# Optimal Speed and Hedging Strategies for Tramp Shipping Operators in Volatile Freight Markets

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**Abstract**

The maritime shipping industry faces significant uncertainties due to the volatility of freight rates, directly impacting business operations. This paper examines the relationship between freight rate uncertainties, hedging policies, and shipping speeds using a novel stochastic optimization framework that integrates practical hedging strategies with operational speed decisions. Unlike traditional models that assume Geometric Brownian Motion (GBM) for price dynamics, our model employs an exponential Ornstein-Uhlenbeck (OU) process to capture the mean-reverting nature of freight rates, providing a more realistic representation of market behavior. Additionally, the model is compatible with Forward Freight Agreement (FFA) hedging practices and allows for partial hedging, aligning closely with real-world risk management strategies. By employing a mean-variance utility function, this research offers a toolkit for risk-averse shipping operators to incorporate risk tolerance into the speed and hedging decision-making process.

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maritime economics, optimal shipping speed, hedging freight risk, risk aversion, tramp shipping, Ornstein-Uhlenbeck process

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**Abstract**

The maritime shipping industry faces significant uncertainties due to the volatility of freight rates, directly impacting business operations. This paper examines the relationship between freight rate uncertainties, hedging policies, and shipping speeds using a novel stochastic optimization framework that integrates practical hedging strategies with operational speed decisions. Unlike traditional models that assume Geometric Brownian Motion (GBM) for price dynamics, our model employs an exponential Ornstein-Uhlenbeck (OU) process to capture the mean-reverting nature of freight rates, providing a more realistic representation of market behavior. Additionally, the model is compatible with Forward Freight Agreement (FFA) hedging practices and allows for partial hedging, aligning closely with real-world risk management strategies. By employing a mean-variance utility function, this research offers a toolkit for risk-averse shipping operators to incorporate risk tolerance into the speed and hedging decision-making process.

Our analysis reveals that the ability to hedge future profits significantly influences current speed choices, uncovering novel insights such as the asymmetric nature of the optimal policy for laden and ballast legs and the sensitivity of the optimal hedge ratio to various risk parameters. We also establish a closed-form relationship between hedging ratios and speed through a newly developed theorem, offering practical guidance for operators. Experimental results demonstrate the model's applicability and effectiveness when tested against real-life data, highlighting its potential to enhance both economic and operational decision-making in maritime shipping.

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## 1. Introduction

Shipping operators face considerable uncertainty in freight rates, which directly impacts business operations and revenues. Figure 1 depicts the *Baltic Exchange dry index* from 2018 to 2023. Although the index is an average, it does illustrate the variability that shipping operators may experience on specific routes.

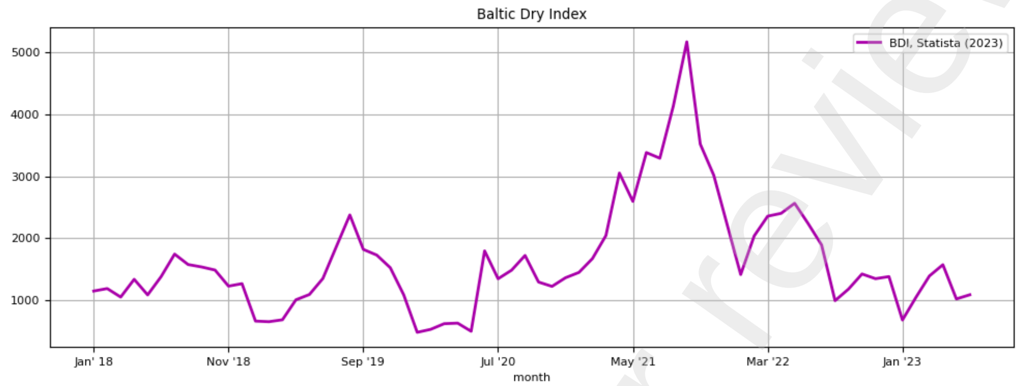


Figure 1: Variation in the Baltic Exchange dry index from 2018 to 2023 (Source: Statista).

Operators may consider that the potential for commensurate long-term rewards is sufficient compensation for such risk. To mitigate exposure to downside risk, however, well-crafted hedging strategies are increasingly used to manage market volatility. A robust market for maritime derivatives, based on Baltic Exchange indices, has been established by brokerage firms such as Clarksons, SSY, and FIS, among others, which facilitate connections between counterparts seeking to manage their exposure. For example, an exporter purchasing grains and transporting them to China may want to limit their exposure to rising freight rates, as this cost represents a liability; thus, they might opt to buy a forward freight agreement (FFA) at a predetermined price (Kavussanos and Nomikos, 2003). Conversely, a shipping operator, as a provider of shipping services, may seek to lock in a fixed rate by selling such an FFA. This manuscript investigates the interplay between optimised hedging and speed optimisation decisions within the maritime shipping context.

Vessel speeds are vital to shape long-term profitability (Stopford, 2009), as they influence fuel consumption and emission rates (Ronen, 1982; Psaraftis and Kontovas, 2013). Furthermore, speeds determine the number of voyages, and consequently the total amount of cargo that can be transported, within any predetermined time interval. Therefore, one can expect a close link between vessel speed and freight rate, as operators will be keen to transport more cargo when the freight rate spot market is favourable, and less inclined to do so when the market is unfavourable. Indeed, an empirical analysis suggests a relationship between market conditions and vessel speeds (Adland and Jia, 2017). The analysis suggests, moreover, that speed decisions may rely more on the ship's operational conditions than on the macroeconomic perspectives. It is also worth mentioning that vessel speed decisions are often constrained by contractual obligations, see also Beullens et al. (2023).

To inform vessel speed decisions, one can choose to maximise the time charter equivalent (TCE) (Stopford, 2009), which leads to the so-called square root rule. While this is a valid heuristic, it does not capture all the essential aspects of the decision-making process, such as future market fluctuations and the overall revenue potential at the destination port. In practice, the freight rate is often considered a stochastic process, as it reflects complex interactions between numerous uncertain factors, leading to unpredictable future profit potential. Thus, shipping operations must be flexible and adapt to each unique circumstance, giving rise to a stochastic sequential decision problem. For a comprehensive treatment of stochastic sequential decision problems (aka Markov decision problems), we refer to Puterman (2014).

To achieve a better characterization of real-world scenarios than the conventional square root rule, our analysis incorporates a stochastic sequential decision problem that results in a more nuanced yet straightforward approach. The square root rule, which suggests adjusting vessel speed based on a simple relationship with freight rates, lacks the flexibility to consider the complexities of future market fluctuations and the variability in revenue potential at different ports. In contrast, our approach intuitively addresses both ballast and laden situations by integrating future profit potential and time charter hire into the decision-making process. While the application of dynamic stochastic optimisation is not new in the maritime economics literature (Magirou et al., 2015; Devanney, 1971; Magirou et al., 1992), the novelty of our approach is that we deal with the ship operator's financial risks jointly with speed optimisation and integrate these topics. In addition, the risks related to the freight rate can be partially hedged in the market using forward freight agreements (FFA) (Kavussanos and Nomikos, 2003). The ship operator will then seek a compromise between hedging or curtailing the risk, and taking the risk in search of a potential risk premium. We model this trade-off via a user-specific risk tolerance parameter.

Given our discussion above, one can expect the freight rate dynamics to be instrumental for both vessel speed and hedging decisions. In this work, we model the freight rate dynamics as an exponential Ornstein-Uhlenbeck (OU) process (Uhlenbeck and Ornstein, 1930). The model is more realistic than the Gaussian Brownian Motion (GBM) often used to model commodities and freight rates (Prokopczuk, 2011; Geman and Smith, 2012; Benth and Koekebakker, 2016), as it can easily account for the market pressure to revert back to previously prevailing levels. Indeed, the economics of the freight rate is such that steep increases in the rate will produce additional demand for ships. Conversely, this will introduce competition, which will lead to a drop in the rates. On the other hand, the business operates with many fixed and variable costs, and these provide a lower limit for the price. Once the price falls below this lower limit, ship operators will have no incentive to transport cargo and that will lead to an increase in freight rates. Our results show that stronger mean reversion dynamics lead to more stable speeds in practice, in contrast to models with high sensitivity to freight rates. This finding aligns with observations in real-world shipping operations. We support the adoption of the OU process through experiments presented in Section 4, which demonstrate its effectiveness in capturing the mean-reverting nature of freight rates. Additionally, the use of the mean reversion for modelling freight rates is not unprecedented; several studies (e.g., Benth and Koekebakker, 2016; Kyriakou et al., 2017*a*) have found it to be a suitable choice for representing price dynamics in volatile markets. For a more detailed discussion on the rationale and supporting evidence for using OU processes, please refer to Section 3.

This work introduces a novel semi-Markov decision process to jointly optimise vessel speed and hedging decisions, while considering that freight rates evolve over time according to an OU stochastic process. The objective is to maximise the discounted long-term reward function, which incorporates a risk aversion term to account for exposure to spot market volatility. Our experiments reveal notable insights into the interplay between freight rate dynamics and speed optimisation. Specifically, we find that the sensitivity of the optimal speed is inversely proportional to the speed of mean reversion of freight rates. This result is intuitive, for when freight rates revert quickly to the mean, increasing speed may not be enough to ensure a favourable price at the next port, hence fuel savings will tend to be prioritised. Conversely, if prices are more persistent and less prone to mean reversion, speeding becomes a more profitable strategy.

Like speed, the hedge ratio is influenced by factors such as risk premium, risk tolerance, and the volatility of the freight rate. Consistent with freight forward brokering practices, the risk premium is determined as the difference between the forward freight rate and the expected spot rate. Our experiments reveal distinct differences in the optimal hedging strategies for the laden and ballast legs, which reflect the varying market exposures: hedging during the laden leg primarily protects against price fluctuations while carrying cargo, whereas the ballast leg ratio acts more like an investment decision, allowing operators to capitalize on market opportunities while traveling without cargo.

This study uncovers several insights that are not immediately intuitive. For example, while it may seem straightforward that hedging can affect optimal speed, our findings show a more nuanced relationship: under certain conditions, aggressive hedging can lead to counterintuitive speed adjustments that deviate from traditional cost-saving approaches. Additionally, the results indicate that the interplay between hedging strategies and speed choices can reveal hidden inefficiencies in the conventional separation of financial and operational decision-making.

Our results also demonstrate the reciprocal relationship between the optimal hedging ratio and vessel speed. Fixing a speed sets the time horizon for hedging, while the chosen hedging ratio affects the reward associated with speed decisions, underscoring the interdependence between economic and operational aspects of maritime operations. This interconnectedness challenges the typical separation of finance and operations in most firms, suggesting new opportunities for integrated decision-making and operational optimisation. For firms not yet considering this approach, our findings provide a case study illustrating the potential benefits of aligning financial risk management with speed optimisation strategies.

To summarize, this paper is the first to propose a novel stochastic sequential decision framework for jointly optimising speed and hedging strategies. Our results demonstrate that shipping operators must balance current and future profit potential by integrating both operational and economic considerations. Flexibility in delivery times at the contract negotiation stage allows operators to optimally choose both speed and hedging strategies. Our analysis shows that optimal vessel speed and hedging decisions are interdependent and should therefore be jointly considered. For example, an experiment considering real-world data for a specific route over the last five years (see Section 4.5) reveals that joint optimisation leads to 10-25% increase in overall profitability compared to scenarios where speed and hedging are optimised separately. Specifically, if speed is fixed based on fuel costs or hedging decisions are made independently, the financial outcomes are significantly less favorable. This finding aligns with and extends existing literature by showing that considering these decisions together is essential in a stochastic freight rate environment. Furthermore, while recent studies (Ge et al., 2021*a*; Beullens et al., 2023) have examined vessel speed optimisation under deterministic freight rates, our work is more general, explicitly considering the stochastic nature of the freight rate market and including hedging strategies to manage market volatility.

In conclusion, this paper introduces a novel stochastic sequential decision framework for jointly optimising vessel speed and hedging strategies in volatile freight markets. Our findings suggest that this joint approach is particularly beneficial under conditions of high freight rate volatility, where the OU process parameters indicate rapid mean reversion or significant market fluctuations. For decision-makers with a low risk tolerance or those frequently exposed to spot market dynamics, integrating speed and hedging decisions can provide substantial gains in overall profitability and risk management. Moreover, our method demonstrates value even in scenarios where constant speeds are maintained throughout all journeys. By using our stochastic model, operators can derive optimised constant speeds for laden and ballast legs that, while fixed in practice, are still aligned with expected market conditions and volatility. This approach offers a more refined alternative to traditional speed-setting methods, allowing for better alignment with long-term financial objectives. Future work could explore how these benefits scale with different levels of market volatility or variations in risk preferences, providing deeper insights into the practical applications of our joint optimisation framework.

The remainder of the paper is organised as follows. Section 2 features a review of the related literature. Section 3 introduces the proposed modelling framework. It includes the model of the freight rate dynamics, as well as the semi-Markov decision process that underpins the optimal hedging and speed decisions. Section 3.7 derives a closed-form solution for the optimal hedging policy and demonstrates the interdependency between hedging and speed decisions. To validate the approach and draw useful insights, Section 4 introduces a set of experiments and analyses the decision-making trade-offs as we vary the system's parameters. Finally, Section 5 concludes the paper.

## 2. Literature review

The shipping industry is multifaceted and must consider both operational and financial aspects. Operationally, it is essential to select an appropriate vessel speed, considering not only the prevailing freight rate but also prospective future earnings and their likely variation. From a financial standpoint, it is crucial to hedge against market variation. The additional challenge of synchronising hedging and speed choices to maximise long-term profits is conducive to a sequential decision-making process under uncertainty (Puterman, 2014). This paper is hence on the intersection of three separate domains in the literature: speed optimisation, hedging freight rate risk and integrated decision making under uncertainty. The remainder of this section addresses each of these separate domains.

### 2.1. *Vessel speed decisions*

Variable speed in maritime shipping is not just an operational detail, it is an economic imperative. A consistent body of literature addresses the complex interplay of cost-efficiency, freight rate timing, and profitability. The impact of external economic pressures, such as oil prices, on vessel speed decisions is well known (Ronen, 1982). Therefore, there is an implicit understanding that vessel speeds should adapt to market volatility. To that end, one can choose to optimise the *time charter equivalent* (TCE), the vessel's average daily revenue within a set of trips (Stopford, 2009), an approach that highlights the impact of freight rates on speed decisions. Building on this foundational understanding, Norstad et al. (2011) investigate the interplay of tramp ship routing and speed. They highlight the potential to accommodate additional spot cargoes with higher speeds, demonstrating the inherent advantage of variable speeds in maximising profitability, especially under fluctuating freight rates. Further expanding on these considerations, Beullens et al. (2023) highlight the importance of the time remaining on time charter contracts as a potentially influencing factor in speed optimisation, demonstrating that the optimal speed choices are not only driven by operational costs and market conditions but are also sensitive to the specific terms and duration of the charter contract. Speed optimisation is also crucial in more intricate situations involving fleet sizing decisions and sustainability measures (Ronen, 2011). Beyond the economic realm, however, the conversation about speed optimisation incorporates other critical dimensions.

In their taxonomy of vessel speed models, Psaraftis and Kontovas (2013) focus on energy efficiency. Cautioning against an excessive focus on cost-cutting, their work suggests that fixed slower speeds may inadvertently heighten operational costs due to freight rate variations. This understanding raises the question: what other external factors influence the speed decision? Among these factors, one finds time-varying environmental conditions (Wang et al., 2018) and vessel safety (Perera and Soares, 2017). These are closely interlinked and reveal the need for a nuanced management of vessel speed. Profitability is also a driving force, as it envelops the financial implications of speed decisions (Adland, Cariou and Wolff, 2020) and the interconnections among logistics, operational costs, and freight rates (Wu et al., 2021). For empirical studies on the relationship between vessel speed, freight rates, bunkers costs and fleet size, we refer to (Açik and Kayiran, 2022).

Despite the significant literature on the importance of adjusting speeds due to various factors, to the best of our knowledge the interplay between vessel speed and hedging decisions remains unexplored in the literature. Bridging this gap, as this paper explores, can be significant, since a method to combine smart hedging with smart speed decisions can help us not only maximise expected revenues under market uncertainty, but also build invaluable insight on the role of integrated hedging and speed management policies in the context of financial risk management.

### 2.2. *Freight risk*

Adapted from financial markets to shipping derivatives markets, the concept of hedging ratio denotes the size of a position in the futures market that is necessary to hedge against the variations of the freight ratio in the spot market (Chen et al., 2003). One can hedge to avoid risks and reduce costs (Johnson, 1960; Goss et al., 1976), to maximise a given utility function (Cecchetti et al., 1988) or to minimise the exposition to risk (Lence, 1995, 1996). Whilst hedging strategies can be static, one often needs dynamic hedging strategies to adapt to market fluctuations (Angelidis and Skiadopoulos, 2008).

A good understanding of the freight rate dynamics is vital to inform hedging policies. Kavussanos and Nomikos (2003) study the correlation between the spot and future freight rate markets, and investigate the effect of incorporating future prices into simple freight rate time series models. Other works provide invaluable insights into pricing, hedging, and the volatility of spot freight rates (Prokopczuk, 2011; Koekebakker et al., 2006), whilst Adland and Cullinane (2006) explore the nonlinear dynamics of these rates. Nonlinear freight rate prediction tools also include the concept of copulas and its application to the freight derivatives market (Shi et al., 2017).

As freight rates vary in time, stochastic modelling tools are a natural way to anticipate rate movements and inform hedging decisions (Benth and Koekebakker, 2016). Kyriakou et al. (2017*b*) extended this approach to the nuanced realm of freight derivatives pricing amidst decoupled mean-reverting diffusion and jumps.

Whilst the literature on freight rate hedging is informative and includes a variety of models to predict future prices and inform hedging, it is generally insular and neglects the operational nuances of the business. One straightforward neglected aspect is that the duration of the trip to be hedged depends upon the vessel speed which, in turn, also determines the future profit potential of the vessel. To bridge this gap, this study integrates established methodologies like mean-variance utility optimisation within multi-period settings into the operational dynamics of businesses, including speed optimisation decisions. We unveil the intricate inter-dependencies and provide a holistic perspective rarely explored in the related literature.

As we do not know a priori and cannot exactly predict the system's next state, the reward (see further in Section 3.2) is only realised at the destination port, and is therefore a random variable at the time of departure. The uncertainty in the reward poses a problem to the shipping operator, and their willingness to carry out the business therefore depends on their level of risk aversion or, equivalently, on their appetite for risk. To account for the decision maker's level of risk aversion, we opt to maximise the expected reward for a given amount of risk, which can be customised by a user-specified parameter. To that end, we apply Markowitz (1952)'s well-known mean-variance optimisation framework and define the utility function as introduced in modern portfolio theory, which uses mean-variance optimisation to construct portfolios that maximise the expected return for a given level of risk.

Samuelson (1963) discusses the concept of risk aversion and its relationship to utility theory and argues that investors are risk-averse because they derive diminishing marginal utility from wealth, and that this aversion to risk can be quantified using tools such as the Sharpe ratio. The application of risk aversion and utility is new in shipping literature. Although in a recent study employs utility as an objective function (Wang, Wen, Yip and Fan, 2021), there is no literature to our knowledge which utilises risk aversion to jointly optimise the maritime decision making process.

### 2.3. Maritime Optimisation

Maritime optimisation has traditionally leveraged dynamic programming, a method introduced by Bellman (1957), to tackle a wide range of problems including scheduling (Norstad et al., 2011), routing (Lo and McCord, 1995), and fleet renewal (Pantuso et al., 2015). Additionally, the application of stochastic programming—pioneered in the maritime sector by Devanney (1971) and expanded by Magirou et al. (1992)—has addressed various uncertainties such as non-linear fuel consumption, charter market decisions, ship positioning, acquisition policies, portfolio analysis, cargo selection, and weather conditions.

Despite these advancements, much of the existing literature focuses on speed and voyage decisions for liner operations under deterministic assumptions, often neglecting uncertainties in the charter markets for tramp vessels. An exception is the work by Magirou et al. (2015), which utilises a Geometric Brownian Motion (GBM) model to represent freight rate evolution, optimising long-term discounted revenue by adjusting speed choices according to market variations. However, this approach does not incorporate hedging strategies to mitigate financial risks, leaving a gap in understanding how speed optimisation and hedging decisions can jointly manage market volatility.

To provide a clearer understanding, Psaraftis (2019) offer a comprehensive categorisation of ship routing and scheduling literature based on several primary assumptions, such as time window constraints, constant speeds without optimisation, independence between fuel consumption and payload, a sole focus on cost without considering revenue, and limited literature on dynamic ship routing and scheduling. While these assumptions help structure specific research areas, they often fail to reflect the complexities of real-world operations, suggesting the need for more integrated approaches.

Recent trends in maritime optimisation emphasise joint optimisation approaches, where multiple decision variables are considered simultaneously. For instance, studies like Dong et al. (2021) focus on minimising costs, including emissions such as $CO_2$ or $SO_2$. While these studies often incorporate uncertainty, they typically concentrate on tactical planning aspects like weather fluctuations rather than integrating comprehensive financial risk management strategies.

Dynamic programming remains a popular method in recent studies, such as Fan et al. (2021) and Yan et al. (2018), which explore speed optimisation for inland vessels under uncertain river flow conditions. These models adapt to external variables, such as changing river speeds, to produce optimal travel speeds. Similarly, Fan et al. (2019) investigate routing problems with speed optimisation, incorporating additional charges for $CO_2$ emissions using a simulated annealing approach. However, these models primarily focus on minimising operational costs and do not account for freight rate volatility or integrate financial hedging strategies.

Further studies, such as Li, Ji, Yu, Zhou and Fang (2022) and Wen et al. (2017), extend existing models by applying advanced optimisation techniques like Branch-and-Price, coupled with speed discrimination (Bektaş and Laporte, 2011). These models aim to minimise fuel consumption while considering demand constraints, but they remain limited to cost minimisation without addressing the revenue implications of different speed choices under uncertain market conditions.

Recent studies like Li, Fagerholt and Schütz (2022) and Li et al. (2023) provide additional insights into speed optimisation under uncertain conditions. For example, Li, Fagerholt and Schütz (2022) examines alternative routes such as the North Sea Route, considering $CO_2$ emissions, while Li et al. (2023) explores manoeuvring aspects of speed optimisation from an engineering perspective, focusing on future energy-saving technologies for autonomous navigation ships. Both studies contribute to understanding cost and environmental considerations but do not integrate stochastic revenue modelling or hedging strategies.

Other studies, such as Li et al. (2020) and Wei et al. (2022), compare optimal speeds with and without voluntary speed loss due to factors like emission control zones, weather, and freight rates. These studies, with more of an engineering focus, address tactical speed optimisation but do not consider broader financial implications or risk management strategies.

Cost optimisation models frequently aim to identify a "sweet spot" for optimal speed that balances fuel consumption and time, as demonstrated in Ma et al. (2021), where joint optimisation of cost and time is considered under constraints like weather optimisation and Emission Control Areas (ECA). Similarly, Mandal et al. (2023) propose a multi-objective model for routing and speed optimisation that minimises both operational costs and total travel time. However, these studies do not integrate broader financial risk considerations or hedging strategies.

Recent contributions, such as those by Ormevik et al. (2023), highlight the significance of weather considerations in operational planning for offshore logistics, advocating for weather-inclusive planning to ensure robust schedules. Meanwhile, Pasha et al. (2021) focus on the dynamic relationship between speed and freight rates in liner shipping, although their deterministic optimisation approach omits stochastic freight rate considerations.

To advance our understanding of the complex relationships between speed, costs, emissions, and financial strategies, recent studies have introduced various rules and algorithms designed to optimise vessel speed under different constraints. For example, Sheng et al. (2019) derive a cubic root rule for determining the optimal speed, suggesting that speed should increase with higher charter rates and inventory costs but decrease with higher fuel costs. This model, based on single-period optimisation and deterministic input variables, aligns with some of the foundational principles explored in our appendix, which also lack stochastic elements.

Several other studies explore how speed optimisation models can adapt to environmental and regulatory constraints. For example, Sun et al. (2023) utilises a single-period optimisation model applying a square root speed model to comply with emission regulations, specifically the Carbon Intensity Indicator (CII) penalty, while maintaining deterministic parameters. In contrast, Wang et al. (2020) and Wang, Fan, Tu and Vladimir (2021) introduce tactical speed optimisation strategies that combine cost minimisation with emission controls, accounting for environmental factors like weather and ocean currents but excluding revenue considerations or hedging strategies. Similarly, Wu et al. (2023) investigate fleet deployment strategies to optimise operational costs and reduce emissions, demonstrating how increasing ship deployment allows for lower speeds and reduced emissions. However, this model also primarily focuses on cost rather than revenue optimisation.

Moving beyond traditional deterministic models, Xie et al. (2023) introduce an innovative machine learning methodology to dynamically define speeds for different segments of a voyage. By incorporating real-time weather parameters such as wave height, ocean stream velocity, and wind conditions, their non-linear model provides a more adaptive and responsive tool for decision-making under uncertain conditions.

These varied approaches underscore the multifaceted nature of maritime optimisation. While some models concentrate on cost and environmental objectives under deterministic assumptions, others, like those using machine learning, push toward more adaptive and comprehensive strategies. However, despite these advancements, there remains a significant gap in integrating speed optimisation with financial hedging to effectively manage market volatility. Addressing this gap, this paper proposes a novel framework that jointly optimises speed and hedging decisions to balance operational efficiency with financial risk management.

In economic decision-making, utility functions linked to risk preferences have been explored by Zhao et al. (2020), who propose a loss aversion mechanism for optimising speed based on fuel consumption, SOx emissions, and delivery delays. While this model offers a novel perspective on risk-based decision-making, it does not integrate speed optimisation with hedging strategies.

Additionally, recent studies by Ge et al. (2021*a*) and Beullens et al. (2023) have developed models of optimal speed within a deterministic framework, revealing a "chain effect" where the optimal speed increases with the number of round trips. These studies illustrate how discounting future revenue and cost cash flows impact speed decisions. However, their reliance on deterministic assumptions limits their applicability under uncertain freight rates and market conditions.

To fill these gaps, this paper introduces a utility-based stochastic optimisation model that explores the complex interplay between freight rate uncertainties, hedging policies, and shipping speeds. By incorporating stochastic elements, we move beyond traditional models focused solely on cost or profit minimisation, proposing a balanced approach that aims to optimise future profits under uncertain conditions. Our model identifies strategies for optimal hedging decisions and economic ship speeds, bridging the gap between operational decision-making and financial risk management.

Furthermore, our framework for the joint optimisation of business logistics and financial operations offers a versatile platform for future research. It could be extended to include hedging carbon footprint costs or fluctuating bunker prices, reflecting the growing need for comprehensive models that address both economic and environmental factors in shipping.

To provide a clearer understanding of the evolution of thought in this area, we have curated a selection of key publications in Table 1. This table categorises important contributions to the field, detailing the core contributions of each work. The chosen studies highlight the progression from early models of speed optimisation under deterministic conditions to more recent efforts that consider financial risk management and market uncertainties.

By bridging these gaps, our study contributes a novel perspective to the literature, advancing the field of maritime optimisation with a comprehensive approach that integrates operational and financial risk management.

Table 1: Related maritime speed optimisation and hedging literature

| Paper | Objectives | Speed | Price | Hedging | Methods |
|---|---|---|---|---|---|
| Norstad et al. (2011) | Ship project value | ✓ | Deterministic | - | LP |
| Alizadeh and Nomikos (2012) | Ship Price Risk | - | Stochastic | ✓ | EcM |
| Psaraftis and Kontovas (2013) | Ship Price Risk | - | Deterministic | ✓ | EcM |
| Magirou et al. (2015) | Ship project value | ✓ | Stochastic | - | DP |
| Wang et al. (2019) | Ship project value | ✓ | Stochastic | - | DP |
| Adland, Ameln and Børnes (2020) | Ship Price Risk | - | Stochastic | ✓ | EcM |
| Adland, Cariou and Wolff (2020) | Elasticity | ✓ | Stochastic | - | EcM |
| Ge et al. (2021*a*) | NPV | ✓ | Deterministic | - | DP |
| Fan et al. (2021) | Cost | ✓ | Stochastic | - | DP |
| Yan et al. (2018) | Cost | ✓ | Deterministic | - | DP |
| Fan et al. (2019) | Cost | ✓ | Stochastic | - | LP |
| Huotari et al. (2021) | Cost | ✓ | Stochastic | - | Convex optimisation |
| Li, Ji, Yu, Zhou and Fang (2022) | Cost | ✓ | Deterministic | - | LP |
| Wen et al. (2017) | Cost | ✓ | Deterministic | - | LP |
| Li, Fagerholt and Schütz (2022) | Cost | ✓ | Deterministic | - | LP |
| Li et al. (2023) | Cost | ✓ | Deterministic | - | PSO |
| Li et al. (2020) | Cost | ✓ | Deterministic | - | LP |
| Wei et al. (2022) | Cost | ✓ | Deterministic | - | LP |
| Ma et al. (2021) | Cost, Time | ✓ | Deterministic | - | LP |
| Mandal et al. (2023) | Cost, Time | ✓ | Deterministic | - | LP |
| Ormevik et al. (2023) | Cost | ✓ | Deterministic | - | LP |
| Pasha et al. (2021) | Cost | ✓ | Deterministic | - | LP |
| Psaraftis (2019) | Review | ✓ | - | - | Vary |
| Sheng et al. (2019) | Cost | ✓ | Deterministic | - | Signle period cost minimisation |
| Sun et al. (2023) | Revenue | ✓ | Deterministic | - | Single period revenue maximisation |
| Wang et al. (2020) | Cost | ✓ | Deterministic | - | LP |
| Wang, Fan, Tu and Vladimir (2021) | Cost | ✓ | Deterministic | - | LP |
| Wu et al. (2023) | Cost | ✓ | Deterministic | - | LP |
| Xie et al. (2023) | Cost | ✓ | Deterministic | - | Non linear model |
| Zhao et al. (2020) | Cost | ✓ | Deterministic | - | Utility based |
| **This paper** | NVP | ✓ | Stochastic | ✓ | DP |

LP - Linear Programming, EcM - Econometrics, DP - Dynamic Programming, PSO - Particle Swarm Optimisation

## 3. Problem formulation

Consider the two-leg shipping problem illustrated in Figure 2. The journey from A to B is called a *ballast leg* and consists of a voyage with no cargo. On a *laden leg*, a journey from B to A, the vessel is always loaded. This assumption reflects common practices in bulk shipping, where vessels frequently operate under long-term fixed routes, such as transporting raw materials between a mine and a processing port (Leite et al., 2020). Prior to commencing a journey, decisions need to be made about (1) the hedging ratio and (2) ship speed. The hedging ratio represents the percentage of the ship's cargo that will be sold at a pre-arranged fixed freight rate, while the ship speed is an average value and a proxy for the ideal travel time to complete a journey from one port to the other. This freight rate depends on the evolution in the futures market at the time of fixing the contract, whereas the speed needs to be selected from a feasible interval according to the ship's technical specifications.

To focus on understanding the core implications of joint speed and hedging optimisation, we consider a simplified A-to-B and back cycle, which provides a clear framework for analysing these decisions. In practice, shipping routes may occasionally change, but incorporating port selection decisions adds another layer of complexity that could obscure key insights. Extending the model to include such factors, while conceptually straightforward, presents additional technical challenges and is left for future research. Our model thus examines how to optimally decide on

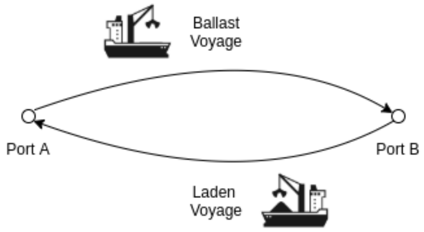


Figure 2: Ballast and Laden legs

the hedging ratio and vessel speed for each journey based on prevailing market conditions to maximise the net present value for the carrier.

The main assumptions can be summarised as follows:

- **Revenue**: The cashflow consists of two parts, the revenue received in the freight spot market and the hedging cashflow which can be positive or negative. Suppose the price in the spot market is $S$, the price in the forward market for the most suitable contract is $F$ and the contract is settled against $S^*$. ($S$, $F$ and $S^*$ are in US$ per metric tonne.) Then the revenue can be written as:

$$\underbrace{wS}_{\text{Spot market revenue}} + \underbrace{\gamma w_h(F - S^*)}_{\text{Hedging cashflow}}, \tag{1}$$

  where $w$ is the actual cargo in metric tonnes, $w_h$ is the cargo which we are hedging, $\gamma$ is the hedge ratio. Here we assume that both the hedge settlement and revenue come in the same time, i.e., the discount factors which apply to both of these payments are the same. For the rest of paper we assume that $w = w_h$.

- **Ornstein-Uhlenbeck (OU)** - The spot freight rate dynamics adhere to an exponential OU process. This rate operates within inherent boundaries that exert a gravitational force, tending to align the price with its long-term equilibrium. Empirical studies suggest significant mean reversion in the freight rate spot market (Adland and Cullinane, 2006; Benth and Koekebakker, 2016).

- **Forward market**: The forward freight agreement in the dry bulk market is settled against the average monthly or average weekly spot prices depending on the length of the settlement window of the contract. For our investigation we assume that the forward price is settled against the spot market on the date. In addition, the forward price is given by:

$$F(t,T) = E(S_T|S_t) + rp = S_t e^{r_0(T-t)}. \tag{2}$$

  The assumption that the underlying spot process follows an exponential Ornstein-Uhlenbeck (OU) model while the forward price $F_{0,T} = F(0,T)$ is expressed as $S_0 e^{r_0 T}$, where $r_0$ is a positive rate reflecting the upward slopping shape of the forward market. This formulation stems from a desire to capture the distinctive mean-reverting behaviour of spot prices in commodity markets while recognising the market's pricing of risk over time. The exponential OU process is particularly suitable for modelling commodities like physical freight because it accounts for the tendency of spot prices to revert to a long-term mean, reflecting market fundamentals and short-term fluctuations due to supply and demand shocks. Our modelling of $F_{0,T}$ incorporates the forward-looking nature of markets, where prices are not only a reflection of expected future spot prices but also of the risk premium required by investors. This form is akin to the geometric Brownian motion (GBM) often used in financial modelling, which simplifies the forward price as a deterministic function of time and initial spot price. This assumption allows for a clear distinction between the expected future spot price, $E(S_T|S_0)$, derived from the mean-reverting OU process, and the additional compensation investors require for bearing risk over the time horizon $T$. The risk premium (rp) embedded in the forward price captures the market's compensation for uncertainty and potential adverse movements in the spot price. Therefore, Eq. (2) encapsulates both the statistical expectation of future spot prices under the OU process and the economic reality of risk aversion. This dual approach provides a robust framework for understanding forward prices in commodity markets, reflecting both mean reversion in spot prices and the influence of risk premiums in forward contracts.

- **Basis**: While the spot market $S$ and the settlement price $S^*$ may significantly diverge in practice, with the settlement price being an average of multiple shipping price arrangements calculated by the Baltic Exchange, we assume that $S = S^*$. In other words we are able to access the exact hedge for the spot market.
- **Fuel consumption**: The fuel consumption function depends on the speed of the vessel (cubic law) as well as cargo loaded and the ship's dry weight (Ronen, 1982). We also apply bounds for the minimum and maximum speeds when we search for an optional solution. For both ballast and laden legs our speed bounds are the same. Other parameters of the ship remain the same throughout the lifetime of the ship. However, they may deteriorate in time as the ship ages.
- **Infinite time horizon**: The lifespan of a ship displays substantial variability and can be prolonged through technological advancements via retrofitting, and most ships are used well over 25 years. Mathematically, it is well known that an infinite horizon discounted problem is equivalent to a finite time horizon problem where the horizon is an exponentially distributed random variable (Bertsekas et al., 2011). Hence, the modelling approach applies to both finite and infinite horizons.

The hedging and vessel speed decisions precede the vessel's departure in either port. This means that the intervals between decision epochs will be variable in time, as their length depends on the selected vessel speeds. As the system's dynamics are stochastic and the intervals between decision epochs vary, we formulate the sequential decision making problem as a semi-Markov decision process (SMDP) (e.g., Puterman, 2014). Next, we introduce the SMDP model components, and present the algorithm to solve the resulting problem. Then, we derive an analytical relationship between optimal hedging and optimal speed. Table 2 summarises the notation.

### *3.1. Probabilistic structure: state space and transition probabilities*

Let $f_t$, $t \geq 0$ be a continuous time stochastic process representing the log price of the freight rate at any time $t \geq 0$, which takes values from the set of positive scalars $\mathbb{R}_+$. In addition, let $l_t \in L$ represent the ship's last visited port at time $t \geq 0$, where $L \triangleq \{0, 1\}$ is the set of ports that the ship visits over time. Here, $l = 0$ corresponds to Port $A$ in Figure 2, whereas $l = 1$ corresponds to Port $B$. The evolution of the system is represented by process $X_t = (f_t, l_t)$, $t \geq 0$, which monitors both the price and the sequence of port visits as time evolves. $S = \mathbb{R}_+ \times L$ is the state space of $X_t$, $t \geq 0$.

We assume that the log price process $f_t$, $t \geq 0$ is governed by the Ornstein-Uhlenbeck equation:

$$df_t = \theta(\bar{f} - f_t)dt + \sigma dW_t, \tag{3}$$

where $\theta > 0$ is a parameter that determines the speed of reversion to the mean, $\bar{f}$ is the long term mean, $\sigma > 0$ is a scalar parameter and $W_t$ is a standard Wiener process (Uhlenbeck and Ornstein, 1930).

To control the system's dynamics, decisions will be taken upon the vessel's arrival to either port in $L$. Hence, the sequence of decision times can be defined as $\tau = \{t \geq 0 : l_t^- \neq l_t^+\}$, where $l_t^-$ and $l_t^+$ denote the ship's last visited port immediately before and immediately after $t$, respectively. Notice that each time $t \in \tau$ corresponds to a change of the last visited port, and consequently to an arrival time at either port. That is when the decision maker must select a combination of vessel speed and hedging ratio for the next trip, which will start at the current port and end at the other port. Let $X_t = s = (f_t, l_t)$, where $s \in S$ is an element of the state space $S$, at some decision epoch $t \in \tau$. The decision maker will select an action $a \in A(s)$ from the set $A(s)$ of feasible actions when in $s$. Action $a \in A(s) = \Upsilon(s) \times \Gamma(S)$ is comprised of a travelling speed $v \in \Upsilon(s) = [v^s_{\min}, v^s_{\max}]$ and a hedging ratio $\gamma \in \Gamma(S) = [\gamma^s_{\min}, \gamma^s_{\max}]$. The parameters $0 < v^s_{\min} < v^s_{\max} < \infty$ (resp. $0 \leq \gamma^s_{\min} \leq \gamma^s_{\max} \leq 1$) correspond to the lower and upper bounds for the vessel speed (resp. hedging ratio) at state $s \in S$. Finally, $A = \bigcup_{s \in S} A(s)$ denotes the set of feasible control actions in the system.

Let $a = (v, \gamma)$ be the control decision at state $X_t = s = (f, l) \in S$ for some $t \in \tau$. Clearly, the next decision epoch $t' \in \tau$, $t' > t$ depends upon the speed decision $v$ at time $t$. We assume that the next decision epoch $t' =$

Table 2: Model parameters

| Variable | Units | Description |
|---|---|---|
| $D_l$ | nm | Distance in nautical miles from port l to port the other port. |
| $w$ | mt | Cargo intake in metric tonnes (Dead Weight Tonnage - DWT). |
| $v$ | knot | Speed of the vessel |
| $v_{min}$ | knot | Minimal speed of the vessel on route applies for both laden . |
| $v_{max}$ | knot | Maximal speed of the vessel on route. |
| $m^{TCH}$ | US$/day | Long time charter hire rate, the charterer is paying |
| $\alpha$ | % /year | Opportunity cost of capital |
| $r_0$ | % /year | Rate of the slope increase of the FFA curve |
| $\gamma$ | % | Hedge ratio as a fraction of the cargo intake |
| $\gamma_{min}$ | % | Minimum hedge ratio |
| $\gamma_{max}$ | % | Maximum hedge ratio |
| $C^u$ | US$ | The cost of unloading of the laden leg |
| $C^l_{0,1}$ | US$ | The cost of fuel as well as the auxiliary costs associated with port charges for ballast (1) and laden (2) legs correspondingly |
| $C^h_{0,1}$ | US$ | Auxiliary costs associated with port charges for ballast (1) and laden (2) legs correspondingly |
| $c^f_{0,1}$ | US$/mt | Fixed cost of fuel in US dollars per tonne, for ballast (1) and laden (2) legs correspondingly |
| $\theta$ | - | The rate of reversion to the mean of the OU process |
| $\sigma$ | US$/mt | The volatility of the OU process |
| $\bar{r}$ | US$/mt | The long-term mean of the freight rate |
| $t_h$ | days | Time spend in harbour, for loading and unloading |
| $\lambda$ | - | Risk aversion, sensitivity to risk |
| $w_0$ | mt | The lightweight of the ship |
| $k$ | - | Normalizing constant |
| $k_w$ | - | Elasticity of fuel consumption of the dead weight tonnage |
| $k_v$ | - | Elasticity of fuel consumption of speed of the vessel |
| $f_0$ | - | Idle engine fuel consumption adjustment |
| $w_b$ | mt | Ballast trip DWT, 25% of $w$ |

$t + g(v, l)$ is a deterministic function of the selected speed and of the current port, with $g(v, l) = \frac{D_l}{24v}$. Let $p^a_{ss'} = \text{Prob}\left( X_{t'} = (f', l') | X_t = (f, l) \right)$ be the probability of transitioning from state $s = (f, l)$ at time $t \in \tau$ to state $s' = (f', l')$ at the next decision epoch $t' = \min\{\rho \in \tau : \rho > t\}$ under action $a \in A(s)$. It follows that :

$$p^a_{ss'} = \begin{cases} \text{Prob}(f_{t'} = f' | f_t = f), & \text{if } \; t' = t + g(v, l), \text{ and } l' \neq l, \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where $\text{Prob}(f_{t'} = f' | f_t = f)$ is obtained from the freight rate dynamics in (3). As we can see from (4), the log-price at the next decision time is determined from (3), whereas the next decision time is a function of the selected speed at the current decision epoch and of the current port of origin $l$ (explained further in Section 3.3). Additionally, if the current port is $l \in L$, then the next port will be the other port $l' \in L \backslash \{l\}$.

To guide the optimal choice of sequential vessel speeds and hedging ratios, the next sections introduce the single-period reward and cost functions. The first characterises the profit from a given trip, whereas the latter includes both the operational costs and the risk penalty.

### 3.2. Rewards

Upon applying an action $a \in A(s)$ at state $s$, the system collects a reward $r(s, s', a)$, which is a function of the origin state $s$, the destination state $s' = (f', l')$ that will be reached at the next decision epoch, and the control action $a = (v, \gamma)$ applied. The reward function is defined as:

$$r(s, s', a) = \begin{cases} we^{f'} + \gamma w \left( e^{r_0 g(v,l)} e^f - e^{f'} \right) & \text{if } l = 0, \\ \gamma w \left( e^{r_0 g(v,l)} e^f - e^{f'} \right), & \text{if } l = 1, \end{cases} \tag{5}$$

where $w > 0$ is the total weight carried by the vessel, $g(v, l)$ is the travel time previously defined. The first expression of (5) quantifies the reward at the destination port (port 1), when the vessel is about to go ballast to port 1 to pick up cargo and the current freight rate at port 1 is $f$. Upon arriving at port 1, the vessel will receive the freight rate revenue

$we^{f'}$, plus the hedging settlement in the second parcel of the expression. The second expression in (5) corresponds to the laden leg from port 1 to port 0. In that situation, the shipping operator can still take potential advantage of shipping rates at port 1 by entering in a cash settled hedging contract, and their revenue will only correspond to the settlement of the hedging contract. We can rewrite (5) as:

$$r(s, s', a) = \begin{cases} \gamma w e^{r_0 g(v,l)} e^f + (1-\gamma) w e^{f'} & \text{if } l = 0, \\ \gamma w \left( e^{r_0 g(v,l)} e^f - e^{f'} \right), & \text{if } l = 1. \end{cases} \tag{6}$$

The effect of the hedging is perhaps more evident in (6). One can see that in case $l = 0$, the shipping operator will receive the hedging agreement upon arrival at port 1 (first term of the right-hand side), in addition to the freight's portion that was left un-hedged and given by the second term of the right-hand side. From port $l = 1$, on the other hand, only the hedging contract will be realised at the end of the voyage.

An example of stochastic revenue cash flows is shown in Figure 3. The x-axis represents time in days, while the y-axis shows the stochastic spot freight rate, which changes according to an exponential mean reversion process. When the ship arrives to the port of loading, the price is determined on the spot and then after the delivery of the cargo back at port A the cash flow is received (red arrows).

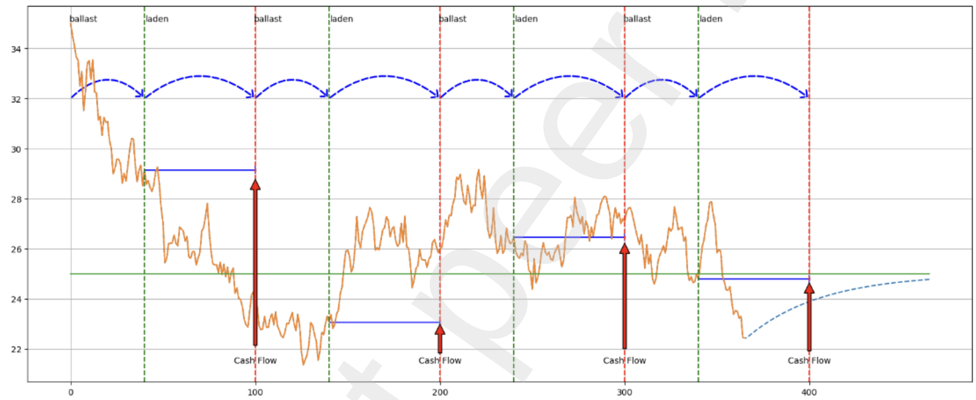


Figure 3: Spot revenue cash flows are a function of the spot rate at the time of agreement

### 3.3. *Costs*

The cost of the shipping operator has several components. There are fixed costs associated with loading and unloading of the cargo: $C_l$ and $C_u$ respectively. The bunker cost is associated with fuel consumption (measured in tonnes per day) and depends on the speed as well as the cargo load as follows (Psaraftis and Kontovas, 2013):

$$\mathcal{F}(v, w) = k(f_0 + v^{k_v})(w + w_0)^{k_w}. \tag{7}$$

The parameters in (7) are presented in Table 2.

For the ballast and laden legs, the total fuel consumption is

$$g(v, l)\mathcal{F}(v, 0)c_f,$$

where scalar $c_f > 0$ is the price per barrel of bunker fuel.

Finally, the charter hire rate $m_{TCH} > 0$ is accrued daily, this yields that the overall charter (i.e., renting) expenses for a given trip can be written as:

$$\int_0^{g(v,l)} e^{-\alpha t} m_{TCH} dt = m_{TCH} \frac{1 - e^{-\alpha g(v,l)}}{\alpha}.$$

Adding up all the expenses previously explained in this subsection, the overall cost per trip becomes:

$$c(s, s', a) = \begin{cases} e^{\alpha g(v,l)}(C_u + c_f \mathcal{F}(v, w_b) g(v, l) + m_{TCH} \frac{1-e^{-\alpha g(v,l)}}{\alpha}) & \text{if } l = 0, \\ e^{\alpha g(v,l)}(C_l + c_f \mathcal{F}(v, w_l) g(v, l) + m_{TCH} \frac{1-e^{-\alpha g(v,l)}}{\alpha}), & \text{if } l = 1. \end{cases} \tag{8}$$

We see that the cost of the trip from $l$ to $l'$ does not depend on the state of arrival $(f', l')$. From Section 3.2, however, it follows that the rewards do depend on the destination state $s' = (f', l')$. Therefore, it follows that the overall profit of either leg depends on both the departure state $(s)$ and the destination state $(s')$.

### 3.4. Utility

As per Levy and Markowitz (1979) we approximate the expected utility function using a mean-variance framework, in which the expected utility of a random profitability variable $x$ in defined in the following way:

$$E[U(x)] = E(x) - \lambda Var(x), \tag{9}$$

where $\lambda > 0$ is the decision maker's risk aversion parameter. Higher values of $\lambda$ imply increased aversion to risk, while lower values of $\lambda$ imply increased appetite for risk. The first part of the equation $E(x)$ is the expected profitability, while the second part of the equation can be regarded as a penalty for the profitability risk. In the next subsection, we will apply this concept to the voyage's revenue in (6).

### 3.5. Value Function

Let us define a decision policy $\pi : S \to A$ as a mapping from states in $S$ to actions in $A$ that specifies, for each state $s \in S$, a single action $\pi(s) \in A(s)$ to be taken each time the controlled process $X_t$, $t \geq 0$ visits state $s$ at a given decision epoch $t \in \tau$, and let $\Pi$ be the set of all feasible policies. For a policy $\pi \in \Pi$, let $H^\pi_\alpha(s)$ denote the infinite horizon discounted reward, given that the process occupies the state $s = (f, l)$, i.e., given that the shipping freight rate at is $f$ and the ship is in Port $l$:

$$H^\pi_\alpha(s_0) = E^\pi_{s_0} \left\{ \sum_{n=0}^{\infty} e^{-\alpha\tau_n} \Big[ U(r(s_n, s_{n+1}, a_n) - c(s_n, s_{n+1}, a_n)) \Big] \right\}, \tag{10}$$

where $\tau_0, \tau_1, ...$ are the times of the successive decision epochs in $\tau$.

The optimal policy $\pi^* \in \Pi$ maximises the value function for each possible initial state $s \in S$, which satisfies:

$$\pi^*(s) = \arg\max_{\pi \in \Pi} H^\pi_\alpha(s). \tag{11}$$

For the shipping operator, the optimal policy will provide for each possible state $s = (f, l)$ - freight rate is $f$ at location $l$, an action that maximises the expected discounted future reward. For example if the shipping rate $f$ is low, it could be optimal to slow down whilst waiting for an increase in the freight rate. Or if the freight rate is high, it could be optimal to hedge the freight rate so when it potentially drops the operator is compensated for the opportunity loss.

The value function in (10) can be presented in the following form as per (Puterman, 2014, Section 11.3.1):

$$H^\pi_\alpha(s_0) = E^\pi_{s_0} \left\{ \sum_{n=0}^{\infty} e^{-\alpha\tau_n} u(s_n, s_{n+1}, a_n) \right\}, \tag{12}$$

where

$$u(s_n, s_{n+1}, a_n) = \Big\{ U(r(s_n, s_{n+1}, a_n) - c(s_n, s_{n+1}, a_n)) \Big\}. \tag{13}$$

Further simplifying notation and using $u_{a_n}(s_n) = E^\pi_{s_n} \Big\{ u(s_n, s_{n+1}, a_n) \Big\}$, we can write:

$$H^\pi_\alpha(s_0) = u_{a_1}(s_0) + E^\pi_s \Big\{ e^{-\alpha\tau_1} H^{\pi'}_\alpha(s_1) \Big\}, \tag{14}$$

where $\pi' = (a_2, a_3, ...)$. As we are interested in the stationary policy, $d^\infty = \pi'$, hence:

$$H_\alpha^{d^\infty}(s_0) = \left[ u_{a_1}(s_0) + E_s^{d^\infty} \left\{ e^{-\alpha\tau_1} H_\alpha^{d^\infty}(s_1) \right\} \right]. \tag{15}$$

For the optimal policy $\pi^*$, the value function is the following:

$$H_\alpha^{\pi^*}(s_0) = \sup_{a \in A(s_0)} H_\alpha^{d^\infty}(s_0) = \sup_{a \in A(s)} \left[ u_a(s_0) + E_s^{\pi^*} \left\{ e^{-\alpha\tau_1} H_\alpha^{\pi^*}(s_1) \right\} \right], \tag{16}$$

where $a^* = (v^*, \gamma^*)$ is the optimal action consisting of the optimal speed and optimal hedge ratio. In a discrete space of states, the expectation can be written in the following way:

$$H_\alpha^{\pi^*}(s_0) = \sup_{a \in A(s_0)} \left[ u_a(s_0) + e^{-\alpha\tau_1} \sum_{s_1 \in S} \left\{ p(s_1|s_0, \pi^*) H_\alpha^{\pi^*}(s_1) \right\} \right], \tag{17}$$

where the sum is taken for all possible states $s_1$, and the probability $p(s_1|s, \pi^*)$ defined in (4).

### 3.6. Solution of the semi-Markov formulation

A classical approach to deriving the optimal policy of a semi-Markov decision process is via the value iteration algorithm. It is this approach that we will employ to find the solution to (10). The algorithm iterates in the space of real-value functions from $\mathcal{H} : S \to \mathbb{R}$ and iteratively refines an estimate $H_b^* \in \mathcal{H}$ of the solution to (10), up to convergence. It is well-known that such an algorithm is guaranteed to converge to the optimal solution under mild assumptions, see Puterman (2014) for further details.

Algorithm 1 shows the pseudo-code of the *value iteration* algorithm utilised in our experiments. The value function update in Step 8 of the algorithm is repeated up to the convergence to the solution of (10), at which point one can retrieve the optimal policy $\pi^* : S \to A$ that comprises a single speed-hedging combination for each state in $S$. For any given state $s \in S$, $\pi^*(s) = (\gamma^*(s), v^*(s))$ describes the optimal speed-hedging strategy for the next voyage.

**Algorithm 1** Value iteration algorithm

1: $H_b^* \leftarrow 0$
2: **while** $d > \epsilon$ **do**
3: $\quad H_f^* \leftarrow SolvingBestAction(H_b^*)$
4: $\quad d \leftarrow \max(|H *_f - H_b^*|)$
5: $\quad H_b^* \leftarrow H_f^*$
6: **end while**
7: **function** SolvingBestAction($v_b^*$)
8: $\quad$ **for all** $s \in S_{grid}$ **do**
9: $\quad\quad H_f^*(s) \leftarrow \max_{\gamma,v} \left\{ U(s,\gamma,v) + e^{-\alpha\frac{D}{24v}} \sum_{s' \in S_{grid}} p(s'|s,v) H_b^*(s') \right\}$
10: $\quad\quad \gamma^*(s), v^*(s) \leftarrow \text{argmax}_{\gamma,v} \left\{ U(s,\gamma,v) + e^{-\alpha\frac{D}{24v}} \sum_{s' \in S_{grid}} p(s'|s,v) H_b^*(s') \right\}$
11: $\quad$ **end for**
12: **end function**

### 3.7. The optimal hedge ratio

Next, we will explore the optimal solution found in Step 8 of Algorithm 1 to derive a closed-form solution of the relationship between optimal hedging and optimal speed.

**Theorem 1.** *Let $\gamma_l$ and $\gamma_b$ denote the hedge ratio at the laden (departing at Port B, $l = 1$) and ballast (departing from Port A, $l = 0$) legs, respectively. Then it follows that:*

$$\gamma_l = \frac{rp}{2\lambda w \sigma_{v^*}^2} + 1, \quad \gamma_b = \frac{rp}{2\lambda w \sigma_{v^*}^2}, \quad rp = e^s - E(e^{s'}|s, v^*) \tag{18}$$

*where $rp$ is defined by the evolution of the forward market (2), and $\sigma^*_v$ is the volatility of the spot market at the time horizon determined by the optimal speed of the vessel $v^*$.*

*Proof.* From (16) one can clearly see that, for an action $a = (v, \gamma)$, only the speed $v$ directly impacts the travel time and, consequently, the arrival time at the next port $\tau_1$. Therefore, we can derive optimal hedge ratios directly through the maximisation of the short-term reward. This yields:

$$\gamma^*(s) = \arg\max_{\gamma\in\Gamma, v^*} \left\{ u_a(s) \right\}. \tag{19}$$

Note that the utility in (13) comprises a reward and a cost term and the latter, given by (8), does not depend upon the hedge ratio $\gamma$. Therefore, for the optimal speed $v^*$, it suffices to find $\gamma^*$ such that:

$$\frac{\partial}{\partial\gamma}\left\{ E_s(r(s, s', a)) - \lambda Var(r(s, s', a)) \right\} = 0. \tag{20}$$

As both the expectation and variance are taken to the state $s'$, to which we transition under optimal speed $v^*$, the time horizon for that expectation will be $T^* = \dfrac{D}{24v^*}$ and hence will depend on the optimal speed. This provides the link between the optimal hedging policy and the speed of the vessel in general.

Substituting (6) into (20), we obtain the following expressions for the ballast and laden leg:

$$\frac{\partial}{\partial\gamma_l}\left\{ wE_s(e^{s'}) + \gamma_l w \underbrace{(F_{s,v^*} - E_s(e^{s'}))}_{rp=risk\ premium} - \lambda w^2(1-\gamma_l)^2 \underbrace{V_s(e^{s'})}_{\sigma^2_{v^*}} \right\} = 0, \tag{21}$$

$$\frac{\partial}{\partial\gamma_b}\left\{ \gamma_b w \underbrace{(F_{s,v^*} - E_s(e^{s'}))}_{rp=risk\ premium} - \lambda w^2\gamma_b^2 \underbrace{V_s(e^{s'})}_{\sigma^2_{v^*}} \right\} = 0 \tag{22}$$

where $F_{s,v^*}$ is the forward price at the state $s$ agreed before the trip, $F_{s,v^*} = F(t, T^*)$ in Eq. (2). Eq. (18) follows immediately from the above. □

The meaning of the theorem is that the optimal hedge ratio can be found simply by maximising the reward rather than the whole reward and the discounted value function. To this extent, the hedging policy is myopic and hedges only immediate risk. The experiments suggest that the value function is still strongly monotonic with respect to the freight rates showing exposure to the freight rates.

## 4. Experiment Results

We examine several aspects of the model in the numerical experiments. The strategy of the experiment is presented in Figure 4. Firstly, we are interested in how the parameters of the OU model and the discount premium in the FFA market impact the value function, the optimal speed and the hedging strategy. Then we study the interaction between optimal speed and optimal hedging, established in Theorem 1, and whether joint optimisation bears any economic significance. For the numerical experiment, we consider the US gulf to China route, where the freight can be hedged in $ per metric ton. Further parameter settings to specify the round-trip characteristics in the model can be found in Table 3.

Table 3: Roundtrip specification

| Variable | Value | Units | Description |
|---|---|---|---|
| $D$ | 13,000 | nm | Distance in nautical miles from port A to port B. |
| $w$ | 150,000 | mt | Cargo intake in metric tonnes (Dead Weight Tonnage - DWT). |
| $v_{min}$ | 10 | knot | Minimal speed of the vessel on route applies for both laden . |
| $v_{max}$ | 20 | knot | Maximal speed of the vessel on route. |
| $m^{TCH}$ | 20,000 | US$/day | Long time charter hire rate, the charterer is paying |
| $\alpha$ | 8 | % | Opportunity cost of capital |
| $r_0$ | 8 | % | Annualized slope increase of the FFA curve |
| $\theta$ | 3 | - | The rate of revertion to the mean of the OU process |
| $\sigma$ | 8 | US$/mt | The volatility of the OU process |
| $\bar{r}$ | 22.5 | US$/mt | The long-term mean of the freight rate |
| $t_h$ | 2 | days | Time spend in harbour, for loading and unloading |
| $\lambda$ | $1.1x10^{-5}$ | - | Risk aversion, sensitivity to risk |
| $C^u$ | 300,000 | US$ | The cost of unloading of the laden leg |
| $c^f_{1,2}$ | 500 | US$/mt | Fixed cost of fuel in US dollars per tonne, for ballast (1) and laden (2) legs correspondingly |
| $w_0$ | 49,000 | mt | The lightweight of the ship |
| $k$ | $3.91 \times 10^{-6}$ | - | Normalizing constant |
| $k_w$ | 0.67 | - | Elasticity of fuel consumption of the dead weight tonnage |
| $k_v$ | 3.1 | - | Elasticity of fuel consumption of speed of the vessel |
| $f_0$ | 381 | - | Idle engine fuel consumption adjustment |
| $w_b$ | 75,000 | mt | Ballast trip DWT, 25% of $w$ |

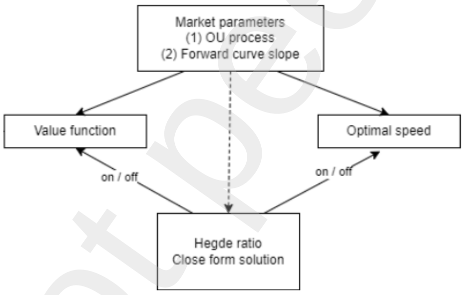


Figure 4: Analysis of Experimental Results and Their Interdependence

### 4.1. Value function and process parameters

The underlying process for the freight rate significantly impacts the value function, as shown in Figure 5 for all the parameters of the underlying freight rate process in Eq. (3). The first plot shows that for different levels of the long-term mean of the process, the value function increases in magnitude while keeping the same shape. This is intuitive as the value function is the long-term profit of the voyages in time; the higher the rate, the better this is for the business. The second plot shows the dependence of the value function on the rate of reversion. Observe that the faster the process reverts to the mean, the more stable the price is, and, therefore, the more stable the present value of the cash flows, i.e., the value function. This is intuitive as a high rate of reversion would mean a nearly instantaneous reversion to the mean, therefore rendering the price deterministic and the value function constant and independent of the initial state. The third plot demonstrates that the volatility of the process also affects the value function, although the differences are not as significant as they were for the two other parameters. The higher the volatility, the higher the risk premium to reward the shipping operator for taking the risk, especially when the rates are lower, as we observe wider gaps for lower rates.

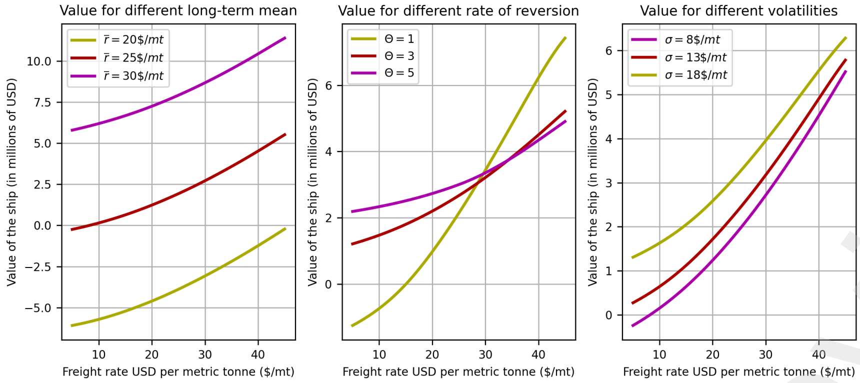


Figure 5: Value function for different parameters of the OU process

One overarching observation for the value functions in Figure (5) is that they are all convex with respect to the freight rate. We will next evaluate the effects of the parameters on the optimal hedging and speed decisions.

### *4.2. Vessel speed and the process parameters*

The optimal ballast and laden speeds are presented in the Figures 6 and 7. The ballast speeds are normally higher than the laden speeds as additional tonnage significantly increases the daily consumption rate as per Eq. (7). Both laden and ballast speeds increase with the freight rate, and this is consistent across all the parameters. The speed growth is not linear but rather consistent with the power laws suggested in the literature (Stopford, 2009). The most visible and interesting change in the speed is related to the ballast leg under different rates of mean reversion. While for slow reversion ($\theta = 1$), the speed stretches from 5 to 20 knots, for faster reversion rates (e.g., $\theta = 5$) the range of speeds is reduced. Hence, speeds are more stable for higher rates of mean reversion, which supports the empirical observation that real-world speeds tend to be less sensitive to freight rates than the theory suggests (Adland and Jia, 2017), see Appendix A.

Clearly, the changes in freight rate volatility have the least impact on the optimal speed. This is somewhat expected, as the purpose of the hedging decisions is to offset such volatility. As a byproduct, this also offsets the effect of the volatility on the speed decisions.

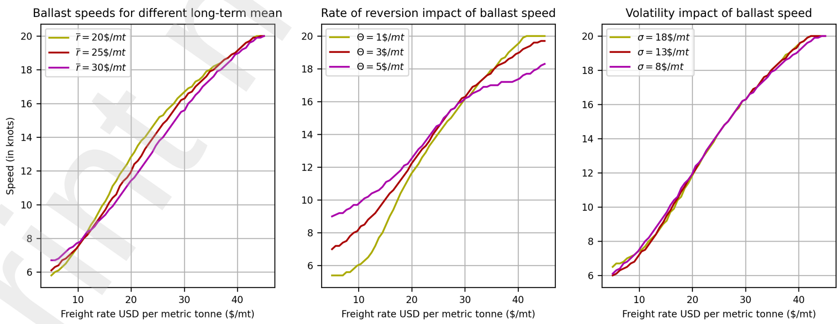


Figure 6: Optimal ballast speed as a function of parameters of the freight process

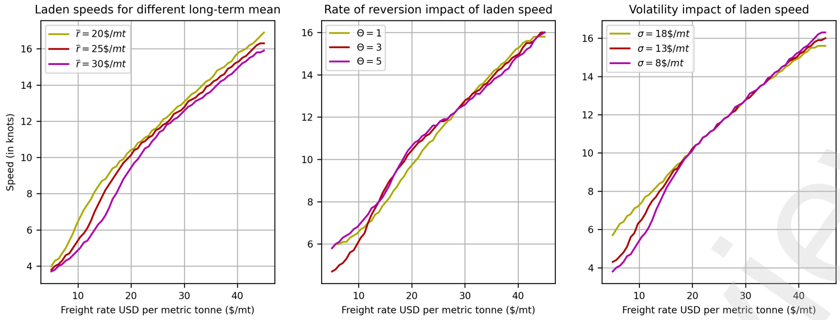


Figure 7: Optimal laden speed as a function of parameters of the freight process

### *4.3. Hedge ratio and the process parameters*

While the dependence between hedge and speed decisions has been established in Theorem 1, Figure 8 explores the influence of different OU parameters into the optimal hedging decisions. As hedging is to offset the volatility of the freight rates, the hedging ratio is limited to the interval $[0, 1]$. Notice that hedging tends to be applied for higher freight rates, and avoided for lower rates. This is to be expected, as hedging becomes uneconomical as the freight rate decreases below the long-term average. Furthermore, hedging ratios are higher in the laden leg regardless of the OU parameters.

The first plot in Figure 8 shows that the optimal hedging decisions have a similar trend regardless of the long-term average ratio. It is also evident that the hedging ratio is inversely proportional to the long-term average, as expected. The middle plot in Fig. 8 indicates that the hedging ratio tends to increase more steeply for higher mean reversion rates. Furthermore, the hedging ratio is inversely proportional to the mean reversion rate. Finally, the third plot in the figure shows that, for any given freight rate, the hedging ratio increases with the volatility rate, as expected. A lower value of $\theta$ indicates a reduced risk premium for a given state, attributable to the slower reversion rate over the journey's duration.

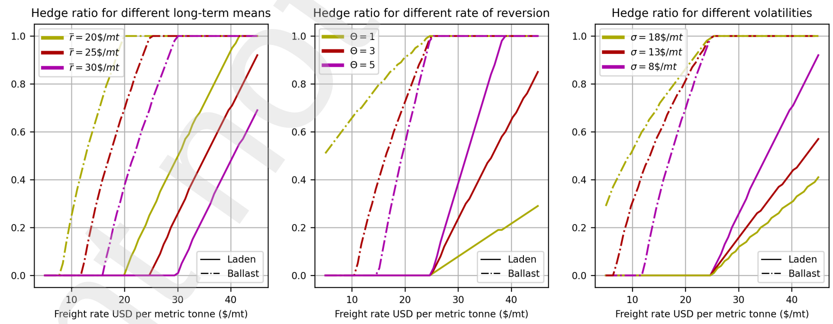


Figure 8: Optimal hedge ratio for different values of parameters and different journey legs

### *4.4. Hedging and speed*

Hedging adds value due to the fact that when the rates are high, hedging them picks up the risk premium and diminishes the risk of the business as reflected in the utility function, see the Figure 9.

Hedging operations also impact the optimal speed, as illustrated by the significant changes in the optimal speed when we allow hedging (hedging on) and when we do not allow it (hedging off). When hedging is off, there is increased variation as the optimal speed is a way to anticipate the market by slowing down in the high market, and speeding up in the low market. When hedging is on, it tackles market volatility, allowing higher speed in favourable market conditions and, consequently, an increased number of round trips.

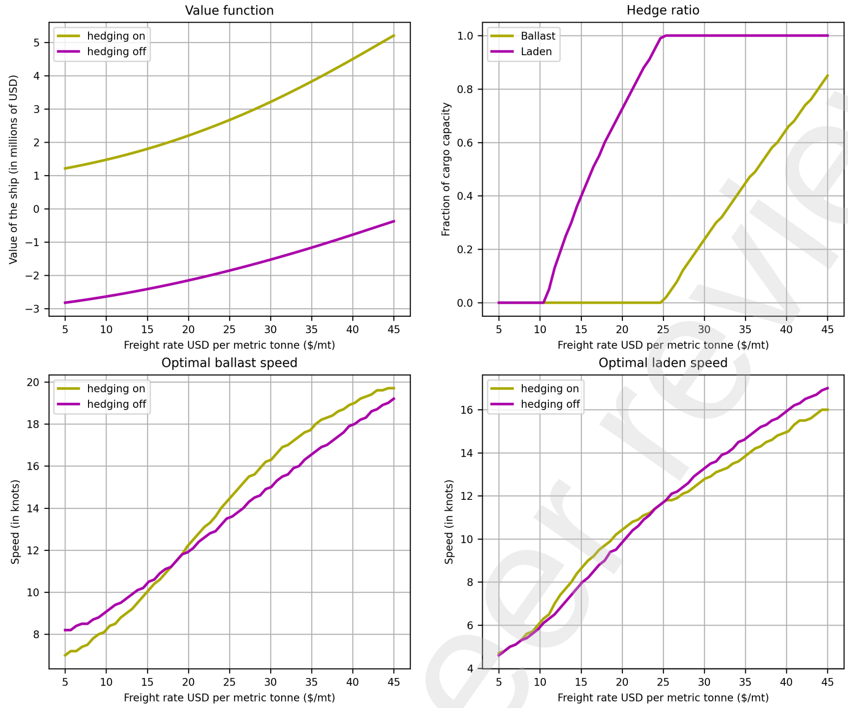


Figure 9: Simulation with hedging and without hedging. Hedging not only adds value but also impacts the optimal speed - more significantly in the ballast leg

The upper left plot in Figure 9 demonstrates that, as expected, hedging adds value to the operation, whereas the upper right plot shows that hedging tends to be more aggressive during the laden leg, and less intense when playing a speculative role in the ballast leg. The penalty for risk which we introduced stimulates hedging decisions even when the price is low, as the model seeks to balance profit and risk.

Notice that the hedging decisions are taken for each individual journey, and depend on the speed and on the parameters of the spot price evolution our approach hedges a single trip (Theorem 1). The effects, however, accumulate over time as the difference in the value functions with hedging on and off demonstrates. A possible avenue for future work is to extend hedging decisions to multiple trips as a way to provide increased stability over long-term operations. This would potentially require an extension of the state space to accommodate decisions that spread across multiple time periods.

To further validate the approach, the next sub-section will evaluate the outcome of our approach in real market conditions. We will apply our speed and hedging decisions and validate their overall utility in the light of retrospective market data for Baltic Route C3, between the Ports of Tubarão (Brazil) and Qingdao (China).

### *4.5. Experiment on Real Data*

This study simulates the journey of a capesize vessel under various operational policies to evaluate and compare different strategies for determining the ship's speed and hedging ratio. The focus of this experiment is the Baltic Route C3, from Tubarão, Brazil to Qingdao, China, with a fully loaded cape vessel carrying 160,000 metric tonnes (mt). A visual representation of this route is shown in Figure 11. The simulation utilises historical spot freight prices from 2019 to 2024, using data denominated in USD per metric tonne (Figure 10).

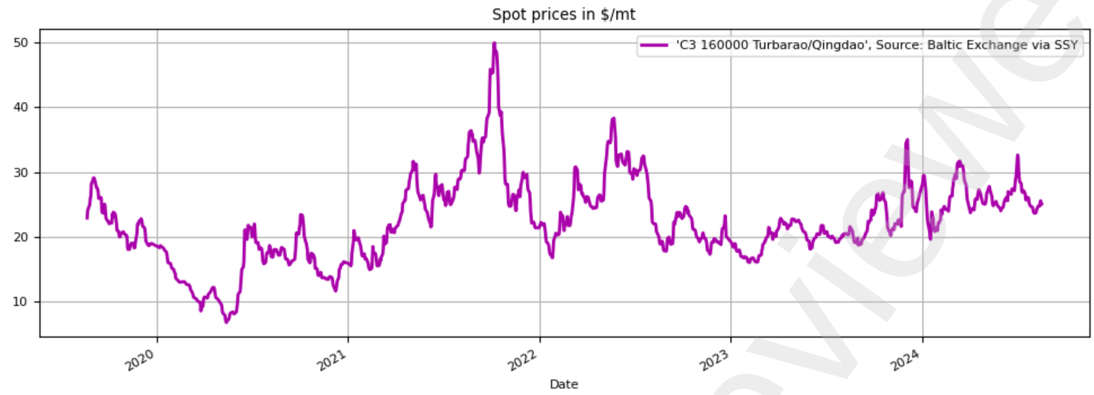


Figure 10: Spot prices for the C3 160,000 mt route from Tubarão to Qingdao.


Figure 11: Route from Tubarão to Qingdao, spanning 10,800 nautical miles, as depicted on the Baltic Exchange website.

For each spot price we fit the exponential OU process please see the methodology **??** , we assume that the estimated parameters of the process persist overtime and are properties of the concrete market. After the parameters have been estimated we apply four distinct policies to assess their effectiveness in adapting to the observed spot price movements. It is important to note that this experiment does not establish the superiority of one policy over another but rather demonstrates the application of each policy under real market conditions. The policies are as follows:

- **Stochastic optimisation policy:** This policy, developed in this research, uses historical data to estimate the parameters of the Ornstein-Uhlenbeck (OU) process that describes the spot freight price dynamics - Eq. (3). The optimal speed and hedge decisions are computed based on the parameters estimated from the data and then applied to the observed freight value a each decision period in the dataset.
- **Myopic policy:** In this approach, the optimal speed maximises the time charter equivalent (TCE): the daily profit within a single journey. The hedging policy is the same as in the stochastic policy.
- **Dummy policy:** In this approach, the vessel maintains a constant speed for both ballast and laden voyages, regardless of the market's spot rate. The ballast (resp. laden) speed is set to the long-term average ballast (resp.

laden) speed under the stochastic optimal policy, ensuring comparability. The hedging policy is the same as in the stochastic optimisation policy.

- **Deterministic policy:** This policy determines the optimal speed and hedge ratio that maximise the value function, assuming a constant spot rate equal to the long-term mean of the OU process.

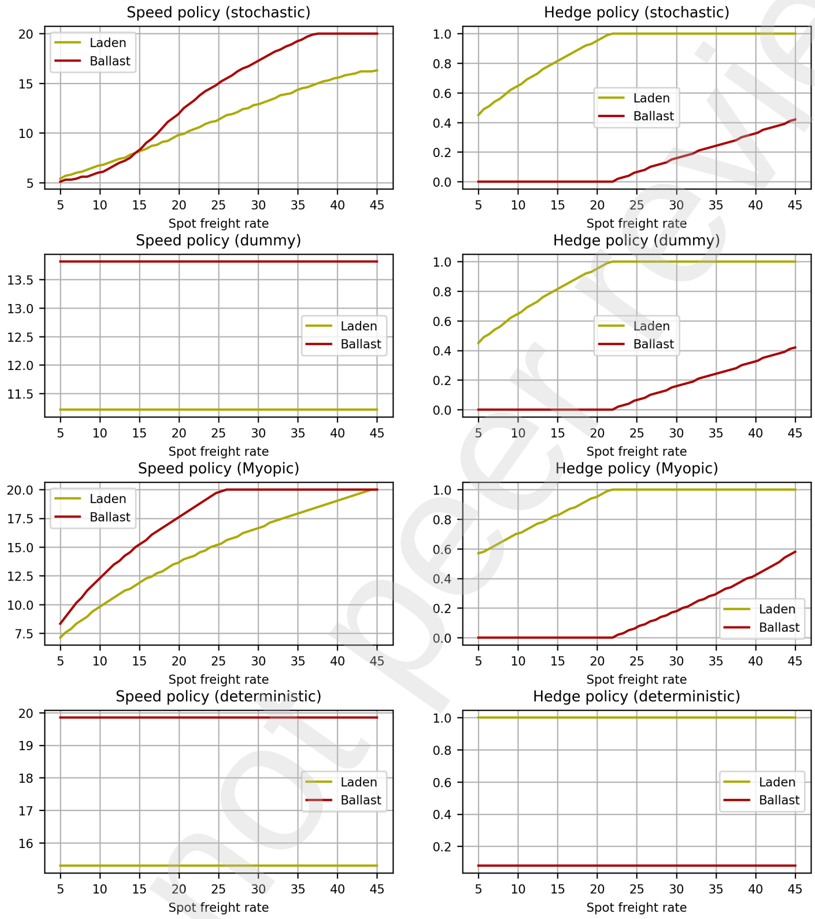


Figure 12: Policies applied at each market state and leg of the journey.

The simulation splits the ship's journey into a series of legs, with each leg representing a decision point. At each decision point, the ship's state is a pair comprising the *current spot freight price* and the *current port*, see Section 3.1.

Additionally, we track the cumulative time elapsed and the cumulative profit accrued up to each decision point. At each decision point, a policy provides an action pair (speed, hedge), where:

- **Speed:** The sailing speed of the ship, which impacts fuel consumption and voyage duration.

- **Hedge Ratio:** The proportion of market exposure that is hedged to mitigate financial risk.

Table 4 presents a summary of the results obtained from applying all three policies to the given route. Detailed analysis of each policy's performance is discussed in Appendix C. Notably, the Stochastic policy significantly outperforms each competing policy. Interestingly, the closest performance is obtained by the *Myopic policy*, with a profit reduction of about 7% with respect to the Stochastic policy. This highlights the importance of properly considering the long-term uncertainty in the model.

Table 4: Summary of Metrics for Turbarao / Qingdao Route

| **Metric** | **Stochastic** | **Dummy** | **Myopic** | **Deterministic** |
|---|---|---|---|---|
| Profit (USD) | 19,992,030 | 18,216,320 | 18,562,990 | 16,003,710 |
| Ballast (USD) | -18,026,000 | -18,084,400 | -25,255,860 | -16,584,190 |
| Laden (USD) | 38,018,040 | 36,300,730 | 43,818,850 | 32,587,900 |
| Roundtrips | 21 | 22 | 27 | 20 |
| Total Days | 1,838 | 1,826 | 1,837 | 1,829 |
| Avg Time Ballast (days) | 40.99 | 36.87 | 29.32 | 40.01 |
| Avg Time Laden (days) | 46.63 | 44.40 | 36.25 | 48.85 |
| Avg Daily Profit (USD/day) | 10,877 | 9,976 | 10,105 | 8,750 |
| Daily Profit Ratio | 1.0000 | 0.9172 | 0.9290 | 0.8045 |

The results of this experiment reveal that adapting the ship's speed according to market conditions enhances overall profit by approximately 10% (Stochastic versus Dummy policy). This improvement is primarily due to the strategic timing of market entry and exit based on market conditions. The combo of optimal speed and hedging decisions result in a 20% increase in profitability (Stochastic versus Deterministic policy). Curiously, the Myopic policy will result in a considerably larger number of trips per unit of time, but with a reduced profit with respect to the Stochastic policy. The fuel consumed applying such a policy is the highest among other policies and this can potentially create a problem with emissions. The policy under-performs due to the implicit assumption that the prevailing freight rate will remain constant.

In particular, the Stochastic policy exhibits longer average ballast times than the Dummy policy, which assumes a constant ballast speed. During periods of low market rates, the Stochastic model suggests reducing speed to wait for favourable market conditions to return. Conversely, in high-market conditions, the model advises accelerating to capitalise on profitable opportunities. Moreover, as the market premium increases, the model recommends increasing hedge ratios, aligning with the goal of maximising risk-adjusted returns.

Overall, these findings underscore the potential value of our dynamic approach compared to deterministic or TCE maximisation policies, highlighting its relevance in real-world shipping operations.

## 5. Concluding remarks

The assumption about the underlying freight rates process significantly impacts the value function and the decision making policy regarding optimal speeds and hedging. The long term mean of the freight rate reflects overall profitability of the business. The rate of the mean reversion plays a significant role in the decision making process as the optimal ballast speed in particular becomes less sensitive to the market conditions. Under strong mean reversion the value function becomes more stable and less dependent on the prevailing market conditions. Hedging behaviour is conditional on the presence of the risk premium in the market for the long run. Hedging is preferable even in the presence of the negative risk premium; and especially when the risk aversion is high, which is typical for a small tramp ship operator. As the hedging impacts the economics of the revenue in the presence of the risk premium, hedging impacts the optimal speeds. Full hedging allows maintaining speed policies that do not need to adjust to time the freight market, but rather is a mechanism to allow the execution of more or less round-trips.

The choice of speed is a balancing act between the fuel cost saving and the number of journeys via time saved. The lower speeds discount heavily the future profit potential, the higher speeds appreciate the future profit potential. While the sailing speed depends on the current costs and the current revenues, it is also affected by the future expected revenues and the stochastic process of spot prices. As these future revenues are uncertain, hedging the freight risk makes a difference on the economics of the business and hence on the speed decisions. The stochastic optimisation allows to calculate the optimal decision based on the current conditions as well as on the expected conditions in the future therefore provides a more optimal decision making toolkit for the shipping operator, because it takes into the account possible states of the future profit potential, which is required to calibrate the balance between the short term cost saving and the long term discount of the future revenues.

Optimal decisions are impacted by the dynamics of the freight rate dynamics; we used an exponential OU process and provide both intuitive arguments and literature studies in support of this choice. We see that the greater is the rate of mean reversion the less sensitive is the speed to the freight rate. The higher is the rate, the greater is the chance to pick up a higher freight rate if we speed up. The speed has a market timing effect which can significantly impact the longer term profits.

As we saw in the theorem, the hedging decisions are impacted by the speed, simply because of time horizon projections. However, the optimal hedging decisions impact the value function by increasing the pick up of a potential risk premium and reducing risk exposure. This impact the long term projections of the revenue and therefore the optimal speed decisions adapt to that new value function. This is the reason the speed is different in the presence of hedging.

In summary, this paper is first to propose a stochastic sequential decision framework for joint speed and hedging optimisation. The results show how shipping operators need to strike a balance between current and future profit potential, considering both operational and economic prospects. This suggests that delivery times should be flexible enough at the contract negotiation stage to allow operators to choose speed and hedging optimally. We demonstrate that optimal vessel speed and optimal hedging are interlinked and therefore should be jointly considered.

In addition to the speed, the optimal hedging also depends on a risk tolerance parameter that represents the operator's trade-off appetite for the volatility related to risk premium earnings.

Future research could explore extending this framework to include other types of risks such as bunker cost and carbon footprint, providing a more comprehensive optimisation model for the maritime industry. This work lays the groundwork for such future developments by integrating hedging and speed decisions under stochastic freight rate dynamics.

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## Appendix A. Speed square root 'rule' derivation

Consider that the ship takes cargo of weight $W$, and is paid for this $R$. The ship travels with a speed $v$ a distance $D$ and carries a dead weight tonnage $W_l$. On a daily basis, the fuel consumption of the ship is given by the cube law $f(v) = k(W_l + L)^h(p + v^3)$, the cost of fuel is $c$, and the total time at sea is $t$, where $t = \frac{D}{24v}$. The Time Charter Equivalent (TCE) then is:

$$TCE(v) = \frac{RW}{D/24v} - k(W_l + L)^h(p + v^3)c$$

Taking the derivative in respect with $v$ and equalizing it with 0, leads to:

$$v = \sqrt{\frac{8RW}{k(W_l + L)^h cD}}$$

Now if you are paid more then the long term market rate do you have to speed up. What if you are travelling on the ballast leg from the port of delivery, what is the speed you have to choose, not zero for sure. To treat these basic flaws in this formulae is what we do in this paper. To get to a simplified version imagine as previously we consume based on the cube law, we also have another cost which is time charter hire which we pay on the daily basis. Finally at the port of destination we be it ballast leg or laden leg (ballast: $l = 0$, laden: $l = 1$) we expect to receive another job, so in the port of the destination we will expect future profit potential. We can write the revenue $P$ which depends on the speed in the following way:

$$P(v) = RW_l - k(W_l + L)^h(p + v^3)c(D/24v) - f_{TCH}(D/24v) + e^{-\alpha(D/24v)}FPP$$

Where the FPP is future profit potential which is determined as an average profit per trip ($\bar{f}_{trip}$) divided by the discount rate $\alpha$ (Ge et al., 2021*b*).

$$P(v) = RW_l - k(W_l + L)^h(p + v^3)c(D/24v) - f_{TCH}(D/24v) + e^{-\alpha(D/24v)}\bar{f}_{trip}/\alpha$$

After some simplifications and approximations, we can see that the optimal speed follows a cubic root rule which is slower than the square root rule the literature normally refers to (Stopford, 2009).

$$v = \sqrt[3]{\frac{f_{TCH} + \bar{f}_{trip}}{2ck(W_l + L)^h}}$$

Magirou et al. (2015) obtains similar solution however without considering the future profits and assuming that in the long-run the time charter rates will grow with the market. However, the implications of the future profits are of most importance while the TCH rates are fixed and agreed, the future profit potential is what keeps the problem dynamic and requires an active speed management. The cubic root could explain much lower sensitivity of the speed to the market conditions which are dictated by the time charter hire rate as well as the future profit potential. The greater is the time charter hire, the greater is the time toll, therefore speeding up is more reasonable, while the higher is the future profit potential, the higher is the profit you can make out of the many trips, so speeding up makes you do more trips.

Rather than relying on the long term future profit potential in our stochastic optimisation paper we make speed decisions based on the current market conditions, therefore we speed up or down more optimally than the cubic root look above. This edge we extract from the optionally the stochastic optimization provides.

## Appendix B. Combined plots

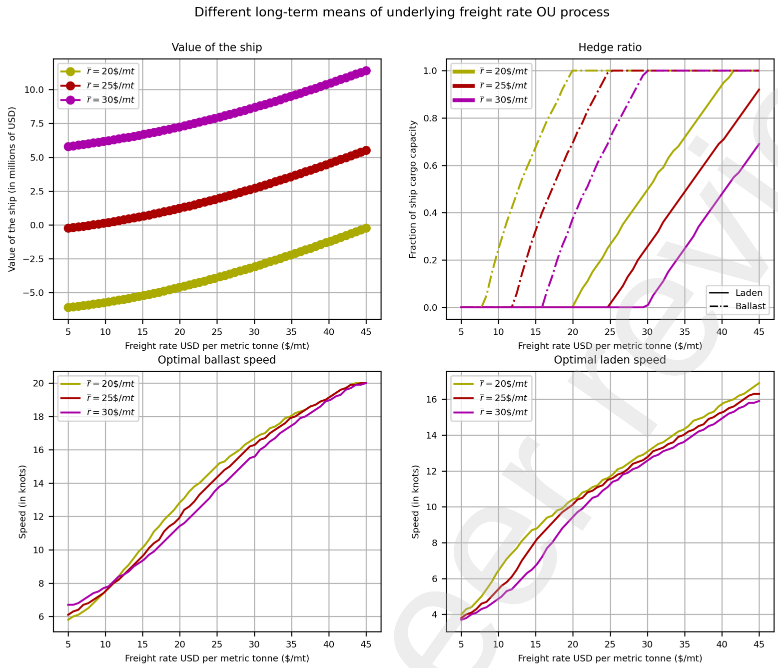


Figure B.13: Combined changes in value function, speeds, and hedge ratio when considering different long-term means

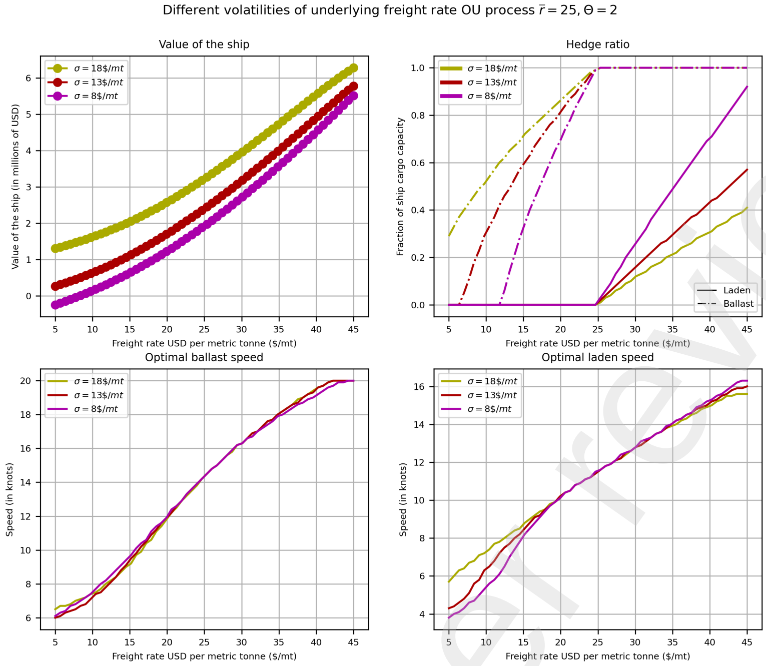


Figure B.14: Combined changes in value function, speeds, and hedge ratio when considering different volatilities

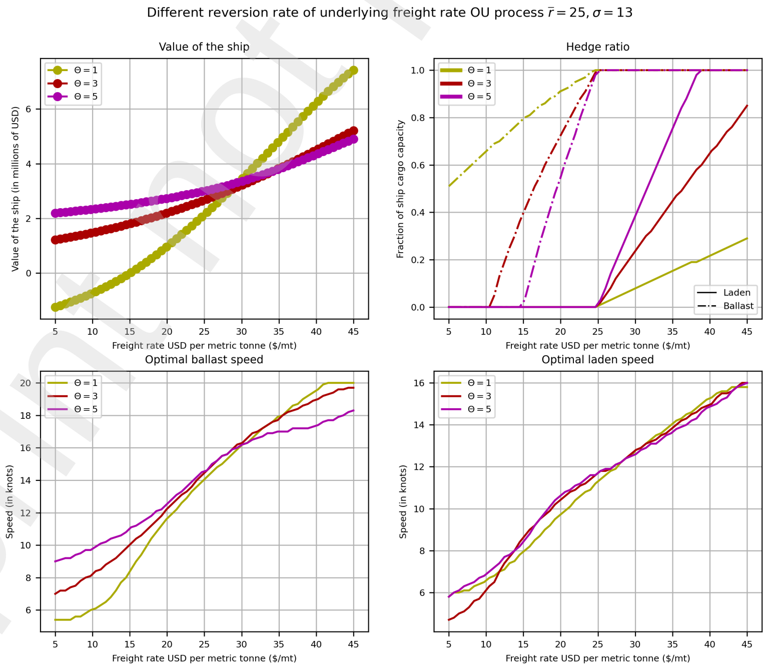


Figure B.15: Combined changes in value function, speeds, and hedge ratio when considering different rates of reversion

## Appendix C. Real data experiment

### *Appendix C.1. Stochastic policy*

Table C.5: Stochastic Policy Simulation Result

| Index | Time | Leg | Speed | Hedge | Profit | Market | Total T | Total Profit |
|---|---|---|---|---|---|---|---|---|
| 0 | 46.74 | 0 | 10.60 | 1.00 | 1,686,451 | 22.87 | 47 | 1,669,168 |
| 1 | 38.13 | 1 | 13.30 | 0.00 | -828,730 | 22.08 | 85 | 855,734 |
| 2 | 51.66 | 0 | 9.50 | 0.93 | 1,226,968 | 18.96 | 137 | 2,046,407 |
| 3 | 44.83 | 1 | 11.10 | 0.00 | -836,040 | 18.39 | 182 | 1,243,061 |
| 4 | 64.29 | 0 | 7.50 | 0.76 | 509,433 | 13.01 | 246 | 1,725,754 |
| 5 | 78.06 | 1 | 6.10 | 0.00 | -1,043,050 | 10.51 | 324 | 754,206 |
| 6 | 51.66 | 0 | 9.50 | 0.93 | 1,242,080 | 19.09 | 376 | 1,898,029 |
| 7 | 47.15 | 1 | 10.50 | 0.00 | -843,881 | 18.02 | 423 | 1,128,869 |
| 8 | 56.02 | 0 | 8.70 | 0.86 | 937,217 | 16.53 | 479 | 1,972,680 |
| 9 | 66.79 | 1 | 7.20 | 0.00 | -959,166 | 13.06 | 546 | 1,121,696 |
| 10 | 53.74 | 0 | 9.10 | 0.90 | 1,081,966 | 17.74 | 600 | 2,070,335 |
| 11 | 35.33 | 1 | 14.50 | 0.04 | -835,407 | 23.68 | 635 | 1,343,469 |
| 12 | 42.43 | 0 | 11.80 | 1.00 | 2,038,355 | 25.98 | 677 | 3,100,736 |
| 13 | 31.56 | 1 | 16.50 | 0.12 | -855,350 | 28.05 | 709 | 2,368,492 |
| 14 | 40.29 | 0 | 12.50 | 1.00 | 2,365,471 | 28.88 | 749 | 4,375,837 |
| 15 | 30.30 | 1 | 17.30 | 0.16 | -865,045 | 30.21 | 779 | 3,646,569 |
| 16 | 31.90 | 0 | 16.30 | 1.00 | 4,578,945 | 48.90 | 811 | 7,479,827 |
| 17 | 33.32 | 1 | 15.50 | 0.08 | -843,614 | 26.15 | 844 | 6,778,686 |
| 18 | 42.11 | 0 | 11.90 | 1.00 | 2,135,713 | 26.82 | 886 | 8,537,446 |
| 19 | 49.75 | 1 | 9.90 | 0.00 | -854,849 | 17.02 | 936 | 7,841,150 |
| 20 | 39.45 | 0 | 12.80 | 1.00 | 2,379,812 | 29.07 | 975 | 9,763,066 |
| 21 | 33.32 | 1 | 15.50 | 0.08 | -843,788 | 26.04 | 1008 | 9,086,541 |
| 22 | 42.11 | 0 | 11.90 | 1.00 | 2,316,809 | 28.22 | 1050 | 11,403,350 |
| 23 | 49.75 | 1 | 9.90 | 0.00 | -858,114 | 16.78 | 1100 | 10,545,236 |
| 24 | 39.45 | 0 | 12.80 | 1.00 | 2,274,599 | 27.09 | 1139 | 12,819,835 |
| 25 | 33.32 | 1 | 15.50 | 0.08 | -844,131 | 26.55 | 1172 | 12,051,508 |
| 26 | 42.11 | 0 | 11.90 | 1.00 | 2,187,354 | 26.73 | 1214 | 14,238,861 |
| 27 | 49.75 | 1 | 9.90 | 0.00 | -860,059 | 16.57 | 1264 | 13,378,802 |
| 28 | 39.45 | 0 | 12.80 | 1.00 | 2,160,731 | 26.54 | 1303 | 15,539,533 |
| 29 | 33.32 | 1 | 15.50 | 0.08 | -844,613 | 25.67 | 1336 | 14,694,920 |
| 30 | 42.11 | 0 | 11.90 | 1.00 | 2,312,997 | 28.14 | 1378 | 17,007,917 |
| 31 | 49.75 | 1 | 9.90 | 0.00 | -861,498 | 17.12 | 1428 | 16,146,419 |
| 32 | 39.45 | 0 | 12.80 | 1.00 | 2,352,822 | 29.09 | 1467 | 18,499,241 |
| 33 | 33.32 | 1 | 15.50 | 0.08 | -845,194 | 25.88 | 1500 | 17,654,047 |
| 34 | 42.11 | 0 | 11.90 | 1.00 | 2,144,496 | 26.93 | 1542 | 19,798,543 |
| 35 | 49.75 | 1 | 9.90 | 0.00 | -863,786 | 16.34 | 1592 | 18,934,758 |
| 36 | 39.45 | 0 | 12.80 | 1.00 | 2,418,511 | 29.23 | 1631 | 21,353,269 |
| 37 | 33.32 | 1 | 15.50 | 0.08 | -845,988 | 26.00 | 1664 | 20,507,281 |
| 38 | 42.11 | 0 | 11.90 | 1.00 | 2,357,297 | 28.66 | 1706 | 22,864,578 |
| 39 | 49.75 | 1 | 9.90 | 0.00 | -865,977 | 16.40 | 1756 | 21,998,601 |
| 40 | 39.45 | 0 | 12.80 | 1.00 | 2,554,987 | 29.49 | 1795 | 24,553,588 |
| 41 | 33.32 | 1 | 15.50 | 0.08 | -846,793 | 26.19 | 1828 | 23,706,795 |

*Appendix C.2. Dummy policy*

| Index | Time | Leg | Speed | Hedge | Profit | Market | Total T | Total Profit |
|---|---|---|---|---|---|---|---|---|
| 0 | 44.40 | 0.0 | 11.22 | 1.00 | 1,680,134 | 22.87 | 44.0 | 1,664,009 |
| 1 | 36.87 | 1.0 | 13.81 | 0.00 | -831,031 | 22.02 | 81.0 | 847,602 |
| 2 | 44.40 | 0.0 | 11.22 | 0.90 | 1,112,055 | 18.03 | 125.0 | 1,929,603 |
| 3 | 36.87 | 1.0 | 13.81 | 0.00 | -831,031 | 18.93 | 162.0 | 1,127,562 |
| 4 | 44.40 | 0.0 | 11.22 | 0.82 | 770,072 | 15.06 | 206.0 | 1,863,638 |
| 5 | 36.87 | 1.0 | 13.81 | 0.00 | -835,092 | 17.20 | 243.0 | 1,328,546 |
| 6 | 44.40 | 0.0 | 11.22 | 0.89 | 1,297,650 | 19.32 | 287.0 | 2,626,196 |
| 7 | 36.87 | 1.0 | 13.81 | 0.00 | -832,657 | 18.11 | 324.0 | 1,793,539 |
| 8 | 44.40 | 0.0 | 11.22 | 0.93 | 1,339,582 | 19.11 | 368.0 | 3,133,121 |
| 9 | 36.87 | 1.0 | 13.81 | 0.00 | -832,324 | 18.85 | 405.0 | 2,300,798 |
| 10 | 44.40 | 0.0 | 11.22 | 0.80 | 727,612 | 15.03 | 449.0 | 3,028,410 |
| 11 | 36.87 | 1.0 | 13.81 | 0.00 | -832,846 | 17.30 | 486.0 | 2,195,564 |
| 12 | 44.40 | 0.0 | 11.22 | 0.85 | 1,088,650 | 17.98 | 530.0 | 3,284,214 |
| 13 | 36.87 | 1.0 | 13.81 | 0.00 | -831,542 | 18.04 | 567.0 | 2,452,672 |
| 14 | 44.40 | 0.0 | 11.22 | 0.84 | 915,120 | 16.49 | 611.0 | 3,367,792 |
| 15 | 36.87 | 1.0 | 13.81 | 0.00 | -832,713 | 17.18 | 648.0 | 2,535,079 |
| 16 | 44.40 | 0.0 | 11.22 | 0.92 | 1,276,982 | 18.45 | 692.0 | 3,812,061 |
| 17 | 36.87 | 1.0 | 13.81 | 0.00 | -832,501 | 18.88 | 729.0 | 2,979,560 |
| 18 | 44.40 | 0.0 | 11.22 | 0.85 | 1,117,218 | 17.72 | 773.0 | 4,096,778 |
| 19 | 36.87 | 1.0 | 13.81 | 0.00 | -831,840 | 18.15 | 810.0 | 3,264,938 |
| 20 | 44.40 | 0.0 | 11.22 | 0.90 | 1,355,505 | 19.56 | 854.0 | 4,620,443 |
| 21 | 36.87 | 1.0 | 13.81 | 0.00 | -832,942 | 18.21 | 891.0 | 3,787,501 |
| 22 | 44.40 | 0.0 | 11.22 | 0.91 | 1,342,134 | 19.03 | 935.0 | 5,129,635 |
| 23 | 36.87 | 1.0 | 13.81 | 0.00 | -832,788 | 18.93 | 972.0 | 4,296,847 |
| 24 | 44.40 | 0.0 | 11.22 | 0.94 | 1,309,605 | 19.38 | 1016.0 | 5,606,452 |
| 25 | 36.87 | 1.0 | 13.81 | 0.00 | -832,897 | 18.28 | 1053.0 | 4,773,555 |
| 26 | 44.40 | 0.0 | 11.22 | 0.95 | 1,391,804 | 19.69 | 1097.0 | 6,165,359 |
| 27 | 36.87 | 1.0 | 13.81 | 0.00 | -832,542 | 18.94 | 1134.0 | 5,332,817 |
| 28 | 44.40 | 0.0 | 11.22 | 0.90 | 1,201,744 | 19.55 | 1178.0 | 6,534,561 |
| 29 | 36.87 | 1.0 | 13.81 | 0.00 | -832,875 | 18.03 | 1215.0 | 5,701,686 |
| 30 | 44.40 | 0.0 | 11.22 | 0.83 | 1,090,214 | 17.55 | 1259.0 | 6,791,900 |
| 31 | 36.87 | 1.0 | 13.81 | 0.00 | -832,571 | 17.22 | 1296.0 | 5,959,029 |
| 32 | 44.40 | 0.0 | 11.22 | 0.92 | 1,425,612 | 20.22 | 1340.0 | 7,384,641 |
| 33 | 36.87 | 1.0 | 13.81 | 0.00 | -832,916 | 18.13 | 1377.0 | 6,551,725 |
| 34 | 44.40 | 0.0 | 11.22 | 0.88 | 1,276,455 | 18.67 | 1421.0 | 7,828,180 |
| 35 | 36.87 | 1.0 | 13.81 | 0.00 | -832,803 | 18.97 | 1458.0 | 6,995,377 |
| 36 | 44.40 | 0.0 | 11.22 | 0.85 | 1,189,307 | 17.80 | 1502.0 | 8,184,684 |
| 37 | 36.87 | 1.0 | 13.81 | 0.00 | -832,691 | 17.98 | 1539.0 | 7,352,251 |
| 38 | 44.40 | 0.0 | 11.22 | 0.93 | 1,374,931 | 20.00 | 1583.0 | 8,727,182 |
| 39 | 36.87 | 1.0 | 13.81 | 0.00 | -832,718 | 18.25 | 1620.0 | 7,894,463 |
| 40 | 44.40 | 0.0 | 11.22 | 0.85 | 1,275,514 | 19.38 | 1664.0 | 9,169,977 |
| 41 | 36.87 | 1.0 | 13.81 | 0.00 | -832,911 | 18.01 | 1701.0 | 8,337,066 |
| 42 | 44.40 | 0.0 | 11.22 | 0.90 | 1,425,000 | 20.33 | 1745.0 | 9,762,066 |
| 43 | 36.87 | 1.0 | 13.81 | 0.00 | -832,927 | 18.09 | 1782.0 | 8,929,139 |
| 44 | 44.40 | 0.0 | 11.22 | 0.87 | 1,315,671 | 18.74 | 1826.0 | 10,244,810 |

Table C.6: Dummy Policy Simulation Result

*Appendix C.3. Myopic Policy*

| Index | Time (days) | Leg | Speed | Hedge Ratio | Profit (USD) | Market | Total Time | Total Profit |
|---|---|---|---|---|---|---|---|---|
| 0 | 35.23 | 0 | 14.55 | 1.00 | 1,585,097 | 22.87 | 35 | 1,572,984 |
| 1 | 27.49 | 1 | 19.39 | 0.05 | -925,412 | 23.95 | 62 | 660,062 |
| 2 | 37.29 | 0 | 13.64 | 0.95 | 1,273,261 | 19.93 | 99 | 1,905,993 |
| 3 | 28.24 | 1 | 18.79 | 0.02 | -910,466 | 22.38 | 127 | 1,020,521 |
| 4 | 37.66 | 0 | 13.48 | 0.94 | 1,161,435 | 18.93 | 165 | 2,140,704 |
| 5 | 33.33 | 1 | 15.50 | 0.08 | -843,614 | 26.15 | 198 | 1,256,368 |
| 6 | 42.11 | 0 | 11.90 | 1.00 | 2,135,713 | 26.82 | 240 | 3,313,890 |
| 7 | 49.75 | 1 | 9.90 | 0.00 | -854,849 | 17.02 | 290 | 2,469,421 |
| 8 | 39.45 | 0 | 12.80 | 1.00 | 2,379,812 | 29.07 | 329 | 4,843,093 |
| 9 | 33.32 | 1 | 15.50 | 0.08 | -843,788 | 26.04 | 362 | 4,004,943 |
| 10 | 42.11 | 0 | 11.90 | 1.00 | 2,316,809 | 28.22 | 404 | 6,319,698 |
| 11 | 49.75 | 1 | 9.90 | 0.00 | -858,114 | 16.78 | 454 | 5,461,698 |
| 12 | 39.45 | 0 | 12.80 | 1.00 | 2,274,599 | 27.09 | 493 | 7,733,669 |
| 13 | 33.32 | 1 | 15.50 | 0.08 | -844,131 | 26.55 | 526 | 6,886,450 |
| 14 | 42.11 | 0 | 11.90 | 1.00 | 2,187,354 | 26.73 | 568 | 9,052,035 |
| 15 | 49.75 | 1 | 9.90 | 0.00 | -860,059 | 16.57 | 618 | 8,193,275 |
| 16 | 39.45 | 0 | 12.80 | 1.00 | 2,160,731 | 26.54 | 657 | 10,354,906 |
| 17 | 33.32 | 1 | 15.50 | 0.08 | -844,613 | 25.67 | 690 | 9,510,293 |
| 18 | 42.11 | 0 | 11.90 | 1.00 | 2,312,997 | 28.14 | 732 | 11,823,290 |
| 19 | 49.75 | 1 | 9.90 | 0.00 | -861,498 | 17.12 | 782 | 10,961,792 |
| 20 | 39.45 | 0 | 12.80 | 1.00 | 2,352,822 | 29.09 | 821 | 13,314,599 |
| 21 | 33.32 | 1 | 15.50 | 0.08 | -845,194 | 25.88 | 854 | 12,469,405 |
| 22 | 42.11 | 0 | 11.90 | 1.00 | 2,144,496 | 26.93 | 896 | 14,613,900 |
| 23 | 49.75 | 1 | 9.90 | 0.00 | -863,786 | 16.34 | 946 | 13,750,114 |
| 24 | 39.45 | 0 | 12.80 | 1.00 | 2,418,511 | 29.23 | 985 | 16,168,626 |
| 25 | 33.32 | 1 | 15.50 | 0.08 | -845,988 | 26.00 | 1018 | 15,322,638 |
| 26 | 42.11 | 0 | 11.90 | 1.00 | 2,357,297 | 28.66 | 1060 | 17,679,935 |
| 27 | 49.75 | 1 | 9.90 | 0.00 | -865,977 | 16.40 | 1110 | 16,813,958 |
| 28 | 39.45 | 0 | 12.80 | 1.00 | 2,554,987 | 29.49 | 1149 | 19,368,945 |
| 29 | 33.32 | 1 | 15.50 | 0.08 | -846,793 | 26.19 | 1182 | 18,522,152 |

Table C.7: Summary of the Myopic Policy for Turbarao / Qingdao Route

*Appendix C.4. Deterministic policy*

| Index | Time | Leg | Speed | Hedge | Profit | Market | Total T | Total Profit |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 48.85 | 0.0 | 10.10 | 1.00 | 1,688,344 | 22.87 | 49.0 | 1,670,309 |
| 1.0 | 40.01 | 1.0 | 12.60 | 0.08 | -828,971 | 23.08 | 89.0 | 857,352 |
| 2.0 | 48.85 | 0.0 | 10.10 | 1.00 | 1,178,518 | 18.56 | 138.0 | 2,000,758 |
| 3.0 | 40.01 | 1.0 | 12.60 | 0.08 | -837,969 | 18.27 | 178.0 | 1,194,851 |
| 4.0 | 48.85 | 0.0 | 10.10 | 1.00 | 514,917 | 12.95 | 227.0 | 1,684,776 |
| 5.0 | 40.01 | 1.0 | 12.60 | 0.08 | -852,205 | 10.66 | 267.0 | 881,012 |
| 6.0 | 48.85 | 0.0 | 10.10 | 1.00 | -192,451 | 6.97 | 316.0 | 701,439 |
| 7.0 | 40.01 | 1.0 | 12.60 | 0.08 | -833,330 | 20.75 | 356.0 | -69,341 |
| 8.0 | 48.85 | 0.0 | 10.10 | 1.00 | 1,105,179 | 17.94 | 405.0 | 941,963 |
| 9.0 | 40.01 | 1.0 | 12.60 | 0.08 | -832,002 | 21.46 | 445.0 | 187,278 |
| 10.0 | 48.85 | 0.0 | 10.10 | 1.00 | 684,070 | 14.38 | 494.0 | 801,151 |
| 11.0 | 40.01 | 1.0 | 12.60 | 0.08 | -842,047 | 16.09 | 534.0 | 52,109 |
| 12.0 | 48.85 | 0.0 | 10.10 | 1.00 | 937,209 | 16.52 | 583.0 | 876,896 |
| 13.0 | 40.01 | 1.0 | 12.60 | 0.08 | -832,787 | 21.04 | 623.0 | 150,402 |
| 14.0 | 48.85 | 0.0 | 10.10 | 1.00 | 2,618,096 | 30.73 | 672.0 | 2,409,938 |
| 15.0 | 40.01 | 1.0 | 12.60 | 0.08 | -822,274 | 26.66 | 712.0 | 1,706,473 |
| 16.0 | 48.85 | 0.0 | 10.10 | 1.00 | 2,415,822 | 29.02 | 761.0 | 3,751,160 |
| 17.0 | 40.01 | 1.0 | 12.60 | 0.08 | -803,549 | 36.67 | 801.0 | 3,076,994 |
| 18.0 | 48.85 | 0.0 | 10.10 | 1.00 | 2,303,447 | 28.07 | 850.0 | 4,988,909 |
| 19.0 | 40.01 | 1.0 | 12.60 | 0.08 | -830,861 | 22.07 | 890.0 | 4,305,295 |
| 20.0 | 48.85 | 0.0 | 10.10 | 1.00 | 1,275,515 | 19.38 | 939.0 | 5,343,551 |
| 21.0 | 40.01 | 1.0 | 12.60 | 0.08 | -819,506 | 28.14 | 979.0 | 4,682,305 |
| 22.0 | 48.85 | 0.0 | 10.10 | 1.00 | 2,103,539 | 26.38 | 1028.0 | 6,361,486 |
| 23.0 | 40.01 | 1.0 | 12.60 | 0.08 | -812,378 | 31.95 | 1068.0 | 5,718,653 |
| 24.0 | 48.85 | 0.0 | 10.10 | 1.00 | 2,499,807 | 29.73 | 1117.0 | 7,675,612 |
| 25.0 | 40.01 | 1.0 | 12.60 | 0.08 | -836,716 | 18.94 | 1157.0 | 7,026,312 |
| 26.0 | 48.85 | 0.0 | 10.10 | 1.00 | 1,482,521 | 21.13 | 1206.0 | 8,164,475 |
| 27.0 | 40.01 | 1.0 | 12.60 | 0.08 | -836,416 | 19.10 | 1246.0 | 7,527,945 |
| 28.0 | 48.85 | 0.0 | 10.10 | 1.00 | 1,029,474 | 17.30 | 1295.0 | 8,303,027 |
| 29.0 | 40.01 | 1.0 | 12.60 | 0.08 | -835,369 | 19.66 | 1335.0 | 7,679,575 |
| 30.0 | 48.85 | 0.0 | 10.10 | 1.00 | 1,534,568 | 21.57 | 1384.0 | 8,812,618 |
| 31.0 | 40.01 | 1.0 | 12.60 | 0.08 | -836,267 | 19.18 | 1424.0 | 8,200,553 |
| 32.0 | 48.85 | 0.0 | 10.10 | 1.00 | 1,346,489 | 19.98 | 1473.0 | 9,175,523 |
| 33.0 | 40.01 | 1.0 | 12.60 | 0.08 | -837,221 | 18.67 | 1513.0 | 8,574,597 |
| 34.0 | 48.85 | 0.0 | 10.10 | 1.00 | 2,026,651 | 25.73 | 1562.0 | 10,013,712 |
| 35.0 | 40.01 | 1.0 | 12.60 | 0.08 | -807,720 | 34.44 | 1602.0 | 9,445,161 |
| 36.0 | 48.85 | 0.0 | 10.10 | 1.00 | 1,818,462 | 23.97 | 1651.0 | 10,711,497 |
| 37.0 | 40.01 | 1.0 | 12.60 | 0.08 | -820,778 | 27.46 | 1691.0 | 10,144,915 |
| 38.0 | 48.85 | 0.0 | 10.10 | 1.00 | 1,863,412 | 24.35 | 1740.0 | 11,417,486 |
| 39.0 | 40.01 | 1.0 | 12.60 | 0.08 | -825,828 | 24.76 | 1780.0 | 10,858,430 |
| 40.0 | 48.85 | 0.0 | 10.10 | 1.00 | 2,354,312 | 28.50 | 1829.0 | 12,435,189 |

Table C.8: Deterministic Policy Simulation Result