# A novel test of economic convergence in time series

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**Abstract**

This paper proposes a novel test for the hypothesis of economic convergence. We extend the standard definition of convergence based on the parity condition and say that two economies converge if the time series of economic output are positively cointegrated and cotrended. With this definition in place, our main contribution is to propose a test of positive cointegration that does not require estimation of the cointegrating relationship, but is able to differentiate between positive and negative cointegration. Once the possibility of positive cointegration is established in a first stage, we test for cotrending in a second stage. Our sequential proposal enjoys an excellent performance in small samples due to the fast convergence of our novel test statistic under positive cointegration. This is illustrated in a simulation exercise where we report clear evidence showing the outperformance of our proposed method compared to existing methods in the related literature that test for economic convergence using cointegration methods. The results are particularly strong for sample sizes between 25 and 50 observations. The empirical application testing for economic convergence between the G7 group of countries over the period 1990-2022 confirms these findings.

*Keywords*: asymptotic theory, hypothesis testing, economic convergence, unit root tests, cointegration.

*JEL codes*: C22; C32; F43; 047

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# 1 Introduction

One of the main areas of interest in economic growth theory in the last decades is the study of the differences in economic output across regions and over time. This topic, that has been broadly labeled as economic convergence, has several interpretations and definitions, see Durlauf et al. (2009). Thus, $\beta-$convergence, see Baumol (1986), DeLong (1988), Barro (1991), and Mankiw et al. (1992), refers to the concept of catching up. This notion is tested using cross-sectional regression analyses and tries to disentangle whether initial levels of income per capita are inversely related to subsequent growth. A second notion is $\sigma-$convergence, that refers to the decrease in the dispersion of income per capita across economies over time. Regression-based studies on this type of convergence are proposed in Friedman (1992) and Cannon and Duck (2000), among many others. This literature connects with a statistical approach to convergence based on time series analysis. Bernard and Durlauf (1995, 1996) were the first to define cross-country output convergence in terms of the limit of expected output gaps. These authors propose two definitions of convergence which directly focus on the transience or permanence of contemporary output differences. Based on these definitions, these authors propose testing for cross-country convergence using cointegration techniques and focused, in particular, on testing the stationarity of the output gap, see also Pesaran (2007). In this setting, the absence of economic convergence is due to the presence of (deterministic or stochastic) trends in the output gap.

A suitable regression approach that captures both scenarios is the Augmented Dickey-Fuller (ADF) test for trend stationarity. The hypothesis of convergence is tested through the composite hypothesis of stationarity of the output gap and the hypothesis of cotrending of the deterministic components. Pesaran (2007) tests these hypotheses sequentially with the unit root hypothesis being tested first and, once rejected, the second stage consists on testing the statistical significance of the time trend. An interesting alternative proposed in Silva-Lopes (2016) is to use the ADF test without linear time trend. This idea takes advantage of the results in Perron (1988) and Campbell and Perron (1991) showing that the ADF test with only an intercept in the set of deterministic regressors has power that goes to zero (as the sample size $T$ increases) in case the output gap is a trend stationary process. Thus, under trend stationarity, the ADF test does not (wrongly) reject the null of unit root. Then, rejection with this ADF test is informative about economic convergence, which makes unnecessary the second step of Pesaran (2007). Note that, however, this approach does not allow us to identify the reason behind the lack of economic convergence (presence of a unit root or trend stationarity of the output gap).

These tests of economic convergence are, however, invalid if the definition of convergence is extended to accommodate the existence of common trends and not only the parity condition in the relationship of log per capita output across economies. In these cases, the cointegration relationship between the output variables needs to be estimated and the critical values of the ADF tests need to be corrected for the presence of estimation effects and/or deterministic trends, see Phillips and Ouliaris (1990) and Hansen (1992). In this context, a suitable method to test for economic convergence is developed in Hansen (1992). This author extends the work in Phillips and Ouliaris (1990) by deriving the asymptotic critical values of two classes of cointegration tests under the presence of deterministic components in the unit root processes and cointegration equation. The unrestricted version of Hansen's (1992) approach estimates simultaneously the coefficients associated to the deterministic and stochastic trends. Cointegration is tested through the stationarity of the residuals and cotrending through the statistical significance of the trend coefficient using a $t$-test based on a fully-modified ordinary least squares (FM-OLS) estimator of the corresponding trend parameter. Note, however, that tests based on the hypothesis of cointegration are necessary, but not sufficient, to test for the presence of economic convergence, which also requires that the time series of economic output commove by means of a positive cointegration parameter (or coefficients of the cointegrating vector with different signs). It could be argued the graphical inspection is informative about the type of cointegration (positive or negative), but the presence of common deterministic characteristics (for example upward trends) might lead to an erroneous conclusion. In this sense, a formal procedure able to differentiate both cases appears to be relevant.

We propose an alternative methodology to test the hypothesis of economic convergence which takes into account the positive nature of the possible cointegrating relationship. Also, our test for positive cointegration is not based on OLS residuals, therefore avoiding the normalization issue, that is the choice of dependent variable in the regression from which residuals are obtained. This user-chosen normalization is critical in residual-based cointegration testing.

Our approach is sequential as in Pesaran (2007); we first focus on testing for the presence of a positive cointegration relationship between the time series of economic output under the possible presence of idiosyncratic deterministic components. If there is positive cointegration, in a second step, we test for the presence of a common deterministic trend between observables. The main idea behind the first step of our method is to exploit the distinct behavior of the empirical correlation coefficient between two unit root processes under positive, negative or trivial cointegration, or under absence of cointegration (see, e.g., Johansen, 2012). In the first scenario, under general conditions, the sample correlation tends to 1 (in probability) as $T \to \infty$;

in the second, it tends to $-1$; in the third, it tends to 0; finally, under no cointegration, the sample correlation converges in distribution to a particular random variable.

Interestingly, our proposed test statistic, which is a simple linear function of a sample correlation, does not require estimation of the cointegration relationship. Thus, an important advantage of the proposed procedure compared to residual-based cointegration tests is that the test statistic is invariant to the presence of serial correlation in the innovations, although, in general, its limiting distribution is not pivotal, so critical values need to be approximated either by a plug-in method or by bootstrap.

The performance of the proposed sequential method is assessed in finite samples in a Monte-Carlo simulation study for a battery of unit root processes with innovations exhibiting mutual dependence and serial correlation and it is compared to other methods proposed in the literature. The empirical size and power of our proposal are very satisfactory, specially in sample samples of 25 to 50 observations. In particular, the comparison with the unrestricted cointegration test developed in Hansen (1992) shows clear advantages under general forms of dependence, but the performance of our method is also comparable and even superior in some instances to Pesaran's (2007) test, which considers the cointegration coefficient to be known.

The small-sample differences in statistical power between the different cointegration methods are also illustrated in an empirical application to GDP data from the G7 group of countries over the period 1990-2022. The tests are applied to all pairwise combinations of the seven countries plus an index containing the average output of OECD countries. The results empirically confirm the lack of power of standard tests for small sample sizes, so the empirical evidence strongly suggests absence of convergence between the GDP levels of major industrialized economies during the last thirty years. These results contrast with the results of our procedure, which are more favourable to the existence of economic convergence between countries with similar economic systems.

The rest of the paper is organized as follows. Section 2 introduces our definition of economic convergence based on the concept of common trends that extends conventional definitions based on the parity condition. The section also reviews existing econometric methodologies to test for economic convergence. Section 3 proposes a novel methodology to test for economic convergence based on a test of positive cointegration that does not require estimating the cointegration relationship. Section 4 presents an exhaustive Monte Carlo simulation exercise that studies its finite-sample performance and is also compared against existing approaches to test for economic convergence in the time series literature. In Section 5, we analyze the possibility of economic convergence for the group of G7 countries using various approaches. Section 6 concludes. The proofs of the main results of the paper are in a mathematical appendix. Tables and figures are collected at the end of the manuscript.

## 2 Convergence in output

This section adapts the definition of economic convergence based on the existence of common trends in Bernard and Durlauf (1995, 1996). Our definition extends the conventional notion of convergence based on the stationarity of the output gap. The section also reviews popular approaches to test for economic convergence between pairs of economies.

### 2.1 Definition of convergence

Bernard and Durlauf (1996) and Pesaran (2007) derive an empirical representation of the dynamics of log per capita output from a stochastic neoclassical growth model. These authors consider the problem of output convergence in a sample of $n$ economies, and suppose that the logarithm of per capita output of unit i at time $t$, $x_{it}$, satisfies the following dynamics

$$x_{it} = c_i + \pi_i t + \lambda_i' f_t + u_{it}, \tag{1}$$

with prime denoting transposition, where $\Delta f_t = v_t$ is a $m \times 1$ vector of common components, $\lambda_i$ is the associated vector of factor loadings and $u_{it}$ is the idiosyncratic component, assumed to be specific to unit $i$. For cross-country output convergence it is necessary that the idiosyncratic components $u_{it}$ are stationary. Pesaran (2007) assumes that $f_t$ and $u_{it}$ are independently distributed with zero means. Hobijn and Franses (2000) propose a similar specification for the dynamics of log per capita output but consider a pair of innovations $(u_{it}, v_t)$ that are covariance stationary and allow for the presence of mutual and serial correlation between the innovations.

To illustrate the concept of convergence in time series, we focus on pairs of observables, $x_{1t}$ and $x_{2t}$. Clearly, if $c_1 = c_2$, $\pi_1 = \pi_2$ and $\lambda_1 = \lambda_2$, we obtain output convergence between $x_{1t}$ and $x_{2t}$ as defined in Bernard and Durlauf (1995, 1996) and Hobijn and Franses (2000). This relationship implies that the two economies are identical almost in every respect, including their saving rates and initial endowments. In this case, $x_{2t} - x_{1t} = \varepsilon_t$, with $\varepsilon_t = u_{2t} - u_{1t}$ a zero-mean stationary sequence. Pesaran (2007) relaxes this definition of convergence and introduces the idea of probabilistic convergence. The processes $x_{1t}$ and $x_{2t}$ converge if for some positive $\delta$ and a tolerance probability $\nu \geq 0$,

$$\lim_{T \to \infty} P\{\, |x_{2,t+s} - x_{1,t+s}| < \delta |\, \Im_t\} > \nu, \tag{2}$$

at all horizons $s = 1, 2, ..., \infty$, where $\Im_t$ is the information set at time $t$. We extend these concepts of convergence as follows.

DEFINITION 1. *The unit root processes* $x_{1t}$ *and* $x_{2t}$ *in* (1) *converge if there exists a positive coefficient* $\beta$ *such that* $x_{2t} - \beta x_{1t}$ *is stationary in levels.*

Thus, there is economic convergence if the dynamics of output have common (stochastic and deterministic) trends and commove ($\beta > 0$).

## 2.2 Convergence tests

The empirical literature of testing for the presence of convergence in output has focused on testing the stationarity of the output gap $d_t = x_{2t} - x_{1t}$, see Bernard and Durlauf (1995, 1996) and Pesaran (2007), that consider different versions of the ADF test. In this setting, the absence of economic convergence is due to the presence of a deterministic or stochastic trend in the output gap. A suitable ADF test regression that captures both scenarios is

$$\Delta d_t = c_0 + \pi_0 t + \rho_d d_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta d_{t-j} + w_t^{(1)}, \tag{3}$$

where $\Delta = 1 - L$, $L$ being the lag operator, with $c_0$ and $\pi_0$ the parameters associated to the deterministic components, $\rho_d$ the autoregressive coefficient, $p$ the number of stationary lags required to remove the effect of serial dependence and $\gamma_j$ the coefficients associated to the stationary components. The hypothesis of convergence is tested through the composite hypothesis of stationarity of the output gap ($\rho_d < 0$) and cotrending of the deterministic components ($\pi_0 = 0$). Pesaran (2007) tests these hypotheses sequentially with the unit root hypothesis being tested first and, once rejected, the second stage consists on testing the statistical significance of the time trend. This test uses the standard Dickey-Fuller critical values obtained under the presence of a linear time trend.

An interesting alternative proposed in Silva-Lopes (2016) is to use the ADF test without linear time trend:

$$\Delta d_t = c_0 + \rho_d d_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta d_{t-i} + w_t^{(2)}. \tag{4}$$

This author takes advantage of the results in Perron (1988) and Campbell and Perron (1991) to exploit the fact that the ADF test with only an intercept in the set of deterministic regressors has power that goes to zero as the sample size increases in case the output gap is a trend stationary process. Thus, under trend stationarity, the ADF test does not (wrongly) reject the null of unit root, which makes unnecessary the second step of Pesaran (2007).

These specifications can be extended to test for economic convergence under the conditions of Definition 1. This implies estimating the cointegration relationship between the nonstationary time series $x_{1t}$ and $x_{2t}$. In this case, the relevant testing procedure is in the same spirit of the original two-stage Engle-Granger (EG) test, see Engle and Granger (1987). However, in contrast to this seminal study, see also Phillips and Ouliaris (1990) that derive the asymptotic critical values of the EG test when the parameters are estimated, the processes $x_{1t}$ and $x_{2t}$ can contain a time trend. In this context a suitable method to test for economic convergence is to apply the unrestricted test of cointegration developed in Hansen (1992), based on estimating by OLS the equation

$$x_{2t} = c_0 + \pi_0 t + \beta x_{1t} + w_t^{(3)}, \tag{5}$$

and checking stationarity of the OLS residuals by means of ADF or Phillips' $Z(\alpha)$, $Z(t)$ tests. Then, if the unit root hypothesis is rejected, cotrending can be assessed by exploring the statistical significance of the FM-OLS estimator of $\pi_0$.

## 3 A new test of economic convergence

We propose a novel sequential strategy for the hypothesis of economic convergence. First, we introduce a new test for the existence of a positive cointegration relationship between $x_{1t}$ and $x_{2t}$ under the presence of deterministic trends. This is the main contribution of the present paper. In a second stage, we test for cotrending between the corresponding time trends.

We generalize the setting of Pesaran (2007) in expression (1) and consider the following data generating process to analyze economic convergence. Let $v_t = (v_{1t}, v_{2t})'$ be a vector of innovations that is defined as a zero mean and $\alpha-$mixing random sequence with mixing coefficients $\alpha_m$ such that $\sum_{m=1}^{\infty} \alpha_m^{(\xi-2)/4\xi} < \infty$, $\sup_t E|v_t' v_t|^{\xi} < \infty$ for some $\xi > 2$, and $\Omega = \lim\limits_{T\to\infty} \frac{1}{T} E[S_T S_T'] > 0$ for $S_t = \sum_{j=1}^{t} v_j$, see Phillips (1986). Then, the observables are generated as

$$x_{it} = c_i + \pi_i t + S_{it}, \text{ for } i = 1, 2, \tag{6}$$

where, either

$$\Delta S_{it} = v_{it}, \text{ for } i = 1, 2, \tag{7}$$

or, for some $\beta \in \mathbb{R}$,

$$\Delta S_{1t} = v_{1t}, \; S_{2t} = \beta S_{1t} + v_{2t}. \tag{8}$$

Clearly, under (7), $x_{1t}$ and $x_{2t}$ are not cointegrated because $\Omega$ is nonsingular, whereas, under (8), $x_{1t}$ and $x_{2t}$ are cointegrated. Conditions $\beta > 0$ and $\beta < 0$ entail positive and negative cointegration, respectively, while if $\beta = 0$ $x_{1t}$ and $x_{2t}$ are trivially cointegrated (after detrending $x_{2t}$ is stationary).

To test for economic convergence we also require cotrending between the deterministic trends. The hypothesis of cotrending is given by $\pi_2 = \beta\pi_1$ in (6), thus, the composite hypothesis of economic convergence (positive cointegration and cotrending) implies that $x_{2t} = c_0 + \beta x_{1t} + v_{2t}$, with $\beta > 0$ and $c_0 = c_2 - \beta c_1$. Note that the case of positive cointegration with a time trend is considered under the hypothesis of no economic convergence and, in this case, the dynamics of $x_{2t}$ are driven by

$$x_{2t} = c_0 + \pi_0 t + \beta x_{1t} + v_{2t}, \tag{9}$$

with $\pi_0 = \pi_2 - \beta \pi_1 \neq 0$.

The above models can be extended to include higher order time trends, see Hansen (1992), as part of the definition of cotrending. However, for consistency with Pesaran (2007), we restrict to the convergence setup discussed by this author.

### 3.1 Testing for positive cointegration

The main advantage of our proposed approach is that the test does not require estimation of the cointegration coefficient $\beta$ and it differentiates between convergence given by $\beta > 0$, and cointegration without convergence given by $\beta \leq 0$. As mentioned before, graphical analysis might be informative about positive or negative cointegration, but the presence of deterministic components may obscure the assessment about the type of cointegration (positive or negative). Let $d_t(\beta) = x_{2t} - \beta x_{1t}$ define a modified output gap with $x_{1t}$ and $x_{2t}$. In the first stage the hypothesis of interest can be written as

$$\begin{aligned} H_0 &: d_t(\beta) \sim UR, \text{ for all } \beta \in \mathbb{R}, \text{ or } d_t(\beta) \sim TS, \text{ for some } \beta \in (-\infty, 0], \\ H_A &: d_t(\beta) \sim TS, \text{ for some } \beta \in \mathbb{R}^+, \end{aligned}$$

with $UR$ denoting the unit root character of the modified output gap and $TS$ denoting trend stationarity (understood in a wide sense, so it covers the case where there is no trend in the modified output gap). The null hypothesis covers two states of nature: Either the observables are not cointegrated, or they are, but cointegration is not positive[1].

In the same spirit of Hansen (1992), Bernard and Durlauf (1995, 1996) and Pesaran (2007), we propose a test of positive cointegration that accommodates time trends as part of the specification of the unit root processes and the cointegration relationship. Let $\widetilde{x}_{it}$ be the detrended process of $x_{it}$ defined as $\widetilde{x}_{it} = x_{it} - \overline{x}_i - \widehat{\pi}_i \left(t - 2^{-1}(T+1)\right)$, for $i = 1, 2$, with $\overline{x}_i$ the sample mean and $\widehat{\pi}_i$ the OLS estimator of $\pi_i$ obtained from the regression of $x_{it}$ on $(1, t)$. Similarly,

[1] $H_0$ is equal to the null hypothesis found in the literature on time series convergence, see Bernard and Durlauf (1995, 1996) and Pesaran (2007) as seminal examples, and usually tested using the ADF test. An alternative is to consider the null hypothesis as convergence between the time series. In this case, the difference between time series is stationary under the null hypothesis. This hypothesis is tested using the KPSS test and multivariate versions of it, as shown in Hobijn and Franses (2000) and Pesaran (2007).

let $\widehat{\sigma}^2_{\widetilde{x}_i} = T^{-1}\sum_{t=1}^T \widetilde{x}^2_{it}$ be the sample variance such that the standardized unit root process is $y_{it} = \widehat{\sigma}^{-1}_{\widetilde{x}_i}\widetilde{x}_{it}$. The proposed test statistic for the hypothesis of positive cointegration is

$$\widehat{D}_T = \frac{1}{T}\sum_{t=1}^{T}(y_{it} - y_{jt})^2 = 2\left(1 - \widehat{\rho}_{ij}\right), \tag{10}$$

with $\widehat{\rho}_{ij} = T^{-1}\sum_{t=1}^T y_{it}y_{jt}$ and $i, j = 1, 2$ used interchangeably, noting that $\widehat{\rho}_{ij}$ is the sample correlation between $\widetilde{x}_{it}$ and $\widetilde{x}_{jt}$.

Proposition 1. Let $x_{it}$, for $i = 1, 2$, be two cointegrated unit root processes defined in expressions (6), (8), with $\beta > 0$. Then, $\widehat{D}_T = O_P(T^{-1})$ as $T \to \infty$.

Nicely, the convergence of the statistic to zero is at a rate $T$, so, as shown in the simulation section below, this feature leads to important power improvements compared to existing cointegration methods allowing for time trends. Also, by construction, the test (10) is robust to the presence of deterministic components. We next study the asymptotic behaviour of $\widehat{D}_T$ under $H_0$.

Proposition 2. Let $x_{it}$, for $i = 1, 2$, be two unit root processes as in Proposition 1, but with $\beta < 0$. Then, $\widehat{D}_T = 4 + O_P(T^{-1})$ as $T \to \infty$.

Proposition 3. Let $x_{it}$, for $i = 1, 2$, be two unit root processes as in Proposition 1, but with $\beta = 0$. Then, $\widehat{D}_T = 2 + O_P(T^{-1/2})$ as $T \to \infty$.

In contrast, if the time series $x_{1t}$ and $x_{2t}$ are non-cointegrated, the test statistic $\widehat{D}_T$ has the following limiting distribution.

Proposition 4. Let $x_{it}$, for $i = 1, 2$, be two unit root processes defined in expressions (6), (7). Then,

$$\widehat{D}_T \xrightarrow{d} 2\left(1 + B_\pi A_{\pi_1} A_{\pi_2} - B_\pi Z_{12}\right), \tag{11}$$

with

$$Z_{12} = \frac{\int_0^1 W_1(r)W_2(r)dr - \int_0^1 W_1(r)dr \int_0^1 W_2(\tau)d\tau}{\left[\int_0^1 W_1(r)^2 dr - \left(\int_0^1 W_1(r)dr\right)^2\right]^{1/2}\left[\int_0^1 W_2(r)^2 dr - \left(\int_0^1 W_2(r)dr\right)^2\right]^{1/2}},$$

$A_{\pi_i} = \sqrt{12}\left[\int_0^1 W_i(r)^2 dr - \left(\int_0^1 W_i(r)dr\right)^2\right]^{-1/2} \int_0^1 \left(r - \frac{1}{2}\right) W_i(r)dr$ for $i = 1, 2$, $B_\pi = (1 - A_{\pi_1}^2)^{-1/2}(1 - A_{\pi_2}^2)^{-1/2}$, and $W_1(r)$ and $W_2(r)$ two Brownian motions such that $Var\left(W_i(r)\right) = r$, $i = 1, 2$, with correlation coefficient $\lambda_{12}$ characterized by the long run correlation between the innovation sequences $v_{1t}$ and $v_{2t}$.

The above result reveals three important insights. First, the limit of the test statistic under the null hypothesis either collapses (to 4 or 2) or depends on the long run correlation between the innovation sequences $v_{1t}$ and $v_{2t}$. Second, if the innovations are mutually independent then the asymptotic distribution in (11) is parameter free and critical values can be tabulated. Finally, the test is robust to the presence of serial correlation in the innovations $v_{it}$ for $i = 1, 2$.

In general, the innovations are mutually correlated invalidating the use of universal critical values. In this setting, plug-in methods and bootstrap techniques can be applied to approximate the distribution of the test under the null hypothesis. The following algorithm shows how to approximate the distribution of the test using bootstrap methods and obtain a valid $p$-value.

**Algorithm**

1. Construct $\widehat{v}_t = \Delta x_t - \overline{\Delta x}$, where $\overline{\Delta x}$ is the sample mean of $\Delta x_t$.

2. Estimate the long run covariance matrix $\Omega$ using the Newey-West estimator, so

$$\widehat{\Omega} = \frac{1}{T}\sum_{t=1}^{T} \widehat{v}_t \widehat{v}_t' + \frac{1}{T}\sum_{j=1}^{M-1} k_j \sum_{t=j+1}^{T} \left(\widehat{v}_t \widehat{v}_{t-j}' + \widehat{v}_{t-j}\widehat{v}_t'\right), \tag{12}$$

   where $M$ denotes the relevant number of lags contributing to the long run variances and covariances and $k_j = 1 - M^{-1}j$ is the Barlett kernel.

3. Generate the error sequence $v_t^{(b)}$ drawn from the distribution $N(0, \widehat{\Omega})$ for $t = 1, ..., T + L$, where $L$ is an integer and for $b = 1, \ldots, B$. Then, setting $z_0^{(b)} = 0$, generate $z_t^{(b)} = z_{t-1}^{(b)} + v_t^{(b)}$ for $t = 1, ..., T + L$ and define the bootstrap observable sequence as $x_t^{(b)} = z_{t-L}^{(b)}$ for $t = 1, ..., T$. Note that the first $L$ observations are discarded to mitigate the effect of the initialization $z_0^{(b)} = 0$.

4. Compute the bootstrap version of the standardized unit root processes $y_{it}^{(b)}$, for $i = 1, 2$ and $t = 1, \ldots, T$. Let $\widetilde{x}_{it}^{(b)}$ be the detrended process of $x_{it}^{(b)}$, defined as $\widetilde{x}_{it}^{(b)} = x_{it}^{(b)} - \overline{x}_{i}^{(b)} - \widehat{\pi}_{i}^{(b)} \left(t - 2^{-1}(T+1)\right)$, for $i = 1, 2$, with $\overline{x}_{i}^{(b)}$ the sample mean of the bootstrap sample $x_{it}^{(b)}$, and $\widehat{\pi}_{i}^{(b)}$ the OLS estimator of $\pi_i$ obtained from the regression of $x_{it}^{(b)}$ on $(1, t)$. Similarly, let $\widehat{\sigma}_{\widetilde{x}_i}^{(b)2} = T^{-1} \sum_{t=1}^{T} \left(\widetilde{x}_{it}^{(b)}\right)^2$ be the sample variance such that the standardized unit root process is $y_{it}^{(b)} = \left(\widehat{\sigma}_{\widetilde{x}_i}^{(b)}\right)^{-1} \widetilde{x}_{it}^{(b)}$.

5. Compute the bootstrap test statistic $\widehat{D}_T^{(b)} = 2\left(1 - \widehat{\rho}_{12}^{(b)}\right)$, with $\widehat{\rho}_{12}^{(b)} = T^{-1} \sum_{t=1}^{T} y_{1t}^{(b)} y_{2t}^{(b)}$ the correlation between the standardized bootstrap samples.

6. Repeat the above steps for $b = 1, \ldots, B$.

The $p$-value obtained from the bootstrap distribution of the test $\widehat{D}_T$, conditional on the realization $(x_{1t}, x_{2t})'$, $t = 1, ..., T$, can be approximated by

$$\widehat{p}_B = \frac{1}{B} \sum_{b=1}^{B} 1\left(\widehat{D}_T^{(b)} \leq \widehat{D}_T\right).$$

The null hypothesis $H_0$ is rejected against $H_A$ if $\widehat{p}_B < \alpha$, with $\alpha$ the significance level of the test.

It is important to note that under $H_1$ the statistic $\widehat{D}_T$ converges in probability to 0, but also the limiting distribution from which critical values are obtained collapses to 0, so the power properties of the test are uncertain. However, we conjecture that the bootstrap critical values converge to zero at a slower rate than $\widehat{D}_T$ under $H_1$. In fact, we believe that the rate of convergence to zero of those critical values is at most the nonparametric convergence rate of $\widehat{\Omega}$ in (12) to $\Omega$. This is slower than $O_P\left(T^{-1}\right)$, hence the test is consistent. Our Monte Carlo results support this conjecture.

### 3.2 Testing for cotrending

Once the hypothesis of no positive cointegration is rejected, the second step of the sequential method to test for economic convergence is to asses for the existence of cotrending. In view of (9), cotrending is equivalent to $\pi_0 = 0$, that is precisely the hypothesis that we test in the second stage by means of the usual Wald test. We adopt Hansen's (1992) approach and base our test statistic ($t$-ratio) on fully-modified OLS estimation of (9) (see Theorem 6 of Hansen, 1992).

# 4 Finite-sample properties of convergence tests

This section carries out a Monte Carlo simulation exercise that studies the finite-sample properties of the tests of economic convergence discussed above for different data generating processes (DGPs). In all cases, we test the null hypothesis of absence of economic convergence against the alternative of economic convergence. The observables $x_{1t}$ and $x_{2t}$ are constructed as in (6), (7), (8) with $v_t = (v_{1t}, v_{2t})'$ being generated as

$$v_t = \Phi v_{t-1} + \varepsilon_t, \tag{13}$$

where $\Phi$ is a diagonal matrix with parameters $\phi_i$, $i = 1, 2$, in the main diagonal and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is an independent and identically normally distributed bivariate vector with $E(\varepsilon_t) = 0$, $Var(\varepsilon_{1t}) = Var(\varepsilon_{2t}) = 1$, and $Cov(\varepsilon_{1t}, \varepsilon_{2t}) = \rho$. Note that, in the cointegrated cases,

$$x_{2t} = c_0 + \pi_0 t + \beta x_{1t} + v_{2t}, \tag{14}$$

with $c_0 = c_2 - \beta c_1$, $\pi_0 = \pi_2 - \beta \pi_1$.

To explore the performance of the above tests of economic convergence under different DGPs, we simulate several processes that are indexed by the persistence parameters $\{\phi_1, \phi_2\}$ and mutual correlation coefficient $\rho$. These parameters are calibrated using the empirical values obtained from the analysis of the time series of log output for the G7 group of industrialized countries, see Table 3. To simplify the simulation section, we consider only two possible values of the persistence parameter $\phi_1 = \{0.5, 0.7\}$ and the cross-correlation coefficient $\rho = \{0.5, 0.8\}$. The persistence of the cointegration error is $\phi_2 = 0.6$. Figures 1-4 present empirical power curves for different combinations of these parameters. The choice of sample size is intended to illustrate the performance of these methods in small samples and closely corresponds to the sample sizes used in the empirical application in Section 5. A more comprehensive simulation exercise including $T = 100$ is available from the authors upon request. Empirical size and power are computed as rejection probabilities at 5% significance level obtained from $M = 1000$ simulations.

The hypothesis of no economic convergence is represented in the simulation exercise as two cointegrated unit roots with cointegrating parameter $\beta = 1$ that do not cotrend, except for $\pi_0 = 0$ (left panels) or as two cotrending processes (so $\pi_0 = 0$) that may cointegrate (if $\beta < 0$) or noncointegrate, which will be labelled as $\beta = 0$ (right panels)[2]. The hypothesis of economic convergence is characterized by positive cointegration and cotrending ($\beta > 0$ and $\pi_0 = 0$) and

[2]The case of two non-cointegrated unit roots that do not cotrend is less interesting because is farther from the alternative hypothesis of economic convergence than the two composite hypotheses that are studied here.

is implemented as the intersection of both hypotheses. DGPs under the null and alternative hypotheses are generated as follows. The left panels cover 12 DGPs; $x_{1t}$ is generated from (6)-(7), with $c_1 = 0.432$ and $\pi_1 = 0.417$ (two random numbers drawn from a Uniform $U[0,1]$ random variable) and $x_{2t}$ is generated from (14) with $\beta = 1$, $c_0 = 0$ and $\pi_0 = (0.5j - 3)\pi_1$ for $j = 1, \ldots, 12$. All processes are generated under the null hypothesis of no convergence except the process characterized by $\pi_0 = 0$ that lies under the alternative hypothesis. The right panels cover 9 DGPs indexed by $j = 1, \ldots, 9$. The process $x_{1t}$ is generated from (6)-(7) with $c_1 = 0.432$ and $\pi_1 = 0.417$, as before, and $x_{2t}$ is generated from (14) with $\beta \equiv \beta(j)$ that takes values between $-1$ and 2; $c_2 = c_2(j) = \beta(j)c_1$ and $\pi_2 = \pi_2(j) = \beta(j)\pi_1$ such that $c_0 = \pi_0 = 0$. For $\beta = 0$, both $x_{1t}$ and $x_{2t}$ are generated from (6)-(7) with $c_2 = c_1$ and $\pi_2 = \pi_1$. The processes with $\beta \leq 0$ are generated under the null hypothesis of no convergence and the processes characterized by $\beta > 0$ lie under the alternative hypothesis. The dependence between the innovations of the processes $x_{1t}$ and $x_{2t}$ is driven by expression (13).

We consider four tests of economic convergence in Figures 1-4: (i) an extension of Pesaran (2007)'s test assuming that $\beta$ is known. In this method we estimate the deterministic trend and test the hypothesis of stationarity using the trend stationary version of the ADF regression equation. The empirical rejection rates are plotted with a dotted line with $+$; (ii) the unrestricted approach of Hansen (1992) where stationarity is checked by ADF on OLS residuals obtained from (9) and no cotrending is taken as statistical significance of the FM-OLS estimator of $\pi_0$ in the same equation. Results for this method are plotted with a dashed line with $\Diamond$; (iii) corresponds to the version of Silva-Lopes' (2016) approach (denoted SL hereinafter) that estimates the cointegration relationship. This is the only one-step approach. Its corresponding results are plotted with a solid line with $\star$; (iv) corresponds to our novel test of economic convergence. This test combines our proposed approach for positive cointegration based on the statistic $\widehat{D}_T$ and the test for the time trend obtained from the unrestricted regression equation (5). Therefore, the second stage of this composite test is the same of method (ii) and obtained from a $t$-test using FM-OLS estimators of the regression coefficients. This method is plotted with a thick solid line with $\circ$.

We proceed to discuss the results of the simulation exercise. Left panels of Figures 1-4 report the empirical size of the different tests for $\pi_0 \neq 0$ and the empirical power of the test for $\pi_0 = 0$. The power curves show that most tests are undersized, the only exception is method (iii), given by the SL approach, that reports an empirical size close to the nominal value of 5%. The power of the tests is low for small sample sizes ($T = 35$) with the only exceptions of the SL approach and our proposed procedure based on the test $\widehat{D}_T$. Unreported results show that most methods exhibit strong power to reject the null hypothesis for $T = 100$. The performance of the tests is also sensitive to the relative degree of persistence in the innovations. Power values for processes characterized by $\phi_1 < \phi_2$ (Figures 1-2) are below those obtained for processes with $\phi_1 > \phi_2$ (Figures 3-4). In fact, a closer inspection to the power properties of the different tests reveals that the only methods with power to detect economic convergence for $\phi_1 = 0.5$ and $\phi_2 = 0.6$ are the SL method (iii) and our proposed approach. This result holds across different values of the cross-correlation coefficient $\rho$.

Right panels of Figures 1-4 report empirical size for $\beta \leq 0$ and power for $\beta > 0$ for the different testing methods. As expected, for $\beta < 0$, all methods but (iv) are heavily oversized since they are not designed to differentiate between positive and negative cointegration. For $\beta > 0$, the performance of the tests varies depending on the persistence of the innovations (as discussed above) increasing for $\phi_1 > \phi_2$ with respect to $\phi_1 < \phi_2$, as noted by a referee. Interestingly, the best performing methods are our proposed approach and the SL method. Empirical power increases with the sample size across testing methods.

One major implication of this simulation exercise is the poor small-sample performance of methods (i) to (ii) under general forms of persistence and correlation between the innovations. The poor performance of these methods is mainly attributed to the low power of the cointegration tests carried out in the first stage. This finding is very relevant for two reasons. First, many empirical applications of economic convergence are based on annual data and, hence, are characterized by small sample sizes. Second, these tests are the main tools used in the empirical time series literature to test for economic convergence using cointegration methods.

## 5 Empirical application

The differences in statistical power across testing methods shown in the simulation section suggest that standard methods to test for economic convergence using cointegration methods may not yield reliable results in small samples. This is illustrated in this section with data on gross domestic product (GDP) from the G7 group of countries over the period 1990-2022 (sample size is 33 observations). This period starts with the reunification of Germany, thus, the group of countries are Canada, France, Germany, Italy, Japan, Great Britain, and USA. Data are collected annually from the OECD website (https://data.oecd.org/gdp/gross-domestic-product-gdp.htm). We consider data on log output in millions of US dollars (top panel of Figure 5) and per capita terms (bottom panel of Figure 5). Both panels show evidence of a positive linear trend and comovements in economic output across all G7 countries.

Table 1 reports the FM-OLS estimates of the cointegration parameter $\beta$ obtained from the unrestricted regression (9) for both types of log output series. This regression is a necessary step for the testing procedures (ii) and (iv) above but not for (i) and (iii). In fact, for the first method given by Pesaran's (2007) convergence test, we assume $\beta = 1$ and test for the parity condition between output series. The top panel presents the results for log GDP in millions of dollars. The parameter estimates are around one for those pairs involving the OECD but not for the remaining pairs. Interestingly, the cointegration coefficient is negative for all pairs of countries that include Germany. This empirical evidence highlights the importance of the deterministic trend in driving the comovements observed in Figure 5 for this region. For other countries such as France, Italy and Japan the cointegration coefficient is far from the parity condition. In contrast, Great Britain reports a coefficient near the parity condition when compared against the USA, Canada and the OECD but not against the remaining countries, including France, Germany and Italy. These results may still provide empirical evidence of economic convergence in our framework, characterized by the presence of positive cointegration even if the cointegration coefficient is different from one. The results for per-capita output are qualitatively similar.

Table 2 presents FM-OLS parameter estimates of the slope coefficient $\pi_0$ obtained from the regression equation (9). The magnitude of the coefficients for $\pi_0$ is very small across pairs of countries except for Germany. As discussed above, the deterministic trend seems to be the driving force of the positive trend in output observed for this country. For the remaining pairs, the magnitude of the linear time trend coefficient is, in general, below 0.02 in absolute value. Nevertheless, once we account for the standard error of the parameter estimates we find overwhelming statistical evidence against the hypothesis of cotrending except in a few cases. This result holds for both the log GDP in millions of dollars and the per-capita counterpart. These results are formalized in Table 3 that reports the rejection outcomes of the different tests of economic convergence discussed above.

The top panel of Table 3 reports the estimates of the persistence parameter of an autoregressive process of order one fitted to the innovations $v_{1t}$ and $v_{2t}$ in (6)-(7) and (9), respectively. There is, indeed, serial correlation in the innovations to the unit root processes (column 1) driving the dynamics of the log GDP in millions dollars. Persistence is low for Italy (0.125) and France (0.190) and quite high for USA (0.627). The remaining cells report the AR(1) parameter estimate characterizing the persistence of the cointegration errors between the different pairs of countries. These errors are rather persistent with values ranging between 0.5 and 0.85 for cointegration relationships between Japan and GBR, USA and OECD. The bottom panel of Table 3 reports the cross-correlations between the pairs $(\varepsilon_i, \varepsilon_j)$ after filtering out the serial dependence in the innovation sequences. The results suggest that there is, indeed, strong dependence between the whitened innovations of nominal log GDP between most industrialized economies. Similar results are obtained for log GDP in per capita terms and omitted for space considerations.

Table 4 presents the results of the four composite tests of convergence introduced above. The table is divided into two panels with the top panel for the analysis of log GDP measured in millions of dollars and the bottom panel for the analysis of log GDP per capita. The cells in Table 4 are given by arrays with four binary entries 0/1 that correspond to the nonrejection/rejection outcomes, respectively, of the tests of economic convergence. Nonrejection of the null hypothesis, reported as a zero in the cell entry, is interpreted as evidence of no convergence. The first entry in the array corresponds to Pesaran's (2007) test of the output gap; the second entry corresponds to Hansen's (1992) test of cointegration and cotrending obtained from the unrestricted residuals. The third entry of the cells uses the ADF test for level stationarity and corresponds to the SL method. The fourth entry corresponds to our proposed procedure given by the test $\widehat{D}_T$ for positive cointegration and a Wald test for the presence of a time trend in the cointegration regression equation (9). All of the tests are computed at a 5% significance level.

The outcomes of the tests provide overwhelming evidence against the presence of pairwise cross-country economic convergence. The results vary depending on whether we consider nominal levels of output or per-capita levels. Furthermore, in line with the findings obtained from the simulation exercise, the only tests reporting empirical evidence of economic convergence are SL and our approach. The first test finds economic convergence between France and Great Britain and Germany and Great Britain. The second test finds empirical evidence of economic convergence between Canada and France, Canada and Great Britain, and Canada and USA. The results for log output in per-capita terms uncover economic convergence between France and Great Britain, and France and the OECD index using the SL test. Our procedure also finds evidence of convergence in per-capita output for the pairs Great Britain, USA and Canada, Japan.

## 6 Conclusion

This paper extends the standard definition of economic convergence given by the parity condition. This is done by allowing a flexible characterization of the convergence hypothesis based on the concept of common trends in output introduced by Bernard and Durlauf (1995, 1996). According to this novel definition, two time series of economic output converge over time if the pairs are positively cointegrated and the corresponding deterministic components cotrend.

With this definition in place, the paper has proposed a composite test of economic convergence given by sequentially testing for these two hypotheses. In the first stage, positive cointegration is tested using a novel statistic exploiting the differences in the asymptotic properties of the sample correlation between cointegrated and non-cointegrated processes. In the second stage, we apply a Wald test for the hypothesis of cotrending in a cointegration regression equation with trend.

The main novelty of our proposed approach resides in the first stage. The proposed test possesses some appealing features such as invariance to the choice of dependent variable, convergence at a rate $T$, and no need of estimation of the cointegration coefficient. The test accommodates very general forms of mutual and serial dependence in the sequence of innovations and exhibits an excellent performance in small samples. The critical values of the test of positive cointegration are obtained either by a plug-in approach or by bootstrap.

A Monte Carlo simulation exercise comparing the finite-sample performance of existent methods to test for economic convergence based on the presence of cointegration confirms the outperformance of our proposed procedure in small samples and the overall poor performance of well behaved methods such as Pesaran (2007) and Hansen (1992). This is illustrated with data on gross domestic product from the G7 group of countries over the period 1990-2022. Existing methods provide overwhelming evidence against the existence of economic convergence among any of the countries in the G7 group. In contrast, the application of our proposed procedure is slightly more favorable to the hypothesis of convergence for countries such as France, Great Britain, Canada, and USA.

# 7 Statements and Declarations

The authors declare no financial or non-financial interests directly or indirectly related to the work in this manuscript.

## Mathematical appendix

**Proof of Proposition 1**. For an arbitrary process $a_t$ let $\overline{a} = T^{-1}\sum_{t=1}^{T} a_t$. Simple algebra shows that

$$x_{it} - \overline{x}_i = \pi_i\left(t - \frac{T+1}{2}\right) + S_{it} - \overline{S}_i, \text{ for } i = 1, 2.$$

Furthermore, let $\widetilde{x}_{it}$ denote the detrended time series such that

$$\widetilde{x}_{it} = x_{it} - \overline{x}_i - \widehat{\pi}_i\left(t - \frac{T+1}{2}\right) = -\left(\widehat{\pi}_i - \pi_i\right)\left(t - \frac{T+1}{2}\right) + S_{it} - \overline{S}_i, \qquad (15)$$

with $\widehat{\pi}_i$ denoting the OLS estimators of $\pi_i$ of the centred variables $x_{it} - \overline{x}_i$ on $t - 2^{-1}(T+1)$. Then,

$$\widetilde{x}_{2t} - \beta\widetilde{x}_{1t} = -(\widehat{\pi}_0 - \pi_0)\left(t - \frac{T+1}{2}\right) + v_{2t} - \overline{v}_2, \qquad (16)$$

with $\pi_0 = \pi_2 - \beta\pi_1$, $\widehat{\pi}_0 = \widehat{\pi}_2 - \beta\widehat{\pi}_1$. By construction of the OLS estimators $\widehat{\pi}_i$, we obtain

$$\widehat{\pi}_i - \pi_i = \frac{\sum_{t=1}^{T}(S_{it} - \overline{S}_i)\left(t - 2^{-1}(T+1)\right)}{\sum_{t=1}^{T}\left(t - 2^{-1}(T+1)\right)^2}, \qquad (17)$$

so, given that,

$$\frac{1}{T^3}\sum_{t=1}^{T}\left(t - \frac{T+1}{2}\right)^2 \to \frac{1}{12}, \text{ as } T \to \infty, \qquad (18)$$

and

$$\sum_{t=1}^{T}(S_{it} - \overline{S}_i)\left(t - 2^{-1}(T+1)\right) = O_p\left(T^{5/2}\right),$$

it immediately follows that $\widehat{\pi}_i - \pi_i = O_p\left(T^{-1/2}\right)$. Also,

$$\begin{aligned}\widehat{\pi}_0 - \pi_0 &= \frac{\sum_{t=1}^{T}[(S_{2t} - \overline{S}_2) - \beta(S_{1t} - \overline{S}_1)]\left(t - 2^{-1}(T+1)\right)}{\sum_{t=1}^{T}\left(t - 2^{-1}(T+1)\right)^2} \\ &= \frac{\sum_{t=1}^{T}(v_{2t} - \overline{v}_2)\left(t - 2^{-1}(T+1)\right)}{\sum_{t=1}^{T}\left(t - 2^{-1}(T+1)\right)^2} = O_p(T^{-3/2}),\end{aligned}$$

because

$$\sum_{t=1}^{T}(v_{2t} - \overline{v}_2)\left(t - 2^{-1}(T+1)\right) = O_p\left(T^{3/2}\right).$$

Noting (10), we examine the behaviour of

$$\widehat{\rho}_{12} = \frac{\sum_{t=1}^{T}\widetilde{x}_{1t}\widetilde{x}_{2t}}{\left(\sum_{t=1}^{T}\widetilde{x}_{1t}^2\sum_{t=1}^{T}\widetilde{x}_{2t}^2\right)^{1/2}} = \frac{\beta\sum_{t=1}^{T}\widetilde{x}_{1t}^2 + \sum_{t=1}^{T}\widetilde{x}_{1t}\left(v_{2t} - \overline{v}_2\right)}{\left(\sum_{t=1}^{T}\widetilde{x}_{1t}^2\sum_{t=1}^{T}\widetilde{x}_{2t}^2\right)^{1/2}} \qquad (19)$$

by (16). Also, using again (16), it can be easily shown that the denominator of (19) is

$$\left(\beta^2\left(\sum_{t=1}^{T}\widetilde{x}_{1t}^2\right)^2 + d_T\right)^{\frac{1}{2}},$$

where $d_T$ is a remainder term such that $d_T = O_p\left(T^3\right)$. Then

$$\begin{aligned}
\widehat{\rho}_{12} &= \frac{\beta T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}^2 + T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}\left(v_{2t}-\overline{v}_2\right)}{\left(\beta^2\left(T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}^2\right)^2 + T^{-4}d_T\right)^{\frac{1}{2}}} \\
&= \left(\frac{\beta T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}^2}{\left(\beta^2\left(T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}^2\right)^2\right)^{\frac{1}{2}}} + \frac{T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}\left(v_{2t}-\overline{v}_2\right)}{\left(\beta^2\left(T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}^2\right)^2\right)^{\frac{1}{2}}}\right) \qquad (20) \\
&\quad \times \frac{\left(\beta^2\left(T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}^2\right)^2\right)^{\frac{1}{2}}}{\left(\beta^2\left(T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}^2\right)^2 + T^{-4}d_T\right)^{\frac{1}{2}}}. \qquad (21)
\end{aligned}$$

Results in the proof of Proposition 4 imply that $T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}^2$ tends in distribution to a positive (almost surely) random variable (see (23) and (24) below). Then, (21) tends in probability to 1, because $T^{-4}d_T = o_p(1)$. Also, because $\beta > 0$, (20) equals

$$1 + \frac{T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}\left(v_{2t}-\overline{v}_2\right)}{\left(\beta^2\left(T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}^2\right)^2\right)^{\frac{1}{2}}} = 1 + O_p\left(T^{-1}\right),$$

because

$$\frac{1}{T^2}\sum_{t=1}^{T}\widetilde{x}_{1t}\left(v_{2t}-\overline{v}_2\right) = O_p\left(T^{-1}\right).$$

Then,

$$\widehat{\rho}_{12} = 1 - O_p\left(T^{-1}\right),$$

so, by (10), the proposition follows.

**Proof of Proposition 2**. The proof is almost identical to that of Proposition 1, with the only difference that $\beta < 0$. This implies that

$$\widehat{\rho}_{12} = -1 + O_p\left(T^{-1}\right),$$

which completes the proof.

**Proof of Proposition 3**. In this case

$$\widetilde{x}_{2t} = -(\widehat{\pi}_0 - \pi_0)\left(t - \frac{T+1}{2}\right) + v_{2t} - \overline{v}_2.$$

Then, by previous results it is simple to show that $\sum_{t=1}^{T}\widetilde{x}_{1t}\widetilde{x}_{2t} = O_p(T)$, whereas $T^{-2}\sum_{t=1}^{T}\widetilde{x}_{1t}^2$ tends in distribution to a positive (almost surely) random variable and $T^{-1}\sum_{t=1}^{T}\widetilde{x}_{2t}^2$ tends in probability to a positive constant (the variance of $v_{2t}$). These results imply that $\widehat{\rho}_{12} = O_p\left(T^{-1/2}\right)$, to conclude the proof.

**Proof of Proposition 4**. In view of (10), the limiting distribution of the test statistic $\widehat{D}_T$ is determined by that of $\widehat{\rho}_{12} = T^{-1}\sum_{t=1}^{T} y_{1t}y_{2t}$. First, letting $S_{it}^0 = S_{it} - \overline{S}_i$, $i = 1, 2$, by (15) and (17),

$$\frac{1}{T}\sum_{t=1}^{T}\widetilde{x}_{1t}\widetilde{x}_{2t} = \frac{1}{T}\sum_{t=1}^{T} S_{1t}^0 S_{2t}^0 - (\widehat{\pi}_1 - \pi_1)\frac{1}{T}\sum_{t=1}^{T} S_{2t}^0\left(t - \frac{T+1}{2}\right) - (\widehat{\pi}_2 - \pi_2)\frac{1}{T}\sum_{t=1}^{T} S_{1t}^0\left(t - \frac{T+1}{2}\right)$$
$$+ (\widehat{\pi}_1 - \pi_1)(\widehat{\pi}_2 - \pi_2)\frac{1}{T}\sum_{t=1}^{T}\left(t - \frac{T+1}{2}\right)^2$$
$$= \frac{1}{T}\sum_{t=1}^{T} S_{1t}^0 S_{2t}^0 - (\widehat{\pi}_1 - \pi_1)(\widehat{\pi}_2 - \pi_2)\frac{1}{T}\sum_{t=1}^{T}\left(t - \frac{T+1}{2}\right)^2. \tag{22}$$

Similarly, we study the sample variance terms, thus, for $i = 1, 2$,

$$\widehat{\sigma}_{\widetilde{x}_i}^2 = \frac{1}{T}\sum_{t=1}^{T}\widetilde{x}_{it}^2 = \frac{1}{T}\sum_{t=1}^{T}(S_{it}^0)^2 - 2(\widehat{\pi}_i - \pi_i)\frac{1}{T}\sum_{t=1}^{T} S_{it}^0\left(t - \frac{T+1}{2}\right) + (\widehat{\pi}_i - \pi_i)^2\frac{1}{T}\sum_{t=1}^{T}\left(t - \frac{T+1}{2}\right)^2$$
$$= \widehat{\sigma}_{S_i^0}^2 - (\widehat{\pi}_i - \pi_i)^2\frac{1}{T}\sum_{t=1}^{T}\left(t - \frac{T+1}{2}\right)^2,$$

where $\widehat{\sigma}_{S_i^0}^2 = T^{-1}\sum_{t=1}^{T}\left(S_{it}^0\right)^2$. Simple algebra shows that $\widehat{\sigma}_{\widetilde{x}_i} = \widehat{\sigma}_{S_i^0}\left(1 - \widehat{A}_{\pi_i}^2\right)^{1/2}$, with

$$\widehat{A}_{\pi_i} = \frac{(\widehat{\pi}_i - \pi_i)}{\widehat{\sigma}_{S_i^0}}\left(\frac{1}{T}\sum_{t=1}^{T}\left(t - \frac{T+1}{2}\right)^2\right)^{\frac{1}{2}},$$

so $\widehat{\sigma}_{\widetilde{x}_1}\widehat{\sigma}_{\widetilde{x}_2} = \widehat{\sigma}_{S_1^0}\widehat{\sigma}_{S_2^0}\left(1 - \widehat{A}_{\pi_1}^2\right)^{1/2}\left(1 - \widehat{A}_{\pi_2}^2\right)^{1/2}$. Then, by (22),

$$\frac{1}{T}\sum_{t=1}^{T} y_{1t}y_{2t} = (1 - \widehat{A}_{\pi_1}^2)^{-\frac{1}{2}}(1 - \widehat{A}_{\pi_2}^2)^{-\frac{1}{2}}\left(\frac{T^{-1}\sum_{t=1}^{T} S_{1t}^0 S_{2t}^0}{\widehat{\sigma}_{S_1^0}\widehat{\sigma}_{S_2^0}} - \widehat{A}_{\pi_1}\widehat{A}_{\pi_2}\right).$$

Using the functional central limit theorem and the continuous mapping theorem, see Phillips (1986),

$$\frac{1}{T^{1/2}}\widehat{\sigma}_{S_i}^0 \xrightarrow{d} \lambda_i\left(\int_0^1 W_i(r)^2 dr - \left(\int_0^1 W_i(r)dr\right)^2\right)^{1/2}, \tag{23}$$

with $\lambda_i$ the long-run standard deviation of the innovation $v_{it}$. Also,

$$\frac{1}{T^2}\sum_{t=1}^{T} S_{1t}^0 S_{2t}^0 \xrightarrow{d} \lambda_1\lambda_2\left(\int_0^1 W_1(r)W_2(r)dr - \int_0^1 W_1(r)dr\int_0^1 W_2(\tau)d\tau\right),$$

so, by the continuous mapping theorem,

$$\frac{T^{-1}\sum_{t=1}^{T} S_{1t}^0 S_{2t}^0}{\widehat{\sigma}_{S_1^0}\widehat{\sigma}_{S_2^0}} = \frac{T^{-2}\sum_{t=1}^{T} S_{1t}^0 S_{2t}^0}{T^{-1/2}\widehat{\sigma}_{S_1^0}T^{-1/2}\widehat{\sigma}_{S_2^0}} \xrightarrow{d} Z_{12}.$$

By similar techniques, in view of (17) and (18),

$$T^{1/2}(\widehat{\pi}_i - \pi_i) \xrightarrow{d} 12\lambda_i\int_0^1\left(r - \frac{1}{2}\right)W_i(r)dr,$$

so

$$\widehat{A}_{\pi_i} \xrightarrow{d} A_{\pi_i} = \frac{\sqrt{12}\int_0^1 \left(r - \frac{1}{2}\right) W_i(r)dr}{\left[\int_0^1 W_i(r)^2 dr - \left(\int_0^1 W_i(r)dr\right)^2\right]^{1/2}}. \tag{24}$$

Collecting these results

$$\frac{1}{T}\sum_{t=1}^{T} y_{1t}y_{2t} \xrightarrow{d} B_\pi(Z_{12} - A_{\pi_1}A_{\pi_2}),$$

where $B_\pi = (1 - A_{\pi_1}^2)^{-1/2}(1 - A_{\pi_2}^2)^{-1/2}$ so the proof follows immediately.

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Table 1: FM-OLS parameter estimates for the coefficient $\beta$ obtained from model (5). Robust standard errors are in brackets.

| log GDP in millions of dollars | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x_{2t}/x_{1t}$ | CAN | FRA | GER | ITA | JPY | GBR | USA | OECD |
| CAN | | 1.412 (0.231) | −0.703 (0.480) | 1.045 (0.188) | 1.204 (0.204) | 1.055 (0.106) | 0.940 (0.076) | 1.600 (0.089) |
| FRA | | | −0.078 (0.349) | 0.743 (0.096) | 0.520 (0.180) | 0.575 (0.101) | 0.451 (0.109) | 0.845 (0.124) |
| GER | | | | −0.007 (0.205) | −0.592 (0.129) | −0.351 (0.151) | −0.380 (0.103) | −0.399 (0.214) |
| ITA | | | | | 0.530 (0.201) | 0.606 (0.100) | 0.544 (0.126) | 0.966 (0.046) |
| JPY | | | | | | 0.674 (0.043) | 0.596 (0.048) | 0.946 (0.031) |
| GBR | | | | | | | 0.849 (0.075) | 1.290 (0.129) |
| USA | | | | | | | | 1.547 (0.171) |

| log GDP per capita | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x_{2t}/x_{1t}$ | CAN | FRA | GER | ITA | JPY | GBR | USA | OECD |
| CAN | | 1.310 (0.269) | −1.070 (0.675) | 0.836 (0.111) | 1.619 (0.290) | 0.899 (0.080) | 1.179 (0.067) | 1.785 (0.137) |
| FRA | | | −0.135 (0.410) | 0.658 (0.069 | 0.590 (0.225) | 0.445 (0.093) | 0.564 (0.115) | 0.870 (0.104) |
| GER | | | | −0.080 (0.164) | −0.443 (0.180) | −0.294 (0.087) | −0.337 (0.119) | −0.245 (0.230) |
| ITA | | | | | 0.555 (0.290) | 0.518 (0.052) | 0.723 (0.061) | 1.056 (0.057) |
| JPY | | | | | | 0.428 (0.053) | 0.505 (0.029) | 0.815 (0.034) |
| GBR | | | | | | | 1.232 (0.097) | 1.429 (0.208) |
| USA | | | | | | | | 1.385 (0.136) |

Table 2: FM-OLS parameter estimates for the coefficient $\pi_0$ obtained from model (5). Robust standard errors are in brackets.

**log GDP in millions of dollars**

| $x_{2t}/x_{1t}$ | CAN | FRA | GER | ITA | JPY | GBR | USA | OECD |
|---|---|---|---|---|---|---|---|---|
| CAN | | −0.014 | 0.070 | 0.011 | 0.013 | −0.001 | 0.003 | −0.025 |
| | | (0.010) | (0.018) | (0.006) | (0.005) | (0.004) | (0.003) | (0.004) |
| FRA | | | 0.044 | 0.017 | 0.028 | 0.017 | 0.021 | 0.005 |
| | | | (0.013) | (0.003) | (0.005) | (0.004) | (0.005) | (0.005) |
| GER | | | | 0.037 | 0.053 | 0.052 | 0.054 | 0.055 |
| | | | | (0.006) | (0.003) | (0.006) | (0.004) | (0.009) |
| ITA | | | | | 0.019 | 0.006 | 0.008 | −0.009 |
| | | | | | (0.005) | (0.004) | (0.005) | (0.002) |
| JPY | | | | | | −0.002 | 0.000 | −0.014 |
| | | | | | | (0.002) | (0.002) | (0.001) |
| GBR | | | | | | | 0.005 | −0.013 |
| | | | | | | | (0.003) | (0.006) |
| USA | | | | | | | | −0.023 |
| | | | | | | | | (0.007) |

**log GDP per capita**

| $x_{2t}/x_{1t}$ | CAN | FRA | GER | ITA | JPY | GBR | USA | OECD |
|---|---|---|---|---|---|---|---|---|
| CAN | | −0.012 | 0.072 | 0.009 | −0.007 | 0.000 | −0.007 | −0.030 |
| | | (0.010) | (0.025) | (0.003) | (0.007) | (0.003) | (0.002) | (0.005) |
| FRA | | | 0.040 | 0.016 | 0.021 | 0.019 | 0.016 | 0.004 |
| | | | (0.015) | (0.002) | (0.006) | (0.003) | (0.004) | (0.004) |
| GER | | | | 0.039 | 0.048 | 0.048 | 0.048 | 0.046 |
| | | | | (0.005) | (0.005) | (0.003) | (0.004) | (0.008) |
| ITA | | | | | 0.016 | 0.011 | 0.005 | −0.009 |
| | | | | | (0.007) | (0.002) | (0.002) | (0.002) |
| JPY | | | | | | 0.010 | 0.008 | −0.003 |
| | | | | | | (0.002) | (0.001) | (0.001) |
| GBR | | | | | | | −0.006 | −0.015 |
| | | | | | | | (0.003) | (0.007) |
| USA | | | | | | | | −0.016 |
| | | | | | | | | (0.005) |

Table 3: Persistence of cointegration errors and cross-correlations for log GDP in $US$ dollars.

| | Autoregressive parameters from AR(1) processes | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\phi_1$ | $\phi_2$ | | | | | | | |
| $x_{2t}/x_{1t}$ | | CAN | FRA | GER | ITA | JPY | GBR | USA | OECD |
| CAN | 0.393 | | 0.561 | 0.536 | 0.748 | 0.583 | 0.531 | 0.364 | 0.185 |
| FRA | 0.190 | | | 0.579 | 0.541 | 0.644 | 0.553 | 0.440 | 0.537 |
| GER | 0.168 | | | | 0.692 | 0.239 | 0.338 | 0.349 | 0.387 |
| ITA | 0.125 | | | | | 0.692 | 0.680 | 0.550 | 0.668 |
| JPY | 0.347 | | | | | | 0.816 | 0.768 | 0.850 |
| GBR | 0.287 | | | | | | | 0.356 | 0.762 |
| USA | 0.627 | | | | | | | | 0.504 |

| | Cross-correlations ($\rho$) between innovations $v_{1t}$ and $v_{2t}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x_{2t}/x_{1t}$ | CAN | FRA | GER | ITA | JPY | GBR | USA | OECD |
| CAN | | 0.307 | 0.679 | 0.327 | 0.588 | 0.405 | 0.526 | 0.255 |
| FRA | | | 0.774 | 0.291 | 0.697 | 0.516 | 0.550 | 0.431 |
| GER | | | | 0.706 | 0.661 | 0.690 | 0.720 | 0.684 |
| ITA | | | | | 0.805 | 0.697 | 0.716 | 0.604 |
| JPY | | | | | | 0.494 | 0.612 | 0.414 |
| GBR | | | | | | | 0.420 | 0.142 |
| USA | | | | | | | | 0.205 |

Top panel reports the autoregressive coefficient of fitting an AR(1) process to the innovation processes. Column 1 ($\phi_1$) presents the persistence parameter of the innovations from fitting process (6)-(7) applied to the time series of log GDP output for each country. The remaining coefficients in the top panel report the autoregressive coefficient $\phi_2$ of fitting an AR(1) process to the cointegration error from process (9). Bottom panels report the cross-correlation coefficient ($\rho$) between the serially uncorrelated innovations ($\varepsilon_{it}, \varepsilon_{jt}$) after filtering out the serial dependence.

Table 4: Rejection outcomes of cointegration tests for G7 countries (1990-2022).

| log GDP in millions of dollars | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x_{2t}/x_{1t}$ | CAN | FRA | GER | ITA | JPY | GBR | USA | OECD |
| CAN | | 0,0,0,1 | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 | 0,0,0,1 | 0,0,0,1 | 0,0,0,0 |
| FRA | | | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 | 0,0,1,0 | 0,0,0,0 | 0,0,0,0 |
| GER | | | | 0,0,0,0 | 0,0,0,0 | 0,0,1,0 | 0,0,0,0 | 0,0,0,0 |
| ITA | | | | | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 |
| JPY | | | | | | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 |
| GBR | | | | | | | 0,0,0,0 | 0,0,0,0 |
| USA | | | | | | | | 0,0,0,0 |
| log GDP per capita | | | | | | | | |
| $x_{2t}/x_{1t}$ | CAN | FRA | GER | ITA | JPY | GBR | USA | OECD |
| CAN | | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 | 0,0,0,1 | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 |
| FRA | | | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 | 0,0,1,0 | 0,0,0,0 | 0,0,1,1 |
| GER | | | | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 |
| ITA | | | | | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 |
| JPY | | | | | | 0,0,0,0 | 0,0,0,0 | 0,0,0,0 |
| GBR | | | | | | | 0,0,0,1 | 0,0,0,0 |
| USA | | | | | | | | 0,0,0,0 |

Each cell is characterized by an array with four binary entries 0/1 that are interpreted as nonrejection/rejection outcomes of the tests of economic convergence. The first entry in the arrays corresponds to Pesaran's (2007) test of stationarity of the output gap. The second entry is for test (ii) corresponding to Hansen's (1992) unrestricted approach. The third entry corresponds to the SL test in (iii). Finally, entry (iv) is the outcome of the test proposed in this paper based on the joint hypothesis of positive cointegration and cotrending.

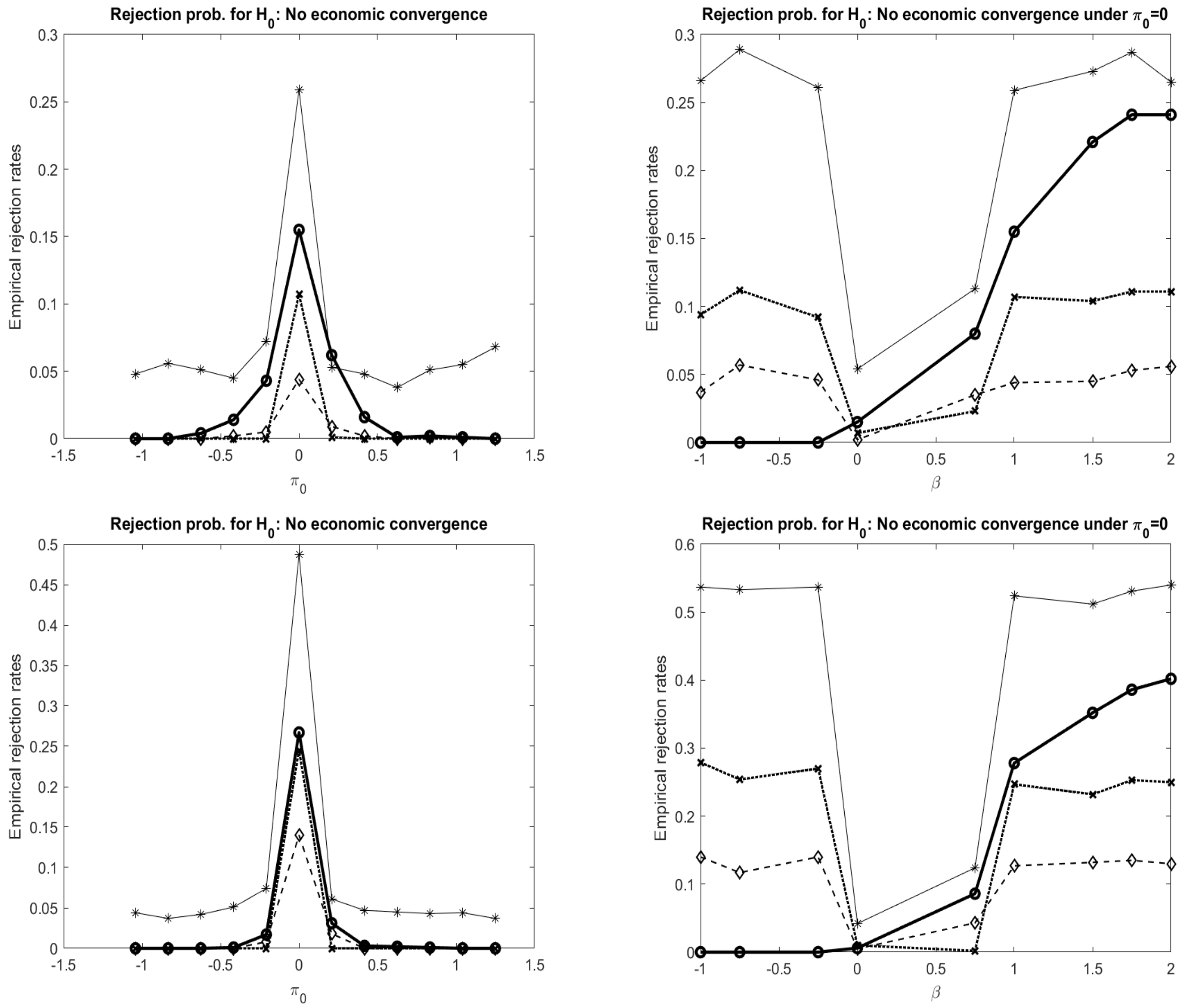


Figure 1: This figure reports the empirical rejection rates out of 1000 simulations of the null hypothesis of no economic convergence at 5% significance level. The innovation process (13) is characterized by the parameters $\phi_1 = 0.5$ and $\rho = 0.5$; $\phi_2 = 0.6$. The left panels consider 12 DGPs; $x_{1t}$ is generated from (6)-(7), with $c_1 = 0.432$ and $\pi_1 = 0.417$ and $x_{2t}$ is generated from (14) with $\beta = 1$, $c_2 = c_1$ and $\pi_0 = (0.5j - 3)\pi_1$ for $j = 1, \ldots, 12$. The right panels consider 9 DGPs indexed by $j = 1, \ldots, 9$; $x_{1t}$ is generated from (6)-(7) as before and $x_{2t}$ is generated from (14) with $\beta(j) = j - 5$ for $j \neq 5$, $c_2(j) = \beta(j)c_1$ and $\pi_2(j) = \beta(j)\pi_1$ such that $c_0 = \pi_0 = 0$. For $j = 5$ ($\beta = 0$), $x_{2t}$ is also generated from (6)-(7) with $c_2 = c_1$ and $\pi_2 = \pi_1$ such that $c_0 = \pi_0 = 0$. Thick solid line with $\circ$ for $\widehat{D}_T$; Dotted line with $+$ for the Pesaran (2007) test given by $\beta$ known; Dashed line with $\Diamond$ for the unrestricted model (5) in Hansen (1992); Solid line with $\star$ for the SL approach. Top panels for $T = 35$ and bottom panels for $T = 50$.

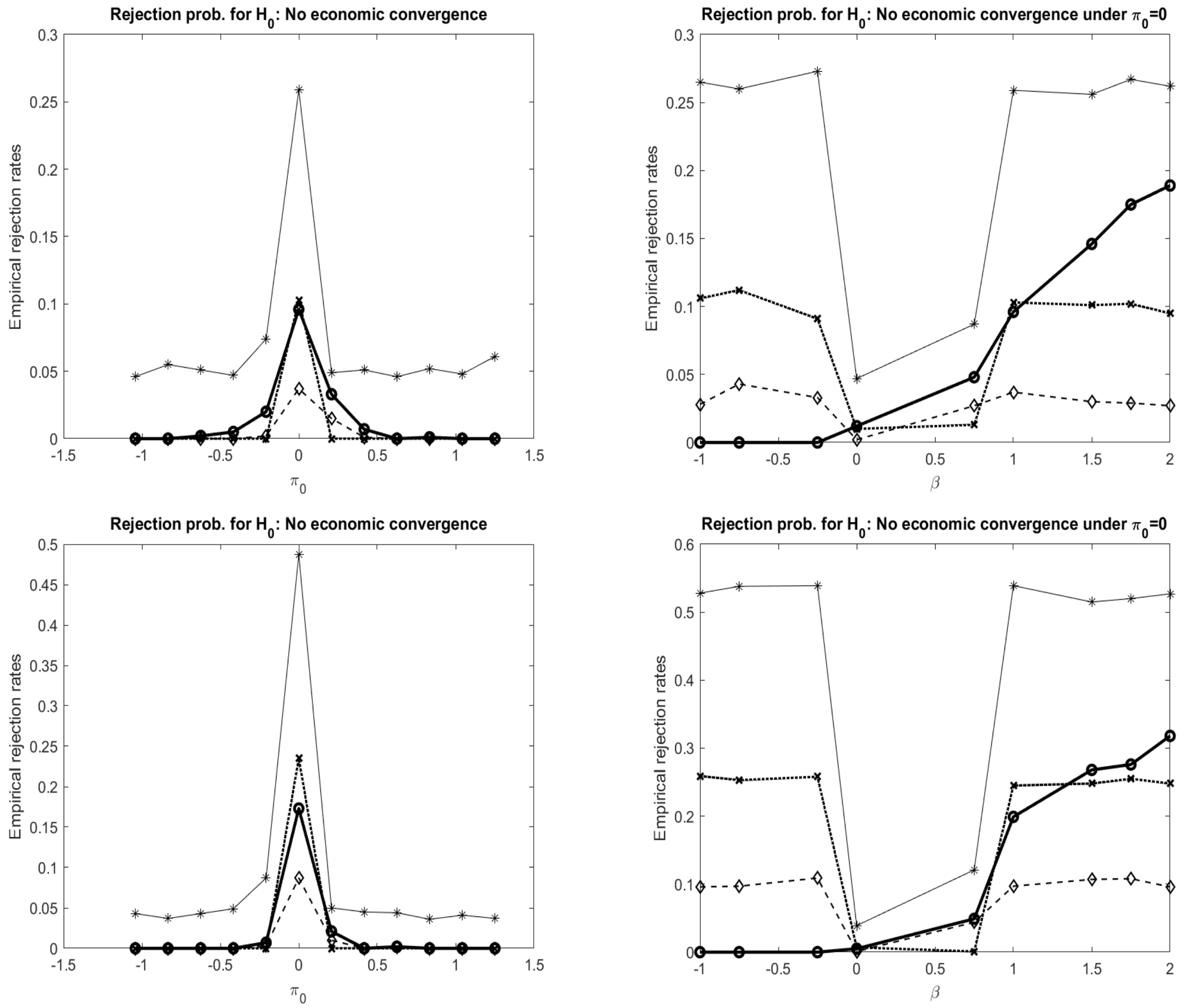


Figure 2: This figure reports the empirical rejection rates out of 1000 simulations of the null hypothesis of no economic convergence at 5% significance level. The innovation process (13) is characterized by the parameters $\phi_1 = 0.5$ and $\rho = 0.8$; $\phi_2 = 0.6$. The left panels consider 12 DGPs; $x_{1t}$ is generated from (6)-(7), with $c_1 = 0.432$ and $\pi_1 = 0.417$ and $x_{2t}$ is generated from (14) with $\beta = 1$, $c_2 = c_1$ and $\pi_0 = (0.5j - 3)\pi_1$ for $j = 1, \ldots, 12$. The right panels consider 9 DGPs indexed by $j = 1, \ldots, 9$; $x_{1t}$ is generated from (6)-(7) as before and $x_{2t}$ is generated from (14) with $\beta(j) = j - 5$ for $j \neq 5$, $c_2(j) = \beta(j)c_1$ and $\pi_2(j) = \beta(j)\pi_1$ such that $c_0 = \pi_0 = 0$. For $j = 5$ ($\beta = 0$), $x_{2t}$ is also generated from (6)-(7) with $c_2 = c_1$ and $\pi_2 = \pi_1$ such that $c_0 = \pi_0 = 0$. Thick solid line with $\circ$ for $\widehat{D}_T$; Dotted line with $+$ for the Pesaran (2007) test given by $\beta$ known; Dashed line with $\Diamond$ for the unrestricted model (5) in Hansen (1992); Solid line with $\star$ for the SL approach. Top panels for $T = 35$ and bottom panels for $T = 50$.

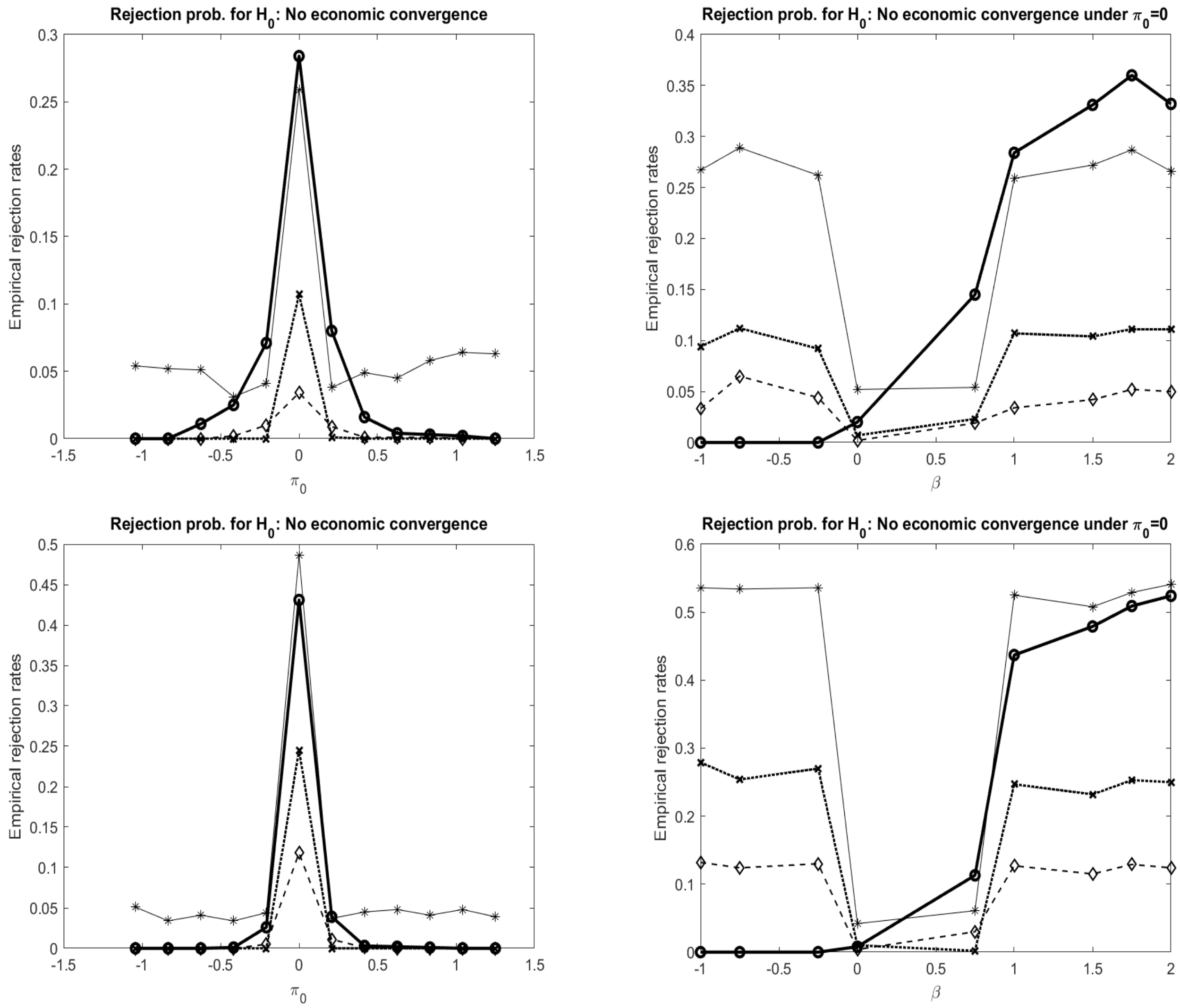


Figure 3: This figure reports the empirical rejection rates out of 1000 simulations of the null hypothesis of no economic convergence at 5% significance level. The innovation process (13) is characterized by the parameters $\phi_1 = 0.7$ and $\rho = 0.5$; $\phi_2 = 0.6$. The left panels consider 12 DGPs; $x_{1t}$ is generated from (6)-(7), with $c_1 = 0.432$ and $\pi_1 = 0.417$ and $x_{2t}$ is generated from (14) with $\beta = 1$, $c_2 = c_1$ and $\pi_0 = (0.5j - 3)\pi_1$ for $j = 1, \ldots, 12$. The right panels consider 9 DGPs indexed by $j = 1, \ldots, 9$; $x_{1t}$ is generated from (6)-(7) as before and $x_{2t}$ is generated from (14) with $\beta(j) = j - 5$ for $j \neq 5$, $c_2(j) = \beta(j)c_1$ and $\pi_2(j) = \beta(j)\pi_1$ such that $c_0 = \pi_0 = 0$. For $j = 5$ ($\beta = 0$), $x_{2t}$ is also generated from (6)-(7) with $c_2 = c_1$ and $\pi_2 = \pi_1$ such that $c_0 = \pi_0 = 0$. Thick solid line with $\circ$ for $\widehat{D}_T$; Dotted line with $+$ for the Pesaran (2007) test given by $\beta$ known; Dashed line with $\Diamond$ for the unrestricted model (5) in Hansen (1992); Solid line with $\star$ for the SL approach. Top panels for $T = 35$ and bottom panels for $T = 50$.

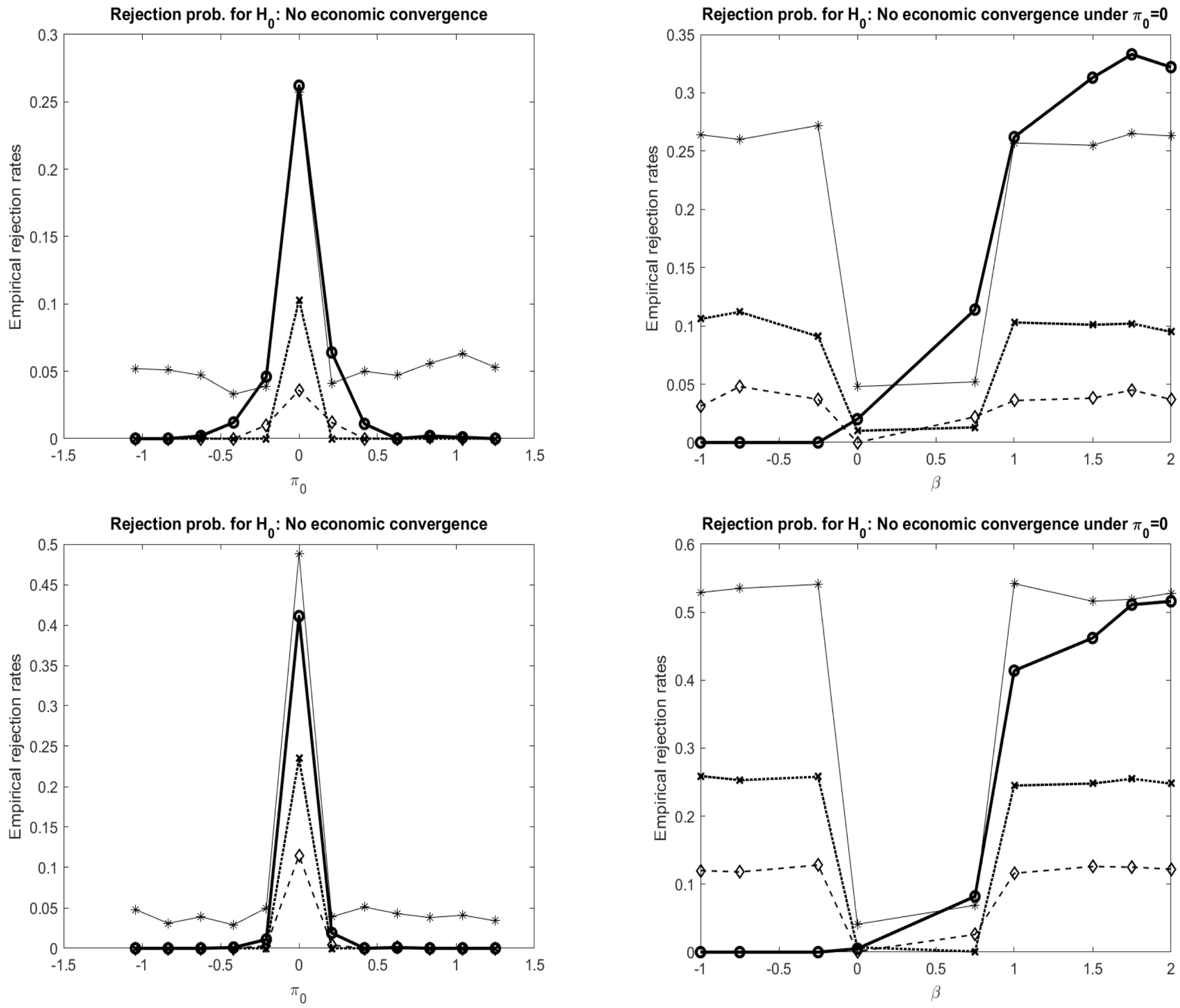


Figure 4: This figure reports the empirical rejection rates out of 1000 simulations of the null hypothesis of no economic convergence at 5% significance level. The innovation process (13) is characterized by the parameters $\phi_1 = 0.7$ and $\rho = 0.8$; $\phi_2 = 0.6$. The left panels consider 12 DGPs; $x_{1t}$ is generated from (6)-(7), with $c_1 = 0.432$ and $\pi_1 = 0.417$ and $x_{2t}$ is generated from (14) with $\beta = 1$, $c_2 = c_1$ and $\pi_0 = (0.5j - 3)\pi_1$ for $j = 1, \ldots, 12$. The right panels consider 9 DGPs indexed by $j = 1, \ldots, 9$; $x_{1t}$ is generated from (6)-(7) as before and $x_{2t}$ is generated from (14) with $\beta(j) = j - 5$ for $j \neq 5$, $c_2(j) = \beta(j)c_1$ and $\pi_2(j) = \beta(j)\pi_1$ such that $c_0 = \pi_0 = 0$. For $j = 5$ ($\beta = 0$), $x_{2t}$ is also generated from (6)-(7) with $c_2 = c_1$ and $\pi_2 = \pi_1$ such that $c_0 = \pi_0 = 0$. Thick solid line with $\circ$ for $\widehat{D}_T$; Dotted line with $+$ for the Pesaran (2007) test given by $\beta$ known; Dashed line with $\Diamond$ for the unrestricted model (5) in Hansen (1992); Solid line with $\star$ for the SL approach. Top panels for $T = 35$ and bottom panels for $T = 50$.

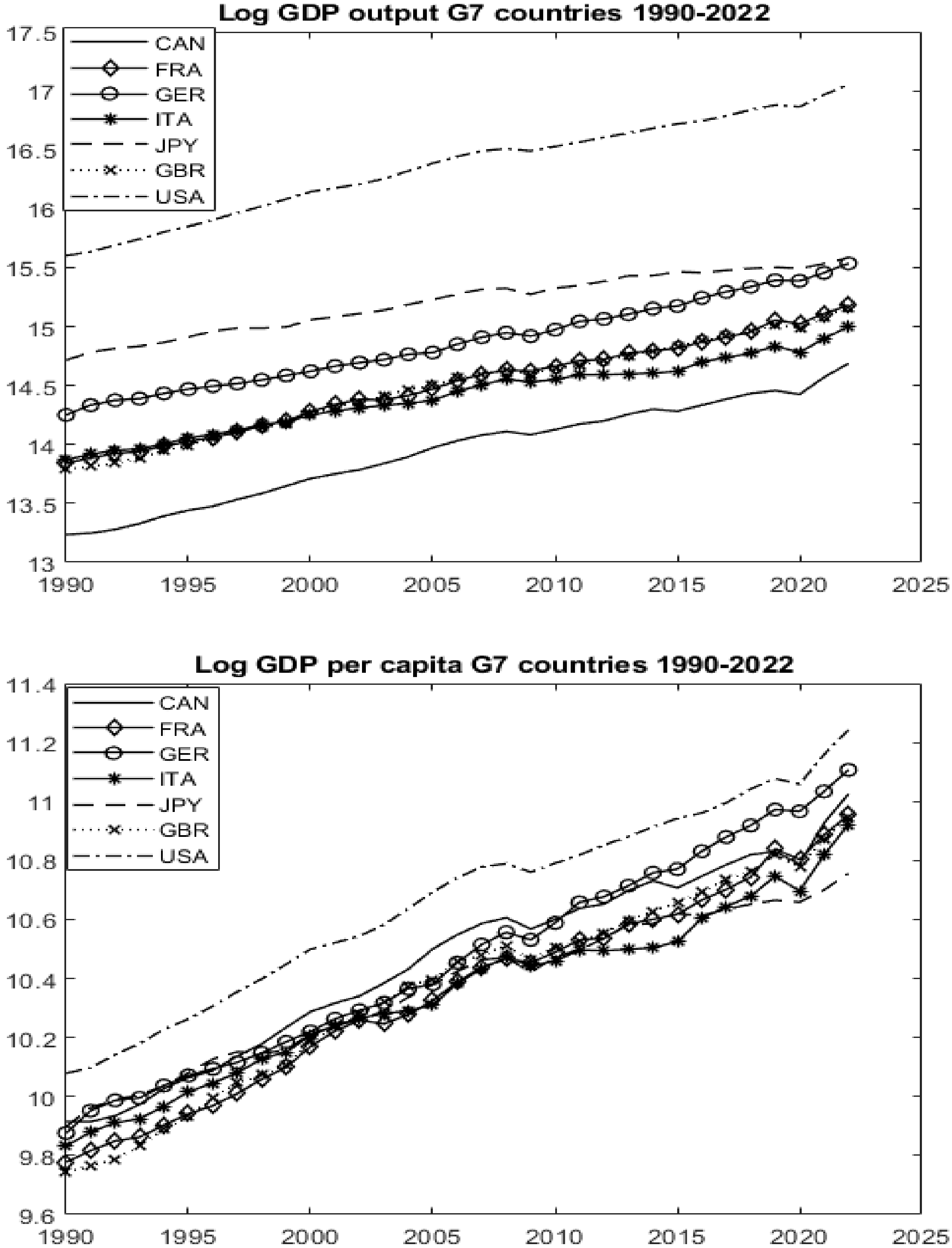


Figure 5: Top panel reports the dynamics of logarithm of Gross Domestic Product measured in millions of dollars for G7 countries over the period 1990 to 2022. Bottom panel reports the dynamics of GDP measured in per-capita terms. Data are obtained from OECD website.