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Effect of sampling locations on reliability of pile groups

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Abstract. Geotechnical site investigations represent an imperative prerequisite in the pursuit of reliable foundation designs. However, site investigations are frequently restricted to a limited number of locations owing to the constraints imposed by budget and time considerations, thereby potentially yielding a range of adverse consequences. Hence, the development of an efficient site investigation plan — one that optimally selects the number and location of tests, is important to gain adequate information for a given cost. This paper proposes a framework to identify optimal investigation locations by minimizing the probability of erroneous decisions (i.e., error types I and II). A pile group was adopted for the demonstration. The best locations corresponding to various numbers of tests were identified based on the derived probabilities of type I and II errors.

1. Introduction

Site investigations, such as cone penetration testing (CPT) and boreholes assume a pivotal role in the domain of geotechnical designs (e.g., pile foundations). Insufficient characterization of subsurface conditions can produce two adverse consequences: the creation of a significantly over-designed system, resulting in a lack of cost-effectiveness, or an under-designed configuration vulnerable to potential failures. Nevertheless, budgetary and time constraints inherently restrict the number of tests conducted for a given project. Consequently, the formulation of an efficient site investigation plan — one that optimally selects the number and location of tests, emerges as a critical imperative for procuring the requisite data.

Several studies have been undertaken to explore site investigation strategies in the context of optimizing geotechnical designs [1-4]. Jiang, *et al.* [5] identified the optimal borehole locations for slope reliability assessment by maximizing the information gained pertaining to soil properties, and revealed that the most favourable sampling location was near the crest of the slope. Similar results have been presented by Yang, *et al.* [1]. Goldsworthy, *et al.* [6] underscored the substantial reduction in the risk associated with foundation designs as the scope of the site investigations expands. Nevertheless, their results also brought to light the existence of an optimal site investigation expenditure threshold that minimizes the financial risk, beyond which supplementary sampling becomes superfluous. Similarly, Yang, *et al.* [7] conducted a comprehensive assessment that combined the costs of site investigation with those associated with slope failures. Their findings indicated the presence of an optimal site investigation scope, beyond which the cost of additional boreholes fails to justify the cost savings attributed to mitigating the slope failure risk. Arsyad, *et al.* [8] focused on the influence of radial distances between CPT soundings and the reliability of pile foundation designs. Their results demonstrated that closer proximity of CPT soundings to the designated pile significantly reduced the probability of over- or under-designing pile foundations. Crisp*, et al.* [9] comprehensively analysed the optimal location of a single borehole, and evaluated its sensitivity with regard to the testing location performance across various variables. Their results revealed that, across all building sizes and soil conditions, the optimal position for a single borehole involved a central location in the presence of a central pile. Likewise, in buildings featuring four piles and horizontal scale fluctuations matching or exceeding the pile spacing, a central borehole is favoured. In other cases the optimal borehole location was at one of the building corners adjacent to one of the four piles. However, it is noteworthy that, the influence of the sampling location on the probability of making incorrect decisions in the context of pile groups has not been explored.

In this study, based on the random field theory, finite difference method FDM), and Kriging method, a rigorous framework was proposed to determine the optimal sampling locations for pile group designs by minimizing the probability of erroneous decisions. To achieve this, the random field theory was utilized to generate spatially variable soil properties, which are then integrated into the FDM to derive a 'real' capacity of the pile group. After conducting the site investigation, the Kriging method was employed to generate a Kriged field based on site investigation data, which was then integrated into the FDM to derive the 'kriged' capacity of the pile group. These two capacities were compared to determine type I and II errors. Then, Monte Carlo simulations were performed to derive the error probabilities. Finally, optimal testing locations were obtained based on the the minimum error probabilities.

2. Methodology

2.1. Simple Kriging

The Kriging method is a technique widely employed for the integration of site investigation data [10]. Its fundamental objective is to provide the best estimation of a random field within a range of known data points. The underlying principle involves estimating the value of this random field at any given point through a weighted linear combination of the observed values of the field at all the data points. Assuming that $X_1, X_2, ..., X_N$ represent the observations of the random field $X(\mathbf{x})$ at the spatial locations $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$, such that $X_k = X(\mathbf{x}_k)$, the Kriged estimate of $X(\mathbf{x})$ at location $\mathbf{x}$ is expressed as follows:

$$\hat{X}(\mathbf{x}) = \sum_{k=1}^{N} \beta_k X_k , \tag{1}$$

where $\mathbf{x}$ is the spatial coordinate of the unobserved value; N is the number of observations; and β_k are unknown weights, established based on the correlation between the observations and unknown locations.

To obtain an unbiased estimation, the condition $\sum_{k=1}^{N} \beta_k = 1$ must be satisfied. The unknown Kriging weights, β_k, are determined through the minimization of the error variance, denoted by $E = (X(\mathbf{x}) - \hat{X}(\mathbf{x}))$. This minimization process leads to the solution of a matrix equation presented as follows:

$$\mathbf{K\beta} = \mathbf{M} , \tag{2}$$

where both $\mathbf{K}$ and $\mathbf{M}$ are contingent upon the covariance structure [11].

$$\begin{pmatrix} C_{11} & C_{12} & \cdots & C_{1N} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{N1} & C_{N2} & \cdots & C_{NN} & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \\ -\eta \end{pmatrix} = \begin{pmatrix} C_{1x} \\ \vdots \\ C_{Nx} \\ 1 \end{pmatrix}, \tag{3}$$

where C_{ij} is the covariance between X_i and X_j and η is a Lagrangian parameter utilized to address the problem of minimizing variance while adhering to the unbiased condition. The covariance C_{ix} featured in the vector on the right-hand side, $\mathbf{M}$, characterizes the covariance between the *i*th observation point and the point located at $\mathbf{x}$, where the best estimate is to be computed. It is to be noted that the Kriging matrix $\mathbf{K}$ is contingent solely upon the spatial coordinates of the observations and their associated covariance values.

2.2. Determining error probability

For a given statistics of soil properties, a 'real' site was generated through the application of random field theory. Subsequently, the capacity of the pile group, referred to as the 'true capacity', was determined based on the 'real' field using the FDM. After conducting the site investigations, the Kriging method was utilized to construct the 'Kriged field'. The capacity of the pile group based on the Kriged field, termed the 'Kriged capacity', was computed using the FDM. It is worth noting that, under a finite number of site investigations, the complete knowledge of a site is unknown, leading to the Kriged capacity deviating from the real capacity. Consequently, two types of errors can occur: 1) type I error, where the pile group is actually unsafe; however, the pile analysis erroneously indicates that it is safe (false safe); and 2) type II error, where the pile group is actually safe; however, the pile analysis erroneously suggests that it is unsafe (false unsafe). In this study, the pile group was deemed as safe if its capacity, taking into account the spatially random properties (i.e., the 'real' field and 'Kriged field'), exceeded the capacity calculated under the assumption of spatially uniform soil properties.

The estimation of type I and type II error probabilities was accomplished by the utilization of Monte Carlo simulations. In general, the procedure can be divided into two key components depending on whether the pile apacity analysis relies on 1) a complete knowledge of a site, or (ii) site investigations. The probabilities of errors can be determined by contrasting the outcomes of these two pile capacity analyses with those stemming from pile capacity calculations under the assumption of uniform soil properties. The optimal site investigation approach yields the lowest values for both type I and type II errors.

3. Example

To illustrate the proposed methodology, a 3×3 free-standing pile group situated within undrained clay was employed. The mean undrained shear strength was 20 kPa; the coefficient of variation was 50%; and the spatial correlation length, θ, ranged from 1 to 100 m. The arrangement of the pile group is presented in Figure 1. As per the guidance provided by Crisp, et al. [12], the placement of the testing locations was executed in proximity to the piles and was sequentially numbered from 1 to 9.

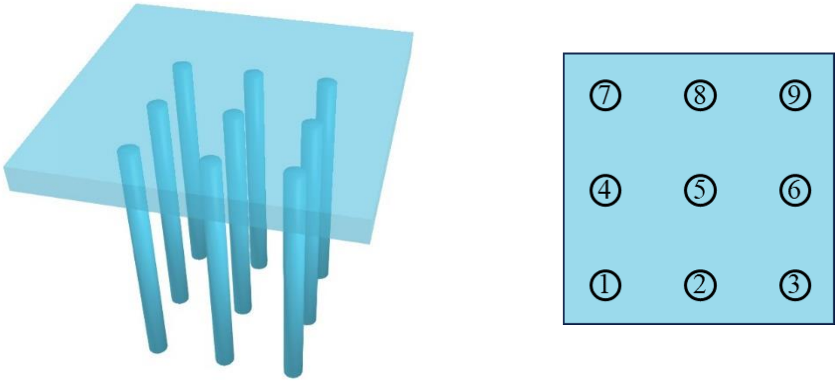

Figure 1. Pile group configuration (left) and plan view of pile locations (right).

Figure 2 depicts the variation in the probability of errors concerning the number of samplings, n, and their corresponding locations. Notably, Figure 2 illustrates a consistent decrease in the probability of errors with an increase in the number of samplings. Furthermore, under the condition of a single sampling, location #5 represented the optimal choice. In the case of two sampling tests, the combination of locations #2 and #8 was found to be the most effective, while with three samplings, locations #7, #9, and #2 proved to be the optimal choices. Additionally, Figure 2 highlights a general trend, wherein the probability of type I error was larger than that of type II error when the same number of tests were conducted in the same locations. For instance, when a single sampling was performed at location #5, the probability of type I error was 0.12, while the probability of type II error was 0.04.

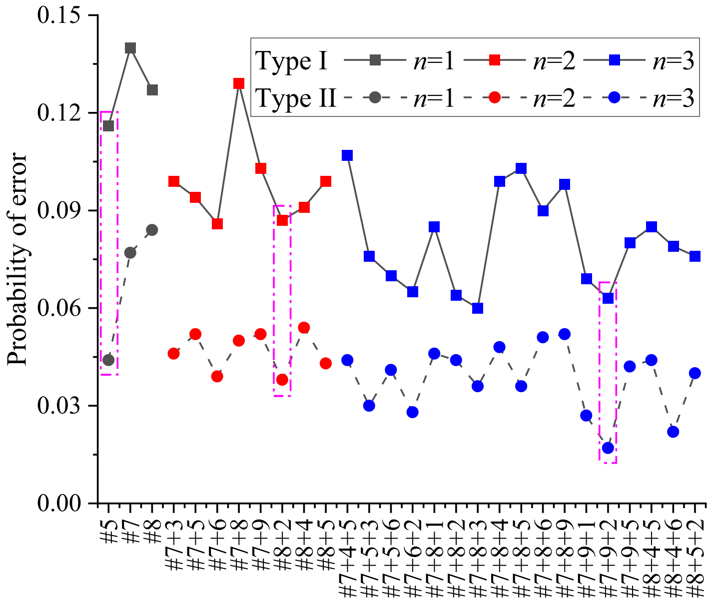

Figure 2. Probability of errors for different numbers of sampling and sampling locations.

Figure 3 illustrates the variation in the probability of type II error with respect to θ. It was assumed that the site investigation was carried out at the optimal location. A distinctive pattern emerges in Figure 3 as the probability of errors initially rose and subsequently declined with an increase in θ. The maximum value was observed when $\theta = 5\text{m}$. Figure 3 also reveals that the probability of type II error converged towards zero as the spatial correlation length approached its lower limit (i.e., $\theta \to 0$), where the site became entirely identical, and its upper limit (i.e., $\theta \to \infty$), where the site exhibited uniform characteristics. In such cases, a comprehensive understanding of the entire site can be obtained through a single sampling effort.

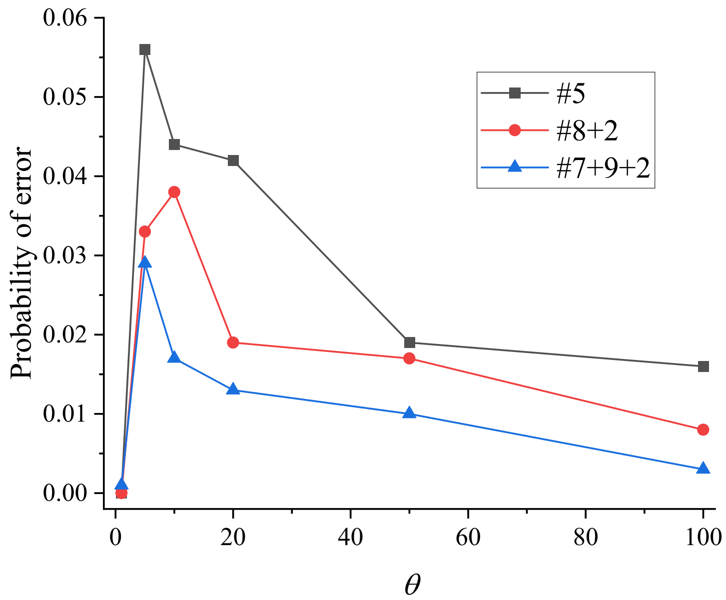

Figure 3. Probability of type II error for different spatial correlation lengths.

4. Conclusions

In this study, a probabilistic approach based on the random field theory, FDM, and Kriging method was proposed to identify the optimal sampling locations for pile group designs. Results indicate a substantial decrease in the error probabilities as the number of samplings increased. Furthermore, for any given number of samplings, specific optimal locations exist, which result in the minimal error probabilities. The results also underscore that the highest error probabilities were encountered in cases of intermediate spatial correlation lengths.

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