

Majorana mass generation, gravitational waves and cosmological tensions^a

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A neutrino Majorana mass generation in the early universe might have left imprints in cosmological observables. It can source the production of a detectable stochastic background of primordial gravitational waves with a spectrum that combines a contribution from a first order phase transition production and from the vibration of global cosmic strings. An intriguing possibility is given by the split seesaw model. In this case, in addition to the traditional high scale seesaw, a low scale neutrino Majorana mass generation can solve a potential primordial deuterium problem and ameliorate the cosmological tensions of the Λ CDM model. At the same time, it can also produce subdominant contribution to the NANOGrav signal of a stochastic background of gravitational waves (GWs), in addition to the astrophysical dominant contribution from supermassive black hole mergers.

1 A neutrino solution to the problem of the origin of matter in the universe

Despite there is no evidence of new physics at colliders (so far)³, the necessity to explain neutrino masses and the origin of matter in the universe requires the existence of new physics. Minimal WIMP dark matter models (those minimal ones realising the WIMP miracle) and electroweak baryogenesis have for long time been regarded as the natural solution to the problems of dark matter and matter-antimatter asymmetry of the universe, respectively. They are nicely both consistent with the idea of new physics close to the electroweak scale, as independently suggested by naturalness. However, the null results at colliders and the strong constraints on WIMPs imposed by direct and indirect searches, have in the last twenty years stimulated the rise of many new ideas and solutions beyond a ‘natural’ solution.

In particular, the discovery of neutrino masses in neutrino oscillation experiments make quite attractive those solutions that rely on extensions of the standard model (SM) able to explain neutrino masses. The most popular one is certainly represented by a minimal type-I seesaw extension of the SM, introducing of $N \geq 2$ right-handed (RH) neutrinos, with Yukawa couplings to the left-handed lepton doublets and Higgs doublet and with a right-right Majorana mass term. This simple extension allows, notoriously, to explain the matter-antimatter asymmetry of the universe with leptogenesis⁴ and even a dark matter solution where the lightest RH neutrino, with a keV mass, plays the role of dark matter^{5,6}. Within this simple bottom-up approach, while the Dirac mass term is generated at the electroweak spontaneous symmetry breaking, the Majorana mass is a given term in the fundamental lagrangian. However, it is reasonable that the Majorana mass term is also generated dynamically, in one or even multiple stages.

^aTalk based on ^{1,2}.

2 Majorana mass generation in the majoron model

There are different ways to describe the generation of a Majorana mass term. In a traditional top-down approach a Majorana mass term generation is described in grandunified models such as $SO(10)$ and it is related to the generation of all other mass terms. A simple bottom-up approach is to consider a majoron model⁷. Introducing a complex scalar field $\phi = (\varphi/\sqrt{2}) e^{i\theta}$, coupling to the RH neutrinos, one has the following simple extension of the SM Lagrangian,

$$-\mathcal{L}_{N_I+\phi} = \left(\overline{L}_\alpha h_{\alpha I} N_I \tilde{\Phi} + \frac{\lambda_I}{2} \phi \overline{N}_I^c N_I + \text{h.c.} \right) + V_0(\phi), \quad (1)$$

where $V_0(\phi)$ is the tree level potential associated to ϕ . At sufficiently high temperatures thermal effects enforce $\langle \phi \rangle = 0$ in a way to have global $U(1)_L$ symmetry restoration. Below a certain temperature T_\star the complex scalar field ϕ undergoes a phase transition at the end of which $\langle \phi \rangle = e^{i\theta_0} v_0/\sqrt{2}$ and $U(1)_L$ is spontaneously broken. This generates a Majorana neutrino mass term, so that RH neutrinos become massive with masses $M_I = \lambda_I v_0/\sqrt{2}$. After spontaneous symmetry breaking the field can be written as

$$\phi = \frac{e^{i\theta_0}}{\sqrt{2}} (v_0 + S) e^{i\frac{J}{v_0}} \simeq \frac{e^{i\theta_0}}{\sqrt{2}} (v_0 + S + iJ), \quad (2)$$

where S is a massive real scalar describing quantum fluctuations in the radial direction ($S = \delta\varphi$) and J is the majoron, a massless Goldstone field (an example of axion-like particle), describing angular quantum fluctuations ($J = v_0 \delta\theta$). Finally, after electroweak symmetry breaking, a Dirac mass term $m_D = v_{\text{ew}} h$ is also generated and a light neutrino mass matrix given by the seesaw formula $(m_\nu)_{\alpha\beta} = -m_D M^{-1} m_D^T$, where v_{ew} is the SM Higgs vev. We will refer to the SM particles as the visible sector while to the RH neutrinos N_I , the majoron J and massive boson S as the dark sector. For definiteness, we have assumed T_\star higher than the temperature of the electroweak symmetry phase transition but the case when Majorana mass generation occurs below the electroweak scale is also possible, we will also consider this option.

3 Gravitational waves from Majorana mass generation

The calculation of the GW spectrum, starting from the lagrangian, proceeds through a few steps. First of all one needs to calculate the *dressed effective potential* including thermal effects at 1 loop and resummed thermal masses that account for higher order effects. From a high temperature expansion, this can be written in the general polinomial form

$$V_{\text{eff}}^T(\varphi) \simeq D (T^2 - T_0^2) \varphi^2 - (AT + \tilde{\mu}) \varphi^3 + \frac{1}{4} \lambda_T \varphi^4, \quad (3)$$

where the different coefficients can be expressed in terms of the parameters of the lagrangian. An important role is played by $\tilde{\mu}$, since its presence creates a barrier at zero temperature and this makes the transition much stronger greatly enhancing the GW production. From the effective potential one can calculate the euclidean action $S_E(T)$ that gives the bubble nucleation rate.

From the euclidean action one can derive different quantities characterising the phase transition and the GW spectrum. First of all, one can calculate the *percolation temperature* that can be identified with the temperature of the phase transition if its duration is short enough, an approximation valid for not too strong phase transitions. Such duration is described by the quantity β/H_\star , giving approximately the age of the universe-to-phase transition duration at the temperature of the phase transition: the higher is β/H_\star the shorter is the duration of the phase transition compared to the age of the universe and vice-versa. One can also calculate the strength of the phase transition parameter $\alpha \equiv \varepsilon(T_\star)/\rho_R(T_\star)$, where ε is the latent heat feed in the phase transition and ρ_R is the total energy density of radiation. If the dark sector is coupled

to the visible sector, then its temperature $T_D = T$, otherwise one has to distinguish the two temperatures and also calculate separately $\alpha_D = \varepsilon(T_D^*)/\rho_{RD}(T_D^*)$ ^{8,9,10}, the strength of the phase transition in the dark sector. From these parameters one can calculate the GW spectrum from first order phase transition due to the nucleation of bubbles. The GW spectrum is defined as

$$h^2\Omega_{\text{GW}0}(f) = \frac{1}{\rho_{c0}h^{-2}} \frac{d\rho_{\text{GW}0}}{d\ln f}, \quad (4)$$

where ρ_{c0} is the critical energy density and $\rho_{\text{GW}0}$ is the energy density of GW, produced during the phase transition, both calculated at the present time. This is given by the sum of three contributions: one from turbulence, one from bubble collisions and one from sound waves¹¹. From current calculations, the sound wave contribution certainly dominates for $\alpha \lesssim 0.3$. In this case one has quite a reliable semi-analytical expression derived within a sound shell model¹², and supported by numerical calculations¹³, given by

$$h^2\Omega_{\text{sw}0}(f) = 3h^2 r_{\text{gw}}(t_*, t_0) \tilde{\Omega}_{\text{gw}} \frac{(8\pi)^{1/3} v_w}{\beta/H_*} \left[\frac{\kappa(\alpha)\alpha}{1+\alpha} \right]^2 \tilde{S}_{\text{sw}}(f) \Upsilon(\alpha, \beta/H_*), \quad (5)$$

where v_w is the bubble wall velocity, $\kappa(\alpha)$ is an efficiency factor giving the fraction of energy transferred in GWs, $\Upsilon(\alpha, \beta/H_*)$ is a suppression factor accounting for the duration of the GW production¹⁴. The normalised spectral shape function is given by $\tilde{S}_{\text{sw}}(f) \simeq 0.687 S_{\text{sw}}(f)$ with

$$S_{\text{sw}}(f) = \left(\frac{f}{f_{\text{sw}}} \right)^3 \left[\frac{7}{4 + 3(f/f_{\text{sw}})^2} \right]^{7/2}, \quad (6)$$

where f_{sw} is the peak frequency given by

$$f_{\text{sw}} = 8.9 \mu\text{Hz} \frac{1}{v_w} \frac{\beta}{H_*} \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{g_{\rho^*}}{106.75} \right)^{1/6}. \quad (7)$$

Finally, the redshift factor $r_{\text{gw}}(t_*, t_0)$, evolving $\Omega_{\text{gw}^*} \equiv \rho_{\text{gw}^*}/\rho_{c^*}$ into $\Omega_{\text{gw}0} \equiv \rho_{\text{gw}0}/\rho_{c0}$, is given by¹⁵

$$r_{\text{gw}}(t_*, t_0) = \left(\frac{a_*}{a_0} \right)^4 \left(\frac{H_*}{H_0} \right)^2 = \left(\frac{g_{S0}}{g_{S^*}} \right)^{\frac{4}{3}} \frac{g_{\rho^*}}{g_\gamma} \Omega_{\gamma 0} \simeq 3.5 \times 10^{-5} \left(\frac{106.75}{g_{\rho^*}} \right)^{\frac{1}{3}} \left(\frac{0.6875}{h} \right)^2. \quad (8)$$

The peak frequency depends linearly on the temperature of the phase transition and, therefore, the experimental identification of such a peak would provide a clear indication of the scale of new physics. For values $\alpha \gtrsim 0.3$, one can expect strong deviations from Eq. (5). Recently, some numerical calculations of $\Omega_{\text{GW},0}$, the total GW contribution to the energy density parameter, have been presented for $\alpha \leq 0.6$ ¹⁶. They show that there is some suppression that is particularly strong (up to three orders of magnitude) in the deflagration case, for $v_w < c_s$. In the case of detonation, for $v_w > c_s$, such suppression is contained within one order of magnitude and the suppression is weaker and weaker for higher values of v_w . For phase transitions in a dark sector the detonation case holds quite reliably.

As a *minimal model* one can consider the usual tree level quartic potential

$$V_0(\varphi) = -\frac{1}{2} \mu^2 \varphi^2 + \frac{\lambda}{4} \varphi^4. \quad (9)$$

In this case one has simply $v_0 \equiv \sqrt{\mu^2/\lambda}$, $m_S^2 = 2\lambda v_0^2$, the majoron J is massless and, importantly, $\tilde{\mu} = 0$. In this case the GW production turns out to be a few orders of magnitude below the experimental sensitivity of any experiment¹⁷.

However, one should also consider a contribution to the GW spectrum from the generation of a global string network during the phase transition². Compared to the Nambu-Goto string-induced almost flat gravitational wave spectrum associated with a gauged symmetry breaking, the contribution from global cosmic string-induced is typically suppressed and one can show that it is $\propto v_0^4$. This makes their detection at interferometers more challenging, unless the symmetry breaking scale v_0 is above 10^{14} GeV.

4 Majorana mass generation and GW production in multiple majoron models

It was already noticed in the case of the electroweak phase transition¹⁸ that the addition of an auxiliary scalar field η coupling to φ can generate a zero temperature barrier, corresponding to $\tilde{\mu} \neq 0$, greatly enhancing the GW production.

The minimal potential in Eq. (9) can be then generalised including the contribution from an auxiliary (real) scalar field η ¹⁷:

$$V_0(\varphi, \eta) = V_0(\varphi) + \zeta \varphi^2 \eta^2 - \frac{1}{2} \mu_\eta^2 \eta^2 + \frac{1}{4} \lambda_\eta \eta^4. \quad (10)$$

If we assume that the scalar field undergoes also a phase transition to a much higher temperature than T_* , settling to its true vacuum prior to the phase transition of φ with a vev $v_\eta \gg v_\varphi$, then at the end of the phase transition of η the effective potential for φ can be written again in the polynomial form Eq. (3) but this time with $\tilde{\mu} = \zeta^2 v_0 / (4\lambda_\eta) \neq 0$. This greatly enhances the strength of the first order phase transition and, therefore, the GW production that can in this case be detectable at future gravitational antennas². It is interesting that if the scale of the Majorana mass generation, that can be identified with v_0 , is lowered at the GeV scale, therefore in this case below the electroweak scale so that Majorana mass is generated after the Dirac mass term, then the GW spectrum peaks at mHZ frequencies, within LISA sensitivity. In this way there is an interesting interplay with collider searches of GeV RH neutrinos.

The general idea of introducing an auxiliary field can be realised, specifically, within a multiple majoron model¹. We can start first considering a two-majoron model. In this case one introduces two complex scalar fields denoted by ϕ_1 and ϕ_3 with their respective global lepton number symmetries $U(1)_{L_1}$ and $U(1)_{L_3}$. The lagrangian can be written as ($I = 1, 2, 3$)

$$-\mathcal{L}_{N_I + \phi_1 + \phi_3} = \left(\overline{L}_\alpha h_{\alpha I} N_I \tilde{\Phi} + \frac{y_1}{2} \phi_1 \overline{N}_1^c N_1 + \frac{y_2}{2} \phi_1 \overline{N}_2^c N_2 + \frac{y_3}{2} \phi_3 \overline{N}_3^c N_3 + \text{h.c.} \right) + V_0(\phi_1, \phi_3). \quad (11)$$

As one can see, ϕ_3 couples only to the RH neutrino N_3 , whereas ϕ_1 couples to both N_1 and N_2 . This can be ensured by giving nonzero $U(1)_{L_1}$ charges to N_1 and N_2 and half of their complementary charge to ϕ_1 , whereas N_3 and ϕ_3 have similar complementary charges under $U(1)_{L_3}$ only. Furthermore, we have chosen a basis where ϕ_1 and ϕ_3 only couple to the diagonal elements of the RH neutrino mass matrix. As in the single majoron model, we can introduce the radial components of the complex fields, writing $\phi_1 = \varphi_1 e^{i\theta_1} / \sqrt{2}$ and $\phi_3 = \varphi_3 e^{i\theta_3} / \sqrt{2}$. Again we can assume that the vacuum expectation values are along the real axis, $\langle \phi_1 \rangle = v_1 / \sqrt{2}$ and $\langle \phi_3 \rangle = v_3 / \sqrt{2}$. After spontaneous breaking of both $U(1)$ symmetries, one has this time two majorons $J_1 = v_1 \delta\theta_1$ and $J_3 = v_3 \delta\theta_3$. We further assume the hierarchy $v_3 \gg v_1$, so that the RH neutrino mass spectrum is hierarchical $M_3 \gg M_1 \simeq M_2 \simeq M$.

The $U(1)_{L_1} \times U(1)_{L_3}$ symmetry allows the usual quadratic and quartic terms for both ϕ_1 and ϕ_3 but also a quartic mixing between the two scalars, so that the tree level potential can now be written as

$$V_0(\phi_1, \phi_3) = -\mu_1^2 |\phi_1|^2 + \lambda_1 |\phi_1|^4 - \mu_3^2 |\phi_3|^2 + \lambda_3 |\phi_3|^4 + \zeta |\phi_1|^2 |\phi_3|^2. \quad (12)$$

This quartic mixing is now exactly the kind of term we have seen to be able to generate a term $\tilde{\mu} \neq 0$ in the effective potential, as we have seen in the generic case of adding an auxiliary scalar field. The role of auxiliary field is now played by φ_3 . This time one has a first high scale phase transition at a temperature $T_{*,3} \sim v_3$ where $U(1)_{L_3}$ is broken and a Majorana mass $M_3 = y_3 v_3$ is generated and then a low scale phase transition occurring at $T_{*,1} \sim v_1$ where $U(1)_{L_1}$ is broken and Majorana masses $M_1 = y_1 v_1$ and $M_2 = y_2 v_1$ are generated. In this way, we obtain GW spectra that at the peak are within the sensitivity of future GW antennas. However, in addition to the GW contribution from sound waves generated by bubble nucleation, one can also have an additional contribution from the network of cosmic strings generated by the $U(1)_{L_3}$ symmetry breaking that is sizeable if $v_3 \gtrsim 10^{14}$ GeV.

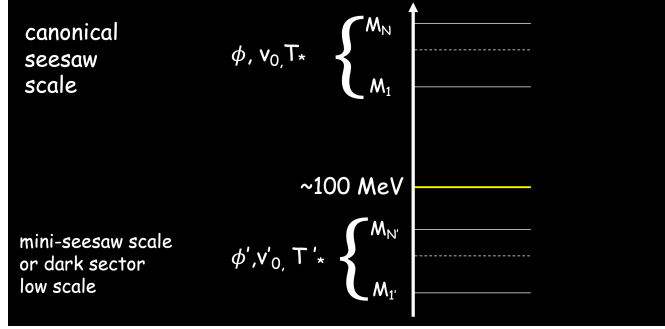


Figure 1 – Split seesaw model.

One can of course make a step further and consider a three-majoron model where each majoron is the leftover of a different $U(1)_{L_I}$ ($I = 1, 2, 3$) symmetry breaking. One introduces now three complex scalar fields ϕ_I and extends the SM with the lagrangian

$$-L_{N_I+\phi_I} = \left(\overline{L}_a h_{aI} H N_I + \frac{y_1}{2} \phi_1 \overline{N}_1^c N_1 + \frac{y_2}{2} \phi_2 \overline{N}_2^c N_2 + \frac{y_3}{2} \phi_3 \overline{N}_3^c N_3 + \text{h.c.} \right) + V_0(\phi_1, \phi_2, \phi_3), \quad (13)$$

imposing a $U(1)_{L_1} \times U(1)_{L_2} \times U(1)_{L_3}$ symmetry. Denoting the vevs by $\langle \phi_I \rangle \equiv v_I$ and assuming $v_3 \gg v_2 \gg v_1$, the tree-level scalar potential is given by

$$V_0(\phi_1, \phi_2, \phi_3) = \sum_{I=1,2,3} \left[-\mu_I^2 \phi_I^* \phi_I + \lambda_I (\phi_I^* \phi_I)^2 \right] + \sum_{I,J,I \neq J}^{1,2,3} \frac{\zeta_{IJ}}{2} (\phi_I^* \phi_I) (\phi_J^* \phi_J). \quad (14)$$

This time there will be a first very high scale phase transition occurring at $T_{*3} \sim v_3$ where $U(1)_{L_3}$ is broken and a Majorana mass $M_3 = y_3 v_3$ is generated. There will be a sizeable GW production from global cosmic strings if $v_3 \gtrsim 10^{14}$ GeV. This will be followed by a second phase transition at $T_{*2} \sim v_2$ where $U(1)_{L_2}$ is broken and a Majorana mass $M_2 = y_2 v_2$ is generated. There will be also an enhanced GW production at bubble nucleation from sound waves with a peak. Finally, a third phase transition occurs at $T_{*1} \sim v_1$ associated to $U(1)_{L_1}$ symmetry breaking with the generation of a Majorana mass $M_1 = y_1 v_1$ and again a GW production from sound waves with a second peak at lower frequencies. In this way the GW spectrum would be certainly quite distinctive with two bumps standing from the smooth spectrum generated by global cosmic strings if $v_3 \gtrsim 10^{14}$ GeV. Finally, we should stress that in all these considerations there is the assumption for the reheat temperature to be sufficiently large that the phase transitions can actually occur, this is a point that is often missed.

5 Split majoron model, cosmological tensions and NANOGrav signal

We have so far considered the case of high scales for the Majorana mass generation, much higher than ~ 100 MeV, that means before the quark-hadron phase transition. In this way the massless majorons, that get thermalised at the phase transition, contribute in a negligible way to the excess radiation compared to the standard model case and one has not to worry about cosmological constraints. Let us now consider a setup that we will refer to as *split majoron model*. This is depicted in the figure. As one can see, we assume that in addition to a traditional seesaw scale(s) with N RH neutrinos, there is a mini-seesaw scale or a dark sector scale, below ~ 100 MeV, where there are additional N' RH neutrinos and a complex scalar field ϕ' . For definiteness we can consider $N = 2$ and $N' = 1$. For example, the lightest RH neutrino could be responsible for the lightest neutrinos mass, as in the ν MSM model⁶.

The high scale Majorana masses are generated by the phase transition of a complex scalar field ϕ . At the end of the phase transition a massless majoron J is left over but again its contribution to radiation is strongly diluted by photon production from all SM particle annihilations

and can be neglected. At some temperature T'_* , the complex scalar field ϕ' undergoes a first order phase transition after which $U(1)_{L'}$ symmetry is broken a low scale Majorana mass M' is generated and a massless majoron J' survives as a cosmological relic. This time its contribution to extra radiation is not negligible. If one considers a phase transition with $T'_* \simeq 10$ MeV, before electroweak interactions decouple, then the extra radiation contribution from J' parameterised in terms of extra-number of effective neutrinos, is given by $\Delta N_\nu = 4/7 \simeq 0.6$. A model producing such a fractional amount of extra neutrinos had been proposed, after first *Planck* satellite results in 2013, as a way to reconcile the Hubble tension, either produced from the decays of a massive particle¹⁹ or just in the form of a Goldstone boson leftover of some $U(1)$ symmetry breaking²⁰, correspondingly exactly to our case.

However, today such a mere injection of extra radiation would reconcile the Hubble tension but it would deform the positions of CMB peaks in an unacceptable way and something more sophisticated needs to be done. Let us consider a case where $T'_* \lesssim T_\nu^{\text{dec}} \sim 1$ MeV. Moreover let us assume that the dark sector has decoupled at high energies so that this time $T_D \neq T$. At temperatures below neutrino decoupling, and prior to the low scale phase transition, ordinary neutrinos interact with majorons J and complex scalar field ϕ' via the effective lagrangian

$$-\mathcal{L}_{\nu+D} = \frac{i}{2} \sum_{i=2,3} \tilde{\lambda}_i \bar{\nu}_i \gamma^5 \nu_i J + \zeta J |\phi'|^2. \quad (15)$$

These interactions can thermalise the ordinary neutrinos to the dark sector after neutrino decoupling (*rethermalisation*), in a way that they reach a common temperature

$$T_{\nu+D} = T_\nu^{\text{SM}}(T) \left(\frac{N_\nu^{\text{SM}}(T)}{N_\nu^{\text{SM}}(T) + N' + 12/7 + 4\Delta g/7} \right)^{\frac{1}{4}}. \quad (16)$$

For example, in the minimal case $N' = 1$ and $\Delta g = 0$, one has $T_{\nu+D} \simeq 0.815 T_\nu^{\text{SM}}$. Prior to rethermalisation, the extra radiation contribution is negligible. After rethermalisation and after the ϕ' phase transition, occurring below 1 MeV, one has

$$\Delta N_\nu \simeq 3.043 \left[\left(\frac{3.043 + N' + 12/7 + 4\Delta g/7}{3.043 + N' + 12/7 + 4\Delta g/7 - N_h} \right)^{\frac{1}{3}} - 1 \right]. \quad (17)$$

In the minimal case one has $\Delta N_\nu \simeq 0.465$. This extra-radiation is produced after neutrino decoupling and, therefore, it does not modify the prediction of the primordial helium abundance in standard BBN. However, if the phase transition occurs at temperatures above nucleosynthesis, for $T \gtrsim T_{\text{nuc}} \simeq 65$ keV, this extra-radiation modifies the primordial deuterium abundance. Current constraints from deuterium on ΔN_ν gives $\Delta N_\nu(t_{\text{nuc}}) \lesssim 0.4$ at 95% C.L.²¹ and so there would be a tension. This can be solved introducing extra-degrees of freedom in the dark sector, so that $\Delta g \neq 0$. However, using theoretical ab-initio energy dependencies of nuclear rates, a group has recently obtained $\Delta N_\nu = 0.3 \pm 0.15$ ²², hinting at the presence of non-standard physics. In this case the split majoron model would perfectly provide the modification of standard BBN able to solve this potential deuterium problem. It is intriguing that for phase transitions temperatures $T'_* \sim 100$ keV the peak of the GW spectrum produced during the phase transition from sound waves peaks exactly within the range of frequencies tested by NANOGrav. The signal is not sufficiently strong to explain the whole signal. However, at the peak and for values sufficiently large, $\alpha \gtrsim 0.4$, the signal can be within the sensitivity of NANOGrav and, therefore, if disentangled from the astrophysical contribution expected from super massive black hole mergers, it could be revealed. As we mentioned, for such large values of α , the GW spectrum from sound waves is still poorly determined. Some recent calculations²³ show that around the peak the GW spectrum can be strongly enhanced for $\alpha \gg 0.1$ with respect to the expression Eq. (5) derived from sound shell model and recovered exactly for $\alpha \lesssim 0.1$. If the deuterium

problem will be confirmed, then the search of such contribution within the NANOGrav signal and a better understanding of the GW spectrum produced from first order phase transitions for $\alpha \sim 0.5$ will become crucial since, in combinations with the cosmological tensions, it would provide a clear signature of the split majoron model.

Finally, let us say that the effect of extra radiation at recombination now can be compensated by the modification of free streaming length of ordinary neutrinos in a way that positions of CMB peaks are unchanged. Actually, such a model has been shown to be able to give a better fit of cosmological observations compared to the Λ CDM model^{24,25} and in this respect it is one of the best performing models in improving the fit of cosmological observations compared to the Λ CDM model²⁶.

6 Conclusions

The generation of Majorana mass can lead to the production of a stochastic background of primordial GWs at the seesaw scale or scales, in the case of a multiple majoron model. The split majoron model can modify pre-recombination era in an interesting way, since it would address a potential emerging deuterium problem and ameliorate cosmological tensions within Λ CDM. At the same time it can give a contribution to the NANOGrav signal.

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