

# A Simplified way to manufacture high-Q microfiber coil resonators by controlling the input/output coupling

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**Abstract:** The dependence of the Q-factor of the microfiber coil resonator on different input/output couplings is investigated and compared. A method of slowly varying input/output coupling is presented to considerably simplify the fabrication of high-Q resonators.

The microfiber coil resonator (MCR) is a promising functional device for compact all-fiber circuits. In particular, with the most recent developments in the manufacturing technology of ultra-low loss microfibers [1-3], MCRs could potentially compete with the highest Q-factors currently achieved only in whispering gallery resonators [4]. An MCR can be created by wrapping an optical microfiber on a low-index dielectric rod and its geometry cannot be achieved by planar technology. However this type of resonator presents a grave issue: the extremely high sensitivity of the Q-factor to the coupling strength, which in turn has an exponential relation with the pitch. If ultra-high Q-factors can be achieved over a large range of coupling coefficients, the manufacture of high-Q MCRs is considerably simplified. In this paper we present a simple method to achieve high-Q MCRs over a wide range of coupling coefficients by tuning the coupling between the coil and the input and output pigtails.

Our analysis of the MCR is based on the solution of coupled wave equations. Consider the propagation of light along an MCR, as illustrated in Fig.1. Along the fiber, we define the amplitude of the field at the  $m^{\text{th}}$  turn as  $A_m(\theta)$ ,  $0 < \theta < 2\pi$ . Ignoring coupling between turns that are not adjacent to each other, the propagation of light along the coil in a  $M$ -turn MCR is described by the coupled wave equations for slowly varying pitches [5,6]:

$$\frac{d}{d\theta} \begin{pmatrix} A_1 \\ A_2 \\ \dots \\ A_{M-1} \\ A_M \end{pmatrix} = \frac{i}{2\pi} \begin{pmatrix} 0 & K(\theta) & 0 & \dots & 0 \\ K(\theta) & 0 & K(\theta+2\pi) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & K(\theta+(M-3)2\pi) & 0 \\ 0 & 0 & 0 & \dots & K(\theta+(M-2)2\pi) \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ \dots \\ A_{M-1} \\ A_M \end{pmatrix} \quad (1)$$

where  $K(\varphi)$  is the coupling coefficient [5] between neighboring turns along the coil and  $\varphi = \theta + (m-1) \cdot 2\pi$ .  $K$  assumes values between 0 (no coupling) and  $K_M$  (maximum coupling for touching fibers). Field continuity between the turns implies that  $A_{m+1}(0) = A_m(2\pi) \exp(i2R\pi\beta)$ ,  $m=1, 2, \dots, M-1$ , where  $\beta$  is the propagation constant and  $R$  is the coil radius. The amplitude transmission coefficient is defined as  $T = A_M(2\pi)/A_1(0) \exp(i2R\pi\beta)$  and the Q-factor as  $Q = \lambda_0/\Delta\lambda$ , where  $\Delta\lambda$  is the full width at half maximum (FWHM) of resonances in  $T$ .

Fig. 2 shows the dependence of the FWHM on constant  $K(\varphi) = K$  near  $\lambda_0 \approx 1.55 \mu\text{m}$  for a MCR with  $M=4$ . The microfiber is assumed to have 800nm radius, 0.02dB/mm loss and  $R=125/2 \mu\text{m}$ . The FWHM fluctuates widely. Therefore, high Q-factors cannot be obtained simply by targeting the highest  $K$ , i.e. by making two adjacent turns touching. In principle, the highest Q-factor can be achieved by selecting a  $K$  for which the FWHM is minimized, but in practice this is difficult to realize because  $K$  is very sensitive to the distance between adjacent turns. For the ease of fabrication it is therefore desirable that the FWHM is at its minimum and changes slowly with  $K$ . To quantify this behavior a new parameter, the tolerance ratio (TR), is introduced and defined as the

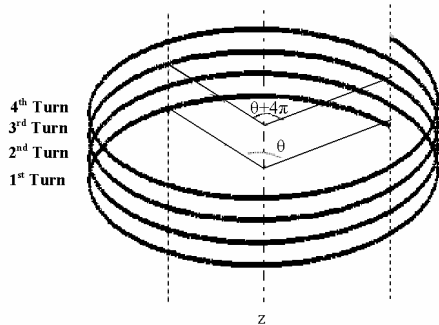


Fig. 1. Illustration of a MCR in cylindrical coordinates

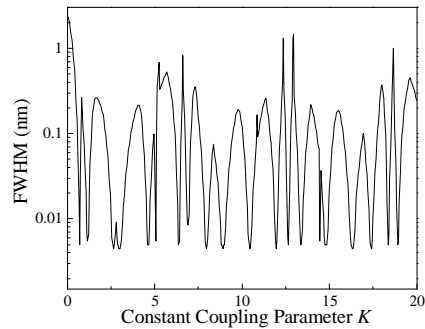


Fig. 2. The FWHM dependency on the constant coupling parameter  $K$  near  $\lambda_0 = 1550 \text{ nm}$  in a uniform 4-turn MCR

fraction of  $K$  values where the FWHM is close to the minimum within a given interval. High TR values imply easier fabrication.

Here we consider the case of constant coupling in the center and variable only at the ends:  $K(\phi) = K_c$  for  $1 < \phi/(2\pi) < M-1$  and  $K(0) = K(2\pi M) = 0$ . Fig 3 shows five profiles of  $K$  for  $M=4$  and the dependence of their FWHM on the central coupling  $K_c$ .  $K$  profiles (a) and (e) are sharp and their TR in proximity of  $K_M$  is poor; the FWHM in  $K$  profile (c) is more flattened than others and the tolerance ratio near  $K_M$  is almost 100%. From Fig. 3 it is easy to assess that high tolerance ratios and flat FWHM profiles can be obtained when the coupling is slowly varying to zero at the input and output sides.

In order to better understand the dependence of TR on the profile smoothness in a 4-turn MCR, we compare linear profiles (similar to the one in Fig. 3c) with different slopes, as shown in Fig. 4(a). Here the coupling parameter is zero for  $0 < \phi < 2\pi x$  and increases linearly thereafter;  $x$  ranges from 0 (minimum slope) to 1 (maximum slope – top hat profile).  $x=1$  corresponds to Fig. 3c, while for  $x=0$  the profile is very similar to that studied in Fig. 3a. Fig. 4(b) summarizes the dependence of TR on  $x$  in the range  $K=0-20$ . TR decreases quickly with  $x$ , meaning that lower coupling slopes are better. This prediction has been confirmed with simulations at  $M=3, 5, 6$  and 7. Although the realization of an MCR with a perfect (c) profile is challenging, a practical and easy method to make an MCR implies wrapping the coils as close as possible and then micro-tune the input and output pigtails to make the coupling turn to zero as slowly as possible.

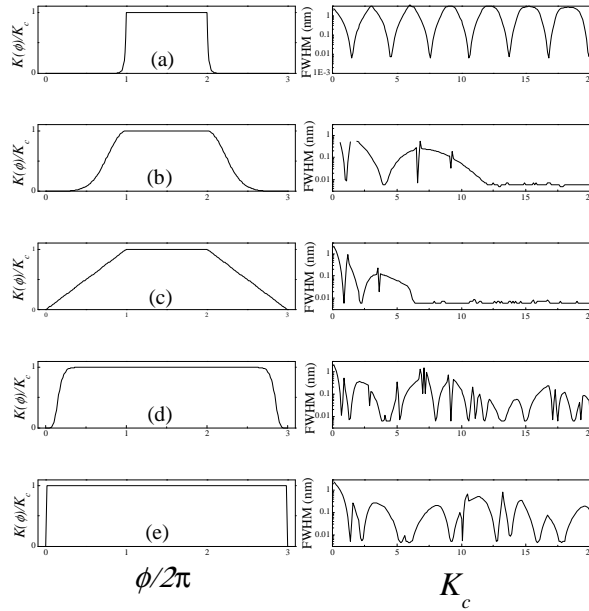


Fig. 3. Various  $K$  profiles and corresponding FWHM dependency on the central coupling parameter  $K_c$  near  $\lambda_0=1550$  nm in 4-turn ONMRs.

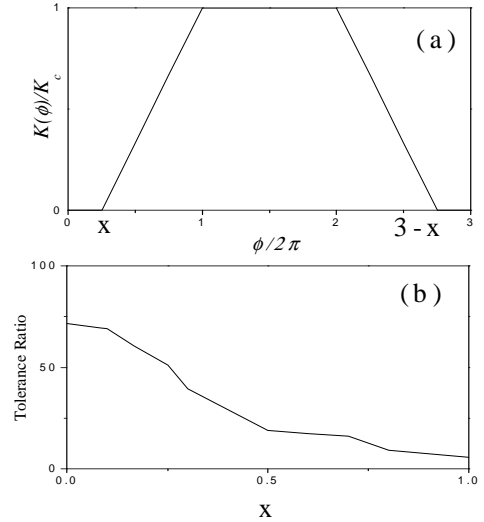


Fig. 4 (a)  $K(\phi)$  with linearly increasing coupling from  $\phi = 2\pi x$ , (b) Tolerance ratio versus  $x$  for  $K=0-20$ .

In order to explain these results, it is useful to remember that a resonance in the transmission spectrum of a MCR occurs when there is a mode with light circulating in the inner rings of the coil while the intensities at the input and at the output end vanish. This requires two conditions to be fulfilled: 1) the circumference of the coil must be a multiple of the wavelength and, 2) the coupling coefficient must be such that upon one roundtrip the light is entirely coupled back to the previous ring. Moreover, the conditions must be fulfilled simultaneously at both sides of the coil. It is believed that slowly varying profiles like those of Figs. 3b and 3c provide a range of different coupling conditions for which condition 2) is more easily fulfilled.

## References

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