liu (2006) liquidity hedging portfolio, namely, ILLIQ2F, and the similar two-factor augmented CAPM with investor protection, namely, IP2F. The final model is a grand asset pricing scheme including all factors, namely CAPM, SMB, HML, OP, INV, UMD, ILLIQ, and IP. The GRS statistic tests whether all intercepts in a set of test portfolios (assets) regressions are zero with the associated p value in square brackets, |a| is the average absolute intercept for a set of regressions, R^2 is the average adjusted-R^2, and SE (model) is the average standard error of the overall models. Huber-White heteroscedasticity-robust standard errors.

over alternatives in explaining average stock returns across the 18 anomalies.

Table 7 reports the regression results from each of the eight models (Panels 1 to 8) applied to the returns-based differences between extreme decile portfolios (D1 and D10) for each of the 18 anomaly variables. At the bottom of each panel is a GRS test statistic testing the likelihood of the regression alphas from all 18 anomaly portfolios in being jointly equal to zero and lacking individual statistical significance. Together, the evidence reveals that the lowest GRS statistics are attributable to FF5F (Panel 4) and FF6F (Panel 5) models which are also associated with the highest average adjusted-R^2 and the lowest |a| or mean absolute alpha. Notably, the evidence alludes to the ILLIQ2F (Panel 6) and IP2F (Panel 7) in being the weakest among all models reflected in the highest GRS statistic, correspondingly lowest average adjusted-R^2 and the highest |a| or mean absolute alpha. This reveals significant statistical support for the superiority of FF5F and FF6F models over and above rival asset pricing models.

Table 8 reports the results from the joint time-series analysis of an overall sample, comprising 18 anomaly sorts of 10-deciles (thus, 180 test portfolios), and a smaller extreme subsample, comprising the three highest and three lowest decile portfolios across the 12 anomalies (thus, 108 test portfolios in all). The MAA is smallest across the overall and

Table 7
Comparison of models applied to anomalies in cross section of stock returns.

	Alpha (%)	Beta: Market (excess return)	Beta: SMB	Beta: HML	Beta: OP	Beta: INV	Beta: UMD	R^2
Panel 1: CAPM								
FF	−0.001 [−0.12]	−0.529 [−9.04]	….	….	….	….	….	0.2309
MCAP	0.025 [4.16]	−0.470 [−5.13]	….	….	….	….	….	0.0883
NSI	−0.003 [−0.75]	0.186 [3.01]	….	….	….	….	….	0.0323
Op	0.003 [0.74]	0.073 [1.13]	….	….	….	….	….	0.0047
P/CF	0.002 [0.33]	0.076 [0.89]	….	….	….	….	….	0.0029
P/E	0.003 [0.47]	−0.153 [−1.76]	….	….	….	….	….	0.0113
DY	0.005 [0.92]	0.199 [2.63]	….	….	….	….	….	0.0247
Asset growth	0.004 [1.04]	−0.312 [−5.19]	….	….	….	….	….	0.0900
CAPX Growth	0.001 [0.20]	0.027 [0.50]	….	….	….	….	….	0.0009
Sales Growth	−0.003 [−0.84]	−0.236 [−3.80]	….	….	….	….	….	0.0503
B/M	−0.021 [−3.77]	0.111 [1.30]	….	….	….	….	….	0.0061
Acc	0.003 [0.70]	−0.066 [−0.93]	….	….	….	….	….	0.0032
β	0.005 [1.06]	−0.908 [−12.65]	….	….	….	….	….	0.3704
Vol	−0.002 [−0.38]	−0.432 [−4.96]	….	….	….	….	….	0.0829
Liu	−0.008 [−2.01]	0.748 [12.35]	….	….	….	….	….	0.3594
Amihud	−0.015 [−2.80]	0.583 [7.12]	….	….	….	….	….	0.1572
To	0.003 [0.61]	−0.708 [−9.84]	….	….	….	….	….	0.2623
Zero	−0.005 [−1.19]	0.590 [8.48]	….	….	….	….	….	0.2089
GRS analysis	R^2	SE (model)	|a|	GRS				
	0.1104	0.00466	0.00611	39.07**				
Panel 2: FF3F								
FF	−0.007 [−1.85]	−0.299 [−0.30]	0.426 [5.39]	0.035 [0.38]	….	….	….	0.3105
MCAP	0.009 [1.74]	0.141 [0.14]	1.084 [9.65]	−0.107 [−0.82]	….	….	….	0.3240
NSI	−0.003 [−0.70]	0.176 [0.18]	−0.014 [−0.16]	0.020 [0.20]	….	….	….	0.0325
Op	−0.001 [−0.12]	0.080 [0.08]	0.078 [0.87]	0.276 [2.64]	….	….	….	0.0365
P/CF	0.001 [0.00]	0.233 [0.23]	0.240 [2.00]	−0.197 [−1.41]	….	….	….	0.0213
P/E	−0.002 [−0.30]	−0.193 [−0.19]	0.032 [0.26]	0.444 [3.15]	….	….	….	0.0491
DY	0.003 [0.60]	0.192 [0.19]	0.017 [0.15]	0.127 [1.02]	….	….	….	0.0289
Asset growth	0.002 [0.60]	−0.475 [−0.48]	−0.180 [−2.20]	0.492 [5.19]	….	….	….	0.1768
CAPX Growth	−0.001 [−0.25]	−0.017 [−0.02]	−0.026 [−0.35]	0.225 [2.55]	….	….	….	0.0246
Sales Growth	−0.004 [−1.04]	−0.293 [−0.29]	−0.054 [−0.62]	0.207 [2.03]	….	….	….	0.0647
B/M	−0.006 [−1.29]	0.107 [0.11]	−0.283 [−2.79]	−1.172 [−9.93]	….	….	….	0.3146
Acc	0.001 [0.14]	−0.006 [−0.01]	0.126 [1.26]	0.073 [0.63]	….	….	….	0.0119
β	−0.004 [−0.99]	−0.545 [−0.54]	0.652 [6.93]	−0.030 [−0.27]	….	….	….	0.4675
Vol	−0.004 [−0.64]	−0.292 [−0.29]	0.213 [1.73]	−0.175 [−1.22]	….	….	….	0.0956
Liu	−0.001 [−0.13]	0.505 [0.50]	−0.457 [−5.63]	−0.066 [−0.70]	….	….	….	0.4329
Amihud	−0.006 [−1.09]	0.293 [0.29]	−0.549 [−4.94]	−0.092 [−0.71]	….	….	….	0.2344
To	−0.003 [−0.66]	−0.552 [−0.55]	0.319 [3.19]	0.150 [1.29]	….	….	….	0.2986
Zero	−0.003 [−0.58]	0.367 [0.37]	−0.340 [−3.52]	0.275 [2.45]	….	….	….	0.2523
GRS analysis	R^2	SE (model)	|a|	GRS				
	0.1765	0.00475	0.00334	16.92				
Panel 3: Carhart 4F								
FF	−0.008 [−2.05]	−0.310 [−4.22]	0.404 [5.03]	0.072 [0.75]	….	….	0.121 [1.39]	0.3155

(*continued on next page*)

Table 7 (*continued*)

	Alpha (%)	Beta: Market (excess return)	Beta: SMB	Beta: HML	Beta: OP	Beta: INV	Beta: UMD	R^2
MCAP	0.009 [1.55]	0.129 [1.24]	1.060 [9.25]	−0.067 [−0.49]	….	….	0.132 [1.07]	0.3269
NSI	−0.001 [−0.21]	0.202 [2.51]	0.039 [0.45]	−0.070 [−0.67]	….	….	−0.293 [−3.07]	0.0653
Op	0.001 [0.05]	0.089 [1.07]	0.098 [1.06]	0.244 [2.23]	….	….	−0.106 [−1.06]	0.0405
P/CF	0.002 [0.38]	0.261 [2.36]	0.297 [2.44]	−0.293 [−2.03]	….	….	−0.313 [−2.39]	0.0416
P/E	−0.001 [−0.12]	−0.179 [−1.59]	0.059 [0.48]	0.397 [2.71]	….	….	−0.151 [−1.13]	0.0536
DY	0.002 [0.40]	0.179 [1.79]	−0.009 [−0.09]	0.171 [1.32]	….	….	0.144 [1.21]	0.0342
Asset growth	0.004 [1.00]	−0.455 [−6.06]	−0.139 [−1.69]	0.423 [4.33]	….	….	−0.224 [−2.52]	0.1958
CAPX Growth	−0.001 [−0.37]	−0.023 [−0.32]	−0.038 [−0.49]	0.244 [2.66]	….	….	0.063 [0.76]	0.0266
Sales Growth	−0.002 [−0.43]	−0.260 [−3.26]	0.012 [0.13]	0.095 [0.91]	….	….	−0.364 [−3.86]	0.1138
B/M	−0.009 [−1.82]	0.075 [0.81]	−0.349 [−3.43]	−1.060 [−8.78]	….	….	0.363 [3.31]	0.3414
Acc	−0.001 [−0.09]	−0.02 [−0.22]	0.097 [0.96]	0.122 [1.01]	….	….	0.158 [1.44]	0.0194
β	−0.006 [−1.28]	−0.562 [−6.46]	0.617 [6.46]	0.029 [0.26]	….	….	0.192 [1.86]	0.4743
Vol	−0.003 [−0.46]	−0.279 [−2.43]	0.239 [1.90]	−0.219 [−1.47]	….	….	−0.145 [−1.07]	0.0995
Liu	−0.001 [−0.27]	0.497 [6.59]	−0.472 [−5.70]	−0.041 [−0.42]	….	….	0.081 [0.90]	0.4346
Amihud	−0.007 [−1.34]	0.275 [2.66]	−0.586 [−5.18]	−0.030 [−0.22]	….	….	0.201 [1.64]	0.2420
To	−0.002 [−0.39]	−0.535 [−5.77]	0.353 [3.47]	0.093 [0.77]	….	….	−0.187 [−1.70]	0.3060
Zero	−0.002 [−0.53]	0.370 [4.12]	−0.334 [−3.39]	0.265 [2.26]	….	….	−0.033 [−0.31]	0.2526
GRS analysis	R^2	SE (model)	|a|	GRS				
	0.1880	0.00479	0.00335	19.99**				
Panel 4: FF5F								
FF	−0.007 [−1.88]	−0.282 [−3.92]	0.448 [5.77]	0.083 [0.90]	0.416 [3.77]	−0.165 [−1.62]	….	0.3452
MCAP	0.010 [1.74]	0.152 [1.45]	1.098 [9.71]	−0.088 [−0.65]	0.218 [1.36]	−0.116 [−0.78]	….	0.3288
NSI	−0.003 [−0.65]	0.178 [2.17]	−0.011 [−0.12]	0.019 [0.18]	0.026 [0.21]	−0.030 [−0.26]	….	0.0328
Op	0.001 [0.19]	0.063 [0.80]	0.047 [0.56]	0.155 [1.54]	−0.775 [−6.46]	0.159 [1.44]	….	0.1718
P/CF	0.002 [0.39]	0.250 [2.24]	0.247 [2.06]	−0.282 [−1.97]	−0.221 [−1.29]	−0.196 [−1.24]	….	0.0413
P/E	−0.002 [−0.44]	−0.156 [−1.48]	0.085 [0.75]	0.579 [4.27]	1.061 [6.59]	−0.369 [−2.48]	….	0.1819
DY	0.003 [0.63]	0.177 [1.78]	−0.004 [−0.04]	0.078 [0.61]	−0.403 [−2.64]	0.149 [1.06]	….	0.0536
Asset growth	0.004 [0.92]	−0.459 [−6.05]	−0.167 [−2.04]	0.460 [4.71]	0.026 [0.22]	−0.179 [−1.67]	….	0.1861
CAPX Growth	−0.001 [−0.22]	−0.028 [−0.40]	−0.041 [−0.55]	0.190 [2.11]	−0.288 [−2.69]	0.110 [1.11]	….	0.0501
Sales Growth	−0.003 [−0.72]	−0.288 [−3.54]	−0.056 [−0.64]	0.146 [1.39]	−0.236 [−1.89]	−0.068 [−0.59]	….	0.0850
B/M	−0.004 [−0.84]	0.134 [1.42]	−0.262 [−2.58]	−1.224 [−10.10]	0.049 [0.34]	−0.295 [−2.22]	….	0.3281
Acc	0.002 [0.34]	−0.002 [−0.02]	0.125 [1.24]	0.027 [0.23]	−0.178 [−1.25]	−0.051 [−0.38]	….	0.0213
β	−0.005 [−1.17]	−0.556 [−6.33]	0.643 [6.79]	−0.006 [−0.05]	−0.012 [−0.09]	0.126 [1.02]	….	0.4698
Vol	−0.005 [−0.84]	−0.299 [−2.60]	0.212 [1.71]	−0.120 [−0.81]	0.186 [1.05]	0.087 [0.53]	….	0.1033
Liu	0.001 [0.03]	0.504 [6.67]	−0.462 [−5.66]	−0.102 [−1.04]	−0.178 [−1.54]	−0.002 [−0.02]	….	0.4390
Amihud	−0.004 [−0.78]	0.302 [2.92]	−0.547 [−4.9]	−0.156 [−1.17]	−0.198 [−1.25]	−0.116 [−0.79]	….	0.2449
To	−0.003 [−0.67]	−0.554 [−5.92]	0.317 [3.14]	0.151 [1.25]	−0.020 [−0.14]	0.022 [0.17]	….	0.2986
Zero	−0.003 [−0.73]	0.356 [3.95]	−0.351 [−3.60]	0.286 [2.47]	−0.076 [−0.55]	0.126 [0.99]	….	0.2551

(*continued on next page*)

Table 7 (*continued*)

	Alpha (%)	Beta: Market (excess return)	Beta: SMB	Beta: HML	Beta: OP	Beta: INV	Beta: UMD	R^2
GRS analysis	R^2 0.2020	SE (model) 0.00478	|a| 0.00352	GRS 15.62				
Panel 5: FF6F								
FF	−0.008 [−2.05]	−0.291 [−4.03]	0.428 [5.42]	0.115 [1.20]	0.413 [3.75]	−0.188 [−1.82]	0.120 [1.39]	0.3499
MCAP	0.009 [1.59]	0.142 [1.35]	1.074 [9.34]	−0.051 [−0.37]	0.216 [1.34]	−0.143 [−0.95]	0.140 [1.11]	0.3318
NSI	−0.001 [−0.27]	0.200 [2.47]	0.040 [0.45]	−0.060 [−0.56]	0.033 [0.26]	0.027 [0.24]	−0.300 [−3.08]	0.0661
Op	0.001 [0.29]	0.068 [0.86]	0.059 [0.69]	0.136 [1.31]	−0.773 [−6.44]	0.173 [1.54]	−0.074 [−0.78]	0.1737
P/CF	0.004 [0.64]	0.269 [2.43]	0.293 [2.41]	−0.353 [−2.41]	−0.215 [−1.27]	−0.143 [−0.90]	−0.271 [−2.04]	0.0561
P/E	−0.001 [−0.26]	−0.143 [−1.35]	0.115 [1.00]	0.531 [3.82]	1.065 [6.62]	−0.335 [−2.23]	−0.179 [−1.42]	0.1880
DY	0.002 [0.46]	0.165 [1.66]	−0.030 [−0.28]	0.119 [0.90]	−0.406 [−2.67]	0.119 [0.84]	0.156 [1.30]	0.0595
Asset growth	0.005 [1.19]	−0.444 [−5.88]	−0.133 [−1.60]	0.407 [4.07]	0.030 [0.26]	−0.140 [−1.29]	−0.203 [−2.24]	0.2011
CAPX Growth	−0.001 [−0.32]	−0.033 [−0.47]	−0.053 [−0.69]	0.209 [2.25]	−0.290 [−2.70]	0.096 [0.96]	0.070 [0.83]	0.0526
Sales Growth	−0.001 [−0.28]	−0.262 [−3.29]	0.002 [0.03]	0.055 [0.52]	−0.228 [−1.87]	−0.001 [−0.01]	−0.346 [−3.61]	0.1277
B/M	−0.007 [−1.34]	0.103 [1.12]	−0.334 [−3.31]	−1.112 [−9.14]	0.040 [0.28]	−0.377 [−2.87]	0.425 [3.85]	0.3634
Acc	0.001 [0.13]	−0.016 [−0.17]	0.093 [0.91]	0.077 [0.62]	−0.182 [−1.28]	−0.087 [−0.65]	0.188 [1.68]	0.0316
β	−0.006 [−1.38]	−0.569 [−6.48]	0.613 [6.38]	0.041 [0.35]	−0.016 [−0.12]	0.092 [0.73]	0.178 [1.69]	0.4754
Vol	−0.004 [−0.67]	−0.285 [−2.48]	0.243 [1.93]	−0.167 [−1.10]	0.190 [1.08]	0.121 [0.74]	−0.181 [−1.31]	0.1090
Liu	−0.001 [−0.11]	0.497 [6.55]	−0.479 [−5.77]	−0.076 [−0.76]	−0.180 [−1.56]	−0.021 [−0.19]	0.099 [1.08]	0.4415
Amihud	−0.006 [−1.03]	0.284 [2.75]	−0.588 [−5.21]	−0.091 [−0.67]	−0.204 [−1.29]	−0.163 [−1.11]	0.245 [1.98]	0.2558
To	−0.002 [−0.45]	−0.540 [−5.77]	0.350 [3.42]	0.100 [0.81]	−0.016 [−0.11]	0.060 [0.45]	−0.196 [−1.74]	0.3065
Zero	−0.003 [−0.66]	0.359 [3.97]	−0.342 [−3.45]	0.273 [2.28]	−0.075 [−0.54]	0.136 [1.05]	−0.051 [−0.47]	0.2557
GRS analysis	R^2 0.2136	SE (model) 0.00480	|a| 0.00349	GRS 18.08*				
Panel 6: ILLIQ2F								
	Alpha (%)	**Beta: Market (excess return)**	**Beta: ILLIQ**	**Beta: IP**				**R^2**
FF	−0.003 [−0.96]	−0.202 [−3.09]	0.438 [8.38]	….	….	….	….	0.3893
MCAP	0.023 [3.94]	−0.311 [−2.74]	0.213 [2.35]	….	….	….	….	0.1065
NSI	−0.002 [−0.59]	0.115 [1.50]	−0.095 [−1.54]	….	….	….	….	0.0408
Op	0.003 [0.69]	0.091 [1.14]	0.025 [0.39]	….	….	….	….	0.0052
P/CF	0.003 [0.48]	−0.022 [−0.21]	−0.131 [−1.54]	….	….	….	….	0.0116
P/E	0.001 [0.02]	0.140 [1.34]	0.392 [4.69]	….	….	….	….	0.0855
DY	0.005 [1.03]	0.135 [1.43]	−0.086 [−1.13]	….	….	….	….	0.0293
Asset growth	0.004 [0.95]	−0.277 [−3.68]	0.047 [0.78]	….	….	….	….	0.0921
CAPX Growth	0.001 [0.05]	0.084 [1.26]	0.077 [1.44]	….	….	….	….	0.0085
Sales Growth	−0.004 [−0.91]	−0.203 [−2.61]	0.044 [0.71]	….	….	….	….	0.0520
B/M	−0.019 [−3.46]	−0.138 [−1.32]	−0.334 [−3.99]	….	….	….	….	0.0613
Acc	0.003 [0.69]	−0.065 [−0.74]	0.001 [0.01]	….	….	….	….	0.0032
β	0.002 [0.39]	−0.522 [−6.46]	0.517 [7.98]	….	….	….	….	0.4902
Vol	−0.002 [−0.44]	−0.395 [−3.63]	0.049 [0.57]	….	….	….	….	0.0840
Liu	– –	– –	– –	….	….	….	….	– –
Amihud	−0.013 [−2.51]	0.384 [3.82]	−0.267 [−3.32]	….	….	….	….	0.1901

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Table 7 (*continued*)

	Alpha (%)	Beta: Market (excess return)	Beta: SMB	Beta: HML	Beta: OP	Beta: INV	Beta: UMD	R^2
To	−0.001 [−0.21]	−0.274 [−3.48]	0.582 [9.25]	….	….	….	….	0.4393
Zero	−0.003 [−0.74]	0.339 [4.07]	−0.337 [−5.05]	….	….	….	….	0.2769
GRS analysis	R^2	SE (model)	|a|	GRS				
	0.1392	0.00460	0.00501	34.97**				
Panel 7: IP2F								
FF	−0.004 [−1.92]	0.090 [2.27]	….	0.825 [25.96]	….	….	….	0.7794
MCAP	0.024 [4.02]	−0.296 [−2.60]	….	0.232 [2.53]	….	….	….	0.1094
NSI	−0.002 [−0.57]	0.053 [0.69]	….	−0.177 [−2.88]	….	….	….	0.0611
Op	0.004 [0.97]	−0.091 [−1.14]	….	−0.217 [−3.41]	….	….	….	0.0455
P/CF	0.002 [0.39]	0.018 [0.17]	….	−0.077 [−0.89]	….	….	….	0.0058
P/E	0.001 [0.26]	0.057 [0.53]	….	0.279 [3.24]	….	….	….	0.0483
DY	0.005 [1.02]	0.111 [1.17]	….	−0.117 [−1.54]	….	….	….	0.0332
Asset growth	0.004 [0.92]	−0.232 [−3.08]	….	0.107 [1.77]	….	….	….	0.1004
CAPX Growth	0.001 [0.25]	−0.007 [−0.11]	….	−0.045 [−0.83]	….	….	….	0.0035
Sales Growth	−0.003 [−0.75]	−0.300 [−3.86]	….	−0.086 [−1.37]	….	….	….	0.0568
B/M	−0.020 [−3.65]	−0.023 [−0.21]	….	−0.179 [−2.07]	….	….	….	0.0216
Acc	0.003 [0.71]	−0.077 [−0.87]	….	−0.016 [−0.22]	….	….	….	0.0034
β	0.003 [0.64]	−0.506 [−6.26]	….	0.536 [8.25]	….	….	….	0.4967
Vol	−0.003 [−0.46]	−0.353 [−3.23]	….	0.105 [1.19]	….	….	….	0.0877
Liu	−0.005 [−1.69]	0.322 [5.11]	….	−0.568 [−11.19]	….	….	….	0.5617
Amihud	−0.014 [−2.67]	0.455 [4.45]	….	−0.171 [−2.08]	….	….	….	0.1704
To	0.001 [0.22]	−0.367 [−4.38]	….	0.455 [6.74]	….	….	….	0.3683
Zero	−0.004 [−0.97]	0.404 [4.72]	….	−0.249 [−3.61]	….	….	….	0.2451
GRS analysis	R^2	SE (model)	|a|	GRS				
	0.1777	0.00446	0.00573	39.07**				

This table presents the regression results (intercept plus time series slopes per factor) along with R-squared for eight asset pricing models (each one per panels one to eight) as applied to the returns-based differences between high and low decile portfolios (D1 and D10) as sorted on each of eighteen anomaly variables. The eight asset pricing models are the CAPM, FF3F (including the additional SMB and HML), Carhart 4F (including the additional momentum factor, UMD), FF5F, based on FF3F plus additional OP ad INV, FF6F, based on FF5F plus additional UMD. Then the two-factor liquidity augmented CAPM by the Liu (2006) liquidity hedging portfolio, namely, ILLIQ2F, and the similar two-factor augmented CAPM with investor protection, namely, IP2F. The final model is a grand asset pricing scheme including all factors, namely CAPM, SMB, HML, OP, INV, UMD, ILLIQ, and IP. Correspondingly, the eighteen anomaly variables are first, the average proportions of FF (free float), MCAP (market capitalization), NSI (net stock issues), then second OP (operating profit), P/CF (price to cash flow ratio), P/E (price to earnings ratio), DY (dividend yield), Asset growth (one year asset growth), CAPX growth (growth in capital expenditure), Sales growth, B/M (book-to-market ratio), ACC (accruals scaled by book equity). The third category of trading anomalies are β (beta), VOL (volatility), MOM (momentum change over 12 months), VOLUM (volume) and P (stock price). The fourth category include The Liu (2006) multidimensional trading speed metric of illiquidity measurement, Amihud's (2002) price impact from a currency unit's traded volume measure, To which is turnover in relation to daily traded volume scaled by number of shares issued and outstanding, and ZERO (daily zero returns, price rigidity, %). The one-month US deposit yield is used as the risk-free rate. Numbers in square brackets are t-statistics. †, *, and ** indicate significance at the 10 %, 5 %, and 1 % levels, respectively. Standard errors are Huber-White heteroscedasticity-robust.

extreme samples for the Carhart 4F model, implying its superiority against rival models. The evidence from the number of regression alphas with $p \leq 0.05$ is largely indeterminate. Similarly, the number of portfolio groups for which a model is not rejected by the chi-square specification test point to the superiority of the Carhart 4F model. In contrast to these findings, the evidence from the cross-sectional constrained R^2 (R^2_c) supports the Carhart 4F model across both the overall ($R^2_c = 0.4687$) and extreme subsamples ($R^2_c = 0.5151$). The relative strength of the Carhart 4F model is further supported by R^2_c being statistically significant at the 1 % confidence margin (*p*-value of 0.000). These results are mirrored in the case of equal-weighted portfolios.[10] An important attribute of the R^2_c statistic is that it can assume negative values, as is evident in the CAPM ($R^2_c = -0.4872$), LIQ2F model ($R^2_c = -0.2499$) and IP2F ($R^2_c = -0.3258$) in the overall sample. This evidence of the lack of importance of the LIQ2F and IP2F model is further reinforced by the R^2_c lacking in statistical significance at the 10 % confidence margin (*p*-value of 1.000). Cooper & Maio (2019b: 1986) argue that such a negative cross-sectional constrained R^2 "means that the model does worse than a simple cross-sectional regression containing just a constant." In summary, at this stage, the evidence points to the superiority of the Carhart 4F model.

Table 9 reports the results of the spanning regression tests. Our starting point is the maximum Sharpe ratios; they are highest for the FF5F (0.4546), FF6F (0.4751) and 8F model (0.4952), followed by the Carhart 4F (0.4400) and FF3F (0.4039) models. Next, the results from the sets of spanning regressions associated with each of the eight asset pricing models are reported between panels 2 to 9 with the CAPM omitted since it only has one factor. A number of observations are apparent. The first is the statistical redundancy of OP, which is visible from an alpha t-statistic lacking significance in both panels 4 and 5 alongside a negligible marginal contribution to the FF5F and FF6F model's $Sh^2(f)$, that is, $a^2/s^2(e)$. Also, both the ILLIQ and IP factors yields

[10] The results are available in Table 13 in the online appendix.

Table 8
Joint time-series tests: cross-sectional analysis.

	CAPM	FF3F	Carhart 4F	FF5F	FF6F	ILLIQ2F	IP2F	8F
Panel 1: overall sample								
MAA, %	0.3236	0.2030	0.1940	0.2180	0.2083	0.2885	0.2978	0.2102
# $p < 0.05$ [$N = 180$]	0	0	0	0	0	0	0	0
# χ^2 [N = 18 groups, 10 test assets]	5	0	0	2	2	3	3	1
R_c^2 [p value]	−0.4872	0.4377	0.4687	0.3564	0.4009	−0.2499	−0.3258	0.4111
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[1.000]	[1.000]	[0.000]
SR^2 [p value]		0.9250	0.9559	0.8436	0.8882	0.2373	0.1614	0.8983
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.003]	[0.000]
Panel 2: extreme deciles								
MAA, %	0.3614	0.2182	0.2138	0.2378	0.2288	0.3218	0.3350	0.2270
# $p < 0.05$ [$N = 108$]	0	0	0	0	0	0	0	0
# χ^2 [N = 18 groups, 6 test assets]	3	0	0	0	0	3	3	0
R_c^2 [p value]	−0.3873	0.5010	0.5151	0.4193	0.4535	−0.1571	−0.2520	0.4867
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.982]	[0.997]	[0.000]
SR^2 [p value]		0.8883	0.9024	0.8066	0.8407	0.2302	0.1353	0.8740
		[0.000]	[0.000]	[0.000]	[0.000]	[0.001]	[0.021]	[0.000]

This table presents the results for the joint time-series tests of the conditional factor models for the African value-weighted universe for the period 2001:01 to 2023:11. The test portfolios are the 18 different sets of 10-decile portfolios for each of the 18 anomalies. The eight asset pricing models are the CAPM, FF3F (including the additional SMB and HML), Carhart 4F (including the additional momentum factor, UMD), FF5F, based on FF3F plus additional OP ad INV, FF6F, based on FF5F plus additional UMD. Then the two-factor liquidity augmented CAPM by the Liu (2006) liquidity hedging portfolio, namely, ILLIQ2F, and the similar two-factor augmented CAPM with investor protection, namely, IP2F. The final model is a grand asset pricing scheme including all factors, namely CAPM, SMB, HML, OP, INV, UMD, ILLIQ, and IP. MAA denotes the mean absolute alpha, in percentage terms. #$p < 0.05$ represents the number of portfolios in which the alphas are significant at the 5 % level. # χ^2 denotes the number of portfolio groups in which the model is not rejected by the chi-square specification test. R_c^2 is the cross-sectional constrained R^2 and the numbers in parentheses represent the respective empirical p-values to test the null that the explanatory ratio is zero (obtained from a bootstrap simulation). The results in Panel 1 are estimated across all 10 deciles for each of the 18 anomaly variables, namely, 180 (10 × 18) portfolios in total, while the results in Panel 2 are for the extreme deciles only, namely, D1, D2, and D3 and then D8, D9, D10, which results in 108 (6 × 18) portfolio test assets in total. Initial time-series regressions based on Huber-White heteroscedasticity-robust standard errors.

similarly minimal marginal contribution to the model's $Sh^2(f)$, that is, $a^2/s^2(e)$ in the case of ILLIQ2F, IP2F and also the 8F models. Together, this evidence supports the redundancy of OP, as well as ILLIQ and IP factors from consideration in asset pricing. This refocuses attention on the relative superiority of Carhart 4F as well as FF5F and FF6F with the latter two propelled by the other factors except OP. However, given the relative statistical support for INV, namely investment or asset growth, and questions relating to the efficacy of momentum returns or persistency in price returns given extreme illiquidity prevalent in smaller developing stock markets, such as those of Africa, and we argue this empirical evidence is more supportive of the FF6F and FF5F models.

4.1. Practitioner implications of results

As an additional exercise, we explore the practitioner implications arising from our preceding differentiation between rival asset pricing models. First, we consider the cost of equity as estimated by each of the eight asset pricing models in succession with this being averaged across all constituent firms with listed equity constituent to each African equity market. Our estimations reveal substantial differences in estimated costs of equity at a national level across equity markets from the application of each of the eight rival models (see Table 10). Notably, CAPM, as well as ILLIQ2F and IP2F all produce the lowest estimates of cost of equity (between 5 % and 7 %), which is arguably due to the inability to sufficiently accommodate the appropriate hedging of risks in the underlying model. However, this rises significantly in the Carhart 4F, FF5F and FF6F models with costs of equity ranging between 9.55 % and 11 %. Importantly, it should be noted that these estimated costs of equity relate to value-weighted valuation factors and therefore differ substantially from earlier estimates based on equal-weighted factors in Hearn and Piesse (2010a, 2010b), Hearn et al., 2012), which placed greater emphasis on the plethora of microcap stocks in African markets.

Also, we explore the risk diversification benefits accruing to minority portfolio investors from the firm raising capital in stock markets. This can be viewed as the flipside to the implications of cost of equity estimation which is centered on the firm-level costs of raising external finance. Table 11 presents evidence relating to the attributes of quadratically optimized minimum variance portfolios for stocks valued through the application of each of the eight respective asset pricing models in terms of returns, standard deviation and Sharpe ratio as well as the asset weights per national equity market (Panel 1). The results reveal that the highest returns, corresponding lowest standard deviation and highest overall Sharpe ratios are for FF5F, FF6F and 8F models indicating their superiority vis-à-vis all other rival models. Notably, the results relating to asset weights are consistently indicative of approximately 45 % to 50 % of portfolios by weight comprising South African assets with much of the remainder being equally distributed across the three North African equity markets of Morocco, Tunisia and Egypt. This evidence has implications for passive regional benchmark track investments, mutual funds and African investment funds, which if using quadratic optimization techniques in stock selection are likely to be polarized between North and South Africa with minimal dispersion across the continent's plethora of smaller equity markets.

As an extension, we also explore two rival portfolio investment strategies in Table 11, Panel 2 based on only French civil code law equity markets and those adhering to English common law (including mixed Roman-Dutch and English common in case of South Africa). The evidence reveals a substantial difference in Sharpe ratio for the minimum variance portfolios formed on each of the two strategies. French civil code strategy has a Sharpe ratio of 0.6947 while that for English common law markets is almost one third higher at 0.9622 indicating much greater diversification benefits. This mirrors lower returns and proportionately higher risk, or standard deviation of the minimum variance portfolio in French civil code as opposed to English common law portfolios. Moreover, the evidence questions the competitiveness of French civil code law jurisdictions in attracting foreign portfolio investment vis-à-vis their English common law counterparts.

As a final exercise we undertake quadratically optimized portfolio simulations based on random asset weights, albeit with their being constrained to sum up in total to unity. The scatterplots from 20,000 optimized portfolio simulations are reported for each of the eight asset pricing models used to model the individual stock returns and standard

Table 9
Spanning regressions.

Panel 1: Max. Sharpe ratios

CAPM	FF3F	Carhart 4F	FF5F	FF6F	ILLIQ2F	IP2F	8F
0.1739	0.4039	0.4400	0.4546	0.4751	0.2012	0.1865	0.4952

Panel 2: Spanning regressions FF3F

LHS	a	R_m-R_f	SMB	HML	t(a)	R^2	s(e)	$Sh^2(f)$	$a^2/s^2(e)$
R_m-R_f	0.013	– –	−0.679	0.350	4.44	0.4152	0.0487	0.1739	0.0712
SMB	0.013	−0.580	– –	0.224	4.63	0.3897	0.045	0.1739	0.0834
HML	0.006	0.221	0.166	– –	2.72	0.0711	0.0387	0.1739	0.0240

Panel 3: Spanning regressions Carhart 4F

LHS	a	R_m-R_f	SMB	HML	UMD	t(a)	R^2	s(e)	$Sh^2(f)$	$a^2/s^2(e)$
R_m-R_f	0.012	– –	−0.694	0.385	0.126	4.01	0.4197	0.0485	0.4400	0.0612
SMB	0.011	−0.577	– –	0.280	0.212	3.84	0.4109	0.0442	0.4400	0.0619
HML	0.008	0.227	0.199	– –	−0.255	3.37	0.1407	0.0372	0.4400	0.0462
UMD	0.007	0.090	0.181	−0.308	– –	2.96	0.0911	0.0409	0.4400	0.0292

Panel 4: Spanning regressions FF5F

LHS	a	R_m-R_f	SMB	HML	OP	INV	t(a)	R^2	s(e)	$Sh^2(f)$	$a^2/s^2(e)$
R_m-R_f	0.012	– –	−0.682	0.363	−0.092	0.182	3.50	0.4206	0.0485	0.4546	0.0612
SMB	0.012	−0.586	– –	0.228	−0.107	0.137	3.44	0.3926	0.0449	0.4546	0.0714
HML	0.008	0.219	0.160	– –	−0.166	−0.141	3.18	0.1228	0.0376	0.4546	0.0452
OP	0.001	−0.039	−0.053	−0.117	– –	0.394	0.11	0.2139	0.0316	0.4546	0.0001
INV	0.006	0.091	0.08	−0.117	0.462	– –	3.43	0.2087	0.0342	0.4546	0.0307

Panel 5: Spanning regressions FF6F

LHS	a	R_m-R_f	SMB	HML	OP	INV	UMD	t(a)	R^2	s(e)	$Sh^2(f)$	$a^2/s^2(e)$
R_m-R_f	0.011	– –	−0.695	0.388	−0.094	0.160	0.105	3.36	0.4228	0.0484	0.4751	0.0516
SMB	0.010	−0.580	– –	0.274	−0.108	0.093	0.203	3.21	0.4112	0.0442	0.4751	0.0511
HML	0.008	0.222	0.188	– –	−0.152	−0.091	−0.216	3.61	0.1697	0.0366	0.4751	0.0477
OP	0.001	−0.040	−0.055	−0.114	– –	0.391	0.013	0.08	0.2112	0.0317	0.4751	0.0001
INV	0.005	0.078	0.055	−0.078	0.447	– –	0.136	2.92	0.2265	0.0339	0.4751	0.0217
UMD	0.006	0.073	0.169	−0.263	0.021	0.193	– –	2.18	0.1162	0.0404	0.4751	0.0220

Panel 6: Spanning regressions ILLIQ2F

LHS	a	R_m-R_f	ILLIQ	t(a)	R^2	s(e)	$Sh^2(f)$	$a^2/s^2(e)$
R_m-R_f	0.010	– –	−0.480	3.27	0.3559	0.0511	0.2012	0.0383
ILLIQ	0.006	−0.747	– –	1.65	0.3559	0.0638	0.2012	0.0088

Panel 7: Spanning regressions IP2F

LHS	a	R_m-R_f	IP	t(a)	R^2	s(e)	$Sh^2(f)$	$a^2/s^2(e)$
R_m-R_f	0.009	– –	−0.487	2.95	0.3631	0.0508	0.1865	0.0314
IP	0.004	−0.750	– –	1.05	0.3631	0.0631	0.1865	0.0040

Panel 8: Spanning regressions 8F

LHS	a	R_m-R_f	SMB	HML	UMD	OP	INV	ILLIQ	IP	t(a)	R^2	s(e)	$Sh^2(f)$	$a^2/s^2(e)$
R_m-R_f	0.012	– –	−0.575	0.256	0.031	0.040	0.036	−0.352	−0.091	3.86	0.5206	0.0441	0.4952	0.0740
SMB	0.010	−0.534	– –	0.273	−0.081	0.104	0.199	−0.117	0.321	3.18	0.4563	0.0425	0.4952	0.0554
HML	0.009	0.174	0.200	– –	−0.126	−0.116	−0.227	−0.065	−0.086	3.84	0.1804	0.0364	0.4952	0.0611
UMD	0.006	0.030	0.178	−0.278	– –	0.045	0.165	−0.068	−0.069	2.30	0.1222	0.0402	0.4952	0.0222
OP	−0.001	0.016	−0.044	−0.092	0.027	– –	0.404	0.121	−0.006	−0.29	0.2332	0.0312	0.4952	0.0010
INV	0.006	0.023	0.065	−0.098	0.114	0.464	– –	−0.090	−0.075	3.27	0.2448	0.0335	0.4952	0.0321
ILLIQ	0.006	−0.403	−0.145	−0.11	−0.094	0.276	−0.179	– –	0.331	2.12	0.3296	0.0472	0.4952	0.0162
IP	0.001	−0.066	0.250	−0.091	−0.060	−0.009	−0.094	0.209	– –	0.12	0.292	0.0375	0.4952	0.0001

This table presents the results for the African universe for the period 2001:01 to 2023:11. The results for the maximum Sharpe ratio, $Sh^2(f)$, are reported in Panel 1 for each of the eight multi-factor asset pricing models. These are the FF3F [CAPM plus SMB, HML], Carhart 4F [FF3F plus UMD], FF5F [FF3F plus OP, INV], FF6F [FF5F plus UMD], then ILLIQ2F and IP2F before a grand asset pricing model, 8F [CAPM plus SMB, HML, OP, INV, UMD, ILLIQ, IP].
Spanning regressions and marginal contributions to each asset pricing model's maximum Sharpe ratio, $Sh^2(f)$, for each individual factor regression for factors contained within each of the 7 models are reported in panels 2 to 8 respectively. Regression intercepts are termed "a", the market excess returns are "R_m-R_f", SMB, HML, OP, INV, UMD, ILLIQ and IP are all formed through a 3 × 3 double sort procedure first on size then on the factor concerned. However, only in the two factor models ILLIQ2F and IP2F are the ILLIQ and IP factors obtained through returns differences between highest (lowest) and lowest (highest) sorted extreme decile portfolios. The table shows intercepts a, t-statistics for the intercepts t(a), slopes, R^2, and residual standard errors s(e) from spanning regressions of each of the factors of a model on the model's other four or five factors. The table also shows $Sh^2(f)$ and each factor's marginal contribution to a model's $Sh^2(f)$, that is, $a^2/s^2(e)$. Standard errors are Huber-White heteroscedasticity-robust.

Table 10
Cost of equity implications.

	Aggregate	CAPM	FF3F	Carhart 4F	FF5F	FF6F	ILLIQ2F	IP2F	8F
North Africa									
Morocco	6.41	3.93	6.18	6.57	7.93	7.95	4.54	5.14	9.03
Tunisia	5.37	2.91	5.99	5.25	7.30	6.64	3.50	3.32	8.08
Egypt	11.61	5.60	19.90	17.96	13.43	12.98	8.57	9.13	5.34
West Africa									
BRVM	7.05	4.83	5.62	4.92	8.52	8.05	6.43	6.83	11.23
Nigeria	12.33	2.67	11.74	15.64	18.17	18.91	8.46	9.70	13.34
East Africa									
Kenya	7.00	3.42	7.87	8.53	8.48	8.33	4.85	4.59	9.92
Southern Africa									
Mauritius	7.49	3.37	8.26	7.96	9.93	9.32	4.24	4.97	11.87
South Africa	10.85	10.46	10.28	8.97	11.96	11.40	9.47	9.15	15.14
Total	8.67	5.49	9.87	9.55	10.55	10.25	6.54	6.72	10.37

This table provides annualized estimates of costs of equity in annualized percentage terms (%) which is the average across stocks constituent to a given national equity market while such average costs of equity are provided per each of the eight asset pricing models in this study, namely CAPM, FF3F, Carhart 4F, FF5F, FF6F, ILLIQ2F, IP2F, 8F.

Table 11
Portfolio asset allocation implications.

	Return, %	Std Dev., %	Sharpe ratio	Optimized asset weights, %							
				Morocco	Tunisia	Egypt	BRVM	Nigeria	Kenya	Mauritius	South Africa
Panel 1: Models											
CAPM	0.5615	1.1189	0.5018	13.37	11.90	7.87	1.54	7.25	5.12	3.04	49.90
FF3F	0.7883	0.7110	1.1087	13.38	12.47	13.46	1.93	7.74	6.08	2.99	41.94
Carhart 4F	0.7585	0.7469	1.0156	13.13	13.26	13.28	2.35	7.81	5.62	3.22	41.33
FF5F	0.8637	0.7479	1.1548	13.20	13.06	12.85	2.54	8.18	5.59	3.13	41.45
FF6F	0.8486	0.7016	1.2096	13.54	13.26	12.97	1.88	7.97	5.84	2.72	41.83
ILLIQ2F	0.5767	1.0077	0.5723	13.70	11.34	7.99	3.40	7.99	5.05	2.45	48.08
IP2F	0.5858	0.9632	0.6082	14.52	11.04	8.61	3.70	9.87	4.94	2.63	44.69
8F	0.8714	0.7289	1.1955	13.03	13.20	13.12	2.28	8.08	5.42	2.94	41.93
Panel 2: Strategies											
French civil code	0.6881	0.9905	0.6947	30.46	36.93	24.13	2.47	– –	– –	6.01	– –
English common law	0.7641	0.7941	0.9622	– –	– –	– –	– –	10.54	7.08	– –	82.38

This table provides the characteristics of minimum variance portfolios based on the expected returns and variance-covariance matrix derived from each of the eight asset pricing models, namely CAPM, FF3F, Carhart 4F, FF5F, FF6F, ILLIQ2F, IP2F, 8F. The estimation time period is from January 2010 to October 2023. Reported statistics are mean and variance of minimum variance portfolio, it's Sharpe ratio and then optimized asset weights per country.

deviations. These are displayed between Figs. 3 to 10. There are some striking differences between the scatter plots. Those produced by models which lack effective hedging of risks, such as CAPM (Fig. 3), ILLIQ2F (Fig. 8) and IP2F (Fig. 9) exhibit a concentration of optimized risk-return points mirroring a flatter ellipsoid. The flatness in their profiles indicates huge increases in portfolio standard deviation, or risk, in the horizontal axis in relation to incremental increases in returns in the vertical axis. Moreover, the density of points in simulations based on CAPM, ILLIQ2F and IP2F is much lower on vertical axis indicating a much-reduced distribution in portfolio returns.

Contrastingly, the ellipsoid distributional scatter plots for portfolio risk-returns for FF3F (Fig. 4), Carhart 4F (Fig. 5), FF5F (Fig. 6), FF6F (Fig. 7) and 8F (Fig. 10) reveal very different profiles. These all have a higher overall position in the vertical returns-axis, indicative of higher returns, while also being notably rounder with a longer vertical profile and less of a horizontal profile. This is indicative of substantially improved returns-risk trade off in terms of incremental increases in returns on vertical axis being associated with equally minimal increases in risk or standard deviation in the horizontal axis. Visually, the relative vertical flatness (as opposed to horizontal flatness) of ellipsoid profile is preferable and can be seen for FF5F (Fig. 6) and FF6F (Fig. 7). This evidence is supportive of the superiority of FF5F and FF6F models.

Finally, we also undertake quadratically optimized portfolio simulations on investment strategies comprising a universe of stocks constituent to French civil code law equity markets (Fig. 11) and then constituent to English common law equity markets (Fig. 12). The two profiles are visibly different with that of English common law being vertically higher and rounder in its ellipsoid shape thereby yielding a better risk-return trade-off than in the universe based on French civil code law equity markets. This French civil code law profile visibly has a much longer horizontal dimension and a lesser vertical dimension in contrast to its English common law counterpart.

5. Conclusions

In this paper, we have undertaken a comprehensive and up-to-date study of the application of multifactor asset pricing methodology in emerging African equity markets. This involved the single market factor, size and book to market value factors of FF3F, additional momentum in Carhart 4F model, then operating profit and asset growth or investment on top of the FF3F model to produce the FF5F model with this itself augmented by momentum to produce FF6F. We also include ILLIQ2F and IP2F models. The findings are of relevance to regulatory authorities in terms of practical implications arising from their recently enacted capital market reforms. They also help practitioners, who seek to include Africa in their investment portfolios, while at same time finding suitable hedges for the risks involved.

Our study has some limitations. The first is our selection of asset pricing models which are all based on modified variants of the CAPM thereby omitting from consideration factor-based models which contain factors derived from the statistical factor decomposition of the variance-covariance matrix of stock returns. Therefore, our study focusses on

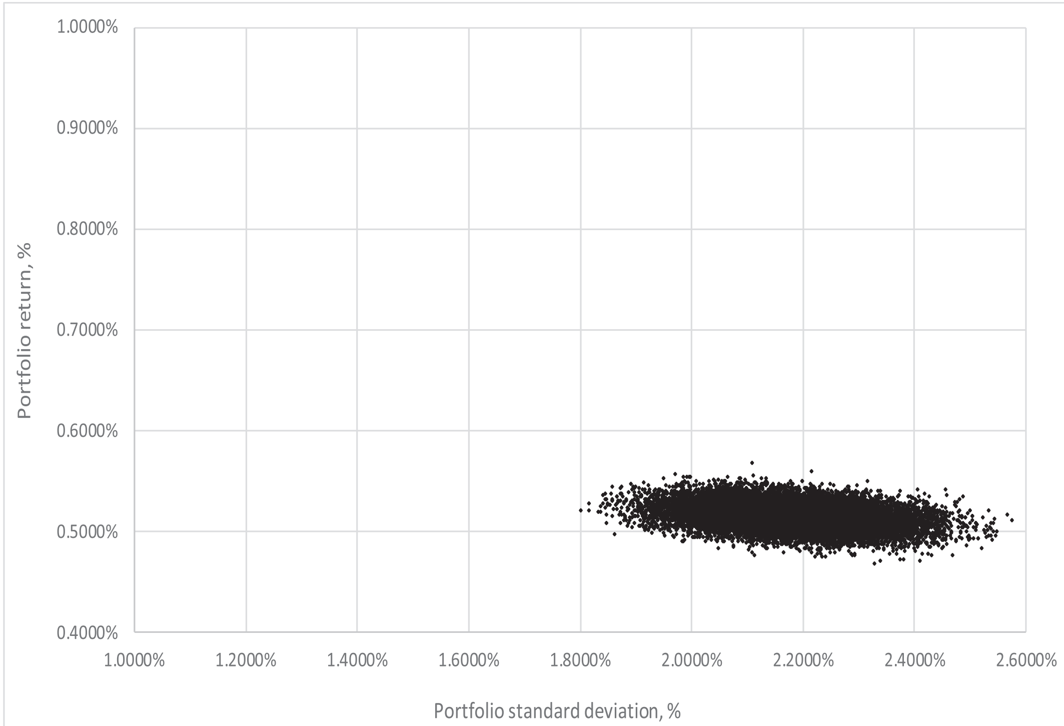

Fig. 3. Opportunity set for CAPM.

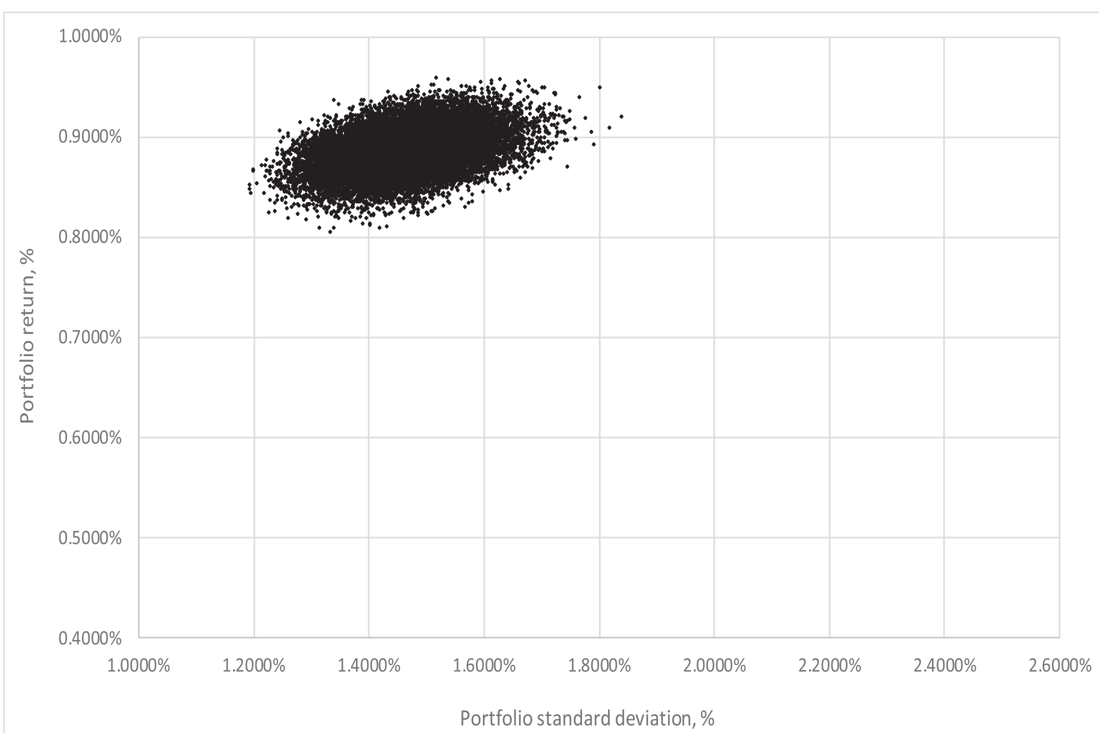

Fig. 4. Opportunity set for FF3F.
*based on 20,000 simulations
*based on 20,000 simulations

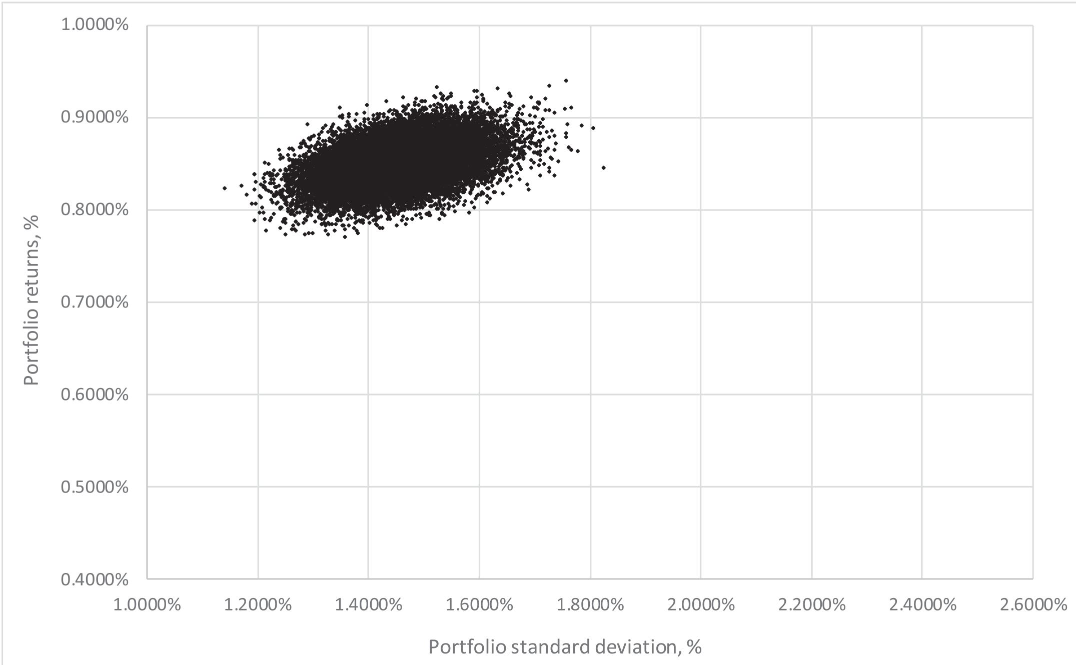

Fig. 5. Opportunity set for Carhart 4F.

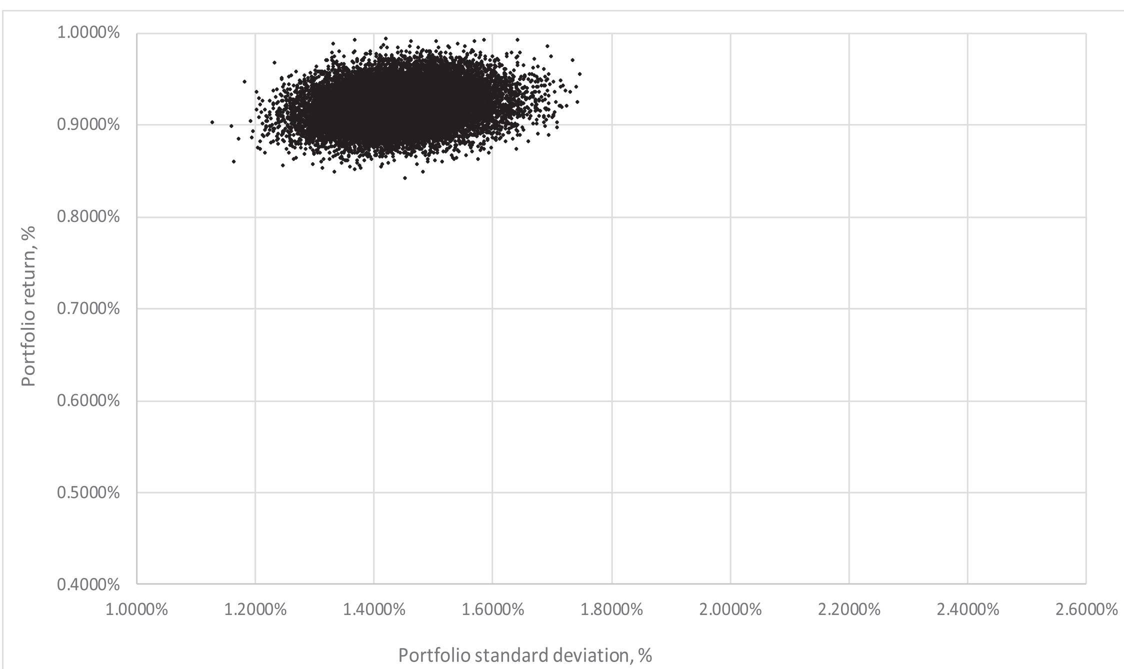

Fig. 6. Opportunity set for FF5F.
*based on 20,000 simulations
*based on 20,000 simulations

factors which are more explicitly tied to the hedging of known risks rather than arising from statistical factor analysis/decomposition techniques. The second is our sample focus on African frontier equity markets with the visible omission from consideration of a wider spectrum of frontier and wider emerging markets. Therefore, our findings are susceptible to being sample-context specific in relation to relevance in the African context. This would motivate further studies focussing on a broader selection of emerging and frontier markets worldwide to verify the external validity of our findings. The third is that our selection of models is not comprehensive in terms of a number of recently developed models in the literature being inappropriate given the significant lack of data necessary for the construction of one or more constituent valuation factors. This situation may change as firm level data reporting becomes more commonplace and widespread across Africa and broader emerging and frontier equity markets worldwide and with data providers successfully backdating their data too.

Our evidence in terms of value-weighted factors and portfolios is that illiquidity and ownership concentration, in the form of investor

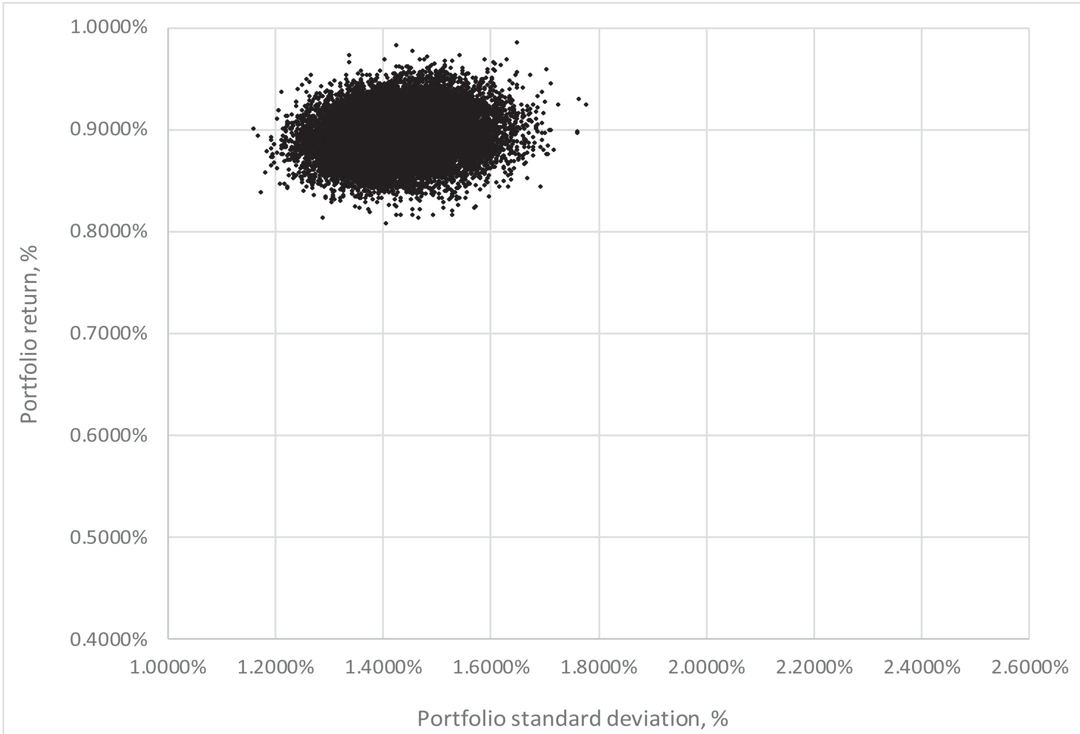

Fig. 7. Opportunity set for FF6F.

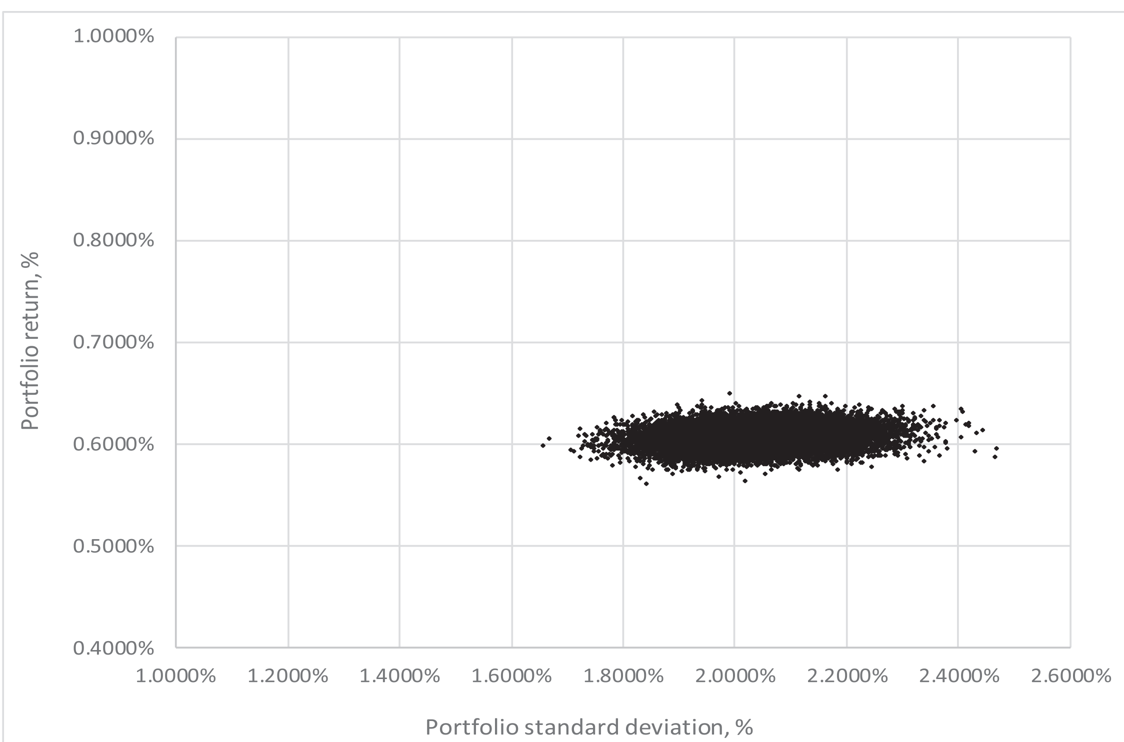

Fig. 8. Opportunity set for ILLIQ2F.
*based on 20,000 simulations
*based on 20,000 simulations

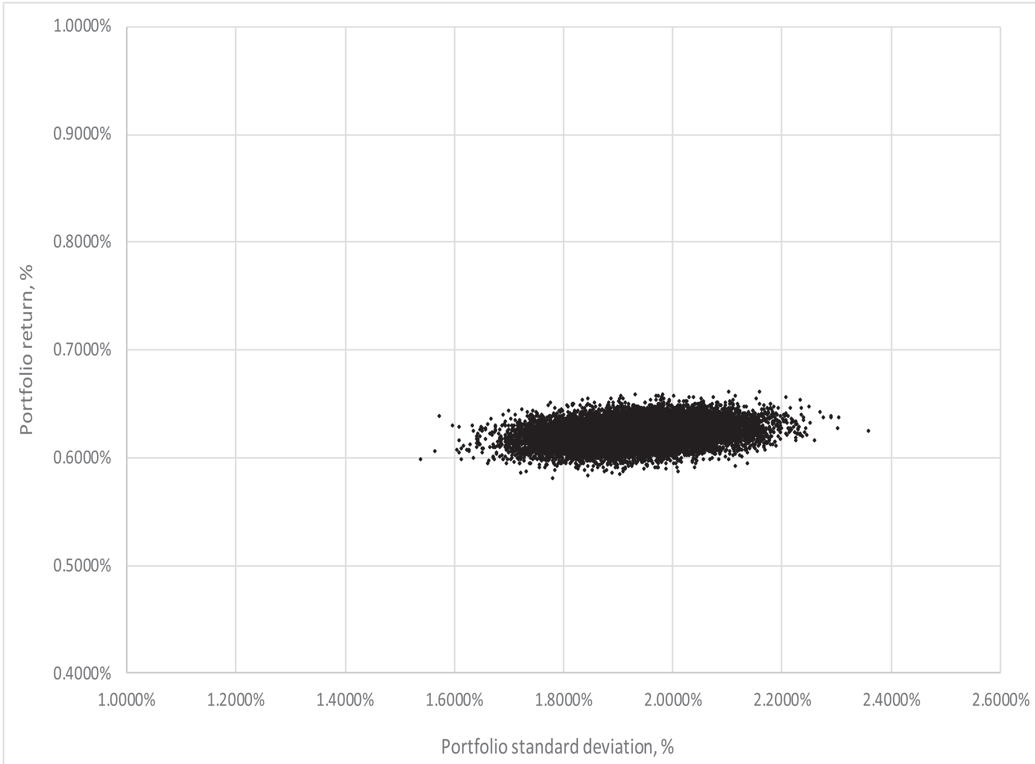

Fig. 9. Opportunity set for IP2F.

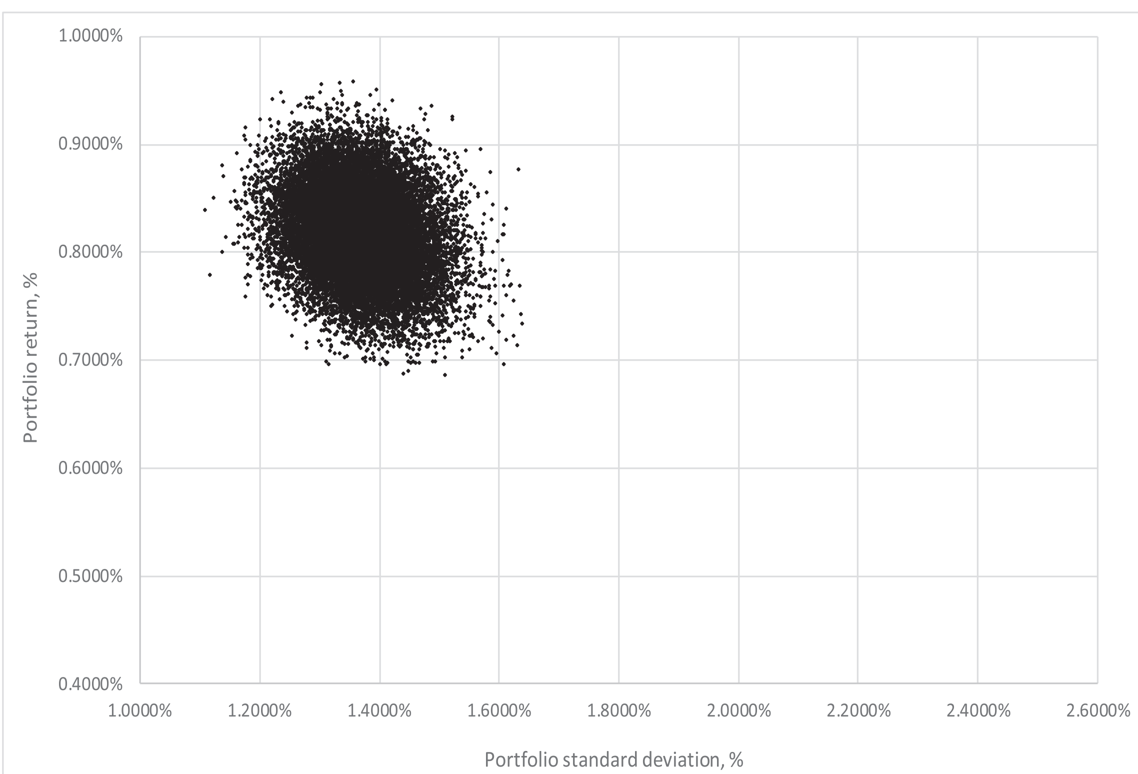

Fig. 10. Opportunity set for 8F.
*based on 20,000 simulations
*based on 20,000 simulations

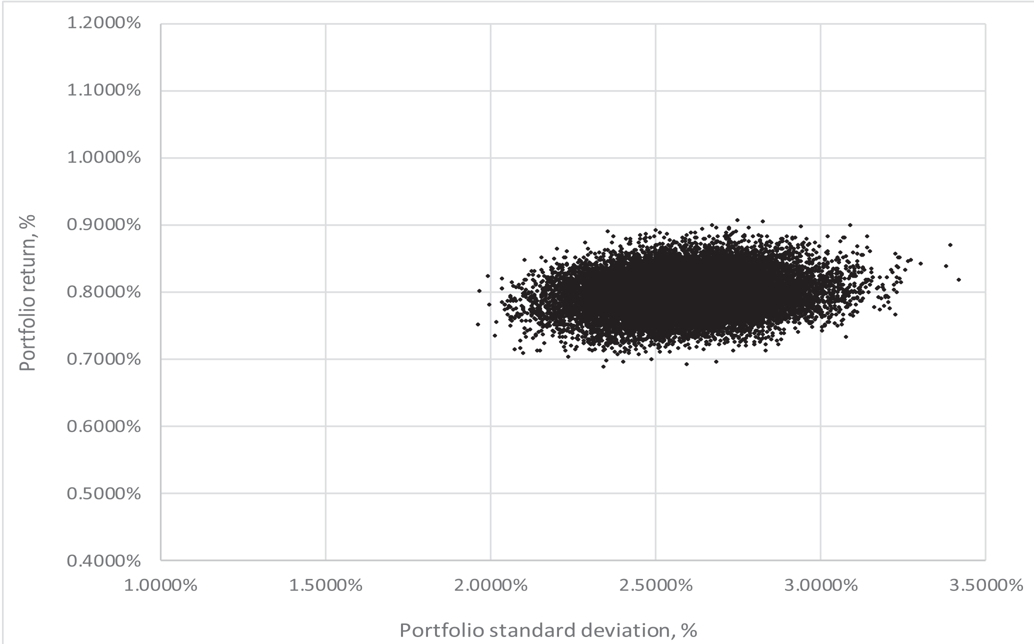

Fig. 11. Opportunity set for French civil code law.

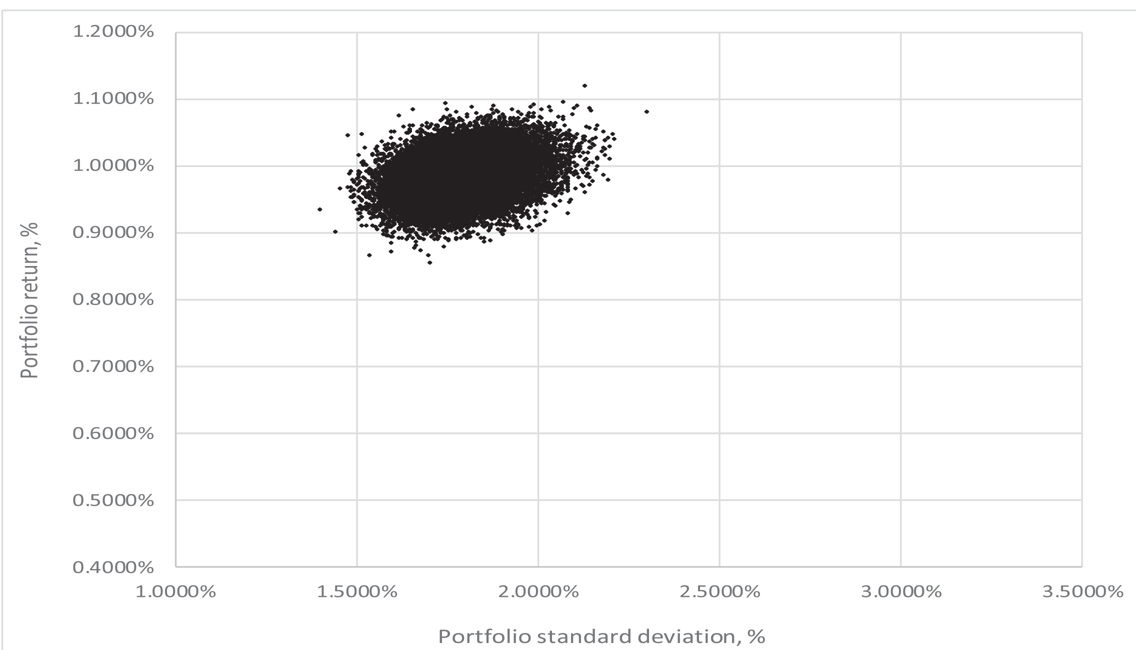

Fig. 12. Opportunity set for English common law.
*based on 20,000 simulations and FF6F model
*based on 20,000 simulations and FF6F model

protection, is not important within asset pricing despite the wider and much publicised importance of these issues across Africa and emerging economies in general. Instead, we find that asset growth or investment and momentum are important factors within asset pricing alongside traditional size and book to market value balance sheet-based factors. These are in a FF5F or FF6F asset pricing model format. In addition to the statistical support for the FF5F and FF6F models over and above rival models, we also find substantial implications arising for firms seeking to raise external capital in the form of cost of equity and minority portfolio investors seeking to invest in firms in relation to their risk diversification opportunities. Our study underscores the profound importance in selecting optimal asset pricing models and in their inclusion of appropriate factors that can effectively hedge against risks within the given investment universe. Our findings have importance to national equity markets and regulators in African and wider emerging economies in terms of the attraction of foreign portfolio investment essential to supplement domestic savings-investment schedules within indigenous economies.

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Appendix A. Appendix

Appendix Table 1
Datastream variable definitions.

Coverage	Index	No. stocks	Description
North Africa			
Algeria		7	
Egypt Hermes	Hermes financial index	43	Source: Thomson Refinitiv via Egyptian SE (EGX)
Morocco	All share index	71	Source: Thomson Refinitiv via Casablanca bourse
Sudan	*All listed equities (no index)	67	Source: Khartoum stock exchange
Tunisia	All share index	86	Source: Thomson Refinitiv via Tunis bourse
West Africa			
BRVM Top10	[daily zero returns <50 %]	10	Source: Thomson Refinitiv
BRVM	All listed equities (no index)	46	Source: Thomson Refinitiv
Ghana	All listed equities (no index)	33	Source: Thomson Refinitiv
Nigeria Ngx30	NGX30 index	30	Source: Thomson Refinitiv via Nigerian stock exchange
Nigeria	All share index	164	Source: Thomson Refinitiv via Nigerian stock exchange
East Africa			
Kenya Top40	[daily zero returns <50 %]	41	
Kenya	All share index	60	Source: Thomson Refinitiv via Nairobi stock exchange
Rwanda	All listed equities (no index)	6	Source: Thomson Refinitiv
Tanzania	All listed equities (no index)	22	Source: Thomson Refinitiv
Uganda	All listed equities (no index)	10	Source: Thomson Refinitiv
Southern Africa			
Botswana	All share index	25	Source: Thomson Refinitiv
Malawi	All share index	14	Source: Thomson Refinitiv
Mauritius Top7	[daily zero returns <50 %]	7	Source: Thomson Refinitiv
Mauritius	Main board all share index	37	Source: Thomson Refinitiv
Namibia	All listed equities (no index)	12	Source: Thomson Refinitiv
South Africa	FTSE/JSE All share index	87	Source: Thomson Refinitiv via FTSE/JSE
Zambia	All listed equities (no index)	22	Source: Thomson Refinitiv
Zimbabwe	All listed equities (no index)	25	Source: Thomson Refinitiv
Africa overall		867	
Africa sample		375	

All data was sourced from Thomson Refinitiv (accessed through Datastream portal) except for Sudan which was obtained from Khartoum stock exchange website. Data for Malawi was additionally augmented with bid and ask prices obtained direct from the Malawi stock exchange.

Appendix B. Variable definitions.

All data are from Refinitiv Datastream and Worldscope (accessed through Datastream).

Variable	Definition	Datastream Mnemonic
Panel 1: essential variables		
Price	This is the adjusted default official daily closing price. It is denominated in primary units of local currency. Prices are generally based on the "last trade" or an official price fixing. The "current" prices taken at the close of the market are stored each day. These stored prices are adjusted for subsequent capital actions, and this adjusted figure then becomes the default price available.	P
Number of shares	This is the total number of ordinary shares that represent the capital of the company. The data type is expressed in thousands.	NOSH
Traded volume	This shows the number of shares traded for a stock on a particular day. The data type is reported in thousands. Daily and non-daily figures are adjusted for capital events. However, if a capital event occurs in the latest period of a non-daily request, then the volume for that particular period is only retrieved as unadjusted.	VO
Bid Ask spread	The bid-ask spread is the average of the available monthly quotes with a minimum of a single month's quote for that month and the average used for the spread, which minimizes outliers that result from monthly sampling. Finally, following Lesmond (2005) negative bid-ask spreads and those that exceed 80 % are removed. The monthly quoted spread is defined as: $\textit{Quoted spread}_M = 1/2\left[\left(\frac{(Ask_M - Bid_M)}{(Ask_M + Bid_M)/2}\right) + \left(\frac{(Ask_{M-1} - Bid_{M-1})}{(Ask_{M-1} + Bid_{M-1})/2}\right)\right]$	
Risk-free rate	Defined as the US$ 1-month deposit middle rate [ECUSD1M] in Datastream. This is expressed annually; thus, it is transformed into a monthly equivalent.	ECUSD1M
Panel 2: Anomaly variables		

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Variable	Definition	Datastream Mnemonic
Free float Number of shares	The percentage of total shares in an issue available to ordinary investors (NOSHFF). This means the total number of shares (NOSH) less the strategic holdings (NOSHST). In general, only holdings of 5 % or more are counted as strategic. Strategic ownership data are collected by the Refinitiv Reuters Ownership team. The data are derived from 11 primary sources, including SEC filings (such as Schedule 13D and Form 13FD) and the UK Register, as well as annual reports, interim reports, stock exchanges, official regulatory bodies, third-party vendors, company websites, approved news sources, and direct contact with company investor relations departments. Ownership updates were obtained at the end of month before August 2009; after this date, values are updated on the 10th and 30th of each month. Strategic holdings are defined as the sum of the following shareholding categories: (1) Government: State (government) or state (government) institution (NOSHGV) (2) Cross Holdings: Holdings by one company in another (NOSHCO) (3) Pension Fund: Pension funds or Endowment funds (NOSHPF) (4) Investment Co.: Investment banks or institutions seeking a long-term return. Note that holdings by hedge funds are not included (NOSHIC) (5) Employees: Employees, or by those with a substantial position in a company that provides significant voting power at an annual general meeting (typically family members) (NOSHEM) (6) Other Holdings: Entities outside one of the above categories (NOSHOF) (7) Foreign Block Holders: Holdings by an institution domiciled in a country other than that of the issuer (NOSHFR).	NOSHFF
Market capitalization	First, the total number of shares issued and outstanding [NOSH] of each firm is rebased from its originally reported in thousands in Datastream to millions. Next, this value [NOSH] is multiplied by the closing price [P]. All data are daily frequency.	– –
Net issues (2 year – 1 year)	At the end of June of year t, we measure net stock issues as the natural log of the ratio of the split-adjusted shares outstanding at the fiscal year ending in calendar year t − 1 to the split-adjusted shares outstanding at the fiscal year ending in t − 2. This is defined as the natural logarithm of the difference in the total shares issued and outstanding for the firm two years before to one year before, namely, Ln(Total shares outstanding [NOSH](−1) − Ln(Total shares outstanding [NOSH](−2).	– –
Operating profit	OP is measured with accounting data for the fiscal year ending in year t − 1 and is revenues minus the cost of goods sold, minus selling, general, and administrative expenses, minus interest expenses, all divided by book equity. This is defined as operating income scaled by total shareholder equity, or book equity, (operating income [WC01250]/ Total shareholder equity [WC03995]).	– –
Price-to-cash- flow ratio	This is defined as the stock price to cash flow per share. It is defined as the market equity at the end of December of t − 1 divided by the cash flows for the fiscal year ending in calendar year t − 1. Cash flows are income before extraordinary items plus depreciation. Firms with nonpositive cash flows are excluded.	WC09604
Price-to-earnings ratio	The ratio for valuing a company that measures its current share price relative to its per-share earnings.	PE
Dividend yield	This is defined as the total dividends paid out from July of year t − 1 to June of year t divided by the market equity (from CRSP) at the end of June of year t. This is a Worldscope data item ["DY"].	DY
Asset growth (1 year)	This is a Datastream-specific term for the change in the total assets of the firm between the total assets reported two years before to one year before, (Total Assets (−2) – Total Assets (−1))/ Total Assets (−2) and is expressed in percentage change terms.	WC08625
CAPX Growth (2 year)	This is calculated from the change in the capital expenditure (CAPX) of the firm between the CAPX reported three years [WC04601] before to one year before [WC04601], (CAPX [WC04601] (−3) – CAPX [WC04601] (−1))/ CAPX [WC04601] (−3) and is expressed in percentage change terms. Capital Expenditure (CAPX) represent the funds used to acquire fixed assets other than those associated with acquisitions. It includes but is not restricted to additions to property, plant and equipment, investments in machinery and equipment but net of disposal.	– –
Sales Growth	This is defined as ((Current Year's Net Sales or Revenues / Net Sales or Revenues six years ago, reduced to a compound annual rate) - 1) * 100	WC08635
Book-to-market value	The ratio of the book value of equity to the market value of equity. Book equity is the total assets for the last fiscal year-end in calendar year t − 1, minus liabilities, plus balance sheet deferred taxes and investment tax credit if available, minus the preferred stock liquidating value if available, or the redemption value if available, or the carrying value, adjusted for net share issuance from the fiscal year-end to the end of December of t − 1. Market equity (market cap) is the price times shares outstanding at the end of December of t − 1. In Datastream, it is reported as the market-to-book value (Worldscope item 03501), implying the inverse of this item should be taken.	BTMV
Accrual/BE	The change in operating working capital per split-adjusted share from t − 2 to t − 1 divided by book equity per split-adjusted share at t − 1. Operating working capital is current assets minus cash and short-term investments minus current liabilities plus debt in current liabilities. Accrual is defined by (Current asset [WC02201] + Short-term investment [WC02001]) – (Current liability [WC03101] + Short-term debt and current portion of long-term debt [WC03051]), while it is scaled by book equity, defined as total shareholder equity [WC03995].	– –
Beta	Measured at the end of each June, it is the sum of the slopes from the regression of monthly returns on the current and first lag of monthly market returns. The regression uses the preceding 60 months of returns. It is calculated as the rolling 60-month (5 years × 12 months) beta from rolling window single-factor capital asset pricing model regression using the African aggregate value-weighted benchmark for each individual African stock.	– –
Volatility (daily, %)	Volatility is measured across the preceding 12 months in terms of the average daily stock price return variance, in terms of daily closing price returns [P]. Stock prices are denominated in local currency units.	– –
Liu liquidity metric	Our measure of illiquidity is the Liu (2006) multidimensional "trading speed" metric, which is the standardized turnover-adjusted number of zero daily trading volumes over the previous 12 months: $LM_{12} = (No.daily\ zero\ volumes\ in\ prior\ 12\ months) + \left(\frac{1}{\frac{12\ month\ turnover}{Deflator}}\right) \times \left(\frac{21\ days*12\ months}{NoTD}\right),$ where the 12-month turnover is calculated as the sum of the daily turnover over the previous 12 months, daily turnover is the ratio of the number of shares traded on a day [VO] to the number of shares outstanding at the end of the day [NOSH], NoTD is the total number of trading days in the market over the previous 12 months [derived from either "VO" or "P" data], and Deflator is chosen such that	– –

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Variable	Definition	Datastream Mnemonic
	$0 < \left(\frac{1}{\frac{12\ month\ turnover}{Deflator}}\right) < 1$ for all sample stocks. Following Liu (2006), a deflator of 480,000 is used in constructing estimates for the Liu metric, which is selected based on $0 < \text{Deflator} < 1$. Given the turnover adjustment (the second term in brackets in the first/top expression), two stocks with the same integer number of zero daily trading volumes can be distinguished. The one with the larger turnover is more liquid. Thus, the turnover adjustment acts as a tie breaker when sorting stocks based on the number of zero daily trading volumes over the previous X months. Because the number of trading days can vary from 15 to 23, multiplication by the factor (21 × 12/ NoTD) standardizes the number of trading days in a month to 21, which makes the liquidity measure comparable over time.	
Amihud	The Amihud price-impact measure is defined as: $1/D_M \sum_{t=1}^{M} (\lvert R_t\rvert / Price_t \times Volume_t)$ where D_M is the number of days in the month, M, and $\lvert R_t\rvert$ is the positive modulus of daily stock returns. $Price_t$ and $Volume_t$ are the daily stock closing prices and traded volumes respectively. The daily security prices are scanned for data errors, omissions and delistings. Following the procedure outlined in Lesmond (2005) the prices are used calculate daily returns. To control for return outliers, a data error filter eliminates daily prices that are +/− 50 % of the prior day's price and that day's price as well as previous day's price are deleted from the sample. Equally if zero volume occurs on day t, then that day is deleted from average. Finally, the measure is multiplied by 10^6 as undertaken in Amihud (2002) in order to provide a common representation of measures and facilitate comparison	
To	The ubiquitous monthly turnover measure is defined as: $1/D_M \sum_{t=1}^{M} (volume_t/\text{shares-outstanding})$ where D_M is the number of days in the month, M. It should be noted that there is considerable variation on an intra-market basis reflecting differences in both liquidity and turnover for many firms within each market. Any turnover statistics that exceed 100 % of the shares outstanding in any month are removed.	
Zero	The proportion of daily zero returns over a period of the total number of trading days in a month is based on the measure introduced by Lesmond, Ogden and Trzcinka (1999), or LOT (1999). It is calculated on a stock-by-stock basis using: $\frac{1}{D_M} \sum_{D=1}^{n} (ZeroDailyReturn)$ where D_M is the number of days in the month, M.	

Appendix C. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.irfa.2024.103752.

Data availability

Data will be made available on request.

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