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# Networks, beliefs, and asset prices

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A B S T R A C T

We set out a novel social communication model of asset prices. An investor's *type* – which depends on their *network* and investment performance – determines their price beliefs. We show how properties of the network such as network centrality and diameter influence the price dynamics, convergence speed, and limiting belief types. For the polar cases of no attention to performance and exclusive attention to performance, we obtain analytically tractable results relating price and belief types to properties of the network, while for intermediate attention to performance we rely on numerical results. As applications, our model can explain price bubbles and price oscillations by network-performance effects, and we also study how price and type dynamics depend on connectedness on a small-world network. Our results shed light on when performance-based updating of beliefs on social networks is stabilising – or destabilising – for asset prices. A key finding is that the impact of network structure on asset prices and beliefs depends on how much attention investors pay to performance.

*[T]he time has come to move beyond behavioral finance to "social finance", which studies the structure of social interactions, how financial ideas spread and evolve, and how social processes affect financial outcomes.* (David Hirshleifer, 2015)

## 1. Introduction

Given the rise of social media and investment platforms, a central question in modern finance is how *social interactions* influence beliefs, investment decisions and asset prices. As noted in the opening quote, the *structure* of social interactions should be central to understanding how financial ideas such as price beliefs spread among investors, undergo evolution and affect financial market outcomes. This "social finance" perspective on asset pricing raises several questions. How do social interactions and the exchange of information – on investor types and their *performance* – influence beliefs and asset prices in the short and long run? What is the impact of *network structure* on the dynamics of beliefs and asset prices, including price stability? Do *initial* belief types have long-lasting consequences?

In this paper, we take a step toward answering these questions by building an asset pricing model in which beliefs spread via *social networks* and long run outcomes, reached through an evolutionary process, depend on how much attention investors pay to the *financial performance* of themselves and their peers. Our focus on networks is motivated by empirical evidence that investment decisions are influenced by close contacts – such as friends, relatives, neighbours or colleagues – and the advice of industry experts; furthermore,

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the relative performance of social contacts is known to affect investment decisions and asset prices.[1] Despite this evidence, relatively little is known about the mechanisms by which network structures and performance influence investor beliefs and price dynamics.

We contribute to the literature by highlighting the roles of the network and performance in shaping beliefs and price dynamics, and by relating these dynamics to properties of the network – such as *eigenvector centrality*, *diameter* and *connectedness* – and initial conditions, such as starting belief types. Our results shed light on when performance-based updating from networks is stabilising – or destabilising – for asset prices, as well as the question of when prices and beliefs will reflect fundamental (i.e. intrinsic) values, and the relative contribution of *network structure* and *initial conditions* to long run outcomes, including price stability.

In our model, an investor's belief depends on their *type* as updated on a social network and their attention to performance is controlled by an exogenous feedback parameter. Belief types are updated every period and are *continuous* on the spectrum from pure fundamentalist to arbitrarily strong chartist; hence, an agent's type determines how *backward-looking* is their forecasting rule for asset prices. A key novelty is that belief types evolve according to repeated-average updating as in the opinion dynamics literature (DeGroot, 1974), but with the difference that weights depend on *past performance* – i.e. trading profit. As a result, investors can adopt more nuanced belief types than the polar cases of pure fundamentalist or strong chartist and may end up reaching a 'melting pot' *consensus* shaped by performance.

Investors are located on an exogenously given (possibly directed) social network and observe only the past investment decisions and returns of those in their network. In contrast to many previous works, we allow for *non-connected* network structures. Hence, our model allows for 'opinion leaders' who influence other investors but are little influenced themselves (e.g. Warren Buffett); *clustering* associated with tight-knit groups of investors such as mutual friends, neighbours and colleagues; and agents who are relatively disconnected in the sense of being pure 'followers' of others, as with a core-periphery network structure.

Based on observation of contacts, agents revise their beliefs, weighing their own success and the success of their contacts. Beliefs and asset prices thus evolve as a system of *coupled dynamics*, with network structure influencing belief types and prices, and prices and dividends determining which contacts investors pay most attention to (performance feedback). Performance-based imitation of peers has been observed in *experimental* asset markets and helps improve the empirical performance of asset pricing models; in such markets, agents' forecasting behaviour is described well by simple rules, or *heuristics*, as in our model.[2]

We characterise the long-run type distribution in the polar cases of exclusive attention to performance and zero attention. With *zero* performance feedback, updating of types is *purely social* via investor networks. Agents in strongly-connected networks reach a long-run *type* consensus and each agent's influence in the consensus is determined by their *network centrality* and their *initial type*, such that initial beliefs of more central agents have greater influence on the consensus. If the type consensus is not too strongly shifted towards chartism, price converges to the fundamental price, and the beliefs of investors will settle on rational expectations. If price convergence does not occur, then the price is explosive, and both price and beliefs move away from fundamental values.

At the other extreme, if investors focus exclusively on performance, only the beliefs of the most profitable traders in each agent's network are adopted at each time step. We show that agents will eventually adopt either the most fundamental or the most chartist *initial type* in their network (as long as dividend shocks remain in certain bounds) and that a type consensus is reached in *finite time*. We characterise the *maximal time* to type convergence and relate it to the *network diameter*. An implication is that network centrality of agents is *irrelevant* when agents attend exclusively to performance: only the most *extreme* initial types matter for the terminal average type and the long run price dynamics, in contrast to the case of pure social dynamics. However, we show that different network structures with the same diameter can lead to quite different consensus types – hence price dynamics – when dividend shocks are large enough.

For intermediate attention to performance, consensus types are not analytically tractable. Here we use a mix of both analytic and numerical results to show that price and type dynamics can be quite diverse relative to the polar cases. We also investigate this case numerically using three applications. The first two applications model 'opinion leaders' (one and two opinion leaders, respectively) who are followed by other agents, while the third application studies asset pricing on a 'small world' social network. In the first application, price bubbles arise endogenously from type updating without any exogenous shocks, and the size of the bubble increases with performance feedback. The second application shows how permanent oscillations in price and average type can arise when the market is buffeted by dividend shocks. Here, performance feedback prevents explosive asset prices, but it is also a source of price oscillations that would otherwise be absent. In the final application we study consensus types and price dynamics on a large social network, with a focus on how *connectedness* of agents affects the consensus type and (hence) price convergence.

*Related literature.* We are not the first to study the implications of social interactions for asset prices. Kirman (1993) and Lux (1995) set out herding models in which investors are more likely to imitate the dominant belief type in the population, and Alfarano and Milaković (2009) add local networks using a mean-field approximation. In a similar vein, Cont and Bouchaud (2000) and Iori (2002) consider models of herding in random networks, whereas Chang (2007) studies social interactions when utility exhibits a preference for social conformity. By comparison, Yang (2009) considers a social network model in which investors differ in trend-following behaviour but ignore performance. Relative to these papers, our model differs in allowing *both* performance-based updating and social network structures.

[1] On social networks and communication, see Shiller and Pound (1989), Arnswald (2001), Hong et al. (2005), Ivković and Weisbenner (2007), Ozsoylev et al. (2014), Steiger and Pelster (2020). On the impact of relative performance on decisions and prices, see Kroll and Levy (1992) and Schoenberg and Haruvy (2012).

[2] See Kroll and Levy (1992), Schoenberg and Haruvy (2012) and Anufriev et al. (2019) for experimental evidence, and Chiarella et al. (2014) and Hommes et al. (2017) for empirical asset pricing models.

The closest papers in the literature are Panchenko et al. (2013) and Gong and Diao (2022), as surveyed by Hatcher and Hellmann (2024). Both of these papers study two-type versions of the Brock and Hommes (1998) model which are augmented with social networks. In Panchenko et al. (2013) local social networks are introduced, such that agents have a probability to change type only if both chartist and fundamental types are present in their network at the previous time step. By comparison, Gong and Diao (2022) model diffusion of chartist and fundamentalist investor types within a SIS model.

Like these papers, we relate asset price dynamics to features of the network. However, we allow *generic* social networks and characterise the dynamics in the polar cases of zero and exclusive attention to performance *analytically*. Since agents in our model take a weighted average of past types as in the opinion dynamics literature (see below), types are *continuous* and *endogenous* on the range from fundamentalist to arbitrarily strong chartist (hence not limited to pre-specified values as in the Brock and Hommes (1998) model). As a result, there is greater *heterogeneity* in the short run – each agent can start with their 'own type' – but a consensus can emerge in the long run, such that heterogeneity 'dies out'. This feature allows us to shed light on when heterogeneous beliefs will persist, as well as the question of how a consensus type (if reached) depends on *initial types* and *network structure*.

In relation to beliefs, our paper sheds light on when price beliefs converge to rational expectations given performance-based updating on social networks. If such convergence occurs, then short run mispricing is reduced over time, and the market is efficient (Fama, 1970, 2014) in the *long run*. Further, if investors focus exclusively on performance and some initial belief types are purely fundamentalist, then mispricing may be eliminated in *finite time*. We thereby show how social networks and initial conditions can influence not just the spread of beliefs among investors, but also the efficiency of financial markets, both when investors ignore financial performance and when they place great emphasis on it.

Finally, relative to the classical literature on opinion dynamics on social networks originated by DeGroot (1974) and extended to multiple settings (e.g. Golub and Jackson, 2010), we go beyond previous attempts to relax the benchmark assumption that updating weights are independent of time and reflect only the network position; see DeMarzo et al. (2003); Jadbabaie et al. (2013); Buechel et al. (2014, 2015) for applications, or Lorenz (2005, 2007) for general convergence conditions. In our model the time-varying updating weights are not exogenous but instead depend on relative performance among one's neighbours in a social network, as measured by past trading profits on a financial market.

## 2. Model

Consider a finite set of risk-averse investors $N = \{1, \dots, n\}$ and discrete time $t \in \mathbb{N}$. At date $t$, agents choose holdings of a risky asset $x_t^i$ with *unknown return* (in zero net supply) and a riskless bond (in flexible supply). Agents buy the risky asset at price $p_t$ and sell it at price $p_{t+1}$ having received stochastic dividends $d_{t+1}$; the riskless bond has a known return $r > 0$ and price of 1. Both price $p_{t+1}$ and realisations of dividends $d_{t+1}$ are unknown in period $t$, so that the unknown *excess return* of the risky asset is $R_{t+1} := p_{t+1} + d_{t+1} - (1+r)p_t$. At each point in time $t \in \mathbb{N}$, agents $i \in N$ are characterised by their current wealth $w_t^i$ and their subjective expectation $\tilde{E}_t^i[.]$ and subjective variance $\tilde{V}_t^i[.]$ about the future asset price $p_{t+1}$ and dividends $d_{t+1}$. Agents are myopic and choose their asset positions to maximise a *mean-variance* utility function over next period wealth $w_{t+1}^i$ given a risk-aversion coefficient $\phi > 0$. Investors can take short positions in the risky asset, so $x_t^i \in \mathbb{R}$.

Therefore, at any $t \in \mathbb{N}$, each investor $i \in N$ solves the problem[3]:

$$\max_{x_t^i} \ \tilde{E}_t^i[w_{t+1}^i] - \frac{\phi}{2}\tilde{V}_t^i[w_{t+1}^i] \quad \text{s.t. } w_{t+1}^i = (p_{t+1} + d_{t+1})x_t^i + (1+r)(w_t^i - p_t x_t^i) \tag{1}$$

where $w_t^i - p_t x_t^i$ denotes the holdings of the riskless asset. The first term in the constraint of the optimisation problem in (1) is the payoff on stocks (dividend plus resale price) and the second term is the gross return on holdings of the riskless asset.

The first-order condition yields the following demand for the risky asset:

$$x_t^i = \delta\left(\tilde{E}_t^i\left[p_{t+1} + d_{t+1}\right] - (1+r)p_t\right) \tag{2}$$

where $\delta := (\phi\tilde{V})^{-1} > 0$ and we make the common assumption that $\tilde{V}_t^i[R_{t+1}] = \tilde{V}$ for all $t \in \mathbb{N}$ and $i \in N$.[4] Demand of agent $i$ is proportional to the expected excess return, $\tilde{E}_t^i\left[R_{t+1}\right]$.

Dividends follow a stochastic process: $d_t = \overline{d} + \varepsilon_t$ where $\overline{d} > 0$ and $\varepsilon_t$ is chosen from an IID distribution with mean 0 and support in an interval $[-d^-, d^+]$ such that $d^-, d^+ > 0$. Our assumption that dividends are drawn from a fixed interval is not restrictive since the interval can be chosen arbitrarily large. (Note that assuming $d^- \leq \overline{d}$ will ensure non-negative dividends.) We assume agents know the dividend process, and hence their subjective expectations coincide with the objective (rational) expectation: $\tilde{E}_t^i\left[d_{t+1}\right] = E_t(d_{t+1}) = \overline{d}$, for all $i \in N$ where $E_t(.)$ is the conditional expectation operator with respect to past dividends.

[3] If we let $b_t^i \in \mathbb{R}$ denote investor $i$'s holdings of the riskless asset then the wealth equation has the form $w_{t+1}^i = (1+r)b_t^i + (p_{t+1} + d_{t+1})x_t^i$ s.t. $p_t x_t^i + b_t^i = w_t^i$, since the price of riskless asset is 1.

[4] We derive optimal demands for the risky asset and give more detail on the assumptions on subjective variances in the *Online Appendix*, Section C.1.1. To obtain a desired $\delta$ in (2) we may either normalise $\tilde{V}$ (say to 1) and set $\phi$, or we may fix $\phi$ and set $\tilde{V}$. Thus, we only report a value for $\delta$ in numerical examples.

### 2.1. Price beliefs

Investors form their price beliefs by taking a weighted average between the price expectation of a fundamentalist and a price expectation of a chartist, where these *polar* beliefs follow Brock and Hommes (1998). Let $g_t^i \in \mathbb{R}_+$ be the weight that investor $i \in N$ attaches to the chartist's price expectation at some point of time $t \in \mathbb{N}$. We call this agent a $g_t^i$-trader.

The price expectation of an $g_t^i$-trader is given by

$$\tilde{E}_t^i[p_{t+1}] = g_t^i \tilde{E}_t^c[p_{t+1}] + (1-g_t^i)\tilde{E}_t^f[p_{t+1}] = g_t^i p_{t-1} + (1-g_t^i)p^f \tag{3}$$

where $\tilde{E}_t^c[p_{t+1}] = p_{t-1}$ is the price expectation of a chartist, $\tilde{E}_t^f[p_{t+1}] = p^f$ is the price expectation of a (pure) fundamentalist, and $p^f = \overline{d}/r$ is the fixed fundamental price.[5]

The price expectation of a $g_t^i$ trader, (3), nests the polar cases of fundamentalist, when $g_t^i = 0$, and chartist when $g_t^i = 1$. More generally, $g_t^i$ traders arrive at a price expectation by taking a weighted average of the fundamentalist and chartist beliefs (if $g_t^i \leq 1$). Note that we also allow $g_t^i > 1$, in which case the agent expects the price to move further away from the fundamental price in the future. We call such agents 'strong chartists'.

Since each agent's price belief is a $g_t^i$-weighted average, there is generic *heterogeneity* of beliefs, in contrast to the *predetermined* predictors in Brock and Hommes (1998). A similar hybrid specification of fundamentalist-chartist beliefs is used in Barberis et al. (2018), but there the types $g_t^i$ differ *exogenously* due to 'wavering', whereas we allow types to be determined by agents' social networks and *endogenous* performance feedback.

Note that while the beliefs in (3) rule out dependence on the current price $p_t$, this assumption can be relaxed while keeping some of the main results intact.[6]

### 2.2. Market clearing and price dynamics

For the asset market to clear, we require $\sum_{i\in N} x_t^i = 0$. Using (2) and (3) and rearranging the market clearing condition, the equilibrium asset price can be written in the form:

$$p_t = \frac{\overline{d} + \left(1 - \sum_{i\in N} \frac{g_t^i}{n}\right)\tilde{E}_t^f[p_{t+1}] + \left(\sum_{i\in N} \frac{g_t^i}{n}\right)\tilde{E}_t^c[p_{t+1}]}{1+r}.$$

This expression can be simplified using (3), the notation of average type $\overline{g}_t := \sum_{i\in N} \frac{g_t^i}{n}$, and deviations from the fundamental value, $\tilde{p}_t := p_t - p^f$, to get the law of motion of the price:

$$\tilde{p}_t = \frac{\overline{g}_t}{1+r}\tilde{p}_{t-1}. \tag{4}$$

From this law of motion, we can already conclude that price converges to the fundamental price if the average type $\overline{g}_t$ converges to a value smaller than $1+r$, while the price will be explosive if the average type is too strongly chartist in the long-run. Note that if average type fluctuates between values above and below $1+r$, then the price will fluctuate as well; we show below that this is possible when there is positive attention to performance.

### 2.3. Return and fitness

To derive excess returns per share, consider the demands $x_t^i$ and investor types $g_t^i$ for all agents $i \in N$ at some point in time $t \in \mathbb{N}$:

$$x_t^i = \delta\left(\tilde{E}_t^i[p_{t+1}] + \overline{d} - (1+r)p_t\right) = \delta\left(g_t^i - \overline{g}_t\right)\tilde{p}_{t-1} \tag{5}$$

where $\overline{d} = rp^f$ and the law of motion of the price (4) have been used. Equation (5) implies that agents more optimistic than the average will buy, and those less optimistic than average will short-sell (a negative position). If last period's price is below the fundamental price, then more fundamental types are more optimistic in expecting the price to increase, while if last period's price exceeds the fundamental price, more chartist types have greater optimism.

Similar algebra can be used to show that the excess return per share is

$$R_t = p_t + d_t - (1+r)p_{t-1} = \left(\frac{\overline{g}_t}{1+r} - (1+r)\right)\tilde{p}_{t-1} + \varepsilon_t. \tag{6}$$

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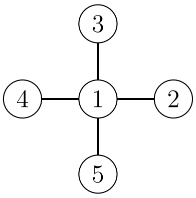
[5] As in Brock and Hommes (1998) the fundamental price is the (hypothetical) price when all in investors are fundamentalists with common rational expectations and speculative bubbles are absent. Their model allows chartists to place some weight on the fundamental price $p^f$, but there is no loss of generality in our specification, (3), since we allow investors to weight the two polar beliefs according to their type $g_t^i$.

[6] We show in the *Online Appendix* (Appendix C) that if the price expectation of $g_t^i$ traders is linear in the current price with a common weight, our main qualitative results are unchanged. If weights are *heterogeneous*, our conclusions may change, but we provide sufficient conditions for our main results to hold.




$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \overline{\mathbf{A}} = \begin{bmatrix} 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 \end{bmatrix}$$

**Fig. 1.** The star network with $n = 5$ (ignoring self-loops), which is used in Examples 1–3, and the associated adjacency matrix $\mathbf{A}$ and adjusted adjacency matrix $\overline{\mathbf{A}}$.

Our fitness measure – net trading profits at time $t$ – can be written as:

$$u_t^i = R_t x_{t-1}^i = \left( \left( \frac{\overline{g}_t}{1+r} - (1+r) \right) \tilde{p}_{t-1} + \varepsilon_t \right) \delta \left( g_{t-1}^i - \overline{g}_{t-1} \right) \tilde{p}_{t-2}. \tag{7}$$

We take profits as the fitness measure because this is consistent with agents $i \in N$ who care about the *realised value* of their objective function (1); it is also a standard assumption in the related literature (see Brock and Hommes, 1998).[7] Since the fitness measure $u_t^i$ in (7) is linear in $i's$ demand $x_{t-1}^i$, it is straightforward to rank agents by performance: if excess return per share is positive, those with highest demand (the most optimistic agents) have the highest fitness, while the least optimistic agent is the best performer if return is negative.

We now explain how the performance ranking relates to the endogenous asset price.

### 2.4. Performance ranking and critical price

As we have seen, performance depends on demand and the *sign* of the realised return. By (5), the more chartist an agent's type, the higher the demand if price deviation $\tilde{p}_t$ is positive while the inverse holds for negative price deviation, i.e.

$$\tilde{p}_{t-2} > 0 \quad \Rightarrow \quad g_{t-1}^i > g_{t-1}^j \Leftrightarrow x_{t-1}^i > x_{t-1}^j.$$

Given the sign of the return $R_t$, there is a *performance ranking*. Returns change sign at a critical price level, so when the price crosses this threshold, a switch in performance ranking occurs. To see this we can rewrite (6) using (4) to obtain:

$$sgn(R_t) = sgn(p_t^{crit}) \cdot sgn(p_t^{crit} - \tilde{p}_t) \qquad \text{s.t. } p_t^{crit} := \frac{\overline{g}_t}{(1+r)^2 - \overline{g}_t} \cdot \varepsilon_t \tag{8}$$

which holds for all $\overline{g}_t \neq (1+r)^2$ where $sgn$ denotes the sign function.[8]

The best-performing agents can then be found among the extreme types (the most chartist or the most fundamental type). We denote the set of best-performing agents from some subset $S \subset N$ at time $t \in \mathbb{N}$ as $U_t^{\max}(S) := \{i \in S \mid u_t^i \geq u_t^j \ \forall j \in S\}$; these must have maximal or minimal type, denoted by $g_t^{\max}(S) := \max\{g_t^i \mid i \in S\}$ and $g_t^{\min}(S) := \min\{g_t^i \mid i \in S\}$. For $S = N$, we simply write $g_t^{\max} := g_t^{\max}(N)$ and $g_t^{\min} := g_t^{\min}(N)$. Analogously, we define the set of agents from $S \subset N$ with maximal or minimal type as $G_t^{\max}(S) := \{i \in S \mid g^i = g_t^{\max}(S)\}$, respectively $G_t^{\min}(S) := \{i \in S \mid g^i = g_t^{\min}(S)\}$. For $S = N$, we analogously drop the argument and write $G_t^{\max} := G_t^{\max}(N)$.

We next consider agents' type updating on a social network.

### 2.5. The network, type updating

We consider a directed network given by a $n \times n$ matrix $\mathbf{A}$ with entries $a_{ij} \in \{0, 1\}$. If $a_{ij} = 0$, then investor $i$ does not observe investor $j$. If, instead, $a_{ij} = 1$, then $i$ observes $j$'s type, and $j$'s returns and fitness $u_t^j$. We assume that $a_{ii} = 1$ for all $i \in N$ such that each agent always observes their own type and returns. Denote by $N^i := \{j \in N : a_{ij} = 1\}$ the set of traders that $i$ observes and by $M^i := \{j \in N : a_{ji} = 1\}$ the set of traders that observe $i$. By above assumption, we have $i \in N^i \cap M^i$. For a subset $S \subset N$ we denote by $M(S) := \{j \in N \mid \exists i \in S : a_{ji} = 1\}$ the set of agents who observe agents in $S$. Further, denote by $\overline{\mathbf{A}}$ the matrix with entries $\overline{a}_{ij} = \frac{1}{|N^i|} a_{ij}$ which is row stochastic. As a *running example* to illustrate our results we consider the star network where agent 1 is connected to all agents while the other agents are just connected to agent 1 and themselves. We illustrate in Fig. 1 the star network, the associated adjacency matrix and matrix $\overline{A}$ which adjusts each row by the number of neighbours. Note that the star is essentially a core-periphery network structure for which the core is a single agent (see e.g. Borgatti and Everett, 2000).

A path from node $i$ to node $j$ of length $k \in \mathbb{N}$ exists if there is a sequence of distinct nodes $(i^1, \dots i^k)$ which are connected, i.e. $a_{i^l, i^{l+1}} = 1$ for all $1 \leq l \leq k-1$, and that starts at $i^1 = i$ and ends at $i^k = j$. Note that a path of length $k$ from $i$ to $j$ exists, if and only if we have $(\mathbf{A}^k)_{ij} > 0$ where $\mathbf{A}^k$ denotes the $k$-th power of the matrix $\mathbf{A}$. We denote the set of nodes that lie on a path that starts in

[7] We explain the link between the objective function and fitness measure in the *Online Appendix*, Appendix B.1.3. In some of the literature, forecast accuracy is taken as the fitness measure rather than profit. Our focus on realised profit is crucial to the extent that the best forecaster need not have the highest profit.

[8] The sign function is defined by $sgn(x) = +1$ if $x > 0$, $sgn(x) = 0$ if $x = 0$, and $sgn(x) = -1$ if $x < 0$.

node $i$ as $\mathcal{P}^i := \{j \in N | \exists k \in \mathbb{N} : (\mathbf{A}^k)_{ij} > 0\}$. Clearly $\mathcal{P}^j \subseteq \mathcal{P}^i$ for all $j \in \mathcal{P}^i$. The distance between two nodes $i$ and $j$ in network $\mathbf{A}$ is defined as the minimal path length denoted by $d(i,j) := \min\{k \in \mathbb{N} : (\mathbf{A}^k)_{ij} > 0\}$. If two nodes are not connected by a path, we set $d(i,j) = \infty$. A network is called strongly connected if $d(i,j) < \infty$ for all $i, j \in N$, i.e. if there exists a path from $i$ to $j$ for all pairs of nodes. We define the distance between two sets of nodes $S, S' \subset N$ by $d(S,S') = \min_{i \in S, j \in S'} d(i,j)$. The diameter of a network, $D(\mathbf{A}) = \max_{i,j \in N} d(i,j)$, is the maximum distance between any two nodes. For any $S \subset N$, we denote the restriction of an $n \times n$ matrix $\mathbf{B}$ to the rows and columns from $S$ by $\mathbf{B}_{SS} = (b_{ij})_{i,j \in S}$, and the restriction of an $n \times 1$ vector to set $S$ by $\mathbf{v}^S = (v_i)_{i \in S}$.

We assume each investor is only influenced by those she observes in her network (including herself). In particular, investors evaluate the performance of the other investors they observe and update their type according to a logit response model such that:

$$g^i_{t+1} = \left( \sum_{k \in N^i} \exp(\gamma u^k_t) \right)^{-1} \sum_{j \in N^i} \exp(\gamma u^j_t) g^j_t, \qquad \forall i \in N. \tag{9}$$

There are two important differences in (9) relative to the Brock and Hommes model. First, in our model agents $i \in N$ do not update from the entire set of agents $N$, but, as in Panchenko et al. (2013), only update *locally* from their neighbours in the set $N^i$. Second, in contrast to Brock and Hommes (1997, 1998), where relative fitness determines the fractions of agents that adopt one of the polar types, here an agent's type in period $t+1$ is a weighted average of *past types* of those in her network, giving higher weight to more successful individuals. The parameter $\gamma$ measures the performance feedback to beliefs. It is similar to the intensity of choice in the Brock and Hommes model which measures how fast agents *switch* between different prediction strategies; here the interpretation is *instead* that willingness to update one's forecasting rule (in a continuous manner) increases with $\gamma$.

Denoting the updating weights by $\tilde{a}_{ij}(t) = \left(\sum_{k \in N^i} \exp(\gamma u^k_t)\right)^{-1} \exp(\gamma u^j_t)$, and the (column) vector of types by $\mathbf{g}_t = (g^1_t, \ldots, g^n_t)'$, the matrix $\tilde{\mathbf{A}}(t) = \left(\tilde{a}_{ij}(t)\right)_{i,j \in N}$ presents the law of motion of the type dynamics in the sense that (9) can be expressed as

$$\mathbf{g}_{t+1} = \tilde{\mathbf{A}}(t)\mathbf{g}_t. \tag{10}$$

Note that $\tilde{\mathbf{A}}(t)$ is always row stochastic by (9) such that each iteration is a weighted average of the type vector of the previous period. Further, for any *finite* $\gamma$, we have $\tilde{a}_{ij}(t) = 0$ if and only if $a_{ij} = 0$ for any $t \in \mathbb{N}$. In the limit $\gamma \to \infty$, instead, some of the weights $\tilde{a}_{ij}(t)$ may converge to 0, even if $a_{ij} > 0$, while the other direction still holds. This means that in the course of repeated updating, agents can only influence each other if they are connected by a path in the network $\mathbf{A}$.

### 2.6. Timing and initial conditions

From time period $t \geq 0$ onwards, the dynamics evolve as described above. At the beginning of each time period $t$, investors' types are given by $g^i_t \in \mathbb{R}_+$ for all $i \in N$ with asset holdings of last period $x^i_{t-1}$ and the last period price deviation given by $\tilde{p}_{t-1} = p_{t-1} - p^f$. Investors then form their demands $x^i_t$ according to (5) such that $\tilde{p}_t$ can be derived from the past price deviation $\tilde{p}_{t-1}$ according to the law of motion in (4). From this, returns $R_t$ are realised and fitness $u^i_t$ of each investor $i \in N$ is given by (7). Investors observe the fitness of others in their network and at the end of period $t$ update their type according to (9).

To have a consistent model, we need assumptions for the period before type updating occurs for the first time, i.e. before $t = 0$. We assume that initially there is a price of the stock $p_{-2}$ (and hence $\tilde{p}_{-2}$) e.g. at the emission of the stock, and there are investors types $g^i_{-1}$ for all $i \in N$. We assume that $p_{-2} \neq p^f$ to allow for price changes over time. Given $g^i_{-1}$ and $\tilde{p}_{-2}$, demand $x^i_{-1}$ can be computed according to (5) yielding equilibrium price $p_{-1}$ such that $\tilde{p}_{-1}$ is determined by (4). At the end of period $t = -1$, we set $g^i_0 = g^i_{-1}$ for all $i \in N$ (updating of the types can only occur once the agents realise differences in performance). In period 0, price $\tilde{p}_0$ and demand $x^i_0$ are determined by (4) and (5), respectively, and performance $u^i_0$ (given the first dividend $d_0$) is evaluated according to (7). The first type updating then occurs such that $g^i_1$ is determined by (9). We can therefore refer to $\tilde{p}_0$ as the initial price.

For all periods $t \geq 1$, demand, price, fitness, and types are determined by (4) – (9).

## 3. Dynamics

We start out by characterising steady states of the model. We then study the dynamics in the polar cases of no performance feedback effect $\gamma = 0$, and exclusive attention to performance, $\gamma \to \infty$, before presenting some results for the case of finite attention to performance, $\gamma \in \mathbb{R}_+$. While we assume in this section that the network is *strongly connected*, we will relax this assumption in Section 4. Proofs of all results appear in the Appendix.

### 3.1. Steady states

In a steady state $(\tilde{p}^*, \mathbf{g}^*)$, the following equations must hold:

$$\left(1 - \frac{\overline{g}^*}{1+r}\right)\tilde{p}^* = 0 \quad \text{and} \quad (\mathbf{I_n} - \tilde{\mathbf{A}})\mathbf{g}^* = \mathbf{0_n}. \tag{11}$$

The first part of (11) is the steady state price which is obtained as a solution to the steady state price condition $\tilde{p}_{t+1} - \tilde{p}_t = 0$ using (4). Similarly, the second part of (11) is the steady state vector of types where (9) is applied to the steady-state types condition $\mathbf{g}_{t+1} - \mathbf{g}_t = \mathbf{0}_n$ with $\mathbf{0}_n$ denoting the $n \times 1$ vector of zeros.

Note that the steady state price $\tilde{p}^*$ depends on average type $\overline{g} = \frac{1}{n}\mathbf{1}_n'\mathbf{g}^*$, where $\mathbf{1}_n$ denotes the $n \times 1$ vector of ones, and the steady state type vector $\mathbf{g}^*$ depends on steady state price indirectly through the updating matrix $\tilde{\mathbf{A}}$. For $\overline{g}^* \neq 1 + r$, (11) is satisfied if and only if $\tilde{p}^* = 0$ (fundamental price); however, if $\overline{g}^* = 1 + r$ then (11) is satisfied by any $\tilde{p}^* \in \mathbb{R}$, and of these all prices except $\tilde{p}^* = 0$ are non-fundamental prices.

**Definition 1.** A price-type vector $(\tilde{p}^*, \mathbf{g}^*)$ is a steady state if it satisfies (11). If, additionally, $\tilde{p}^* = 0$, then a steady state is fundamental, and else it is non-fundamental.

We refer to steady states with a fundamental price as fundamental steady states and we refer to any other steady states as non-fundamental. At a fundamental steady state, price is equal to the fundamental price $p^f$ and agents' price beliefs coincide with the fundamental price. At a non-fundamental steady state, neither of these conditions holds.

The second part of (11) is the type equation. Aside from the trivial solution $\mathbf{g}^* = \mathbf{0}$, this equation allows for other solutions only if $\det(\mathbf{I_n} - \tilde{\mathbf{A}}) = \mathbf{0}$, i.e. if $\tilde{\mathbf{A}}$ has a unit eigenvalue and corresponding eigenvector(s) $\mathbf{g}^*$. Since $\tilde{\mathbf{A}}$ is row stochastic and strongly connected, the principal eigenvalue is equal to 1 and the corresponding eigenspace is one-dimensional with all eigenvectors being real and all their components of the same sign by the Perron-Frobenius theorem. This implies that all types must be identical which is commonly referred to as consensus. Hence, at any steady state, both the weights matrix $\tilde{\mathbf{A}}$ and the fitness vector $\mathbf{u}$ are steady, too.

**Proposition 1.** *The following holds in any steady state* $(p^*, \mathbf{g}^*)$*:*

1. *All types coincide,* $g^{i,*} = \overline{g}\ \forall i \in N$.
2. *The steady-state weights matrix satisfies* $\tilde{\mathbf{A}}^* = \overline{\mathbf{A}}$ *and* $\mathbf{u}^* = \mathbf{0}$.

Proposition 1 characterises steady states where both price and types are invariant to the law of motion. Only if all agents types are identical, a steady state is reached. The focus of this result is on the steady state types $\mathbf{g}^*$ as the steady state price $p^*$ follows from the consensus type which trivially coincides with the average type $\overline{g}$. The second part of Proposition 1 holds since for consensus types, (7) implies that agents' fitness is equal to zero while (9) implies that $\tilde{a}_{ij} = \overline{a}_{ij}$.

Note that types can even be steady when prices move. We will see later that there exist initial conditions such that the price diverges while types converge to consensus. We, therefore, refer to a *type* steady state if only $\mathbf{g}^*$ satisfies the second part of (11). In this case, it is straightforward to see that Property 1 of Proposition 1 still holds. Once a consensus is obtained, the types will remain in this state forever due to the nature of updating defined in (9). On the other hand, types can only be steady when a consensus is obtained.

### 3.2. $\gamma = 0$: pure network-based updating

We first take a brief look at the dynamics when $\gamma = 0$. In this case, agents simply update their own type through their network independently of how others perform such that the weights in the updating matrix $\tilde{\mathbf{A}}(t)$ in (10) are given by $\tilde{a}_{ij}(t) = \overline{a}_{ij}$ for all $t \in \mathbb{N}$. Since this law of motion of the type dynamics is time-invariant, it can be written as

$$\mathbf{g}_{t+1} = \tilde{\mathbf{A}}(t)\mathbf{g}_t = \left(\overline{\mathbf{A}}\right)^{t+1}\mathbf{g}_0. \tag{12}$$

In other words, agents just take the weighted average over all neighbours since $\overline{\mathbf{A}}$ just adjusts the matrix $\mathbf{A}$ by the number of neighbours. In particular, agents update independently of how each of their neighbours performs since $\gamma = 0$. Such a dynamic model is closely related to a model of opinion dynamics first formulated by DeGroot (1974). Since agents observe their own type, the matrix $\overline{\mathbf{A}}$ is aperiodic. By standard results, the type dynamics converge and we can characterise the terminal types and the price dynamics.

**Proposition 2.** *For any realisation of the dividends, we get the following:*

1. *Types converge to a steady state type vector* $\mathbf{g}^{\mathbf{v}}$ *such that*
$$g^{i,\mathbf{v}} = \sum_{j\in N} v^j g_0^j \qquad \forall i \in N$$
*where* $v^j$ *is the* $j$*-th entry of the (unique) left-unit eigenvector* $\mathbf{v}$ *of* $\overline{\mathbf{A}}$ *with* $\sum_{i\in N} v^i = 1$.
2. *Prices converge to the fundamental price if* $\overline{g}^{\mathbf{v}} < 1 + r$*, prices converge to some other steady state if* $\overline{g}^{\mathbf{v}} = 1 + r$*, and price diverges to* $\pm\infty$ *if* $\overline{g}^{\mathbf{v}} > 1 + r$.

From Proposition 2, we can conclude that all agents reach a consensus and that this consensus is a weighted average of initial opinions (i.e. types). The social influence weights correspond to the entries in the left-unit eigenvector which sums to one (implying that it is unique by Perron-Frobenius). The left-unit eigenvector of the adjacency matrix is often used as a measure of centrality in the network, also called *eigenvector centrality*. Hence, how much agents' initial types influence the steady state consensus type depends on their network centrality. The more central an agent, the higher will be the social influence.

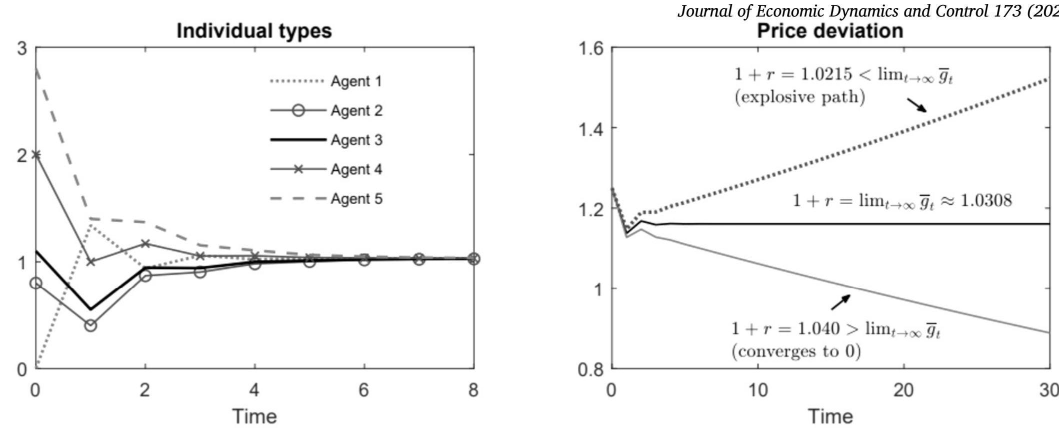



**Fig. 2.** Dynamics of individual types and price in the star network for $\gamma = 0$ where initial values are as in Example 1 and are chosen such that for small variations of the interest rate $r$ (as shown in the figure), price can converge to the fundamental price, can converge on some other price, or diverge to $+\infty$.

Given terminal types, the price dynamics immediately follow. The group type steady state vector $\overline{g}^{\mathbf{v}}$ only depends on eigenvector centrality and initial types. The price will converge to the fundamental price if and only if the consensus type is less than $1 + r$. An agent with an initial type that exceeds 1 (strong chartist) believes the price will move away from the fundamental price. If there are not many strong chartists with types exceeding $1 + r$ or if such agents are not central, then the price will converge to the fundamental price.

**Example 1.** Consider a star network with $n = 5$ agents, as illustrated in Fig. 1. The vector of initial types is $g_0 = (0, 0.8, 1.1, 2, 2.8) = g_{-1}$ and we set the initial price deviation at $\tilde{p}_0 = 1.25$, which implies an initial price in levels of $p_0 = p^f + 1.25$ where $p^f = \overline{d}/r$ is the fundamental price, $\overline{d} = 0.5$ is the expected dividend, and $r$ is the net interest rate.

Given the star network structure, Agent 1 initially updates their type in the direction of the initial types of the other agents (left panel), while the other agents update their types toward the initial type of Agent 1 (which is a pure fundamental type). Updating narrows the initial heterogeneity in types but also changes the relative optimism, with Agent 1 becoming relatively 'bullish' and Agent 2 being the most pessimistic at date 1; there are 'type reversals' for Agents 1 and 4 in period 2. These fluctuations in type – which are due entirely to the social network – mean that the average type $\overline{g}_t$ fluctuates somewhat in the early periods and this induces some price fluctuations in the first three periods (right panel).

By period 8, types have essentially converged to a consensus type. The consensus type is around 1.0308 (see left panel), compared to the initial average type of 1.34. The reason is that, as shown in Proposition 2, the consensus depends on initial types weighted by *network centrality*, such that more central agents have greater influence on the consensus. In this case, the most central agent is Agent 1 (core), whose initial type is 0, so the consensus type is somewhat smaller than the unweighted average of the initial types.[9]

The long run price dynamics depend on where the gross interest rate $1 + r$ lies in relation to the consensus type. By varying the interest rate $r$ in the right panel of Fig. 2, we trace out each possible case of the price dynamics set out in Proposition 2. Price is explosive when $\lim_{t\to\infty} \overline{g}_t > 1 + r$ (dashed line), converges to the fundamental price when $\lim_{t\to\infty} \overline{g}_t < 1 + r$ (grey line), and converges to a non-fundamental price if $\lim_{t\to\infty} \overline{g}_t = 1 + r$ (black line) which only occurs for a single value of interest rate $r$, underlining that this is a knife-edge case.

### 3.3. $\gamma \to \infty$: pure performance-based updating

We now consider the other polar case of exclusive attention to performance, $\gamma \to \infty$. In this case, agents are still restricted to update from those they observe in their social network, but they only update from the best-performers within that set. Hence, we only have that $a_{ij} = 0 \Rightarrow \tilde{a}_{ij}(t) = 0$ for any $t \in \mathbb{N}$, but the other direction does not hold anymore.

We first study the case where dividends are non-stochastic; we then relax this assumption and present additional analytical results and a numerical example.

#### 3.3.1. The case of non-stochastic dividends

If dividends are non-stochastic, then depending on the prevailing average type, either the more fundamental types are doing better in terms of fitness (if $\overline{g}_t < (1 + r)^2$) or more chartist types are (if $\overline{g}_t > (1 + r)^2$). This can be seen by rewriting (7) and using (4) to get:

$$\varepsilon_t = 0 \;\Rightarrow\; u_t^i = \delta\left(\overline{g}_t - (1 + r)^2\right)\left(\frac{g_{t-1}^i}{\overline{g}_{t-1}} - 1\right)\left(\tilde{p}_{t-1}\right)^2. \tag{13}$$

[9] The social influence weights correspond to the (left) eigenvector centrality of $(\frac{5}{13}, \frac{2}{13}, \frac{2}{13}, \frac{2}{13}, \frac{2}{13})$. Thus, Agent 1 has highest centrality and the most influence, while the other agents all have equal influence.

Note that $\varepsilon_t = 0$ also implies that the critical price is zero, such that no switch in performance ranking can occur in this case if the average type stays below (above) the $(1+r)^2$ threshold; see (8). Intuitively, if the initially best-performing agents remain the best performers, then eventually their type should be adopted by all other agents along paths in the network, which is shown in Proposition 3.

**Proposition 3.** *Suppose $d_t = \overline{d}$ for all $t \in \mathbb{N}$. For $\gamma \to \infty$, we get the following:*

1. *If $\overline{g}_0 < (1+r)^2$, then all agents form a consensus on the most fundamental type in finite time, i.e. $g_t^i \to g_0^{\min}$ for all $t \geq 2D(\mathbf{A}) - 1$, $i \in N$. Price converges to the fundamental price if $g_0^{\min} < (1+r)$, price converges to a non-fundamental price if $g_0^{\min} = (1+r)$, and price diverges to $\pm\infty$ if $g_0^{\min} > (1+r)$*
2. *If $\overline{g}_0 > (1+r)^2$, then all agents form a consensus on the most chartist type in finite time, i.e. $g_t^i \to g_0^{\max}$ for all $t \geq 2D(\mathbf{A}) - 1$, $i \in N$. Price diverges to $\pm\infty$.*

Proposition 3 relates consensus and price convergence to initial types when there is exclusive attention to performance. As noted in (13), if initial average type is above $(1+r)^2$, then higher types initially perform better, implying that the maximal types are adopted. This means that average type is not decreasing, such that $\overline{g}_t > (1+r)^2$ for all $t \in \mathbb{N}$. Intuitively, strong chartist beliefs become reinforcing in the sense that strong chartists expect the price to move away from the fundamental price which indeed happens if there are sufficiently many strong chartists with types exceeding $(1+r)^2$, i.e. if the initial average type is large. As a result, types will converge to the maximal type and price diverges for any network structure.

On the other hand, if initial average type is small enough, then fundamental expectations are doing better and these beliefs become reinforcing. Note that price may still diverge in this case (which occurs for sure if the initial average type is below $(1+r)^2$ and the initial minimal type is above $(1+r)$), but more fundamental expectations are still yielding a higher fitness in every period, ensuring convergence to the minimal type. Hence, only the minimal initial type determines the consensus and, thus, whether price converges. So, if investors focus strongly on performance, one weak chartist (type $< 1+r$ is enough) will be enough to stabilise asset prices, *irrespective* of the network centrality of that agent.

Compared to pure social updating (Proposition 2), network centrality is *irrelevant* for the consensus type as the influence of the initially best-performing agent is 1 (and 0 for all others). The network structure does, however, affect time to convergence which is finite and does not exceed $2D(\mathbf{A}) - 1$, where $D(\mathbf{A})$ is the diameter of the network. The reason is that if the best-performing type is adopted by some agent $i \in N$ at time $t \in \mathbb{N}$, then all agents $j$ who directly observe $i$ (such that $a_{ji} = 1$) will have adopted this type themselves at latest by time step $t+2$ and will remain with this type forever. Since the first updating occurs in period 1, the maximal convergence time is given by the twice the length of the longest path in the network reduced by 1, i.e. $2D(\mathbf{A}) - 1$. If all agents observe all other agents $a_{ij} = 1$ for all $i, j \in N$ (i.e. a complete network), then convergence will obtain after 1 period.

Lastly, note that if there is a pure fundamentalist and we are in Part 1 of Proposition 3, then this type is adopted in at most $2D(\mathbf{A}) - 1$ periods, and the fundamental price is reached in at most $2D(\mathbf{A}) - 1$ periods, such that *mispricing is eliminated in finite time*. This result speaks to a common notion that stock markets are inefficient in the short run but efficient in the long-run, but with the extra observation that the *network* can influence how *quickly* mispricing is eliminated via the diameter $D(\mathbf{A})$. If instead, there is initially no pure fundamentalist, reaching the fundamental price is not possible in finite time.

**Example 2.** Consider again the star network from Example 1. We now let $\gamma \to \infty$ and set $r = 0.04$, $\varepsilon_t = 0$ for all $t$ (no dividend shocks) while leaving unchanged all other parameters from Example 1, so we have a comparable case to Fig. 2 except that agents now attend exclusively to the best-performing types who they observe and dividend shocks are no longer irrelevant. The results in this case – see Fig. 3 – relate to Proposition 3 above.

Since initial average type exceeds $(1+r)^2$, higher types perform better and the initial best performer is Agent 5. As a result, Agent 1 – who observes the types of all others – adopts the high initial type of Agent 5 in period 1 (Fig. 3, left panel). Agents 2–4 observe the type of Agent 1 in period 0 and the resulting (worst) performance in periods 0 and 1 and therefore stick with their own initial type in periods 1 and 2.[10] In period 3, however, they all adopt the type of Agent 1 (whose period 2 performance reflects their period 1 type). Therefore, we see consensus on the highest initial type in period 3, and Agent 5 has a weight of 1 in the consensus (and all others zero). Consensus on the highest type and the time to consensus are consistent with Proposition 3 (network diameter is 2 in this example). Since agents reach a consensus on the highest type, the price diverges to $+\infty$ (see Fig. 3, right panel).

### 3.3.2. The case of stochastic dividends

To characterise the terminal types in Proposition 3 we assumed dividends were non-stochastic. Clearly, for small enough dividend shocks, the same conclusions should hold. We now show this and characterise the relevant bounds on the shocks. Consider the following bounds on shocks given by

$$\underline{\varepsilon}(\mathbf{g}_0, r) = \min_{0 \leq t \leq 2D(\mathbf{A}_C)-2} \left|\sigma_t(\mathbf{g}_0, r)\right| \quad \text{and} \quad \overline{\varepsilon}(\mathbf{g}_0, r) = \max_{0 \leq t \leq 2D(\mathbf{A}_C)-2} \left|\sigma_t(\mathbf{g}_0, r)\right|$$

[10] Recall that there is a lag from type to performance to type updating: the next period type depends on the last realised profit, which in turn depends on the past demand determined by the lagged type; see (7).

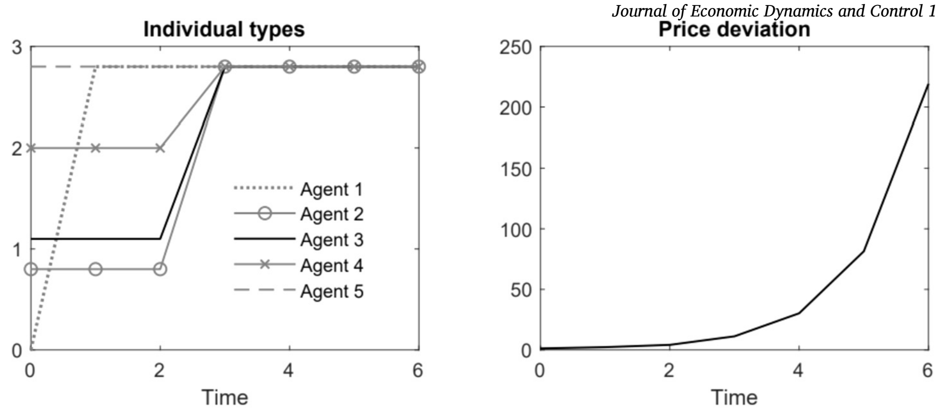



**Fig. 3.** Dynamics of individual types and price in the star network for $\gamma \to \infty$ and non-stochastic dividends. Initial values are as in Example 2 and were chosen to illustrate that even for interest rate $r = 0.04$, where we observed convergence to the fundamental price in Example 1 (case: $\gamma = 0$), price diverges to $+\infty$.

where $\sigma_t(\mathbf{g}_0, r) = \left((1+r) - \frac{\overline{g}_t}{1+r}\right) \tilde{p}_0 \frac{1+r}{\overline{g}_0} \prod_{j=0}^{t-1} \frac{\overline{g}_j}{1+r}$. Note that all these bounds depend on the concrete sequence of type updating via $\overline{g}_t$. Since we only characterise two particular sequences of type updating (updating from the maximal types only or from the minimal types only), these bounds are fully defined by the initial vector of types $\mathbf{g}_0$ such that $\overline{g}_t$ is recursively defined by

$$\overline{g}_t = \begin{cases} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{\left|G_{t-2}^{\min}(N^i)\right|}\sum_{j\in G_{t-2}^{\min}(N^i)} g_{t-1}^j\right) & \text{if } \overline{g}_0 < (1+r)^2 \\ \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{\left|G_{t-2}^{\max}(N^i)\right|}\sum_{j\in G_{t-2}^{\max}(N^i)} g_{t-1}^j\right) & \text{if } \overline{g}_0 > (1+r)^2 \end{cases},$$

where $G_t^{\min}(S)$ and $G_t^{\max}(S)$ refer to the set of investors with minimal and maximal type from set $S \subset N$ at time $t \in \mathbb{N}$ as defined in Section 2.

The next result shows that if shocks are small then the result of Proposition 3 still holds.

**Proposition 4.** *Suppose shocks are small enough such that $d^+ < \underline{\epsilon}(\mathbf{g}_0, r)$ and $-d^- > -\overline{\epsilon}(\mathbf{g}_0, r)$. For $\gamma \to \infty$, we get the following:*

1. *If $\overline{g}_0 < (1+r)^2$, then $g_t^i \to g_0^{\min}$ for all $t \geq 2D(\mathbf{A}) - 1$, $i \in N$.*
2. *If $\overline{g}_0 > (1+r)^2$, then $g_t^i \to g_0^{\max}$ for all $t \geq 2D(\mathbf{A}) - 1$, $i \in N$.*

The basic message of Proposition 4 is that the result in Proposition 3 holds if dividend shocks are small enough. We showed this by restricting the interval of admissable shocks such that $[-d^-, d^+] \subseteq [-\overline{\epsilon}(\mathbf{g}_0, r), \underline{\epsilon}(\mathbf{g}_0, r)]$. Alternatively, one could assume the distribution of shocks has zero mass outside this interval, implying that Proposition 4 holds almost surely.

For dividend processes $d_t$ for which $\epsilon_t$ does not satisfy the conditions on the bounds $d^-, d^+$ in Proposition 4 – or the zero mass condition above – we can still provide a lower bound on the *probability* that consensus is reached on one of the extreme types by $t \geq 2D(\mathbf{A}) - 1$, i.e. this is the probability that the dividends are drawn from within the set of admissible bounds. For instance for $\overline{g}_0 < (1+r)^2$, we would get that types converge to minimal type by period $2D(\mathbf{A}) - 1$ with at least probability

$$Pr(g_t^i = g_0^{\min} | \forall t \geq 2D(\mathbf{A}) - 1, i \in N) \geq \prod_{t=0}^{2D(\mathbf{A})-2} \Pr(-\sigma_t(\mathbf{g}_0) < \epsilon_t < \sigma_t(\mathbf{g}_0)). \tag{14}$$

The case of $\overline{g}_0 < (1+r)^2$ and convergence to the maximal type is fully analogous and yields the same probability. This illustrates that a lower bound for the probability to reach the consensus on the extreme type can be related to the diameter of the network $D(\mathbf{A})$. Note that (14) presents only a lower bound for two reasons. First, the concrete sequence of updating depends on the initial vector of types (not only the extreme types). So the consensus may be reached a lot faster than by period $2D(\mathbf{A}) - 1$ in which case the following shocks can be arbitrarily large, increasing the probability of this event (given the same network structure). Second, as long as the extreme type is not eliminated, even shocks that exceed the bounds and occur before the consensus is reached may be admissible. Since this both depends on the given network structure and on the initial vector of types, this becomes analytically intractable. We therefore revert to simulation in what follows.

**Example 2** *(continued).* We now add stochastic dividends in Example 2 for $\gamma \to \infty$.

In Fig. 4 we show that if dividend shocks are large enough, they may change the consensus type and date. As a simple example, we draw a strongly *negative* dividend shock in period 0, which makes low types the initial best performers (the initial return is now negative, not positive). The impact on the price-type dynamics is substantial: since all agents observe the best-performing Agent 1,

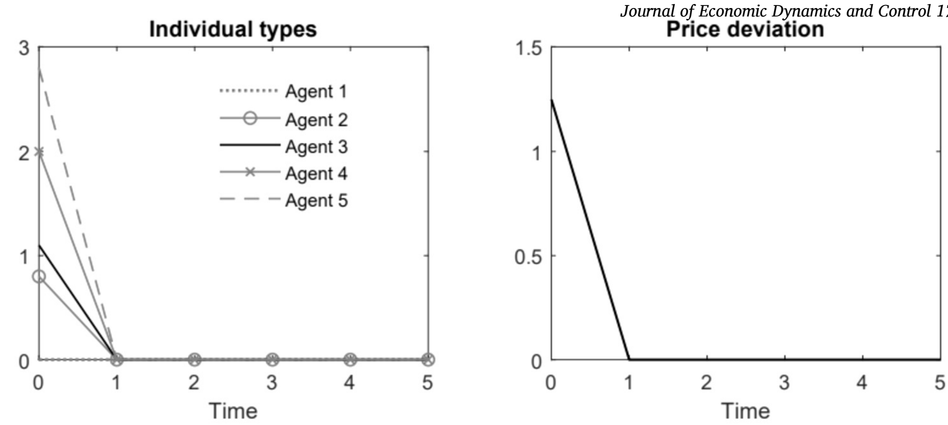



**Fig. 4.** Dynamics of individual types and price in the star network for $\gamma \to \infty$, $\tilde{p}_0 = 1.25$ and a *large negative* dividend shock $\varepsilon_0 < 0$ in period 0 that reverses the initial performance ranking. Initial values are as in Example 2. While in the absence of dividend shocks, price diverges to $+\infty$ as illustrated in Fig. 3, this simulation illustrates that price can instead converge to the fundamental price when dividends are stochastic.

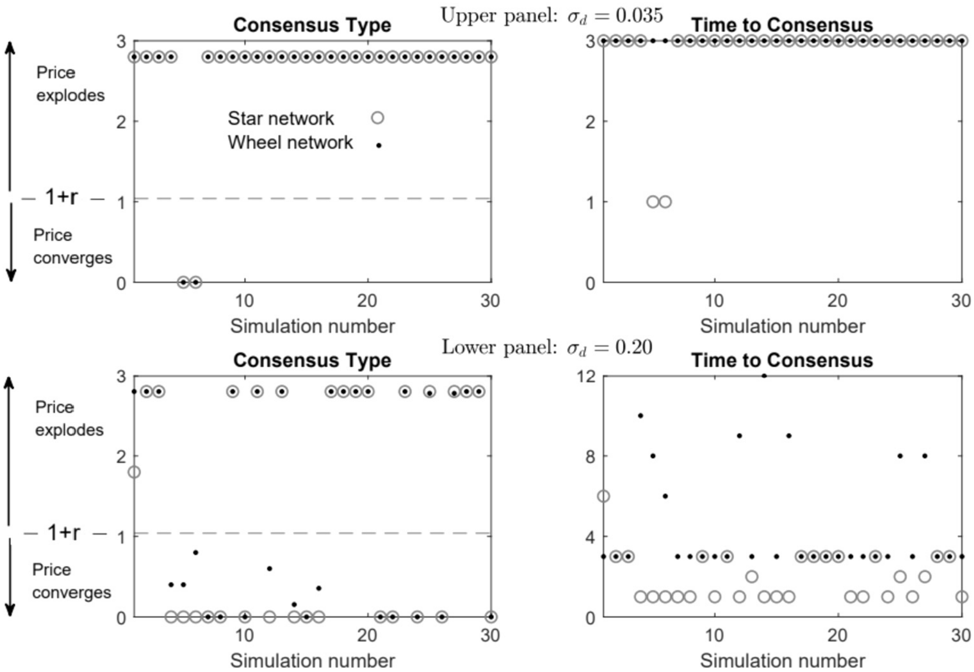



**Fig. 5.** Consensus type and time to consensus in the star (∘) and wheel (⋅) networks when $\tilde{p}_0 = 0.25$, $r = 0.04$ and $\gamma \to \infty$: results from 30 different stochastic simulations. Upper panel: small variance ($\sigma_d = 0.035$); lower panel: large variance ($\sigma_d = 0.20$). Other parameters and initial values as in Example 2.

they all adopt her pure fundamental type of 0 in period 1, so a consensus is reached in period 1 and price equals the fundamental price from period 1 onwards (hence, in these periods, price and beliefs coincide with the rational expectations outcome).

More nuanced changes in the type dynamics are also possible. For example, if the dividend shock in period 0 is relatively small (leaving the sign of return unchanged) but the period 1 dividend shock is negative and substantial enough to make the date 1 return negative, then Agent 1 will have adopted the highest type in period 1, but will then want to adopt a lower type in period 1. As a result, a consensus will form on a non-extreme type and it may be several periods before a consensus is reached (depending on subsequent dividend shocks).

Fig. 5 illustrates the diversity of outcomes which are possible. It reports the consensus type and time to consensus for 30 different sequences of dividend shocks, given an initial price of $\tilde{p}_0 = 0.25$ (all other parameters and initial conditions are the same as in Example 2). We draw the shocks $\varepsilon_t$ from a truncated-normal distribution with standard deviation $\sigma_d$ and support $[-\overline{d}, \overline{d}]$, to ensure that dividends are bounded and non-negative. We consider both a 'small' variance $\sigma_d = 0.035$ (upper panel) and a 'large' variance

$\sigma_d = 0.20$ (lower panel). We consider two different network structures – the star network already studied and a wheel network [11] – which have the *same diameter* for $n = 5$ and thus identical long-run price and type behaviour in the deterministic case by Proposition 3.

In the 'small variance' case, 28 of 30 simulations coincide with the predictions of Proposition 3 in the deterministic case (see Fig. 5, upper panel); intuitively we expect most shocks to lie in the bounds in Proposition 4 in this case. The two outlier simulations – due to large negative dividend shocks – have a consensus type of zero (for both networks), but in the case of the star network, the consensus is reached in period 1, as in Fig. 4 above.

In the 'large variance' case, there is a much greater diversity of outcomes (lower panel), as well as substantial differences when comparing across the two networks. We see consensus types that lie *between* the extreme initial types, and there are many more such cases for the wheel network than the star network (bottom left). The latter result arises because the 'fast' 1-period convergence does not happen on realising a large negative dividend shock. For the wheel network, 8 simulations of 30 have time to consensus of 6 periods or more, as compared to only 1 such simulation for the star network.

In short, for the case of stochastic dividends, both the time to consensus and the consensus type are quite difficult to predict if dividend shocks have high variance or are *not* restricted to narrow bounds. Furthermore, we see that although network *centrality* does not determine the consensus type when $\gamma \to \infty$, it is clear that network *structure* can influence not just the time to consensus, but also the consensus itself when dividend shocks are large enough.

### 3.4. $\gamma > 0$: *intermediate attention to performance*

We now examine the case where $\gamma$ is positive but finite, leading to a setting where type and price dynamics are analytically complex. Here, asset prices depend on agents' beliefs, which in turn are influenced by performance feedback that varies with stochastic dividends. Consequently, belief updating is shaped by both network structure and performance. Nevertheless, we can establish concrete results regarding price and type convergence.

**Proposition 5.** *Let $\gamma > 0$ and let dividends $d_t$ be stochastic with support $[\overline{d} - d^-, \overline{d} + d^+]$.*

1. *If $g_0^{\max} < 1 + r$, then the price converges to the fundamental price for all realisations of $d_t$. Conversely, if $g_0^{\max} > 1 + r$ and the parameters $\gamma$, $d^-$, and $d^+$ are sufficiently large, there is a positive probability that the price will diverge to $\pm\infty$.*
2. *If $g_0^{\min} > 1 + r$, then the price diverges to $\pm\infty$ for all realisations of $d_t$. However, if $g_0^{\min} < 1 + r$ and $\gamma$, $d^-$, and $d^+$ are sufficiently large, then there exists a positive probability that the price converges to the fundamental price.*
3. *If $\lim_{t\to\infty} \tilde{p}_t$ exists, then both the price and types converge to a steady state $(p^*, \mathbf{g}^*)$.*

The first part of each of the first two statements Proposition 5 follows intuitively from the nature of weighted-average updating. Since the convex hull of types does not expand over time, the average type $\overline{g}_t$ at any point $t \in \mathbb{N}$ will always lie within the interval defined by the initial extreme types, $[g_0^{\min}, g_0^{\max}]$. Therefore, if $g_0^{\max} < 1 + r$, then $\overline{g}_t < 1 + r$ for all $t \in \mathbb{N}$, leading the price to converge to its fundamental value. Similarly, if $g_0^{\min} > 1 + r$, then price divergence is straightforward.

Although this may seem trivial, the second part of each of the first two statements shows that these conditions are quite tight in the sense it is not possible to relax the conditions on the initial type distribution without additional conditions on the performance feedback parameter $\gamma$, or the dividend shocks $\varepsilon_t$, for ensuring convergence or divergence, respectively.

While the coupled price-type dynamics become analytically intractable even in the absence of stochastic dividends, it is still possible to establish convergence to a steady state under weak conditions in the final part of Proposition 5. The proof of type convergence draws on the boundedness of price and dividends within the interval $[\overline{d} - d^-, \overline{d} + d^+]$, along with finite $\gamma$, which ensure that the profit terms $u_t^i$ are bounded.[12] Hence, if $i$ observes $j$ in the network, i.e. $a_{ij} > 0$, then $i$ will also update from $j$ such that the corresponding updating weights in the law of motion can be bounded away from 0, i.e. there exists a $\zeta > 0$ such that $\tilde{a}_{ij}(t) \geq \zeta$ for all $t \in \mathbb{N}$. The resulting ergodicity property ensures types converge to a consensus. If price convergence is not guaranteed, then this ergodicity property cannot be established. Indeed in the small world network simulations in Section 5.3, we encountered numerical instances of non-convergence of types when price diverges (see Footnote 22).

The consensus itself is analytically intractable due to path dependency. To see why, note that solving forward the equation for the type dynamics yields:

$$\mathbf{g}_{t+1} = \left( \prod_{s=0}^{t} \tilde{\mathbf{A}}(s) \right) \mathbf{g}_0 \tag{15}$$

where the entries of the $\tilde{\mathbf{A}}(s)$ matrices depend on *performance* via the endogenous market price. Although the matrix product is analytically intractable, (15) shows how to compute the type dynamics for any finite $t$ using numerical simulations (a feature we exploit below).

[11] The adjacency matrix $\mathbf{A}$ of the wheel network has entries $a_{ij} = 1$ if $|i - j| \in \{1, n-1\}$ and $a_{ij} = 0$ else, such that its graph looks like a wheel with agents connected to only the next and previous agents in order.

[12] The assumption that dividend shocks are drawn from a bounded interval is useful here. If, instead, this interval is allowed to be unbounded, then the conclusion of Proposition 5 will still hold almost surely.

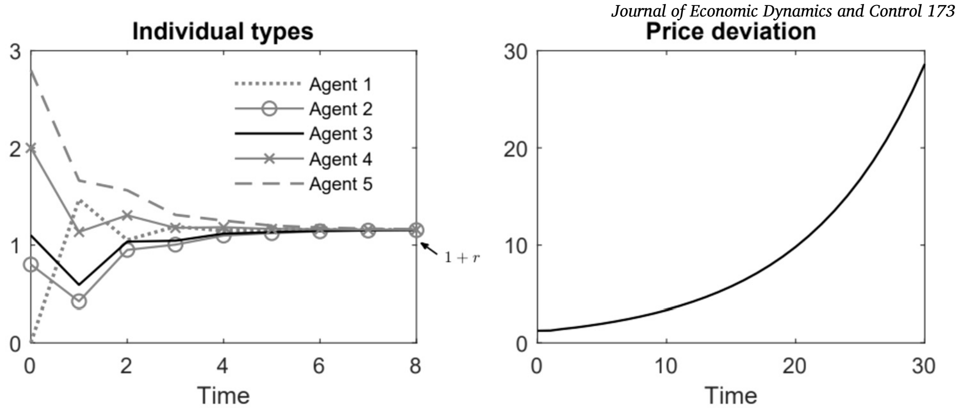



**Fig. 6.** Dynamics of individual types and price in the star network: $\gamma = 0.3$, $r = 0.04$, $\epsilon_t = 0\ \forall t$, $\tilde{p}_0 = 1.25$. With no dividend shocks, price diverges in this example (Fig. 7 below allows stochastic dividends).

By Proposition 5, the limiting type vector exists when $\lim_{t\to\infty} \tilde{p}_t$ exists and is given by

$$\mathbf{g}_\infty := \lim_{t\to\infty} \left( \prod_{s=0}^{t} \tilde{\mathbf{A}}(s) \right) \mathbf{g}_0, \qquad \mathbf{g}_\infty^i = \mathbf{g}_\infty^j \quad \forall i, j \in N. \tag{16}$$

Hence, for $\gamma > 0$, an agent's *performance-adjusted* influence on the consensus type can be found by approximating the limit in (16) (whose row entries are the influence weights) using numerical simulation. What complicates matters is that the consensus type as determined by (16) may not be in the interval whose endpoints are given by the consensus type for $\gamma = 0$, as determined by Proposition 2, and the consensus type for $\gamma \to \infty$ as determined by Proposition 3. As we show using a counterexample in Appendix B.3 for a bipartite network, the consensus need not be a weighted average of the consensuses of the pure social dynamics and exclusive attention to performance cases, and the consensus type can be *non-monotonic* in $\gamma$ even in the absence of random influences like dividend shocks. Both results are related to the 'double lag' between the future type update and the past type that determines realised performance (see (7), (9)); in turn, this double-lag structure is a key obstacle which prevents an analytical characterisation of consensus types for finite attention.

In light of this result, the price-type dynamics which will prevail in a market with finite attention to performance $\gamma > 0$ – including price stability – can be difficult to predict and *cannot* be inferred simply by studying the polar cases. We now give an example where the consensus and performance-adjusted influence of each agent are obtained numerically.

**Example 3.** Consider the star network of Example 1 for finite performance feedback $\gamma > 0$ and $r = 0.04$, $\epsilon_t = 0$ (no dividend shocks), and $\tilde{p}_0 = 1.25$. For $\gamma$ sufficiently close to zero, the type and price dynamics are similar to the case of zero performance feedback ($\gamma = 0$) in Fig. 2: the consensus type is smaller than $1 + r$, so the price converges to the fundamental price; see the grey line in the right panel of Fig. 2. However, increasing the feedback parameter $\gamma$ further leads to a qualitative change in the price dynamics.

For $\gamma \approx 1/45$ or larger, the consensus type exceeds $1 + r$ and hence the price becomes explosive (i.e. diverges to $+\infty$). We provide an example Fig. 6 (above), which is based on a feedback parameter of $\gamma = 0.3$. As before, we plot individual types in the left panel and the price deviation in the right panel. The key difference is that, because the initial average type exceeds $(1 + r)^2$, higher types are performing better and these performance differences are taken into account at the first update. As a result, individual types at date 1 are higher than in the case of no performance feedback (i.e. when $\gamma = 0$, cf. Fig. 2).

The largest difference in period 1 is for Agent 5 – the best performer – because they compare their own strong performance against that of Agent 1, the worst performer in period 0. In particular, they attach a much higher weight to their own past type than to the fundamental type of Agent 1, giving them a noticeably higher period 1 type (dashed line, left). The other agents also update to higher types (but to less extent) and so types move toward a more strongly chartist consensus after several updates. Thus, in this example the performance feedback reduces influence of the core (Agent 1) on the consensus type and increases the influence of agents in the periphery, such as Agent 5, who have better profit performance.

In the above example, price diverges to $+\infty$ but there is a type consensus despite the unbounded price dynamics. By Part 3 of Proposition 5, a type consensus is guaranteed if price converges, and by Part 2 of Proposition 5 the price will converge to the fundamental price with positive probability if $\gamma$ is large enough and the bounds for *dividend shocks* are 'wide enough'. We now add dividend shocks and show this result in Fig. 7.

In Fig. 7 we keep $\gamma = 0.3$, so the only difference compared to Fig. 6 is the dividend shocks. We set a 'large' negative dividend shock $\epsilon_0$ in period 0, such that the critical price becomes positive and large enough to reverse the initial performance ranking relative to Fig. 6; see (8).[13] Type updating now leads to much lower types in period 1 than if performance weighting were absent, since the

[13] We draw dividend shocks from a truncated-normal distribution with standard deviation 0.05 and support $[-\overline{d}, \overline{d}]$. We set the initial dividend shock at $\epsilon_0 = -\overline{d} + c$, where $c$ is a small positive number.

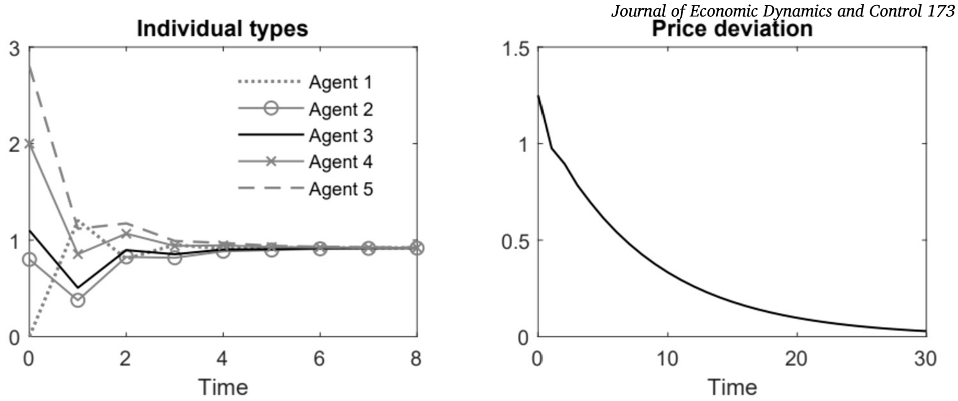



**Fig. 7.** Dynamics of individual types and price in the star network: $\gamma = 0.3$, $r = 0.04$, $\tilde{p}_0 = 1.25$, dividend shocks and a *large negative* dividend shock $\epsilon_0 < 0$ at date 0 that reverses the initial performance ranking.

period 0 return (inclusive of dividends) is now *negative* so that the pessimistic short-sellers (agents 1,2,3) are the best initial performers (in that order).

As a result, we see that the highest type in period 1 is for Agent 1 – who updates from all others – while Agents 4 and 5 reduce their types very sharply because their performance in period 0 is now much worse than the best-performing Agent 1 who they weigh their performance against. The sharp reduction in type for Agents 2–5 in period 1 lowers average type substantially and subsequent updating leads to a consensus type much smaller than $1 + r$ (around 0.92), so the price converges on the fundamental price (in contrast to Fig. 6) and converges at a much faster rate than if performance is ignored ($\gamma = 0$, Fig. 2, where dividend play no role). Due to performance feedback, Agent 1 has more influence on the consensus compared to $\gamma = 0$ (agents 2 and 3, too), while agents 4 and 5 become less influential.[14]

In Fig. 6, where $\gamma = 0.3$ and there are *no* dividend shocks, the consensus type is just above $1 + r = 1.04$ and therefore lies between the consensus of approx. 1.031 for $\gamma = 0$ (Fig. 2) and the consensus of 2.8 when $\gamma \to \infty$ (Fig. 3). Likewise, the consensus type for finite attention to performance lies between the consensus types for $\gamma = 0$ and $\gamma \to \infty$ in the case of a large *negative* dividend shock in period 0 (see Figs. 2, 4 and 7).[15] Although it is quite intuitive that consensus lies between the polar cases this is not always the case even in the absence of dividend shocks, as shown in Appendix B.3.

## 4. Extension to non-connected networks

Our discussion thus far has been limited to strongly connected networks, where all agents are interlinked, i.e., $d(i, j) < \infty$ or equivalently $j \in \mathcal{P}^i$ for all $i, j \in N$. This assumption facilitates the exposition, yet it is also useful to understand how our results extend to non-connected networks.

To begin, we introduce additional notation. A subset of agents $C \subset N$ is defined as *strongly connected* if any two agents $i$ and $j$ within $C$ can communicate directly or indirectly, i.e., $j \in \mathcal{P}^i$ for all $i, j \in C$. Thus, information can propagate between any pair of agents within a strongly connected subset. A subset of agents $C \subset N$ is termed *closed* if there is no communication path from any $i \in C$ to any external agent $j \in N \setminus C$, i.e., $\mathcal{P}^i \subseteq C$ for all $i \in C$. This concept induces a partition of agents into distinct communication classes $\Pi(N, \mathbf{A}) = \{C_1, C_2, \ldots, C_K, \mathcal{R}\}$, where each $C_k$ is strongly connected and closed, and $\mathcal{R}$ denotes the (potentially empty) *Rest of the World* comprising agents not included in any strongly connected, closed group. Notably, each network includes at least one non-empty, strongly connected, and closed subset $C$.

Fundamentally, the type dynamics results derived for strongly connected networks extend to each strongly connected and closed subset $C$. Since agents only listen to peers within their closed group, the presence of other agents outside this group affects price dynamics hence may affect the updating weights, but not the core updating mechanics. Qualitatively, all results continue to hold in this broader framework, as demonstrated in Appendix A.2. Our primary results are thus presented and proven in their generalised form in Appendix A.2, applicable to both connected and non-connected networks.

Only agents in the Rest of the World exhibit distinct behaviour, as they may listen to individuals outside their group. Here, we summarise the main extensions to our results, emphasizing implications for agents in the Rest of the World. First, similar to Proposition 1, we demonstrate in Proposition 1a that in any steady state, agents in each closed, strongly connected group reach a consensus, while agents in the Rest of the World adopt a convex combination of the types from the closed, strongly connected groups. In a fundamental steady state, we can precisely characterise the types in the Rest of the World by:

$$\mathbf{g}^{\mathcal{R},*} = \left(\mathbf{I}_{|\mathcal{R}|} - \overline{\mathbf{A}}_{\mathcal{R}\mathcal{R}}\right)^{-1} \overline{\mathbf{A}}_{\mathcal{R}(N\setminus\mathcal{R})}\mathbf{g}^{N\setminus\mathcal{R},*}. \tag{17}$$

[14] The performance-adjusted social influence weights are approx. (0.409, 0.174, 0.164, 0.137, 0.117) (see (16)). Thus, compared to $\gamma = 0$ (Example 1, Fn. 9), Agent 1 remains most influential and has a larger influence on the consensus, while agents 4,5 become less influential and agents 2,3 become more influential.

[15] Recall that for $\gamma = 0$, dividends are irrelevant for the type dynamics, so Fig. 2 applies in this case.

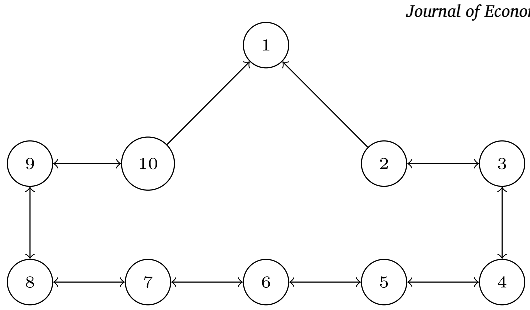



**Fig. 8.** The network configuration of Section 5.1 for $n = 10$ omitting the self loops.

Intuitively, in a fundamental steady state, all agents perform uniformly, so the updating matrix is given by $\overline{\mathbf{A}}$. Thus, types in the Rest of the World can be characterised accordingly.

For pure social updating ($\gamma = 0$), the consensus within each closed and strongly connected set aligns with the eigenvector centrality of the adjacency matrix restricted to that set, analogous to Proposition 2 but just applied to each closed and strongly connected set separately (see Proposition 2a). Terminal types of agents in the Rest of the World are then also determined by (17) since updating in the pure social case ($\gamma = 0$) adheres to $\overline{\mathbf{A}}$.

The case of pure performance-based updating ($\gamma \to \infty$) generalises smoothly to non-connected networks. Proposition 3, which states that all agents adopt the highest performing type in the network, generalises in the sense that each agent adopts the best-performing type along their path $\mathcal{P}^i$ (see Proposition 3a). Thus, closed and strongly connected groups adopt the top-performing type from within their group, while agents in the Rest of the World adopt the best-performing type available along their path. This contrasts sharply with (17), where agents from the Rest of the World average across all types from closed and strongly connected groups to which they are connected. For pure performance updating, initial types in the Rest of the World can persist over time which occurs if the initially best-performing agent originates from the Rest of the World since this agent will not change the type even if outside the convex hull of types in closed and strongly connected groups. Notably, the best-performing agents remain determined by the initial type distribution: extreme fundamental types prevail if $\overline{g}_0 < (1+r)^2$, while extreme chartist types prevail if $\overline{g}_0 > (1+r)^2$. As before, the long-run price behaviour follows from the terminal types in Propositions 2a and 3a.

The other main results, i.e. Propositions 4 and 5 extend to non-connected networks with minor modification (Proposition 4a) or no modification at all (Proposition 5). Appendix A.2 includes proofs of these results without requiring the network to be strongly connected.

## 5. Applications

We close the paper with three applications – price bubbles, price fluctuations, and asset pricing on a 'small world' network. These numerical applications highlight some concrete implications of network-performance effects for asset pricing when analytic results are not possible, as well as showing the usefulness of some of the extensions covered in Section 4.

### 5.1. Price bubbles

We first consider an example with asset price 'bubbles' in the sense that positive deviations from the fundamental price initially grow to reach a peak before the price collapses and converges on the fundamental (i.e. intrinsic) value. Following Smith et al. (1988), such 'bubbly' price dynamics have been documented in numerous studies of experimental asset markets, and adding social communication influences the incidence of such bubbles.[16] In the application here, the bubble dynamics are generated by the network with *no* exogenous disturbances to dividends. Besides highlighting the possibility of price bubbles, our example has the pure network effect and the performance feedback either competing against one another or reinforcing one another, depending on the value of the average type $\overline{g}_t$.

Consider a network with a 'die-hard' pure fundamentalist, Agent 1, who does not listen to any other agent. The remaining agents, 2 to $n$, start out as strong chartists but update their type based on their network, with weights depending upon performance when $\gamma > 0$. For convenience we set $g_0 = (0, 2, ..., 2)$ and $n = 10$, so that $\overline{g}_0 = 1.8$. Agents $i = 2, \dots, 10$ listen to each of their nearest 'neighbours' on either side, so the network structure is similar to a wheel network, except that Agent 1 does not listen to agents 2 or 10 (see Fig. 8).

In the terminology of the paper, Agent 1 is the only closed and strongly connected set while the Rest of the World consists of the remaining agents 2 to 10. Note that this example can be interpreted as a world with a 'die-hard' fundamentalist (Agent 1) and many followers who either follow Agent 1 directly (agents 2,10) or follow her indirectly by following either her followers or her followers' followers; intuitively, we may think of this example as a market with 'one Warren Buffet and many sheep'.

We set $r = 0.04$, $\delta = 1$, $d_t = \overline{d} = 0.02$ for all $t$ (so $p^f = 1/2$, $\varepsilon_t = 0$) and $\tilde{p}_0 = 0.1 p^f$. Initially we set $n = 10$ so that chartists ('sheep') outnumber the fundamentalist by 9 to 1; later we allow $n$ to vary. When $\gamma = 0$, the fitness ranking is irrelevant and the updating of

[16] See, for example, Oechssler et al. (2011), Schoenberg and Haruvy (2012), Steiger and Pelster (2020).

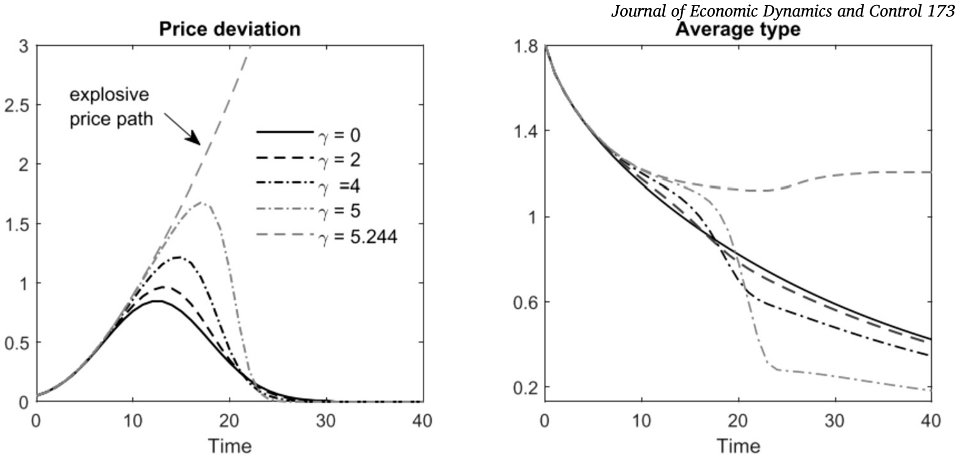



**Fig. 9.** Asset price bubbles and type dynamics: $\tilde{p}_0 = 0.05$ and various $\gamma$. In this example, increasing the performance feedback parameter $\gamma$ increases the size of bubbles and price explodes for large enough $\gamma$.

agents 2 to 10 depends on the (local) average computed from their own type and their neighbour on either side, whereas agent 1 always keeps $g_t^1 = 0$. As a result, the type dynamics for $\gamma = 0$ are *guaranteed* to converge to a consensus of zero, giving us a benchmark to compare against the case where both a network effect and performance feedback are present ($\gamma > 0$).

Fig. 9 shows that the price follows a stylised 'bubble' dynamic, first increasing for several periods to reach a peak before collapsing and falling toward the fundamental price. The right panel shows the corresponding average type dynamics. Price initially increases because $\overline{g}_0 = 1.8 > 1 + r$, and it goes on increasing for several periods while $\overline{g}_t$ remains above $1 + r$. Only after several rounds of updating has fundamentalist 'thinking' spread sufficiently through the population to lower the average type below $(1 + r)$, so that price starts falling and the bubble collapses. Hence, the price 'bubble' here is generated by a network effect.

Once we turn on the performance feedback ($\gamma > 0$), the bubble is prolonged and peaks at a higher value. At the same time, the bubble becomes strongly *asymmetric*, with price collapsing quickly after reaching its peak (left panel). Intuitively, when the average type satisfies $\overline{g}_t > (1 + r)^2 \approx 1.082$, more optimistic agents earn higher returns and hence chartist beliefs outperform the pure fundamentalist one; however, once $\overline{g}_t$ is below $(1 + r)^2$, the performance ranking is reversed, and there is a shift toward more fundamental beliefs, which can be seen in the dramatic decline of $\overline{g}_t$ soon after period 10 (dashed lines, right panel). This decline is especially evident in the case of $\gamma = 5$. Note that price quickly collapses once $\overline{g}_t < (1 + r)$ (left panel) because a falling price makes chartist expectations more conservative and is reinforced by a fall in average type $\overline{g}_t$, so that price declines are exacerbated.

Though the network effect wins the 'battle' against performance feedback up to $\gamma = 5$, this result is *reversed* in the final case ($\gamma = 5.244$). That is, once $\gamma$ is large enough, the performance feedback is strong enough that $\overline{g}_t$ always exceeds $(1 + r)^2$. In this case the reversal in performance ranking does not happen – see right panel – and since the terminal average type exceeds $(1 + r)$, we have a *perpetual bubble* where price diverges to $+\infty$.[17] In short, the attention placed on performance, as controlled by feedback parameter $\gamma$, has important qualitative and quantitative and implications for the price and type dynamics.

The findings here relate neatly to our theoretical results discussion in Section 4. The agents 2–10 form the Rest of the World and (17) implies that for $\gamma = 0$, the Rest of the World will adopt a weighted average of the consensuses in the closed and strongly connected sets. The same is true for any finite $\gamma > 0$ if the price converges to the fundamental price by Proposition 5 since the result also applies to non-connected networks as shown in the proof. Since the only closed and strongly connected set is the singleton set containing Agent 1, Propositions 2a and 5 imply that the types of agents 2–10 will converge to the fundamental type of agent 1 if price converges. By contrast, for $\gamma \to \infty$, Proposition 3a implies the Rest of the World will converge to the maximal type on their path (initial average type is above $(1 + r)^2$), which is clearly their initial type. Hence, for $\gamma \to \infty$, the initial types of agents 2– 10 will never change, giving an explosive price path.

*How sensitive are these results to the mass of chartists?* Fig. 10 varies the number of agents $n$ at three different values of $\gamma$, including the case $\gamma = 0$ (left panel).

Increasing $n$ raises the magnitude and persistence of the price bubble; intuitively, a large population of chartists corresponds to greater initial optimism among investors. At the same time, the *diameter* of the network increases for greater $n$ so it takes longer for the fundamental belief to spread. For the cases with positive $\gamma$ (Fig. 10, middle and right), the bubble is amplified and more persistent when investors are more focused on performance, and even small increases in $n$ have substantial effects on the bubble size and duration. Due to the switch in the performance ranking noted above, a strong *asymmetry* develops as $n$ is increased: the price bubbles build over many periods but collapse very sharply.

We only plot cases where price converges in Fig. 10, but $\gamma = 1.33, n = 11$ (right panel) is near the knife-edge, so a small increase in $\gamma$ or $n$ would lead to an explosive path – i.e. a perpetual price bubble. Recall that if $\gamma = 0$ (no attention to performance), then regardless of how large $n$ is, types will converge on a consensus of 0 (pure fundamentalist) and hence the price bubble will always

[17] Clearly, consensus does not obtain in this case since the 'die hard' fundamentalist has type of 0.

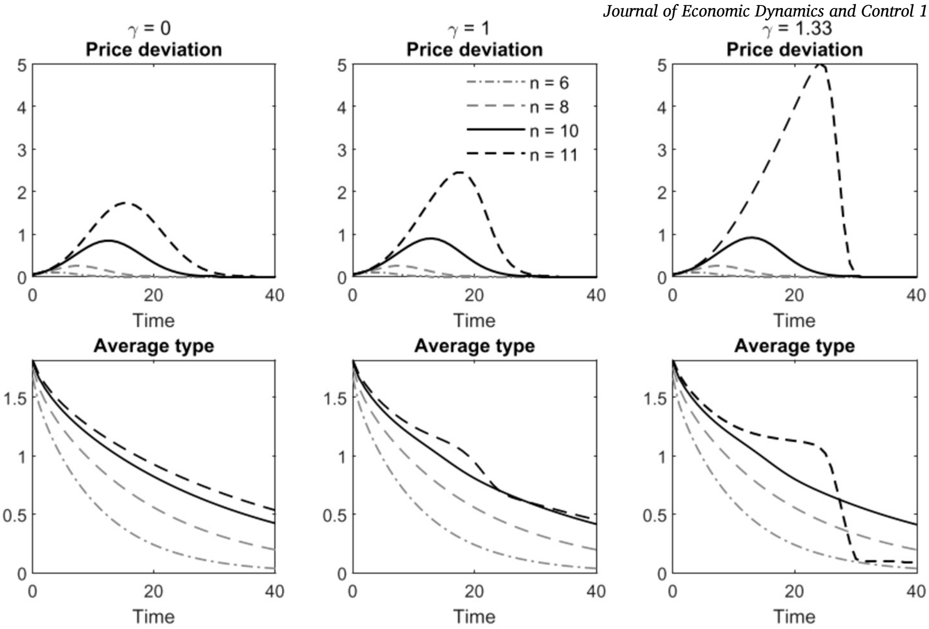
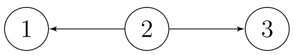



**Fig. 10.** Sensitivity of bubble paths to increasing the mass of chartists ($\tilde{p}_0 = 0.05$). As the number of agents $n$ rises, the share of chartists also increases, causing bubbles to peak at a higher price before collapse.




$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

**Fig. 11.** The 3-agent network (ignoring self-loops) and its adjacency matrix.

collapse, giving convergence to the fundamental price. However, if $\gamma > 0$ then for large enough $n$ we may see average type settle above $(1+r)$ such that the bubble is permanent and price explodes. Thus, we see once again that attention to performance has important implications for the price and type dynamics.

### 5.2. *Price fluctuations*

We now study price fluctuations for a network of $n = 3$ agents illustrated in Fig. 11. Agents 1 and 3 are 'die-hards' who listen only to themselves, i.e. $a_{ij} = 1$ if $j = i$ and 0 otherwise. Agent 2 listens to both agents 1 and 3, so $a_{2,j} = 1$ for $j \in \{1,2,3\}$ and Agent 2 belongs to the Rest of the World, while agents 1 and 3 each form a singleton closed and strongly connected set.

Agent 1's initial type is $g_0^1 = 1$ and Agent 3's initial type is $g_0^3 = (1+r+\nu)^2$, where $\nu = 0.001$. Since these agents update only from themselves, $g_t^1 = 1$ and $g_t^3 = (1+r+\nu)^2$ for all $t \in \mathbb{N}$. Agent 2's initial type is $g_0^2 = 1+r$, and their subsequent types $g_t^2$, for all $t \geq 1$, will depend on their updates from past types (of all agents), which are weighted according to their relative performance. Thus, changes in the average type $\overline{g}_t$ must come from changes in Agent 2's type. The other parameters are $\delta = 2$, $r = 0.04$, $\overline{d} = 0.5$, so $p^f = \overline{d}/r = 12.5$. The initial price deviation is $\tilde{p}_0 = 1$ and dividend shocks have standard deviation $\sigma_d$.

#### 5.2.1. *Finite* $\gamma$

We start by showing the price deviation $\tilde{p}_t$ for $\gamma \in \{0, 5, 10, 45, 75\}$, $\sigma_d = 0$ in Fig. 12. For $\gamma = 0$, performance is irrelevant and there is an explosive price that diverges to $+\infty$ (left panel). Intuitively, pure social updating implies that agent 2 will continually adopt a higher type, such that the average type $\overline{g}_t$ exceeds $1+r$ in all periods. For *any* $\gamma > 0$, explosive price is avoided because average type starts smaller than $(1+r)^2$, such that agent 2 puts a higher weight on the low type of agent 1 and continues to do so. The performance feedback thus dominates the social influence effect, preventing an explosive price path; the reason is that if average type and price do increase, then agent 2 puts even less weight on the high type of agent 3 because when price becomes large, so does the performance differential.[18]

For $\gamma = 5$ or $\gamma = 10$, price initially increases, but the increase is reversed as time passes, due to the performance effect offsetting the social influence effect (see right panel). For $\gamma = 45$, price increases in the first period, but all later updates have average type and

[18] As a check, we ran some long simulations of 5,000 periods for several very small (but positive) $\gamma$ values and found that price convergence occurs. The figure is provided in the *Online Appendix* (see Appendix C).

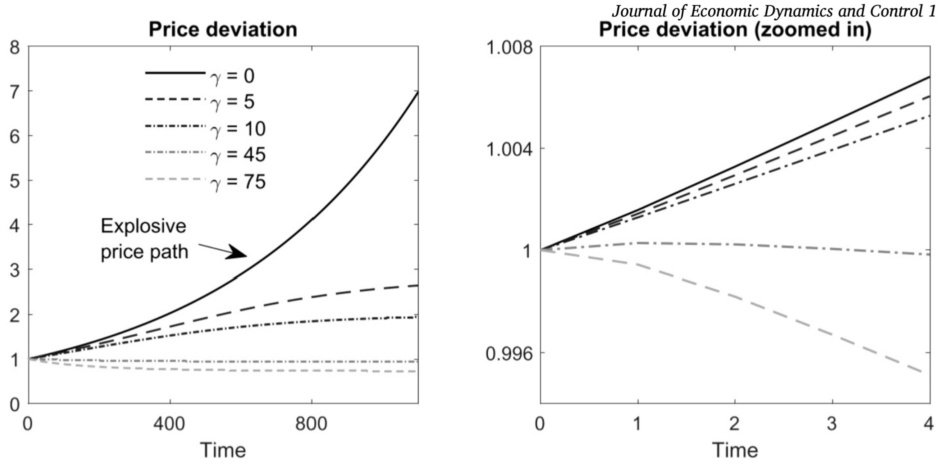



**Fig. 12.** Price deviation $\tilde{p}_t$ for selected values of the performance parameter $\gamma$ when dividends are deterministic: $d_t = \overline{d}$ for all $t$. Left panel: simulation of 1,100 periods; right panel: zoomed in on periods 0 to 4.

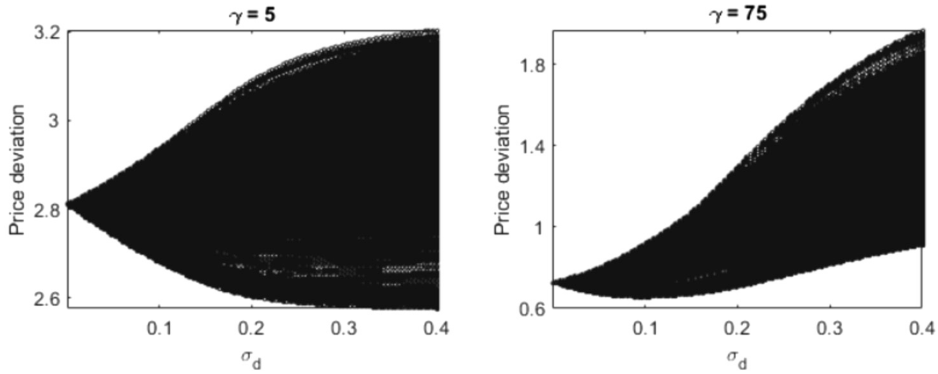



**Fig. 13.** Price deviation after 2,100 periods. For each standard deviation of dividend shocks $\sigma_d$, we simulated with 30 different sequences of dividend shocks; the last 50 values from each simulation are plotted.

price falling because the performance feedback dominates. For $\gamma = 75$, price falls in all periods because the social influence effect is dominated even at the first update, given that initial average type is smaller than $(1+r)^2$ and performance now receives extra attention.

In Fig. 13 we add dividend shocks from a truncated normal distribution with mean zero and standard deviation $\sigma_d > 0$. For each $\sigma_d$, we plot a sample of terminal values of the price from many long simulations with different shocks. We see that price fluctuates, since large dividend shocks imply switches in performance ranking between agent 1 and agent 3, such that average type moves above and below $1+r$. As $\sigma_d$ is increased the fluctuations and differences across simulations grow, but there is some symmetry around the deterministic outcome at $\gamma = 5$ (left panel; see also Fig. 12). Intuitively, a sequence of large dividend shocks of the same sign may cause price to rise somewhat at the start of a simulation, or to fall somewhat if average type is pushed below $1+r$ for several periods. When the feedback parameter is increased to $\gamma = 75$ (right panel), price fluctuations are highly asymmetric and strongly skewed toward higher values, as compared to the relative symmetry for $\gamma = 5$. In short, attention to performance matters not just for the long run average price, but also for its volatility, with fluctuations not necessarily occurring 'around' the deterministic outcome.

### 5.2.2. *Exclusive attention to performance: $\gamma \to \infty$*

We now consider the case $\gamma \to \infty$, for which there are some non-trivial qualitative differences in the price and type dynamics. In this case, Agent 2 will adopt either the type of Agent 1 or Agent 3, depending on which is the best performer (Section 3.3). This can result in substantial price fluctuations, since switches in average type can be large and sudden, as they will happen *immediately* following a switch in performance ranking.

Fig. 14 gives an illustration. Here we plot the price deviation and average type for a given sequence of dividend shocks. When shocks cause the price deviation to cross the critical price, updating by Agent 2 is based on the bast performer among agents 1 and 3, so we see sizeable shifts in average type (right panel), and price fluctuates as average type shifts above and below $1+r$ (left panel). Note that switches in average type become more frequent after price has fallen (right panel) because the latter 'scales down' the performance differentials (see (6)–(7)), so that given shocks are more likely to switch performance ranking.

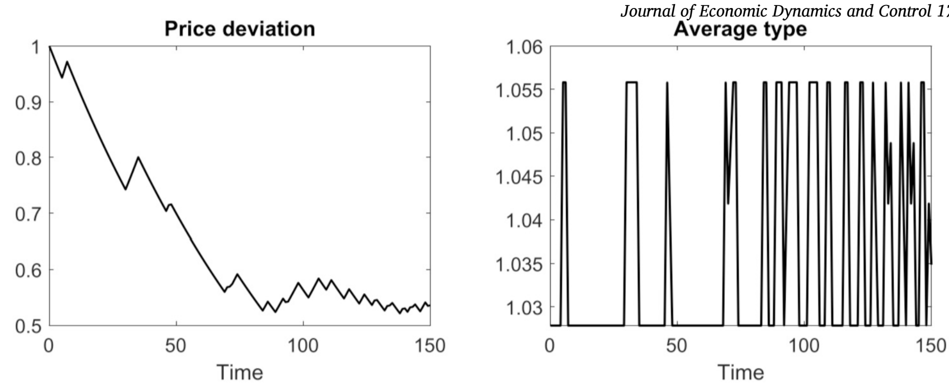



**Fig. 14.** Price deviation $\tilde{p}_t$ and average type $\overline{g}_t$ for $\gamma \to \infty$ and a particular sequence of dividend shocks. Note that there are 'sharp' fluctuations in the average type that cause the price to fluctuate.

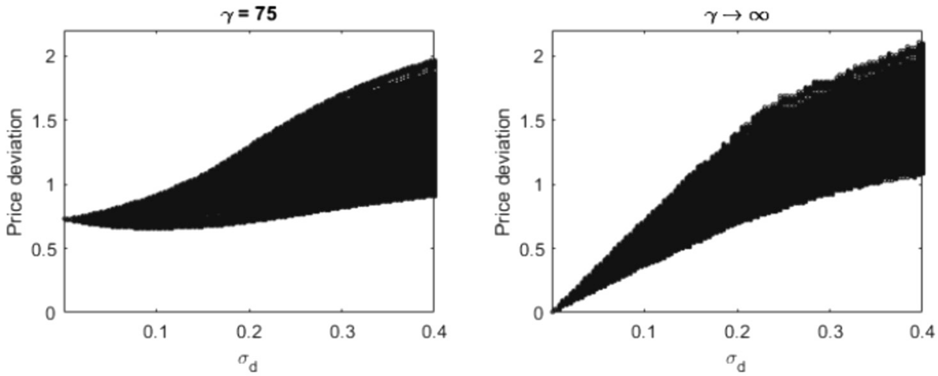



**Fig. 15.** Price deviation after 2,100 periods. For each standard deviation of dividend shocks $\sigma_d$, we simulated with 30 different sequences of dividend shocks; the last 50 values from each simulation are plotted.

*What happens as the variance of dividend shocks is increased?* Fig. 15 plots the terminal values of the price from many long simulations when $\gamma \to \infty$ (see right panel); here we use the same sequences of dividend shocks as in Fig. 13 and the results for $\gamma = 75$ are shown in the left panel. If shock variance is near zero, exclusive attention to performance pushes the long run price close to the fundamental price, consistent with the pattern in the deterministic case when $\gamma$ was increased (Fig. 12). Intuitively, for $\gamma \to \infty$, Agent 2 adopts the best-performing type rather than a weighted average, so average type is quite responsive even to small shocks (low values of $\sigma_d$), unlike in the finite $\gamma$ case (see left and right panels).

As $\sigma_d$ is increased, the price deviation increases and we also see a wider spread of prices, reflecting both the history dependence of the price (scale effect) and the introduction of non-trivial shocks that affect performance ranking. In particular, price is 'held up' relative to the deterministic case because falling price may be interrupted (or reversed) by dividend shocks that favour adoption of the higher type of agent 3 (right panel), and a higher frequency of large shocks means more switches in performance ranking that lead to price fluctuations.

Although price can fluctuate even if $\sigma_d$ is relatively low, the price drops sufficiently close to zero in these cases that fluctuations in the average type have little impact; that is, because price depends on average type *scaled* by the past price, a low price reduces the sensitivity of price to fluctuations in average type – the *history dependence* mentioned above. This scale effect is one reason the price is quite similar in terms of level and fluctuations at relatively high shock variances for $\gamma = 75$ and $\gamma \to \infty$ (see both panels): in such cases there are many reversals in performance, so the long run price does not get close to the fundamental price.

### 5.3. Asset pricing on a small-world network

As a last application, we investigate how *connectedness* on a social network influences type consensus and price dynamics when the performance feedback parameter $\gamma$ is held *fixed*. Social networks exhibit sparsity, clustering and small diameter, properties which motivate the 'small world' network of Watts and Strogatz (1998). In a small world, each agent is connected to a small fraction of the network; agents are more likely to be connected if they have a connection in common; and distance between any two agents is small. Examples include networks of the US corporate elite and partnerships of investment banks in Canada.[19]

[19] See, for example, Panchenko et al. (2013, p. 2625) and the references therein.

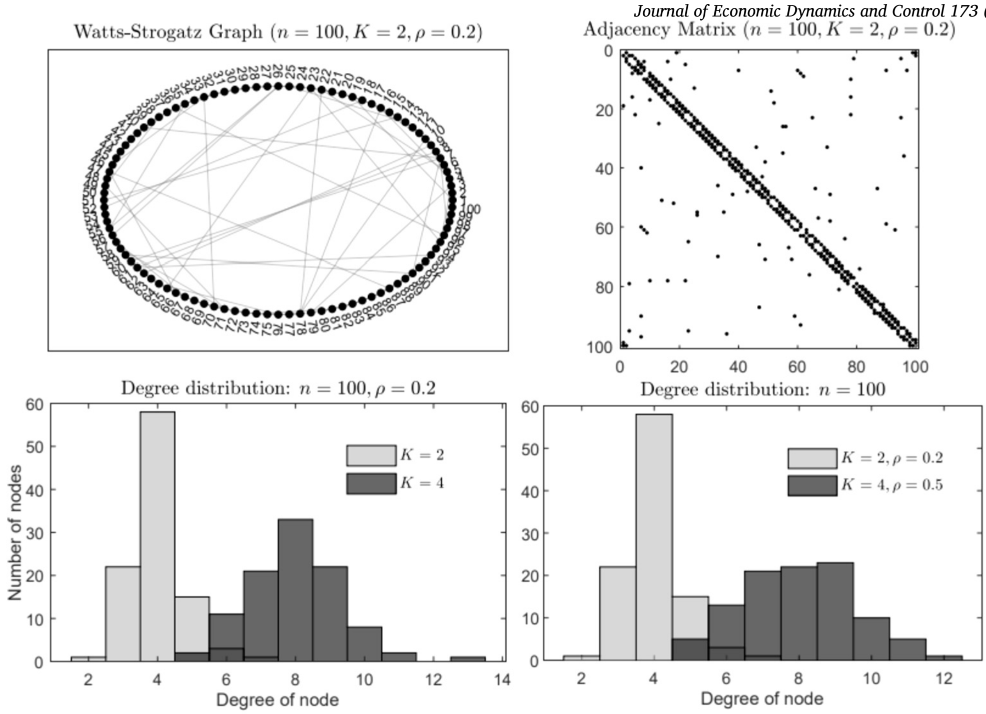



**Fig. 16.** A small world network (top panel) and some example degree distributions (lower panel). In all panels and computations in the figure, self loops are ruled out as in the baseline Watts-Strogatz model.

From a technical perspective, a small world is a hybrid network between a regular lattice (e.g. the 'wheel' in Example 2) and a random graph in which all links are random. In the first step, a regular lattice is constructed in which each node is connected to the $K$ nearest neighbours on each side; in the second step, each link is rewired with probability $\rho \in (0, 1)$ to a different node in the network. The 'small world' properties emerge at around $\rho = 0.1$ for $n = 100$, which we take as our number of agents in this section.[20]

We start by plotting the graph of a particular small world network and its adjacency matrix in Fig. 16 (top panel) and by showing how the degree distribution changes as the network parameters $K$ and $\rho$ are varied (lower panel). The graph (top left) runs anti-clockwise starting from Agent 1 at three o'clock and ending at Agent 100, and the adjacency matrix is sparse but with clusters of connected agents (top right). The top row sets $K = 2$ (implying a mean degree of 4) and the rewiring probability is set at $\rho = 0.2$. The bottom panel of Fig. 16 shows some example degree distributions. The degree distribution is centred around $2K$ and there is less variation in connectedness across agents the larger the rewiring probability $\rho$. The bottom left panel holds $\rho$ fixed while doubling the average degree by increasing $K$ from 2 to 4; the bottom right panel also doubles $K$ from 2 to 4, but the rewiring probability is increased from $\rho = 0.2$ to $\rho = 0.5$, which gives a more uniform degree distribution and fewer agents concentrated at the average degree.[21]

As a first exercise, we hold the parameters $K, \rho$ *fixed* and investigate the impact of different random draws of the network on the price and type dynamics. We set $n = 100$, $K = 2$, $\rho = 0.2$ as in the top panel of Fig. 16 and the initial conditions are a price deviation $\tilde{p}_0 = 0.1$, an expected dividend of $\overline{d} = 1$, a scaling $\delta = 2.5$ in the demands (2), an interest rate of $r = 0.04$, and initial types linear on the interval $[0, 1.95]$, so initial average type is $\overline{g}_0 = 0.975$. The performance feedback parameter is set at $\gamma = 180$. Dividend shocks $\varepsilon_t$ are drawn from truncated-normal distribution with standard deviation $\sigma_d = 1/3$ and support $[-\overline{d}, \overline{d}]$, to ensure that dividends are bounded and non-negative. The dividend shocks $\varepsilon_t$ are held fixed across simulations (identical sequences), so the only difference between simulations is the difference in the realised networks which are drawn from an identical distribution.

Fig. 17 shows the time paths of price and average type over 30 periods for 4 different random 'draws' of the network; one of these, shown with the dashed line, is the network shown in the top panel of Fig. 16. For these networks that differ only in the random links, the average type eventually settles on quite different values (left panel).

In three of the networks, the average type settles at a small enough value for price to converge on the fundamental price (Networks 1–3), while for Network 4 (dotted line) average type settles above $1 + r$, so that the price explodes (right panel). Since average type initially increases to exceed $1 + r$ in all cases, we first see a growing price deviation, but this growth is halted in the cases of Networks 1–3 (since lower types gain a performance advantage given the sequence of dividend shocks) and price then falls toward the fundamental value. These price paths are reminiscent of the 'price bubbles' seen in Fig. 9 but they arise from random differences

[20] For $\rho = 0$ (no rewiring), a regular lattice is obtained; for $\rho = 1$ we obtain the random graph of Erdös and Rényi (1959). For a discussion of the emergence of small world properties, see Albert and Barabási (2002).

[21] As before, we ignore self-loops in all figures including Fig. 16. Note, however, that for all simulations we have set the *diagonal* of the adjacency matrix equal to 1.

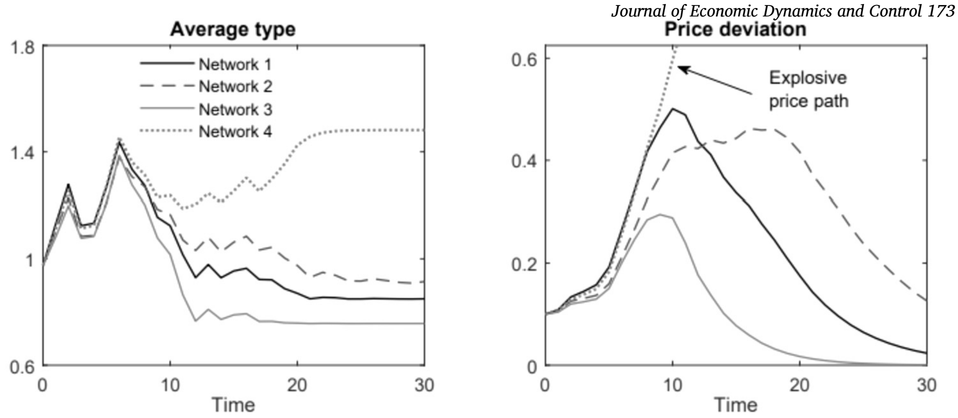



**Fig. 17.** Average type and price deviation in four different small world networks, all drawn from an identical distribution with parameters $n = 100$, $K = 2$, $\rho = 0.2$. In each case $\gamma = 180$ and $\tilde{p}_0 = 0.1$. We see that price dynamics can differ substantially due to differences in the random links in the network.

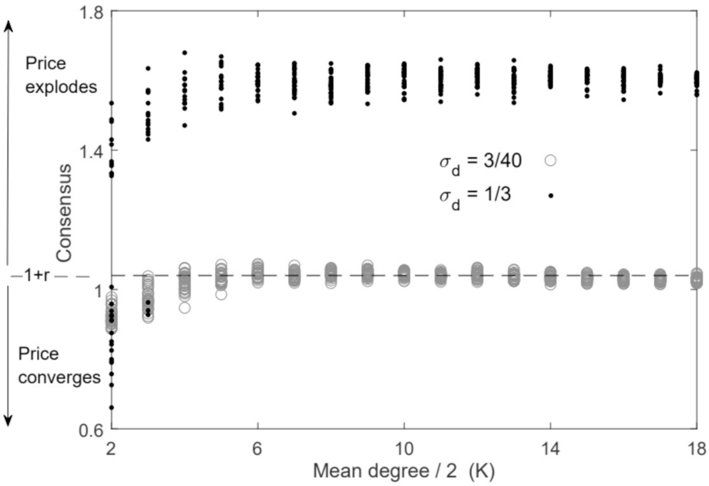



**Fig. 18.** Consensus type and price stability as $K$ (mean degree/2) is increased: $\rho = 0.2$ and two values of $\sigma_d$. Each marker plots the consensus in one of 30 simulations; in each $n = 100$, $\gamma = 180$ and only the random links vary. A marker is plotted in the figure only if a consensus was found in that particular simulation.

in the 'small world' interacting with a given sequence of dividend shocks, without any change in the performance feedback parameter $\gamma$ (which is fixed at 180).

We now investigate the impact of changing the network parameters, in particular, the parameter $K$ that determines expected degree and the rewiring probability $\rho$. We start with the degree parameter $K$. In Fig. 18 we continue with the same parameter values and initial conditions as in the last example, except that we allow the mean degree $2K$ to increase and we record the consensus type (when applicable) in 30 simulations for which only the random links in the network differ, by varying the random seed.[22] We plot consensus values for two different variances of dividend shocks, one where the standard deviation is $\sigma_d = 3/40$ (grey circles) and the other for which $\sigma_d = 1/3$ (black dots), as in Fig. 17.

The main finding is more variation in the consensus type when the mean degree is low (agents less connected). As the mean degree is increased from relatively low values, the consensus type increases and values are quite concentrated, despite differences in random link formation. As seen in Fig. 18, the price explodes to $+\infty$ if the consensus exceeds $1 + r = 1.04$. Therefore, lower connectedness favours price stability in this example, as does a smaller variance of dividend shocks (for $\sigma_d = 3/40$, most simulations have non-explosive price dynamics – the grey circles – but *not* when $\sigma_d = 1/3$, black dots). Intuitively, higher variance increases the likelihood returns will change sign, and increased connectedness means that, if such a switch occurs, many agents can move toward a better-performing type.

Next, we vary the rewiring probability $\rho$. For each $\rho$ in Fig. 19, we plot the consensus type from 30 simulations in which only the random links in the network vary; this is done for both a 'low' dividend shock variance ($\sigma_d = 3/40$, grey circles) and a higher shock variance ($\sigma_d = 3/20$, black dots). For the low shock variance, there is relatively little variation in the consensus across simulations, and

[22] We did not find a numerical consensus in some simulations when $\sigma_d = 1/3$ and $K$ is between 2 and 9. In these cases price did not converge, so there is *no* guarantee a consensus will be reached (see Proposition 5). Simulation length is 800 periods, but we 'break' simulations immediately once a (near) consensus is reached.

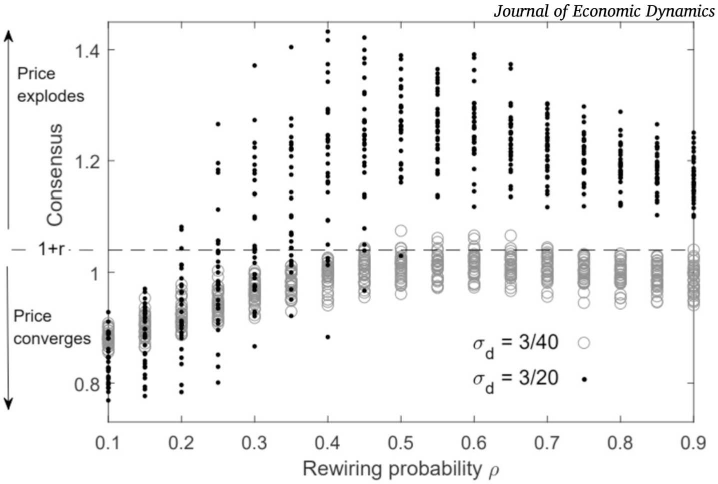



**Fig. 19.** Consensus type and price stability as rewiring probability $\rho$ is increased: $K = 2$ and two values of $\sigma_d$. Each marker plots the consensus in one of 30 simulations; in each $n = 100$, $\gamma = 180$ and only the random links vary. A marker is plotted in the figure only if a consensus was found in that particular simulation.

price converges with the exception of some simulations at intermediate rewiring probabilities (and one for $\rho = 0.9$). When the variance of dividend shocks is doubled (black dots), only very low rewiring probabilities lead to price convergence in all simulations. As the $\rho$ is increased, the consensus type become highly variable across simulations and hence difficult to predict. At intermediate rewiring probabilities, there are some simulations for which price converges to the fundamental price, but when the rewiring probability $\rho$ is large enough we see that all simulations had price exploding to $+\infty$.

In summary, we see that changes in network structure had non-trivial effects on consensus types and price dynamics for performance feedback and initial types held *fixed*. Notably, even when networks were drawn from a fixed distribution, there were non-trivial effects on the price-type dynamics, and increasing the mean degree (connectedness) in this example tends to raise the consensus type, leading to price divergence.

## 6. Conclusion

In this paper we presented a novel model of “social finance” in which prices and belief types evolve as a system of coupled dynamics that depend on social networks and relative performance of neighbours. Our model allows investors to be connected via arbitrary social networks and to adopt continuous types from fundamentalist to arbitrarily strong chartist.

The influence of the social network on price and type dynamics depends on investors’ attention to performance when they update their type. We characterised the long-run type distribution for the polar cases where (i) updates are purely social, and (ii) agents attend exclusively to performance. For pure social updating, a consensus type is reached and this is a ‘melting pot’ of initial types (with influence weights given by eigenvector centrality), while for exclusive attention to performance only the best performer(s) in each agent’s network are imitated, such that – depending on the initial average type – either the most fundamental initial type or the most chartist initial type is adopted in finite time, and price will converge only if these extreme types are not too strongly chartist.

For exclusive attention to performance, the network affects *time to consensus* and price may coincide with the rational expectations solution (fundamental price) if there are one or more pure fundamentalists, in which case mispricing will be eliminated in *finite time*. For intermediate attention to performance, the network influences the consensus type, and we gave conditions such that a long-run consensus is reached, although the consensus itself is analytically intractable. Our three numerical applications – price bubbles, price fluctuations, and asset pricing on a ‘small world’ network – illustrated some implications of network-performance effects both with and without dividend shocks.

Our results show how price and type dynamics depend on concrete features of networks and market conditions – such as distance between agents (diameter), network centrality, expected degree (connectedness), and the initial distribution of types. We thereby provide an understanding of when performance-based updating from a social network is stabilising – or not – for asset prices. An implication of our results is that policymakers concerned with financial market stability will require information not just on social networks, but also on the *extent* to which performance comparisons among peers affect investment decisions.

Future research could proceed in three main directions. First, it would be instructive to relax some key assumptions in the model, by allowing memory of earlier trading performance or chartist beliefs with multiple time lags. Second, while agents take into account the trading profits and types of others, we did not model private information of agents and potential sources of misinformation. Extending a framework of opinion dynamics like Della Lena (2024) to our setting could yield useful insights into investment behaviour and information updating. Finally, it would be interesting to investigate empirical performance of our model. One important finding from experimental asset markets is that after several rounds with a particular group of participants, beliefs seem to coordinate on a common predictor (see Hommes et al., 2005, 2008; Bao et al., 2017). Since the standard framework of discrete types cannot replicate such belief-type consensus (except as an extreme case), it is an open question whether our model of continuous types would provide a good fit to empirical and experimental data while helping to explain this stylised fact.

**Acknowledgements**

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## Appendix A. Proofs of the main results

In this section, we do not assume the networks to be strongly connected such that there may exist several closed and strongly connected groups $\mathcal{C}_i$ and possibly empty Rest of the World $\mathcal{R}$ as defined in Section 4. We start by showing two important Lemmas in Appendix A.1 before stating and proving more general versions of the results shown in the paper which only hold for strongly connected networks in Appendix A.2.

### A.1. Two lemmas for the case of $\gamma \to \infty$

**Lemma 1.** *Let $\gamma \to \infty$ and suppose there exists $\bar{t} \in \mathbb{N}$ such that $sgn(R_t) = sgn(R_0)$ or $\overline{g}_t = 0$ for all $t \le \bar{t}$. Then the following holds:*

1. *If $\tilde{p}_0 R_0 < 0$, then $g_t^i \ge g_{t-1}^{\min}(N^i) \ge g_{t+1}^i$ for all $i \in N$, for all $t \le \bar{t}$.*
2. *If $\tilde{p}_0 R_0 > 0$, then $g_t^i \le g_{t-1}^{\max}(N^i) \le g_{t+1}^i$ for all $i \in N$, for all $t \le \bar{t}$.*

**Proof.** First, note that $sgn(\tilde{p}_0) = sgn(\tilde{p}_t)$ holds for all $t \in \mathbb{N}$ such that $\overline{g}_t \neq 0$ by (4). With the assumption of the Lemma, this implies $sgn(R_0\tilde{p}_0) = sgn(R_t\tilde{p}_t)$ for all $t \le \bar{t}$.

1. Consider the case $\tilde{p}_0 R_0 < 0$, implying $\tilde{p}_t R_t < 0$ or $\overline{g}_t \neq 0$ for all $t \le \bar{t}$. Note that if there exists $t' \le \bar{t}$ such that $\overline{g}_{t'} = 0$ then $g_{t'}^i = 0$ for all $i \in N$, and by weighted average updating (by row stochasticity of $\tilde{\mathbf{A}}$) there is nothing to show since then $g_{t'}^i = 0$ for all $i \in N$, $t' \ge t$ is implied.
   Therefore, suppose for the remainder that $\tilde{p}_t R_t < 0$ holds for all $t \le \bar{t}$. Note that $sgn(R_t) = -sgn(\tilde{p}_t)$ implies $u_t^i > u_t^j$ if and only if $g_{t-1}^i < g_{t-1}^j$ by (7) (since $sgn(\tilde{p}_t) = sgn(\tilde{p}_{t-1}) = sgn(\tilde{p}_{t-2})$). Hence, more fundamental types perform better for any $t \le \bar{t}$.
   First, consider $t = 0$ and let $\gamma \to \infty$. Since each agent $i \in N$ only updates from those with maximal fitness in their neighbourhood, i.e. from those with minimal type. Note, $g_{-1}^i = g_0^i$ for all $i \in N$ by assumption on initial conditions. Hence, $U_0^{\max}(N^i) = G_{-1}^{\min}(N^i) = G_0^{\min}(N^i)$ for all $i \in N$ recalling that $U_t^{\max}(N^i) := \arg\max_{j \in N^i}\{u_t^j\}$ and, similarly, $G_t^{\min}(N^i) := \arg\min_{j \in N^i}\{g_t^j\}$.
   We therefore get the following:
   $$g_1^i = \frac{1}{\left|U_0^{\max}(N^i)\right|} \sum_{j \in U_0^{\max}(N^i)} g_0^j = \frac{1}{\left|G_0^{\min}(N^i)\right|} \sum_{j \in G_0^{\min}(N^i)} g_0^j = \min_{j \in N^i} g_0^j \le g_0^i \quad \forall i \in N,$$
   since $i \in N^i$ for all $i \in N$. Thus, $\tilde{p}_0 R_0 < 0$ implies $g_1^i \le g_0^i$ for all $i \in N$.
   Further suppose for some $t \le \bar{t}$ we have $g_t^i \le g_{t-1}^i$ for all $i \in N$. We show that this implies $g_{t+1}^i \le g_t^i$ for all $i \in N$. Since $R_t\tilde{p}_t < 0$, we get similar to above:
   $$g_{t+1}^i = \frac{1}{\left|G_{t-1}^{\min}(N^i)\right|} \sum_{j \in G_{t-1}^{\min}(N^i)} g_t^j \le \frac{1}{\left|G_{t-1}^{\min}(N^i)\right|} \sum_{j \in G_{t-1}^{\min}(N^i)} g_{t-1}^j = \min_{j \in N^i} g_{t-1}^j \quad \forall i \in N$$
   By weighted average updating (by row stochasticity of $\tilde{\mathbf{A}}$) we have $g_t^i \ge \min_{j \in N^i} g_{t-1}^j$ for all $i \in N$. Hence, we have shown that if $\tilde{p}_0 R_0 < 0$, then $g_1^i \le g_0^i$ and if $\tilde{p}_t R_t < 0$ and $g_t^i \le g_{t-1}^i$, then
   $$g_{t+1}^i \le \min_{j \in N^i} g_{t-1}^j \le g_t^i. \quad \forall i \in N, \tag{A.1}$$
   Induction implies that (A.1) holds for all $t \le \bar{t}$ which is what we had to show.
2. Now, consider the case $\tilde{p}_0 R_0 > 0$ which is completely analogous to above. $\tilde{p}_0 R_0 > 0$ implies $\tilde{p}_t R_t > 0$ for all $t \in \mathbb{N}$ for which there exists $i \in N$ such that $g_t^i \neq 0$. Note that we will show that for all such $t \in \mathbb{N}$, we have $g_t^i \le g_{t+1}^i$. Hence, if there exists a $i \in N$ such that $g_0^i > 0$, then this will hold for all $t \le \bar{t}$.
   Note that $sgn(R_t) = sgn(\tilde{p}_t)$ implies $u_t^i < u_t^j$ if and only if $g_{t-1}^i < g_{t-1}^j$ by (7). Hence more chartist types are always performing better at any point in time $t \le \bar{t}$.
   Analogously to above for $t = 0$, we get because of the initial assumptions,
   $$g_1^i = \frac{1}{\left|U_0^{\max}(N^i)\right|} \sum_{j \in U_0^{\max}(N^i)} g_0^j = \frac{1}{\left|G_0^{\max}(N^i)\right|} \sum_{j \in G_0^{\max}(N^i)} g_0^j = \max_{j \in N^i} g_0^j \ge g_0^i \quad \forall i \in N.$$
   Assuming $g_t^i \ge g_{t-1}^i$ for all $i \in N$, we get from $R_t\tilde{p}_t > 0$ that:

$$g^i_{t+1} = \frac{1}{\left|G^{\max}_{t-1}(N^i)\right|} \sum_{j\in G^{\max}_{t-1}(N^i)} g^j_t \geq \frac{1}{\left|G^{\max}_{t-1}(N^i)\right|} \sum_{j\in G^{\max}_{t-1}(N^i)} g^j_{t-1} = \max_{j\in N^i} g^j_{t-1} \quad \forall i \in N$$

and, hence,

$$g^i_{t+1} \geq \max_{j\in N^i} g^j_{t-1} \geq g^i_t. \quad \forall i \in N, t \leq \bar{t}. \quad \square \tag{A.2}$$

**Lemma 2.** *Let $\gamma \to \infty$ and suppose there exists $\bar{t} \in \mathbb{N}$ such that $sgn(R_t) = sgn(R_0)$ or $\bar{g}_t = 0$ for all $t \leq \bar{t}$. Then the following holds:*

1. *If $\tilde{p}_0 R_0 < 0$, then $g^i_t \to g^{\min}_0(\mathcal{P}^i)$ $\forall i \in N$, $t \in \mathbb{N} : 2d(i, G^{\min}_0(\mathcal{P}^i)) - 1 \leq t \leq \bar{t}$.*
   *If for a closed and strongly connected group $C$ we have $\bar{t} \geq 2D(\mathbf{A}_C) - 1$, then $g^i_t \to g^{\min}_0(C)$ for all $t \geq 2D(\mathbf{A}_C) - 1$.*
2. *If $\tilde{p}_0 R_0 > 0$, then $g^i_t \to g^{\max}_0(\mathcal{P}^i)$ $\forall i \in N$, $t \in \mathbb{N} : 2d(i, G^{\max}_0(\mathcal{P}^i)) - 1 \leq t \leq \bar{t}$.*
   *If for a closed and strongly connected group $C$ we have $\bar{t} \geq 2D(\mathbf{A}_C) - 1$, then $g^i_t \to g^{\max}_0(C)$ for all $t \geq 2D(\mathbf{A}_C) - 1$.*

**Proof.** For some agent $i \in N$, recall that $\mathcal{P}^i := \{j \in N | \exists k \in \mathbb{N} : (\mathbf{A}^k)_{ij} > 0\}$ denotes the set of agents to which there exists a path from $i$. Since $a_{jj} = 1$ for all $j \in N$, we have $j \in \mathcal{P}^j$ for all $j \in N$. Further recall that for any $M \subseteq N$, we denote by $g^{\min}_t(M) := \min\{g^j_0 | j \in M\}$ the minimal initial type of all agents in the set $M$ and by $g^{\max}_t(M) := \max\{g^j_0 | j \in M\}$ the maximal initial type of all agents in the set $M$ for some point in time $t \in \mathbb{N}$. Clearly, $\mathcal{P}^j \subseteq \mathcal{P}^i$, and hence, $g^{\min}_t(\mathcal{P}^i) \leq g^{\min}_t(\mathcal{P}^j)$ while $g^{\max}_t(\mathcal{P}^i) \geq g^{\max}_t(\mathcal{P}^j)$ for all $j \in \mathcal{P}^i$, $t \in \mathbb{N}$. Note that by weighted average updating (by row stochasticity of $\tilde{\mathbf{A}}$), we have $g^j_{t+1} \geq g^{\min}_t(\mathcal{P}^j) \geq g^{\min}_t(\mathcal{P}^i) \geq g^{\min}_{t-1}(\mathcal{P}^i)$ and $g^j_{t+1} \leq g^{\max}_t(\mathcal{P}^j) \leq g^{\max}_t(\mathcal{P}^i) \leq g^{\max}_{t-1}(\mathcal{P}^i)$ for all $j \in \mathcal{P}^i$, $t \in \mathbb{N}$.

1. Consider the case $\tilde{p}_0 R_0 < 0$ such that (A.1) holds for all $t \leq \bar{t}$ by Lemma 1. Hence, the first inequality in (A.1) must be satisfied with equality if $\min_{k\in N^j} g^k_{t-1} = g^{\min}_{t-1}(\mathcal{P}^i)$ for some $j \in \mathcal{P}^i$. Thus, all agents $j \in \mathcal{P}^i$ who listen to agents within the set $G^{\min}_{t-1}(\mathcal{P}^i)$ must adopt $g^{\min}_{t-1}(\mathcal{P}^i)$ at latest by period $t + 1$. Hence,

   $$G^{\min}_{t+1}(\mathcal{P}^i) \supseteq \bigcup_{j\in G^{\min}_{t+1}(\mathcal{P}^i)} M^j(G^{\min}_{t-1}(\mathcal{P}^i)) \quad \forall t \in \mathbb{N}. \tag{A.3}$$

   Note that (A.1) also implies $G^{\min}_{t+1}(\mathcal{P}^i) \supseteq G^{\min}_t(\mathcal{P}^i)$ for all $t \in \mathbb{N}$. Since, further, by assumption on initial conditions, $G^{\min}_{-1}(\mathcal{P}^i) = G^{\min}_0(\mathcal{P}^i)$, (A.3) implies

   $$G^{\min}_{t+1}(\mathcal{P}^i) \supseteq \begin{cases} G^{\min}_t(\mathcal{P}^i) & \text{if } t \text{ is even} \\ \bigcup_{j\in G^{\min}_t(\mathcal{P}^i)} M^j(G^{\min}_t(\mathcal{P}^i)) & \text{if } t \text{ is odd} \end{cases}$$

   Since $j \in M^j$ for all $j \in N$, the set $G^{\min}_t(\mathcal{P}^i)$ just expands over the path $\mathcal{P}^i$ at latest at every odd time-step by the neighbours of the previous set. Thus, each agent $j \in \mathcal{P}^i$ within distance $d(j, G^{\min}_0(\mathcal{P}^i))$ of the agents with initial minimal types of $\mathcal{P}^i$ has adopted this minimal type at latest at time step $2d(j, G^{\min}_0(\mathcal{P}^i)) - 1$ and will keep it from there for all $t \geq 2d(j, G^{\min}_0(\mathcal{P}^i)) - 1$ as long as $t \leq \bar{t}$.
   Now, for a closed and strongly connected set $C$, we have by definition $\mathcal{P}^i = \mathcal{P}^j$ for all $i, j \in C$. Thus if $\bar{t} \geq 2D(\mathbf{A}_C) - 1$, then all agents in $C$ obtain a consensus on $g^{\min}_0(C)$ after at most $2D(\mathbf{A}_C) - 1$ steps where, as before, $\mathbf{A}_C$ is the matrix $\mathbf{A}$ restricted to the set $C$ and $D(\mathbf{A}_C)$ is the length of the longest path within the network $\mathbf{A}_C$. Since $g^i_{2D(\mathbf{A}_C)-1} = g^{\min}_0(C)$ for all $i \in C$ if $\bar{t} \geq 2D(\mathbf{A}_C) - 1$, these will not change henceforth by the nature of weighted average updating. Hence, $g^i_t = g^{\min}_0(C)$ for all $t \geq 2D(\mathbf{A}_C) - 1$.
2. Now, consider the case $\tilde{p}_0 R_0 > 0$ such that (A.2) holds for all $t \leq \bar{t}$ by Lemma 1. Analogously to above, we conclude that for any $i \in N$,

   $$G^{\max}_{t+1}(\mathcal{P}^i) \supseteq \begin{cases} G^{\max}_t(\mathcal{P}^i) & \text{if } t \text{ is even} \\ \bigcup_{j\in G^{\max}_t(\mathcal{P}^i)} M^j(G^{\max}_t(\mathcal{P}^i)) & \text{if } t \text{ is odd} \end{cases}$$

   Thus, for a closed and strongly connected set $C$, all agents in $C$ obtain a consensus on $g^{\max}_0(C)$ after at most $2D(\mathbf{A}_C) - 1$ steps and do not change types after that. $\square$

### *A.2. Proofs of the main results*

**Proposition 1a** *(For non-strongly connected networks). The following holds in any steady state $(p^*, \mathbf{g}^*)$:*

1. *The types in any closed and strongly connected set $C$ coincide: $g^{i,*} = g^{C,*}$, $\forall i \in C$.*
2. *The steady-state weights matrix satisfies $\tilde{\mathbf{A}}^*_{(N\setminus\mathcal{R})(N\setminus\mathcal{R})} = \overline{\mathbf{A}}_{(N\setminus\mathcal{R})(N\setminus\mathcal{R})}$ and the fitness vector is fixed at some $\mathbf{u}^* \in \mathbb{R}^{\mathbf{n}}$. In a fundamental steady state, $\tilde{\mathbf{A}}^* = \overline{\mathbf{A}}$ and $\mathbf{u}^* = \mathbf{0}$.*

3. *The types of investors belonging to the Rest of the World are convex combinations of the types of the closed and strongly connected sets, i.e.* $g^j \in conv\{g^{i,*}|i \in N \setminus \mathcal{R}\}$ *for all* $j \in \mathcal{R}$*. In a fundamental steady state, Rest of the World types satisfy*

$$\mathbf{g}^{\mathcal{R},*} = \left(\mathbf{I}_{|\mathcal{R}|} - \overline{\mathbf{A}}_{\mathcal{R}\mathcal{R}}\right)^{-1} \overline{\mathbf{A}}_{\mathcal{R}(N\setminus\mathcal{R})}\mathbf{g}^{N\setminus\mathcal{R},*}. \tag{A.4}$$

**Proof of Proposition 1a.** 1. Suppose that $\hat{\mathbf{g}}$ is a steady state type vector and suppose to the contrary that for some closed and strongly connected set $C$ there exists $i, j \in C$ such that $\hat{g}^i \neq \hat{g}^j$. Let $\hat{g}^{\max}(C) = \max_{k\in C} \hat{g}^k$. Since $C$ is strongly connected, there must exist $i, j \in C$ with $\hat{g}^i = \hat{g}^{\max}(C)$, $\hat{g}^j \neq \hat{g}^{\max}(C)$, and $a_{ij} = 1$. But since $C$ is closed, we have $a_{ik} = 0$ for all $k \in N$ such that $\hat{g}^k > \hat{g}^i$. Hence, $\sum_{l\in N^i} \tilde{a}_{il}\hat{g}^l < \hat{g}^i$ contradicting that $\hat{\mathbf{g}}$ is a steady state type vector.

2. Consider a closed and strongly connected group $C$. Then, by part 1 we have $g^{i,*} = g^{j,*}$ for all $i, j \in C$ in any steady state, implying $u^{i,*} = u^{j,*}$ by (7), and thus $\tilde{a}^*_{ij} = \overline{a}_{ij}$. Further since by (7), fitness only depends on price and types, $\mathbf{u}^*$ is invariant to the law of motion in a steady state $(p^*, \mathbf{g}^*)$. In a fundamental steady state, $\tilde{p}^* = 0$, thus $\mathbf{u}^* = \mathbf{0}$ by (7), implying $\tilde{a}^*_{ij} = \overline{a}_{ij}$ for all $i, j \in N$.
3. Suppose to the contrary that there exists a $j \in \mathcal{R}$ with $g^j \notin conv\{g^i|i \in N \setminus \mathcal{R}\}$ where *conv* denotes the convex hull, i.e. wlog let $g^{\max}(\mathcal{R}) > g^{\max}(N \setminus \mathcal{R})$. Analogously to proof of part 1, this leads to a contradiction.
In a fundamental steady state, we have $\tilde{\mathbf{A}}^* = \overline{\mathbf{A}}$ by part 2. Thus,

$$g^{i,*} = \sum_{j\in N\setminus\mathcal{R}} \overline{a}_{ij} g^{j,*} + \sum_{k\in\mathcal{R}} \overline{a}_{ik} g^{k,*} \quad \forall i \in \mathcal{R}.$$

With $\mathbf{g}^{S,*} = \left(g^{i,*}\right)_{i\in S}$ for some $S \subset N$ this implies

$$\mathbf{g}^{\mathcal{R},*} = \overline{\mathbf{A}}_{\mathcal{R}(N\setminus\mathcal{R})}\mathbf{g}^{(N\setminus\mathcal{R}),*} + \overline{\mathbf{A}}_{\mathcal{R}\mathcal{R}}\mathbf{g}^{\mathcal{R},*}$$

$$\mathbf{g}^{\mathcal{R},*} = \left(\mathbf{I}_{|\mathcal{R}|} - \overline{\mathbf{A}}_{\mathcal{R}\mathcal{R}}\right)^{-1} \overline{\mathbf{A}}_{\mathcal{R}(N\setminus\mathcal{R})}\mathbf{g}^{(N\setminus\mathcal{R}),*}. \quad \square$$

**Proof of Proposition 1.** 1. Since for strongly connected networks, $N$ is a closed and strongly connected set, the statement following from part 1 Proposition 1.

2. Since in a steady state, all types of agents $i \in N$ are identical, we have $u_i = u_j = 0$, implying $\tilde{a}^*_{ij} = \overline{a}_{ij}$ for all $i, j \in N$. □

**Proposition 2a** *(For non-strongly connected networks). Suppose* $\gamma = 0$*. For any realisation of the dividends, types converge to a steady state group type vector* $\mathbf{g}^{\mathbf{v}}$ *such that for any closed and strongly connected group* $C$*,*

$$g^{i,\mathbf{v}} = \sum_{j\in C} v^j_C g^j_0 \qquad \forall i \in C$$

*where* $v^j_C$ *is the* $j$*-th entry of the (unique) left-unit eigenvector* $\mathbf{v}_C$ *of* $\overline{\mathbf{A}}_C$ *with* $\sum_{i\in C} v^i_C = 1$*. Further, the Rest of the World converges to*

$$\mathbf{g}^{\mathcal{R},\mathbf{v}} = \sum_{j\in N\setminus\mathcal{R}} \left(\mathbf{I}_{|\mathcal{R}|} - \overline{\mathbf{A}}_{\mathcal{R}\mathcal{R}}\right)^{-1} \overline{\mathbf{A}}_{\mathcal{R}j}\mathbf{g}^{j,\mathbf{v}}.$$

*There is convergence to a fundamental steady state if* $\overline{g}^{\mathbf{v}} < 1 + r$*, while convergence to a non-fundamental steady state is obtained if* $\overline{g}^{\mathbf{v}} = 1 + r$*. Price diverges to* $\pm\infty$ *if* $\overline{g}^{\mathbf{v}} > 1 + r$*.*

**Proof of Proposition 2a.** The type dynamics characterization follows from standard results of the DeGroot model (see e.g. Golub and Jackson, 2010; Buechel et al., 2015). If the limit average type $\overline{g}^{v}$ is below $1 + r$, then there exists a $t' \in \mathbb{N}$ such that $\overline{g}_{t'} < 1 + r$ for all $t \geq t'$ which implies by Eq (4) that $\lim_{t\to\infty} \tilde{p}_t = 0$, i.e. price converges to the fundamental price. If the limit average type $\overline{g}^{v}$ is above $1 + r$, then there exists a $t' \in \mathbb{N}$ such that $\overline{g}_{t'} > 1 + r$ for all $t \geq t'$ which implies by Eq (4) that price diverges (to $+\infty$ if $\tilde{p}_t > 0$ and to $-\infty$ if $\tilde{p}_t < 0$). If $\overline{g}^{v} = 1 + r$, then for any $\nu > 0$ there exists $t_\nu$ such that $|\overline{g}_t - (1 + r)| < \nu$ for all $t \geq t_\nu$. By Eq (4), price will settle on some value. □

**Proof of Proposition 2.** The result follows straightforwardly from Proposition 2a by noting that in case of a strongly connected network the set $N$ is the only closed and strongly connected set and the Rest of the World is empty. □

**Proposition 3a** *(For non-strongly connected networks). Suppose* $d_t = \overline{d}$ *for all* $t \in \mathbb{N}$*. For* $\gamma \to \infty$*, we get the following*

1. *If* $\overline{g}_0 < (1 + r)^2$*, then any agent adopts the most fundamental type on their path in finite time, i.e. for all* $i \in N$*,* $t \geq 2d(i, G^{\min}_0(\mathcal{P}^i)) - 1$*, we have* $g^i_t \to g^{\min}_0(\mathcal{P}^i)$*.*
2. *If* $\overline{g}_0 > (1 + r)^2$*, then any agent adopts the most chartist type on their path in finite time, i.e. for all* $i \in N$*,* $t \geq 2d(i, G^{\max}_0(\mathcal{P}^i)) - 1$*, we have* $g^i_t \to g^{\max}_0(\mathcal{P}^i)$*.*

3. *Price converges to the fundamental price if $\overline{g}_0 < (1+r)^2$ and $\sum_{i\in N} g_0^{\min}(\mathcal{P}^i) < n(1+r)$, price converges (to some finite limit) if $\overline{g}_0 < (1+r)^2$ and $\sum_{i\in N} g_0^{\min}(\mathcal{P}^i) = n(1+r)$, and price diverges if $\overline{g}_0 < (1+r)^2$ and $\sum_{i\in N} g_0^{\min}(\mathcal{P}^i) > n(1+r)$, or $\overline{g}_0 > (1+r)^2$.*

**Proof of Proposition 3a.** By (8), we get $sgn(R_t) = -sgn(\tilde{p}_t)$ if $0 < \overline{g}_t < (1+r)^2$ while $sgn(R_t) = sgn(\tilde{p}_t)$ if $\overline{g}_t > (1+r)^2$. If $0 = \overline{g}_t$ for some $t \in \mathbb{N}$ then $g_t^i = 0$ for all $i \in N$ and by weighted average updating convergence to the fundamental belief is obtained in which case there is nothing to show.

1. Suppose now $\overline{g}_0 < (1+r)^2$ then $sgn(R_0) = -sgn(\tilde{p}_0)$ and by part 1 of Lemma 1, we get that $g_0^i \geq g_1^i$ for all $i \in N$. Hence, $\overline{g}_1 < (1+r)^2$, implying $sgn(R_1) = -sgn(\tilde{p}_1)$. Repeatedly applying part 1 of Lemma 1 implies that $g_{t+1}^i \leq g_t^i < (1+r)^2$ for all $t \in \mathbb{N}$ and hence, $sgn(R_t) = -sgn(\tilde{p}_t)$ for all $t \in \mathbb{N}$.
   Lemma 2 then implies that for beliefs we have $g_t^i \to g_0^{\min}(\mathcal{P}^i))$ for $\gamma \to \infty$ for all time steps $t \geq 2d(i, G_0^{\min}(\mathcal{P}^i)) - 1$ for all $i \in \mathbb{N}$. In particular, each closed and strongly connected group $C$ obtains a group consensus on $g_t^i \to g_0^{\min}(C)$ for all $i \in C$ and each agent in the Rest of the World $j \in \mathcal{R}$ does not change after obtaining the belief $g_0^{\min}(\mathcal{P}^j)$ since $R_t\tilde{p}_t < 0$ for all $t \in \mathbb{N}$ such that $\exists i \in N$: $g_t^i \neq 0$ implying that Lemma 2 holds for all $t \in \mathbb{N}$.
2. If instead $\overline{g}_0 > (1+r)^2$ then $R_0\tilde{p}_0 > 0$ and by part 2 of Lemma 1, we get, completely analogously to above, that $g_0^i \leq g_1^i$ for all $i \in N$. Hence, $\overline{g}_1 > (1+r)^2$, implying $R_1\tilde{p}_1 > 0$. Repeatedly applying part 2 of Lemma 1 implies that $(1+r)^2 < g_t^i \leq g_{t+1}^i$ for all $t \in \mathbb{N}$ and hence, $R_t\tilde{p}_t > 0$ for all $t \in \mathbb{N}$.
   Lemma 2 then implies that for beliefs we have $g_t^i \to g_0^{\max}(\mathcal{P}^i)$ for $\gamma \to \infty$ for all time steps $t \geq 2d(i, G_0^{\min}(\mathcal{P}^i)) - 1$. In particular, each closed and strongly connected group $C$ obtains a group consensus on $g_t^i \to g_0^{\max}(C)$ for all $i \in C$ and each agent in the Rest of the World $j \in \mathcal{R}$ does not change does not change after obtaining the belief $g_0^{\max}(\mathcal{P}^j)$ since $R_t\tilde{p}_t > 0$ for all $t \in \mathbb{N}$ such that $\exists i \in N$: $g_t^i \neq 0$ and hence Lemma 2 holds for all $t \in \mathbb{N}$.
3. Clearly, if $\overline{g}_0 > (1+r)^2$ price diverges since from Case 2 we have $\overline{g}_t > (1+r)^2$ for all $t \in \mathbb{N}$, implying price divergence by (4). Instead, suppose that $\overline{g}_0 > (1+r)^2 < 0$. By Case 1, $g_t^i = g_0^{\min}(\mathcal{P}^i)$ for all $i \in N$, $t \geq 2D(\mathbf{A}) - 1$. Hence, $\overline{g}_t = \frac{1}{n}\sum_{i\in N} g_0^{\min}(\mathcal{P}^i)$ for all $t \geq 2D(\mathbf{A}) - 1$ implying that price converges to the fundamental price, i.e. $\tilde{p}_t \to 0$, if $\frac{1}{n}\sum_{i\in N} g_0^{\min}(\mathcal{P}^i) < 1 + r$, price converges to some finite limit price, if $\frac{1}{n}\sum_{i\in N} g_0^{\min}(\mathcal{P}^i) = 1 + r$ and price diverges if $\frac{1}{n}\sum_{i\in N} g_0^{\min}(\mathcal{P}^i) > 1 + r$, see (4). □

**Proof of Proposition 3.**

1. For strongly connected networks, Part 1 of Proposition 3a implies that all agents's types have converged to the minimal initial type $g_0^{\min}$ by time $t \geq 2D(\mathbf{A}) - 1$ since $N$ is the single closed and strongly connected set. Since average type thereby satisfies $\overline{g}_t = g_0^{\min}$ for all $t \geq 2D(\mathbf{A}) - 1$, the statement on price follows from the law of motion of the price dynamics (4).
2. Similarly, Part 2 of Proposition 3a implies that all agents's types have converged to the maximal initial type $g_0^{\max}$ by time $t \geq 2D(\mathbf{A}) - 1$ since $N$ is the single closed and strongly connected set. In this case, price diverges since $\overline{g}_t > (1+r)$ for all $t \in N$. □

**Proposition 4a** *(For non-strongly connected networks). For $\gamma \to \infty$, we get the following:*

- *If $\overline{g}_0 < (1+r)^2$, then for any closed and strongly connected $C$ we have $g_t^i \to g_0^{\min}(C)$ for all $t \geq 2D(\mathbf{A}_C) - 1$, $i \in C$ if $\tilde{p}_0 > 0$ and $d^+ < \min_{0\leq t\leq 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r)$ or $\tilde{p}_0 < 0$ and $d^- > \max_{0\leq t\leq 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r)$.*
- *If $\overline{g}_0 > (1+r)^2$, then for any closed and strongly connected $C$ we have $g_t^i \to g_0^{\max}(C)$ for all $t \geq 2D(\mathbf{A}_C) - 1$, $i \in C$ if $\tilde{p}_0 > 0$ and $d^- > \max_{0\leq t\leq 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r)$ or $\tilde{p}_0 < 0$ and $d^+ < \min_{0\leq t\leq 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r)$.*

**Proof of Proposition 4a.** First note that by (6), we have that

$$\begin{aligned} sgn(R_t) &= sgn\left(\varepsilon_t - \left((1+r) - \frac{\overline{g}_t}{1+r}\right)\tilde{p}_{t-1}\right) \\ &= sgn\left(\varepsilon_t - \left((1+r) - \frac{\overline{g}_t}{1+r}\right)\tilde{p}_0 \frac{1+r}{\overline{g}_0}\prod_{j=0}^{t-1}\frac{\overline{g}_j}{1+r}\right) = sgn(\varepsilon_t - \sigma_t(\mathbf{g}_0, r)) \end{aligned} \tag{A.5}$$

since $\tilde{p}_t = \tilde{p}_{t-1}\frac{\overline{g}_t}{1+r}$ and $\sigma_t(\mathbf{g}_0, r) = \left((1+r) - \frac{\overline{g}_t}{1+r}\right)\tilde{p}_0 \frac{1+r}{\overline{g}_0}\prod_{j=0}^{t-1}\frac{\overline{g}_j}{1+r}$.

Hence, by (A.5), we have that $R_t < 0$ if and only if $\varepsilon_t < \sigma_t(\mathbf{g}_0, r)$. If the bound of the dividend shocks is such that $d^+ < \min_{0\leq t\leq 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r)$, then clearly $\varepsilon_t < \sigma_t(\mathbf{g}_0, r)$ and hence also $R_t < 0$ for all $0 \leq t \leq 2D(\mathbf{A}_C) - 2$. Moreover, we get from (A.5) that $R_t > 0$ if and only if $\varepsilon_t > \sigma_t(\mathbf{g}_0, r)$. If the bound of the dividend shocks is such that $d^- > \max_{0\leq t\leq 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r)$, then clearly $\varepsilon_t > \sigma_t$ and hence also $R_t < 0$ for all $0 \leq t \leq 2D(\mathbf{A}_C) - 2$.

For $\overline{g}_0 < (1+r)^2$, we then conclude from the proof of Proposition 3 that convergence to the minimum type $g_0^{\min}(C)$ obtains if either $\tilde{p}_0 > 0$ and $d^+ < \min_{0\leq t\leq 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r)$ since this implies $R_t < 0$ for all $0 \leq t \leq 2D(\mathbf{A}_C) - 2$ or $\tilde{p}_0 < 0$ and $d^- > \max_{0\leq t\leq 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r)$ since this implies $R_t > 0$ for all $0 \leq t \leq 2D(\mathbf{A}_C) - 2$.

Similarly, for $\overline{g}_0 > (1+r)^2$ and $\tilde{p}_0 > 0$, we conclude from the proof of Proposition 3 that convergence to the minimal type $g_0^{\max}(C)$ obtains if either $\tilde{p}_0 > 0$ and $d^- > \max_{0\le t\le 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r)$ since this implies $R_t > 0$ for all $0 \le t \le 2D(\mathbf{A}_C) - 2$ or $\tilde{p}_0 < 0$ and $d^+ < \min_{0\le t\le 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r)$ since this implies $R_t < 0$ for all $0 \le t \le 2D(\mathbf{A}_C)-2$. Note that $\max_{0\le t\le 2D(\mathbf{A}_C)-2} \sigma_t(\mathbf{g}_0, r) = \sigma_0(\mathbf{g}_0)$ for $\overline{g}_0 > (1+r)^2$ and $\tilde{p}_0 > 0$ since $\overline{g}_t$ is increasing and $\overline{g}_t > (1+r)$ in this case. □

**Proof of Proposition 4.** The result follows straightforwardly from Proposition 4a by noting that in case of a strongly connected network the set $N$ is the only closed and strongly connected set. □

**Proof of Proposition 5.** In the proof of this result, we allow the network to be non-strongly connected and use the notation introduced in Section 4. The result straightforwardly also holds for strongly connected networks.

1. Note $g_{t+1}^{\max} \le g_t^{\max}$ and $g_{t+1}^{\min} \ge g_t^{\min}$ for all $t \in \mathbb{N}$ by row stochasticity of $\tilde{\mathbf{A}}$. Hence if $g_0^{\min} > 1+r$, then $\overline{g}_t > 1+r$ for all $t \in \mathbb{N}$, implying $\prod_{k=1}^t \frac{\overline{g}_k}{1+r} \overset{t\to\infty}{\longrightarrow} \infty$ and, hence, price diverges by (4). On the other hand, if $g_0^{\max} < 1+r$, then $\overline{g}_t < 1+r$ for all $t \in \mathbb{N}$ implying $\prod_{k=1}^t \frac{\overline{g}_k}{1+r} \overset{t\to\infty}{\longrightarrow} 0$ and, hence, price converges by (4).
   Now suppose some vector of types $\mathbf{g}_0 \in \mathbb{R}_+^n$ with $1+r < g_0^{\max}$ and, wlog, consider $\gamma \to \infty$ such that agents only update from best-performing agents that they observe.[23] Let $\tilde{p}_0 > 0$ which together with (8) implies $u_t^i > u_t^j$ if and only if $\tilde{p}_t < p_t^{crit}$. In particular, we can show completely analogously to Proposition 3 that the maximal type $g_0^{\max}$ will be adopted by all agents if $\tilde{p}_t < p_t^{crit}$ for all $0 \le t \le 2D(\mathbf{A}) - 1$ since $\mathbf{A}$ is strongly connected. In the remainder of the proof, we will show that choosing $d^+$ large enough, ensures a positive probability that the realizations of $\varepsilon_t$ in periods $0 \le t \le 2D(\mathbf{A}) - 1$ are large enough to ensure this.
   First, note that due to (4), price deviation $\tilde{p}_t$ is bounded by $|\tilde{p}_t| \le \left(\frac{g_0^{\max}}{1+r}\right)^t |\tilde{p}_0|$ and that a lower bound for average type $\overline{g}_t$ is $\overline{g}_t \ge \overline{g}_0$ given that $\tilde{p}_k < p_k^{crit}$ for all $k \le t-1$. Thus, we have,
$$\tilde{p}_t < p_t^{crit} \Leftrightarrow \varepsilon_t > \left(\frac{(1+r)^2}{\overline{g}_t} - 1\right)\tilde{p}_t$$
$$\Leftarrow \varepsilon_k > \left(\frac{(1+r)^2}{\overline{g}_0} - 1\right)\left(\frac{g_0^{\max}}{1+r}\right)^k \quad \forall k \le t$$
   This is satisfied if $\varepsilon_t > \left(\frac{(1+r)^2}{\overline{g}_0} - 1\right)\left(\frac{g_0^{\max}}{1+r}\right)^{2D(\mathbf{A})-1} =: \overline{\varepsilon}$ for all $t \le 2D(\mathbf{A}) - 1$. Clearly, if $d^+ > \overline{\varepsilon}$ and $d^-$ is bounded from below, the probability that the realization of $\varepsilon_t$ exceeds $\overline{\varepsilon}$ in any period $t$ is positive, i.e. $P(\varepsilon_t > \overline{\varepsilon}) = P(\varepsilon_0 > \overline{\varepsilon}) > 0$ for all $t \in \mathbb{N}$. Hence, also the probability that the first $2D(\mathbf{A}) - 1$ realizations of $\varepsilon_t$ exceed $\overline{\varepsilon}$ is positive, since $P(\varepsilon_t > \overline{\varepsilon} | 0 \le t \le 2D(\mathbf{A}) - 1) = \left(P(\varepsilon_0 > \overline{\varepsilon})\right)^{2D(\mathbf{A})-1} > 0$.
   The proof for the cases of $\tilde{p}_0 < 0$ is analogous and requires small enough $d^-$.
2. This case of $g_0^{\min} < 1+r$ can be fully analogously shown to part 1.
3. Suppose $\tilde{p}_t$ is bounded for all $t \in \mathbb{N}$. Since dividends are bounded, $d_t \in [\overline{d} - d^-, \overline{d} + d^+]$, and $\gamma$ is some (finite) non-negative real number, we get that $u_i^t$ is bounded for all $i \in N$ for all $t \in \mathbb{N}$. Hence, there exists $\zeta > 0$ such that
$$a_{ij} > 0 \quad \Rightarrow \quad (\tilde{a}(t))_{ij} := \left(\sum_{k\in N^i} \exp(\gamma u_t^k)\right)^{-1} \exp(\gamma u_t^j) > \zeta \quad \forall t \in \mathbb{N}$$
   (otherwise as stated before, $(\tilde{a}(t))_{ij} = 0$).
   Now, for all strongly connected and closed groups $C_k$, denote by $\mathbf{A}_{C_k}$ the restriction of $\mathbf{A}$ to $C_k$. Hence, $\tilde{\mathbf{A}}_{C_k}(t)$ is strongly connected with a positive diagonal and each entry is bounded below by $\zeta$. Thus, for each $t \in \mathbb{N}$, the matrix $\tilde{\mathbf{A}}_{C_k}(t, t+n)$, defined by
$$\tilde{\mathbf{A}}_{C_k}(t, t+n) := \tilde{\mathbf{A}}_{C_k}(t+n) \cdot \tilde{\mathbf{A}}_{C_k}(t+n-1) \cdot \ldots \cdot \tilde{\mathbf{A}}_{C_k}(t),$$
   is strictly positive with all entries bounded below by $\zeta^n$ for every closed and strongly connected group $C_k$. Finally note that $\tilde{\mathbf{A}}_{C_k}(t, t+n)$ is still row stochastic since the product of two row stochastic matrices is also row stochastic. If such a sequence of sub-accumulations $\tilde{\mathbf{A}}_{C_k}(t, t+n)$ appears infinitely often, then by Theorem 3.2.33 in Lorenz (2007), convergence to consensus in all closed and strongly connected groups is obtained. Hence, denoting by $\mathbf{g}_t^{C_k}$ the type vector in period $t \in \mathbb{N}$ restricted to $C_k$, we get that

[23] By (9), the updating weights are a differentiable function of $\gamma$. Thus, for any $\mathbf{u}_t$ and $\delta > 0$ there exists a $\overline{\gamma} \in \mathbb{R}$ such that $\left|\tilde{a}_{ij}(\mathbf{u}_t|\gamma) - \lim_{\gamma'\to\infty} \tilde{a}_{ij}(\mathbf{u}_t|\gamma')\right| < \delta$ for all $\gamma > \overline{\gamma}$ implying that this proof works for any finite but large enough $\gamma$.

$$\lim_{t\to\infty} \mathbf{g}_t^{C_k} = \lim_{T\to\infty} \tilde{\mathbf{A}}_{C_k}(0,T)\mathbf{g}_0^{C_k}$$

exists, and is such that consensus is achieved, i.e. $g_\infty^i = g_\infty^j$ for all $i, j \in C_k$. Since all $\tilde{\mathbf{A}}(t)$ are row stochastic, the consensus must be such that $g_\infty^{C_k} \in [g_{C_k}^{\min}, g_{C_k}^{\max}]$ such that the consensus in the interior of the interval if $g_{C_k}^{\min} \neq g_{C_k}^{\max}$.

Finally, if $\lim_{t\to\infty} \tilde{p}_t = 0$, then expectations of all types converge to the fundamental price, $\lim_{t\to\infty} \tilde{E}_t^i[\tilde{p}_{t+1}] = 0$. Hence, $\lim_{t\to\infty} u_t^i = \lim_{t\to\infty} u_t^j$ for all $i, j \in N$. Thus, $\lim_{t\to\infty} \tilde{a}_{ij}(t) = \frac{1}{|N_i|} =: \overline{a}_{ij}$ for all $i \in R$. Thereby $\lim_{t\to\infty} \tilde{\mathbf{A}}_{\mathcal{R}\mathcal{R}}(t) = \lim_{t\to\infty} \overline{\mathbf{A}}_{\mathcal{R}\mathcal{R}}(t) = 0$, and hence $\lim_{t\to\infty} \tilde{\mathbf{A}}_{\mathcal{R}k}(0,t) = \left(I - \overline{\mathbf{A}}_{\mathcal{R}\mathcal{R}}\right)^{-1} \overline{\mathbf{A}}_{\mathcal{R}k}$. Thus, $\lim_{t\to\infty} \mathbf{g}_t^{\mathcal{R}} = \left(I - \overline{\mathbf{A}}_{\mathcal{R}\mathcal{R}}\right)^{-1} \overline{\mathbf{A}}_{\mathcal{R}k}\mathbf{g}^{C}$. □

## Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jedc.2025.105059.

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