Biases in the Perceived Area of Different Shapes: A Comprehensive Account and Model

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This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 765121 "DyViTo". An earlier version of this work was presented as a poster at the 43rd European Conference on Visual Perception (ECVP), online, 22–27 August 2021. We have no conflicts of interest to disclose.

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Abstract

Common daily tasks require us to estimate surface area. Yet, area judgements are substantially and consistently biased: For example, triangles appear larger than same-area squares and disks. Previous work has explored small subsets of shapes, and related biases in area perception to one or two geometric features, such as height or compactness. However, a broader understanding of shape-related biases is lacking. Here we quantify biases in area perception for a wide variety of shapes and explain them in terms of geometric features. In four online experiments (each N = 35), typical adult observers made 2AFC judgements ("which stimulus has larger area?") for pairs of stimuli of different shape, orientation, and / or area. We found clear shape-related biases that replicate known biases and extend them to novel shapes. We provide a multi-predictor model (R^2 = .96) that quantitatively predicts biases in perceived area across 22 shape / orientation combinations.

Keywords: size perception, area perception, perceptual biases

Public Significance Statement

If you had to choose between two pizza slices, you may compare their surface area to decide which one is bigger. Surprisingly, our ability to judge area is influenced by shape: For example, triangles appear bigger than same-area disks and squares. Here we ask: Can we predict how big a certain surface will appear, given its shape? Answering this question has implications that go well beyond lunch choices, and can help mitigate (or exploit) the effect of these biases in a variety of contexts, from visual design (creating size-matched displays), to marketing (choosing a product shape), and surgery (judging the size of bodily structures).

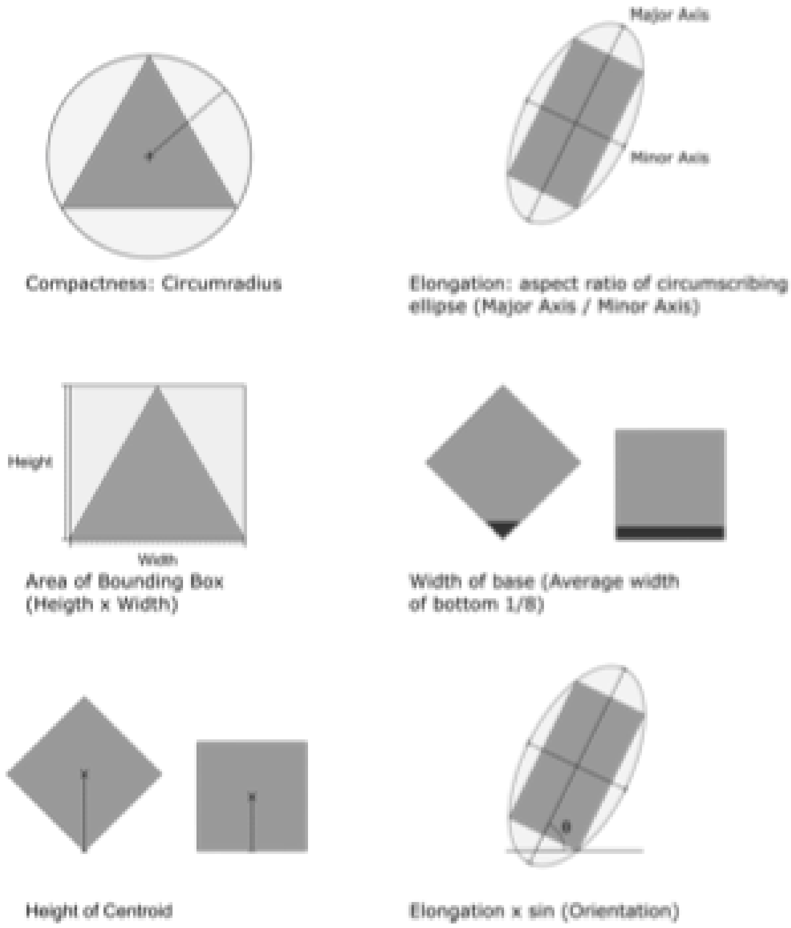
Introduction

Estimating surface area is integral to many daily tasks, such as choosing a pizza slice for lunch (Krider et al., 2001), or perhaps more critically, judging the surface area / volume of a bodily structure when performing surgery (Schuld et al., 2012). As in the pizza case, judging area is an integral step in estimating volume and inferring weight in preparation for grasping and lifting. It is somewhat surprising, therefore, that humans exhibit substantial and systematic biases in area perception. For example, triangles appear larger than same-area squares or disks (Anastasi, 1936; Dresslar, 1894; Fisher & Foster, 1968; Martinez & Dawson, 1973; Warren & Pinneau, 1955), and rectangles appear larger than same-area squares (Krider et al., 2001). However, little is known about shape-related biases beyond a small number of simple shapes. Moreover, there has been little effort to quantify biases in area perception and no models exist to generate predictions of perceived area for novel shapes.

In previous studies, triangles are consistently perceived as larger than same-area disks and squares. This suggests that compactness (e.g., the ratio of a shape's perimeter to its area) may be negatively associated with perceived area (Dresslar, 1894). Various metrics of compactness (see Figure ***1***, Supplementary Table S8) all produce the same ordering across these shapes: disks > squares > triangles. However, inconsistent findings muddy the waters: disks have been reported as perceptually smaller (Anastasi, 1936; Di Maio & Lansky, 1990) or larger (Fisher & Foster, 1968; Warren & Pinneau, 1955) than same-area squares. Across other sets of shapes, compactness has been reported to correlate either negatively (Dresslar, 1894; Owen, 1970) or positively (Foster, 1976; Smith, 1969) with perceived area.

Figure 1

Shape Metrics Featured in the Best Model



Note. The best model is described below. See Supplementary Table S8 for the full list of predictors with definitions. Elongation corresponds to the aspect ratio of the ellipse circumscribing the shape; orientation corresponds to the angle between the x axis and the major axis of the circumscribing ellipse.

In volume perception, elongation is associated with increased perceived volume: the "elongation bias" (Krishna, 2006a). Whether this generalises to area perception is unclear, as more elongated rectangles have been reported as perceptually larger than (Holmberg & Holmberg, 1969, as cited in Krider, Raghubir, & Krishna, 2001; Mates et al., 1992), smaller than (Martinez & Dawson, 1973), or not different from (Holmberg & Wahlin, 1969, as cited in Krider et al., 2001) less elongated rectangles and squares. Orientation also plays a role in perceived area: squares appear larger when they are presented in a ‘diamond’ orientation (Mach, 1897). However, the wider effects of orientation, and their interaction with elongation, are unclear.

As described above, many authors have identified a single geometric feature (e.g., height, elongation or compactness) as the source of shape-related biases. Others have suggested that observers use multiple geometric features but combine these via an incorrect rule to infer area (Carbon, 2016; Krider et al., 2001; Yousif & Keil, 2019). For example, Krider and colleagues (2001) propose that observers compare shapes according to the ratio of their most and least salient dimensions. However, saliency is ill-defined, and the model gives no quantitative predictions. Other authors have suggested that observers sum stimulus height and width (“additive area”) to estimate area (Yousif, Aslin, & Keil, 2020; Yousif & Keil, 2019).

In summary, our understanding of biases in area perception is limited to a small number of simple shapes; few studies have included polygons with more than four sides or concave polygons such as stars and crosses (Anastasi, 1936; Martinez & Dawson, 1973; Owen, 1970; Warren & Pinneau, 1955). In addition, discrepancies are hard to reconcile due to limitations such as the absence of formalised descriptions of geometric features (e.g. Anastasi, 1936; Martinez & Dawson, 1973; Owen, 1970). We lack a quantitative account of area perception that characterises biases across a varied set of shapes and provides testable predictions for novel shapes.

The current work seeks to address these limitations: Across four experiments we measured biases in area perception for a wide range of shapes / orientations. To preview the key results: Shape related biases are substantial; the perceptually largest shape (three-point star) was perceived to be around 41%, or 2 JNDs (Just Noticeable Differences) larger than the perceptually smallest shape (disk). We present a quantitative model that captures variations in area perception (R^2 = .96), with a combination of shape metrics including compactness, elongation, and orientation. The model captures previously reported area biases (e.g., disks < squares < triangles; squares < rectangles; squares < ‘diamonds’) but extends to 22 different shapes/orientations.

General methods

Participants

Effect sizes for shape-related biases could only be calculated from Warren & Pinneau (1955). These were large (*d* = 5.07, 4.57, 1.06 for disks vs triangles, squares v. triangles, disks vs squares, respectively) suggesting 10 observers to give .8 power with alpha = .05. In order to quantify biases across a larger set of shapes, we chose a sample of 35. We did not address a specific participant population. Different sets of participants were recruited in 2020 and 2021 via Prolific (www.prolific.co) for each experiment. All participants were financially compensated. Mean ages (*SD*) in experiments 1–4 were 29.2 (*7.5*), 27.7 (*7.4*), 31.5 (*11.6*), 29.4 (*9.0*) years. Genders (female, male, non-binary) were represented as follows: 10 f, 24 m, 1 n-b; 14 f, 21 m; 17 f, 18 m; 20 f, 15 m. The study complies with the APA ethical standards for research with human participants and was approved by the University of Southampton Psychology Ethics Committee. All participants gave informed prior consent.

Setup

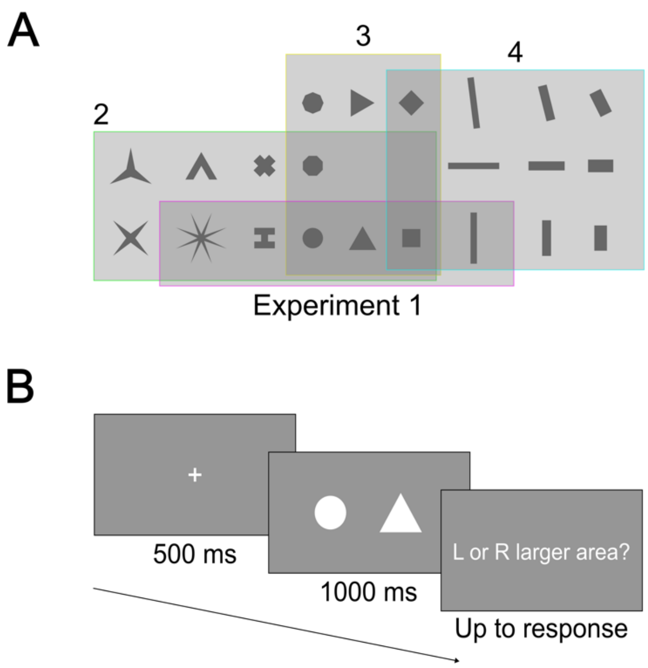
Experiments were conducted online, hosted on a JATOS (Lange et al., 2015) server at the University of Southampton and accessed via Prolific. The experimental software was written in HTML, CSS, and JavaScript using jsPsych (de Leeuw, 2015). The experiment was rendered in a fixed screen partition (width: 800 px) and required a physical keyboard. To control stimulus size across different displays, we used the jsPsych 'resize' plugin: participants adjusted a rectangle on the screen to match the size of a credit card. Participants were asked to sit at a viewing distance of approximately 57 cm. Some variation in viewing distance was deemed acceptable; as expected, shape related area biases proved to be broadly scale-invariant. We have no reason to believe that our results depend on other characteristics of the participants, materials, or context.

Stimuli

Each experiment employed a different subset of stimulus conditions (geometric shape-orientation combinations) from a total of 22 (Figure 2 A). Stimuli were presented in four different sizes (5–10 cm^2 in equal steps: 5, 6.67, 8.33, 10 cm^2). All stimuli were pre-rendered in white on a grey background using MATLAB (The MathWorks Inc., 2020).

Figure 2

Stimuli and Trial Structure for All Four Experiments



Note. (A) Stimuli are presented here with equal area. (B) Trial Structure not to scale.

Procedure

On each trial, participants were presented with a central fixation cross (500ms), followed by two stimuli, ± 5 cm from fixation (1000 ms, see Figure 2 B). Participants reported which of the two stimuli appeared larger in area, via arrow keys (left vs right). An opportunity to take a break was given every 20 trials. The number of trials varied across experiments, see below.

Each experiment was divided into two blocks (each included one repetition of the full trial set), separated by a break. Stimulus left and right positions were assigned randomly in block 1 and swapped in block 2. Trial order was randomised within blocks. The first experimental block was preceded by a variable number of practice trials (2 × number of shapes) which were identical to the experimental trials except that feedback was given ("Correct!" or "Wrong."). To avoid providing information about shape-related biases, stimulus pairs in practice trials had very different sizes (5 and 10 cm^2). The pre-task instructions included a definition of area. Average completion times in minutes (*SD*) for experiments 1–4 were 40 (*15*), 56 (*14*), 48 (*17*), 49 (*9*).

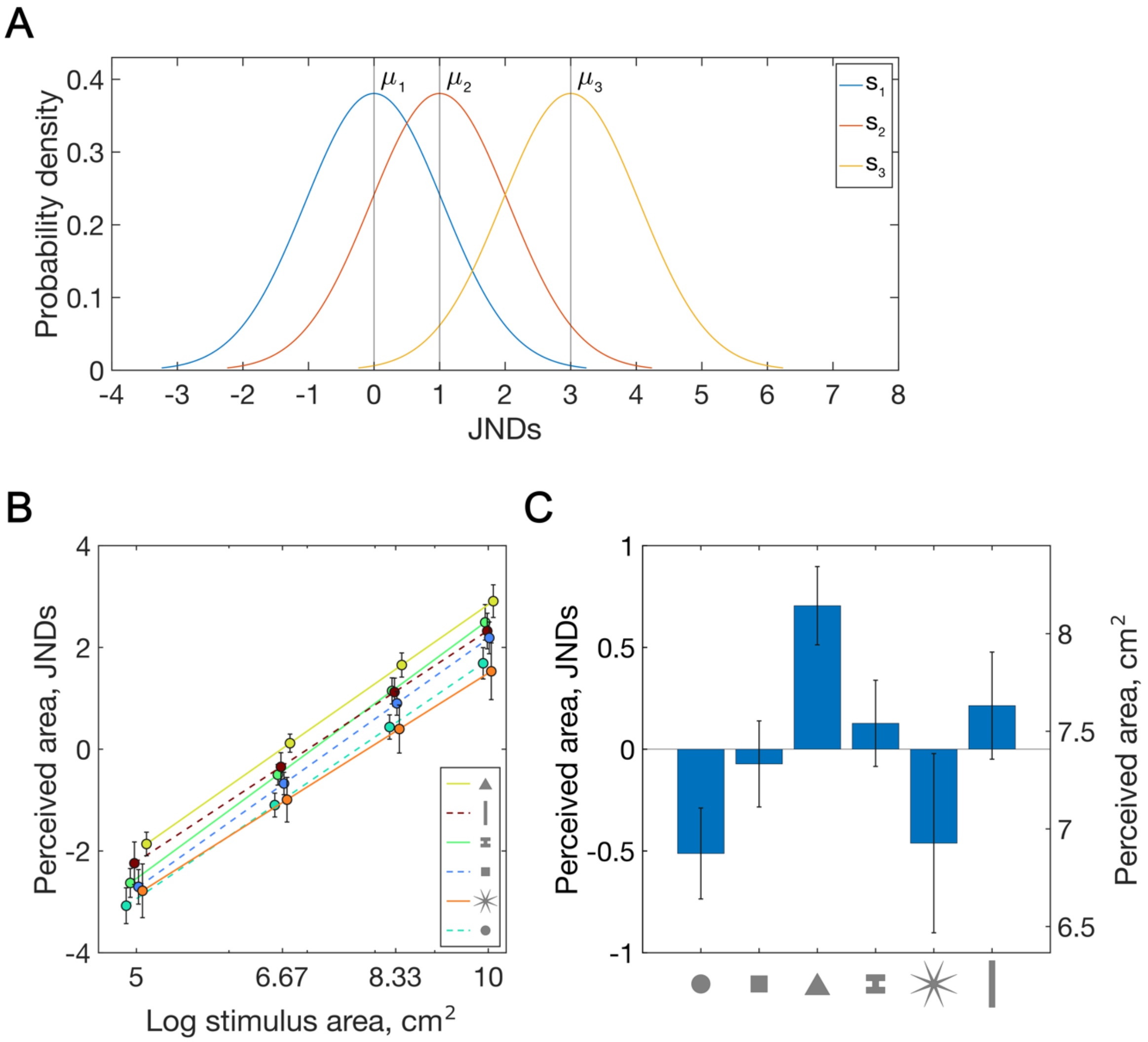
Data analysis

We screened participants' data for control trial errors (same stimulus, size 1 vs 4) and all participants exceeded threshold performance (85% correct). Control trials were excluded from subsequent analyses.

Each participant's 2AFC judgements were converted to estimates of relative perceived area in just noticeable differences (JNDs) using Thurstone case V scaling (Thurstone, 1927; see Adams et al., 2018; Perez-Ortiz & Mantiuk, 2017). Each unique stimulus (combination of shape, orientation, and size) is assumed to invoke perceived area with a unimodal mean (μ), perturbed by Gaussian noise with standard deviation σ. Figure 3 A illustrates the method: For every pair of unique stimuli (s_1, s_2), the distance between the corresponding means gives the probability of perceiving s_1 as larger than s_2. In the examples given in Figure 3 A, p($\hat{s}_2 > \hat{s}_1) = 0.75$, p($\hat{s}_3 > \hat{s}_2) = 0.83$, p($\hat{s}_3 > \hat{s}_1) = 0.99$. Thus, for each stimulus pair we calculate the probability of the observed responses (number of trials, number of $s_1 > s_2$ responses), given values of μ_1 and μ_2. Using gradient descent (fminsearch, MATLAB), we find the set of μ values for all unique stimuli that maximises the probability of each participant's complete dataset.

Figure 3

Thurstonian Scaling and Data for Experiment 1.



Note. (A) A simplified scenario with only three unique stimuli. Perceived area for each stimulus is represented by a Gaussian with σ = 1.05 JNDs. (B) Perceived area in JNDs for each condition as a function of stimulus size. (C) Data summarised by averaging across stimulus size. Error bars give 95% confidence intervals.

The Thurstonian scaling approach allowed us to compare a larger number of shapes in each experiment compared to previous studies, while keeping the trial number appropriate for online experiments. Although the method assumes a common noise parameter for all stimuli, estimates of mean bias (averaged across size, as in Figure 3 C) are minimally affected even by substantial deviations from this assumption (see Simulations, Supplementary material).

We compared two nested models of perceived area. In the first, JNDs were fitted independently to all unique stimuli (degrees of freedom = $n_{conditions} \times n_{sizes} - 1$). The second model assumes Weber's law within each condition, i.e. that perceived area in JNDs increases linearly with log(area) (degrees of freedom = $n_{conditions} \times 2 - 1$); see the straight line fits in Figure 3 B. For all experiments, the second model was clearly preferable (likelihood-ratio tests, all $p > .99$).

Transparency and openness

The study was not preregistered. Determination of sample size, all experimental manipulations and all measures were reported; no data were excluded from the sample. Raw data are available at https://eprints.soton.ac.uk/482307/. All analyses were performed in MATLAB; versions and relevant packages are reported and referenced where appropriate.

Experiment 1: Common shapes

In Experiment 1 we quantified relative biases in perceived area for shapes commonly used in the literature (disk, triangle, square, 8:1 ratio rectangle). In addition, we included two concave shapes (i.e., at least one interior angle greater than 180°): an h-shape and an eight-pointed star. This set decoupled potential correlates of perceived area such as compactness, elongation, height and perimeter length. For example, the h-shape and square have similar height and width but different compactness. The star and rectangle are both low in compactness with similar height but differ in elongation.

Trials

Stimuli were presented in all four sizes. Experimental trials (N = 516) excluded the least informative stimulus pairings (same condition and size; same condition with size difference greater than one step; we expected the latter to be unambiguous). All other possible pairings were included, in two repetitions. Control trials (N = 12) were intermingled with experimental trials. Twelve practice trials preceded the first experimental block.

Results

Figure 3 B shows perceived area in relative JNDs as a function of true area. Our method gives relative, not absolute JNDs; the mean JND value (across conditions and sizes) was set to 0 for each participant. The fitted lines for each condition are roughly parallel, suggesting that biases in area perception are broadly scale-invariant; triangles were perceived as 19% larger than same-area disks, irrespective of absolute size. Figure 3 C summarises the same data, averaged across size. To provide a more intuitive representation of the area biases, we converted JNDs to perceived area in cm^2 (righthand y-axis) under the simple assumption that squares (common to all experiments) were perceived veridically. Selecting a different shape would simply scale the values up or down, without affecting the relative biases between shapes in percentage terms.

Close inspection of Figure 3 B reveals that two shapes (rectangle, eight-point star) have slightly shallower slopes than the others. There are two possible explanations: First, biases may not be entirely scale invariant: at one viewing distance, a rectangle might appear larger than an equal area ‘h’, but on moving closer, this relationship is reversed. Alternatively, the rectangle and star might be associated with greater uncertainty than other shapes, i.e., a violation of the equal noise assumption of our fitting method (see Simulations, Supplementary Figure S1). The latter presents a more likely explanation for the observed small slope variations; note that the star and rectangle are also associated with the largest inter-observer variation (error bars in Figure 3 C). For this reason, we focus our analyses and interpretation on the mean shape-related biases (averaged across size, treated as scale invariant), whose estimation is negligibly affected by noise variations across stimuli.

A regression analysis revealed that biases in perceived area were significantly modulated by shape ($p < .001$, see Supplementary material, Table S1 for the full model). Disks were perceived as significantly smaller ($b = -0.44$, $p = .03$) and triangles as significantly larger ($b = 0.78$, $p < .001$) than squares. See Supplementary material, Figure S2 for effect sizes (Cohen's *d*) for all experiments.

Interim discussion

As expected, triangles were perceived as larger than squares or disks (Anastasi, 1936; Dresslar, 1894; Fisher & Foster, 1968; Martinez & Dawson, 1973; Warren & Pinneau, 1955). In accordance with the elongation bias (Krishna, 2006), rectangles were perceptually larger than squares, although this did not reach significance. That triangles were perceived to be the largest shape suggests that height, elongation, or compactness alone cannot explain the biases. Surprisingly, the eight-point star – the widest, tallest and least compact shape in the set – was perceived as one of the smallest. This is at odds with suggestions that less compact or taller shapes are perceived as larger (Dresslar, 1894; Owen, 1970; Smets, 1970). One possibility is that shapes with a circular, or near circular, convex hull (such as the star) are underestimated, in a similar way to disks. However, perception of the star was more variable across observers than other shapes (see error bars in Figure 3 C), suggesting that observers may use different strategies to assess this shape.

Experiment 2: Convex hull and compactness

In Experiment 2 we explore how the form of a shape's convex hull affects perceived area and how this interacts with compactness. The stimuli included four convex shapes (disk, equilateral triangle, square, octagon) plus two subsets of concave shapes whose convex hulls matched the polygons (triangle, square, octagon). The two subsets had moderate (rotated 'v-' and 'h-shape', and 'x-shape') and low compactness (three-, four-, eight-pointed star shapes).

Trials

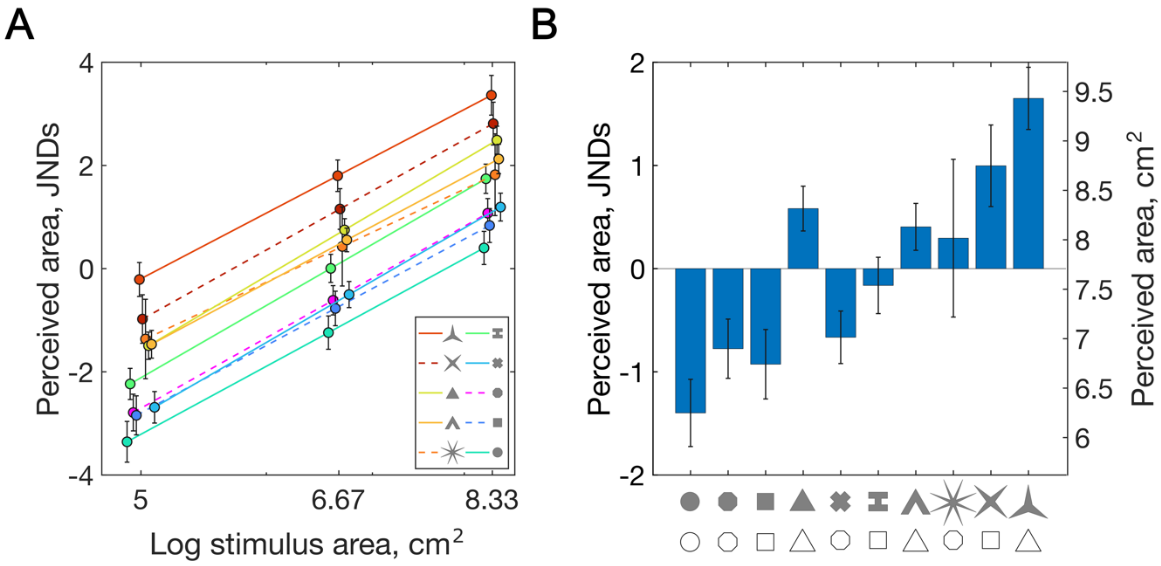
Stimuli were presented in sizes 1–3 in the experimental trials. Participants completed 870 trials (850 experimental, 20 control) in two blocks. Selection criteria for trials were identical to those of Experiment 1.

Results

Figure 4 shows the results of Experiment 2. We uncovered larger relative biases than in Experiment 1. The perceptually largest shape (three-point star) was perceived to be considerably larger (52%) than the perceptually smallest shape (disk). For shapes common to Experiments 1 and 2 (disk, triangle, square, eight-point star), the biases are broadly consistent except that the eight-point star was perceived as smaller than the square in Experiment 1, but this was reversed in Experiment 2. However, the star was again associated with the most variability.

Figure 4

Data for Experiment 2



Note. (A) Perceived area in JNDs for each condition as a function of stimulus size. (B) Data summarised by averaging across stimulus size. Error bars give 95% confidence intervals. Open symbols on the x axis bottom row represent the shape of the convex hull for each stimulus.

We fit biases in perceived area as a function of convex hull shape (octagon, square, triangle) and compactness (high, moderate low) (see Supplementary material, Table S2 for the full regression model). Compactness was modelled quantitatively as the average compactness (area / area of circumdisk) within each subset. This analysis confirmed that triangular shapes were perceptually larger than square shapes (b = .81, p = .001). In addition, less compact shapes were perceptually larger than more compact shapes (b = -2.83, p = < .001), with no significant interactions between convex hull shape and compactness.

Interim discussion

As for Experiment 1, simple models of area perception based on height, width or their combination (Krider et al., 2001; Yousif et al., 2020; Yousif & Keil, 2019) cannot explain the pattern of biases.

Within both subsets of concave stimuli, shapes with a triangular convex hull were perceived as larger than their square and octagonal counterparts (i.e., ‘v’ shapes larger than ‘h’ and ‘x’ shapes, and three-point stars larger than four- and eight-point stars). This pattern replicates the bias observed for convex shapes and is consistent with previous reports for star-shaped stimuli (Martinez & Dawson, 1973). Within each convex hull shape, perceived area increased with decreasing compactness (e.g. triangle < v-shape < three-point star). Thus, compactness negatively correlates with perceived area but is not the whole story; the three-point star is more compact, but perceptually larger than the eight-point star.

Experiment 3: Orientation

Experiments 1 and 2 suggest that height, width or their combination cannot fully explain area biases. Nonetheless, changes in orientation (with corresponding changes to height and width) do affect perceived area, as when rotating a square by 45° (Mach, 1897). Here we explore the effects of orientation by presenting stimuli in their canonical orientation or with their longest linear length vertical to maximise stimulus height. Stimuli included the four convex shapes from Experiment 2, with the three polygons presented in both orientations (Figure 2 A).

Trials

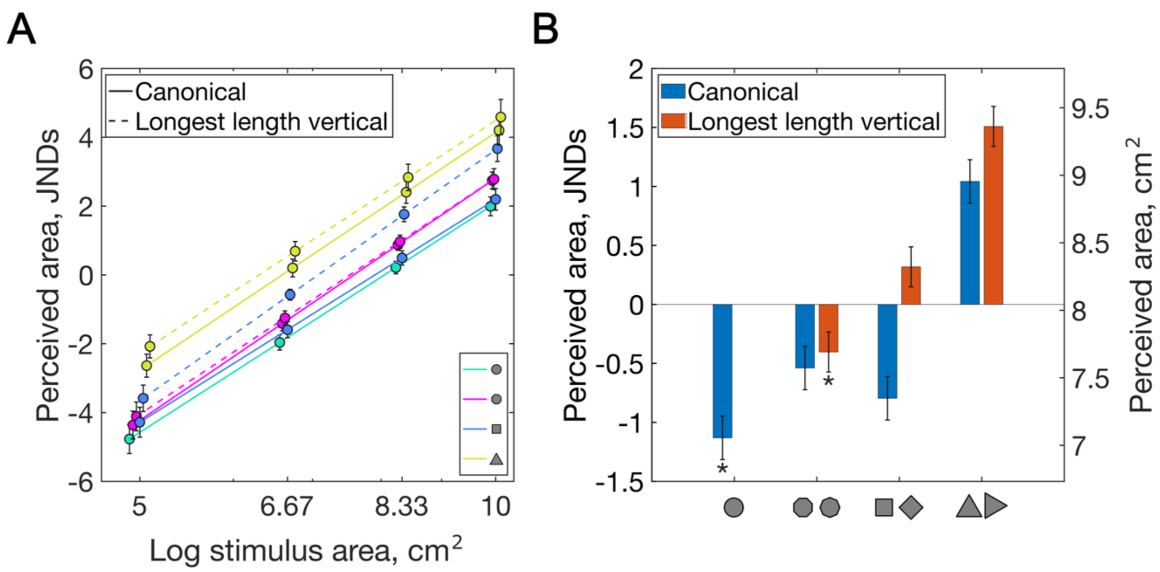
Stimuli were presented in all four sizes. Participants completed 704 trials (690 experimental, 14 control) across two blocks. Experimental trials included all possible pairings except (i) same shape and orientation: only stimuli separated by one step were compared, (ii) same shape and different orientation: same-size and one size step away were compared. The 14 practice trials featured stimuli only in their canonical orientations (sizes 1 and 4).

Results

The results of Experiment 3 are shown in Figure 5. Rotated triangles (perceptually largest) were perceived as 33% larger than disks (perceptually smallest). The relative biases for shapes in their canonical orientations are consistent with those observed in Experiments 1 and 2.

Figure 5

Data for Experiment 3



Note. (A) Perceived area in JNDs for each condition as a function of stimulus size. Line colours represent stimulus shape; line types represent stimulus orientation. (B) Data summarised by averaging across stimulus size. Bar colours represent shape orientation. Error bars give 95% confidence intervals.

A regression analysis of perceived area as a function of stimulus shape and orientation (canonical vs longest length vertical) confirmed that triangles were perceived as significantly larger than squares (b = 1.84, p < .001), whilst octagons and squares were perceptually similar in size (b = 0.26, p = .11). In addition, orienting shapes with their longest length vertical increased their perceived area (b = 1.11, p < .001). This orientation effect was larger for squares than triangles (b = -

.64, p = .004) or octagons (b = -.98, p < .001). (See Supplementary material, Table S3 for the full regression model).

Interim discussion

To summarise, all shapes were perceived as larger when presented with the longest length vertical. However, this orientation effect varied across shapes: a greater change in stimulus height, or the height of centroid, was associated with a greater change in perceived area.

Experiment 4: Orientation and elongation

Here we further probe the effects of orientation and determine how this interacts with elongation. The stimulus set (Figure 2 A) included rectangles in four aspect ratios (1:1, 2:1, 4:1, 8:1), presented in three orientations. These were (i) longest edge vertical, (ii) longest length vertical and (iii) longest edge horizontal. (Orientations (i) and (iii) are equivalent for squares).

Trials

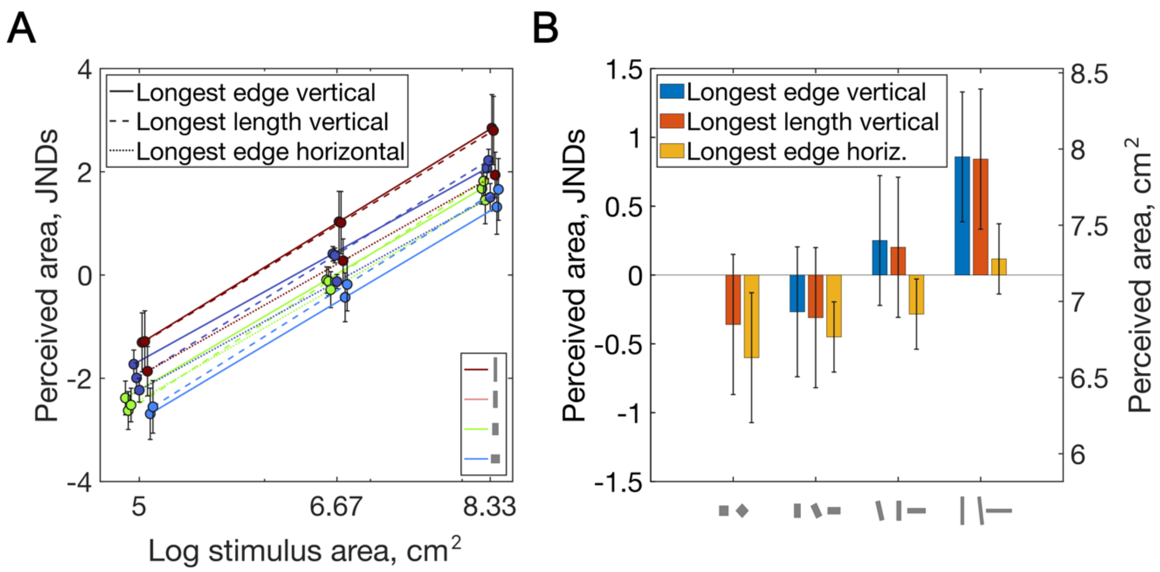
Experimental trials employed stimulus sizes 1–3. Participants completed 800 trials (778 experimental, 22 control) across two blocks. Comparisons of identical stimuli (same size and condition) were excluded. All remaining condition pairs were compared up to one size away. The 12 practice trials employed stimuli with their ‘longest edge vertical’ orientation (sizes 1 and 4).

Results

Experiment 4 results are shown in Figure 6. Vertical 8:1 rectangles were perceived as 22% larger than canonical squares. A regression analysis (perceived area as a function of aspect ratio (four levels) and orientation (longest length vertical vs longest edge horizontal) confirmed that perceived area increased with elongation (b = 0.18, p < .001), in accordance with the elongation bias previously observed for cuboids (Krishna, 2006). Similarly to Experiment 3, orientation also affected perceived area: all rectangles were perceived as smaller when presented horizontally than vertically, with a larger orientation effect for more elongated shapes, although these effects were not significant in Experiment 4 (see Supplementary material, Table S4 for the full regression model). The difference between the two vertical orientations (longest edge vertical versus longest length vertical) was small and inconsistent.

Figure 6

Data for Experiment 4



Note. (A) Perceived area in JNDs for each condition as a function of stimulus size. Line colours represent stimulus shape; Line types represent stimulus orientation. (B) Data summarised by averaging across stimulus size. Bar colours represent shape orientation. Error bars give 95% CIs.

Interim discussion

We found a clear effect of elongation, independent of orientation. Whilst the interaction between elongation and orientation did not reach significance, the observed pattern across Experiments 3 and 4 is consistent with the idea that perceived area is positively correlated (albeit imperfectly) with height (or negatively correlated with width). This is broadly consistent with the relative overestimation of vertical line length seen in the vertical-horizontal illusion (Valentine, 1912; Wolfe et al., 2005).

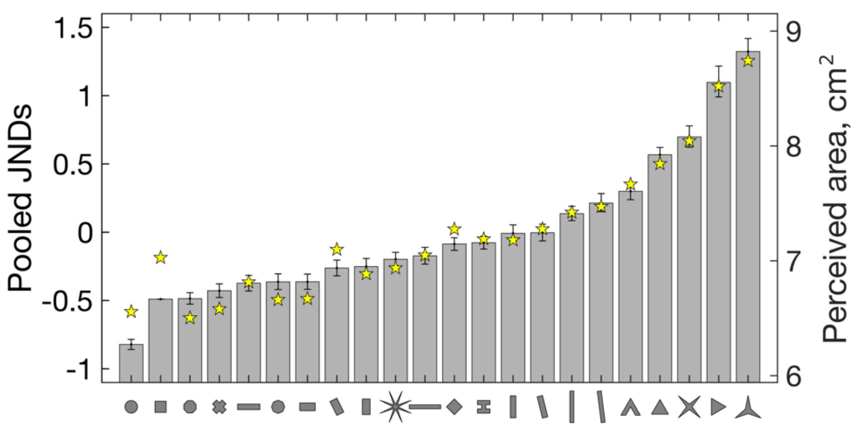
Modelling biases in area perception

Our four experiments demonstrate that biases in perceived area are substantial and are associated with various shape metrics including compactness, elongation, orientation and width or height. Here we determine how different shape metrics combine to predict perceived area, across all stimulus conditions.

Raw data (2AFC responses) for stimulus sizes 1–3 (common to all four experiments) were pooled across observers and experiments. This allowed us to determine the maximum likelihood set of JND values (i.e. perceived area for each condition in units of discrimination), given all data. The results are shown in Figure 7. Note that the range of JND values is compressed relative to the JNDs derived from individual observers (Figures 3–6). This is because inter-observer variation is conflated with uncertainty within the pooled data. Pooling, therefore, underestimates relative biases in units of discrimination (JNDs). However, relative biases in percentage terms (A is perceptually X% larger than B) are preserved. As above, JNDs were converted into perceived area in cm^2 by assuming that the common shape (canonical square) is perceived veridically (see righthand y-axis of Figure 7).

Figure 7

JNDs for Each Condition, for Data Pooled Across Observers and Experiments, Converted to Perceived Area and Averaged Across Size



Note. Error bars give 95% CIs from bootstrapping (10,000 samples). Yellow stars show the model fit for each condition.

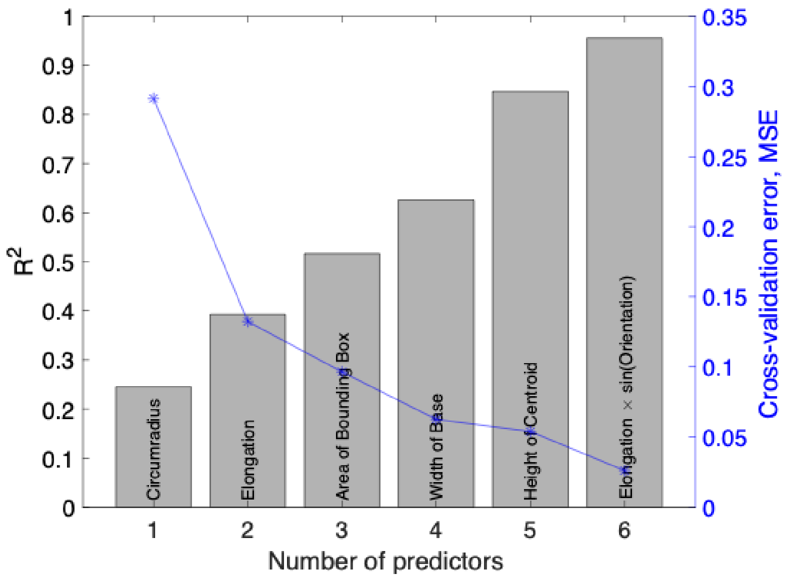
Perceived area was averaged across sizes, to capture scale-invariant biases in perceived area (as in Figure 7). We compared linear regression models that characterise these biases as a function of various shape metrics (these were calculated for the middle stimulus size, for each shape). Our set of candidate predictors included shape descriptors previously proposed in the literature (e.g. compactness, height, additive area, aspect ratio), enabling us to test earlier models, in addition to some of our own design (e.g. width of object base, height of centroid, see Figure ***1***). As we fitted *biases* in perceived area, predictors did not include true area (whose regression coefficient would be undefined). The full list of candidate predictors, with definitions, is given in Supplementary Table S8.

Regression models were evaluated and compared via leave-one-out cross-validation (leaving out each of the 22 stimulus conditions, in turn). This approach prevents overfitting by quantifying how well each model generalises to novel stimuli. Whilst many candidate predictors are partially correlated, cross-validation only rewards the addition of predictors that explain additional systematic variance in the data. Further to this, we evaluated regression models using PCA (Principal Component Analysis) and PLS (Partial Least Squares) components as candidate predictors; two methods often employed with partially correlated predictors. Neither of these provided more parsimonious accounts of biases in perceived area. Cross-validation results, alternative candidate models, and model comparisons are presented in the Supplementary material (S3 Model selection; S4 Alternative models).

The selected model includes six predictors (Figure ***1***), and accounts for 96% of the variance in perceived area across the 22 unique stimuli (combinations of shape and orientation), see Figure 8. The influence of each predictor on every condition is given in Supplementary Figure S4.

Figure 8

Variance In Perceived Area (cm²) Explained by the Addition of Each Predictor in the Model



Note. The presented order of predictors follows the maximum increase in R^2 (left-hand y axis) with each addition. See
Figure **1** for a visual representation of each predictor. Each bar gives the best N factor model (for N=1:6), given the 6 predictors in the final model. The blue asterisks (right-hand y axis) show the cross-validation error (MSE) for each model.

The first predictor, circumradius, is a measure of compactness. This is positively correlated with perceived area; less-compact shapes are perceived as larger (b = 3.36, 95% CI [2.87, 3.84]. The second and third predictors can be interpreted as modulating the relationship between compactness and perceived area (both are positively correlated with compactness but have negative coefficients). Elongation (the aspect ratio of the circumscribing ellipse, b = -0.31, 95% CI [-0.44, -0.18]) reflects the observation that the most elongated shapes (1:8 and 1:4 rectangles) were perceived as smaller than shapes of a comparable compactness but with aspect ratio close to 1:1. The area of the bounding box is also negatively associated with perceived area in the model (b = -0.03, 95% CI [-0.05, -0.01]), such that some of the least compact shapes (the stars) and in particular the eight-point star are associated with reduced perceived area than predicted by compactness alone.

The remaining predictors (fourth to sixth) are all orientation dependent, and together capture the observation that more bottom-heavy / wide shapes are perceived as smaller than tall / top-heavy ones. Base width (average width of bottom 1/8 of stimulus' convex hull) has a negative coefficient (b = -0.71, 95% CI [-0.84, -0.57]), reflecting the observation that the triangle and square both appear larger when presented with their longest edge vertical (and resting on a corner). It also captures the orientation effect for rectangles, i.e. that they are perceived as smaller when presented horizontally. The sixth predictor is elongation × sin(orientation) (see Figure ***1***), reflecting the larger orientation effect for elongated stimuli (b = -2.59, 95% CI [-3.50, -1.68]). The fifth predictor, height of centroid, can be thought of as a suppressor variable: it is negatively correlated with base width (r = -0.44), and with elongation × sin(orientation) (r = -0.45). It thus suppresses some of the (over-inflated) effects introduced by the fourth and sixth predictors, while reflecting the fact that the star is perceived as smaller than other shapes of low compactness (b = -2.03, 95% CI [-2.45, -1.62]).

General discussion and conclusions

We quantified biases in area perception for a wide range of shapes and orientations. These biases were substantial and highly consistent across participants and experiments, with the exception of one shape: the eight-point star. We used an assumption-free method, cross-validation, to derive a model that provides an excellent account of these biases, explaining 98% of response variance in terms of simple shape metrics. Our model quantifies biases that have previously been reported in qualitative terms (e.g. triangles > disks). Simultaneously, the model accounts for biases demonstrated here with novel shapes.

Consistent with previous suggestions (Dresslar, 1894; Owen, 1970), compactness is strongly correlated with perceived area, with less compact shapes appearing to be larger. Within squares and rectangles, perceived area also increases with elongation, as previously suggested for volumetric judgments of cuboids (Krishna, 2006). However, elongation has a negative coefficient in the model:

shapes that are concentrated along one axis (i.e. rectangles) are not perceptually enlarged to the same degree as stimuli with similar compactness but an aspect ratio close to 1. Elongation interacts with orientation in its effects on perceived area: when very elongated shapes are rotated there is a bigger change in perceived area than for less elongated stimuli. Note that this elongation × orientation interaction did not reach significance when considering data from Experiment 4 alone. More generally, shapes that are wider or more bottom-heavy appear to be smaller.

Previous work has, in general, focussed on a small set of objects and identified a single feature that correlates with perceived area. For example, one might assume that triangles look larger because they are taller (and wider) than squares and disks (Krider et al., 2001), but this rule fails when comparing equilateral triangles and rectangles. Similarly, 'additive area' (Yousif & Keil, 2019; Yousif et al., 2020) captures the difference in perceived area for squares presented in different orientations but fails to explain the effect of orientation for other common shapes, such as triangles and elongated rectangles, or the difference in perceived area between disks and squares in their canonical orientation.

Unlike these previous studies, our model does not provide a simple rule or heuristic, representing how we estimate area. A simple heuristic could involve the model's first predictor only: circumradius (i.e., compactness): 'imagine a circle enclosing the shape – its diameter is an estimate of area'. Shapes with a greater circumradius (i.e., less compact shapes) are generally perceived as larger, as previously suggested (Dresslar, 1894; Owen, 1970). Indeed, the best single predictor of all candidates (identified via cross-validation, see Supplementary Figure S3 A, Supplementary Table S9) is an alternative compactness metric (area-to-area of circumdisk ratio). These measures of compactness have lower (i.e., better) cross-validation error (0.35 and 0.29) than additive area (0.53). However, these single predictors are all inferior to our multi-predictor model in terms of predicting novel shapes (i.e. cross-validation error) or explaining variation in perceived area across shapes (see Figure 8 and Supplementary Figure S3 A). For example, compactness alone does not explain orientation effects and overestimates the perceived area of shapes with low compactness but high elongation. Thus, whilst these simple heuristics might appear more elegant, they fail to capture systematic biases in area perception and therefore have limited utility.

In the current study, we used leave-one-out cross-validation over shapes to assess the model's generalisability. Whilst this is a standard approach to avoid over-fitting, it cannot test how well our model will generalise to all possible novel shapes, particularly those that are more dissimilar to the current set than our shapes are to each other. Further studies could, therefore, use a similar approach to ours to test and refine our model with new classes of shapes. These might be randomly generated blobby shapes, or silhouettes of recognisable objects. The latter may introduce additional, higher-level biases, if perceived area is biased towards the surface area of the corresponding known 3D objects.

A big unanswered question remains: why do biases in perceived area occur? Our model provides an excellent fit to the data but does not provide insight into why area biases exist in general, or why they correlate with the particular predictors of our model. Here we consider three classes of explanation that might apply to area biases: (i) misapplied constancy scaling, (ii) affordances, and (iii) limitations of visual system.

Gregory (1963) has proposed that misapplied size constancy scaling can explain a variety of 2D illusions including the Ponzo, Müller-Lyer, and Hering illusions. In essence, when we ask observers to judge some aspect of a proximal image, they can't help but perceive the corresponding aspect of the inferred distal object, i.e., the object that is most likely to have given rise to the image. This after all is the goal of perception – to understand and interact with the world around us. Whilst this explanation has been questioned for the Müller-Lyer illusion (DeLucia & Hochberg, 1991), it remains a plausible explanation for the Ponzo illusion (Yildiz et al., 2007). A similar explanation has been proposed for the horizontal-vertical illusion. Armed with LiDAR, Howe and Purves (2002) looked at the relationship between the extent of 3D objects in natural scenes and the length of the contours that they project to in the image. They found that vertical contours arose from longer objects than horizontal ones, due to a difference in the orientations of the corresponding sets of objects. In essence, we overestimate the length of vertical lines (relative to horizonal ones) because their source in the world is likely to be longer.

Can this be generalised to the analogous bias in area perception, i.e., that vertically extended objects are perceived as having greater area than horizonal ones? Not easily: the relative overestimation of vertical contours should apply equally to the long edge on a vertical rectangle and the short edge on a horizontal one, and thus does not predict an orientation effect for area. Nonetheless, we have tested this flavour of hypothesis in a further study (not yet published) in which observers compared the surface area of the front face of 3D prisms with various cross sections (e.g., square, triangular, rectangular, circular). The same biases remained, including orientation effects, even though 3D attitude relative to the image plane was unambiguous, due to structure from motion cues.

Can we explain other aspects of our findings with misapplied constancy scaling? Bottom heavy objects (e.g. trees) are more common in nature than top-heavy ones, for obvious reasons. Thus, we might argue that a bottom-heavy triangle is likely to invoke a more voluminous bottom-heavy distal object, i.e. a pyramid, than that evoked by a rotated triangle resting on its corner. Moreover, the corresponding surface of a pyramid is slanted away from the observer, and thus has a larger surface area than a fronto-parallel one. Unfortunately, this line of reasoning predicts the opposite of the observed bias: bottom-heavy stimuli are perceived as smaller in area than rotated ones.

Secondly, biases in area perception might relate to affordances, i.e. planned or potential interactions with the 3D objects corresponding to the stimuli. Proffitt and colleagues have proposed affordance-based perceptual illusions: hills look steeper when we wear a heavy backpack (Bhalla & Proffitt, 1999); doorways look narrower to broad-shouldered individuals (Stefanucci & Geuss, 2009). Some of these findings have faced strong criticism, however, as being due to response bias, rather than perceptual bias (Firestone, 2013). In terms of our stimuli, less compact objects will, in general, require a larger grasp aperture and a larger space if we imagine lifting and re-locating them, relative to more compact objects. However, this is a highly speculative explanation and is at odds with a key goal of perception: to accurately perceive and interact with the world.

A final class of explanation is that some illusory size biases may be caused by constraints of the visual system. For example, the Müller-Lyer Illusion has been explained by spatial pooling. Morgan and colleagues (1990) suggested that the visual system employs spatial pooling via large, overlapping cortical receptive fields in order to locate objects in space (see also Whitaker et al., 1996). Whilst beneficial for rapid object localisation, spatial pooling sacrifices positional acuity, as local features are blurred. The Müller-Lyer illusion would arise when segment ends and flankers are spatially pooled, causing perceptual displacement of the endpoints' centroids (Bulatov et al., 2011; Morgan et al., 1990). How would this translate into biases in area perception? Spatial pooling is broadly equivalent to Mates and colleagues' (1992) proposal that observers estimate area from a shape's blurred contours. This hypothesis predicts that biases in perceived area are correlated with perimeter length. Unfortunately, perimeter length is a poor predictor of our reported biases. It does not, for example, capture orientation effects, or the biases observed for concave shapes (e.g., that three-point stars are perceptually larger than four- and eight-point stars).

Given finite neural resources, what might a 'quick and dirty' algorithm to efficiently compute area look like? One might approximate area by using an alternative 'reference' shape. One could find the smallest disk or rectangle that encompasses a shape and use its diameter or area as a proxy for the shape's area. Indeed, as noted above, the single best predictor of the area biases reported here is compactness (as quantified by the circumdisk's area). Thus, any algorithm that correlates with compactness can explain some (but not all) of the variance in perceived area across different shapes. Spatial pooling, or a mechanism that approximates area via a circumscribing disk (or ellipse or bounding box) might therefore constitute a partial explanation for area biases, but we suspect that multiple factors may be at play to explain the full gamut of biases.

In general, our sensorimotor system recalibrates in order to reduce perceptual biases (Adams et al., 2001, 2010; Burge et al., 2008). However, this mechanism appears to fail in the case of area perception. There is some evidence that biases in area perception have correlates in volume perception: Tetrahedrons are perceived to be larger than cubes and spheres (Kahrimanovic et al., 2010), and elongated cuboids appear larger than cubes (Krishna, 2006), and own work with 3D prisms found similar biases to those reported here. This apparent translation of area biases into erroneous volume and weight perception (Kahrimanovic et al., 2011) makes it all the more peculiar that these perceptual biases are not corrected during everyday object handling. On the other hand, it is well known that grasping forces adapt rapidly in the size-weight illusion, while the perceptual illusion persists (Flanagan & Beltzner, 2000).

Although we have no clear explanation of why these systematic biases in area perception occur, we can try to mitigate their effects. When selecting a partner for a wife-carrying contest (https://en.wikipedia.org/wiki/Wife-carrying), go for a tall, skinny one rather than a deceptively compact stout one. When subsequent hunger strikes, if in doubt, choose the circular pizza.

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