



Original Article

Dynamic feed scheduling for optimised anaerobic digestion: An optimisation approach for better decision-making to enhance revenue and environmental benefits



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ABSTRACT

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Anaerobic digestion (AD) offers a sustainable solution for clean energy production, with the potential for significant revenue enhancement through enhanced decision-making. However, the complexity and limited flexibility of AD systems pose challenges in developing reliable optimisation methods. Changing feeding strategies provides opportunities for efficient feedstock utilisation and optimal gas production, especially in volatile gas markets.

To provide better decision-making tools in AD for energy production, we propose an integrated site model for the dynamic behaviour of the AD process in a biomethane-to-grid system and optimise production based on predicted gas prices. The model includes methods for optimal feed co-digestion strategies and integrates these results into a scheduling model to identify the optimal feedstock acquisition, feeding pattern, and potential gas storage operation considering feedstock availability, properties, sustainability, and fluctuating gas demand under different pricing variations.

The methodology was tested on a 150 tonnes per day farm-scale AD plant in the UK, processing energy crops and manure considering both environmental (global warming potential) and economic objectives. The results showed strong adaptability of the proposed feeding schedule to the general trend of gas prices over time. To address the challenge of immediate price peaks, typically unattainable due to the system's sluggish behaviour and high retention times, the impacts of on-site storage were explored, leading to annual revenue increases ranging from 2 % to 7.4 %, depending on the pricing scheme, which translates to a significant boost in terms of revenue.

1. Introduction

Anaerobic digestion (AD) is an established approach for sustainable waste management and renewable energy production from renewable feedstocks. It provides a potential alternative to address environmental problems and meet energy demands, while also having the potential to deliver overall negative emissions through integration with carbon capture and storage (CCS). Negative emissions will be required to offset hard-to-abate emissions for net-zero (IPCC, 2023; Tan et al., 2022). Recently, the UK Biomass Strategy has set ambitions to maximise the

utilisation of sustainable biomass supporting the UK's net-zero target and enhancing benefits including food and agricultural waste recycling and biogas production through AD (Biomass Strategy, 2023). As it can produce uninterrupted energy, AD can also help address the issue of intermittent energy supply that is commonly associated with other renewable sources such as wind and solar (Lafratta et al., 2021; Mauky et al., 2017).

AD systems can be designed to be demand-oriented, meaning they can be adjusted to meet the varying demands (price) of the gas grid. For instance, the produced biogas can be stored and used when energy

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demand is high, or when other renewable sources are not producing energy (Lafratta et al., 2021). However, there is a lack of extensive study on the application of feed scheduling and techno-economic feasibility of storage for large-scale flexible AD operations, as well as data on how operating AD more flexibly will impact feedstock acquisition and plant revenue. The development of demand-oriented models, in which an AD plant can flexibly and sustainably respond to changes in gas price and/or feedstock availability, can be of significant industrial interest. Such models may increase the profitability of an industry that typically relies on government subsidies to remain viable (Weinrich and Nelles, 2021).

This study proposes an integrated scheduling model to optimise AD processes through considering whole-site integration including feedstock seasonality, cost, and availability, co-digestion, storage, gas price prediction, and global warming potential (GWP) within a single optimisation model. Currently, there is no such tool available to aid operators with key decisions. AD operators operate conservatively based on experience and simple models to ensure operation with minimal perturbations and continuous high gas production. They are not, however, able to consider the impacts of varying feedstock prices, GWPs, process dynamics, gas and electricity demand, etc. simultaneously to obtain optimal operation. Such integration is therefore crucial for optimising and realising the potential of AD and to help operators make strategic decisions about feedstock selection and scheduling, while striking a balance between biogas potential, costs, and reducing emissions.

Detailed mechanistic models of AD, most commonly the Anaerobic Digestion Model Number 1 (ADM1) proposed by Batstone et al. (2002), describe the physiochemical and biochemical reactions involved in AD for representing the production of biogas. However, the model is known to be challenging for real-time optimisation implementation due to the large number of parameters that have low identifiability, slow solution speed, and reliance on data that may not be collected in industrial operation (Mata-Alvarez et al., 2011). Moreover, the ADM1 does not account for co-digestion, which refers to the complex synergistic interactions when various substrates are combined to enhance biogas production (Pagés-Díaz et al., 2014). Co-digestion enhances biogas production and process efficiency by reducing process inhibition, improving digestibility, increasing the nutritional content of the generated digestate, and strengthening process stability (Karki et al., 2021; Xie et al., 2016). Extensions to the ADM1 that incorporate the impacts of co-digestion have been explored by researchers such as Mudzanani et al. (2023), Liu et al. (2020). However, these models still require a large number of calibrating parameters and are computationally challenging to solve when included in optimisation models.

There are also simpler AD models which have been extended for demand-oriented models, proposed by Barchmann et al. (2016), Körber et al. (2022), Ohnmacht et al. (2021). These demand-oriented models in these studies consider flexible gas production as demand profiles and storage capacities, alongside simplified AD models considering only the biomethane potential (BMP) of individual mono-digested feedstocks, ignoring co-digestion effects, and assuming constant feedstock availability and neglecting the environmental footprint of the system in decision-making (Ó Céileachair et al., 2022). However, it is important to consider the effect of co-digestion, sustainability assessments, as well as variable nature of feedstocks availability and costs.

The degree to which models can accurately predict whether demand can be met, is reduced by this simplicity. Ohnmacht et al. (2021) presented a framework which optimised the biogas supply chain, determining the optimal capacities of AD and biogas storage systems based on known demand profiles, however, they did not consider price changes relating to storing and selling/convertng at peak prices, nor did they consider variation in solid retention times and its effect on storage requirements. Willeghems and Buysse (2019) explored the potential profitability of an AD site, by participating in the day-ahead electricity market and proposed modifications that could enhance profitability. However, there was no consideration of biomethane or bio-LNG costs, storage and demands, nor of developing a whole-site scheduling model

that can be adapted for different feedstock availabilities. To provide accurate and optimal biogas production, co-digestion, feedstock availability and digester feed scheduling must be considered simultaneously. Feedstock availability is crucial in AD operations, with varying composition, and seasonal pricing and availability fluctuations posing challenges to continuous and efficient digestion. To meet varying energy demands, it is necessary to simultaneously consider how feeding schedules impact the AD process for flexible biogas output.

Co-digestion is commonly employed in many commercial AD facilities, as it allows for a diversification of feedstock sources that can mitigate supply chain risks, while potentially providing synergistic effects for enhanced biogas production (Aichinger et al., 2015). A new digital twin of AD proposed by Moretta et al. (2021) offers a reliable and simplistic approach for better predictions of the impacts of co-digestion on biogas output; however, feedstock acquisition, scheduling, and mixing were not considered. Liu et al. (2021) optimised co-digestion in a demand-oriented biogas supply chain with one-day modelling timescale and simplified system boundaries. However, they did not account for various feedstock scheduling, nor did they consider the effects of GWP and seasonality on the response to demand.

Efficient storage management plays a pivotal role in balancing supply and demand dynamics in AD facilities (Dolat et al., 2024). utilising storage not only ensures uninterrupted operation but also enables strategic inventory management to capitalise on market demand and gas pricing fluctuations. On a gas-to-grid AD site, biogas could be stored on-site for later use, combusted in a combined heat and power (CHP) engine, primarily for on-site electricity and heat generation, or injected into the grid after upgrading to biomethane (Liu et al., 2020). Moreover, the integration of GWP as a factor within decision-making offers a holistic approach to environmental sustainability, incentivising emission-reducing AD (Chang et al., 2012; Zhang et al., 2024). Fig. 1 shows a gradual increase in the annual publications annual number of publications on Scopus for key word search 'Anaerobic Digestion Modelling' refined further to key words: 'Global Warming Potential', 'Co-digestion', 'Scheduling', 'Demand Oriented' and 'Other'.

The aim of this study is to improve the efficiency of the AD process by enabling better on-site decision-making that connects up- and downstream processes in an integrated scheduling optimisation model that simultaneously considers dynamic pricing mechanisms to bolster revenue. To the best of our knowledge, no models consider the dynamic interplay and balance between feedstock availability, storage capacities, gas production, energy demand and dynamic pricing, which requires a comprehensive approach for effective management and profitability maximisation.

This paper introduces a comprehensive model that incorporates diverse feedstocks with varying properties and availabilities, aiming to determine the optimal blending patterns and feeding schedules in response to predicted gas prices over a specified time horizon. The model addresses critical factors such as co-digestion, demand fluctuations, and seasonality while considering different storage capacities to enhance profitability and the sustainability of AD operations. By tackling these challenges within a unified optimisation framework, this study advances the state-of-the-art in industrial AD optimisation, facilitating more integrated and informed decision-making for operators.

The paper is structured as follows: The methodology section provides a detailed explanation of the modeling approach and the mathematical formulation for the co-digestion and scheduling functions. A comprehensive case study is presented, focusing on the optimal feed scheduling for a full-scale AD plant using real feedstock data and exploring various scenarios, including gas price variability, GWP considerations, and storage options. The results and discussion section analyses the optimisation outcomes and the insights derived from the different scenarios. Finally, the conclusions summarise the key findings and implications of the study.

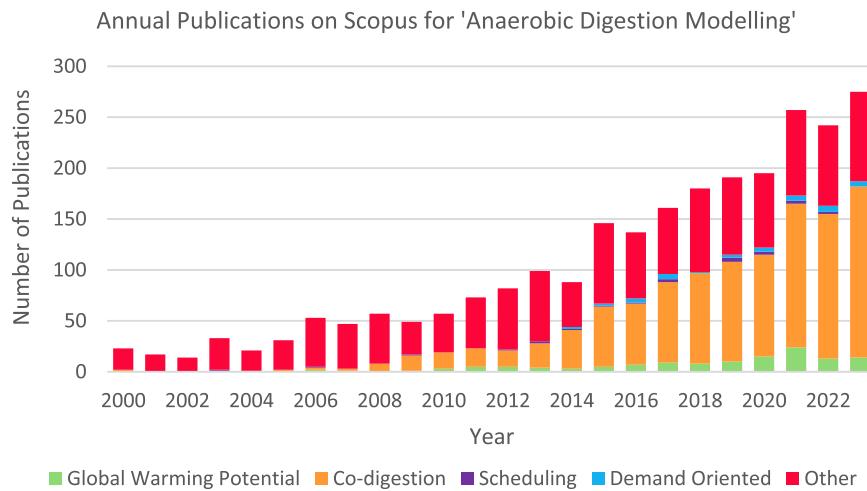


Fig. 1. Annual number of publications on Scopus for key word search 'Anaerobic Digestion Modelling' refined further to key words: ' Global Warming Potential', 'Co-digestion', 'Scheduling', 'Demand Oriented' and 'Other'.

2. Methodology

To formulate the optimal feeding plan for the AD reactor, several critical considerations must be addressed. This process involves optimising feed consumption through an optimal blending procedure to exploit synergies, thereby minimising feeding rates while maximising biomethane yield. Concurrently, the feeding rate must be regulated to consider feedstocks' supply chain constraints (such as seasonality, availability and transportation), operational considerations (solids content, carbon-to-nitrogen ratio, etc.) and to closely align with fluctuating and uncertain market demands and price changes. Additionally, environmental concerns, specifically GWP factors, must be incorporated into the decision-making process to ensure that relevant sustainability criteria are met.

The model includes two optimisation stages, both implemented in Pyomo (Bynum et al., 2021), a Python-based optimisation package. The first stage (referred to as the *CoD* model) calculates the optimal co-digestion recipe, using a method proposed by Moretta et al. (2022). The function is formulated as a Nonlinear Programme (NLP) and the primary objective of the model is to identify the substrate blending patterns that results in the highest biomethane potential due to their synergistic effect (B_{CoD}).

The second optimisation function, referred to as the *Scheduler*, integrates various essential data, including the availability and seasonality of substrates, their chemical and physical compositions, prices, and distances from the feed origin. Using the optimal blending ratios determined in the first optimisation stage, together with the predicted gas prices for the upcoming days as input data, the Scheduler generates the optimal feeding and gas storage strategy through a Mixed-Integer Linear Program (MILP) problem. This comprehensive approach ensures that the feeding plan not only meets production targets, but also adapts to market conditions and minimises environmental impact.

The separation of the co-digestion and scheduling models serves two key objectives. First, it prevents the integration of the nonlinear programming (NLP) formulation of the co-digestion model with the mixed-integer linear programming (MILP) problem of the scheduler, thereby minimising the resulting computational complexity and enabling fast, efficient solutions. Second, while this approach may lead to suboptimal economic solutions due to the decoupling of blending and scheduling decisions, it improves on the state of the art by providing higher short-term gas yields and increased reactor productivity under any combination of feedstocks. This method offers a practical and scalable solution that advances the integration of co-digestion optimisation into scheduling models, a feature not previously incorporated in other studies.

2.1. Co-Digestion model (f_{CoD})

The model takes experimental data such as the biomethane potential (EBMP) and elemental composition of individual substrates as input and calculates the optimal blending ratios leading to the maximum Co-digestion biomethane yield (B_{CoD}) for every possible two or three-component system. The number of possible substrates has been limited to three for several reasons. BMP tests rarely use more than three substrates, and substrates can often be grouped into fewer categories due to their similarities. Additionally, limiting the blends to triplets reduces computational time and increases efficiency.

It is important to note that the proposed model does not account for complex co-digestion phenomena, such as the inoculum and microbial community effects, nor does it consider kinetic parameters of biomethane production. However, due to the high retention times in the process and the complexity involved in modeling these microbial activities, we consider this model a suitable surrogate for scheduling purposes.

The objective function of the NLP model is to maximise the biomethane potential (BMP) of the feed blend. This model returns the optimal feedstock blending ratios in terms of mass fractions of the chosen substrates (Eq. (2)).

$$\text{Objective Function} = \text{Max } [B_{CoD}] \quad (1)$$

$$B_{CoDF} = \sum_{i \in F} x_i B_i + \sum_{\substack{\emptyset \neq T \subseteq F \\ |T| > 1}} \left(\prod_{i \in T} x_i \right) B_{mixF} \quad (2)$$

$$\forall F = (j, k) \vee (j, k, m)$$

Here, B_{CoDF} is the BMP of co-digestion process for any desired feed blend F . The model supports either two-component (j, k) , or three-component feed blends (j, k, m) . x_i is feeding ratio of component i in the blend, B_i (mL/gvs) is the cumulative methane yield of single component i obtained in lab-scale tests (EBMP). Here B_{mixF} quantifies the degree of synergistic correlation between substrates which is characterised by two key parameters: the weighted average carbon to nitrogen ratio $\left(\left(\frac{C}{N} \right)_{mix} \right)$ and the weighted average biodegradability (BD_{mix}) of the blend. These parameters are calculated according to Eqs. (3) and (4), respectively.

$$\left(\frac{C}{N} \right)_{mix} = \sum_{i \in F} x_i \left(\frac{C}{N} \right)_i \quad (3)$$

$$BD_{mixF} = \sum_{i \in F} x_i BD_i \quad (4)$$

Here, $\left(\frac{C}{N}\right)_i$ represents the ratio of carbon to nitrogen content of each substrate i in the blend and BD_i is the biodegradability of the substrate i and is the ratio of experimental to theoretical biomethane yield (Eq. (5)).

$$BD_i = \frac{B_i}{TB_i} \quad (5)$$

The Theoretical biomethane yield for each substrate i (TB_i) can be evaluated using the modified Buswell equation (Eq. (6)). This equation calculates the theoretical BMP of the biomass with the hypothetical chemical formula $C_nH_aO_bN_cS_d$ as follows:

$$TB = \frac{\left(\frac{n}{2} + \frac{a}{8} - \frac{b}{4} - \frac{3c}{8} - \frac{d}{4}\right) 22415}{12n + a + 16b + 14c + 32d} \quad (6)$$

The equation, originally introduced by Buswell and Mueller (2002) and later modified by Boyle (1977), is used to predict biomethane yield based on the elemental composition of waste materials. Boyle's modification includes nitrogen and sulphur to account for ammonia and hydrogen sulphide content in the biogas. Although this equation assumes perfect conversion of biomass to methane—leading to an over-estimation of the BMP—it remains widely used due to its simplicity and reliance on straightforward input data (elemental composition). It provides valuable insights and comparisons regarding the potential of various biomass feeds (Achinas and Euverink, 2016).

Through response surface prediction method, the correlation of B_{mix} and the weighted parameters are identified according to Eq. (7):

$$B_{mixF} = \beta_0 + \beta_1 \left(\frac{C}{N}\right)_{mixF} + \beta_2 BD_{mixF} + \beta_3 \left(\frac{C}{N}\right)_{mixF}^2 + \beta_4 BD_{mixF}^2 \quad (7)$$

where β_0 , β_1 , β_2 , β_3 , and β_4 are regression coefficients equal to 21.7, 1.26, 445.7, -0.02 and -7.82 respectively according to Moretta et al. (2022). The authors validated their model using experimental BMP data and reported a good reliability with the Root Mean Square Error (RMSE) values less than 20 mL/gvs for two-substrate mixtures and less than 30 mL/gvs for three-substrate mixtures.

It is important to recognise that the accuracy of the regression parameters—and, by extension, the precision of the response surface model—is significantly limited by the database on which the model is based. To achieve more accurate results, it is advisable to develop specific regression parameters tailored to the particular feed database in use. However, in the absence of sufficient data, the current method and parameters serve as a reasonable approximation.

As an input to this model, a comprehensive set of feed blends is generated, encompassing all possible combinations of substrate pairs or triplets. The model is then executed multiple times to determine the optimal proportion for each blend within this set, ensuring maximum yield and minimal waste by identifying the blending patterns with the highest synergistic effect. These optimally blended feed combinations are subsequently utilised as inputs for the second model, the scheduler, which manages the scheduling of these blends.

It is important to note that the optimal feed proportions can range between 0 and 1. A fraction close to 0 or 1 indicates that the pair or triplet does not exhibit a synergistic effect, effectively suggesting monodigestion.

2.2. Scheduling model ($f_{scheduler}$)

The second step in the proposed approach is the site scheduling model, which suggests an optimal feeding timetable aimed at maximising profits from selling the produced bio-methane to the grid while considering the system's GWP impacts. This model selects feeding

strategies based on the optimal recipes generated in the first stage and accounts for constraints such as substrate availability, cost, location, and physical-chemical properties. It is designed to integrate with a real-time gas price prediction mechanism, which serves as an input for regulating feed scheduling and subsequent gas production. Additionally, the model can utilise gas storage as an extra degree of freedom, enhancing the adaptability of gas supply to the grid under various pricing schemes. The model is formulated as MILP for efficient solution time and performance.

2.2.1. Production model

The objective function maximises the revenue from selling the biomethane to the grid while minimising the expenditure incurred to purchase the feed from suppliers ($Cost_{Total}$) and minimising the GWP of the entire process (Eq. (8)).

$$\max \sum_{d \in \text{days}} (Sell_d \cdot pr_d) - \gamma' GWP - \gamma'' Cost_{Total} \quad (8)$$

In this equation, $Sell_d$ represents the amount of produced biomethane sold on day (or time) d , pr_d is the gas price on day d , and γ' and γ'' are weighting factors for GWP and total cost respectively.

The model accounts for the potential to store gas in on-site storage facilities. The produced biomethane (P_d) can be either stored (S_d), in part or in full, or sold based on the methane price at that specific time (pr_d) and the maximum storage capacity (S_{max}).

$$Sell_d = P_d + S_{d-1} - S_d \quad (9)$$

$$Sell_d \leq P_d + S_{d-1} \quad (10)$$

$$S_d \leq S_{max} \quad (11)$$

To model changes in gas production due to the addition of various substrates with different biomethane potentials to the digester, a simplified correlation is introduced as follows:

$$P_d - P_{d-1} = [(r_{td}^+ \cdot y_{td}^+) + (r_{cd}^+ \cdot y_{cd}^+)] - [(r_{td}^- \cdot y_{td}^-) + (r_{cd}^- \cdot y_{cd}^-)] \quad (12)$$

$$y_{td}^+ + y_{td}^- + y_{cd}^+ + y_{cd}^- = 1 \quad (13)$$

The term $P_d - P_{d-1}$ indicates the change to the production between time ($d-1$) and d . The variables r_{td}^+ and r_{td}^- specify the production ramping up and ramping down when switching between different feeds. Each ramp-up and ramp-down has two modes: transition and continuous, indexed as t or c . The transition mode (r_{td}^+ or r_{td}^-) occurs when a new feed is introduced to the digester and continuous mode (r_{cd}^+ , r_{cd}^-) occurs when the feeding continues based on the previous new feed. The activation or de-activation of these modes are controlled via the relevant binary variables (y). To reduce computational time, the non-linear terms in Eq. (12) are linearised through piecewise linearisation method.

The values of the ramping variables are calculated based on the changes that occur to the biomethane potential of the whole reactor as a new feed is introduced. For the transition mode, when a new feed is selected for time d , depending on whether the new feed potential (θ_d) is higher or lower than the production rate of the substrates already in the reactor (P_{d-1}), the upward or downward transition values are calculated according to Eq. (14). Their corresponding binary variables, y_{td}^+ and y_{td}^- become active depending on the direction of change to the biomethane potential (Eqs. (15)–(18)).

$$\frac{(\theta_d - P_{d-1})}{SRT} = r_{td}^+ - r_{td}^- \quad (14)$$

$$\theta_d - \theta_{d-1} \leq \epsilon + M y_{td}^+ \quad (15)$$

$$\theta_d - \theta_{d-1} \geq -\epsilon - M y_{td}^- \quad (16)$$

$$\theta_d - \theta_{d-1} - \epsilon \geq -M(1 - y_{td}^+) \quad (17)$$

$$\theta_d - \theta_{d-1} + \epsilon \leq M(1 - y_{td}^-) \quad (18)$$

In these equations, ϵ and M refer to relatively small and large values respectively, to ensure the logical consistency of the inequality constraints. Solid retention time (SRT), determines the maximum feeding rate (Eq. (21)) based on the volume of the digester (V_{react}), which in turn affects the ramp values. Specifically, a higher SRT allows for a higher feeding rate and larger ramping increments. In other words, when a particular feed is continuously fed into the reactor, it takes SRT duration for the digester to reach equilibrium, where the feed potential (θ_d) aligns with the production rate (P_d).

The biomethane potential of the feed selected to be introduced at time d (θ_d) is calculated according to Eq. (20).

$$B_{CoDF} = \sum_{i \in F} B_{CoDF} (x_{CoDF,i} \cdot VS_i \cdot TS_i) \quad \forall F \in J \quad (19)$$

$$\theta_d = \sum_{F \in J} B_{CoDF} \cdot W_{F,d} \quad F = (j, k) \vee (j, k, m) \quad (20)$$

Here, J represents the total set of pairs (j, k) or triples (j, k, m) feed blends that are exported from the CoD model. The parameter $x_{CoDF,i}$ represents the portion of substrate i within the feed blend F , while B_{CoDF} refers to the biomethane yield of that blend. Both of these parameters are output from the CoD model. The feeding rate of that particular feed blend on time d of the process is denoted by $W_{F,d}$.

The total feed rate into the digester, which is the sum of the weight rate of blend and pertinent supplied water ($W_{F,d} + W_{F,d}^{water}$), is controlled by the solid retention time (SRT) and the total solid content of the mixed substrate. This is expressed in Eq. (21):

$$W_{F,d} + W_{F,d}^{water} = \left(\frac{V_{react} \rho}{SRT} \right) y_{F,d} \quad \forall F \in J \quad (21)$$

The water content ($W_{F,d}^{water}$) is calculated based on the maximum allowable solid content of the feeds when entering the reactor (TS_{max}), as shown in Eq. (22):

$$W_{F,d}^{water} = \frac{W_{F,d}(TS_F - TS_{max})}{TS_{max}}; \text{ if } TS_F \geq TS_{max}, \text{ else : } W_{F,d}^{water} = 0 \quad (22)$$

The selection of feed mixtures F for time d is identified by the binary variable $y_{F,d}$, ensuring that only one feed mixture is chosen for each time increment, as expressed in Eq. (23):

$$\sum_{F \in J} y_{F,d} = 1 \quad \forall d \in D \quad (23)$$

After a transition, while the reactor continues to be fed with the same feed until a new combination is selected, the ramping will operate in continuous mode. This mode follows the same ramping increments as the transition mode, as described by Eqs. (24)–(26):

$$P_{d-1} - P_{d-2} = r_{cd}^+ - r_{cd}^- \quad (24)$$

$$r_{cd}^+ \leq M(1 - y_{cd}^-) \quad (25)$$

$$r_{cd}^- \leq M(1 - y_{cd}^+) \quad (26)$$

In these equations, θ_d represents the biomethane potential of the feed at time d . The SRT influences the feed rate, ensuring that the total solid content remains below the maximum threshold (TS_{max}). The binary variable y_{cd} ensures the selection of a specific feed mixture for each time period. During continuous mode, the ramping adjustments maintain consistency in feed transitions.

In our proposed model, the ramping effect (as described in Eq. (14)) is assumed to be linear, despite the actual changes in production following a nonlinear pattern. More rigorous approaches, such as the first-order kinetic model described by Brûlé et al. (2014) and modified Gompertz models, such as the ones proposed by Abudi et al. (2022), Dumitrel et al. (2017), better mimic organic matter degradation through

exponential terms. However, these models come at the cost of significant computational complexity and time. Also, in practical feeding scenarios, operators typically adjust feeds gradually from the previous pattern to the desired new one, which mitigates deviations from linearity. Thus, to streamline the optimisation process and eliminate non-linear terms, we have formulated the ramping effects in a linear manner. This simplification reduces computational demands while still providing a practical approximation of the system's response to feed changes.

2.2.2. Scheduling and availability constraints

As explained earlier, the binary variable $y_{F,d}$ is responsible for selecting a feed mixture F to be processed at time d . The model incorporates two important availability constraints: maximum acquirable feed weight and seasonality (availability time period). The constraints are formulated as follows:

$$d + M(1 - y_{F,d}) \geq \max_{i \in F} (d_{release,i}) \quad \forall F \in J, \forall d \in D \quad (27)$$

$$d + M(1 - y_{F,d}) \leq \min_{i \in F} (d_{end,i}) \quad \forall F \in J, \forall d \in D \quad (28)$$

$$\sum_{F \in J} \sum_{i \in F} \sum_{d \in D} W_{F,d} \cdot x_{CoDF,i} \leq W_{av,i} \quad \forall i \in I \quad (29)$$

$$\sum_{i \in I} \sum_{F \in J, i \in F} \sum_{d \in D} W_{F,d} \cdot x_{CoDF,i} \cdot cost_i = Cost_{Total} \quad (30)$$

Eqs. (27) and (32) ensure that the selection and processing of the feed mixture F occur within the availability period of each feed component (from $d_{release}$ to d_{end}). Additionally, Eq. (28) imposes a quantitative limit on the feed, ensuring that the sum of the weights of components used throughout the process does not exceed the total available weight of each component (W_{av}). The ratio of each feed component (x_{CoDF}) is linked to the optimal feed ratio derived from the first optimisation model (f_{CoD}).

2.2.3. Global warming potential constraints

The GWP formulations are mainly built on the basis of the calculation model presented in Zhang et al. (2024). Total GWP is calculated as the sum of GWPs from cultivation, transportation, plant's external energy consumption, leakage and Combined Heat and Power (CHP) utilisation (Eq. (31)).

$$GWP = \sum_{F \in J} GWP_{CF} + \sum_{F \in J} GWP_{TF} + GWP_E + GWP_L + GWP_{CHP} \quad (31)$$

The cultivation GWP (GWP_C) is calculated based on the total weight of the feed blend F consumed over the time period D ($W_{Total,F}$) and the cultivation burdens parameters (α_C) which are listed in Table 1. The

Table 1
GWP parameters list.

Cultivation (Styles et al., 2016)	Crop type	α_C (GWP kg _{CO2e})	Ref. unit
	Grass silage	0.39	kg dry
	Maize silage	0.19	matter
	Other cereal silage	0.31	
Transportation (Wernet et al., 2016)	Truck type	α_T (GWP kg _{CO2e})	Ref. unit
	lorry 3.5–7.5 t	0.51617	t.km
	lorry 7.5–16 t	0.21658	
	lorry 16–32 t	0.1612	
	lorry >32 t	0.08955	
Electricity & Heat (UK.GOV, 2023)	Energy	$\alpha_{E/H}$ (GWP kg _{CO2e})	Ref. unit
	Electricity generation	0.207	kWh
	Heat generation	0.180	
CHP Units (UK.GOV, 2023)	CHP combustion	$\alpha_{E/H}$ (GWP kg _{CO2e})	Ref. unit
	Biogas combustion	0.00022	kWh
	energy	0.0015884	Nm ³
	Biogas combustion		
	volume		

fixed coefficient of 1.1 represents the impact of the crop loss:

$$GWP_{CF} = 1.1 \sum_{i \in F} W_{TotalF} (x_{CoDF,i} T_{Si} \alpha_{Ci} \hat{y}_i) \quad \forall F \in J \quad (32)$$

The (user specified) *binary parameter* \hat{y} identifies the feed exclusively cultivated for the AD processes.

The transportation impact (GWP_T) is calculated as sum of the GWP for shipping feeds to the plant and transporting the digestate to where it is utilised.

$$GWP_T = \sum_{i \in F} W_{TotalF} x_{CoDF,i} L_i \alpha_{Ti} + \sum_{i \in F} W_{TotalF} x_{CoDF,i} L'_i \alpha'_{Ti} \quad \forall F \in J \quad (33)$$

The parameters of L and L' are the distances from the feed origin and digestate consumption area to the AD plant respectively, W_{TotalF} is the total weight of mixture F that converted to digestate, and α_{Ti} and α'_{Ti} are vehicle conversion factors (as a function of feed or digestate weight) listed in Table 1. If the electricity and heat required for the plant is provided from the grid (not from the inside-plant heat and power units) the resultant GWP will be calculated based on the kWh electricity and heat consumed multiplied by the GWP factor for the electricity and heat respectively according to Table 1.

$$GWP_E = \alpha_E E_{elect} + \alpha_H E_{heat} \quad (34)$$

If the heat and electricity for the process are provided by the CHP units within the AD plant, the resultant GWP (GWP_{CHP}) will be calculated according to the conversion factors listed in Table 1.

These factors are based on the units in which heat and electricity are reported.

$$GWP_{CHP} = \alpha_E E_{elect} + \alpha_H E_{heat} \quad (35)$$

If system leakage information is available (expressed as a percentage of biogas production α_L), the GWP of the leakage is calculated according to the below correlation:

$$GWP_L = 18.09 \alpha_L \sum_{d \in D} P_d \quad (36)$$

Of all the GWP components, only the cultivation and transportation GWPs influence feed selection and decision-making. Other GWPs are calculated and reported based on the AD plant's existing performance and infrastructure.

3. Case study

To demonstrate the performance of the model, we selected a case study as a farm-scale anaerobic digestion (AD) plant with a digester capacity of 10,000 m³, designed to process energy crops and manures. Table 2 details the physicochemical properties, availability, cost, and sourcing of the various feedstocks. The feeding rate is constrained by an SRT of 70 days, with the maximum total solids content of the digester

feed limited to 35 %. All specified capacities and feed information are adapted from real site data provided by industrial partners with some minor changes to the feed cost.

The objective of this case study is to establish the optimal schedule aimed at maximising the revenue from selling bio-methane to the grid. This involves considering the availability and seasonality of various feed sources while accounting for fluctuating gas prices over time. The GWP can also be integrated into the optimisation process or reported separately at the discretion of process management.

To reduce computational time, the time frame is set on a weekly basis, denoted by the subscript d in the mathematical formulations presented in the methodology section. With changes to parameters, finer time discretisation is easily considered in the model. Additionally, co-digestion is limited to a two-component system. This approach generates the optimal blending ratio for feed pairs (j,k) and the corresponding estimated biomethane potential ($B_{CoD,j,k}$) using the *CoD* model. The second model then optimises the scheduling of the available pairs to maximise revenue, minimise feed cost, and reduce the GWP of the entire process.

The study aims to optimise the production according to the predicted gas price (parameter pr_d) over a limited time horizon with the ability of quick re-scheduling based on an updated prediction set. The predicted gas price scenarios used in the study are according to the real weekly price of biogas reported by the Office of Gas and Electricity Market, Ofgem, for natural gas price in 2023 (Ofgem, 2024) as depicted in Fig. 2. The figure is divided into three distinct sections: low, middle and high-range price frequency zones. The model was tested for each zone to identify the effect of different pricing schemes on the scheduling. The additional revenue gained through utilisation of gas storage facilities for various pricing frequencies is also evaluated. We assume perfect foresight of gas prices to demonstrate the approach, however, any forecasting approach may be used.

Many current AD plants typically lack the storage facilities necessary to accommodate large-scale storage of produced gas, and they generally rely on small ambient-storage facilities that can support only 20–30 % of daily production. As a result, the base-case optimisation scenarios do not

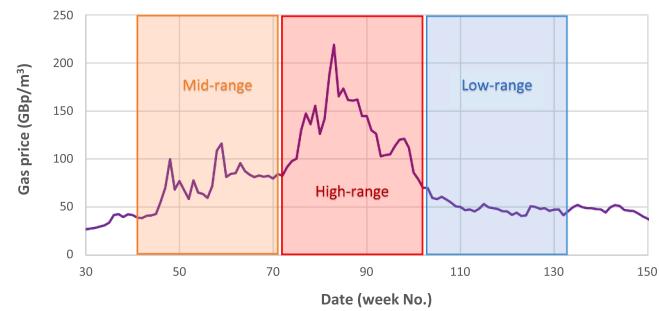


Fig. 2. Weekly price of natural gas from February 2021 to February 2024 acc. to Ofgem (2024).

Table 2

Physical-chemical properties and availability of feedstocks (provided by project industrial partners).

Substrate Property	Unit	GFC	DMS	RS	GS	WG	MS	PS
Distance from plant	km	15	15	15	15	15	15	15
Release date (Week No.)	–	0	0	0	0	0	10	0
Release end (Week No.)	–	20	20	10	20	20	20	20
Weight available	tonnes	4000	4000	4000	4000	3000	10,000	10,000
Cost	£/t _{feed}	30	80	40	20	100	50	0
Total Solid (TS)	kg _{TS} /t _{feed}	105	523	328	453	874	220	72
Volatile Solid (VS)	kg _{VS} /t _{TS}	875	959	939	962	983	941	863
EBMP	m ³ _{CH4} /t _{VS}	315	397	363	326	392	377	404
TBMP	m ³ _{CH4} /t _{VS}	433	446	435	435	436	442	508
C/N	–	14.7	36.8	26.7	34.3	25.4	31.3	12.5

GFC: Grass Fresh-Cut, DMS: Dried Maize Silage, RS: Rye Silage, GS: Grass Silage, WG: Wheat Grain, MS: Maize Silage, PS: Pig Slurry.

account for the effect of storage. However, for scenarios where we consider storage, the mass storage capacities of up to 10,000 m³/day (70,000 m³/week) is considered. The techno-economic impact of integrating storage facilities is also included in the result analysis.

The starting point for all optimisation tasks is set to a baseline production of 140,000 m³ of biomethane, representing the current output of the digester. The time horizon for price prediction and optimisation is set to 20 weeks, meaning that price predictions provide estimates for the upcoming 20 weeks. The model may be re-run in a rolling-horizon scheduling optimisation method with updated data from production output. For GWP calculations, only the cultivation and transportation stages are considered, while other factors such as heat, electricity, and CHP units are excluded. All system losses and leakages, including those associated with storage facilities, are considered negligible and have been excluded from the scope of this study as they do not affect the optimisation trade-offs. In this case study, dried maize silage is the sole feedstock cultivated for the AD process (i.e., all other energy crops are assumed to be “bought-in”). Additionally, the distance from all feed suppliers to the AD plant is uniformly set at 15 km. This uniformity is intended to highlight the trade-offs related to the potential of various feedstocks, rather than complicating the analysis with additional constraints such as transportation costs and the environmental impacts of the decision-making process.

The modeling is performed in Pyomo, with the first model (CoD) being solved using NLP solver IPOPT (Wächter and Biegler, 2006) and the second model (Scheduler) using GUROBI (Gurobi Optimization, 2024) as the external solvers.

4. Results and discussions

The results of the first optimisation model (CoD) are presented in Table 3, which identifies the optimal feed blending ratios and the resultant bio-methane yield potential for various two-component feeding system from the available substrates. In this table, the columns x_j and x_k represent the weight percentages of components j and k in the co-digestion process, respectively, and B_{CoD} indicates the maximum bio-methane yield achievable from the synergistic effect of these blending ratios. The column $TS_{j,k}$ shows the total solid percentage, and the column $W_{j,k}/W_{total}$ denotes the ratio of substrate (j, k) weight over the total feed weight (the sum of supplied water and the substrate weight). The weight fraction for each feed pair ($W_{j,k}/W_{total}$) is calculated based on the total solid content of that particular feed and the amount of

supply water that needs to be added to maintain the maximum total solid content for the digester inlet, which is set at 35 %. It should be noted that feed mixtures where substrates share less than 5 % can be considered as blends with potentially no synergistic effect for optimal co-digestion.

The results obtained from the Co-Digestion function can also be visualised in a graph, shown in Fig. 3. This graph provides an insight into the energy density (m³_{CH4}/t_{feed}) and unit price for each feed blend according to the co-digestion function results. Specifically, the figure illustrates the unit weight bio-methane potential of each feed pair (m³_{CH4}/t_{total}) plotted against the unit price (f/t_{j,k}) for each feed combination. It should be noted that the energy density of each feed pair (y-axis) considers the biomethane yield of the *total feed*, i.e., the amount of water needed to be supplied per tonne of feed blend (j, k) has also taken into account. This visualisation helps in understanding the trade-offs between bio-methane yield and cost for different feed blends.

Fig. 4 illustrates the optimised feed scheduling aimed at maximising net revenue from selling the produced biomethane to the grid across three different gas price regions. The figure shows how the feeding regime adjusts in response to predicted price variations and peaks. As the pricing shifts from the low-range to the high-range, the feeding pattern adapts accordingly to the new pricing scenarios, effectively managing price fluctuations over time.

To minimise the GWP while maximising the revenue, the scheduling and production of bio-methane adjusted as shown in Fig. 5. Similar to the previous optimisation results (Fig. 4), the feeding schedule and supply pattern are being adjusted to three different price frequency levels and the feeding schedule is optimised for each of these pricing schemes over the 20-weeks' time frame.

As observed from the production figures, the generated schedules generally follow the gas price trends, with production gradually adjusting to target the main price peaks. However, the system's response to rapid price spikes is inadequate, and the designated feeding schedule fails to address these sharp peaks effectively. This sluggish behaviour is primarily attributed to the low organic loading rate (OLR) resulting from the high solid retention time (SRT) of the process. The high SRT rates (typically around 70 days in many UK plants) limits the flexibility of AD plants to respond quickly to sudden price fluctuations.

This inflexibility is partly due to regulatory requirements, such as those for AD plants with PAS110 specifications, which mandate that digestates meet certain residual BMP levels (WRAP, 2014). Additionally, biological concerns, such as the risk of microbial population wash-out, limit the ability to reduce the SRT below certain thresholds (Dicks and Blasé, 1982). Consequently, feeding rates cannot be significantly increased without alternative measures to lower the BMP levels of the digestates to acceptable standards while maintaining high microbial activity.

Comparing Figs. 4 and 5 demonstrates that the inclusion of GWP minimisation within the optimisation toward increasing revenue has further limited the system's flexibility in responding to gas price variations. Dried maize silage (DMS), as the only cultivated crop in this particular case study, and as a high potential energy crop (as shown in Table 3 and Fig. 3) necessary for adjusting production to address price peaks, has been deselected from all feed blends. This limitation reduced the degree of freedom in the optimisation, resulting in a further production-to-price mismatch compared to the non-GWP optimisation. This further highlights the significant contribution of cultivation to the overall GWP of the process.

To overcome this intrinsic inflexibility, storage is integrated and tested as an auxiliary solution to improve the flexibility of AD processes. Incorporating storage capabilities into the production process significantly alters the supply pattern of produced biomethane to the grid. As shown in Fig. 6, during periods when prices are predicted to be low, the biomethane supply to the grid is minimised. Conversely, when prices peak, the supply to the grid is maximised, matching the storage facility capacities. This strategic use of storage ensures optimal revenue by

Table 3
Optimal feed blending pattern (output of the CoD function).

Feed (j, k)	x_j (%)	x_k (%)	B_{CoD} (m ³ _{CH4} /t _{VS})	$TS_{j,k}$ (%)	$W_{j,k}/W_{total}$ (t _{j,k} / t _{total})
GFC, DMS	2 %	98 %	403	52 %	0.70
GFC, RS	8 %	92 %	389	31 %	1.00
GFC, GS	8 %	92 %	353	42 %	0.83
GFC, WG	0 %	100 %	394	87 %	0.42
GFC, MS	12 %	88 %	412	21 %	1.00
GFC, PS	59 %	41 %	440	9 %	1.00
DMS, RS	77 %	23 %	464	48 %	0.73
DMS, GS	68 %	32 %	464	50 %	0.70
DMS, WG	25 %	75 %	475	79 %	0.45
DMS, MS	86 %	14 %	446	48 %	0.73
DMS, PS	94 %	6 %	420	50 %	0.70
RS, GS	38 %	62 %	430	41 %	0.86
RS, WG	12 %	88 %	433	81 %	0.44
RS, MS	68 %	32 %	456	29 %	1.00
RS, PS	89 %	11 %	407	30 %	1.00
GS, WG	14 %	86 %	435	81 %	0.43
GS, MS	75 %	25 %	411	39 %	0.88
GS, PS	88 %	12 %	376	41 %	0.83
WG, MS	92 %	8 %	424	82 %	0.43
WG, PS	95 %	5 %	413	84 %	0.42
MS, PS	87 %	13 %	427	20 %	1.00

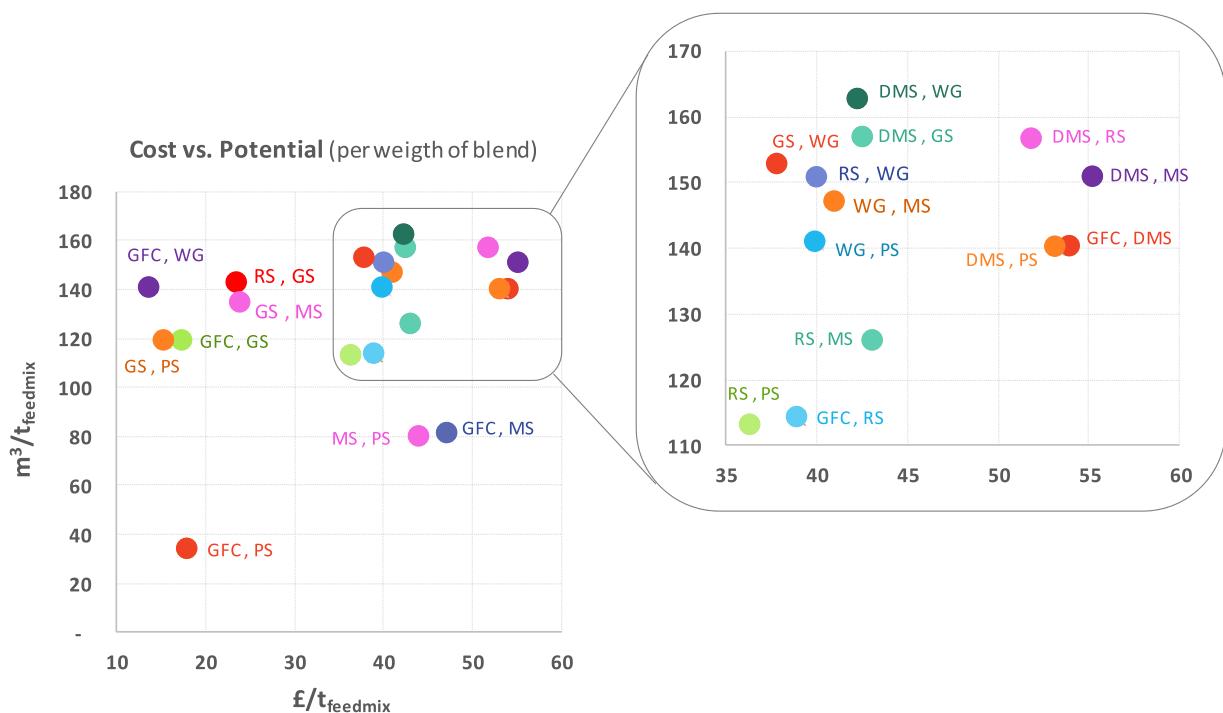


Fig. 3. Distribution of cost and biomethane potential of various feed blends.

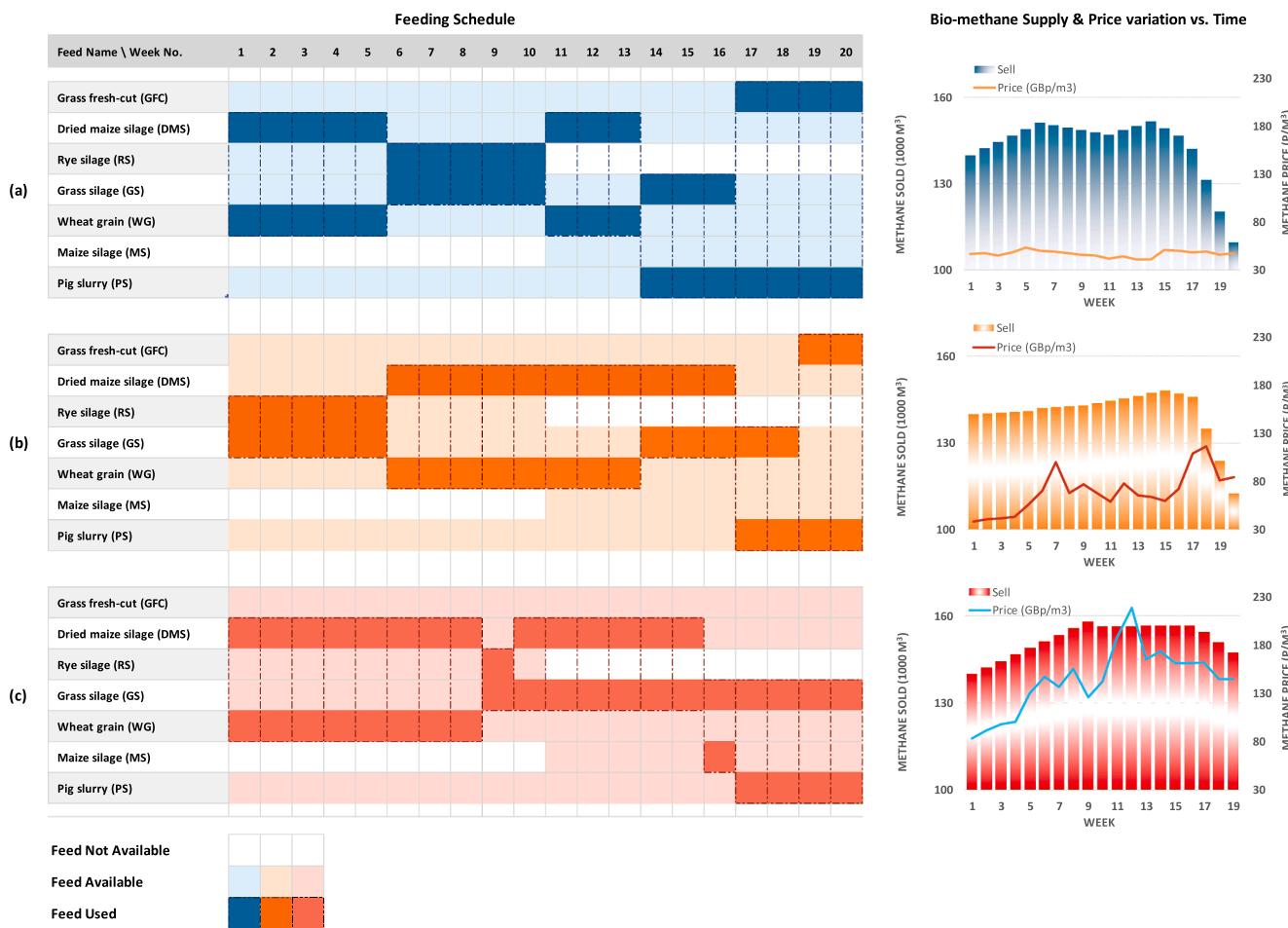


Fig. 4. The optimal feeding schedule and bio-methane supply chart based on three different pricing zones (a) Low-range (b) Middle-range (c) High-Range.

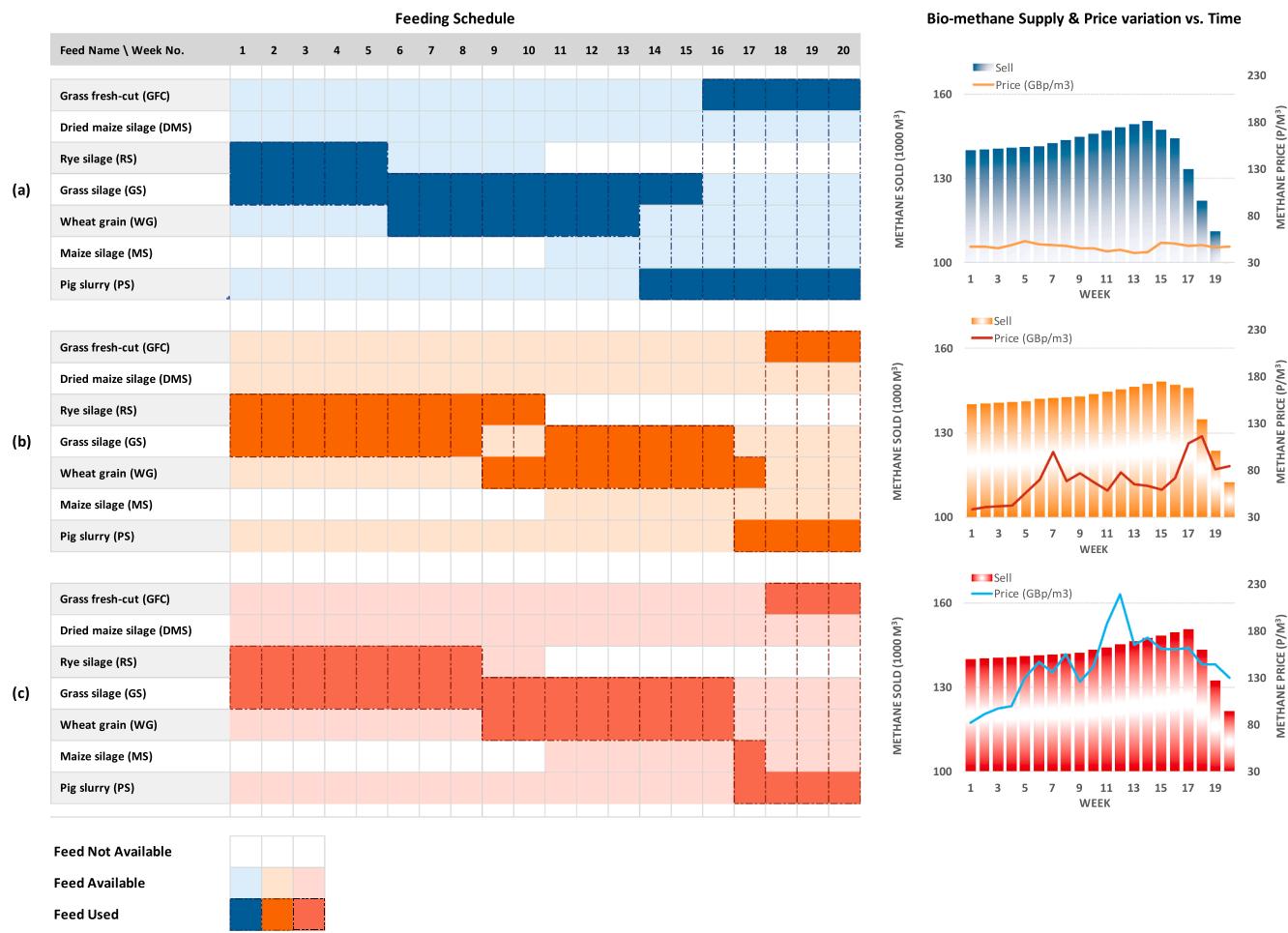


Fig. 5. The optimal feeding schedule and bio-methane supply chart with minimum GWP based on three different pricing zones (a) Low-range (b) Middle-range (c) High-Range.

aligning supply with favourable market conditions.

The inclusion of storage in the optimisation scenarios does not change the feeding schedules hugely, as these schedules were designed to follow the overall price trends. Instead, the storage capacities were effectively utilised during periods of price fluctuations, particularly when prices experienced a decline.

The detailed optimisation results for different objective scenarios are summarised in Table 4. Scenarios S1 to S3 focus on optimising the schedule to maximise bio-methane selling revenue, considering the feedstock prices alone across three different pricing schemes, as illustrated in Fig. 4. Scenarios S4 to S6 aim to minimise GWP in addition to the revenue-maximising objectives from the first three scenarios, with the corresponding feeding schedules and gas selling graphs shown in Fig. 5. Scenarios S7 to S9 examine the impact of gas storage on optimisation without GWP concerns, while scenarios S10 to S12 incorporate the effect of gas storage with GWP considerations.

According to Table 4, transitioning from the low-range (S1) to the medium (S2) and high-range (S3) gas price schemes resulted in revenue increases of approximately £660,000 and £2900,000, respectively, over a 20-week period. However, this revenue growth comes with an increase in feed costs of £35,000 and £131,000, and a rise in GWP of 85,000 kg_{CO2e} and 508,000 kg_{CO2e}, respectively. Additionally, incorporating GWP-reduction scenarios leads to a revenue decrease of 1.9 %, 2.9 %, and 4.7 % for low, middle, and high-range pricing, respectively, while significantly decreasing the GWP by approximately 90 %, 93 %, and 97 % for these cases.

According to the feeding schedule for non-GWP scenarios S1 to S3 (Fig. 4), the feed blends [RS,GS] and [DMS,WG] are frequently selected

for low and mid-range gas price schemes, being used for mild and sharp production increases, respectively. In the high-range price scenario, [DMS,WG] continues to serve this purpose, while [RS,GS] is mainly replaced by [DMS,GS]. For the GWP-included scenarios S4 to S6 (Fig. 5), [DMS,WG] and [DMS,GS] are replaced mainly by [GS,WG] and, to some extent, [RS,WG] to address the necessary changes in methane production.

Fig. 3 provides insight into these trends. Among all possible feedstocks, [DMS,GS] and [DMS,WG] blends exhibit the highest methane potential while maintaining a moderate price per unit weight. This is why they are predominantly used in non-GWP scenarios to increase production and target high price rises. [GS,WG] and [RS,WG] are the next two high-potential options, slightly cheaper in price, followed by [RS,GS], which has the highest yield but at a much lower cost. These blends are mainly used for targeting mild price peaks and in GWP minimisation scenarios, where DMS is avoided. Blends like [DMS,RS] and [DMS,MS] are considered more expensive substitutes for [DMS,GS] and [RS,WG], making them less attractive for GWP reduction or revenue increase.

It must be emphasised that the above-mentioned results are interpreted and discussed in the context of the particular case study of this research with all its assumptions, data and parameters provided. The quality of the optimisation outcomes and production patterns is significantly influenced by the physicochemical properties of the substrates, their availability, and the co-digestion results, especially in constructing a potential-price distribution graph as shown in Fig. 3. However, the approach presented here can be applied as a general methodology for other plants with different assumptions, feed data, and parameters.

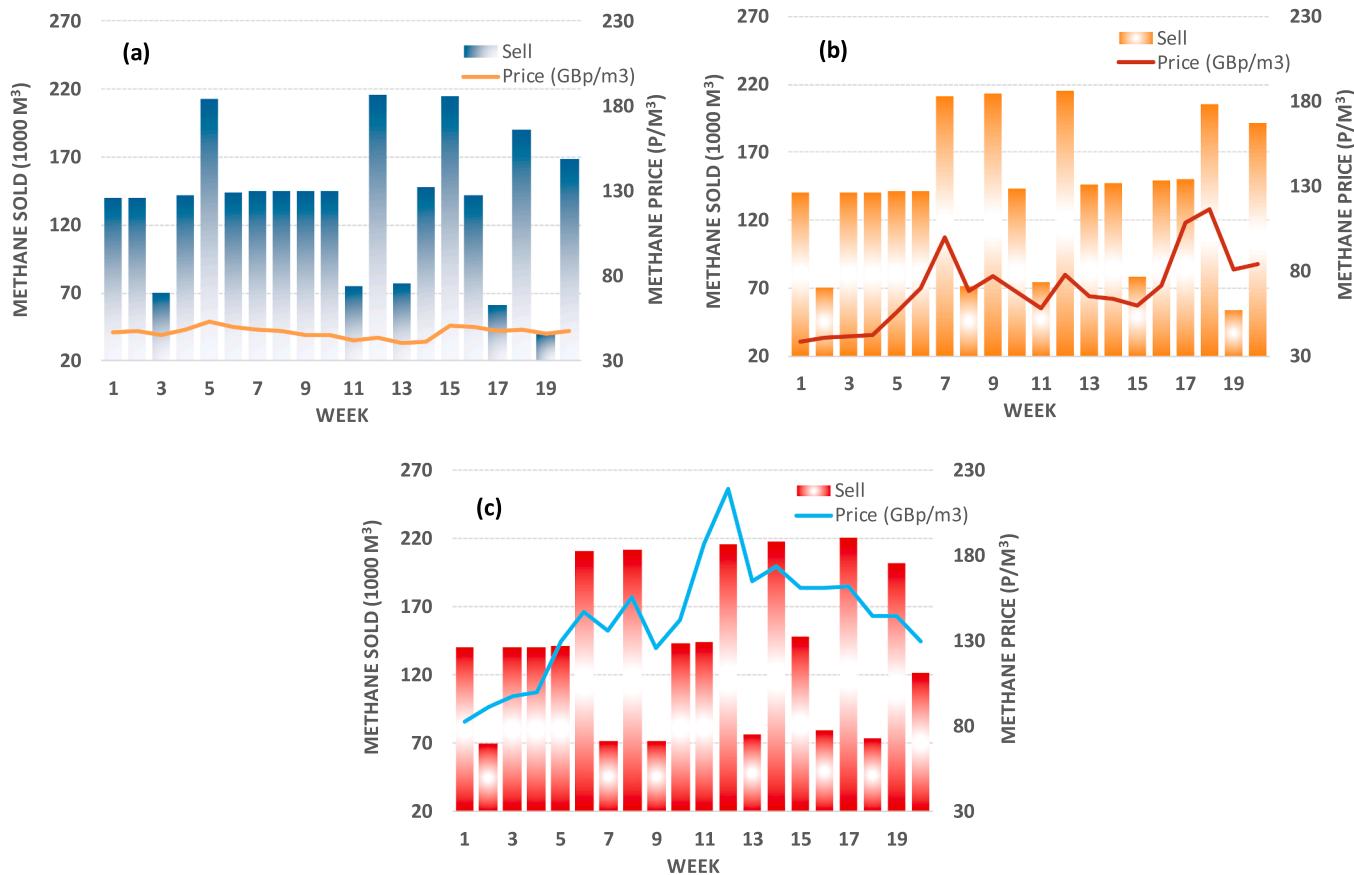


Fig. 6. The effect of gas storage (up to 10,000 m³/day) on the supply of bio-methane to the grid for (a) Low-range (b) Middle-range (c) High-Range pricing schemes.

Table 4
Optimisation results for different scenarios.

No.	Scenario	Biomethane Selling Revenue (£)	Feed Cost (£)	Net Revenue (£)	GWP (kg _{CO2e})	Revenue Increase (%)
S1	Scheduling - Low Gas Price	1344,120	574,018	178,106	770,102	–
S2	Scheduling - Mid Gas Price	2034,089	608,953	262,635	1425,136	–
S3	Scheduling - High Gas Price	4364,593	705,047	685,626	3659,546	–
S4	S1 + Min. GWP	1297,034	541,639	19,392	755,395	–1.9 %
S5	S2 + Min. GWP	1944,024	559,951	19,247	1384,073	–2.9 %
S6	S3 + Min. GWP	4078,133	589,806	19,286	3488,328	–4.7 %
S7	S1 + Gas Storage	1359,859	574,018	178,106	785,841	2.0 %
S8	S2 + Gas Storage	2188,365	661,489	516,461	1526,876	7.1 %
S9	S3 + Gas Storage	4488,430	705,047	685,626	3783,383	3.4 %
S10	S4 + Gas Storage	1331,727	560,877	19,239	770,850	2.0 %
S11	S5 + Gas Storage	2046,777	559,951	19,247	1486,826	7.4 %
S12	S6 + Gas Storage	4201,970	589,806	19,286	3612,164	3.6 %

The revenue increase due to storage, although modest in percentage terms, represents significant earnings for AD plants. If the same price changes occur over a year (instead of 20 weeks), earnings could rise by approximately £40k, £267k, and £322k (2 %, 7.4 % and 3.6 %) for low, mid, and high price change scenarios, respectively.

Utilising LNG facilities for storing the produced bio-methane in liquid form is an appealing option. Liquefaction of the huge volume of biogas produced per day and storage of bio-methane in relatively smaller volumes can be a viable storage option. In addition to the reduction of the storage volume, LNG cycles can be utilised in the purification step and leveraging the cryogenic cycle to liquify the CO₂ content of the biogas alongside with the liquefaction of the methane. This in turn can eliminate the considerable costs related to conventional purification methods such as membrane processes in addition to the potential to store the CO₂ (instead of venting it, which is done in most of the AD plants in the UK).

Small scale LNG units with a liquefaction capacity of less than 15 tonnes per day are regarded as suitable options for liquifying the daily production of such farm scale AD plants. Cost evaluation of such units in the literature (specific capital cost) include £350/tonnes per annum of LNG (TPA) (Pasini et al., 2019), £950/TPA (Capra et al., 2019; Lee et al., 2020) up to £1450/TPA (Gustafsson et al., 2020). For the current case study AD plant storage capacity (i.e., up to 10,000 m³_{CH4}/day), this can lead to annualised cost of £70k to £280k (on 20 years cost breakdown basis). However, due to the large carbon credit potential of CO₂ storage in such processes (credits ranging from \$35 to \$250/t_{CO2}, depending on the pricing scenario in future (bp Energy Outlook, 2020)) in addition to the aforementioned earnings from optimised scheduling, the cost of LNG units can be fully compensated and the AD plants can benefit from extra income.

While a detailed techno-economic assessment of biomethane storage as LNG is beyond the scope of this research, heuristic calculations, such

as those provided above, clearly demonstrate the viability of cryogenic storage, warranting further investigation.

5. Conclusion

This study introduced a streamlined two-step optimisation method aimed at enhancing feed and storage scheduling in anaerobic digestion processes. The method integrates a Co-Digestion optimisation model that replicates the synergistic effects of feed blends, coupled with a linear model that simulates the impacts of dynamic ramping between feeding recipes. This approach was tested under three distinct pricing schemes, each characterised by different levels of price variability and frequency of gas price fluctuations. The method was evaluated not only for maximising revenue, but also for minimising global warming potential.

The results indicate that, with reliable gas price prediction mechanisms, the proposed method can suggest optimal feeding blends and schedules, along with storage schedules. Although the schedules generated alone are not able to fully capitalise on adaptations to short-term price peaks due to high retention times, they effectively followed the main trends of price changes. The optimiser demonstrated a clear preference for high energy density feeds when substitutions were necessary. In regions with low gas prices, the substitution was limited to lower cost, lower energy density blends. However, in higher price zones, the optimiser prioritised energy density over cost. When the optimisation was further constrained to minimize GWP alongside increasing revenue, it restricted the optimiser to a limited number of feed candidates, leading to more misalignments with price fluctuations.

Maximum residual BMP level standards and biological limitation have been significant factors in preventing the reduction of SRT, thereby contributing to the low flexibility of AD plants. To mitigate the impact of low OLR (i.e., high SRT) on gas supply flexibility and maximise revenue by leveraging price fluctuations, the study found gas storage to be a promising solution. Implementing storage capacities of 10,000 m³ led to annual revenue increases of 2 %, 7.4 %, and 3.6 % for low, mid, and high price change scenarios, respectively. These increases correspond to significant annual earnings of £40k, £267k, and £322k. Additionally,

utilising LNG facilities for biogas storage presents an opportunity for extra revenue through CO₂ capture and storage, further enhancing the economic viability of AD operations.

CRediT authorship contribution statement

Meshkat Dolat: Writing – original draft, Software, Methodology, Investigation, Conceptualization. **Rohit Murali:** Writing – original draft, Software, Methodology. **Mohammadamin Zarei:** Writing – original draft, Software, Methodology, Conceptualization. **Ruosi Zhang:** Methodology, Data curation. **Tararag Pincam:** Writing – review & editing, Validation, Methodology. **Yong-Qiang Liu:** Writing – review & editing, Validation, Supervision, Project administration, Methodology, Funding acquisition. **Jhuma Sadhukhan:** Writing – review & editing, Supervision, Project administration, Funding acquisition. **Angela Bywater:** Writing – review & editing, Validation, Methodology, Conceptualization. **Michael Short:** Writing – review & editing, Writing – original draft, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Michael short reports financial support was provided by Engineering and Physical Sciences Research Council. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Nomenclature

Symbol	Definition	Unit
B	ultimate experimental biomethane yield (EBMP)	$\frac{m^3}{tonnes\ vs}$
B_{CoDF}	BMP for co-digestion of feed blend F (per tonnes of VS)	$\frac{m^3}{tonnes\ vs}$
B_{CoDTF}	BMP for co-digestion of feed blend F (per tonnes of blend)	$\frac{m^3}{tonnes\ vs}$
BD	biodegradability	–
$\left(\frac{C}{N}\right)$	carbon to nitrogen content ratio	–
$Cost_{total}$	total purchasing cost of feeds	£
$cost_i$	purchasing cost for substrate i	£
D	total number of steps (days/weeks) in the time horizon	days, weeks, etc.
$d_{end\ i}$	the time when supply of substrate i is ended	day, week, etc.
$d_{release\ i}$	the time when substrate i is supplied	day, week, etc.
E_{elect}	electrical energy consumption of AD plant	kWh
E_{heat}	heating energy consumption of AD plant	kWh
F	feed blend of (j, k) or (j, k, m)	–
GWP	global warming potential (Total)	kg_{CO2e}
GWP_{CF}	cultivation GWP	kg_{CO2e}
GWP_{CHP}	combined heat and power (CHP) GWP	kg_{CO2e}
GWP_E	electrical consumption GWP	kg_{CO2e}
GWP_L	leakage GWP	kg_{CO2e}
GWP_{TF}	transportation GWP for feed mix F	kg_{CO2e}
I	set of single feed substrates	–

(continued on next page)

(continued)

Symbol	Definition	Unit
L_i	distance of substrate i origin from the AD plant	km
L_d	distance of the digestate i (pertained to substrate i) utilisation location from the AD plant	km
P_d	biomethane production on time d	m^3
pr_d	price of biomethane on time d	£
r_{cd}^+	upward gas production ramp – Continuous mode	m^3
r_{cd}^-	downward gas production ramp – Continuous mode	m^3
r_{td}^+	upward gas production ramp – Transition mode	m^3
r_{td}^-	downward gas production ramp – Transition mode	m^3
S_d	amount of biomethane stored on time d	m^3
S_{max}	maximum gas storage capacity	m^3
$Sell_d$	amount of biomethane sold to the grid on time d	m^3
SRT	solid retention time	d
TB	theoretical biomethane yield	$\frac{m^3}{tonnes\ vs}$
TS_F	total solid content for feed mix F	$\frac{kg\ TS}{tonnes\ feed}$
TS_{max}	maximum total solid content of the feed	%
V_{react}	digester volume	m^3
VS	volatile solid	$\frac{kg\ vs}{tonnes\ TS}$
$W_{F,d}$	feeding rate of the reactor	$\frac{tonnes\ feed}{d}$
W_{TotalF}	total feed weight (for the whole time horizon D)	$tonnes_{feed}$
$W_{F,d}^{water}$	rate of water supply alongside with feed F	$\frac{tonnes_{water}}{d}$
$W_{av,i}$	available weight of substrate i	$tonnes_{feed}$
$x_{CoDF,i}$	fraction of substrate i in blend F in co-digestion	–
x_i	fraction of substrate i	–
y_{cd}^+	activation of upward gas production ramp – Continuous mode (Binary variable)	–
y_{cd}^-	activation of downward gas production ramp – Continuous mode (Binary variable)	–
y_{td}^+	activation of upward gas production ramp – Transition mode (Binary variable)	–
y_{td}^-	activation of downward gas production ramp – Transition mode (Binary variable)	–
$y_{F,d}$	selection of blend F for feeding in time d	–
\hat{y}_i	cultivated / non-cultivated substrate (user-defined parameter)	–
α_{ci}	cultivation GWP parameter	$\frac{kg_{CO_2e}}{kg_{TS}}$
α_L	leakage GWP parameter	$\frac{kg_{CO_2e}}{m^3}$
α_{Ti}	transportation GWP parameter for substrate i	$\frac{kg_{CO_2e}}{tonnes\ km}$
α'_{Ti}	transportation GWP parameter for substrate i digestate	$\frac{kg_{CO_2e}}{tonnes\ km}$
α_E	GWP parameter for externally supplied electricity	$\frac{kg_{CO_2e}}{kWh}$
α_H	GWP parameter for externally supplied heat	$\frac{kg_{CO_2e}}{kWh}$
α_E'	GWP parameter for electricity supplied by CHP	$\frac{kg_{CO_2e}}{kWh}$
α_H'	GWP parameter for heat supplied by CHP	$\frac{kg_{CO_2e}}{kWh}$
β	regression parameters for co-digestion model	$\frac{m^3}{tonnes\ vs}$
γ	penalty factor	–
θ_d	gas production potential for time d	m^3

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