Similarity based nonlinear settlement predictions of circular surface footings on clay

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ABSTRACT

In 1951, Skempton introduced the concept of similarity to obtain predictions of non-linear settlement of rigid circular footings on deep clay deposits under undrained conditions. This approach is based on the premise that the pressure-settlement curve of the footing and a stress-strain curve from a characteristic point in the soil can be linearly scaled to collapse into a single “master” curve. The method has been extended to predict deflections of axially and laterally loaded piles and is widely used in the offshore industry. Despite the theoretical and practical appeal of the method as well as its wide application in a range of geotechnical problems, limited investigation and validation exists in the literature. In this work, (1) existing “classical” similarity methods are reviewed, including a Boussinesq solution for elastic soil and the Mobilisable Strength Design (MSD) method by Bolton and co-workers. (2) The similarity factors derived from these methods are compared with those obtained from a novel non-linear cone model solution. (3) The resulting expressions are evaluated against rigorous numerical analyses undertaken by the authors in FLAC. These are based on two different non-linear constitutive models calibrated against triaxial tests from three clay deposits. Two alternative families of similarity methods are also compared with classical similarity. (4) Firstly, a “two-part” similarity technique (based on separate scaling factors for elastic and plastic strains) and (5) secondly, a “stiffness” similarity approach introduced by Atkinson (based on secant stiffness degradation). Finally, (6) three field test results are evaluated as case studies to demonstrate the applicability of the method in real-life problems. It is concluded that similarity approaches offer a rational yet approximate tool for non-linear settlement analysis of footings.

Keywords: surface footings, settlement, non-linear analysis, soil/structure interaction, similarity

1. INTRODUCTION

Improved understanding of the non-linear pressure-settlement response of surface footings on clay would enable more efficient design to prevent excessive settlements. Simple analytical solutions are available to determine both the "fully-elastic" initial slope of the pressure-settlement curve as well as the "perfectly-plastic" failure load (e.g., Skempton 1951, Brinch Hansen 1970 – see recent summary by Salgado 2022). Of particular interest is the elastic solution for the stiffness of a rigid footing on the surface of an elastic half-space established by Boussinesq (Poulos and Davis 1974, Davis and Selvadurai 1996). Some empirical solutions for the pressure-settlement response of surface footings (e.g., Jardine *et al.* 1995, Lehane 2003, Agaiby and Ahmed 2022) are available in the literature, as well as some numerical solutions (e.g., Osman and Bolton 2005, Ghosh Dastider *et al.* 2021). However, these solutions are limited to specific soil-footing configurations and may require site-specific studies that are costly and time consuming to undertake. Alternatively, non-linear pressure-settlement curves can be determined using theoretical models such as the cavity expansion theory introduced by Bishop *et al.* (1945) for metals and later extended by Gibson (1950) to clay soils (also employed for penetration resistances in sand, e.g., Salgado *et al.* 1997 and Salgado and Prezzi 2007). This method has been employed by McMahon *et al.* (2013) using an energy approach to estimate a nonlinear pressure-settlement curve for a surface footing on an elastic-perfectly plastic half space and has been further extended by McMahon *et al.* (2014) to incorporate the non-linear soil constitutive model. Alternatively, Klar and Osman (2008) developed a non-linear pressure-settlement curve by combining an elastic and an elastoplastic mobilisable strength design (MSD) solution using an energy method to weight the contributions of the two mechanisms. However, despite the frequency this problem is encountered in routine engineering practice and its importance in settlement estimation, limited analytical solutions are available to determine the full non-linear pressure-settlement curve.

A simple approach to obtain a non-linear pressure-settlement curve for footings was introduced by Skempton (1951), who suggested that a pair of linear scaling factors for stresses and strains can be used to transform a stress-strain curve directly into a pressure-settlement curve and vice-versa. This similarity approach (which is referred to in the ensuing as "classical similarity") is based on the premise that there is similarity in shape between a stress-strain curve from a laboratory test and the foundation pressure-settlement curve (Figure 1). In the realm of this approach, the non-linear pressure-settlement curve of a vertically loaded footing can be obtained directly from a routine laboratory test using two linear transformation factors, one scaling each axis.

Classical similarity has been employed for surface footings by Elhakim (2005), Osman *et al.* (2007) and Agaiby and Ahmed (2022). An analogous similarity has been utilised to obtain "*t-z*" curves for axially-loaded piles (e.g., Seed and Reese 1957, Fu *et al.* 2020, Bateman *et al.* 2022a), as well as "*p-y*" curves for laterally-loaded piles (e.g., McClelland and Focht 1956, Matlock 1970, Kagawa and Kraft 1981, Bransby 1999, Reese and Van Impe 2011) and associated "*m-θ*" curves (e.g., Fu *et al.* 2020, Bateman *et al.* 2023). While the cost and time benefits from this approach can hardly be overstated, there is no guarantee that such similarity exists for each case considered and the resulting predictions should be considered as approximate.

For the classical similarity method (as originally suggested by Skempton 1951) to be usable in routine design of vertically loaded circular footings, suitable values of the scaling factors must be determined. Furthermore, the accuracy and limits of the similarity approach should be established. This could be done through either numerical modelling (e.g., finite element analysis), or field and laboratory testing where both stress-strain and pressure-settlement curves are obtained.

1.1 Alternative Similarity Approaches

The classical similarity approach has also been extended using a "two-part" similarity method that consists of individual scaling factors applied individually on the elastic and plastic portions of the curve. Previously, this approach has been employed for "*t-z*" curves for axially loaded piles by Fu et al. (2020); "*p-y*" curves by Jeanjean et al. (2017), Zhang and Anderson (2017; 2019) and Fu et al. (2020); and base curves for laterally loaded piles by Fu et al. (2020) and Lai et al. (2020). This approach has also been used implicitly by Jakub (1977) who assumed that a secant stiffness-stress curve can be given in the same form as a secant stiffness-load curve for a strip footing under dynamic horizontal and moment loading.

Additionally, Atkinson (2000) suggested a "stiffness" similarity approach based on the shapes of the secant stiffness-strain ($G - \gamma$) curve from a triaxial soil test and a secant stiffness-settlement ($K - w_b$) curve of a footing. Employing similar arguments to those of Skempton, Atkinson (2000) proposed a linear transformation factor to relate between these two curves.

1.2 This Paper

Despite the theoretical importance and practical appeal of these simplified methods, their existence for a long period of time and their applicability in a wide range of geotechnical problems, limited validation has been carried out, and some authors have even questioned some of the fundamental assumptions (Burland *et al.* 1966, Randolph and Wroth 1978). More importantly, there is currently limited understanding of the underlying principles and the way these methods relate to and differ from one another.

Motivated by this gap in knowledge, this paper investigates the similarity proposal and its variants as applied to obtain a pressure-settlement curve of a vertically loaded (circular) surface footing on clay from a corresponding stress-strain curve of a soil element test. This involves (1) a review of existing methods related to the similarity approach, (2) reformulating these solutions into a consistent framework, and (3) developing and validating the novel expressions for the required transformation factors using both analytical and numerical methods. Specifically,

- the classical similarity proposal by Skempton (1951) to directly relate stress-strain and pressure-settlement curves is first reviewed. To this end, two related methods, an elastic stiffness approach based on the Boussinesq solution and the MSD method, are reformulated in a consistent framework to derive linear-transformation factors.
- a novel non-linear solution using a cone model for pressure-settlement curves is derived, inspired by related elastic solutions to dynamic footing problems. This is used to derive linear-transformation factors for specific non-linear soil constitutive models.
- the above methods are compared and validated by means of rigorous numerical solutions in the finite difference software FLAC 2D. Two different non-linear soil constitutive models are used, calibrated against three different types of clay.
- the alternative two-part similarity approach is applied to the vertically loaded foundation problem for the first time. An analytical solution, in conjunction with further numerical results, is employed to derive novel linear-transformation factors for this method.
- the stiffness similarity approach proposed by Atkinson (2000) to directly relate secant stiffness-strain with secant stiffness-settlement curves is reviewed. A novel, closed form expression for the similarity factor for an elastic-perfectly plastic material is derived and compared with the original values from Atkinson (2000) and those obtained from the FLAC results.
- the three similarity methods are compared and the appropriate choice of linear transformation factors is discussed for different loading ranges. These factors are applied to predict the pressure-settlement curve for three case study examples and demonstrate the use and limitations of these approaches.

2. CLASSICAL SIMILARITY

The classical similarity approach is demonstrated in Figure 1. Employing this method requires the selection of two linear transformation factors, one for each axis. Given the two curves are similar in shape, the linear transformation factor of the y-axis can be obtained by comparing the ultimate capacity of each curve, which naturally bounds both curves between 0 and 1. Specifically, the pressure-settlement curve approaches the ultimate capacity of the footing, q_u and the stress-strain curve approaches the undrained soil shear strength, s_u. It is well known that the ultimate capacity of a footing in clay can be given by a dimensionless bearing capacity factor, N_c, multiplied by s_u. Therefore, the scaling factor on the y-axis is simply N_c (values for which are discussed later). Secondly, the x-axis of the pressure-settlement curve should be normalised by a characteristic dimension, with the aim of collapsing the two curves into a single “master” curve. This characteristic dimension is selected here to be proportional to the footing diameter, D, with a dimensionless proportionality constant, defined here as a linear transformation factor, c_q. Therefore, the linear transformation of the x-axis can be expressed by:

$$\gamma_{rep} = \frac{w_b}{c_q D} \tag{1a}$$

where γ_{rep} is a representative “average” shear strain of the soil under the footing.

Inverting this equation gives the footing settlement, w_b, obtained by scaling the representative strain by the characteristic dimension $c_q D$ as follows:

$$w_b = \gamma_{rep}\, c_q\, D \tag{1b}$$

The key idea behind this approach is that γ_{rep} can be established from a pertinent soil element test under the same level of normalised stress (i.e., $\tau_{rep} = q/N_c$). Therefore, after appropriate N_c and c_q values have been selected, the following simple steps should be followed to employ this approach in design:

1. Divide q, the pressure applied to the foundation, by N_c to get the corresponding τ_{rep}, the shear stress on the representative soil sample.
2. Use a representative soil element test (or an assumed constitutive model) to obtain γ_{rep}, the strain in the representative soil sample at τ_{rep}.
3. Use Equation 1b to obtain w_b, the foundation displacement, under the applied pressure.

It should be noted that the selection of the footing diameter to normalise settlement is an arbitrary decision and alternative selections (e.g., the footing radius) can be equally valid, and merely scale the transformation factor c_q. Furthermore, Equation 1 is defined with a “representative shear strain” that is obtained from a soil element test undertaken on a representative soil sample. To employ the similarity approach, the location of a representative soil sample under the footing must be identified and a suitable soil element test (e.g., triaxial compression) selected. Using Finite Element Analysis (FEA), Osman and Bolton (2005) suggest that this representative sample should be taken from a depth of $0.3D$ beneath the centre of the footing. However, a greater understanding of the relevance of the location of the representative soil sample is required before this approach can be adopted in design. Additionally, the stress-strain curve of the representative soil sample may depend on which type and shear mode of element test is chosen. Within the original similarity proposal, Skempton (1951) suggested an undrained compression (triaxial) test would be suitable. Since the choice of sample location and test type are outside the scope of this work, an idealised isotropic homogeneous clay is considered. This means that any element test will produce identical results for a test in any location.

The value of N_c at the surface has been considered by many authors and is dependent on the foundation shape and roughness. Shield (1955) and Eason and Shield (1960) calculated N_c for a circular rigid footing to be 5.69 and 6.05 for a perfectly smooth and rough footing conditions, respectively. Alternative N_c values for footings are available (e.g., Ishlinky 1944, Skempton 1951, Meyerhof 1951, Cox *et al.* 1961, Brinch Hansen 1970, Tani and Craig 1995, Salgado et al. 2004, Gourvenec *et al.* 2006). These solutions vary between $5.58 < N_c < 6.23$. However, the solutions by Shield (1955) and Eason and Sheild (1960) are both lower and upper bounds and have subsequently been verified by Houlsby and Wroth (1983) as essentially exact values (Martin and Randolph 2001).

Similarly, some solutions for c_q in various forms can be found in the literature. Notably, the mobilisable strength design (MSD) method used by Osman and Bolton (2005) is a form of classical similarity. These authors derive a coefficient M_c (the reciprocal of the linear transformation factor) as the average shear strain within an assumed displacement mechanism. Any M_c value can be converted to a c_q value that follows the definition used in Equation 1 (discussed below). In fact, any method that obtains a pressure-settlement curve from a soil stress-strain curve, including numerical analysis and experimental data, can be reformulated as a c_q value. Therefore, the methods to obtain c_q can be broadly split into two main categories: firstly, those which obtain c_q directly, without employing a pressure-settlement curve; and secondly, those which derive c_q by comparing a pressure-settlement curve with the respective stress-strain curve. While a single c_q value would suggest perfect similarity exists, for most cases, c_q will vary with applied load as well as soil properties.

2.1 Elastic Stiffness Approach

Skempton (1951) suggested a method to analytically obtain c_q for a circular surface footing by assuming an elastic half-space and matching the stiffnesses of the two curves. To this end, the linear-elastic soil constitutive model can be expressed in normalised form as:

$$\frac{\tau}{s_u} = \left(\frac{G}{s_u}\right) \gamma \tag{2}$$

where τ and γ are the shear stress and strain, respectively, G is the soil shear modulus and s_u is the soil undrained shear strength.

The elastic settlement of a rigid circular footing can be established using the Boussinesq solution (Poulos and Davis 1974, Davis and Selvadurai 1996). The resulting pressure-displacement relationship can be normalised by the ultimate bearing pressure $q_u (= N_c s_u$ for undrained conditions), to yield the dimensionless equation:

$$\frac{q}{q_u} = \frac{K_i w_b}{q_u} = \frac{8}{\pi(1-\nu_s)} \left(\frac{G}{N_c\, s_u}\right) \frac{w_b}{D} \tag{3}$$

where q is the mean pressure acting on the soil-footing interface, q_u is the corresponding ultimate bearing pressure, K_i is the elastic stiffness of the footing, ν_s is the Poisson's ratio of the soil, and N_c is the bearing capacity factor (values for which are discussed above).

The above solution was developed assuming a smooth footing-soil interface. An alternative solution is available for a rough footing-soil interface (Spence 1968); however, for undrained conditions, this is equivalent to Equation 3, subject to the selection of appropriate N_c values.

For soft soil, the left-hand sides of Equations 2 and 3 are naturally bounded between 0 and 1. Therefore, equating the right-hand side of these equations and introducing c_q in the form of Equation 1 yields the linear-transformation factor:

$$c_q = \frac{\pi}{8}(1-\nu_s)\, N_c \approx 1.1 - 1.2 \tag{4}$$

which, remarkably, is independent of G and s_u.

The essentially exact N_c values for smooth and rough circular footings produced by Shield (1955) and Eason and Shield (1960) - 5.69 and 6.05 - and considering $\nu_s = 0.5$, result in a c_q of 1.12 and 1.19, respectively. This value is roughly equivalent to the factor of 2 (applied to normal strain instead of shear strain) obtained by Skempton (1951), dependent on the selected N_c. The full range of available N_c mentioned in this paper result in c_q values of $1.10 < c_q < 1.22$.

2.2 Mobilisable Strength Design (MSD) Method (Osman and Bolton 2005)

The mobilisable strength design (MSD) method was introduced by Bolton and Powrie (1988) for earth pressures and has primarily been used in the design of deep excavations (e.g., Osman and Bolton 2004). The method has been extended by Osman and Bolton (2005) to obtain c_q values for vertically loaded circular footings. The resulting values have been compared against numerical and field data (Osman *et al.* 2007).

Osman and Bolton (2005) employed a displacement field where the outer boundaries are defined using a Prandtl-like failure mechanism modified for axisymmetric loading. Within the boundaries, three regions are defined: the active, fan and passive zones, in which the displacement field was chosen such that shear strains and displacements remain compatible (Figure 2). Either side of the fan zone (boundaries OF and OG) vertical and radial displacements are equal in magnitude and direction. Beyond the mechanism boundaries (boundary FGP) the soil is assumed perfectly rigid. Finally, as the loading conditions are undrained, no volume change is assumed. The soil strains are therefore constrained by the following equation:

$$\varepsilon_r + \varepsilon_\theta + \varepsilon_z = -\frac{\partial u}{\partial r} - \frac{u}{r} - \frac{\partial v}{\partial z} = 0 \tag{5}$$

where u, v are the radial and vertical displacements, respectively. $\varepsilon_r = -\partial u/\partial r$, $\varepsilon_\theta = -u/r$, $\varepsilon_z = -\partial v/\partial z$ are the normal strains in the cylindrical coordinate system defined by r, θ, z as illustrated in Figure 2, respectively. Additionally, shear strains in axisymmetric conditions are $\gamma_{r\theta} = 0$, $\gamma_{\theta z} = 0$ and $\gamma_{zr} = -(\partial v/\partial r + \partial u/\partial z)$.

Regarding the selection of the displacement mechanism, Osman and Bolton (2005) assume the variation of vertical displacements along the centre line (CF) can be given by a quadratic polynomial. They also assume that within the active zone, the vertical displacements are independent of radial distance. Thus, by considering Equation 5 and applying boundary conditions ($u = 0$ at $r = 0$; $v = \delta$ at $r = 0$ and $z = 0$; $u = v$ along boundary OF) u and v can be derived as shown in Table 1. It should be noted that these assumptions correspond to a smooth footing (i.e., there are non-zero radial soil displacements at the footing-soil interface). Also note that, contrary to Prandtl's mechanism, soil is not at a state of failure so the displacement field is continuous and displacements are zero along the outer boundary PGF.

Following the assumption that the radial and vertical displacements either side of each zone boundary are equal, the u and v in the fan and passive zones can be calculated, also given in Table 1. Note that to ensure zero volume change, the total displacement in the fan and passive zones ($\sqrt{u^2 + v^2}$) must decay proportional to $1/r$ (Osman and Bolton 2005).

The radial and vertical displacements in Table 1 can be converted into normal and shear strains which are employed to calculate the principal strains, ε_1, ε_2, ε_3. The resulting mobilised engineering shear strain ($\varepsilon_1 - \varepsilon_3$) can then be averaged over the mechanism and set equal to the representative shear strain γ_{rep} in Equation 1, as follows (Osman and Bolton 2005):

$$\gamma_{rep} = \frac{\int_{vol} |\varepsilon_1 - \varepsilon_3| dvol}{\int_{vol} dvol} = M_c \frac{w_b}{D} = \frac{w_b}{c_q D} \tag{6}$$

This approach yields a single value of $c_q = 0.74$ (equivalent to $M_c = 1/c_q = 1.35$ in the notation of Osman and Bolton 2005) that is independent of the footing dimension and developing settlement. This value implies a smaller characteristic dimension (lower c_q) than the elasticity solution of Equation 4, which is associated with the confined area of plastic strain concentration, compared to strain distribution across a wider area in the elastic half-space solution. Thus, it is thought that this value may pertain better to situations of higher load levels, where significant plastic deformation has taken place.

The MSD method has the advantage that it calculates c_q directly and does not depend on the selection of a soil constitutive model and the level of applied load. However, it is dependent on the geometry (size) of the chosen deformation mechanism, which, indirectly relates to the level of concentration of high strains near the footing.

3. CONE MODEL

Cone models have been widely used in footing displacement calculations. Considering a surface footing, of diameter, D, and area, A_b, loaded by a vertical traction, q, vertical stress attenuates with depth based on a selected cone opening line $f(z)$, assumed here to be linearly varying with gradient $1/m_{cone}$, as shown in Figure 3. Original applications often refer to this approach as the "2:1 method" and set m_{cone} between 1 and 2 (Bowles 1997). Wolf and Deeks (2004) also provide static solutions using the cone model for lateral and rocking modes. This paper applies a cone model to determine novel solutions for the non-linear vertical pressure-settlement curves of footings from which c_q values are derived.

Following the cone model logic, it is assumed that the vertical strain, ε, can be integrated over the depth, z, to furnish the settlement of the footing, w_b:

$$w_b = \int_0^\infty \varepsilon(z)\, dz \tag{7}$$

The vertical strain, ε, can be written as a function of the normalised deviatoric stress, $g\left(\sigma_q/2s_u\right)$, by introducing a pertinent soil constitutive model in flexibility form. The normalised deviatoric stress within the soil, $\sigma_q(z)/2s_u$, is taken as equal to the normalised vertical stress at depth $q_z(z)/q_u$ due to the footing load. This is arguably a similarity assumption itself. Additionally, vertical equilibrium is assumed between horizontal layers of the cone itself and the stress at depth, z, which is considered to be uniform over the area $A_z(z)$. This can be written as:

$$q_z(z) A_z(z) = q A_b \tag{8}$$

where $A_z(z)$ depends on the chosen m_{cone}. This key assumption implies that tractions developing along the cone boundary are horizontal. It also means m_{cone} must have a dependency on the Poisson's ratio of the soil in order to match the overall elastic stiffness of the foundation. Therefore, by using a constant m_{cone} value over the full range of stresses, it is effectively assumed that the Poisson's ratio of the soil remains constant.

Based on these assumptions, the vertical strain is given by:

$$\varepsilon = g\left(\frac{\sigma_q}{2s_u}\right) = g\left(\frac{q_z(z)}{q_u}\right) = g\left(\frac{q}{q_u}\left(\frac{A_b}{A_z(z)}\right)\right) \tag{9}$$

Firstly, assuming the soil is described using a linear-elastic soil constitutive model (Equation 2) [$\varepsilon = \gamma/(1+\nu_s)$, $\sigma_q = 2\tau(z)$]:

$$\varepsilon = \frac{\sigma_q}{2G(1+\nu_s)} = \frac{s_u}{G(1+\nu_s)}\left(\frac{q_z(z)}{q_u}\right) \tag{10}$$

Substituting this function into Equation 7 and evaluating the integral using the depth-varying area $A_z(z)$ shown in Figure 3, yields the elastic settlement of the footing as a function of the applied stress:

$$\frac{w_b}{D} = \frac{m_{cone}}{2(1+\nu_s)}\left(\frac{s_u}{G}\right)\left(\frac{q}{q_u}\right) \tag{11}$$

By employing the concept of similarity and comparing this equation with the normalised shear stress-strain curve (Equation 2), c_q simplifies to:

$$c_q = \frac{m_{cone}}{2(1+\nu_s)} \tag{12}$$

which is again independent of the footing dimension and the soil stiffness and strength. In addition, the proportionality with m_{cone} indicates that when the cone is assumed to be narrower and strains are distributed over a larger depth, the characteristic length $c_q D$ increases. The unknown gradient coefficient m_{cone} can be calculated to ensure compliance with other similarity models. For instance, in the case of a linear-elastic model where c_q is known (Equation 4), m_{cone} can be calculated by equating Equation 4 with Equation 12, for a smooth ($N_c = 5.69$) and a rough ($N_c = 6.05$) footing, respectively:

$$m_{cone} = \frac{\pi}{4}(1-\nu_s^2)N_c \approx 3.4 - 3.6 \tag{13}$$

This value of m_{cone} is used in the numerical applications below. Note that this calibration of m_{cone} is higher than that given by Wolf and Deeks (2004), who derive a value of $m_{cone} = \pi(1-\nu_s) \approx 1.6$ for the vertical mode in incompressible soil. However, this value was calibrated for elastic settlement prediction and not in a similarity context.

Equation 9 also enables nonlinear stress-strain functions to be employed to obtain analytical non-linear pressure-settlement curves. For example, it can be assumed that the soil can be modelled using a hyperbolic soil constitutive model in the form:

$$\frac{G_s}{G_i} = \left[1 + \frac{\gamma G_i}{s_u}\right]^{-1} \tag{14}$$

where $G_s (= \tau/\gamma)$ is the secant shear modulus and G_i is the initial (low-strain) shear modulus. Substituting this function, rearranged into vertical strains, into Equation 7 and evaluating the integral, yields the non-linear pressure-settlement curve in flexibility form:

$$\frac{w_b}{D} = \frac{s_u m_{cone}}{2\, G_i\, (1+\nu_s)} \sqrt{\frac{q}{q_u}} \left[\operatorname{arctanh}\left(\sqrt{\frac{q}{q_u}}\right)\right] \tag{15}$$

Comparison of the constitutive model (Equation 14) and the pressure-settlement curve in Equation 15 enables c_q to be obtained:

$$c_q = \frac{m_{cone}\left(\frac{q_u}{q}-1\right)}{2\,(1+\nu_s)} \sqrt{\frac{q}{q_u}} \left[\operatorname{arctanh}\left(\sqrt{\frac{q}{q_u}}\right)\right] \tag{16}$$

Remarkably, once again, c_q is independent of soil parameters G_i and s_u, but it depends on the geometry of the cone, the soil's Poisson's ratio and, most significantly, on the intensity of loading, q/q_u. This is plotted in Figure 4 for three example soils, assuming m_{cone} from Equation 13, as estimated above. Example parameters for the hyperbolic soil model have been determined by Bateman et al. (2022b), by fitting this model to two triaxial tests from Soga (1994) in (1) Pisa clay, and (2) kaolinite. A third example, London clay, has been fitted here using the same approach as that employed by Bateman et al. (2022b) using a triaxial test from Gasparre (2005). The parameters for the three examples are shown in Figure 4b.

Additionally, a hyperbolic tangent (tanh) soil constitutive model is considered in the form:

$$\gamma = \frac{\tau}{G_i} + \gamma_r \operatorname{arctanh}^2\left(\frac{\tau}{s_u}\right) \tag{17}$$

Substituting this equation into Equation 7 (rearranged into vertical strains) yields an integral whose solution could not be established in closed form. A numerical solution is presented in Figure 5 for the same three example soils considered for the hyperbolic soil constitutive model. This results in the more complex c_q plot shown in Figure 5b, that, in addition to the aforementioned parameters, q/q_u, ν_s and m_{cone}, the c_q value for this constitutive model also depends on G_i/s_u and γ_r. This result is unsurprising due to the addition of a parameter in the model.

The first point to observe is that for both the hyperbolic and tanh models, c_q varies with the applied load. At nearly zero load, both models start at a $c_q = 1.12$, as per the elastic solution that m_{cone} was calibrated to, followed by a decrease of c_q with increased loading. This result aligns with the idea that a higher applied footing load results in increased strength mobilisation and strain concentration in the area close to the footing, thus decreasing the characteristic dimension ($c_q D$). The dependence of c_q on load intensity is a calibration parameter of the model and implies that perfect similarity does not exist, and c_q is dependent on load intensity. However, an appropriate c_q may still be determined for a given range of q/q_u. The second observation involves the dependence of the transformation factor on the adopted soil model. In the case of a hyperbolic stress-strain relationship, there is no additional effect of soil parameters. However, in the case of the tanh model, a further variation of c_q is demonstrated for the different types of clay examined.

4. DISCUSSION

A summary of the derived c_q values is shown in Figure 6a, for the classical similarity methods examined so far. Evidently, as shown by results from the novel cone model, c_q is dependent on the load intensity, the constitutive model and parameters, and the footing roughness.

Firstly, for approaching zero applied load, the elastic stiffness approach (Section 2.1) gives a single value of c_q - Equation 4 ($1.12 - 1.19$ for smooth and rough footings, respectively). In this work, these values are derived as closed-form expressions based on the original assumptions made by Skempton (1951). For higher stress regions, selection of an appropriate c_q value is more uncertain. Since geotechnical design practice usually involves safety factors equivalent to around 2 to 3, the main stress region of interest is $q/q_u < 0.5$. While significant variation of c_q values can be seen in this stress region on Figure 6a, the curves start from the aforementioned elastic value and decrease with increasing q/q_u values to an approximate range of $0.5 < c_q < 0.8$. For a higher stress range ($q/q_u > 0.5$), c_q can vary significantly and appears to approach zero. This implies that the normalised stress-strain curve for a given soil specimen asymptotes faster than the pressure-settlement curve that incorporates the response of soil over a wider area underneath the footing. At this stress range similarity is unlikely to be applicable and more complex analysis considering plasticity and failure should be sought.

5. NUMERICAL ANALYSIS

To explore the values of the linear transformation factor, c_q, in a more rigorous manner, idealised element tests can be compared to the pressure-settlement curves obtained from non-linear numerical analysis. Osman *et al.* (2007) considered vertical, horizontal and moment loading on a pad foundation using this approach and updated the $M_c = 1.35$ value obtained in Osman and Bolton (2005) to $M_c = 1.25$, corresponding to $c_q = 0.8$.

In this work, non-linear numerical analysis was carried out in FLAC 2D (Itasca Consulting Group Inc, 2011) using (1) a hyperbolic soil constitutive model (Equation 14) and (2) a hyperbolic tangent (tanh) soil constitutive model (Equation 17). These models were implemented in FLAC using the CPPUDM (user-defined) option using isotropic shear hardening. To this end, a Tresca yield surface was defined according to the mobilised soil shear strength. The evolution of the yield surface is controlled by a hyperbolic or tanh relationship, expressed in terms of plastic shear strain. This approach is undertaken for the three example soils discussed in Section 3. Numerical element-level undrained direct simple shear tests were initially conducted for the different soil types examined (model parameters and results shown in black in Figure 7a and Figure 8a). These tests were undertaken to validate the accuracy of the model implementation at element level. Further validation of the boundary value problem examined herein was obtained by comparing the initial stiffness and ultimate values against available analytical solutions.

The footing pressure-settlement curves were obtained by applying a constant settlement rate to a rigid footing in large-scale axisymmetric-mode analyses (results shown in grey in Figure 7a and Figure 8a). The model is set up with ~ 850 rectangular zones, with 10 grid points along the footing radius and the boundaries sufficiently extended to have negligible effect on the results. As expected, the pressure-settlement curve asymptotes towards an ultimate bearing capacity, q_u, which can be used to obtain the bearing capacity factor, N_c ($q_u = N_c s_u$ for undrained conditions). N_c of $5.58 - 5.61$ (smooth) and $6.03 - 6.08$ (rough) were obtained for both the hyperbolic and tanh models, all within 2% of the exact theoretical values from Shield (1955) and Eason and Shield (1960).

To obtain c_q, the y-axis of the stress-strain and the pressure-settlement curve are normalised by their ultimate capacities, taken from the numerical results (s_u and q_u, respectively). Comparing the two normalised curves enables a c_q to be obtained as a function of load intensity. Figure 7 and Figure 8 show the numerical results for the hyperbolic and tanh models, respectively. Comparison with the corresponding results from the elastic and cone models (Figure 4) indicate a very good match between the initial values of c_q and its variation with load intensity. The general trend of decreasing c_q with load intensity indicates that the size of the mechanism is decreasing which aligns with the lower c_q value obtained from Osman and Bolton (2005) based on a smaller plastic displacement mechanism. Notably, the hyperbolic c_q results are essentially independent of soil properties.

Evidently, c_q is dependent on the roughness of the footing-soil interface. From the elasticity approach the c_q at low q/q_u values for a perfectly rough footing are approximately 5% larger than that of a smooth footing. The numerical results indicate that the c_q for a rough footing decreases slower with load intensity than that for a smooth footing.

5.1 The elastic-perfectly-plastic model "paradox"

Both soil constitutive models employed in the FLAC analyses above asymptote towards an undrained shear strength s_u. However, numerical models with an elastic-perfectly-plastic response are often used in geotechnical practice (e.g., the Mohr-Coulomb soil model). If such a model is selected, the c_q factor at zero loading starts from an elastic value that remains consistent with the results discussed in the sections above. This is shown in Figure 9 which summarises the results of FLAC analyses with the Mohr-Coulomb model. The elastic c_q value is maintained until a loading intensity of approximately $0.3\,q/q_u$ at which point yielding of soil elements under the footing starts occurring. From this point onwards, the pressure-settlement curve asymptotes towards the ultimate bearing capacity of the footing, but the stress-strain curve remains linear-elastic until the undrained shear strength is reached. The transformation factor c_q increases towards a maximum value of:

$$c_{q,u} = \frac{w_{b,u}}{\gamma_u D} \tag{18}$$

where $w_{b,u}$ is the settlement at failure of the footing, γ_u is the failure strain of the soil and $c_{q,u}$ is the c_q value at $q/q_u = 1$. As a result, when γ_u is finite (as per the Mohr-Coulomb model) and the pressure-settlement curve asymptotes towards infinite settlement, c_q approaches infinity at large applied loads.

Evidently, any elastic-perfectly-plastic model would significantly underpredict the failure strain, or, alternatively overpredict the initial stiffness G_i. Therefore, this increase of the transformation factor c_q is unrealistic and such models should be avoided in the context of similarity.

5.2 Sensitivity to N_c

To employ the similarity approach, the applied pressure is factored by N_c to get the shear stress to input into the representative soil sample. While the N_c values provided by Shield (1955) are exact, the solutions for non-circular or slightly embedded foundations are not. This, in addition to soil heterogeneity and the non-linearity of the foundation response means these "exact" values may not match field test results. Therefore, the effect of selecting an inaccurate N_c has been investigated with an example analysis in FLAC, using both a hyperbolic and a tanh soil constitutive model. As shown in Figure 10a, the same pressure-settlement curve from the surface footing in FLAC is normalised against N_c values that have been under-predicted or over-predicted by 10%. Figure 10b demonstrates that the error in the c_q propagates to the prediction of the elastic, low-load value of the transformation factor, with the underestimation of N_c resulting in underpredictions of c_q (and consequently foundation settlement) by an equal percentage, and vice versa.

6. "TWO-PART" SIMILARITY

As discussed in the above sections, it is evident from the results that perfect similarity across the full loading range is unlikely, and instead, c_q appears to decrease with increased load intensity. To tackle this issue, one possible solution is a "two-part" similarity procedure that employs separate scaling factors for the elastic and plastic components of strain in the normalised stress-strain curve, to produce the corresponding elastic and plastic components of displacement in the normalised pressure-settlement curve. Comparable to Equation 1, this can be written as:

$$\frac{w_b}{D} = \frac{w_{b,e}}{D} + \frac{w_{b,p}}{D} = c_{q,e}\,\gamma_e + c_{q,p}\,\gamma_p \tag{19}$$

where γ_e and γ_p are the elastic and plastic components of the soil shear strains, $w_{b,e}$ and $w_{b,p}$ are the elastic and plastic components of settlement and $c_{q,e}$ and $c_{q,p}$ denote the corresponding elastic and plastic linear transformation factors, respectively. This approach is shown in Figure 11.

In a similar way to the "classical" similarity approach, to employ this method in design (after appropriate $c_{q,e}$ and $c_{q,p}$ values are selected), the following simple steps should be followed:

1. Divide q, the pressure applied to the foundation, by N_c to get τ_{ref}, the equivalent shear stress on the representative soil sample.
2. Split the representative soil element test into the elastic and plastic components using G_i.
3. Use this soil element test to obtain the elastic strain, γ_e, and the plastic strain, γ_p in the representative soil sample at τ_{ref}, the equivalent shear stress.
4. Use Equation 19 to obtain the foundation displacement, w_b, under the applied pressure, q.

Note that step 2 requires the value of G_i to be known, a soil parameter often hard to determine in the laboratory without special equipment (e.g., a resonant column or bender element tests). However, this is typically easier to obtain with in-situ methods, such as correlating with CPT results or through geophysical testing (such as SASW, MASW) – see Foti et al. (2015) for more details.

Equivalent values for $c_{q,e}$ and $c_{q,p}$ have previously been derived for curves relating to axially- and laterally- loaded piles (e.g., Fu et al. 2020; Jeanjean et al. 2017). This approach has also been used implicitly by Jakub (1977) who assumed that a secant stiffness-stress curve can be given in the same form as a secant stiffness-load curve for a strip footing under dynamic horizontal/moment loading. Since the two-part similarity approach has not been explicitly applied to a vertically loaded footing in axisymmetric mode this paper will go on to extend the method employed by Jakub (1977) to obtain novel $c_{q,e}$ and $c_{q,p}$ values for the particular case.

6.1 Jakub-Roesset Method

Working with Roesset, Jakub (1977) suggested that lateral load-displacement curves and moment-rotation curves for strip footings can be given in the same functional form as a stress-strain curve. Jakub (1977) employed a Ramberg-Osgood soil constitutive model, given by:

$$\frac{G_s}{G_i} = \frac{\tau}{\gamma G_i} = \frac{1}{1+a\left(\frac{\tau}{s_u}\right)^{b-1}} \tag{20a}$$

$$\gamma = \gamma_e + \gamma_p = \frac{\tau}{s_u}\frac{s_u}{G_i} + a\frac{s_u}{G_i}\left(\frac{\tau}{s_u}\right)^b \tag{20b}$$

where G_s and G_i are the secant and initial (or low-strain) shear modulus, respectively, $a = \gamma_{pf} G_i / s_u$ is a fitted model parameter corresponding to the plastic shear strain at failure, γ_{pf}, and b is a model exponent. Note that this model does not asymptote to an ultimate value but requires a cap at s_u (see Section 5.1).

Following the assumption of Jakub (1977), the corresponding pressure-settlement curve is given by:

$$\frac{K_s}{K_i} = \frac{q}{w_b K_i} = \frac{1}{1+\chi a\left(\frac{q}{q_u}\right)^{b-1}} \tag{21a}$$

$$w_b = \frac{q}{q_u}\frac{N_c s_u}{K_i} + \chi a \frac{N_c s_u}{K_i}\left(\frac{q}{q_u}\right)^b \tag{21b}$$

where K_s and K_i are the secant and initial stiffness of the pressure-settlement curve, respectively ($K_s = q/w_b$) and χ is a fitting parameter discussed below.

Evidently, both Equations 20b and 21b are naturally split into the elastic and a plastic portion of the curves. Furthermore, the assumption that Equations 20a and 21a can be given in the same form is equivalent to assuming a “two-part” similarity. Therefore, $c_{q,e}$ and $c_{q,p}$ can be calculated directly (substituting in $K_i = 8G_i/(\pi(1-\nu_s)D)$ from Equation 3):

$$c_{q,e} = \frac{w_{b,e}}{\gamma_e D} = \frac{\pi}{8}(1-\nu_s)N_c \tag{22a}$$

$$c_{q,p} = \frac{w_{b,p}}{\gamma_p D} = \chi\left[\frac{\pi}{8}(1-\nu_s)N_c\right] = \chi c_{q,e} \tag{22b}$$

As expected, $c_{q,e}$ (Equation 22a) is identical to the elasticity solution for c_q in Equation 4.

Jakub (1977) originally suggested determining χ by fitting Equation 21 to numerical pressure-settlement curves, obtained using a Ramberg-Osgood model simplified by setting $b = 2$. This is undertaken here using FLAC 2D following the same method as discussed in Section 5. Following the assumption that the pressure-settlement curve can be given in the form of Equation 21, plotting $K_i w_b/q$ against aq/q_u would be expected to result in a straight line with a gradient χ and an intercept at $q = 0$ defined by $K_i = K_s$ (shown in Figure 12a). Evidently, this assumption is not perfect, but a simple linear regression can be applied to obtain χ. The results are plotted in Figure 12a for different a values, with interpreted trend lines shown. These give $\chi = 0.45$ and $\chi = 0.43$ for rough and smooth footings, respectively which correspond to $c_{q,p} = 0.53$ and $c_{q,p} = 0.48$ (see Equation 22b). For preliminary analysis, Jakub (1977) suggested that these values can be also used in cases with alternative b values or even for different constitutive models.

Given $c_{q,e}$ and $c_{q,p}$ in the form of Equations 22a and 22b, an equivalent value of the "classical" c_q can be obtained:

$$c_q = \frac{w_b}{\gamma D} = \frac{c_{q,e}\,\gamma_e + c_{q,p}\,\gamma_p}{\gamma_e + \gamma_p} \tag{23}$$

Equation 22 suggests that $c_{q,e}$ and $c_{q,p}$ are constant with applied load. The low stress region is governed by $c_{q,e}$ since $\gamma_p = 0$ as the applied load approaches zero. However, the variation of c_q with increased applied loads is governed by the value of $c_{q,p}$. Evidently, if $c_{q,p} < 1$ (suggested by the fit in Figure 12a), this would suggest that c_q decreases with increased applied load. Remarkably, this agrees with the results presented in Sections 3, 4 and 5 and has the additional benefit of being controlled by a constant $c_{q,p}$.

Applying Equation 23 to the hyperbolic and tanh models (given by Equation 14 and 17, respectively) results in c_q values that can be compared with those obtained previously. Assuming that χ can be given by those obtained in Figure 12a, the resulting values are plotted in Figure 6b.

6.2 Representative soil sample

Jakub (1977) also proposed rewriting the footing secant stiffness (Equation 21a) in an alternate form:

$$\frac{K_S}{K_i} = \frac{q}{w_b K_i} = \frac{1}{1 + a\left(\frac{\tau_{ref}}{s_u}\right)^{b-1}} \tag{24}$$

where τ_{ref} is the shear stress at a reference location at a certain depth below the edge of the footing. This is illustrated in Figure 12b and allows converting χ into a reference location (for $b = 2$), resulting in:

$$\chi = N_c \frac{\tau_{ref}}{q} = N_c \psi\left(\frac{z}{D}\right) \tag{25}$$

where $\psi(z/D) = \tau_{ref}/q$ describes the dimensionless attenuation of shear stress with depth.

For lateral loading in plane strain conditions, Jakub (1977) proposed that the representative soil element is located at $z = 0.25D$ under the edge of the footing. This is notably similar to a depth of $z = 0.3D$ for the vertical mode suggested independently by Osman and Bolton (2005) in the context of the MSD method. Assuming a depth of $z = 0.3D$ in the problem examined here, the corresponding dimensionless attenuation can be obtained from Poulos and Davis (1974) as $\psi(z/D) = 0.23$, leading to $c_{q,p} = 1.6$. This is substantially higher than the values of 0.48 to 0.53 presented above. On the other hand, the values of χ obtained in Figure 12a correspond to attenuation coefficients approximately $\psi = 0.08$, which would apply to locations of the representative soil sample between $z = 0.9D$ and $1D$ below the edge of the footing. This is much deeper than the representative soil element location suggested by Osman and Bolton (2005), from the MSD approach.

6.3 Numerical analysis

As a comparison to the $c_{q,p}$ values determined using the Jakub-Rosett method above, $c_{q,p}$ can also be interpreted directly from the numerical results obtained in Section 5. As expected, the elastic component $c_{q,e}$ is consistent with the value of c_q at zero loading. To obtain $c_{q,p}$, the y-axis of the stress-strain and the pressure-settlement curve are normalised by their ultimate capacities, taken from the numerical results (s_u and q_u, respectively), similar to what was done in Section 5. However, the predicted elastic component of the corresponding strain/displacement is also subtracted from the original x-axis value for the hyperbolic and tanh models calibrated to the three example soils, respectively. Comparing the two normalised curves (with elastic portions removed), enables $c_{q,p}$ to be obtained as a function of load intensity. This is done for rough and smooth footings, shown in Figure 13.

The numerical results shown in Figure 13 are compared to the values obtained using the Jakub-Roesset method. The numerical results in Figure 13 show less variation of $c_{q,p}$ with load intensity than observed for c_q in the "classical" similarity method (see Figure 7b and Figure 8b). This good agreement applies over a wider range of load intensity when compared to the classical similarity solutions, possibly as high as $q/q_u = 0.8$.

7. "STIFFNESS" SIMILARITY

An alternative similarity method has been proposed by Atkinson (2000), who suggested that similarity in shape exists between (1) the secant shear modulus degradation of a soil element with increasing strain ($G_s - \gamma$) and (2) the secant stiffness decay of a surface foundation with increasing normalised settlement ($K_s - w_b/D$). This will be denoted herein as "stiffness similarity" and is employed in a similar manner to the classical similarity method suggested by Skempton (1951).

Firstly, the two curves are normalised by their ultimate values, naturally bounding the curves between 0 and 1 on the y-axis. These are the initial (low-strain) shear modulus, G_i, and the initial stiffness of the pressure-settlement curve, K_i, respectively. This is given by the Boussinesq solution in Equation 3 when $G = G_i$. Therefore, the linear transformation factor of the y-axis is given by:

$$\frac{K_i}{G_i} = \frac{8}{\pi(1-\nu_s)} \tag{26}$$

Secondly, the abscissa (x-axis) of the $G_s - \gamma$ curve is factored (stretched or compressed) by a characteristic dimension, typically selected to be proportional to the footing diameter, D. This method is illustrated in Figure 14. The linear transformation of the x-axis can be expressed by:

$$\gamma_{rep} = \frac{w_b}{c_{q,s}\, D} \tag{27}$$

This linear transformation factor, $c_{q,s}$, appears to be in the same form as Equation 1, namely defining a characteristic dimension, $c_{q,s}\,D$, normalising the footing settlement, w_b. However, the derived transformation factors using the two similarity approaches (c_q from Section 2 and $c_{q,s}$ here) cannot be directly compared since the form of the soil element test that is scaled is not the same.

Unlike the previous similarity approaches discussed, the "stiffness" similarity approach does not allow an engineer to start with an applied foundation pressure and estimate the settlement. Instead, after an appropriate $c_{q,s}$, is determined, the following simple steps should be employed (see Atkinson 2000 for more details):

1. Choose an allowable settlement, w_b/D (normalised by the footing diameter).
2. Divide the normalised settlement by $c_{q,s}$ to calculate the representative shear strain within the soil (Equation 27).
3. Use a representative soil element test (or assumed constitutive model) to obtain the secant shear modulus in the representative soil sample, G_s, at this representative shear strain.
4. Use the Boussinesq equation (Equation 3) to calculate the allowable pressure that can be applied to the foundation.

If the settlement at a known applied pressure is desired instead, these steps can be applied iteratively. Note that Step 4 is equivalent to applying the scaling factor in Equation 26 to the secant shear modulus, G_s, to calculate the secant stiffness of the footing, K_s, and then multiplying by the normalised footing settlement, w_b/D.

Atkinson (2000) compared empirical settlement values for surface (and piled raft) footings on London clay with triaxial tests undertaken in the same material (for $0.05 < K_s/K_i < 0.25$). Atkinson (2000) did this by calculating equivalent undrained secant Young's modulus values for the footing using the bearing pressure and observed settlement in Equation 3. This is equivalent to the method discussed above. From this comparison, Atkinson (2000) established that the normalised foundation settlement was three times larger than the corresponding axial strains from the triaxial test. This is equivalent to $c_{q,s} \approx 2$ for an undrained material as the linear transformation factor in this work is applied to shear strain rather than axial strain. This value was then verified by Atkinson (2000) using centrifuge modelling on kaolin clay (for $0.05 < K_s/K_i < 0.6$) and model plate load tests in sand (for $0.05 < K_s/K_i < 0.75$). Further validation was subsequently provided by Osman et al. (2007), using nonlinear FEA analysis.

In the classical similarity approach the y-axis is normalised by the capacity, which means an elasticity solution is used to derive a c_q value. However, in the stiffness similarity approach the y-axis is normalised by an elasticity solution, which means it cannot be employed to analytically derive $c_{q,s}$ and the predicted capacity can be used instead. In both the classical and stiffness similarity solutions, these analytical methods are both equivalent to matching the intersection between the elasticity solution and the capacity, i.e. the yield point in an elastic-perfectly plastic model. In the stiffness similarity case, the normalised stiffnesses for the soil element and the footing after yield are:

$$\frac{G_s}{G_i} = \frac{s_u}{G_i}\left(\frac{1}{\gamma}\right) \tag{28a}$$

$$\frac{K_s}{K_i} = \frac{q_u}{K_i}\left(\frac{1}{w_b}\right) \tag{28b}$$

where K_i can be found using the Boussinesq solution (Equation 3). Therefore:

$$c_{q,s} = \frac{w_b}{\gamma D} = \frac{\pi}{8}(1-\nu_s)N_c \approx 1.1 - 1.2 \tag{29}$$

More specifically, this would yield a $c_{q,s}$ of 1.12 and 1.19 for smooth and rough footings, respectively. Although this value is identical to the c_q calculated in Equation 4, there cannot be a direct comparison between these cases. Firstly, this is because classical similarity is performed on the basis of stress-strain and pressure-settlement curves, while stiffness similarity is applied on secant shear modulus and foundation stiffness degradation. Secondly, the transformation factor in Equation 4 refers to small load intensities $q/q_u \to 0$ while Equation 29 corresponds to loading close to failure $q/q_u \to 1$.

In addition to the analytical validation presented above, this paper proceeds to evaluate the applicability of the stiffness similarity method by calculating $c_{q,s}$ values from the FLAC analysis conducted (both for Hyperbolic and tanh model) in Section 5. The results are shown in Figure 15. As it can be observed, the $c_{q,s}$ value rapidly approaches infinity at low strain ranges (where G_s is still close to G_i, i.e. $G_s/G_i > 0.9$), where the classical similarity approach may be more applicable. For $0.2 < G_s/G_i < 0.8$, $c_{q,s}$ can be seen to be in the range $0.8 < c_{q,s} < 1.5$ for the hyperbolic model and obtain a slightly higher range of approximately $1.2 < c_{q,s} < 2$ for the tanh model. Interestingly, in general, the two models are bounded by the elastic perfectly plastic solution and the proposed value from Atkinson (2000).

8. CASE STUDY

Three case study examples are provided to illustrate the use and applicability of the three similarity methods investigated in this paper: (1) classical similarity – Sections 2, 3, 4 and 5, (2) two-part similarity – Section 6 and (3) stiffness similarity – Section 7. The various similarity factors determined in the above sections have been employed and compared. The three examples considered include both pressure-settlement curves from vertically loaded footings as well as triaxial test data from the same site. Details about each case are discussed below.

8.1 Bothkennar

Firstly, Jardine et al. (1995) obtained vertical pressure-settlement curves from rigid pad foundations in Bothkennar (Scotland) on clays and silts. Full details about the material are provided in the original paper and other publications from the site (Hight et al. 1992a, 1992b, Allman and Atkinson 1992, Nash et al. 1992). Jardine et al. (1995) conducted tests on two reinforced concrete foundations cast at a depth of 0.8m. "Pad A" was loaded to failure under short term loading conditions and thus, is selected for use in this case study. The footing is 2.2m square, with an estimated equivalent diameter of 2.48m (Jardine et al. 1995) and is assumed perfectly rough. Following the original paper, an N_c value of 6.1 (Eason and Shield 1960) can be corrected for a depth of 0.8m using Brinch Hansen's (1970) depth correction factor $[1 + 0.4z/D]$ giving an overall $N_c = 6.9$. The pressure-settlement curve is shown in Figure 16a (in black) which approaches an ultimate stress, q_u, of 138 kPa.

Undrained triaxial compression and extension tests were undertaken by Hight et al. (1992a) in Bothkennar clay at multiple depths. As the s_u value increases with depth, it is important to select a representative soil sample. Given $N_c = 6.9$ a representative undrained shear strength of $s_u \approx 20kPa$ should be employed. This occurs at a depth of approximately $1.6\text{m} - 2.7\text{m}$ $(0.3 < z/D < 0.8)$. Therefore, an undrained triaxial compression test using a Sherbrooke sampler at a depth of 2.67m is selected as the only test within this depth region. However, $s_u = 16kPa$ was obtained from this test, resulting in a predicted capacity of $q_u = 108kPa$ (used to normalise the results). Where relevant, an initial shear modulus, G_i, value of 3MPa is used, as obtained from pressuremeter tests detailed in Hight et al. (1992b), and K_i is determined theoretically from Equation 3.

The predicted N_c value (6.9) is multiplied by the s_u obtained from the soil element test to predict the capacity of the footing. This value is used to normalise both the field test and the predicted pressure-settlement curve ($q_{u,pred} = N_c s_u$).

8.2 Kinnegar

Secondly, a vertically loaded footing test was undertaken by Lehane (2003) at Kinnegar in Northern Ireland. The footing was cast at 1.6m depth on a silty stratum. Full details of the material properties are provided by Lehane (2003). The footing consisted of a 2m square, 1.7m thick reinforced concrete footing, which is assumed here to have an equivalent circular diameter of 2.26m (Osman and Bolton 2005). Following the original paper, an N_c value of 6.2 (using a shape correction factor of 1.2 and an inclination factor of 0.98) can be corrected for a depth of 1.6m using Brinch Hansen's (1970) depth correction factor $[1 + 0.4z/D]$ giving an overall $N_c = 7.8$. The pressure-settlement curve is shown in Figure 16b, which approaches an ultimate stress, $q_u = 96.5$kPa.

Lehane (2003) also presents an undrained triaxial compression test on the silt, presented in secant stiffness form, normalised by the initial mean effective stress (30kPa as per the original paper). A triaxial test from the recommended depth beneath the footing is not available. This interpretation results in an s_u of $9.2kPa$, resulting in a predicted capacity of $q_{u,pred} = 60kPa$ (used to normalise both the field test results and predicted pressure-settlement curve). Where relevant, an initial shear modulus, $G_i = 11.8$MPa was selected as the maximum measured shear modulus in the triaxial test, and the corresponding K_i is determined theoretically from Equation 3.

8.3 Ballina

Finally, two vertically loaded footing tests were undertaken by Gaone et al. (2018) at the Australian National Field Testing Facility (NFTF), near Ballina. Full details of the site investigation are provided by Doherty et al. (2018a). The footings consisted of a 1.8m square (assumed equivalent to 2.04m diameter circular footing) constructed at a depth of 1.5m in a pit on soft clay. Doherty et al. (2018b) interpreted a $q_u = 63kPa$. The pressure-settlement curves are shown in Figure 16c.

Undrained triaxial compression tests from the site are available from Doherty et al. (2018a,b). A triaxial test taken at a depth below the footing of $0.3D$ (as suggested by Osman and Bolton 2005) is selected, which resulted in $s_u = 10.5kPa$. Taking $N_c = 5.69$ and applying a shape factor of 1.2 gives $N_c = 6.8$ and yields $q_u = 67kPa$, used to normalise the results. Where relevant, $G_i = 1600kPa$ is assumed (Doherty et al. 2018b), and the corresponding K_i is determined from Equation 3.

8.4 Application of Similarity

For each example, the three similarity approaches are employed. For classical similarity, the pressure-settlement and stress-strain curves are normalised by N_c (discussed above). The shear strain of the triaxial test is then scaled by different c_q values discussed in Section 4. Firstly, the strain is scaled by c_q from Equation 4 suggested for very low stress ($q/q_u < 0.05$), and secondly, by a range of $0.5 < c_q < 0.8$, as suggested for medium stress levels ($q/q_u \approx 0.5$) – see the discussion in Section 4. The resulting transformed normalised stress-strain curves are compared to the corresponding normalised pressure-settlement curves in part (i) of Figure 16 for the three case study examples.

Two-part similarity is employed by separating the elastic and plastic components of the stress-strain curve. The elastic portion (calculated using G_i) and plastic portions (remaining after subtracting the elastic component) are scaled by Equations 22a and 22b (setting $\chi = 0.45$), respectively. The results are shown in Part (ii) of Figure 16 for the three case studies.

Finally, stiffness similarity is performed by converting the triaxial test stress-strain curve into the stiffness space ($G_{sec} = \tau/\gamma$) and scaling the shear strain of the triaxial test by the different $c_{q,s}$ values discussed in Section 7 to get w_b/D. The curves are scaled by $c_{q,s}$ from Equation 29 (suggested in this work for very high stress, $q/q_u = 1$), and by $c_{q,s} = 2$ (as suggested by Atkinson 2000 and in this work for medium stresses, $q/q_u = 0.5$). To get the corresponding applied pressure, q, the Boussinesq solution (Equation 3) is applied to each G_{sec} value from the triaxial test and the resulting K_{sec} value is multiplied by w_b. The results are shown in Part (iii) of Figure 16 for the three case studies.

The three similarity approaches employed to predict pressure-settlement curves of the field tests for the three case study examples demonstrate reasonable results in the loading range considered. The absolute percentage errors of the predicted settlement against the measured value are shown in Table 2 at $q/q_{u,pred} = 25\%$ and $q/q_{u,pred} = 50\%$. The classical similarity method provides remarkably good results for both the Bothkennar and Ballina sites. The best results were obtained with $c_q = 0.8$, which showed a maximum error of 15%, increasing to 67% when including the Kinnegar site. Two-part similarity works well for the Bothkennar site (less that 25% error). The remaining errors are higher and increase to over 100% for $c_{q,s} = 2$ as suggested by Atkinson (2000) for stiffness similarity.

It is worth noting that as additional complexities, the pressure-settlement curve itself will be affected by the rate of loading, and behaviours such as creep or consolidation are not considered by the simplified approach of similarity presented herein.

The errors obtained from this approach should also be taken in context. Doherty et al. (2018b) conducted an international competition to predict the footing displacement of the two Ballina footing field-tests. Out of the 50 submissions, they found that around 15% of submissions predicted the footing settlement to be within 50% of the measured value for $q/q_u = 0.25$ and 22% for $q/q_u = 0.5$. It is also worth noting that Doherty et al. (2018b) refer to the two field-tests as “almost identical foundations”. Using the same method as above, if Test 1 from the Ballina site is used to predict Test 2, percentage errors of 53% ($q/q_u = 0.25$) and 25% ($q/q_u = 0.5$) are obtained. This demonstrates the variability and uncertainty inherent in geotechnical design, even when a comprehensive site investigation is conducted. This also indicates that a simplified method such as similarity is well-suited for settlement estimation in routine design.

9. SUMMARY AND CONCLUSIONS

A simplified approach to obtain non-linear pressure-settlement curves of vertically loaded, rigid, circular footings on clay has been presented. The "classical" similarity approach, originally suggested by Skempton (1951), relates the x-axis of a normalised stress-strain curve with that of a normalised pressure-settlement curve (Figure 1). This transformation factor is defined in this work using a dimensionless linear-transformation factor c_q, defined by Equation 1. In the original work, Skempton (1951) suggested that the stress-strain curve should be obtained from a routine soil element test (undrained triaxial compression) undertaken on a representative soil sample. Despite the theoretical importance and practical appeal of this simplified approach as well as its wide application in a range of geotechnical problems, limited investigation and validation exists in the literature. Motivated by this lack of knowledge, this paper initially investigated the classical similarity approach (Section 2, 3, 4 and 5). To this end:

1. Three related methods - an elastic stiffness approach based on the Boussinesq solution in Equation 4 (Skempton 1951), the existing MSD method in Equation 6 (Osman and Bolton 2005), and a novel cone model solution in Equation 16 - are reviewed and extended to derive c_q. A summary of c_q values obtained is shown in Figure 6a and discussed in Section 4.
2. The novel cone model solution demonstrates that c_q depends on the pressure applied to the foundation and gives a simple approach to determine this non-linear function. The resulting c_q values are shown in Figure 6a.
3. It was found that for low stresses ($q/q_u < 0.05$), the elasticity value of $c_q = 1.2$ (Equation 4) would be sufficient to "stretch" a stress-strain curve.
4. For higher stress levels, $q/q_u \approx 0.5$, (applicable in geotechnical engineering where safety factors of 2 to 3 are common) values in the range of $0.5 < c_q < 0.8$ are needed to "compress" a stress-strain curve (range from Figure 6a).
5. For even higher stress regions the c_q value appears to approach zero (Figure 6a). At this stress range classical similarity is unlikely to be applicable and more complex analysis considering soil plastic flow and failure should be sought. These results indicate that a higher applied footing pressure invariably

results in increased strength mobilisation and strain concentration in the area close to the footing, thus decreasing the characteristic dimension ($c_q D$).

6. For a rough footing, c_q was shown to be approximately 5% larger than that of a smooth footing, which indicates a marginally larger area of influence around the footing, increasing $c_q D$.

Contrary to the implied assumption in classical similarity, this paper has demonstrated that perfect similarity is unlikely for the problem at hand, and instead, c_q depends on load intensity. As an alternative, a “two-part” similarity procedure that consists of individual scaling factors on both the elastic, $c_{q,e}$, and plastic, $c_{q,p}$, portions of the stress-strain curve is investigated and applied to vertically loaded foundations for the first time. To this end (Section 6):

7. $c_{q,e}$ and $c_{q,p}$ can be obtained from the Ramberg-Osgood model, applied for both the stress-strain curve and the pressure-settlement curve (Equation 22) and calibrated with the aid of numerical analyses. According to Jakub (1977) and as further validated herein by comparison with numerical analyses, the elastic and plastic transformation factors obtained from the Ramberg-Osgood model can be generalised to other models as well.
8. The two-part similarity approach yields $c_{q,e} = 1.2$ and $c_{q,p} = 0.5\ c_{q,e}$ (Section 6.1). Whilst these results remain dependent on footing roughness, the dependency on load intensity is reduced and can be applicable possibly as high as $q/q_u = 0.8$ (Figure 13).
9. $c_{q,e}$ and $c_{q,p}$ can be converted into a single c_q value using classical similarity (Equation 23). At low stress levels c_q is naturally governed by $c_{q,e}$; however, the variation of c_q with increased applied loads is governed by the value of $c_{q,p}$. This paper recommends $c_{q,p} < 1$ (Section 6.1) which would suggest c_q decreases with an increasing applied load. Remarkably, this agrees with the classical similarity results but has the additional benefit of being controlled by a constant $c_{q,p}$.

As another alternative to the classical similarity method, Atkinson (2000) proposed a “stiffness” similarity approach that suggests similarity exists between the shear modulus reduction curve of a soil element with increasing strain ($G_s - \gamma$) and the stiffness reduction curve of a surface foundation with increasing normalised settlement ($K_s - w_b/D$). The transformation factor, $c_{q,s}$, and the application of this approach has been investigated in this paper. To this end (Section 7):

10. Once again, perfect similarity does not exist and the similarity factor, $c_{q,s}$ is dependent on the applied load intensity (Section 7; Figure 15).

11. For a perfectly plastic material, a value $c_{q,s} = 1.2$ can be analytically established (Equation 29). This value is applicable at high stress ranges ($q/q_u = 1$).
12. For lower applied stresses, values in the range $1.5 < c_{q,s} < 3$ would be applicable. This agrees with the $c_{q,s}$ suggested by Atkinson (2000) in the original work. However, it is evident that $c_{q,s}$ is dependent on the load intensity and thus, a single value of $c_{q,s}$ is hard to determine. The stiffness similarity approach does not work well for low strains but, contrary to the other methods discussed, accuracy may improve with increased load intensity.

The results for all three approaches have been validated using numerical solutions in FLAC 2D using hyperbolic and tanh soil constitutive models and have been applied to three case study examples in Figure 16.

It is important to mention that the similarity methods discussed are approximate solutions to obtain a non-linear pressure-settlement curve of a vertically loaded circular footing. The transformation factors determined are (to varying extents) dependent on soil properties, applied load and soil constitutive models. Although the methods are fundamentally approximate and accuracy in the results cannot be guaranteed, this should be considered in the context of the wider uncertainties present when predicting foundation settlements. These approaches enable simple, easy to understand solutions with clear assumptions, which can be easily obtained from standard site investigation tests.

DATA AVAILABILITY STATEMENT

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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