Optimal Monetary Policy Mix at the Zero Lower Bound*

Dario Bonciani[†] Joonseok Oh[‡]

November 19, 2024

Abstract

We study the optimal mix of forward guidance and quantitative easing at the ZLB. The welfare loss function depends on inflation, output, and consumption heterogeneity (which we label as *inequality*) between different households. When solely focusing on inflation and output, the central bank excessively expands its balance sheet, thereby increasing inequality. Forward guidance is more effective at stabilising inflation, and quantitative easing at stabilising output. The two tools are, therefore, complementary. Since neither instrument can fully neutralise adverse demand shocks, the optimal policy combines both, resulting in a shorter ZLB duration and milder balance-sheet expansion than if the central bank relied on one policy instrument alone.

Keywords: Optimal monetary policy, Unconventional monetary policy, Forward guidance, Quantitative easing, Zero lower bound

JEL Classification: E52, E58, E61

*This project was initiated while Dario Bonciani was employed at the Bank of England. The views expressed in this paper are solely those of the authors and do not necessarily reflect the views of the Bank of England or its committees.

†Department of Economics and Law, Sapienza University of Rome, Via del Castro Laurenziano 9, 00161 Rome, Italy. E-mail: `dario.bonciani@uniroma1.it`.

‡Department of Economics, School of Economic, Social and Political Sciences, University of Southampton, Murray Building (58), Southampton SO17 1BJ, United Kingdom. E-mail: `joonseok.oh@soton.ac.uk`.

1 Introduction

In the wake of the 2008 Global Financial Crisis, nominal short-term interest rates in the U.S. and other advanced economies approached the zero lower bound (ZLB), prompting central banks to implement various forms of unconventional policies. These policies included public communications regarding the future path of the policy rate, known as forward guidance (FG), as well as balance-sheet policies involving large-scale asset purchases, commonly referred to as quantitative easing (QE).

While the literature has analysed these policy instruments separately (see, e.g., Bilbiie, 2019, on FG or Sims et al., 2023, on QE), there is relatively little work on their optimal combination. In this paper, we fill this gap by studying optimal policy at the ZLB in the context of a tractable two-agent New Keynesian (TANK) model along the lines of Sims et al. (2023), in which the central bank can carry out both FG and QE. The model features patient (savers) and impatient (borrowers) households. The former save in short-term bonds, whereas the latter borrow resources from a financial intermediary via long-term bonds. Frictions in the financial sector allow QE to impact the real economy. Differently from the original model, we assume that households feature bounded rationality (Gabaix, 2020), which mitigates the excessive power of FG under rational expectations (FG Puzzle, Del Negro et al., 2015). In response to shocks, the central bank can either adjust the short-term rate, subject to a ZLB constraint, or implement QE, by purchasing long-term bonds. In its log-linearised form, this stylised model boils down to an extension of the basic three-equation model (Woodford, 2003 and Galí, 2015), which allows accounting for QE.

In this context, we derive the social welfare loss function.[1] In contrast with the canonical three-equation model, where the welfare loss is solely a function of inflation and output volatility, in our model it includes an additional term related to fluctuations in consumption heterogeneity between savers and borrowers (we label this term as "inequality"). This last term stems from the TANK structure of the model, as seen in related literature such as Bilbiie (2008, 2024), Bilbiie and Ragot (2021), and Debortoli and Galí (2024). The rationale behind the inclusion of this extra term is that when the central bank expands its balance sheet to improve credit conditions, it induces an increase in the consumption of borrowers, who hold long-term bonds, relative to the consumption of savers. As a result, this policy has unequal effects on the consumption of these two groups of households. Therefore, a welfare-maximising central bank should avoid large countercyclical changes in its balance sheet to mitigate excessive swings in relative consumption between savers

[1] which focuses on optimal monetary policy under discretion using an ad-hoc (dual-mandate) objective function for the central bank, we derive the actual social welfare loss function. Additionally, we consider both optimal monetary policies under discretion and commitment, allowing us to study the interaction between FG and QE.

and borrowers.

While away from the ZLB, the central bank can fully stabilise inflation and output in response to a fall in demand using a single policy instrument, namely the short-term interest rate, this is not the case when the economy reaches the ZLB. In other words, neither FG nor QE separately can fully neutralise the effects of adverse demand shocks. The reason for this lies in the fact that FG and QE, unlike conventional monetary policy, have impacts on both aggregate demand and aggregate supply. Specifically, FG, which involves maintaining the short-term interest rate at zero for an extended period, boosts inflation expectations, positively impacting demand, and negatively affecting supply. On the other hand, QE expands both aggregate demand, by relaxing financing conditions, and aggregate supply, by introducing a negative wealth effect for lenders, prompting them to supply more labour. As a result, FG is more effective in stabilising inflation, while QE is better suited for stabilising output. Therefore, these two policies are imperfect substitutes and partially complement each other.

Following an adverse demand shock that drives the policy rate to the ZLB, the optimal monetary policy under commitment involves a combination of FG and QE.[2] Specifically, FG sustains current demand by raising expectations about inflation and output, while QE further eases the initial decline in demand. A subsequent contraction of the central bank's balance sheet, known as quantitative tightening (QT), mitigates any overshooting of prices and real activity caused by FG. This optimal mix of policies significantly enhances welfare, requiring a shorter ZLB duration and milder balance-sheet expansion compared to a scenario where the central bank can only rely on a single instrument. Specifically, absent balance-sheet policies (assuming a constant size of the central bank's balance sheet), the central bank would solely rely on FG and would need to keep the short-term interest rate at zero for a considerably longer period in response to a decline in demand. Conversely, without FG, the central bank's optimal policy would require a larger expansion of the balance sheet on impact.

Next, we study how the degree of bounded rationality impacts the optimal policies under both discretion and commitment, following the approach of Nakata et al. (2019). Under discretion, whether QE is implemented or not, a higher degree of bounded rationality necessitates a quicker return of the short-term rate to its steady-state value once the ZLB constraint is no longer binding. This is because bounded rationality reduces

[2]Under discretion (see Section 3) the optimal policy does not imply any FG and, hence, the short-term rate lifts off as soon as the ZLB constraint is not binding. In the absence of QE, the optimal policy under discretion coincides with a strict inflation-targeting rule for the short-term rate. This policy implies a severe recession at the ZLB in response to a negative demand shock. QE helps mitigate the decline in inflation and output. However, the welfare losses under discretion are substantially larger than under commitment.

the negative impact of the shock, as agents place less emphasis on a future decline in expected macroeconomic conditions. Under commitment, without QE, greater bounded rationality, weakening the effectiveness of FG, requires a longer ZLB duration to sufficiently stabilise inflation and output. Therefore, in the absence of QE, the central bank should maintain the short-term rate at zero longer than it would under fully rational expectations. Conversely, in the presence of QE, weaker FG results in a shorter ZLB duration and a more significant expansion of the central bank's balance sheet. In other words, as FG becomes less effective, the central bank compensates by using QE to stimulate demand.

To further highlight the trade-off between stabilising inflation, output, and inequality, we consider the case in which the central bank aims to minimise a simple loss function, only accounting for inflation and output volatility, as in the canonical three-equation model. An important reason to consider this case is that, in practice, central banks' mandates entail a limited set of objectives, such as stabilising inflation and (sometimes) a measure of economic slack (Svensson, 2010 and Debortoli et al., 2019). We find that when monetary policy aims to minimise a simple ad-hoc loss function, disregarding household inequality, welfare losses in response to a negative demand shock are substantially larger than with the optimal policy under commitment. This is because the central bank ends up excessively expanding its balance sheet, which strongly increases inequality. Moreover, we show that the benefits of QE significantly depend on the relative weight associated with output stabilisation. When this weight is large (i.e., low weight on inflation), QE improves welfare. This is because output stabilisation requires relatively small adjustments in the central bank's balance sheet (compared to inflation stabilisation). As a result, the welfare gains from QE in terms of reducing inflation and output volatility outweigh the costs associated with higher inequality. Conversely, when the weight on output is small (i.e., a large weight on inflation), QE worsens welfare as the costs associated with balance-sheet volatility become very large.

Finally, we consider the case where the central bank sets the short-term interest rate and the size of its balance sheet using simple policy rules. In this case, we find that if the central bank sets the short-term rate according to a strict price-level-targeting rule and QE based on an output-targeting rule, it can achieve welfare outcomes close to the optimal monetary policy under commitment.[3] The result that the optimised QE rule should only target output is in line with Sims et al. (2023).

[3] The reason for considering a price-level targeting rule for the short-term rate, is that this policy implies history dependence in the central bank's reaction function, similar to FG. In contrast, a strict inflation-targeting rule does not imply history dependence of the short-term rate and welfare losses are, therefore, larger.

Related Literature This paper is strictly related to the strand of the macroeconomic literature studying the optimal conduct of monetary policy when nominal short-term rates are at the ZLB. Eggertsson and Woodford (2003) examines the implications of the ZLB on the ability of a central bank to counteract deflation. They show that a credible commitment to the right sort of history-dependent policy can largely mitigate the distortions created by the ZLB. Jung et al. (2005) show that at the ZLB, the optimal monetary policy response implies policy inertia, i.e., a zero interest rate policy should be continued for a while even after the natural rate returns to a positive level. Adam and Billi (2006, 2007) study optimal monetary policy under commitment and discretion at the ZLB. Agents anticipate the possibility of reaching the lower bound in the future, which amplifies the effects of adverse shocks before the ZLB is actually reached. This result calls for a more aggressive response by the central bank. Under the discretionary monetary policy, output losses and deflation are much larger than under commitment. Werning (2012), Schmidt (2013), and Nakata (2016) study the joint optimal monetary and fiscal (government spending) policy problem when the economy is at the ZLB. They find that the optimal policy under commitment is a mix of monetary and fiscal stabilisation. Bilbiie (2019) studies how long a central bank should keep interest rates at a low level after a liquidity trap ends. The paper argues that the optimal duration is approximately half the time the economy spends in a liquidity trap. Nakata et al. (2019) show that in a framework where the stimulating ability of FG is relatively muted, and the economy is in a liquidity trap, the monetary policy authority should commit to keeping the policy rate at zero for a significantly long time.[4]

Our work also relates to the literature on optimal monetary policy in models with heterogeneous agents. In particular, Bilbiie (2008) studies, among other things, optimal monetary policy in the context of a stylised TANK model, in which a share of the agents is hand-to-mouth, meaning they have limited participation in asset markets. Cúrdia and Woodford (2016) and Nisticò (2016) study optimal monetary policy in models with infrequent participation and borrowers and savers. Challe (2020) analyses optimal monetary policy in a heterogeneous-agent New Keynesian (HANK) model with labour market frictions and idiosyncratic unemployment risk. In such a context, contractionary cost-push or productivity shocks lead to a rise in precautionary savings and a fall in inflation, calling for an accommodative monetary policy. Bilbiie and Ragot (2021) show that price stability is not optimal when households self-insure against idiosyncratic risk using scarcely available liquid assets. Acharya et al. (2023) study optimal monetary policy in a HANK framework, where the central bank's objective function accounts for consumption inequality, in addition to stabilising inflation and output. When income risk is countercyclical, they find that policy curtails the fall in output

[4] A non-exhaustive list of papers dealing with monetary policy at the ZLB is Nakov (2008), Christiano et al. (2011), Nakata (2017), Nakata and Schmidt (2019), Masolo and Winant (2019), and Bonciani and Oh (2023a,b).

in recessions to alleviate the increase in inequality.

Our paper builds on the literature analysing QE within DSGE models. Gertler and Karadi (2011, 2013) and Carlstrom et al. (2017), among others, incorporate QE into medium-scale DSGE models. Cui and Sterk (2021) analyse the impact of QE in a New Keynesian model with heterogeneous agents and find that QE is highly stimulative and successfully mitigated the drop in demand during the Great Recession. However, their paper suggests that QE could, as a byproduct, significantly increase inequality and thereby reduce welfare. Lee (2021) develops a HANK model to study how QE affects households' welfare across the wealth distribution. Sims and Wu (2021) build on Gertler and Karadi (2013)'s modelling of the financial sector to analyse the impact and interaction of the main forms of unconventional monetary policy (FG, QE, and negative interest rates). Sims et al. (2023) develop a tractable four-equation New Keynesian model that accounts for QE, whereas Sims and Wu (2020) deploy this framework to study the degree of substitutability between conventional monetary policy and QE. Unlike Sims et al. (2023), we focus on the interaction between FG and QE. We also explicitly derive the exact welfare loss, which is a function not only of inflation and output but also inequality. Karadi and Nakov (2021) study optimal QE in a model with banks and occasionally binding constraints. The paper shows that balance-sheet policies are effective in offsetting large financial shocks. However, it also highlights that these policies could be ineffective against non-financial shocks and may be in general an imperfect substitute for conventional interest rate policies. Within a tractable New Keynesian model, Wu and Xie (2022) study the interaction of monetary policies, such as conventional short-term rate policy and QE, and fiscal policies, namely transfers and government spending. One crucial difference to our paper is that in their paper monetary policy is not constrained by the ZLB. Cardamone et al. (2023) show that during times when the production sector is facing significant cash-flow shortages, such as the COVID-19 crisis, QE should be aimed at lending to non-financial corporations rather than banks.

Finally, a closely related paper to ours is Harrison (2017), which studies optimal QE in a model with portfolio adjustment costs. The paper focuses on the optimal policy under discretion and models QE such that it is a perfect substitute for conventional monetary policy. In our paper, instead, we study the optimal monetary policy both under commitment and under discretion. QE affects both the demand and the supply side of the economy and, as a result, is an imperfect substitute for conventional monetary policy. By studying optimal monetary policy under commitment at the ZLB, we analyse the interactions between FG and QE and show the implications of mitigating the power of FG. Furthermore, we study cases where the central bank targets simplified loss functions or follows simple policy rules.

The remainder of the paper is structured as follows. In Section 2, we describe the basic model. Section 3 studies the welfare implications of optimal monetary policy under discretion and under commitment. Section 4 provides an analytical discussion of the implications of optimal monetary policies. Section 5 analyses numerical results, namely welfare implications and impulse response functions. In Section 6, we consider the case where the central bank targets a simpler (ad-hoc) loss function rather than the welfare-based loss function. Section 7 studies the case where the central bank sets the short-term interest rate and the size of its balance sheet according to simple policy rules. Finally, Section 8 presents some concluding remarks.

2 Model

Our model is based on Sims et al. (2023), which extends the basic three-equation framework to allow for QE. In its nonlinear form, the model includes two types of agents, patient and impatient, and financial intermediaries (modelled along the lines of Gertler and Karadi, 2011) subject to a risk-weighted leverage constraint. It features short and long-term bonds, which, combined with the credit frictions, allows QE to affect real activity. Besides setting the short-term interest rate, the central bank can purchase long-term bonds by expanding its balance sheet. The increase in the price of long-term bonds, as a consequence of the unconventional monetary policy, relaxes the financial intermediary's leverage constraint, easing the supply of credit. Therefore, in such a framework, QE is equivalent to an increase in credit supply.

For the purpose of our analysis, we deviate from the original model in two ways. First, to match empirically plausible inflation dynamics at the ZLB, we assume industry-specific labour markets rather than aggregate labour markets, in line with Woodford (2003).[5] This assumption has important implications for real rigidities. Secondly, we assume that both types of households feature boundedly-rational expectations (Gabaix, 2020) to alleviate the FG puzzle (Del Negro et al., 2015). To simplify the analysis, we assume that the decision-making process at the firm level remains fully rational.[6] In practice, these assumptions imply that the IS curve will feature an additional discounting term, which could be also rationalised by assuming incomplete markets (McKay et al., 2016, 2017), portfolio adjustment costs (Cantore and Freund, 2021), or wealth in the utility function (Michaillat and Saez, 2021). For the sake of conciseness, we leave the derivations and the details of the model to Appendices A and B.

[5] This is a common assumption in the ZLB literature (see, e.g., Eggertsson, 2011, Schmidt, 2013, Eggertsson and Singh, 2019), implying a flatter NKPC, which allows matching the "missing deflation" during the 2008 Global Financial Crisis.

[6] Assuming firms to be also boundedly rational would imply an additional discounting term in the NKPC. However, discounting the IS curve is sufficient to solve the FG puzzle (see e.g., McKay et al., 2016). The main conclusions of the paper would remain unaffected by this assumption, as shown in Appendix G.

The first two equations of the linear version of the model are a discounted IS curve and a New Keynesian Phillips curve (NKPC), augmented to account for changes in the central bank's balance sheet:

$$y_t = (1-\alpha) E_t y_{t+1} - \frac{1-z}{\sigma} \left(r_t^s - E_t \pi_{t+1} - r_t^n\right) + z\bar{b}_{cb} \left(qe_t - (1-\alpha) E_t qe_{t+1}\right), \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \left(\chi + \frac{\sigma}{1-z}\right) y_t - \frac{\gamma \sigma z \bar{b}_{cb}}{1-z} qe_t. \tag{2}$$

Lowercase variables denote log deviations around the deterministic steady state. π_t is inflation and y_t is output. r_t^s is the nominal policy rate, whereas qe_t denotes the real value of the central bank's long-term bond holdings. r_t^n captures exogenous fluctuations in the natural real rate of interest, which can also be interpreted as a demand shock. This shock is commonly used in the literature to achieve a binding ZLB constraint.

The parameter $\alpha \in [0, 1]$ captures the degree of cognitive discounting under the assumption of bounded rationality of households.[7] In particular, when $\alpha = 0$, households feature fully rational expectations. The parameter σ represents the inverse elasticity of intertemporal substitution, β is the discount factor of the patient households, and χ is the inverse labour supply elasticity. γ is a convolution of deep parameters defined as $\gamma \equiv \frac{(1-\phi)(1-\phi\beta)}{\phi(1+\chi\varepsilon)}$, where ϕ is the price rigidity parameter (Calvo, 1983) and ε is the elasticity of the intermediate good's demand.[8] The parameter z is the share of impatient households, and $\bar{b}_{cb}$ is the steady-state share of long-term bonds that the central bank holds.

Finally, one important distinction between conventional monetary policy and QE, in this model, is that qe_t enters both the IS curve and NKPC.[9] The intuition behind this feature of the model is as follows. Keeping the aggregate level of output fixed, improved credit conditions (due to QE) induce a reallocation of resources from the patient (savers) to the impatient (borrowers) households. Since only the patient households supply labour, (similarly as in, e.g., Carlstrom and Fuerst, 1997 and Iacoviello, 2005), the reallocation of resources causes a negative wealth effect, putting downward pressure on wages and real marginal costs.

3 Optimal Monetary Policy

This section presents the welfare loss function implied by the model and studies two types of possible optimal monetary policy conducts. Specifically, we define the optimal monetary policies under discretion

[7] It bears noting that the discounting term appears only in front $E_t y_{t+1}$ and $E_t qe_{t+1}$ but not in front of $E_t \pi_{t+1}$. Following Gabaix (2020), this is because inflation expectations enter the time-t real rate $r_t^r \equiv r_t^s - E_t \pi_{t+1}$.

[8] In Sims et al. (2023)'s original model, assuming aggregate labour markets, instead, $\gamma \equiv \frac{(1-\phi)(1-\phi\beta)}{\phi}$.

[9] This makes the effects of QE akin to a government spending stimulus. Typically, government spending shocks enter the IS and the NKPC with a similar sign as QE in our model.

and commitment.

3.1 Welfare-Theoretic Loss Function

To understand the relevant policy trade-offs, we derive a second-order approximation to the aggregate welfare loss function (see Appendix C), similarly as in Woodford (2003). We assume that in the steady state, a subsidy corrects the distortion stemming from monopolistic competition and that trend inflation is equal to zero. Deriving the aggregate welfare loss function entails calculating the welfare of the two households and weighting them by their shares in the population.[10] Households' welfare at time t is given by the expected discounted sum of current and future utility flows. Following Gabaix (2020), boundedly-rational households are assumed to experience utility from consumption and leisure like rational households. Hence, the welfare criterion is invariant to the expectations formation mechanism. The resulting welfare function writes as:

$$\mathbb{W} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t + t.i.p. + O\left(||\xi||^3\right), \tag{3}$$

where $t.i.p.$ stands for the terms independent of monetary policy, and $O\left(||\xi||^3\right)$ denotes all relevant terms that are of third or higher order. We express welfare losses in terms of the equivalent permanent consumption decline, i.e., as a fraction of steady-state consumption. The period loss function $\mathbb{L}_t$ is equal to:

$$\mathbb{L}_t = \underbrace{\frac{\varepsilon}{\gamma} {\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z}\right) {y_t}^2}_{\text{Standard}} + \underbrace{\frac{\sigma z \bar{b}_{cb}}{1-z} \left(\bar{b}_{cb} q {e_t}^2 - 2 q e_t y_t\right)}_{\text{Balance Sheet}}. \tag{4}$$

The first two terms denote the welfare loss from higher inflation and output volatility. These are standard components in the loss function obtained in the representative-agent New Keynesian (RANK) model. The last term represents the welfare loss from higher volatility and (counter)cyclicality of the central bank's balance sheet. In other words, this model's welfare loss function also implies a social preference for small and not excessively countercyclical deviations in the central bank's balance sheet from its steady state.

[10] We assume that the social welfare function discounts future utility losses with the patient households' discount factor β.

The loss function above can also be written as:[11]

$$\mathbb{L}_t = \underbrace{\frac{\varepsilon}{\gamma}{\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z}\right){y_t}^2}_{\text{Standard}} + \underbrace{\frac{\sigma z}{1-z}\left({c_{b,t}}^2 - 2c_{b,t}y_t\right)}_{\text{Inequality}}. \tag{5}$$

In this case, the last part of the expression represents the welfare loss resulting from increased volatility and countercyclicality of the impatient household's consumption. This term, derived from the TANK model structure, represents the welfare loss attributable to consumption heterogeneity, which we label as inequality (see, e.g., Bilbiie, 2008, 2024, Bilbiie and Ragot, 2021, and Debortoli and Galí, 2024). Intuitively, a benevolent central bank would aim to distribute the costs of fluctuations evenly among all agents, thereby eliminating fluctuations in consumption heterogeneity.

To sum up, balance-sheet expansions induce a reallocation of resources from patient to impatient households. Large changes in the balance sheet cause a higher volatility of the impatient households' consumption, who hold long-term debt. Countercyclical movements in the balance sheet (e.g., a balance-sheet expansion in response to a fall in output) induce a widening in the consumption gap between patient and impatient households. Therefore, welfare-maximising central banks should aim at stabilising inflation and output, and avoid large countercyclical movements in their balance sheet.

3.2 Optimal Monetary Policy under Discretion

Under discretion, the central bank seeks to minimise the period-t social welfare loss:

$$\min_{\pi_t, y_t, r_t^s, qe_t} \frac{1}{2}\mathbb{L}_t, \tag{6}$$

subject to Equations (1), (2), and the ZLB constraint on the policy rate:

$$r_t^s \geq -\frac{R^s - 1}{R^s}. \tag{7}$$

R^s is the steady-state value of the gross short-term interest rate. The instantaneous welfare loss function $\mathbb{L}_t$ is defined in Equation (4). Appendix D.1 provides further details on the optimisation problem and the associated first-order conditions.

[11] This result stems from the "full bailout" assumption, i.e., each period transfers from the patient to the impatient households payoff their outstanding debt obligations. Because of this assumption, the real value of long-term bonds equals the impatient households' consumption $zC_{b,t} = Q_t\frac{B_t}{P_t} = Q_t\frac{B_{cb,t}}{P_t} + Q_t\frac{B_{FI,t}}{P_t} = QE_t + t.i.p.$ The long-term bonds held by the financial intermediaries $B_{FI,t}$ are assumed to be exogenous and constant. They are therefore not affected by monetary policy interventions.

Throughout the rest of the paper, we consider two possible cases:

Case A, namely the optimal monetary policy under discretion (OMP-D) in the absence of QE. In this case, the central bank cannot respond to negative demand shocks when the short-term rate is at the ZLB.

Case B, i.e., the OMP-D in the presence of QE. In this case, the central bank can expand its balance sheet to ease the negative consequences of an adverse demand shock at the ZLB.

3.3 Optimal Monetary Policy under Commitment

When the central bank follows the optimal policy under commitment (OMP-C), it aims to maximise welfare by minimising the expected discounted lifetime social welfare loss:

$$\min_{\{\pi_t, y_t, r_t^s, qe_t\}_{t=0}^{\infty}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t, \tag{8}$$

subject to Equations (1), (2), and (7). Further details on the optimisation problem and the associated first-order conditions are provided in Appendix D.2. Unlike the discretionary case, commitment allows the central bank to carry out FG in addition to QE.

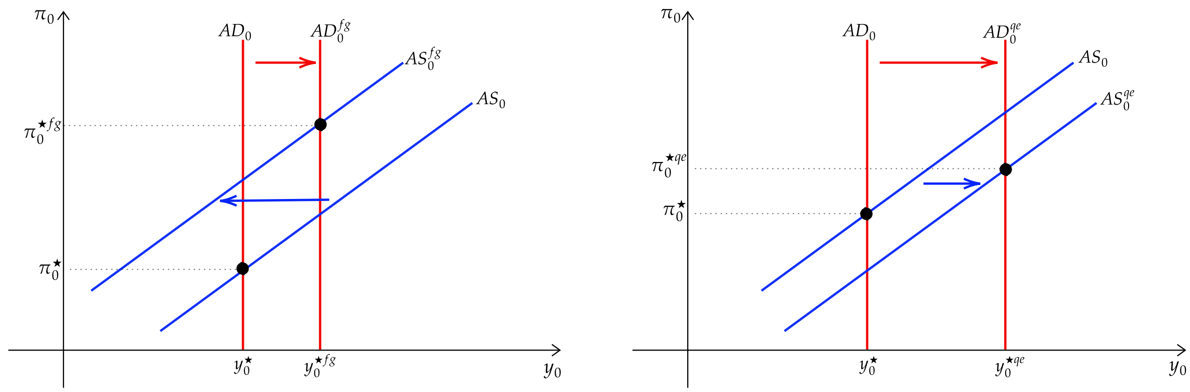
Similarly as for the OMP-D, also for the OMP-C, we consider two possible cases:

Case C, we consider the OMP-C without QE. In this case, at the ZLB the central bank can respond to negative demand shocks by keeping the short-term rate lower for longer, i.e., FG.

Case D, we consider the OMP-C with QE. In this case, the central bank can carry out both FG and balance-sheet policies to mitigate the effects of demand shocks while the policy rate is at the ZLB.

4 Analytical Insights

In this section, we explore analytically some of the key properties of FG and QE through the lens of a two-period version of the model by using the dynamic IS curve (1), NKPC (2), and the first-order conditions derived in Appendix D. In Sections 4.1 and 4.2, we consider the effects of FG and QE separately (Cases A-C). In Section 4.3, instead, we discuss their optimal combination (Case D). To solve the two-period model,



(a) Forward Guidance (Case C) (b) Quantitative Easing (Case B)

Figure 1: The Different Effects of Unconventional Monetary Policies

Note: The figures display the effects of FG (left panel) and QE (right panel) at the ZLB in the two-period model. AD_0 and AS_0 represent aggregate demand and supply in the absence of FG and QE, whereas AD_0^{fg}, AS_0^{fg}, AD_0^{qe}, and AS_0^{qe} are aggregate demand and supply when the central bank can use FG and QE, respectively. The y-axis is inflation (π_0), while the x-axis is output (y_0).

we make the following simplifying assumptions: (i) Shock: $r_0^n < 0$, $r_0^s = -\frac{R^s-1}{R^s}$, $r_1^n = 0$, $-\frac{R^s-1}{R^s} < r_1^s \leq 0$ and (ii) Perfect foresight: $E_t x_{t+1} = x_{t+1}$, $\forall t$.

4.1 The Effects of Forward Guidance

In a two-period version of the model, the optimal monetary policies labelled as Case A and Case C are described by the aggregate demand (AD) and supply (AS), given in Equations (9) and (10):

$$AD: \quad \underbrace{\underbrace{y_0 = \frac{1-z}{\sigma}\left(\frac{R^s-1}{R^s} + r_0^n\right)}_{AD_0 \text{ (Case A)}} - \frac{1-z}{\sigma}\left(1 - \alpha + \gamma\left(1 + \frac{\chi(1-z)}{\sigma}\right)\right) r_1^s,}_{AD_0^{fg} \text{ (Case C)}} \tag{9}$$

$$AS: \quad \underbrace{\underbrace{y_0 = \frac{1}{\gamma}\left(\chi + \frac{\sigma}{1-z}\right)^{-1}\pi_0}_{AS_0 \text{ (Case A)}} + \frac{1-z}{\sigma}\beta r_1^s.}_{AS_0^{fg} \text{ (Case C)}} \tag{10}$$

Comparing Cases C and A allows us to isolate the effect of FG (i.e., a future expected change in the policy rate) in our model. From Equations (9) and (10), we see how an expected reduction in the policy rate (i.e., a fall in r_1^s) expands demand and contracts supply. Figure 1(a) provides a graphical representation of the effects FG has on the AD and the AS curves. On the one hand, FG shifts the AD curve to the right. All things equal, this effect increases π_0 and y_0. On the other hand, FG also shifts the AS curve to the left, which, all things equal, has a negative effect on output y_0 and a positive effect on inflation π_0. Intuitively, this is because a reduction in r_1^s increases π_1 and y_1. Expected rises in inflation reduce the real rate today but also lead profit-maximising firms to raise their price today. The combination of the two effects implies that FG has a more substantial effect on inflation than on output.

Below, we prove this result more formally. In particular, let $\widehat{\pi}_0^{fg}$ and $\widehat{y}_0^{fg}$ be the change in the equilibrium values of π_0 and y_0 as a result of FG, which can be written as:

$$\widehat{\pi}_0^{fg} \equiv \left|\frac{\pi_0^{\star fg} - \pi_0^\star}{\pi_0^\star}\right| = \left|-\frac{\left(1-\alpha+\beta+\gamma\left(1+\frac{\chi(1-z)}{\sigma}\right)\right) r_1^{s\star}}{\frac{R^s-1}{R^s}+r_0^n}\right|, \tag{11}$$

$$\widehat{y}_0^{fg} \equiv \left|\frac{y_0^{\star fg} - y_0^\star}{y_0^\star}\right| = \left|-\frac{\left(1-\alpha+\gamma\left(1+\frac{\chi(1-z)}{\sigma}\right)\right) r_1^{s\star}}{\frac{R^s-1}{R^s}+r_0^n}\right|. \tag{12}$$

It can then be shown that FG has a relatively larger impact on inflation, i.e.:

$$\frac{\widehat{\pi}_0^{fg}}{\widehat{y}_0^{fg}} = \left|\frac{1-\alpha+\beta+\gamma\left(1+\frac{\chi(1-z)}{\sigma}\right)}{1-\alpha+\gamma\left(1+\frac{\chi(1-z)}{\sigma}\right)}\right| > 1 \iff \widehat{\pi}_0^{fg} > \widehat{y}_0^{fg}. \tag{13}$$

4.2 The Effects of Quantitative Easing

Similarly as above, to isolate the impact of QE, we now compare Cases B and A, which can be described by the following AD and AS equations:

$$AD: \quad \underbrace{\underbrace{y_0 = \frac{1-z}{\sigma}\left(\frac{R^s-1}{R^s}+r_0^n\right)}_{AD_0 \text{ (Case A)}} + z\bar{b}_{cb} q e_0}_{AD_0^{qe} \text{ (Case B)}}, \tag{14}$$

$$AS: \quad \underbrace{\underbrace{y_0 = \frac{1}{\gamma}\left(\chi+\frac{\sigma}{1-z}\right)^{-1}\pi_0}_{AS_0 \text{ (Case A)}} + z\bar{b}_{cb}\left(1+\frac{\chi(1-z)}{\sigma}\right)^{-1} q e_0}_{AS_0^{qe} \text{ (Case B)}}. \tag{15}$$

Equations (14) and (15) show how a balance-sheet expansion (i.e., an increase in qe_0) expands both demand and supply. We display the two effects of QE in Figure 1(b). The shift of the AD to the right, all things equal, increases both inflation and output. The shift of the AS curve to the right, instead, has a positive effect on output and a negative one on inflation. The underlying intuition behind the impact of QE is as follows. On the one hand, QE relaxes the bank lending constraints, which favours borrowers' consumption, thereby expanding demand. On the other hand, the reallocation of resources from the lender to the borrower induced by QE leads to a negative wealth effect for the lender, who will therefore supply more labour. The combination of the two effects implies that QE has a more substantial effect on output than on inflation.

To prove this result more formally, we compute the change in the equilibrium values of π_0 and y_0 as a result of QE:

$$\widehat{\pi}_0^{qe} \equiv \left| \frac{\pi_0^{\star qe} - \pi_0^{\star}}{\pi_0^{\star}} \right| = \left| \frac{\frac{\chi}{\chi + \frac{\sigma}{1-z}} z \bar{b}_{cb} q e_0^{\star}}{\frac{1-z}{\sigma} \left(\frac{R^s - 1}{R^s} + r_0^n \right)} \right|, \tag{16}$$

$$\widehat{y}_0^{qe} \equiv \left| \frac{y_0^{\star qe} - y_0^{\star}}{y_0^{\star}} \right| = \left| \frac{z \bar{b}_{cb} q e_0^{\star}}{\frac{1-z}{\sigma} \left(\frac{R^s - 1}{R^s} + r_0^n \right)} \right|. \tag{17}$$

It follows that QE has a relatively larger impact on output, i.e.:

$$\frac{\widehat{\pi}_0^{qe}}{\widehat{y}_0^{qe}} = \left| \frac{\chi}{\chi + \frac{\sigma}{1-z}} \right| < 1 \Longleftrightarrow \widehat{\pi}_0^{qe} < \widehat{y}_0^{qe}. \tag{18}$$

4.3 The Optimal Interaction of Unconventional Monetary Policies

Having discussed the effects of each individual tool, this section now analyses their interaction.[12] To that end, consider the expressions for QE in period 0 under discretion (Case B) and under commitment (Case D):

$$qe_0^{\star, B} = \Gamma_B \left(-\frac{R^s - 1}{R^s} - r_0^n \right) > 0, \tag{19}$$

$$qe_0^{\star, D} = \Gamma_D^1 \left(-\frac{R^s - 1}{R^s} - r_0^n \right) + \Gamma_D^2 r_1^{s\star} > 0, \tag{20}$$

where $\Gamma_B > 0$, $\Gamma_D^1 > 0$, and $\Gamma_D^2 > 0$ are convolutions of deep parameters (detailed derivations are left to Appendix E.1.2). Γ_B is independent of cognitive discounting α, whereas Γ_D^1 and Γ_D^2 are both decreasing in α:[13]

$$\frac{d\Gamma_D^1}{d\alpha} < 0, \tag{21}$$

$$\frac{d\Gamma_D^2}{d\alpha} < 0. \tag{22}$$

[12] In Appendix E.2, we present analogous analytical results for the infinite-period model. The two-period model, however, has the advantage of allowing us to derive an explicit and intuitive expression for the optimal QE as a function of the future interest rate.

[13] The reason why Γ_B is independent of α, unlike Γ_D^1 is that, under discretion, all the variables return to their steady state in period 1. In other words, all the variables expressed in deviations from steady state become zero. This is not the case under commitment.

Table 1: Baseline Quarterly Calibration

Parameter	Description	Value
α	Cognitive discounting parameter	0.20
β	Discount factor of patient households	0.997
σ	Inverse elasticity of intertemporal substitution	1.00
χ	Inverse labour supply elasticity	1.00
z	Share of impatient households	0.33
$\bar{b}_{cb}$	Steady-state share of central bank's long-term bond holdings	0.30
ε	CES parameter	11.00
ϕ	Probability of keeping price unchanged	0.75
ρ_n	Natural rate shock persistence	0.83
σ_n	Natural rate shock volatility	0.0501

Period-1 equilibrium QE in Cases B and D are respectively given by:

$$qe_1^{\star,B} = 0, \tag{23}$$

$$qe_1^{\star,D} = -\frac{(1-\alpha)}{\beta} qe_0^{\star,D}. \tag{24}$$

Comparing Equations (19) and (20), we observe that in both cases, the central bank expands its balance sheet in response to a negative demand shock ($r_0^n < 0$). Under commitment, however, QE also depends on the period-1 policy rate, reflecting the interaction between the balance-sheet size and FG. A lower $r_1^{s,\star}$ (indicating more FG) results in a smaller balance sheet expansion, while a higher $r_1^{s,\star}$ (indicating less FG) requires a larger expansion to address the fall in demand.

Expression (21) indicates that the optimal balance sheet size under commitment is inversely related to the strength of FG. Specifically, when the degree of bounded rationality α is large (i.e. weak FG), Γ_D^2 is smaller, meaning that for a given $r_1^{s\star} \leq 0$ (i.e., the central bank keeps the policy rate below its steady state) we have a larger balance-sheet expansion. Intuitively, when FG is less effective, the central bank will rely more heavily on QE to mitigate the welfare losses from the fall in demand. Moreover, expression (22) highlights that the impact of the shock is overall smaller when households are more myopic, as they internalise less the expected future effects of the shock.

Comparing Equations (23) and (24), we note that under discretion the central bank's balance sheet returns to steady state already in period 1. Under commitment, instead, the central bank cuts the balance sheet below its steady-state value (quantitative tightening).

5 Numerical Results

Next, we provide numerical results under the various optimal policies (Cases A-D). Specifically, we analyse their welfare implications and the impulse response functions. Last, we study how changing the power of FG affects the optimal monetary policy mix.

5.1 Calibration and Solution Method

For the numerical exercises, we parameterise the model using standard values in the literature, as listed in Table 1. The cognitive discounting parameter α is set to 0.2, in line with Gabaix (2020). Some parameters specific to the four-equation model are taken from Sims et al. (2023). In particular, we set the share of impatient households z to 0.33. The steady-state share of the central bank's long-term bond holdings, $\bar{b}_{cb}$, is set to 0.3. The shock's persistence and standard deviation are calibrated to $\rho_n = 0.83$ and $\sigma_n = 0.0501$. The calibrated shock induces a 3 percentage-point decline in inflation, a 10 per-cent fall in output, and the ZLB constraint binds for 16 quarters in Case A (see Section 3.2).

We solve the model via a piecewise linear perturbation method, as suggested by Guerrieri and Iacoviello (2015). The numerical simulations allow us to compute the welfare gains of the optimal policies in response to the negative demand shock. The impulse response analysis in the infinite-period setup allows us to show how the presence of balance-sheet policies affects the duration of the ZLB. This interaction could not be shown in the two-period model analytical exercise.

5.2 Welfare

In Table 2, we present the welfare implications of the four optimal policy cases defined in Sections 3.2 and 3.3, in response to a strong decline in the natural rate that causes the ZLB to bind.[14] Each row of Table 2 presents the changes in aggregate welfare and its subcomponents, i.e., changes in welfare due to the volatility of inflation and output, and balance-sheet volatility and cyclicality. This decomposition of welfare would not have been possible with a more complex model.

In Cases A and B (discretion), the central bank does not implement FG. Specifically, in Case A, the central bank does not have the tools (neither FG nor QE), to address the adverse demand shock at the zero lower bound (ZLB). Consequently, Case A exhibits the largest welfare losses, measured in consumption-

[14] It is important to note that this welfare analysis is valid in this specific "case study." We are concerned with the level effects of a particular given shock.

Table 2: Evaluation of the Optimal Policy Mix

Welfare	Case A	Case B	Case C	Case D
Aggregate	−15.02%	−11.15%	−7.80%	−5.50%
Inflation Volatility	−11.63%	−8.02%	−5.09%	−2.32%
Output Volatility	−3.40%	−1.12%	−2.71%	−0.83%
Balance-Sheet Volatility	0%	−1.04%	0%	−1.85%
Balance-Sheet Cyclicality	0%	−0.96%	0%	−0.49%

Note: We evaluate the welfare implications of the optimal monetary policies in Cases A-D, as defined in Sections 3.2 and 3.3. Case A of OMP under discretion absent QE is the benchmark case.

equivalent terms, primarily due to significant declines in both inflation and output. Introducing QE in Case B significantly improves welfare by reducing inflation and output volatility. In line with the analytical discussion, QE proves to be more effective in mitigating output volatility compared to inflation volatility. However, the enhanced stabilisation of macroeconomic conditions comes at the expense of increased balance-sheet volatility and countercyclicality, which negatively impact the inequality component of welfare. In Cases C and D (commitment), welfare losses are significantly smaller than in the first two cases. Compared to Case C, the presence of QE in Case D further mitigates aggregate welfare losses. In other words, from Table 2 it is evident that the optimal monetary policy response to a negative demand shock at the ZLB is given by a mix of FG and QE, highlighting their complementary roles.

5.3 Impulse Response Functions

Next, we present the dynamic responses to an exogenous decline in the natural rate under the four optimal policy cases discussed earlier. The results are displayed in Figure 2. In Cases A and B (discretion), the central bank does not carry out FG, as it cannot commit to a pre-specified path for the short-term rate.[15] Consequently, we observe that the policy rate path is identical in these two cases, with the central bank raising the short-term rate as soon as the ZLB constraint is no longer binding. In particular, in Case A, the central bank does not have any tools to counteract the adverse shock at the ZLB. For this reason, inflation expectations fall, leading to an increase in the real rate and exacerbating the decline in demand. Consequently, in this case, we observe the most substantial decline in both inflation and output. On the other hand, when the central bank can make use of QE in Case B, we observe a significant mitigation of the fall in output and a milder effect on inflation.

In Cases C and D (commitment), where the central bank carries out FG, the ZLB duration is significantly

[15] As in the analytical section, comparing Cases A and B helps us explain the marginal contribution of QE in the absence of FG.

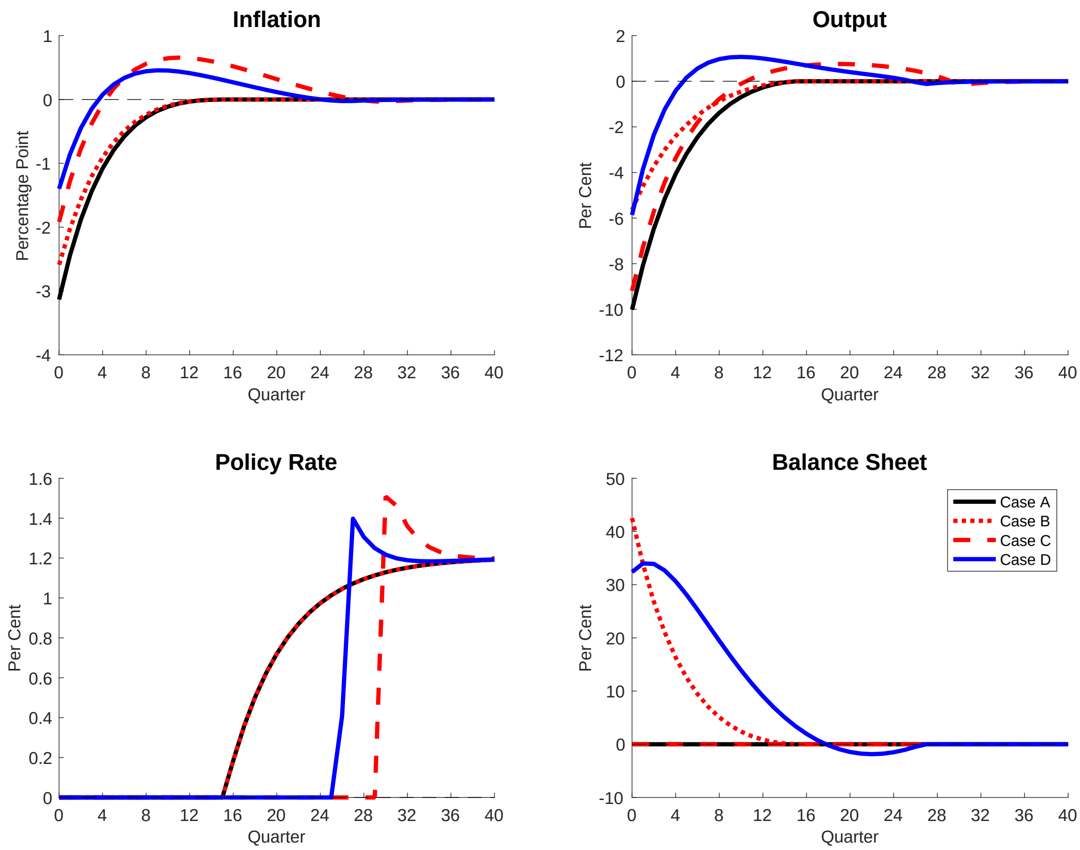


Figure 2: Optimal Monetary Policy Mix

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policies (Cases A-D, described in Sections 3.2 and 3.3). Case A of OMP under discretion absent QE is the benchmark case. Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

longer compared to the previous two cases. Comparing Case C to A or B, we observe that FG has the effect of boosting inflation expectations, thereby reducing the real rate and easing the initial decline in macroeconomic conditions. Moreover, when comparing Cases C and A, we note that FG mostly mitigates the drop in inflation. In Case D, where the central bank employs both FG and QE, we observe the smallest decline in inflation. Compared to Case C, the optimal policy in Case D requires a shorter duration of the ZLB. This is because the central bank partially substitutes FG with QE.

Compared to Case B, Case D implies a smaller (yet more persistent) balance-sheet expansion due to the presence of FG. In other words, the optimal monetary policy mix at the ZLB requires less FG and less QE compared to the cases when the central bank has access to only one policy instrument. Moreover, in case D, the balance sheet expansion is followed by a contraction (QT) to mitigate the overshoot in output and

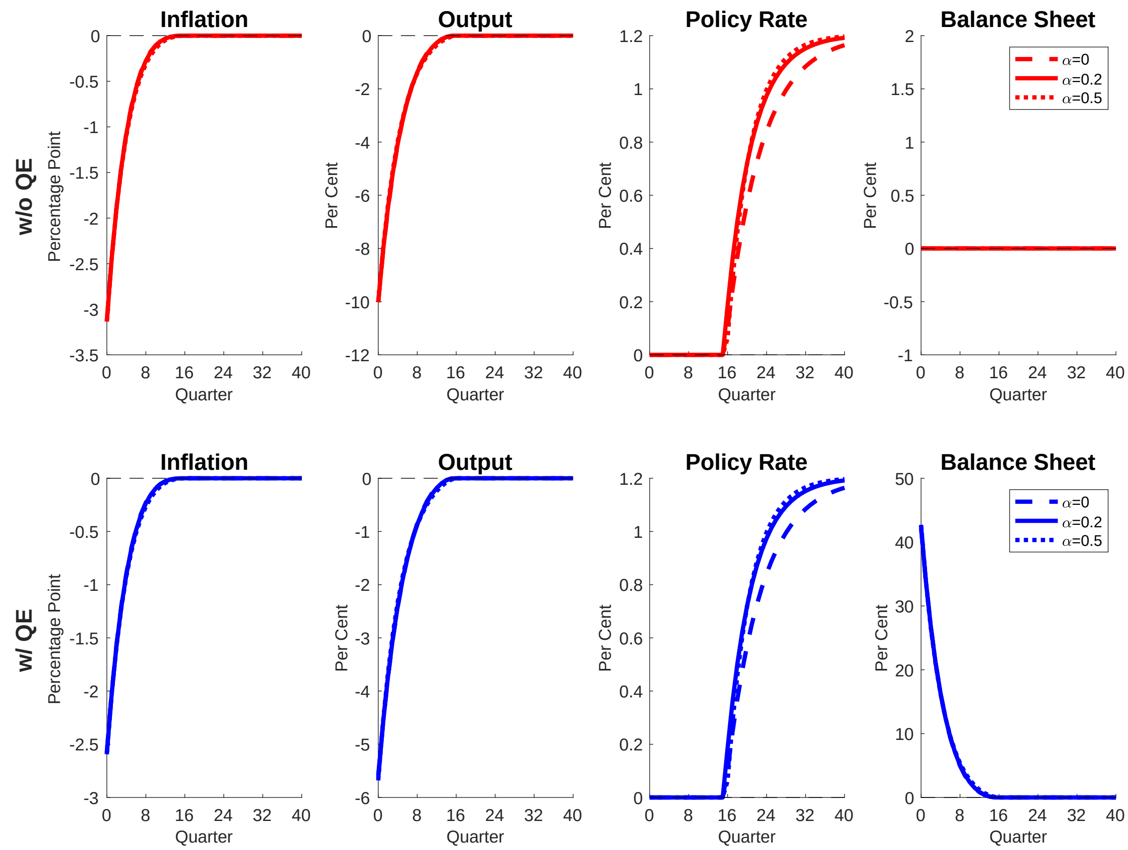


Figure 3: Varying the Degree of Bounded Rationality: Optimal Discretion

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under discretion (Cases A and B defined in Section 3.2) for different values of cognitive discounting. The shock is calibrated to cause a 3 percentage-point decline in inflation and a 10 per cent fall in output when the central bank conducts the optimal monetary policy under discretion absent QE. Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

inflation.[16]

5.4 The Impact of Bounded Rationality

We examine how the dynamic responses vary for different values of the cognitive discounting parameter α under the optimal policies without and with commitment. As α increases, agents become more myopic ($\alpha = 0$ corresponds to rational expectations), attach less weight to expected changes in future macroeconomic conditions, and the FG puzzle weakens. Conditional on the value of α, we adjust the persistence ρ_n and size σ_n of the natural rate shock such that inflation and output fall by 3 percentage points and 10

[16]These results are akin to those found in some papers on optimal fiscal and monetary policy at the ZLB (Werning, 2012, Schmidt, 2013, and Nakata, 2016). Similarly as for QE in our paper, optimal government spending stimulus is larger under discretion. Moreover, at the end of the ZLB, the optimal policy under commitment implies a reversal in government spending.

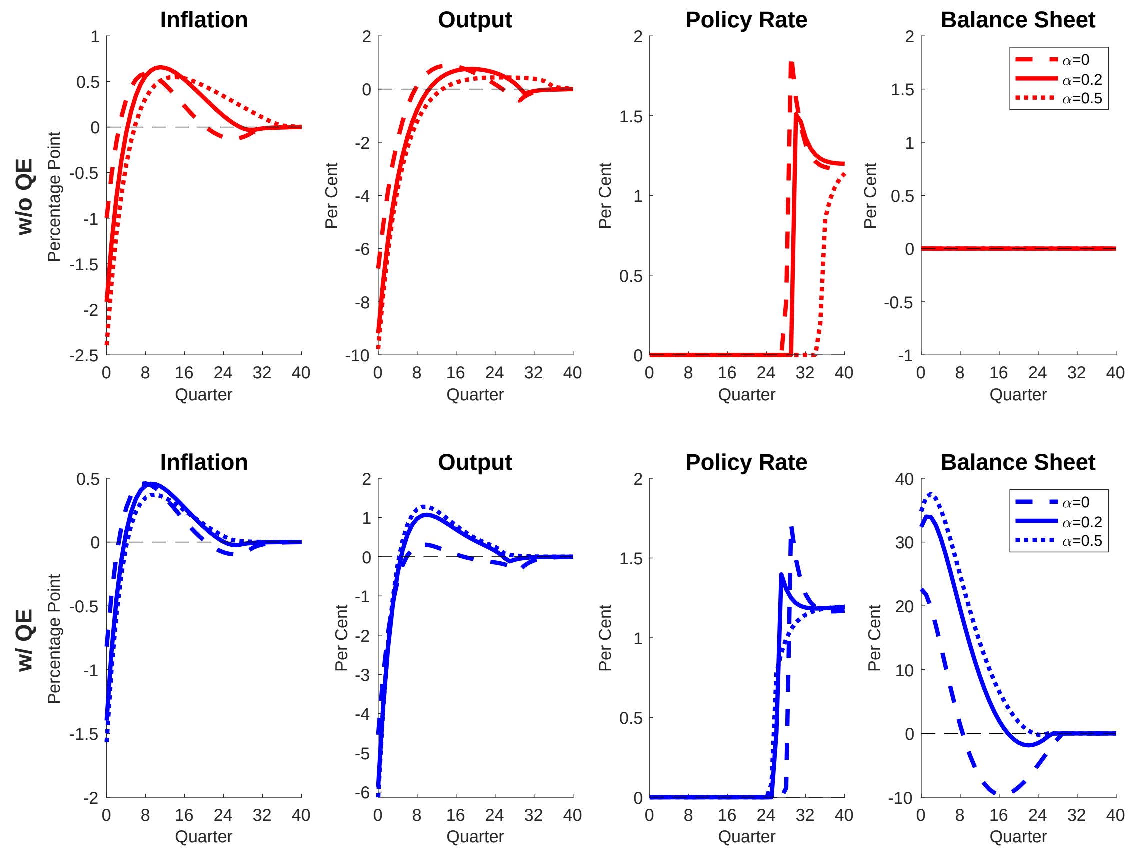


Figure 4: Varying the Degree of Bounded Rationality: Optimal Commitment

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under commitment (Cases C and D defined in Section 3.3) for different values of cognitive discounting. The shock is calibrated to cause a 3 percentage-point decline in inflation and a 10 per cent fall in output when the central bank conducts the optimal monetary policy under discretion absent QE. Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

per cent respectively, leading the ZLB constraint to bind for 16 quarters when the central bank follows the OMP-D without QE (i.e., Case A).[17]

Figure 3 shows the results for Cases A and B (discretion). In Case A (first row of Figure 3), one can see that the responses of inflation and output do not change, as by construction we readjust the shock to cause the same recession size. However, after the ZLB constraint is not binding anymore, the policy rate increases more the more myopic agents are. In other words, if the central bank reacted the same way irrespective of α, we would have a less severe recession under bounded rationality (i.e., for larger values of α). This can also be seen in Figure I.1 in Appendix I, in which the shock is not readjusted for each value of α. In that

[17] The approach of keeping the severity of the downturn constant as one varies the model's parameter values is adopted by Boneva et al. (2016), Hills and Nakata (2018), and Nakata et al. (2019).

case, we see that for larger values of α the falls in inflation and output become less severe, as agents become less and less worried about the expected future decline in macroeconomic conditions.

Case B (second row of Figure 3) displays similar results to Case A. The presence of QE mitigates the decline in inflation and output. However, changing α does not affect per se the response of the central bank's balance sheet.[18] The policy rate's response, however, depends on α as in Case A. When we do not readjust the shocks (Figure I.1 in Appendix I), the falls in inflation and output become less severe for larger values of α and the central bank needs to expand its balance sheet less substantially.

Figure 4 displays the results for Cases C and D (commitment). In Case C (first row of Figure 4), when the central bank does not deploy QE, the duration of the ZLB is longer the weaker is FG (i.e., for larger α). Intuitively, this is because more FG is required to ease the fall in inflation and output. This result aligns with Nakata et al. (2019). By contrast, in Case D (second row of Figure 4), a weaker FG leads to a shorter ZLB duration (i.e., less FG) and a stronger expansion of the central bank's balance sheet. In other words, when FG is relatively weak, QE becomes relatively more effective at stabilising macroeconomic conditions. Consequently, it is optimal for the central bank to implement more QE and raise the policy rate earlier.

6 Simple Mandates

In this section, we analyse the consequences of the central bank following a simple mandate, aiming to minimise a weighted average of inflation and output volatility. In other words, we study the implications of omitting the balance-sheet volatility and cyclicality from the policy objective function. The reason for this exercise is that, in line with the RANK literature, central banks around the world have mostly focused on inflation and output stabilisation. Under the simple mandate, the central bank has the following objective function:

$$\min_{\{\pi_t, y_t, r_t^s, qe_t\}_{t=0}^{\infty}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t^{sm}, \tag{25}$$

$$\mathbb{L}_t^{sm} = {\pi_t}^2 + \vartheta {y_t}^2, \tag{26}$$

subject to Equations (1), (2), and (7). ϑ is the relative weight the monetary policy authority puts on output stabilisation. Equation (26) is the relevant objective function for a welfare-maximising central bank in the RANK literature. However, as discussed above, the simple mandate does not coincide with the optimal

[18]This can also be seen from Equation (19) (and also from Equation (E.36) in the Appendix), showing how the optimal response of QE under discretion is independent of α.

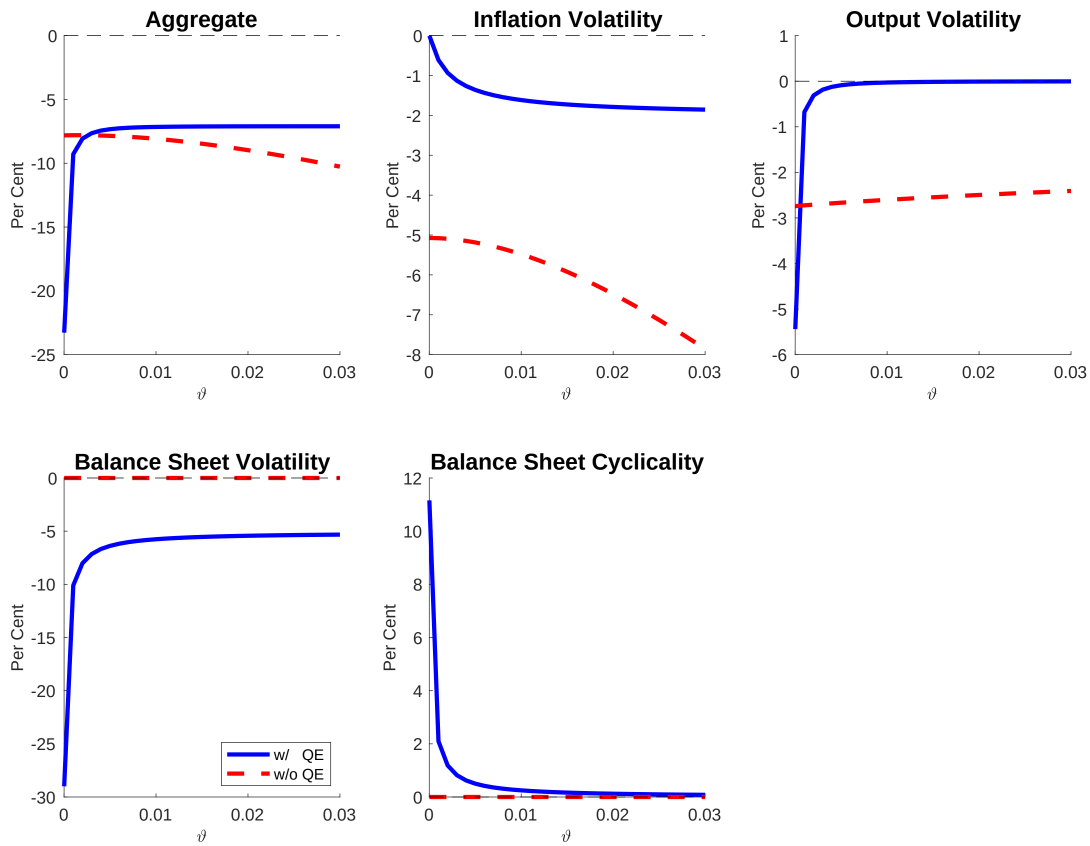


Figure 5: Welfare under Simple Mandates

Note: The figure displays the aggregate welfare and its subcomponents for different weights on output in the central bank's objective function. We assume that the central bank follows a simple mandate, aiming to stabilise only inflation and output.

policy objective in the four-equation model, defined above in Equation (4). It is important to note that, although the objective of the central bank is Equation (26), Equation (4) is still the relevant welfare measure we use to assess the policy implications. Further details on the optimisation problem and the associated first-order conditions are provided in Appendix D.3.

We now study the welfare implications of changing ϑ in the simple loss function (26). Figure 5 shows how aggregate welfare and its subcomponents vary as a function of ϑ. Red dashed (blue solid) lines represent the case in the absence (presence) of QE.[19] In the absence of QE, a larger ϑ (i.e., smaller weight on inflation) worsens aggregate welfare. For larger values of ϑ, the central bank increasingly focuses on stabilising output and allows more inflation volatility. This results in a stronger decline in the inflation component of welfare and a smaller decline in the output component. However, the fall in the inflation component of welfare

[19] It is important to note that the central bank's objective function differs from the social welfare function. The latter, in fact, includes QE volatility and cyclicality, which together represent the inequality component of welfare.

Table 3: Evaluation of the Simple-Mandate Policy Mix

	SM-O		SM-D	
Welfare	w/o QE	w/ QE	w/o QE	w/ QE
Aggregate	−7.80%	−8.40%	−15.91%	−7.11%
Inflation Volatility	−5.09%	−0.82%	−13.71%	−1.93%
Output Volatility	−2.71%	−0.41%	−2.20%	−0.01%
Balance-Sheet Volatility	0%	−8.62%	0%	−5.21%
Balance-Sheet Cyclicality	0%	1.45%	0%	0.04%

Note: We evaluate the welfare implications of the simple-mandate policies with optimal (SM-O) and equal weight (SM-D) on annual inflation and output. We consider both the cases without (w/o) and with (w/) balance sheet policies.

significantly outweighs the improvement in the output component.

In the presence of QE, instead, welfare increases with ϑ. As mentioned above, QE is less effective than conventional policy (and FG) at stabilising inflation and more effective at stabilising output. Therefore, a small ϑ (i.e., a large weight on inflation) requires a significant balance-sheet expansion, which worsens the component of welfare related to the balance sheet's volatility. Moreover, if ϑ is very small QE rises strongly, boosting output and causing a rise in output volatility (negatively affecting welfare). The rise in output leads to a positive co-movement between the balance sheet and output, which improves the balance-sheet cyclicality component of welfare. For larger values of ϑ, the central bank needs to expand its balance sheet significantly less. At the cost of slightly higher inflation volatility and reduced welfare gains from balance-sheet procyclicality, the larger ϑ implies a significantly smaller increase in the volatility of output and QE, which leads to less severe welfare losses (compared to smaller values of ϑ).

Two particular cases of interest are the simple mandate with $\vartheta = \frac{\gamma}{\varepsilon}\left(\chi + \frac{\sigma}{1-z}\right) \approx 0.007$, in line with the OMP, and the simple mandate with $\vartheta = 1/16 = 0.0625$, i.e., equal weight on annualised inflation and output. We label the two policies SM-O (O for optimal weight) and SM-D (D for dual mandate). Table 3 reports the welfare implications of the SM-O or SM-D mandates. Under SM-O, ϑ is relatively small, and QE causes larger welfare losses due to higher balance-sheet volatility. By contrast, under SM-D, ϑ is larger than under SM-O, and QE improves aggregate welfare, as the gains from stabilising inflation and output outweigh the welfare costs from higher balance-sheet volatility. It bears noting that both mandates perform significantly worse than the optimal monetary policy mix in Case D. Appendix J discusses the impulse responses generated under the SM-O and SM-D mandates.

Table 4: Evaluation of Simple Rules

	QE Rule		Welfare				
	ξ_π	ξ_y	Aggregate	Infl. Vol.	Outp. Vol.	BS Vol.	BS Cyc.
SI-PLT Rule							
$\eta_p = 1.5$	0	15	-6.50%	-3.36%	-0.46%	-1.86%	-0.83%
$\eta_p = 5.0$	0	15	-6.42%	-3.28%	-0.46%	-1.85%	-0.82%
$\eta_p = +\infty$	0	15	-6.38%	-3.25%	-0.46%	-1.85%	-0.82%

Note: Conditional on η_p, we select the values of ξ_π and ξ_y that maximise aggregate welfare. We consider natural values of ξ_π and ξ_y on the interval $[0, 100]$. BS Vol. and BS Cyc. stand for balance-sheet volatility and cyclicality.

7 Simple Policy Rules

This section studies the welfare implications of the central bank following simple policy rules for setting the short-term interest rate and the real value of its balance sheet. We then compare the outcomes to those under the optimal monetary policies. For the short-term interest rate, we consider a price-level-targeting rule:[20]

$$r_t^\star = \eta_p p_t + \eta_y y_t, \tag{27}$$

where $\pi_t \equiv p_t - p_{t-1}$ and set $\eta_y = 0$.[21] The short-term rate is constrained by the ZLB:

$$r_t^s = \max\left\{ r_t^\star, -\frac{R^s - 1}{R^s} \right\}. \tag{28}$$

It is important to notice that Equation (27) implies the short-term interest rate is history-dependent. In other words, under price-level-targeting, a fall in inflation today implies that the central bank will keep its rate lower for longer and allow for an overshoot in inflation in the future. In addition to the policy rule governing the short-term interest rate, the central bank chooses the amount of long-term bond holdings according to the following rule:

$$qe_t = -\xi_\pi \pi_t - \xi_y y_t. \tag{29}$$

Next, we consider the welfare implications of combining rule (29) with (27) for different values of η_p, ξ_π, and ξ_y.[22]

Table 4 reports how welfare changes in response to a negative demand shock under different policy-rule

[20] In Appendix L, we also consider an inflation targeting rule.

[21] We also compute the optimal price-level targeting rules in the absence of QE, and find that the optimal weight on output η_y is zero. This is because we are considering the optimal reaction function in response to a negative demand shock.

[22] Conditional on η_p, we select the welfare-maximising combination of ξ_π and ξ_y. To this end, we perform a grid search on a discrete interval $[0, 100]$ with unit steps.

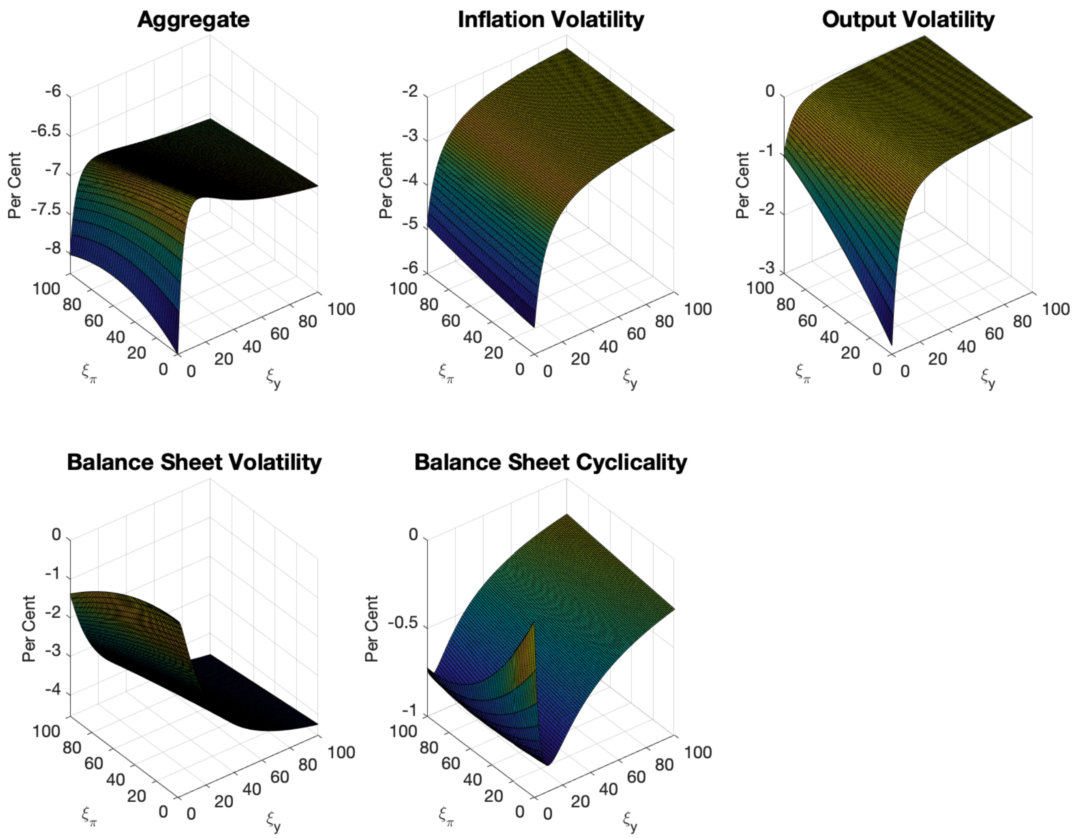


Figure 6: Strict Price-Level-Targeting Rule and Welfare

Note: The figure displays the aggregate welfare and its subcomponents when the central bank sets the short-term interest rate following a strict price-level-targeting rule and adjusts its balance sheet according to a policy rule with parameters ξ_π and ξ_y.

combinations and for different parameterisations. In particular, for each value η_p, we report the QE-rule coefficients that minimise the welfare losses. We label the mix of Equations (27) and (29) as SI-PLT (Short-term Interest rate, Price Level Targeting). Under SI-PLT, the optimal QE-rule coefficients are $\xi_\pi = 0$ and $\xi_y = 15$. We find that higher values of η_p mitigate the welfare losses by reducing the volatility of inflation and, to a lesser extent, that of the balance sheet. Moreover, the policy mix, including the optimal QE rule, brings the welfare losses significantly closer to those in Case D.

Finally, Figure 6 displays how aggregate welfare and its subcomponents change, as we vary the parameters in the QE rule ξ_π and ξ_y. In particular, the figure represents the case of an SI-PLT with $\eta_p \to \infty$. One can see how aggregate welfare is maximised for $\xi_\pi = 0$ and $\xi_y = 15$. Despite attenuating the volatility of inflation and output, and making the balance sheet less countercyclical, setting $\xi_\pi > 0$ and $\xi_y > 15$ would reduce welfare due to a rise in the volatility of the central bank's balance sheet. In other words, since QE is significantly more effective at stabilising output than inflation (as discussed in Sections 4.1, 4.2, 5.2, and 5.3), the marginal benefit of putting a higher weight on inflation is relatively low and comes at the cost of larger balance sheet adjustments. The intuition is akin to the case of a small ϑ in Section 6. If a central bank focuses solely on stabilising inflation and output while disregarding balance sheet volatility, QE can enhance welfare, provided the emphasis on inflation stabilisation is not too large. Therefore, the exercise discussed in this section highlights once more the trade-off between stabilising inflation, output, and the size of the central bank's balance sheet.

8 Concluding Remarks

In this paper, we study the optimal conduct of monetary policy at the ZLB when the central bank can employ both FG and balance-sheet policies. To do so, we consider a stylised model that enables us to derive a second-order approximation of the social welfare loss function. In contrast to the canonical result, which suggests that welfare is a negative function of the volatility of inflation and output, we find that in our model there is also a social preference for small and moderately countercyclical changes in the central bank's balance sheet.

The reason is that long-term asset purchases by the central bank affect asset prices, which impact the consumption of savers and long-term debt holders unequally. Consequently, following a negative demand shock at the ZLB, the optimal monetary policy under commitment involves a combination of FG and modest adjustments in the size of the balance sheet. Specifically, FG boosts expectations about inflation and output, an initial expansion of the balance sheet further eases the initial drop in demand, and a subsequent contraction alleviates the overshoots in prices and real economic activity. The presence of balance-sheet policies reduces the optimal duration of the ZLB needed to stabilise inflation and output. In contrast, under discretion, the central bank is unable to implement FG and can solely rely on balance-sheet policies to stabilise inflation and output. Thus, compared to the optimal policy under commitment, the optimal policy under discretion leads to lower levels of inflation and output stabilisation and necessitates a larger expansion of the balance sheet on impact. Consequently, welfare losses are significantly greater under discretion.

The optimal level of balance-sheet policies and the optimal ZLB duration crucially depend on the power of FG. Specifically, when households are assumed to be less responsive to FG compared to rational expectations, balance-sheet policies become relatively more effective in stabilising inflation and output. Therefore, the central bank should pursue a more aggressive expansion of its balance sheet and lift off the short-term interest rate earlier.

When the central bank's objective is solely focused on stabilising inflation and output, rather than maximising social welfare, it tends to excessively expand its balance sheet in response to a negative demand shock. This excessive expansion results in a significant increase in inequality, partially offsetting the welfare gains achieved through inflation and output stabilisation. Additionally, we find that balance-sheet policies may further diminish welfare if the relative weight assigned to output in the central bank's objective function is small. In our model, as balance-sheet policies are relatively less effective at stabilising inflation compared to output, a smaller weight on output (i.e., a larger weight on inflation) requires greater balance-sheet expansions following negative demand shocks. Consequently, when the weight on output is small, the welfare costs in terms of inequality outweigh the benefits of reducing inflation and output volatility.

Lastly, we examine the welfare implications of the central bank following simple policy rules. In this scenario, we show that a central bank that sets the short-term interest rate according to a strict price-level targeting rule and adjusts the balance sheet based on a flexible output-targeting rule can achieve welfare outcomes similar to the optimal policy under commitment.

The findings presented in this paper underscore that the introduction of unconventional monetary policies into the central bank's toolkit can potentially introduce additional trade-offs beyond the standard considerations of inflation and output. Understanding the implications of these policies is particularly important given the likelihood of more frequent binding periods at the ZLB due to the secular decline in interest rates and inflation.

Acknowledgements

We are grateful to Giovanni Caggiano, Efrem Castelnuovo, Sunghun Cho, Jaedo Choi, Richard Harrison, Yoon Joo Jo, Hyunju Kang, Chanwoo Kim, Mira Kim, Eunseong Ma, Riccardo Masolo, Taeha Oh, Yeonu Oh, Evi Pappa, Giovanni Pellegrino, Ricardo Reis, Myungkyu Shim, Vincent Sterk, and an anonymous referee for the BoE Staff Working Paper series for insightful discussions and helpful suggestions. All errors are our own.

References

Acharya, Sushant, Edouard Challe, and Keshav Dogra, "Optimal Monetary Policy According to HANK," *American Economic Review*, 2023, *113* (7), 1741–1782.

Adam, Klaus and Roberto M. Billi, "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*, 2006, *38* (7), 1877–1905.

_ **and** _ , "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates," *Journal of Monetary Economics*, 2007, *54* (3), 728–752.

Benigno, Pierpaolo and Michael Woodford, "Linear-Quadratic Approximation of Optimal Policy Problems," *Journal of Economic Theory*, 2012, *147*, 1–42.

Bilbiie, Florin O., "Limited Asset Market Participation, Monetary Policy and (Inverted) Aggregate Demand Logic," *Journal of Economic Theory*, 2008, *140*, 162–196.

_ , "Optimal Forward Guidance," *American Economic Journal: Macroeconomics*, 2019, *11* (4), 310–345.

_ , "Monetary Policy and Heterogeneity: An Analytical Framework," *Review of Economic Studies*, 2024, *Forthcoming.*

_ **and Xavier Ragot**, "Optimal Monetary Policy and Liquidity with Heterogeneous Households," *Review of Economic Dynamics*, 2021, *41* (71-95).

Billi, Roberto M. and Jordi Galí, "Gains from Wage Flexibility and the Zero Lower Bound," *Oxford Bulletin of Economics and Statistics*, 2020, *82* (6), 1239–1261.

Bonciani, Dario and Joonseok Oh, "Monetary Policy Inertia and the Paradox of Flexibility," *Journal of Economic Dynamics and Control*, 2023, *151*, 104668.

_ **and** _ , "Uncertainty Shocks, Innovation, and Productivity," *B.E. Journal of Macroeconomics*, 2023, *23* (1), 279–335.

Boneva, Lena Mareen, R. Anton Braun, and Yuichiro Waki, "Some Unpleasant Properties of Loglinearized Solutions When the Nominal Rate Is Zero," *Journal of Monetary Economics*, 2016, *84*, 216–232.

Calvo, Guillermo, "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics*, 1983, *12*, 383–398.

Cantore, Cristiano and Lukas B. Freund, "Workers, Capitalists, and the Government: Fiscal Policy and Income (Re)Distribution," *Journal of Monetary Economics*, 2021, *119*, 58–74.

Cardamone, Dario, Eric R. Sims, and Jing Cynthia Wu, "Wall Street vs. Main Street QE," *European Economic Review*, 2023, *156*, 104475.

Carlstrom, Charles T. and Timothy S. Fuerst, "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, 1997, *87* (5), 893–910.

_ , _ , and Matthias Paustian, "Targeting Long Rates in a Model with Segmented Markets," *American Economic Journal: Macroeconomics*, 2017, *9* (1), 205–242.

Challe, Edouard, "Uninsured Unemployment Risk and Optimal Monetary Policy in a Zero-Liquidity Economy," *American Economic Journal: Macroeconomics*, 2020, *12* (2), 241–283.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo, "When Is the Government Spending Multiplier Large?," *Journal of Political Economy*, 2011, *119* (1), 78–121.

Cui, Wei and Vincent Sterk, "Quantitative Easing with Heterogeneous Agents," *Journal of Monetary Economics*, 2021, *123*, 68–90.

Cúrdia, Vasco and Michael Woodford, "Credit Frictions and Optimal Monetary Policy," *Journal of Monetary Economics*, 2016, *84* (30-65).

Debortoli, Davide and Jordi Galí, "Heterogeneity and Aggregate Fluctuations: Insights from TANK Models," 2024. NBER Working Papar 32557.

_ , Jinill Kim, Jesper Lindé, and Ricardo Nunes, "Designing a Simple Loss Function for Central Banks: Does a Dual Mandate Make Sense?," *Economic Journal*, 2019, *129* (621), 2010–2038.

Del Negro, Marco, Marc Giannoni, and Christina Patterson, "The Forward Guidance Puzzle," 2015. Federal Reserve Bank of New York Staff Report N0. 574.

Eggertsson, Gauti B., "What Fiscal Policy is Effective at Zero Interest Rates?," in Daron Acemoglu and Michael Woodford, eds., *NBER Macroeconomics Annual 2010*, Vol. 25, The University of Chicago Press, 2011, pp. 59–112.

_ and Michael Woodford, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, *2003* (1), 139–211.

— **and Sanjay R. Singh**, "Log-linear Approximation versus an Exact Solution at the ZLB in the New Keynesian Model," *Journal of Economic Dynamics and Control*, 2019, *105*, 21–43.

Gabaix, Xavier, "A Behavioral New Keynesian Model," *American Economic Review*, 2020, *110* (8), 2271–2327.

Galí, Jordi, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications*, Princeton University Press, 2015.

Gertler, Mark and Peter Karadi, "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 2011, *58* (1), 17–34.

— **and** — , "QE 1 vs. 2 vs. 3...: A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool," *International Journal of Central Banking*, 2013, *9* (1), 5–53.

Guerrieri, Luca and Matteo Iacoviello, "OccBin: A Toolkit for Solving Dynamic Models with Occasionally Binding Constraints Easily," *Journal of Monetary Economics*, 2015, *70*, 22–38.

Harrison, Richard, "Optimal Quantitative Easing," 2017. Bank of England Staff Working Paper No. 678.

Hills, Timothy S. and Taisuke Nakata, "Fiscal Multipliers at the Zero Lower Bound: The Role of Policy Inertia," *Journal of Money, Credit and Banking*, 2018, *50* (1), 155–172.

Iacoviello, Matteo, "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review*, 2005, *95* (3), 739–764.

Jung, Taehun, Yuki Teranishi, and Tsutomu Watanabe, "Optimal Monetary Policy at the Zero-Interest-Rate Bound," *Journal of Money, Credit and Banking*, 2005, *37* (5), 813–835.

Karadi, Peter and Anton Nakov, "Effectiveness and Addictiveness of Quantitative Easing," *Journal of Monetary Economics*, 2021, *117*, 1096–1117.

Lee, Donggyu, "Quantitative Easing and Inequality," 2021. Working Papar.

Masolo, Riccardo M. and Pablo E. Winant, "The Stochastic Lower Bound," *Economics Letters*, 2019, *180*, 54–57.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson, "The Power of Forward Guidance Revisited," *American Economic Review*, 2016, *106* (10), 3133–3158.

— **,** — **, and** — , "The Discounted Euler Equation: A Note," *Economica*, 2017, *84* (336), 820–831.

Michaillat, Pascal and Emmanuel Saez, "Resolving New Keynesian Anomalies with Wealth in the Utility Function," *Review of Economics and Statistics*, 2021, *103* (2), 197–215.

Nakata, Taisuke, "Optimal Fiscal and Monetary Policy with Occasionally Binding Zero Bound Constraints," *Journal of Economic Dynamics and Control*, 2016, *73* (C), 220–240.

— , "Uncertainty at the Zero Lower Bound," *American Economic Journal: Macroeconomics*, 2017, *9* (3), 186–221.

— **and Sebastian Schmidt**, "Gradualism and Liquidity Traps," *Review of Economic Dynamics*, 2019, *31*, 182–199.

— **, Ryota Ogaki, Sebastian Schmidt, and Paul Yoo**, "Attenuating the Forward Guidance Puzzle: Implications for Optimal Monetary Policy," *Journal of Economic Dynamics and Control*, 2019, *105*, 90–106.

Nakov, Anton, "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," *International Journal of Central Banking*, 2008, *4* (2), 73–127.

Nisticò, Salvatore, "Optimal Monetary Policy and Financial Stability in a Non-Ricardian Economy," *Journal of the European Economic Association*, 2016, *14* (5), 1225–1252.

Schmidt, Sebastian, "Optimal Monetary and Fiscal Policy with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*, 2013, *45* (7), 1335–1350.

Sims, Eric R. and Jing Cynthia Wu, "Are QE and Conventional Monetary Policy Substitutable?," *International Journal of Central Banking*, 2020, *16* (1), 195–230.

— **and** — , "Evaluating Central Banks' Tool Kit: Past, Present, and Future," *Journal of Monetary Economics*, 2021, *118*, 135–160.

— **,** — **, and Ji Zhang**, "The Four-Equation New Keynesian Model," *Review of Economics and Statistics*, 2023, *105* (4), 931–947.

Svensson, Lars E.O., "Inflation Targeting," in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, Vol. 3, Elsevier, 2010, chapter 22, pp. 1237–1302.

Werning, Ivan, "Managing a Liquidity Trap: Monetary and Fiscal Policy, manuscript," 2012.

Woodford, Michael, "Fiscal Requirements for Price Stability," *Journal of Money, Credit and Banking*, 2001, *33* (3), 669–728.

—, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.

Wu, Jing Cynthia and Yingxi Xie, "(Un)Conventional Monetary and Fiscal Policy," 2022. Working Papar.

Appendices

A Full Nonlinear Model

The model we consider follows Sims et al. (2023). Unlike their paper, and in line with Woodford (2003), we assume industry-specific labour in the utility function, rather than aggregate labour. We assume there is a share $1-z$ of patient households and z of impatient households, whereas in the original model z represents the impatient households' steady-state share of consumption. This assumption allows us to simplify the calculation of the aggregate welfare function, equalizing the consumption of the two households in steady state (Benigno and Woodford, 2012), as detailed in Section C. This assumption does not affect the equilibrium conditions.

A.1 Patient Households

The representative patient household maximises its discounted lifetime utility:

$$\max E_0^{BR} \sum_{t=0}^{\infty} \beta^t \left(\frac{{C_{s,t}}^{1-\sigma} - 1}{1-\sigma} - \psi \int_0^1 \frac{L_{s,t}(i)^{1+\chi}}{1+\chi} di \right), \tag{A.1}$$

where E_0^{BR} is the subjective (behavioural) expectation operator, and $C_{s,t}$ is a Dixit-Stiglitz aggregate:

$$C_{s,t} = \left(\int_0^1 C_{s,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A.2}$$

and $L_{s,t}(i)$ is the quantity of labour supplied to the firm producing good i. The parameter σ is the coefficient of relative risk aversion, ε is the demand elasticity of good i, χ is the inverse of the Frisch elasticity of labour, ψ is a normalising constant, and β is the discount factor for the impatient households. The patient household demand for good i is given by:

$$C_{s,t}(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_{s,t}, \tag{A.3}$$

where $P_t(i)$ is the price of good i. The aggregate price level P_t therefore writes as:

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}, \tag{A.4}$$

so that:

$$P_t C_{s,t} = \int_0^1 P_t(i) C_{s,t}(i) di. \tag{A.5}$$

The patient household maximises its expected discounted lifetime utility (A.1) subject to the following budget constraint:

$$\int_0^1 P_t(i)\, C_{s,t}(i)\, di + \frac{S_t}{1-z} \leq R^s_{t-1}\zeta_{t-1}\frac{S_{t-1}}{1-z} + \int_0^1 W_t(i)\, L_{s,t}(i) di + \int_0^1 \frac{D_t(i)}{1-z} + P_t\frac{D_{FI,t}}{1-z} + P_t\frac{T_t}{1-z} - P_t\frac{X_{b,t}}{1-z} - P_t\frac{X_{FI,t}}{1-z}, \tag{A.6}$$

where S_t is a one period risk-free bond, paying a gross nominal interest rate R^s_t. $W_t(i)$ is the nominal wage rate in the ith industry in the economy and $D_t(i)$ are the nominal profits from the sale of good i. The household owns the financial intermediaries and receives dividends $D_{FI,t}$. T_t is a lump-sum transfer from the central bank. ζ_t is the risk-premium shock. Finally, $X_{b,t}$ and $X_{FI,t}$ are transfers to the impatient household and the financial intermediaries. The resulting optimality conditions are standard:

$$1 = E^{BR}_t \Lambda^s_{t,t+1}\frac{R^s_t\zeta_t}{\Pi_{t+1}}, \tag{A.7}$$

$$\Lambda^s_{t-1,t} = \beta\left(\frac{C_{s,t}}{C_{s,t-1}}\right)^{-\sigma}, \tag{A.8}$$

where $\Lambda^s_{t-1,t}$ is the patient household's stochastic discount factor. $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

A.2 Impatient Households

The impatient household can borrow/save with long term bonds B_t. Similarly as in Woodford (2001), long-term bonds are modelled as perpetuities with geometrically decaying coupon payments. The decaying rate of the coupon payments is denoted by $\kappa \in [0,1]$. The agent that issues the bond in time t needs to pay 1, κ, κ^2, ... in the following periods. The new bond issuance CB_t equals:

$$CB_t = B_t - \kappa B_{t-1}. \tag{A.9}$$

Given the market price of newly issued bonds Q_t, the total value of the bond portfolio equals $Q_t B_t$. Moreover, define the gross return on the long bond as:

$$R^b_t = \frac{1+\kappa Q_t}{Q_{t-1}}, \tag{A.10}$$

and the gross yield-to-maturity as:

$$Q_t = \frac{1}{RL^b_t} + \frac{\kappa}{{RL^b_t}^2} + \frac{\kappa^2}{{RL^b_t}^3} + \cdots, \tag{A.11}$$

Consequently,

$$RL_t^b = \frac{1}{Q_t} + \kappa. \tag{A.12}$$

The impatient household does not work and derives utility only from its consumption C_t^b. It maximises its lifetime utility:

$$\max E_0^{BR} \sum_{t=0}^{\infty} {\beta_b}^t \left(\frac{{C_{b,t}}^{1-\sigma} - 1}{1-\sigma} \right), \tag{A.13}$$

where $C_{b,t}$ is a Dixit-Stiglitz aggregate:

$$C_{b,t} = \left(\int_0^1 C_{b,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A.14}$$

and β_b is the discount factor for the impatient households. The impatient household demand for good i is given by:

$$C_{b,t}(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_{b,t}, \tag{A.15}$$

and

$$P_t C_{b,t} = \int_0^1 P_t(i) \, C_{b,t}(i) \, di. \tag{A.16}$$

The impatient household maximises its expected discounted lifetime utility (A.13) subject to the following budget constraint:

$$\int_0^1 P_t(i) \, C_{b,t}(i) \, di + \frac{B_{t-1}}{z} \leq Q_t \left(\frac{B_t}{z} - \kappa \frac{B_{t-1}}{z} \right) + P_t \frac{X_{b,t}}{z}. \tag{A.17}$$

The optimality condition for the impatient household is, therefore:

$$1 = E_t^{BR} \Lambda_{t,t+1}^b \frac{R_{t+1}^b}{\Pi_{t+1}}, \tag{A.18}$$

where $\Lambda_{t-1,t}^b$ denotes stochastic discount factor, defined as:

$$\Lambda_{t-1,t}^b = \beta_b \left(\frac{C_{b,t}}{C_{b,t-1}} \right)^{-\sigma}. \tag{A.19}$$

A.3 Financial Intermediaries

A representative financial intermediary is born each period and exits the industry in the subsequent period. It receives an exogenous amount of net worth from the patient household, $P_t X_{FI,t}$, which equals:

$$P_t X_{FI,t} = P_t \bar{X}_{FI} + \kappa Q_t B_{FI,t-1}. \tag{A.20}$$

$\bar{X}_{FI}$ is a fixed amount of new equity, whereas $\kappa Q_t B_{FI,t-1}$ is the value of outstanding long-bonds inherited from past intermediaries. The balance sheet of the financial intermediary is given by:

$$Q_t B_{FI,t} + RE_{FI,t} = S_{FI,t} + P_t X_{FI,t}. \tag{A.21}$$

where the left-hand side are the assets (long-term lending to impatient households $Q_t B_{FI,t}$ and reserves $RE_{FI,t}$), whereas the right-hand side are the liabilities (short-term deposits from the patient households $S_{FI,t}$ and the transfer $P_t X_{FI,t}$). The financial intermediary, pays interest R_t^s on the deposits, earns interest, R_t^{re}, on its reserves, and earns a gross return R_{t+1}^b on long-term bonds.

When the financial intermediary exits the market, gives dividends $P_{t+1} D_{FI,t+1}$ (in nominal terms) to the patient households:

$$P_{t+1} D_{FI,t+1} = \left(R_{t+1}^b - R_t^s\right) Q_t B_{FI,t} + \left(R_t^{re} - R_t^s\right) RE_{FI,t} + R_t^s P_t X_{FI,t}. \tag{A.22}$$

In time t the financial intermediary maximises the expected $t+1$ dividends, discounted by the nominal stochastic discount factor of the patient households $\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}$, subject to a leverage constraint:

$$Q_t B_{FI,t} \leq \Theta P_t \bar{X}_{FI}. \tag{A.23}$$

The condition states that the value of the long-term loans to the impatient households cannot be larger than a multiple Θ of the value of its equity. The first-order conditions with respect to $B_{FI,t}$ and $RE_{FI,t}$ write as:

$$E_t \Lambda_{t,t+1} \frac{R_{t+1}^b - R_t^s}{\Pi_{t+1}} = \Omega_t, \tag{A.24}$$

$$E_t \Lambda_{t,t+1} \frac{R_t^{re} - R_t^s}{\Pi_{t+1}} = 0, \tag{A.25}$$

where Ω_t is the Lagrangian multiplier associated with the leverage constraint.

A.4 Production

A monopolistically competitive firm produces good i using the following production function:

$$Y_t(i) = L_t(i) = (1-z) L_{s,t}(i). \tag{A.26}$$

Each firm faces a downward-sloping demand function given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t. \tag{A.27}$$

Following Woodford (2003), the labour employed by each monopolistically competitive firm corresponds to a particular type of the labour variety supplied by the households. The firm takes the wage rate as given and its period profits are given by:

$$D_t(i) = P_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t - (1-\tau) W_t^I \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t, \tag{A.28}$$

where W_t^I should be interpreted as an industry specific wage for good variety i. W_t^I can then be related to the price level of good i via the first-order labour condition of the households as:

$$\frac{W_t^I}{P_t} = \psi\left(\frac{L_t(i)}{1-z}\right)^{\chi} {C_{s,t}}^{\sigma} = \psi\left(\frac{Y_t\left(\frac{P_t^I}{P_t}\right)^{-\varepsilon}}{1-z}\right)^{\chi} {C_{s,t}}^{\sigma}, \tag{A.29}$$

where P_t^I is the industry-wide common price. We write then the period profit function of a firm producing good i as $D\left(P_t(i), P_t^I, P_t, Y_t\right)$.

As in Calvo (1983), a fraction $1-\phi$ of randomly picked firms can reset their price. Let P_t^* be the optimal reset price in period t. A supplier that changes its price in period t chooses its newly-adjusted price $P_t(i)$ to maximise the its expected discounted lifetime profits, taking as given the industry level wage W^I, expressed in terms of P_t^I:

$$E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \frac{D\left(P_t(i), P_{t+j}^I, P_{t+j}, Y_{t+j}\right)}{P_{t+j}}, \tag{A.30}$$

The first-order condition for optimal price setting is:

$$E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \left((1-\varepsilon)\left(\frac{P_t^*}{P_{t+j}}\right)^{1-\varepsilon} Y_{t+j} + \varepsilon(1-\tau)\psi\left(\frac{Y_{t+j}\left(\frac{P_{t+j}^I}{P_{t+j}}\right)^{-\varepsilon}}{1-z}\right)^{\chi} {C_{s,t+j}}^{\sigma}\left(\frac{P_t^*}{P_{t+j}}\right)^{-\varepsilon} Y_{t+j}\right) = 0. \tag{A.31}$$

Following Woodford (2003), all firms in industry I reset the price in period t:

$$E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \left((1-\varepsilon) \left(\frac{P_t^*}{P_{t+j}} \right)^{1-\varepsilon} Y_{t+j} + \varepsilon (1-\tau) \psi \left(\frac{Y_{t+j} \left(\frac{P_t^*}{P_{t+j}} \right)^{-\varepsilon}}{1-z} \right)^{\chi} {C_{s,t+j}}^{\sigma} \left(\frac{P_t^*}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \right) = 0. \tag{A.32}$$

This implies that the optimal reset price is:

$$\frac{P_t^*}{P_t} = \left(\frac{\varepsilon (1-\tau)}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \psi (1-z)^{-\chi} {C_{s,t+j}}^{\sigma} \left(\frac{P_t}{P_{t+j}} \right)^{-\varepsilon(1+\chi)} {Y_{t+j}}^{1+\chi}}{E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \left(\frac{P_t}{P_{t+j}} \right)^{1-\varepsilon} Y_{t+j}} \right)^{\frac{1}{1+\chi\varepsilon}}, \tag{A.33}$$

where P_t indicates the aggregate price level. We can re-write the expression in recursive form:

$$p_t^* = \left(\frac{\varepsilon (1-\tau)}{\varepsilon - 1} \frac{F_{1,t}}{F_{2,t}} \right)^{\frac{1}{1+\chi\varepsilon}}, \tag{A.34}$$

$$F_{1,t} = \psi (1-z)^{-\chi} {Y_t}^{1+\chi} + \phi \beta E_t {\Pi_{t+1}}^{\varepsilon(1+\chi)} F_{1,t+1}, \tag{A.35}$$

$$F_{2,t} = {C_{s,t}}^{-\sigma} Y_t + \phi \beta E_t {\Pi_{t+1}}^{\varepsilon - 1} F_{2,t+1}, \tag{A.36}$$

where inflation Π_t evolves according to:

$$\phi {\Pi_t}^{\varepsilon-1} = 1 - (1-\phi) {p_t^*}^{1-\varepsilon}. \tag{A.37}$$

A.5 Central Bank

The monetary authority creates reserves to finance the purchase of long bonds $B_{cb,t}$. Its balance sheet, therefore, writes as:

$$Q_t B_{cb,t} = RE_t. \tag{A.38}$$

The real value of long-term bonds held by the central bank is denoted as:

$$QE_t = Q_t b_{cb,t}, \tag{A.39}$$

where $b_{cb,t} = \frac{B_{cb,t}}{P_t}$. Potential profits made by the central bank are then transferred lump-sum to the patient households:

$$P_t T_t = R_t^b Q_{t-1} B_{cb,t-1} - R_{t-1}^{re} RE_{t-1}. \tag{A.40}$$

A.6 Aggregation and Equilibrium

Market clearing requires the following conditions:

$$RE_t = RE_{FI,t}, \tag{A.41}$$

$$S_t = S_{FI,t}, \tag{A.42}$$

$$B_t = B_{FI,t} + B_{cb,t}, \tag{A.43}$$

$$Y_t = C_t = (1 - z)\, C_{s,t} + zC_{b,t}, \tag{A.44}$$

$$P_t X_{b,t} = (1 + \kappa Q_t)\, B_{t-1}, \tag{A.45}$$

$$P_t C_{b,t} = Q_t \frac{B_t}{z}. \tag{A.46}$$

Given that we abstract from productivity shocks, the level of output that arises in the flexible-price version of the model is constant and equal to the steady-state value, i.e., $Y_t^f = Y$. Hence, we obtain the natural rate of interest:

$$R_t^n = \frac{\Pi}{\beta \zeta_t}. \tag{A.47}$$

The risk-premium shock is assumed to follow AR(1) process:

$$\log \zeta_t = \rho_\zeta \log \zeta_{t-1} + \sigma_\zeta \epsilon_t^\zeta. \tag{A.48}$$

B Full Linearised Model

B.1 Bounded Rationality

In order to mitigate the FG puzzle highlighted in Del Negro et al. (2015), we assume that households are partially myopic as in Gabaix (2020). By reacting myopically to distant events, such as future interest rate changes, FG becomes significantly less powerful than in the canonical rational-expectations NK model. In particular, for any variable $z(X_t)$ with $z(0) = 0$, we have that, for all $k \geq 0$:

$$E_t^{BR} z(X_{t+k}) = (1-\alpha)^k E_t z(X_{t+k}), \tag{B.1}$$

where E_t^{BR} is the subjective (behavioural) expectation operator, and E_t is the rational one. $\alpha \in [0,1]$ captures the degree of attention to the future and, when $\alpha = 0$, agents have rational expectations.

We assume that both patient and impatient households are affected by cognitive discounting and their linearised Euler equations write as:

$$c_{s,t} = E_t^{BR} c_{s,t+1} - \frac{1}{\sigma}\left(r_t^s - E_t \pi_{t+1} + \widehat{\zeta}_t\right), \tag{B.2}$$

$$c_{b,t} = E_t^{BR} c_{b,t+1} - \frac{1}{\sigma}\left(E_t^{BR} r_{t+1}^b - E_t^{BR} \pi_{t+1}\right). \tag{B.3}$$

Using Equation (B.1), we can re-write the Euler equations above as:

$$c_{s,t} = (1-\alpha) E_t c_{s,t+1} - \frac{1}{\sigma}\left(r_t^s - E_t \pi_{t+1} + \widehat{\zeta}_t\right), \tag{B.4}$$

$$c_{b,t} = (1-\alpha) E_t c_{b,t+1} - \frac{1-\alpha}{\sigma}\left(E_t r_{t+1}^b - E_t \pi_{t+1}\right). \tag{B.5}$$

B.2 System of Equations

The equilibrium conditions in log-linearised form are summarised below. The following variables denote percentage change deviations from their steady-state values, e.g., $c_{s,t} \equiv \frac{C_{s,t} - C_s}{C_s}$. We use the "hat" notation when the variable in the nonlinear model is labelled with a lower-case letter, e.g., $\widehat{b}_{FI,t} \equiv \frac{b_{FI,t} - b_{FI}}{b_{FI}}$.

$$c_{s,t} = (1-\alpha) E_t c_{s,t+1} - \frac{1}{\sigma}\left(r_t^s - E_t \pi_{t+1} + \widehat{\zeta}_t\right), \tag{B.6}$$

$$r_t^b = \frac{\kappa}{R^b} q_t - q_{t-1}, \tag{B.7}$$

$$rl_t^b = -\frac{1}{1+\kappa Q} q_t, \tag{B.8}$$

$$c_{b,t} = (1-\alpha) E_t c_{b,t+1} - \frac{1-\alpha}{\sigma} \left(E_t r_{t+1}^b - E_t \pi_{t+1} \right), \tag{B.9}$$

$$q_t + \widehat{b}_{FI,t} = 0, \tag{B.10}$$

$$Qb_{FI} (1-\kappa) q_t + Qb_{FI} \widehat{b}_{FI,t} - \kappa Q b_{FI} \widehat{b}_{FI,t-1} + \kappa Q b_{FI} \pi_t + re \widehat{re}_t = s \widehat{s}_t, \tag{B.11}$$

$$-\sigma \left(E_t c_{s,t+1} - c_{s,t} \right) - E_t \pi_{t+1} + \frac{R^b}{sp} E_t r_{t+1}^b - \frac{R^s}{sp} r_t^s = \omega_t, \tag{B.12}$$

$$r_t^{re} = r_t^s, \tag{B.13}$$

$$\widehat{p}_t^* = \frac{1}{1+\chi\varepsilon} \left(f_{1,t} - f_{2,t} \right), \tag{B.14}$$

$$f_{1,t} = (1-\phi\beta)(1+\chi) y_t + \phi\beta\varepsilon (1+\chi) E_t \pi_{t+1} + \phi\beta E_t f_{1,t+1}, \tag{B.15}$$

$$f_{2,t} = -(1-\phi\beta) \sigma c_{s,t} + (1-\phi\beta) y_t + \phi\beta (\varepsilon - 1) E_t \pi_{t+1} + \phi\beta E_t f_{2,t+1}, \tag{B.16}$$

$$(1-z) c_{s,t} + z c_{b,t} = y_t, \tag{B.17}$$

$$\pi_t = \frac{1-\phi}{\phi} \widehat{p}_t^*, \tag{B.18}$$

$$q_t + \widehat{b}_{cb,t} = \widehat{re}_t, \tag{B.19}$$

$$\widehat{b}_t = \frac{b_{FI}}{b} \widehat{b}_{FI,t} + \frac{b_{cb}}{b} \widehat{b}_{cb,t}, \tag{B.20}$$

$$c_{b,t} = q_t + \widehat{b}_t, \tag{B.21}$$

$$qe_t = \widehat{re}_t, \tag{B.22}$$

$$r_t^n = -\widehat{\zeta}_t, \tag{B.23}$$

$$\widehat{\zeta}_t = \rho_\zeta \widehat{\zeta}_{t-1} + \sigma_\zeta \epsilon_t^\zeta. \tag{B.24}$$

B.3 Deriving the IS Curve

First, we begin by adding the Euler equations of the patient and impatient households, i.e., Equations (B.6) and (B.9):

$$\begin{aligned} (1-z) c_{s,t} + z c_{b,t} = (1-\alpha) E_t \left((1-z) c_{s,t+1} + z c_{b,t+1} \right) - \frac{1-z}{\sigma} \left(r_t^s - E_t \pi_{t+1} + \widehat{\zeta}_t \right) \\ - \frac{z(1-\alpha)}{\sigma} \left(E_t r_{t+1}^b - E_t \pi_{t+1} \right). \end{aligned} \tag{B.25}$$

Using the resource constraint (B.17), we get a first version of the IS curve:

$$y_t = (1-\alpha) E_t y_{t+1} - \frac{1-z}{\sigma} \left(r_t^s - E_t \pi_{t+1} + \widehat{\zeta}_t \right) - \frac{z(1-\alpha)}{\sigma} \left(E_t r_{t+1}^b - E_t \pi_{t+1} \right). \tag{B.26}$$

Second, we combine Equations (B.21) and (B.20):

$$c_{b,t} = q_t + \bar{b}_{FI} \hat{b}_{FI,t} + \bar{b}_{cb} \hat{b}_{cb,t} = \bar{b}_{FI} \left(q_t + \hat{b}_{FI,t} \right) + \bar{b}_{cb} \left(q_t + \hat{b}_{cb,t} \right), \tag{B.27}$$

where $\bar{b}_{FI} \equiv \frac{b_{FI}}{b}$ and $\bar{b}_{cb} \equiv \frac{b_{cb}}{b}$. Using Equations (B.10), (B.19), and (B.22), the last equation rewrites as:

$$c_{b,t} = \bar{b}_{cb} q e_t. \tag{B.28}$$

Using this in the impatient households' Euler equation (B.9), we have:

$$\bar{b}_{cb} \left((1-\alpha) E_t q e_{t+1} - q e_t \right) = \frac{1-\alpha}{\sigma} \left(E_t r_{t+1}^b - E_t \pi_{t+1} \right). \tag{B.29}$$

This last result can be used to rewrite the IS curve as a function of qe_t:

$$y_t = (1-\alpha) E_t y_{t+1} - \frac{1-z}{\sigma} \left(r_t^s - E_t \pi_{t+1} + \widehat{\zeta}_t \right) + z \bar{b}_{cb} \left(q e_t - (1-\alpha) E_t q e_{t+1} \right). \tag{B.30}$$

The latter can be written in terms of the natural rate of interest:

$$y_t = (1-\alpha) E_t y_{t+1} - \frac{1-z}{\sigma} \left(r_t^s - E_t \pi_{t+1} - r_t^n \right) + z \bar{b}_{cb} \left(q e_t - (1-\alpha) E_t q e_{t+1} \right). \tag{B.31}$$

B.4 Deriving the NKPC

We combine Equations (B.14)-(B.16), and (B.18) to obtain:

$$\pi_t = \frac{(1-\phi)(1-\phi\beta)}{(1+\chi\varepsilon)\phi} \left(\chi y_t + \sigma c_{s,t} \right) + \beta E_t \pi_{t+1}. \tag{B.32}$$

Define $\gamma \equiv \frac{(1-\phi)(1-\phi\beta)}{(1+\chi\varepsilon)\phi}$ and use the resource constraint, Equation (B.17), to replace $c_{s,t}$:

$$\pi_t = \gamma \left(\left(\chi + \frac{\sigma}{1-z} \right) y_t - \frac{\sigma z}{1-z} c_{b,t} \right) + \beta E_t \pi_{t+1}. \tag{B.33}$$

We use Equation (B.28) to write the NKPC as a function of qe_t:

$$\pi_t = \gamma \left(\left(\chi + \frac{\sigma}{1-z} \right) y_t - \frac{\sigma z \bar{b}_{cb}}{1-z} qe_t \right) + \beta E_t \pi_{t+1}. \tag{B.34}$$

Finally, we rearrange:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \left(\chi + \frac{\sigma}{1-z} \right) y_t - \frac{\gamma \sigma z \bar{b}_{cb}}{1-z} qe_t. \tag{B.35}$$

C Utility-Based Welfare Criterion

We follow Woodford (2003) in deriving the utility-based loss function. We take a Taylor expansion of each term of the following utility function:

$$\overline{U}_t \equiv U\left(C_{s,t}, C_{b,t}\right)+\int_0^1 V\left(L_t\left(i\right)\right)di = \left(1-z\right)\left(\frac{{C_{s,t}}^{1-\sigma}-1}{1-\sigma}\right)+z\left(\frac{{C_{b,t}}^{1-\sigma}-1}{1-\sigma}\right)-\left(1-z\right)\psi\int_0^1 \frac{\left(\frac{L_t(i)}{1-z}\right)^{1+\chi}}{1+\chi}di. \tag{C.1}$$

Recall that the economy's resource constraint is:

$$Y_t = C_t = \left(1-z\right)C_{s,t} + zC_{b,t}. \tag{C.2}$$

We can rewrite:

$$U\left(C_{s,t}, C_{b,t}\right) = U\left(Y_t, C_{b,t}\right) = \left(1-z\right)\left(\frac{\left(\frac{Y_t - zC_{b,t}}{1-z}\right)^{1-\sigma}-1}{1-\sigma}\right)+z\left(\frac{{C_{b,t}}^{1-\sigma}-1}{1-\sigma}\right). \tag{C.3}$$

Taking a second-order expansion around the steady state, we obtain

$$\begin{aligned} U\left(Y_t, C_{b,t}\right) = U\left(Y, C_b\right) + U_Y\left(Y_t - Y\right) + \frac{1}{2}U_{YY}\left(Y_t - Y\right)^2 + U_{C_b}\left(C_{b,t} - C_b\right) + \frac{1}{2}U_{C_bC_b}\left(C_{b,t} - C_b\right)^2 \\ + U_{YC_b}\left(Y_t - Y\right)\left(C_{b,t} - C_b\right) + t.i.p. + O\left(||\xi||^3\right), \end{aligned} \tag{C.4}$$

where $O\left(||\xi||^3\right)$ represents all relevant terms that are of third or higher order, and $t.i.p.$ denotes all the terms independent of monetary policy. Then, we take a second-order Taylor expansion of Y_t and $C_{b,t}$:

$$Y_t = Y\left(1 + y_t + \frac{1}{2}{y_t}^2\right) + O\left(||\xi||^3\right), \tag{C.5}$$

$$C_{b,t} = C_b\left(1 + c_{b,t} + \frac{1}{2}{c_{b,t}}^2\right) + O\left(||\xi||^3\right), \tag{C.6}$$

where $y_t \equiv \log Y_t - \log Y$ and $c_{b,t} \equiv \log C_{b,t} - \log C_b$. This implies:

$$Y_t - Y = Yy_t + \frac{1}{2}Y{y_t}^2 + O\left(||\xi||^3\right), \tag{C.7}$$

$$C_{b,t} - C_b = C_bc_{b,t} + \frac{1}{2}C_b{c_{b,t}}^2 + O\left(||\xi||^3\right). \tag{C.8}$$

Substituting Equations (C.7) and (C.8) into Equation (C.4) gives:

$$U\left(Y_t, C_{b,t}\right) = U\left(Y, C_b\right) + U_Y Y y_t + \frac{1}{2} U_Y Y {y_t}^2 + \frac{1}{2} U_{YY} Y^2 {y_t}^2 + U_{C_b} C_b c_{b,t} + \frac{1}{2} U_{C_b} C_b {c_{b,t}}^2 + \frac{1}{2} U_{C_b C_b} {C_b}^2 {c_{b,t}}^2 + U_{Y C_b} Y C_b y_t c_{b,t} + t.i.p. + O\left(||\xi||^3\right). \tag{C.9}$$

Note that $U\left(Y, C_b\right)$ is independent of monetary policy. We rewrite (C.9) as:

$$U\left(Y_t, C_{b,t}\right) = U_Y Y \left(y_t + \frac{1}{2} {y_t}^2 + \frac{1}{2} \frac{U_{YY} Y}{U_Y} {y_t}^2 + \frac{U_{C_b} C_b}{U_Y Y} \left(c_{b,t} + \frac{1}{2} {c_{b,t}}^2 \right) + \frac{1}{2} \frac{U_{C_b C_b} {C_b}^2}{U_Y Y} {c_{b,t}}^2 + \frac{U_{Y C_b} C_b}{U_Y} y_t c_{b,t} \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.10}$$

From the utility function, we have $\frac{U_{YY} Y}{U_Y} = -\frac{\sigma}{1-z}$, $\frac{U_{C_b} C_b}{U_Y Y} = 0$, $\frac{U_{C_b C_b} {C_b}^2}{U_Y Y} = -\frac{\sigma z}{1-z}$, and $\frac{U_{Y C_b} C_b}{U_Y} = \frac{\sigma z}{1-z}$. Thus, we obtain:

$$U\left(Y_t, C_{b,t}\right) = U_Y Y \left(y_t + \frac{1}{2} \left(1 - \frac{\sigma}{1-z} \right) {y_t}^2 - \frac{1}{2} \frac{\sigma z}{1-z} \left({c_{b,t}}^2 - 2 y_t c_{b,t} \right) \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.11}$$

Using the resource constraint, we know $U_C C = U_Y Y$. Finally, we then rewrite:

$$\frac{U\left(C_{s,t}, C_{b,t}\right)}{U_C C} = y_t + \frac{1}{2} \left(1 - \frac{\sigma}{1-z} \right) {y_t}^2 - \frac{1}{2} \frac{\sigma z}{1-z} \left({c_{b,t}}^2 - 2 y_t c_{b,t} \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.12}$$

Now, we also take a second-order Taylor expansion of $V\left(L_t\left(i\right)\right)$.

$$V\left(L_t\left(i\right)\right) = V\left(L\right) + V_L\left(L_t\left(i\right) - L\right) + \frac{1}{2} V_{LL}\left(L_t\left(i\right) - L\right)^2 + t.i.p. + O\left(||\xi||^3\right). \tag{C.13}$$

The second-order approximation of $L_t\left(i\right)$ is:

$$L_t\left(i\right) = L\left(1 + l_t\left(i\right) + \frac{1}{2} l_t\left(i\right)^2 \right) + O\left(||\xi||^3\right), \tag{C.14}$$

where $l_t\left(i\right) \equiv \log L_t\left(i\right) - \log L$. This implies:

$$L_t\left(i\right) - L = L l_t\left(i\right) + \frac{1}{2} L l_t\left(i\right)^2 + O\left(||\xi||^3\right). \tag{C.15}$$

Substituting Equation (C.15) into Equation (C.13) gives:

$$V\left(L_t\left(i\right)\right) = V\left(L\right) + V_L L l_t\left(i\right) + \frac{1}{2} V_L L l_t\left(i\right)^2 + \frac{1}{2} V_{LL} L^2 l_t\left(i\right)^2 + t.i.p. + O\left(||\xi||^3\right). \tag{C.16}$$

Note that $V(L)$ is independent of monetary policy. We rewrite (C.16) as:

$$V(L_t(i)) = V_L L\left(l_t(i) + \frac{1}{2}l_t(i)^2 + \frac{1}{2}\frac{V_{LL}L}{V_L}l_t(i)^2\right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.17}$$

Since $\frac{V_{LL}L}{V_L} = \chi$, we rewrite Equation (C.17) as:

$$V(L_t(i)) = V_L L\left(l_t(i) + \frac{1}{2}(1+\chi)\, l_t(i)^2\right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.18}$$

From the production function, we have:

$$l_t(i) = y_t(i). \tag{C.19}$$

Substituting Equation (C.19) into Equation (C.18), we obtain:

$$V(L_t(i)) = V_L L\left(y_t(i) + \frac{1}{2}(1+\chi)\, y_t(i)^2\right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.20}$$

By integrating Equation (C.20), we obtain:

$$\int_0^1 V(L_t(i))\, di = V_L L\left(E_i y_t(i) + \frac{1}{2}(1+\chi)\left((E_i y_t(i))^2 + var_i y_t(i)\right)\right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.21}$$

Taking a second-order approximation of the aggregators gives:

$$y_t = E_i y_t(i) + \frac{1}{2}\left(1 - \frac{1}{\varepsilon}\right) var_i y_t(i) + O\left(||\xi||^3\right), \tag{C.22}$$

which implies

$$E_i y_t(i) = y_t - \frac{1}{2}\left(1 - \frac{1}{\varepsilon}\right) var_i y_t(i) + O\left(||\xi||^3\right), \tag{C.23}$$

$$(E_i y_t(i))^2 = {y_t}^2 + O\left(||\xi||^3\right). \tag{C.24}$$

We substitute Equations (C.23) and (C.24) into Equation (C.21) obtaining:

$$\int_0^1 V(L_t(i))\, di = V_L L\left(y_t + \frac{1}{2}(1+\chi)\, {y_t}^2 + \frac{1}{2}\left(\chi + \frac{1}{\varepsilon}\right) var_i y_t(i)\right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.25}$$

Now, recall that $L = Y = C$. From the household's labour supply relation, we have:

$$-\frac{V_L L}{U_C C} = 1, \tag{C.26}$$

Then, we rewrite:

$$\frac{\int_0^1 V\left(L_t\left(i\right)\right)di}{U_C C} = -y_t - \frac{1}{2}\left(1+\chi\right){y_t}^2 - \frac{1}{2}\left(\chi + \frac{1}{\varepsilon}\right) var_i y_t\left(i\right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.27}$$

Combining Equations (C.12) and (C.27), we finally obtain:

$$\begin{aligned}\frac{\overline{U}_t}{\overline{U}_C C} \equiv \frac{U\left(C_{s,t}, C_{b,t}\right) + \int_0^1 V\left(L_t\left(i\right)\right)di}{U_C C} &= -\frac{1}{2}\left(\chi + \frac{\sigma}{1-z}\right){y_t}^2 - \frac{1}{2}\frac{\sigma z}{1-z}\left({c_{b,t}}^2 - 2y_t c_{b,t}\right) \\ &\quad -\frac{1}{2}\left(\chi + \frac{1}{\varepsilon}\right) var_i y_t\left(i\right) + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{C.28}$$

We take the expected discounted sum over time, we obtain:

$$\begin{aligned}\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\overline{U}_t}{\overline{U}_C C} = E_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2}\left(\chi + \frac{\sigma}{1-z}\right){y_t}^2 - \frac{1}{2}\frac{\sigma z}{1-z}\left({c_{b,t}}^2 - 2y_t c_{b,t}\right) - \frac{1}{2}\left(\chi + \frac{1}{\varepsilon}\right) var_i y_t\left(i\right)\right] \\ + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{C.29}$$

The demand for $Y_t\left(i\right)$ is given by:

$$Y_t\left(i\right) = \left(\frac{P_t\left(i\right)}{P_t}\right)^{-\varepsilon} Y_t. \tag{C.30}$$

Then, we get:

$$y_t\left(i\right) = -\varepsilon\left(p_t\left(i\right) - p_t\right) + y_t. \tag{C.31}$$

This implies that:

$$var_i y_t\left(i\right) = \varepsilon^2 var_i p_t\left(i\right), \tag{C.32}$$

where $\Delta_t \equiv var_i p_t\left(i\right)$ is a measure of price dispersion. When prices are staggered as in the discrete time Calvo (1983) fashion, Woodford (2003) shows that:

$$\Delta_t = \phi\Delta_{t-1} + \frac{\phi}{1-\phi}{\pi_t}^2 + O\left(||\xi||^3\right) = \phi^{t+1}\Delta_{-1} + \sum_{k=0}^{t}\phi^{t-k}\left(\frac{\phi}{1-\phi}\right){\pi_k}^2 + O\left(||\xi||^3\right). \tag{C.33}$$

If a new policy is conducted from $t \geq 0$, the first term, $\phi^{t+1}\Delta_{-1}$ is independent of policy. If we take the discounted sum over time, we obtain:

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{\phi}{\left(1-\phi\right)\left(1-\phi\beta\right)}\sum_{t=0}^{\infty}\beta^t{\pi_t}^2 + t.i.p. + O\left(||\xi||^3\right). \tag{C.34}$$

Now, we consider:

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \left(\chi + \frac{\sigma}{1-z} \right) {y_t}^2 - \frac{1}{2} \frac{\sigma z}{1-z} \left({c_{b,t}}^2 - 2 y_t c_{b,t} \right) - \frac{1}{2} \left(\chi + \frac{1}{\varepsilon} \right) \varepsilon^2 \Delta_t \right] + t.i.p. + O\left(||\xi||^3 \right). \tag{C.35}$$

Therefore, we can rewrite:

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \left(\chi + \frac{\sigma}{1-z} \right) {y_t}^2 - \frac{1}{2} \frac{\sigma z}{1-z} \left({c_{b,t}}^2 - 2 y_t c_{b,t} \right) - \frac{1}{2} \frac{\phi \varepsilon (1 + \chi \varepsilon)}{(1-\phi)(1-\phi\beta)} {\pi_t}^2 \right] + t.i.p. + O\left(||\xi||^3 \right). \tag{C.36}$$

Then, we rearrange:

$$\mathbb{W} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\phi \varepsilon (1 + \chi \varepsilon)}{(1-\phi)(1-\phi\beta)} {\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z} \right) {y_t}^2 + \frac{\sigma z}{1-z} \left({c_{b,t}}^2 - 2 c_{b,t} y_t \right) \right] + t.i.p. + O\left(||\xi||^3 \right). \tag{C.37}$$

From the market clearing conditions, we have:

$$c_{b,t} = \bar{b}_{cb} q e_t. \tag{C.38}$$

Substituting Equation (C.38) into Equation (C.37) gives:

$$\mathbb{W} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\phi \varepsilon (1 + \chi \varepsilon)}{(1-\phi)(1-\phi\beta)} {\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z} \right) {y_t}^2 + \frac{\sigma z \bar{b}_{cb}}{1-z} \left(\bar{b}_{cb} q {e_t}^2 - 2 q e_t y_t \right) \right] + t.i.p. + O\left(||\xi||^3 \right). \tag{C.39}$$

The average welfare loss per period is thus given as:

$$\mathbb{L}_t = \frac{\phi \varepsilon (1 + \chi \varepsilon)}{(1-\phi)(1-\phi\beta)} {\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z} \right) {y_t}^2 + \frac{\sigma z \bar{b}_{cb}}{1-z} \left(\bar{b}_{cb} q {e_t}^2 - 2 q e_t y_t \right). \tag{C.40}$$

D Linear-Quadratic Approaches

D.1 Optimal Monetary Policy under Discretion

The central bank is assumed to choose π_t, y_t, r_t^s, qe_t in order to minimise the period losses:

$$\frac{1}{2}\left(\frac{\varepsilon}{\gamma}{\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z}\right){y_t}^2 + \frac{\sigma z \bar{b}_{cb}}{1-z}\left(\bar{b}_{cb}{qe_t}^2 - 2qe_t y_t\right)\right), \tag{D.1}$$

subject to the sequences of constraints and $r^s \geq -\frac{R^s-1}{R^s}$:

$$y_t = (1-\alpha)E_t y_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - E_t\pi_{t+1} - r_t^n\right) + z\bar{b}_{cb}\left(qe_t - (1-\alpha)E_t qe_{t+1}\right), \tag{D.2}$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma\left(\chi + \frac{\sigma}{1-z}\right)y_t - \frac{\gamma\sigma z\bar{b}_{cb}}{1-z}qe_t, \tag{D.3}$$

where the terms $E_t\pi_{t+1}$, $E_t y_{t+1}$, $E_t qe_{t+1}$, and r_t^n are taken as given by the central bank. The Lagrangian for the above problem takes the form:

$$\begin{aligned} L = \frac{1}{2}\left(\frac{\varepsilon}{\gamma}{\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z}\right){y_t}^2 + \frac{\sigma z\bar{b}_{cb}}{1-z}\left(\bar{b}_{cb}{qe_t}^2 - 2qe_t y_t\right)\right) \\ +\xi_{1,t}\left(y_t - (1-\alpha)y_{t+1} + \frac{1-z}{\sigma}\left(r_t^s - \pi_{t+1} - r_t^n\right) - z\bar{b}_{cb}\left(qe_t - (1-\alpha)qe_{t+1}\right)\right) \\ +\xi_{2,t}\left(\pi_t - \beta\pi_{t+1} - \gamma\left(\chi + \frac{\sigma}{1-z}\right)y_t + \frac{\gamma\sigma z\bar{b}_{cb}}{1-z}qe_t\right), \end{aligned} \tag{D.4}$$

where $\xi_{1,t}$ and $\xi_{2,t}$ are Lagrangian multipliers.

Differentiating the Lagrangian with respect to π_t, y_t, r_t^s, and qe_t yields the optimality conditions:

$$\frac{\partial L}{\partial \pi_t} = \frac{\varepsilon}{\gamma}\pi_t + \xi_{2,t} = 0, \tag{D.5}$$

$$\frac{\partial L}{\partial y_t} = \left(\chi + \frac{\sigma}{1-z}\right)y_t - \frac{\sigma z\bar{b}_{cb}}{1-z}qe_t + \xi_{1,t} - \gamma\left(\chi + \frac{\sigma}{1-z}\right)\xi_{2,t} = 0, \tag{D.6}$$

$$\frac{\partial L}{\partial r_t^s}\left(r_t^s + \frac{R^s-1}{R^s}\right) = \frac{1-z}{\sigma}\xi_{1,t}\left(r_t^s + \frac{R^s-1}{R^s}\right) = 0, \quad \xi_{1,t} \geq 0 \text{ and } r_t^s \geq -\frac{R^s-1}{R^s}, \tag{D.7}$$

$$\frac{\partial L}{\partial qe_t} = \frac{\sigma z\bar{b}_{cb}}{1-z}\left(\bar{b}_{cb}qe_t - y_t\right) - z\bar{b}_{cb}\xi_{1,t} + \frac{\gamma\sigma z\bar{b}_{cb}}{1-z}\xi_{2,t} = 0, \tag{D.8}$$

that must hold for $t = 0, 1, 2, \ldots$ and where $\xi_{1,-1} = \xi_{2,-1} = 0$, because Equations (D.2) and (D.3) corresponding to period -1 is not an effective constraint for the central bank choosing its optimal plan in period

0. In sum, the equilibrium conditions under the optimal discretionary policy are then given by Equations (D.2), (D.3), (D.6), (D.5), (D.7), and (D.8).

D.2 Optimal Monetary Policy under Commitment

The central bank is assumed to choose a state-contingent sequence $\{\pi_t, y_t, r_t^s, qe_t\}_{t=0}^{\infty}$ that minimises:

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{\varepsilon}{\gamma}{\pi_t}^2+\left(\chi+\frac{\sigma}{1-z}\right){y_t}^2+\frac{\sigma z\bar{b}_{cb}}{1-z}\left(\bar{b}_{cb}{qe_t}^2-2qe_ty_t\right)\right), \tag{D.9}$$

subject to the sequences of constraints and $r^s \geq -\frac{R^s-1}{R^s}$:

$$y_t=(1-\alpha)E_ty_{t+1}-\frac{1-z}{\sigma}\left(r_t^s-E_t\pi_{t+1}-r_t^n\right)+z\bar{b}_{cb}\left(qe_t-(1-\alpha)E_tqe_{t+1}\right), \tag{D.10}$$

$$\pi_t=\beta E_t\pi_{t+1}+\gamma\left(\chi+\frac{\sigma}{1-z}\right)y_t-\frac{\gamma\sigma z\bar{b}_{cb}}{1-z}qe_t. \tag{D.11}$$

In order to solve the previous problem it is useful to write down the associated Lagrangian, which is given by:

$$\begin{aligned}L=E_0\sum_{t=0}^{\infty}\beta^t\Bigg[&\frac{1}{2}\left(\frac{\varepsilon}{\gamma}{\pi_t}^2+\left(\chi+\frac{\sigma}{1-z}\right){y_t}^2+\frac{\sigma z\bar{b}_{cb}}{1-z}\left(\bar{b}_{cb}{qe_t}^2-2qe_ty_t\right)\right)\\&+\xi_{1,t}\left(y_t-(1-\alpha)y_{t+1}+\frac{1-z}{\sigma}\left(r_t^s-\pi_{t+1}-r_t^n\right)-z\bar{b}_{cb}\left(qe_t-(1-\alpha)qe_{t+1}\right)\right)\\&+\xi_{2,t}\left(\pi_t-\beta\pi_{t+1}-\gamma\left(\chi+\frac{\sigma}{1-z}\right)y_t+\frac{\gamma\sigma z\bar{b}_{cb}}{1-z}qe_t\right)\Bigg],\end{aligned} \tag{D.12}$$

where $\{\xi_{1,t},\xi_{2,t}\}_{t=0}^{\infty}$ are sequences of Lagrangian multipliers, and where the law of iterated expectations has been used to eliminate the conditional expectation that appeared in each constraint.

Differentiating the Lagrangian with respect to π_t, y_t, qe_t, and r_t^s yields the optimality conditions:

$$\frac{\partial L}{\partial \pi_t}=\frac{\varepsilon}{\gamma}\pi_t+\xi_{2,t}-\frac{1-z}{\beta\sigma}\xi_{1,t-1}-\xi_{2,t-1}=0, \tag{D.13}$$

$$\frac{\partial L}{\partial y_t}=\left(\chi+\frac{\sigma}{1-z}\right)y_t-\frac{\sigma z\bar{b}_{cb}}{1-z}qe_t+\xi_{1,t}-\gamma\left(\chi+\frac{\sigma}{1-z}\right)\xi_{2,t}-\frac{1-\alpha}{\beta}\xi_{1,t-1}=0, \tag{D.14}$$

$$\frac{\partial L}{\partial r_t^s}\left(r_t^s+\frac{R^s-1}{R^s}\right)=\frac{1-z}{\sigma}\xi_{1,t}\left(r_t^s+\frac{R^s-1}{R^s}\right)=0, \quad \xi_{1,t}\geq 0 \text{ and } r_t^s\geq-\frac{R^s-1}{R^s}, \tag{D.15}$$

$$\frac{\partial L}{\partial qe_t}=\frac{\sigma z\bar{b}_{cb}}{1-z}\left(\bar{b}_{cb}qe_t-y_t\right)-z\bar{b}_{cb}\xi_{1,t}+\frac{\gamma\sigma z\bar{b}_{cb}}{1-z}\xi_{2,t}+\frac{z\bar{b}_{cb}(1-\alpha)}{\beta}\xi_{1,t-1}=0, \tag{D.16}$$

that must hold for $t=0,1,2,\ldots$ and where $\xi_{1,-1}=\xi_{2,-1}=0$, because Equations (D.10) and (D.11) corresponding to period −1 is not an effective constraint for the central bank choosing its optimal plan in period 0. In sum, the equilibrium conditions under the optimal commitment policy are then given by Equations (D.10), (D.11), (D.14), (D.13), (D.15), and (D.16).

D.3 Simple Mandate

The central bank is assumed to choose a state-contingent sequence $\{\pi_t, y_t, r_t^s, qe_t\}_{t=0}^{\infty}$ that minimises:

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left({\pi_t}^2+\vartheta{y_t}^2\right), \tag{D.17}$$

subject to the sequences of constraints and $r^s \geq -\frac{R^s-1}{R^s}$:

$$y_t=(1-\alpha)E_t y_{t+1}-\frac{1-z}{\sigma}\left(r_t^s-E_t\pi_{t+1}-r_t^n\right)+z\bar{b}_{cb}\left(qe_t-(1-\alpha)E_t qe_{t+1}\right), \tag{D.18}$$

$$\pi_t=\beta E_t\pi_{t+1}+\gamma\left(\chi+\frac{\sigma}{1-z}\right)y_t-\frac{\gamma\sigma z\bar{b}_{cb}}{1-z}qe_t. \tag{D.19}$$

In order to solve the previous problem it is useful to write down the associated Lagrangian, which is given by:

$$\begin{aligned} L=E_0\sum_{t=0}^{\infty}\beta^t\Bigg[\frac{1}{2}\left({\pi_t}^2+\vartheta{y_t}^2\right)\\ +\xi_{1,t}\left(y_t-(1-\alpha)y_{t+1}+\frac{1-z}{\sigma}\left(r_t^s-\pi_{t+1}-r_t^n\right)-z\bar{b}_{cb}\left(qe_t-(1-\alpha)qe_{t+1}\right)\right)\\ +\xi_{2,t}\left(\pi_t-\beta\pi_{t+1}-\gamma\left(\chi+\frac{\sigma}{1-z}\right)y_t+\frac{\gamma\sigma z\bar{b}_{cb}}{1-z}qe_t\right)\Bigg], \end{aligned} \tag{D.20}$$

where $\{\xi_{1,t}, \xi_{2,t}, \xi_{3,t}\}_{t=0}^{\infty}$ are sequences of Lagrangian multipliers, and where the law of iterated expectations has been used to eliminate the conditional expectation that appeared in each constraint.

Differentiating the Lagrangian with respect to π_t, y_t, qe_t, and r_t^s yields the optimality conditions:

$$\frac{\partial L}{\partial \pi_t}=\pi_t+\xi_{2,t}-\frac{1-z}{\beta\sigma}\xi_{1,t-1}-\xi_{2,t-1}=0, \tag{D.21}$$

$$\frac{\partial L}{\partial y_t}=\vartheta y_t+\xi_{1,t}-\gamma\left(\chi+\frac{\sigma}{1-z}\right)\xi_{2,t}-\frac{1-\alpha}{\beta}\xi_{1,t-1}=0, \tag{D.22}$$

$$\frac{\partial L}{\partial r_t^s}\left(r_t^s+\frac{R^s-1}{R^s}\right)=\frac{1-z}{\sigma}\xi_{1,t}\left(r_t^s+\frac{R^s-1}{R^s}\right)=0,\quad \xi_{1,t}\geq 0 \text{ and } r_t^s\geq-\frac{R^s-1}{R^s}, \tag{D.23}$$

$$\frac{\partial L}{\partial qe_t}=-z\bar{b}_{cb}\xi_{1,t}+\frac{\gamma\sigma z\bar{b}_{cb}}{1-z}\xi_{2,t}+\frac{z\bar{b}_{cb}(1-\alpha)}{\beta}\xi_{1,t-1}=0, \tag{D.24}$$

that must hold for $t = 0, 1, 2, \ldots$ and where $\xi_{1,-1} = \xi_{2,-1} = 0$, because Equations (D.18) and (D.19) corresponding to period -1 is not an effective constraint for the central bank choosing its optimal plan in period 0. In sum, the equilibrium conditions under the optimal policy are then given by Equations (D.18), (D.19), (D.22), (D.21), (D.23), and (D.24).

E Analytical Analysis

E.1 Two-Period Model

In this section, we derive the equations of the two-period model when the central bank carries out either the optimal policy under discretion or under commitment. The aim is to explicitly show how inflation and output respond to FG and balance-sheet policies. To solve the model, we make the following simplifying assumptions: (i) Shock: $r_0^n < 0$, $r_0^s = -\frac{R^s-1}{R^s}$, $r_1^n = 0$, $-\frac{R^s-1}{R^s} < r_1^s \leq 0$ and (ii) Perfect foresight: $E_t x_{t+1} = x_{t+1}$, $\forall t$. Thus, the IS equation and NKPC in the periods 0 and 1 write as:

$$y_0 = (1-\alpha)\, y_1 - \frac{1-z}{\sigma}\left(-\frac{R^s-1}{R^s} - \pi_1 - r_0^n\right) + z\bar{b}_{cb}\left(qe_0 - (1-\alpha)\, qe_1\right), \tag{E.1}$$

$$\pi_0 = \beta\pi_1 + \gamma\left(\chi + \frac{\sigma}{1-z}\right) y_0 - \frac{\gamma\sigma z\bar{b}_{cb}}{1-z} qe_0, \tag{E.2}$$

$$y_1 = -\frac{1-z}{\sigma} r_1^s + z\bar{b}_{cb} qe_1, \tag{E.3}$$

$$\pi_1 = \gamma\left(\chi + \frac{\sigma}{1-z}\right) y_1 - \frac{\gamma\sigma z\bar{b}_{cb}}{1-z} qe_1. \tag{E.4}$$

E.1.1 AD-AS Analysis

Case A Using the first-order conditions of the optimal discretion problem without QE, we obtain the following AD and AS equations:

$$y_0 = \frac{1-z}{\sigma}\left(\frac{R^s-1}{R^s} + r_0^n\right), \tag{E.5}$$

$$\pi_0 = \gamma\left(\chi + \frac{\sigma}{1-z}\right) y_0. \tag{E.6}$$

We, therefore, obtain the following solution:

$$\pi_0^\star = \gamma\left(1 + \frac{\chi(1-z)}{\sigma}\right)\left(\frac{R^s-1}{R^s} + r_0^n\right), \tag{E.7}$$

$$y_0^\star = \frac{1-z}{\sigma}\left(\frac{R^s-1}{R^s} + r_0^n\right). \tag{E.8}$$

Case B Using the first-order conditions of the optimal discretion problem with QE, we obtain the following AD and AS equations:

$$y_0 = \frac{1-z}{\sigma}\left(\frac{R^s-1}{R^s} + r_0^n\right) + z\bar{b}_{cb} qe_0, \tag{E.9}$$

$$\pi_0 = \gamma \left(\chi + \frac{\sigma}{1-z} \right) y_0 - \frac{\gamma \sigma z \bar{b}_{cb}}{1-z} q e_0. \tag{E.10}$$

The equilibrium in this case is given by:

$$\pi_0^\star = \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) \left(\frac{R^s - 1}{R^s} + r_0^n \right) + \gamma \chi z \bar{b}_{cb} q e_0^\star, \tag{E.11}$$

$$y_0^\star = \frac{1-z}{\sigma} \left(\frac{R^s - 1}{R^s} + r_0^n \right) + z \bar{b}_{cb} q e_0^\star. \tag{E.12}$$

Case C Using the first-order conditions of the optimal commitment problem without QE, we obtain the following AD and AS equations:

$$y_0 = \frac{1-z}{\sigma} \left(\frac{R^s - 1}{R^s} - \left(1 - \alpha + \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) \right) r_1^s + r_0^n \right), \tag{E.13}$$

$$\pi_0 = \gamma \left(\chi + \frac{\sigma}{1-z} \right) y_0 - \beta \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) r_1^s. \tag{E.14}$$

The equilibrium can be written as:

$$\pi_0^\star = \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) \left(\frac{R^s - 1}{R^s} - \left(1 - \alpha + \beta + \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) \right) r_1^{s\star} + r_0^n \right), \tag{E.15}$$

$$y_0^\star = \frac{1-z}{\sigma} \left(\frac{R^s - 1}{R^s} - \left(1 - \alpha + \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) \right) r_1^{s\star} + r_0^n \right). \tag{E.16}$$

Case D Using the first-order conditions of the optimal commitment problem with QE, we obtain the following AD and AS equations:

$$y_0 = \frac{1-z}{\sigma} \left(\frac{R^s - 1}{R^s} - \left(1 - \alpha + \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) \right) r_1^s + r_0^n \right) + z \bar{b}_{cb} \left(q e_0 + \frac{\gamma \chi (1-z)}{\sigma} q e_1 \right), \tag{E.17}$$

$$\pi_0 = \gamma \left(\chi + \frac{\sigma}{1-z} \right) y_0 - \beta \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) r_1^s - \gamma z \bar{b}_{cb} \left(\frac{\sigma}{1-z} q e_0 - \beta \chi q e_1 \right). \tag{E.18}$$

We obtain the following solution:

$$\begin{aligned} \pi_0^\star = \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) & \left(\frac{R^s - 1}{R^s} - \left(1 - \alpha + \beta + \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) \right) r_1^{s\star} + r_0^n \right) \\ & + \gamma \chi z \bar{b}_{cb} \left(q e_0^\star + \left(\beta + \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) \right) q e_1^\star \right), \end{aligned} \tag{E.19}$$

$$y_0^\star = \frac{1-z}{\sigma} \left(\frac{R^s - 1}{R^s} - \left(1 - \alpha + \gamma \left(1 + \frac{\chi (1-z)}{\sigma} \right) \right) r_1^{s\star} + r_0^n \right) + z \bar{b}_{cb} \left(q e_0^\star + \frac{\gamma \chi (1-z)}{\sigma} q e_1^\star \right). \tag{E.20}$$

E.1.2 Optimal QE under Discretion and Commitment

Case B Under discretion, we have:

$$qe_0^{\star} = \frac{\chi\left(1-z\right)\left(1+\frac{\chi(1-z)}{\sigma}\right)\left(1+\varepsilon\gamma\left(\chi+\frac{\sigma}{1-z}\right)\right)}{\sigma\left(\chi+\sigma\right)\bar{b}_{cb}\left(1+\frac{\chi^2 z(1-z)}{\sigma(\chi+\sigma)}\left(1+\varepsilon\gamma\left(\chi+\frac{\sigma}{1-z}\right)\right)\right)}\left(-\frac{R^s-1}{R^s}-r_0^n\right). \tag{E.21}$$

We define:

$$\Gamma_B \equiv \frac{\chi\left(1-z\right)\left(1+\frac{\chi(1-z)}{\sigma}\right)\left(1+\varepsilon\gamma\left(\chi+\frac{\sigma}{1-z}\right)\right)}{\sigma\left(\chi+\sigma\right)\bar{b}_{cb}\left(1+\frac{\chi^2 z(1-z)}{\sigma(\chi+\sigma)}\left(1+\varepsilon\gamma\left(\chi+\frac{\sigma}{1-z}\right)\right)\right)}. \tag{E.22}$$

We can then express $qe_0^{\star}$ as:

$$qe_0^{\star} = \Gamma_B\left(-\frac{R^s-1}{R^s}-r_0^n\right). \tag{E.23}$$

Case D Under commitment, instead, we have:

$$qe_0^{\star} = \frac{\chi\left(1-z\right)\left(1+\frac{\chi(1-z)}{\sigma}\right)\left(1+\varepsilon\gamma\left(\chi+\frac{\sigma}{1-z}\right)\right)\left(-\frac{R^s-1}{R^s}-r_0^n+\left(1-\alpha+\frac{\beta\varepsilon\gamma\left(\chi+\frac{\sigma}{1-z}\right)}{1+\varepsilon\gamma\left(\chi+\frac{\sigma}{1-z}\right)}+\gamma\left(1+\frac{\chi(1-z)}{\sigma}\right)\right)r_1^{s\star}\right)}{\sigma\left(\chi+\sigma\right)\bar{b}_{cb}\left(1+\frac{\chi z}{\chi+\sigma}\left(\left(1+\frac{\chi(1-z)}{\sigma}\right)\left(1-\frac{(1-\alpha)\gamma\chi(1-z)}{\beta\sigma}+\varepsilon\gamma\chi\left(1-\frac{1-\alpha}{\beta}\left(\beta+\gamma\left(1+\frac{\chi(1-z)}{\sigma}\right)\right)\right)\right)-1\right)\right)}. \tag{E.24}$$

We define δ_1, δ_2, and δ_3 as:

$$\delta_1 \equiv \chi\left(1-z\right)\left(1+\frac{\chi\left(1-z\right)}{\sigma}\right)\left(1+\varepsilon\gamma\left(\chi+\frac{\sigma}{1-z}\right)\right), \tag{E.25}$$

$$\delta_2 \equiv \left(1-\alpha+\frac{\beta\varepsilon\gamma\left(\chi+\frac{\sigma}{1-z}\right)}{1+\varepsilon\gamma\left(\chi+\frac{\sigma}{1-z}\right)}+\gamma\left(1+\frac{\chi\left(1-z\right)}{\sigma}\right)\right), \tag{E.26}$$

$$\begin{aligned}\delta_3 \equiv \sigma\left(\chi+\sigma\right)\bar{b}_{cb}\Bigg(1+\frac{\chi z}{\chi+\sigma}\Bigg(\left(1+\frac{\chi\left(1-z\right)}{\sigma}\right)\Bigg(1-\frac{\left(1-\alpha\right)\gamma\chi\left(1-z\right)}{\beta\sigma} \\ +\varepsilon\gamma\chi\left(1-\frac{1-\alpha}{\beta}\left(\beta+\gamma\left(1+\frac{\chi\left(1-z\right)}{\sigma}\right)\right)\right)\Bigg)-1\Bigg)\Bigg).\end{aligned} \tag{E.27}$$

Thus, we can define the parameters Γ_D^1 and Γ_D^2, used in Section 4.3 of the main text:

$$\Gamma_D^1 \equiv \frac{\delta_1}{\delta_3} > 0, \tag{E.28}$$

$$\Gamma_D^2 \equiv \frac{\delta_1\delta_2}{\delta_3} > 0. \tag{E.29}$$

We can then express $qe_0^{\star}$ as:

$$qe_0^{\star} = \Gamma_D^1\left(-\frac{R^s-1}{R^s}-r_0^n\right)+\Gamma_D^2 r_1^{s\star}. \tag{E.30}$$

It is easy to derive that:

$$\frac{\partial \delta_1}{\partial \alpha} = 0, \tag{E.31}$$

$$\frac{\partial \delta_2}{\partial \alpha} = -1 < 0, \tag{E.32}$$

$$\frac{\partial \delta_3}{\partial \alpha} = \sigma\left(\chi+\sigma\right)\bar{b}_{cb}\left(\frac{\chi z}{\chi+\sigma}\right)\left(1+\frac{\chi\left(1-z\right)}{\sigma}\right)\left(\frac{\gamma\chi\left(1-z\right)}{\beta\sigma}+\frac{\varepsilon\gamma\chi}{\beta}\left(\beta+\gamma\left(1+\frac{\chi\left(1-z\right)}{\sigma}\right)\right)\right) > 0. \tag{E.33}$$

Thus Γ_D^1 and Γ_D^2 are both decreasing in α:

$$\begin{aligned}\frac{d\Gamma_D^1}{d\alpha} &= \frac{1}{\delta_3}\frac{\partial \delta_1}{\partial \alpha} - \frac{\delta_1}{\delta_3^2}\frac{\partial \delta_3}{\partial \alpha}\\ &= -\frac{\delta_1}{\delta_3^2}\sigma\left(\chi+\sigma\right)\bar{b}_{cb}\left(\frac{\chi z}{\chi+\sigma}\right)\left(1+\frac{\chi\left(1-z\right)}{\sigma}\right)\left(\frac{\gamma\chi\left(1-z\right)}{\beta\sigma}+\frac{\varepsilon\gamma\chi}{\beta}\left(\beta+\gamma\left(1+\frac{\chi\left(1-z\right)}{\sigma}\right)\right)\right) < 0,\end{aligned} \tag{E.34}$$

$$\begin{aligned}\frac{d\Gamma_D^2}{d\alpha} &= \frac{\delta_2}{\delta_3}\frac{\partial \delta_1}{\partial \alpha} + \frac{\delta_1}{\delta_3}\frac{\partial \delta_2}{\partial \alpha} - \frac{\delta_1\delta_2}{\delta_3^2}\frac{\partial \delta_3}{\partial \alpha}\\ &= -\frac{\delta_1}{\delta_3} - \frac{\delta_1\delta_2}{\delta_3^2}\sigma\left(\chi+\sigma\right)\bar{b}_{cb}\left(\frac{\chi z}{\chi+\sigma}\right)\left(1+\frac{\chi\left(1-z\right)}{\sigma}\right)\left(\frac{\gamma\chi\left(1-z\right)}{\beta\sigma}+\frac{\varepsilon\gamma\chi}{\beta}\left(\beta+\gamma\left(1+\frac{\chi\left(1-z\right)}{\sigma}\right)\right)\right) < 0.\end{aligned} \tag{E.35}$$

E.1.3 Model Responses

In this section, parameters are calibrated as presented in Table 1 in the main text. Exceptionally, we assume $\alpha = 0.5$, $\rho_n = 0$, and $\sigma_n = 0.005$. Figure E.1 displays the responses following a one-time decline in the natural rate under the various optimal policies (Cases A-D defined in Section 3.2 and 3.3). The negative demand shock causes a significant decline in inflation and output. The fall in inflation leads the central bank to cut its policy rate, which hits the ZLB constraint. In Case A, the central bank cannot respond to the fall in demand due to the binding ZLB. In Case B, the central bank expands its balance sheet and, as a result, the fall in inflation and output is less severe. Moreover, as mentioned above, we note that the presence of QE in Case B has a larger stabilising effect on output than inflation.

In Cases C and D, the central bank is able to deploy FG as a policy tool, i.e., commit to keeping the nominal policy rate lower for longer than implied by current macroeconomic conditions. As a result, compared to Cases A and B, the policy rate in period 1 is kept below steady state. FG has the effect of boosting inflation expectations, which lowers the real rate and eases the drop in output.

In Case D, the central bank adjusts its balance sheet to mitigate the initial fall in demand and the subsequent overshoot in inflation and output. This can be observed by comparing Cases D and C. More specifically, in period 0, the policy rate is at the ZLB, and the central bank expands its balance sheet ($qe_0 > 0$). In

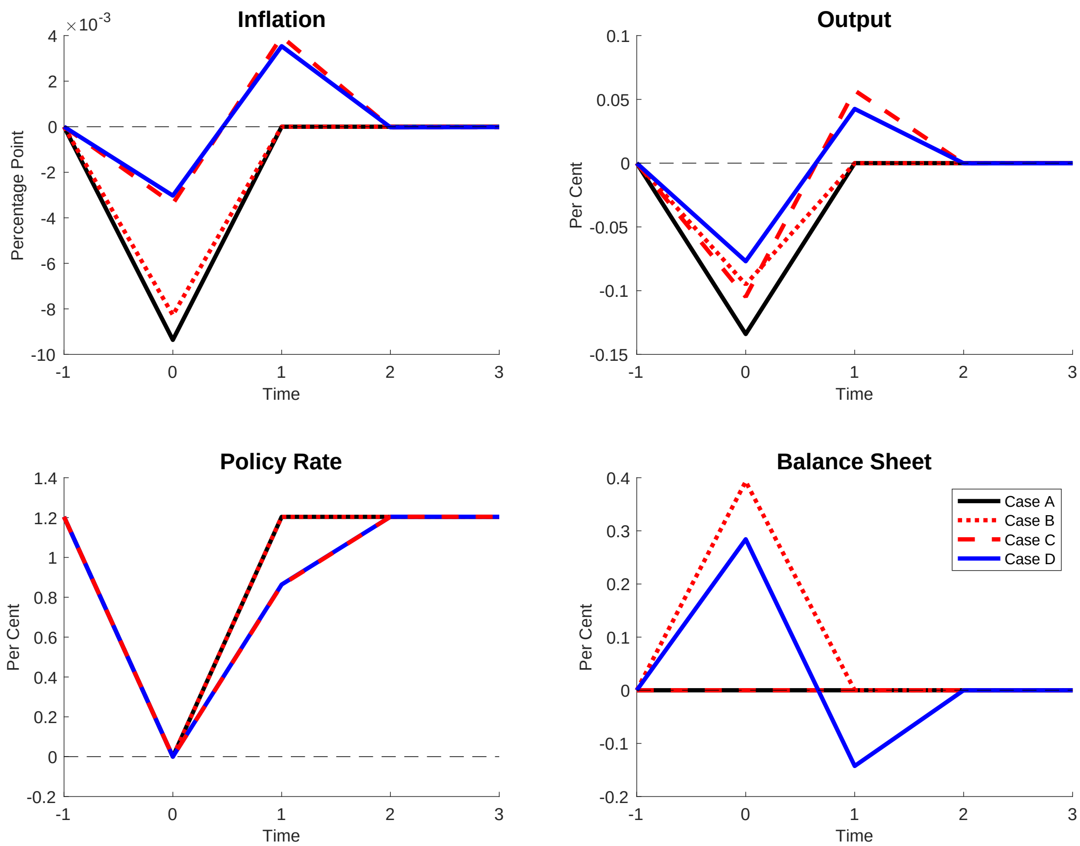


Figure E.1: Optimal Monetary Policy Mix in the Two-Period Model

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy in the two-period version of the model. Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

period 1, the policy rate is gradually adjusted (FG) and the central bank's balance sheet is actively reduced (QT, $qe_1 < 0$). As can be seen in Equations (E.19) and (E.20), the expectation about the balance-sheet contraction has a slight negative impact on π_0 and y_0. However, overall, the combination of balance sheet and interest rate policies significantly eases the initial decline in inflation and output compared to cases A and B.

E.2 Infinite-Period Model

In this section, we show that similar analytical results and intuition can be obtained in the infinite-horizon version of the model, although we cannot derive an explicit relationship between current QE and future interest rates. More specifically, consider the optimal QE responses under discretion (Case B, Equation

(E.36)) and commitment (Case D, Equation (E.37)):

$$qe_t = \frac{\chi(1-z)}{\sigma(\chi+\sigma)\bar{b}_{cb}}\xi_{1,t}, \tag{E.36}$$

and:

$$qe_t = \frac{\chi(1-z)}{\sigma(\chi+\sigma)\bar{b}_{cb}}\xi_{1,t} - \underbrace{\frac{(1-\alpha)\chi(1-z)}{\beta\sigma(\chi+\sigma)\bar{b}_{cb}}\xi_{1,t-1}}_{\text{History Dependence}}. \tag{E.37}$$

In the expressions above, $\xi_{1,t}$ is the Lagrangian multiplier associated with the IS equation. By Karush-Kuhn-Tucker conditions, $\xi_{1,t} = 0$ when the ZLB constraint is not binding and $\xi_{1,t} > 0$ when it is binding.

Under discretion, the optimal QE depends solely on the contemporaneous multiplier $\xi_{1,t}$. The positive coefficient means that the central bank expands its balance sheet when the ZLB constraint is binding. When the ZLB constraint is not binding, instead, the central bank does not use QE as a stabilisation tool and only relies on the conventional short-term rate policy.

Under commitment, the optimal QE also depends on the lagged value of $\xi_{1,t-1}$ (as well as $\xi_{1,t}$), introducing history dependence, which implies FG, as discussed in Billi and Galí (2020). To be specific, when the ZLB constraint is binding, the monetary authority cannot utilise the short-term rate, and will instead use a mix of FG and QE. The positive coefficient on the contemporaneous multiplier implies that the policymaker expands the balance-sheet above its steady-state value when the nominal policy rate hits the ZLB. However, unlike the discretionary case, the balance expansion is followed by a balance-sheet contraction (QT), as shown by the negative coefficient on the $\xi_{1,t-1}$. Hence, the presence of history dependence means that the central bank's balance sheet will react less to contemporaneous macroeconomic conditions than under discretion. This is because, under commitment, the central bank also uses FG (i.e., it keeps the rate at the ZLB for longer). Last, as shown in the two-period model, the optimal mix of FG and QE also depends on their relative strength. This can be seen by considering a rise in α, i.e., the degree of agents' myopia, which reduces the power of FG. In this case, the optimal monetary policy will include a larger balance-sheet expansion. When the ZLB constraint is not binding, $\xi_{1,t} = \xi_{1,t-1} = 0$ and the central bank can set the interest rate such that $\pi_t = y_t = 0$ and, hence, $qe_t = 0$.

F Optimised QE Taylor-Type Rule

In this section, we consider the implications of the optimal discretionary policy (OMP-D) with QE. We compare this policy to Case B (see Section 3.2), where we use an optimised Taylor-type rule for QE. The short-term rate is in both cases given by the OMP-D subject to the ZLB constraint.

It is important to notice that a simple rule, such as the one in Case B, implies a form of commitment that significantly improves welfare over the OMP-D with QE. This can be observed in Table F.1. In response to a negative demand shock, the OMP-D with QE implies larger welfare losses from inflation and output volatility and smaller losses from balance-sheet volatility. In aggregate, this policy induces larger welfare losses compared to Case B.

Table F.1: Evaluation of Optimised QE Taylor-Type Rule

Welfare	Case B	Optimised QE Rule
Aggregate	−11.15%	−9.52%
Inflation Volatility	−8.02%	−4.66%
Output Volatility	−1.12%	−0.05%
Balance-Sheet Volatility	−1.04%	−4.38%
Balance-Sheet Cyclicality	−0.96%	−0.42%

Note: We evaluate the welfare implications of the optimal monetary policy under discretion (OMP-D) and commitment (OMP-C) with (w/) and without (w/o) balance sheet policies.

Figure F.1 displays the IRFs under the two policies. Following the OMP-D with QE, the central bank expands its balance sheet significantly less than in Case B. As a result, we observe a substantially larger output contraction and a slightly larger fall in inflation. Since in both cases the policy rate is set according to the OMP-D, the short-term rate path is the same under both policies.

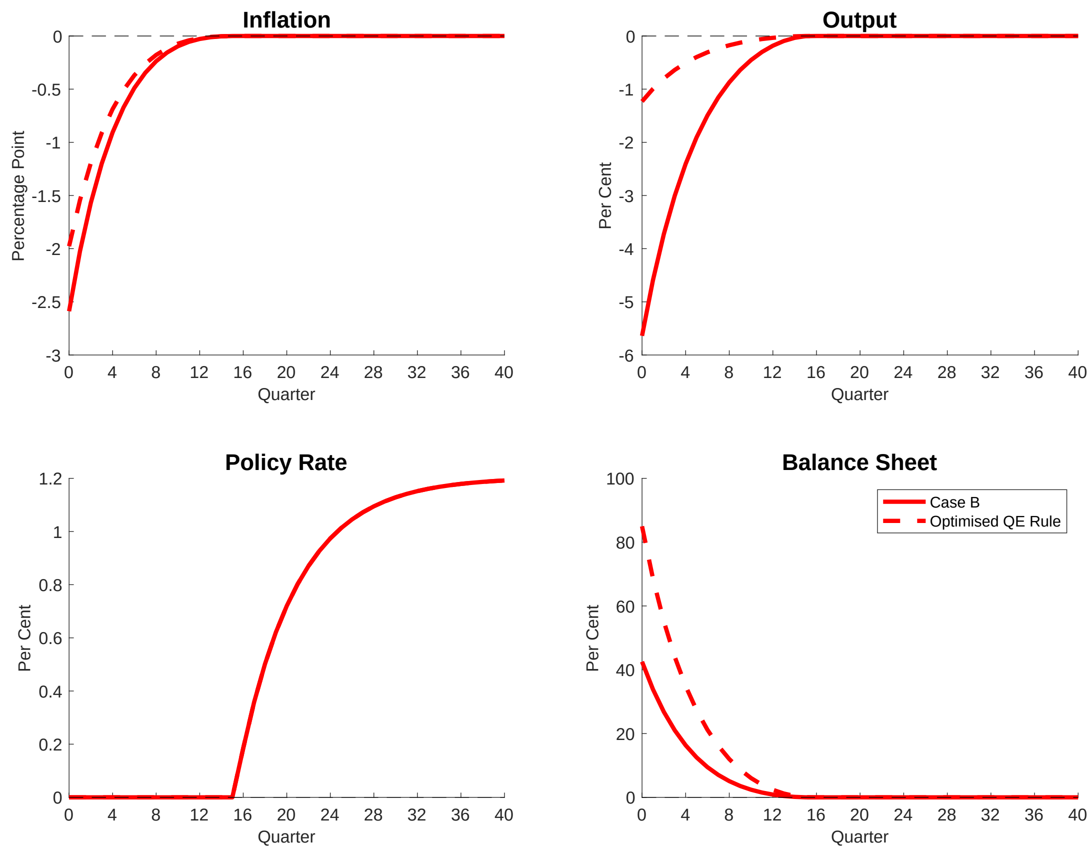


Figure F.1: Optimal Discretion vs Optimised QE Taylor-Type Rule

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under discretion (first row) and commitment (second row). Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

G Firms' Discounting

In this section, we assume that also firms are affected by the behavioural discounting, in line with Gabaix (2020).[23] In this case, the NKPC writes as:

$$\pi_t = \beta (1-\alpha) E_t \pi_{t+1} + \gamma \left(\chi + \frac{\sigma}{1-z} \right) y_t - \frac{\gamma \sigma z \bar{b}_{cb}}{1-z} q e_t. \tag{G.1}$$

As can be seen in Figure G.1, the impulse responses under the OMP-D and the OMP-C are nearly identical to the baseline case, displayed in Figure 2.

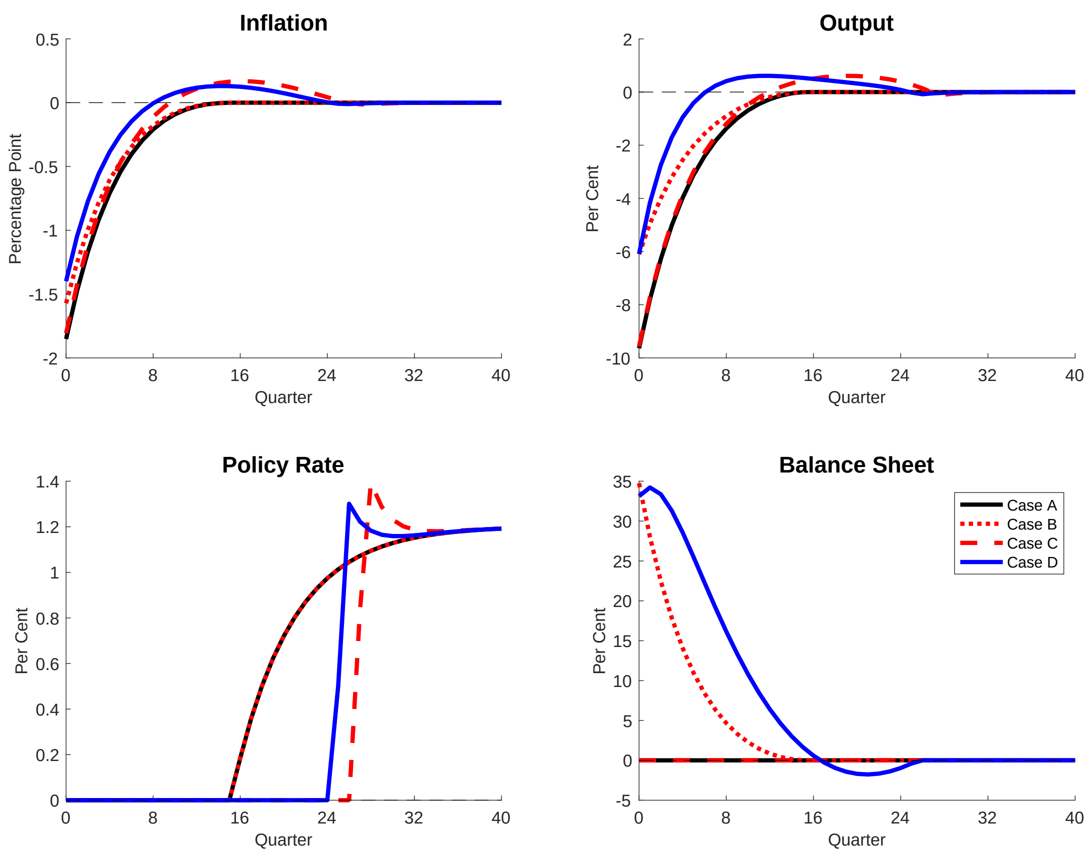


Figure G.1: Discounting in the New Keynesian Philips Curve

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policies (Cases A-D defined in Sections 3.2 and 3.3). Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

[23] In the main text, we assumed firms to be perfectly rational. That resulted in a non-discounted NKPC and a discounted IS curve as in McKay et al. (2016, 2017).

H Sensitivity Analysis

In this section, we consider the sensitivity of our baseline results with respect to certain parameters. In particular, Figures H.1-H.4 display how our results change for different values of σ (coefficient of relative risk aversion), χ (inverse labour supply elasticity), z (share of impatient households), and $\bar{b}_{cb}$ (steady-state share of central bank's longer-term bond holdings).

Reducing σ to 0.5 (see Figure H.1), i.e., increasing the elasticity of intertemporal substitution, amplifies the fall in inflation and output on impact, and leads to a stronger overshoot in subsequent periods. The balance-sheet expansion is significantly larger for $\sigma = 0.5$ than the baseline case $\sigma = 1$. The stronger monetary policy reaction, mitigates the initial drop and the overshoot of inflation and output. As a result, in the presence of QE, the responses of inflation and output are quantitatively closer to those in the baseline scenario. Conversely, increasing σ to 2 makes the economy less volatile.

Reducing χ to 0.5 (see Figure H.2) has a significant impact on inflation, which falls more on impact and overshoot more strongly in the subsequent periods. The response of output instead is not substantially impacted by the change in χ. Conversely, when $\chi = 2$, the responses in inflation and output are more muted. It bears noting that when labour supply is less elastic, the central bank find it optimal to expand its balance sheet more significantly and shortens the duration of the ZLB (i.e., carries out less FG).

Changing the share of impatient households (see Figure H.3) significantly affects the model with QE. When $z = 0.2$, the central bank expands its balance sheet more strongly which mitigates the initial fall in inflation and output.Conversely, when $z = 0.4$, the balance sheet expands less (compared to $z = 0.33$).

Changing the steady-state value of long-term bonds held by the central bank (see Figure H.4) does not affect the responses of inflation, output, and the policy rate. It only affects the response of the balance sheet (express in percentage deviation from steady state).

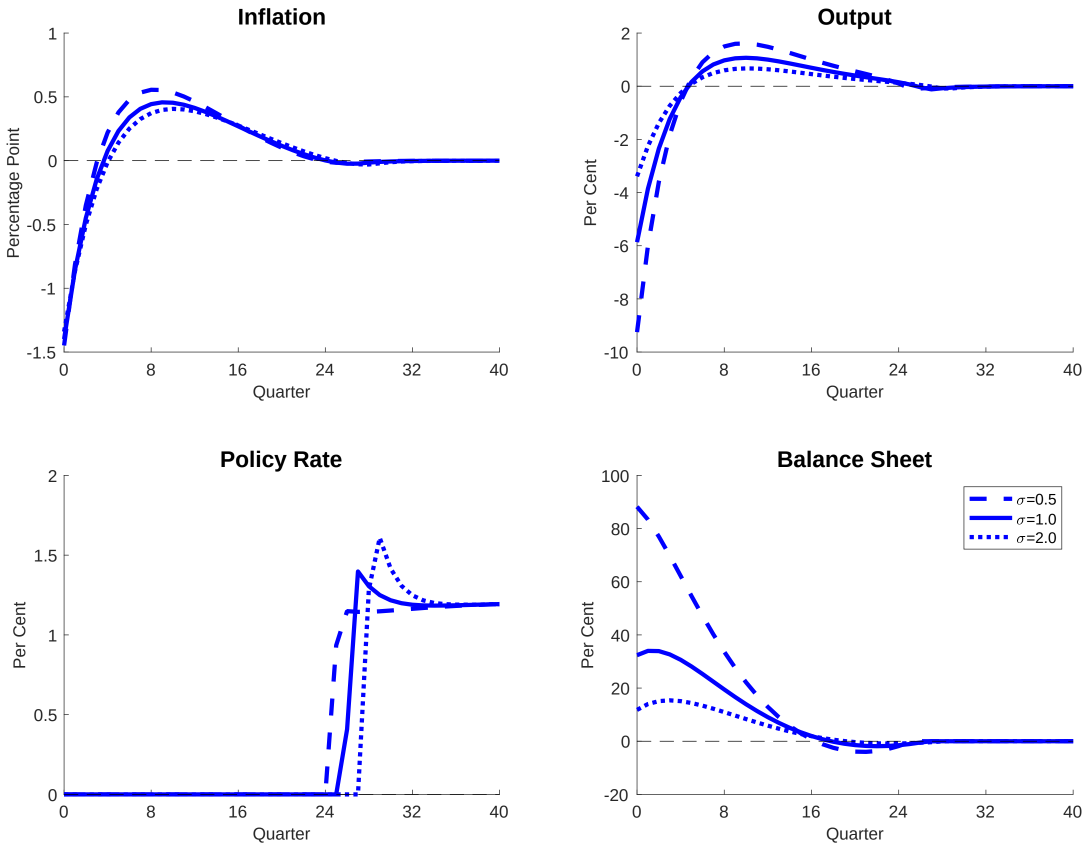


Figure H.1: Different Values of the Relative Risk Aversion Coefficient

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under commitment (Case D defined in Section 3.3). Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

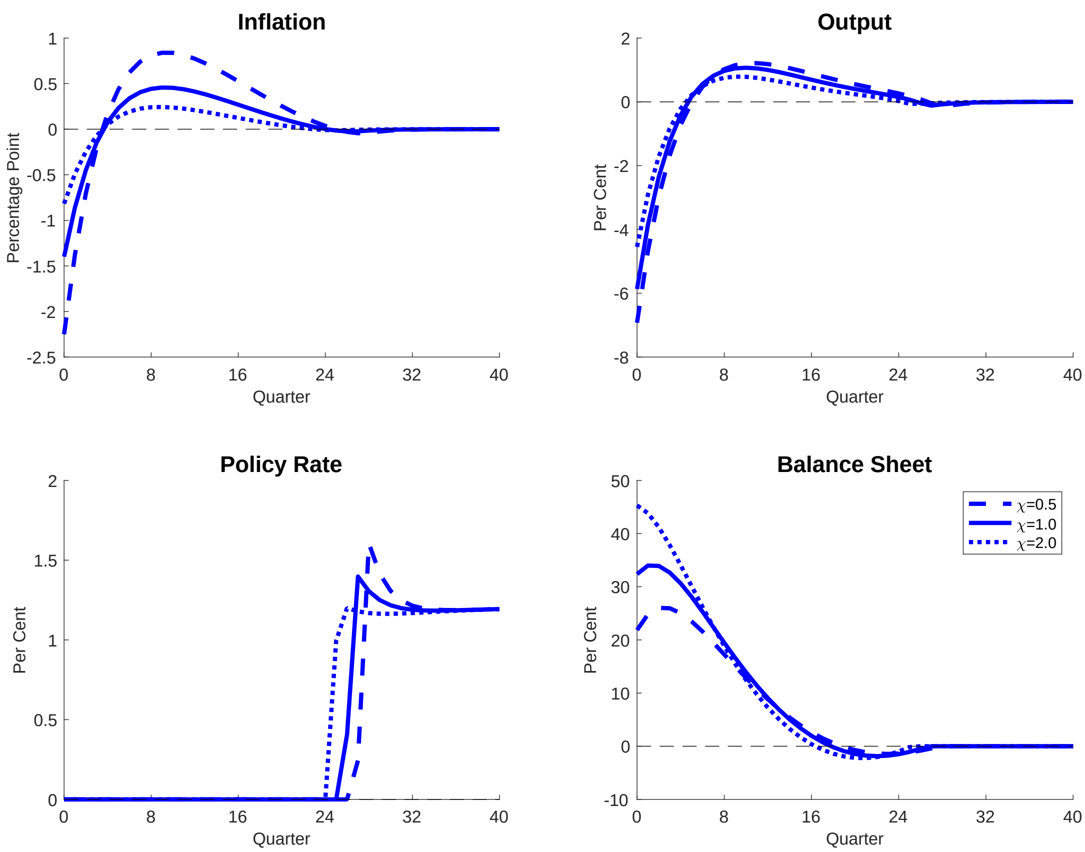


Figure H.2: Different Values of the Inverse Labour Supply Elasticity

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under commitment (Case D defined in Section 3.3). Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

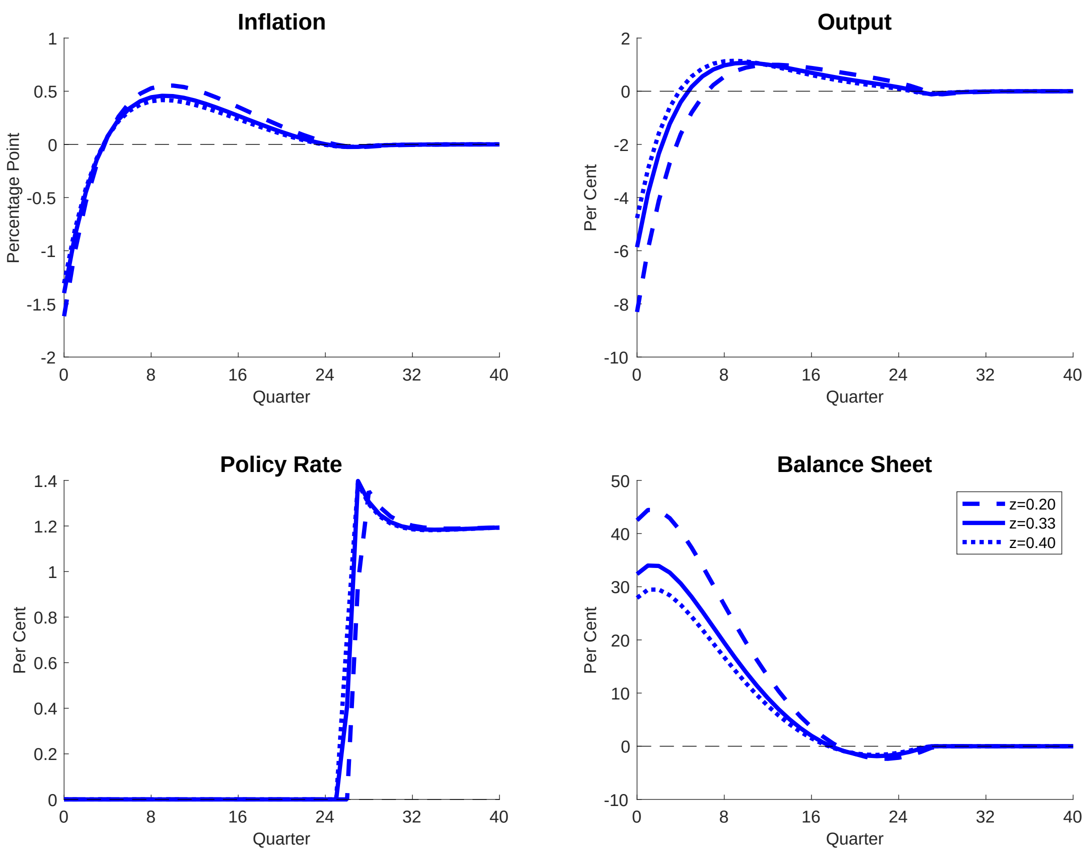


Figure H.3: Different Values of the Share of Impatient Households

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under commitment (Case D defined in Section 3.3). Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

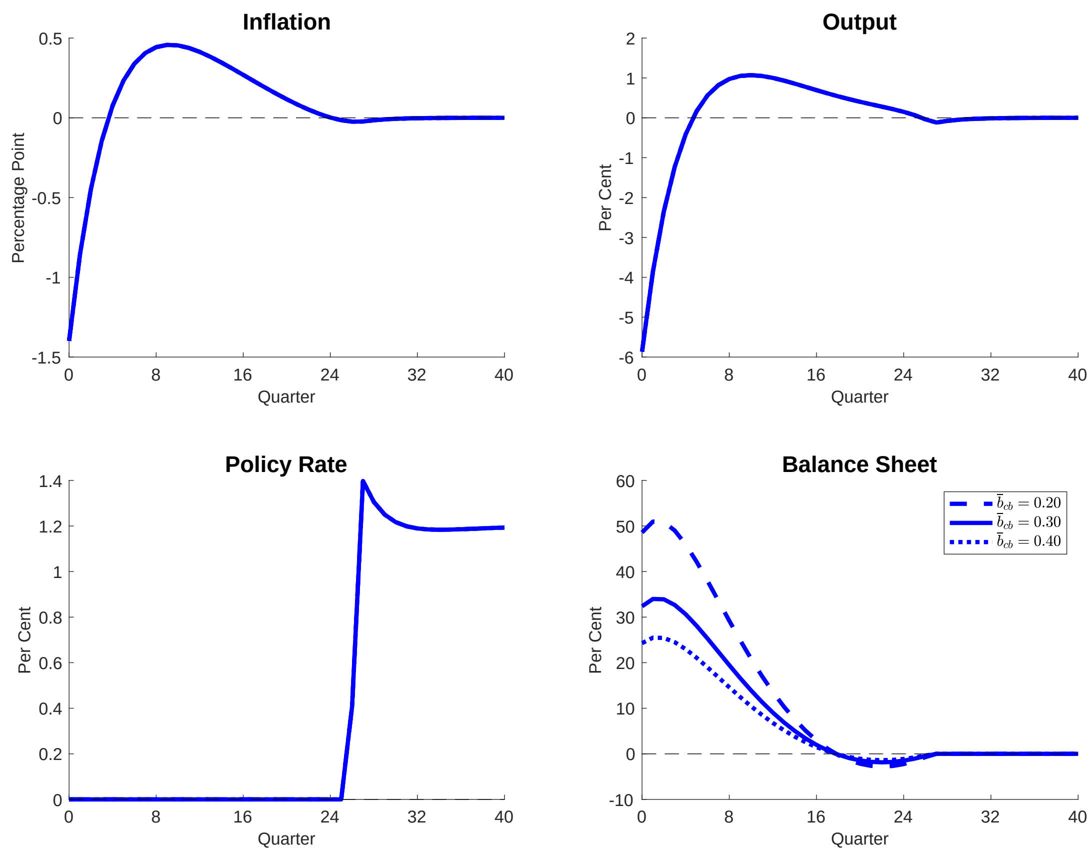


Figure H.4: Different Steady-State Values of the Central Bank's Long-Term Bond Holdings

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under commitment (Case D defined in Section 3.3). Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

I The Impact of Cognitive Discounting: Fixed Shock

In this section of the appendix, we conduct a similar exercise to that in Section 5.4. However, in this case, we do not recalibrate the shock for each value of the cognitive discounting parameter α. In other words, we keep the calibration of the shock fixed as in Table 1 and study how varying α affects the results in Cases A - D (described in the main text). Results for Cases A and B (optimal discretion without and with QE) are displayed in Figure I.1. We see that a higher α mitigates the macroeconomic effects of the negative demand shock, as households give less importance to an expected future decline in inflation and output. Figure I.2 displays the results for Cases C and D (optimal commitment without and with QE). Under commitment, increasing α has two effects. On the one hand, it dampens the effects of the demand shock. On the other, it also makes FG less effective. In Case C (absent QE), the shock causes the largest fall in macroeconomic conditions under fully rational expectations ($\alpha = 0$). However, we also see the largest overshoots in inflation, since forward guidance is much more powerful. In Case D, the central bank mitigates the falls and overshoots in output and (to a lesser extent inflation) by adjusting its balance sheet.

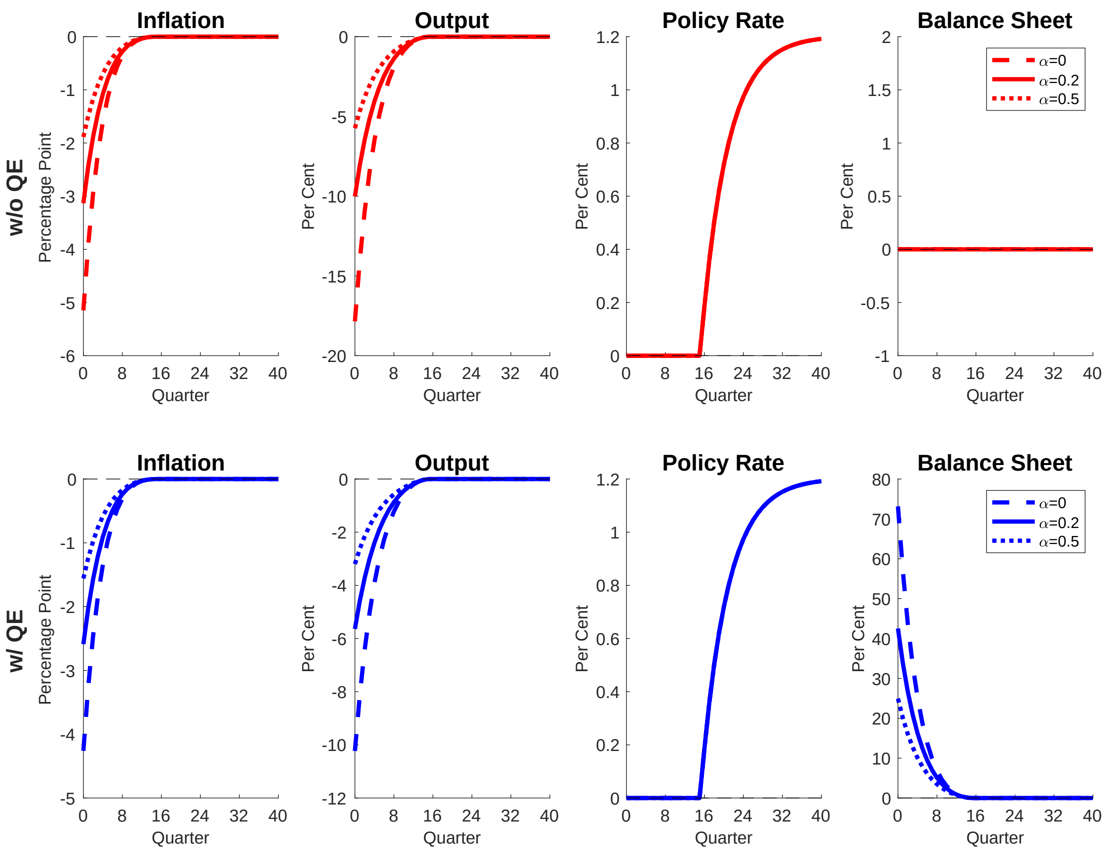


Figure I.1: Varying the Degree of Bounded Rationality: Optimal Discretion

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under discretion (Cases A and B defined in Section 3.2) for different values of cognitive discounting. The shock is calibrated to cause a 3 percentage-point decline in inflation and a 10 per cent fall in output when the central bank conducts the optimal monetary policy under discretion absent QE. Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

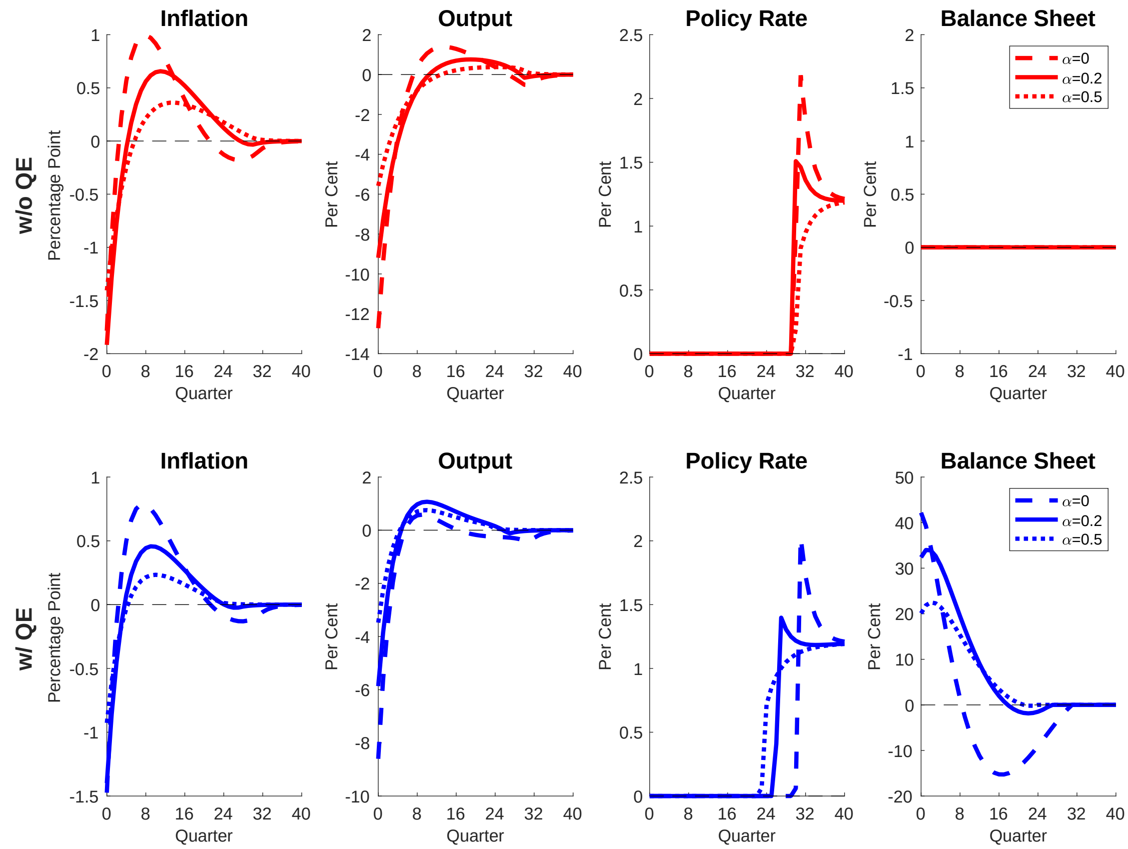


Figure I.2: Varying the Degree of Bounded Rationality: Optimal Commitment

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under commitment (Cases C and D defined in Section 3.3) for different values of cognitive discounting. The shock is calibrated to cause a 3 percentage-point decline in inflation and a 10 per cent fall in output when the central bank conducts the optimal monetary policy under discretion absent QE. Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

J Impulse Response Functions under Simple Mandates

In this section, we discuss the impulse responses generated in a model where the central bank follows a simple mandate, as defined in Section 6. In particular, we consider two alternative weights on output. First, we consider the one used in the OMP, i.e., $\vartheta = \frac{\gamma}{\varepsilon}\left(\chi + \frac{\sigma}{1-z}\right) \approx 0.007$. Second, we consider an equal weighting between annualised inflation and output, i.e., $\vartheta = 1/16 = 0.0625$.

Optimal Weighting The first row of Figure J.1 displays the dynamic responses of key model variables to a negative demand shock under SM-O. In the absence of QE (red dashed line), the responses are the same as those in Figure 2. When the monetary authority can carry out QE (pe line), additionally to FG, it aggressively expands its balance sheet. This is because the central bank puts a large weight on inflation stabilisation. This is reflected in a milder drop in inflation by about one percentage point compared to the scenario without QE. Furthermore, the strong balance-sheet expansion has the effect of fully offsetting the negative impact on output, which increases on impact.

Equal Weighting The second row of Figure J.1 displays the dynamic responses of key model variables to a negative demand shock under SM-D. In this case, the central bank puts a larger weight on output stabilisation than the SM-O. In the absence of QE (red dashed line), the policy rate is kept at zero for 23 quarters, 7 quarters longer than under the OMP-D policy without QE and 7 quarters less than under SM-O without QE. At the end of the ZLB, the central bank adjusts the policy rate gradually. The FG policy and the gradual lift-off of the policy rate boost inflation and inflation expectations. As a consequence, the real rate declines, which mitigates the drop in output. Under this policy, output declines by 8 per cent on impact, 1 percentage point less than under SM-O without QE. When we allow for QE (blue solid line), the central bank aggressively expands its balance sheet by about 90 per cent (10 percentage points less than under SM-O). This policy slightly increases output, while inflation declines by about 1.2 percentage points. It is important to notice that, under this policy, the central bank is more concerned about stabilising output than under SM-O. To this end, after the ZLB, the central bank raises the interest rate significantly more (nearly 5 percentage points) compared to the SM-O case (2.7 percentage-point increase).

Compared to the OMP-C discussed in the previous section, both SM-O and SM-D imply a significantly larger balance-sheet expansion, as the central bank does not internalise the welfare costs associated with higher QE volatility. Unlike the OMP-C case, output increases on impact, and the fall in inflation is more muted.

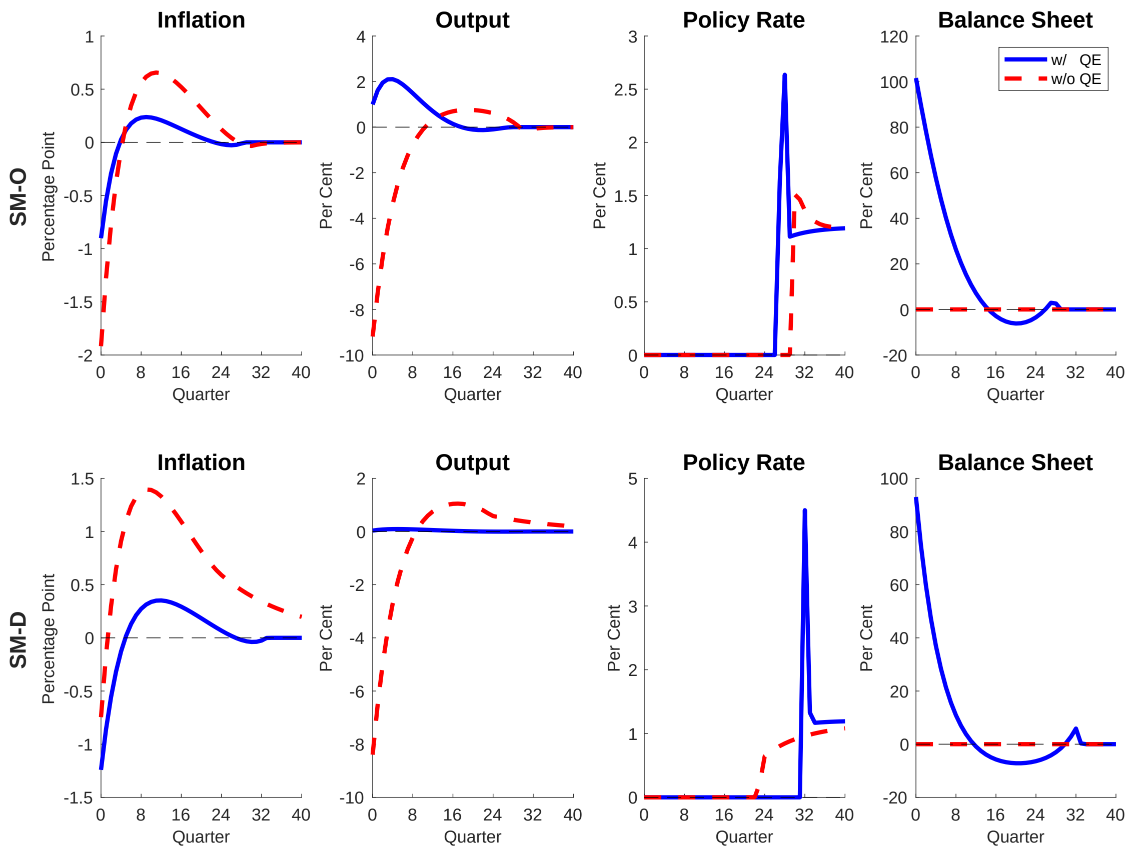


Figure J.1: Simple Mandates

Note: The figure displays the model responses to a negative natural rate shock under the simple mandates with optimal weight (first row), i.e., $\vartheta = \frac{\gamma}{\varepsilon}\left(\chi + \frac{\sigma}{1-z}\right)$, and with equal weight (second row), i.e., $\vartheta = 1/16$. Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

K Impulse Response Functions under Simple Rules

In this section, we discuss how the economy responds to a negative demand shock when the central bank follows simple implementable policy rules, as described in Section 7. In particular, we assume that the short-term rate follows either a strict inflation-targeting (SIT) rule or a strict price-level-targeting (SPLT) rule. Conditional the rule for the short-term rate, QE is set following an optimised rule, as per Table L.1.

Figure K.1 displays the results. Under SIT, the central bank lifts off the policy rate as soon as the ZLB is not binding. In the absence of QE, the decline in inflation and output is substantial. When the central bank can carry out QE, the falls in inflation and output are significantly more muted. Under SPLT, the short-term rate is kept at zero several quarters longer. This induces a rise in inflation expectations, which puts downward pressure on the real rate and helps ease the fall in demand. Also in this case, QE substantially mitigates the drop in output and, to a minor extent, that inflation. Last, we observe that, under a SPLT, the optimised balance-sheet expansion is about 10 percentage points smaller than under SIT.

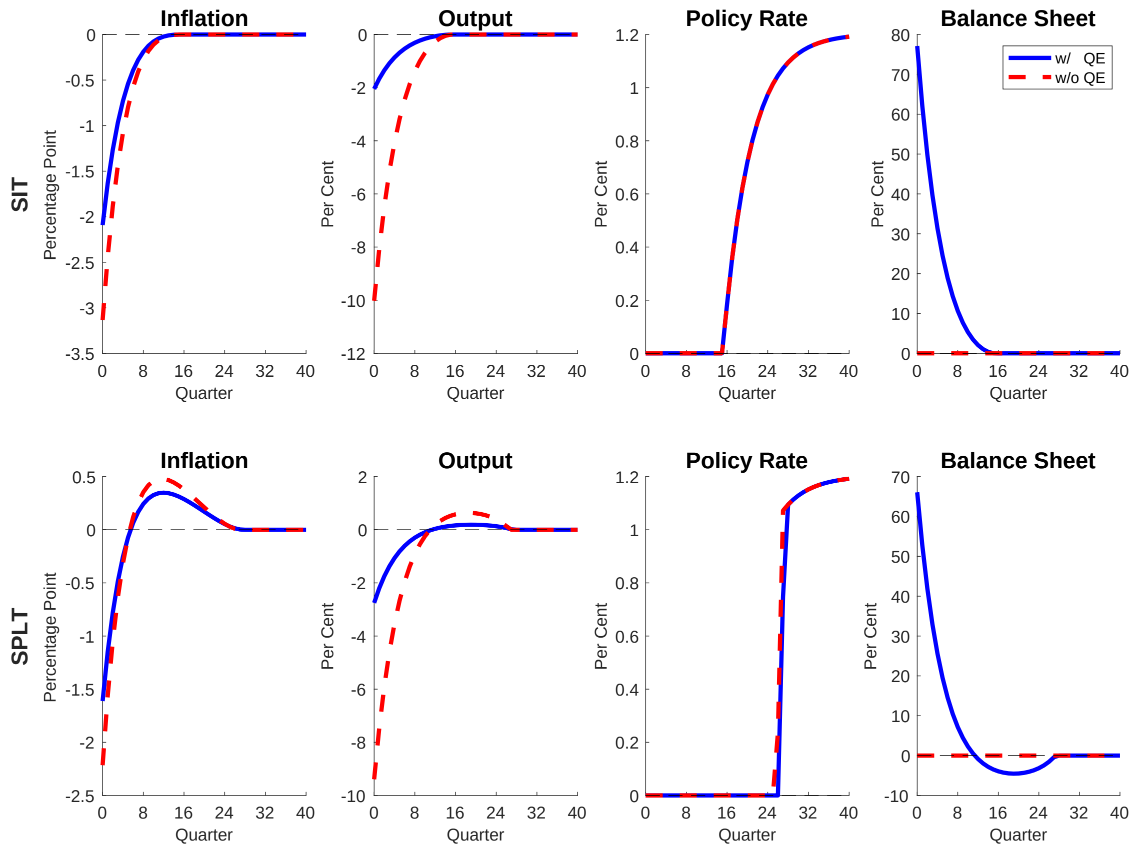


Figure K.1: Simple Rules

Note: The figure displays the model responses to a negative natural rate shock when the short-term rate is set according to either a strict inflation-targeting rule (first row) or a strict price-level-targeting rule (second row). QE is set according to an optimised rule, conditional on the rule for the short-term rate. Inflation is expressed as an annualised percentage-point deviation from the steady state. Output and the balance sheet are expressed as percentage deviations from their steady-state values. The policy rate is annualised.

L Simple Policy Rules: Strict Inflation Targeting

In this section, we consider the possibility that the central bank sets the short-term interest rate following an inflation-targeting rule:

$$r_t^\star = \eta_\pi \pi_t. \tag{L.1}$$

and QE according to Equation (29).

We label the mix of Equations (L.1) and (29) as SI-IT (Short-term Interest rate, Inflation Targeting). Under SI-IT, a higher value of η_π significantly improves welfare. When $\eta_\pi \to \infty$, the optimal QE rule should target just output. This policy combination is equivalent to the optimal policy in Case B (see Table 2). Compared to Cases C and D, SI-IT performs significantly worse. Compared specifically to Case D, the SI-IT rule leads to more balance-sheet volatility.

Table L.1: Evaluation of Simple Rules

	QE Rule				Welfare		
	ξ_π	ξ_y	Aggregate	Infl. Vol.	Outp. Vol.	BS Vol.	BS Cyc.
SI-IT Rule							
$\eta_\pi = 1.5$	100	100	−15.13%	−8.58%	−0.01%	−6.38%	−0.16%
$\eta_\pi = 5.0$	100	100	−12.30%	−6.42%	−0.01%	−5.71%	−0.17%
$\eta_\pi = +\infty$	0	69	−9.52%	−4.66%	−0.05%	−4.38%	−0.42%

Note: Conditional on the parameter η_π, we select the values of ξ_π and ξ_y that maximise aggregate welfare. We consider natural values of ξ_π and ξ_y on the interval $[0, 100]$. BS Vol. and BS Cyc. stand for balance-sheet volatility and cyclicality.

Figure L.2 displays how aggregate welfare and its subcomponents change, as we vary the parameters in the QE rule ξ_π and ξ_y. In particular, the figure considers the case of an SI-IT rule with $\eta_\pi \to \infty$. In Figure L.2, we see that increasing ξ_π and ξ_y can significantly improve inflation and output stabilisation and substantially mitigate welfare losses due to a negative demand shock. However, for large values of these coefficients, the welfare losses due to higher balance-sheet volatility become significant and substantially offset the welfare gains from inflation and output stabilisation. The welfare contribution of balance-sheet cyclicality is relatively minor compared to the other three components. The relationship between the balance-sheet cyclicality component of welfare and ξ_y is not monotonic. For values of $\xi_y > 10$, the welfare losses due to balance-sheet cyclicality decrease with ξ_y, as output and the central bank's balance sheet become less negatively correlated.

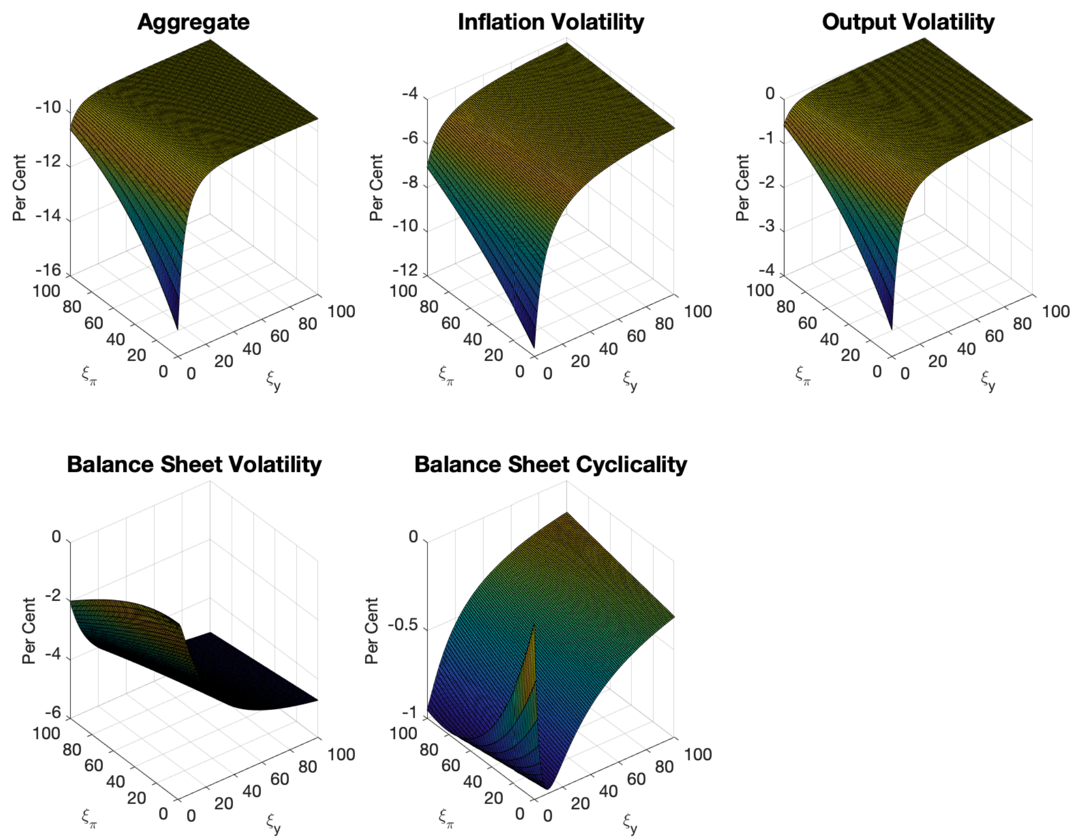


Figure L.2: Strict Inflation-Targeting Rule and Welfare

Note: The figure displays the aggregate welfare and its subcomponents when the central bank sets the short-term interest rate following a strict inflation-targeting rule and adjusts its balance sheet according to a policy rule with parameters ξ_π and ξ_y.