Journal of Physics:
Conference Series

PAPER • **OPEN ACCESS**

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To cite this article: M Kara and N S Ferguson 2024 *J. Phys.: Conf. Ser.* **2909** 012031

Journal of Physics: Conference Series **2909** (2024) 012031 doi:10.1088/1742-6596/2909/1/012031

The effect of joint uncertainty on scattering properties using a hybrid methodology

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Abstract. Joint uncertainty is a subject of much interest in structural dynamics' research, as joint behaviour potentially becomes more significant at higher frequencies. Therefore, an accurate determination of the scattering properties of uncertain joint elements for wave-based methods becomes more important on the subsequent prediction sensitivity. For this reason, this study examines the resulting scattering properties (reflection and transmission efficiencies) due to uncertain joints using a hybrid methodology. The scattering is calculated for a beam-to-beam joint via a combined hybrid Wave Finite Element and Finite Element (abbreviated as hybrid WFE) model, while Polynomial Chaos Expansion is utilized for the uncertainty modelling. It is assumed that the joint has a uniformly distributed uncertain thickness and loss factor. The results are compared to analytical transmission efficiencies and Monte Carlo simulations. The results show that the uncertainty in the joint does not become more evident as the frequency increases as expected, and the proposed methodology successfully models the joint uncertainty problem.

1. Introduction

The Finite Element (FE) method typically requires at least six elements per wavelength for an accurate dynamic analysis and hence the number of elements required in the FE analyses and the computational load (memory usage and time consumption) increases drastically with the frequency of interest. For that reason, alternative numerical methodologies must be utilized for high frequency predictions. For high frequency vibration, Statistical Energy Analysis (SEA) is a common method and can rapidly estimate the spatially averaged and frequency averaged response of the vibratory uncertain system [1]. However, success of this method also depends upon accurate determination of SEA parameters namely, coupling loss factor and power input. Coupling loss factor is a parameter that is related to the joint transmission efficiency.

Wave Finite Element (WFE) might be utilized to determine scattering properties of the joint. When WFE was first introduced by Mace et al. [2], the wavenumber of different structural waveguides was of interest. Then, Duhamel et al. [3] and Waki et al. [4,5] calculated the forced vibration response using WFE for different individual structures. Renno and Mace [6,7] extended the application of WFE for the forced vibrations of jointed structures, the so-called hybrid WFE. In hybrid WFE, the joint is modelled differently, namely, analytically or numerically, and scattering properties of the joint are combined with the waves propagating in the waveguides to determine the forced response. In analytical modelling of the joint, the joint can be assumed to be rigid and it is applicable only for simple types of joints. For the high frequency vibrations or relatively complex joints, the model of the joint is constructed by FE and

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a dynamic stiffness matrix of the joint is obtained after dynamic condensation between the nodes of interfacing with the waveguides. Then, the dynamic stiffness matrix is utilized together with continuity and equilibrium equations to determine the scattering properties of the joint. In the literature, there are studies on determining the transmission efficiencies of different structures by hybrid WFE [6–12]. Fan et al. [8] divided the periodic structure into three wave fields i.e., near field, periodic field and far field, and estimated the vibration energy transmission by modelling the near field with FE and other fields with WFE. Mitrou et al. [10] specifically focused on the joint orientation (L-type, lap and line joint) and modelled the wave transmission efficiencies when two-dimensional structures are connected to each other with different joint types using the WFE/FE approach. Recently, Aimakov et al. [12] determined the power scattering coefficients of laminated plate junctions by hybrid WFE.

The studies referred to Refs. [6–12], deal with the deterministic joints. However, in NVH and engineering structures in general, the uncertainty may arise due to local attachments such as hydraulic pipes, cable harnesses, instrumentation, etc. Therefore, joint uncertainty must be taken into account. In the literature, there are limited number of studies considering WFE and uncertainty together [13–18]. Henneberg et al. [17] modelled a globally uncertain metamaterial by the combined WFE and Polynomial Chaos Expansion (PCE). They modelled the uncertainty in a single cell by PCE then projected it into the whole structure by using the periodicity assumption in WFE to evaluate uncertainty of the bandgap and centre frequency of the bandgap. Kara and Ferguson [18] examined the uncertainty of a complex joint via combined hybrid WFE and PCE (shortly WFE-PCE) and calculated the forced vibration response of an L-type jointed beams up to 1000 Hz [18]. However, there is no study that considers scattering corresponding to uncertain joint properties. This study fills this gap. Here, the thickness and damping of a L-type joint is assumed to be uncertain and the resulting reflection and transmission efficiencies are calculated by WFE-PCE. In the analysis, hybrid WFE is utilized to determine scattering properties and PCE models the uncertainty of the joint. The uncertain parameters are assumed to have uniform distributions. The results are verified by running hybrid WFE computations together with Monte Carlo simulations. The results show that WFE-PCE can be an efficient alternative numerical method and the scattering uncertainty does not always increase with frequency for structures possessing local uncertainty.

2. Scattering properties obtained using a hybrid Wave Finite Element (WFE) method

In this section, the calculation of the scattering properties of a joint is briefly described. Note that, before starting to calculate the scattering properties, one needs to analyse the wavenumber and wavemodes of the jointed waveguides by employing classical WFE methodology. As the calculation of wavenumber and wavemodes of waveguides are well-reported in literature, it is not presented in this paper. One may refer to Ref. [2] for more information on the calculation of wavenumber and wavemodes.

Assume that the wavenumber and the left and right wavemodes of the waveguide are denoted as $\boldsymbol{k}$, $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$, respectively. In figure 1, a schematic is presented for two waveguides joined together. Each waveguide in figure 1 has its local coordinate system where their respective waveguide axis points towards the joint. The rotation matrix ($\mathbf{R}_j$) transforms the local coordinate system (x_j,y_j) to the global coordinate system (X,Y). The main aim in hybrid WFE is to correlate the wave amplitudes propagating on the joint i.e., $\mathbf{a}_j^-$ and $\mathbf{a}_j^+$ in figure 1. In figure 1, the positive going waves are always incident on the joint. This relation may be constructed as follows:

$$\{\mathbf{a}^-\} = [\mathbf{s}]\{\mathbf{a}^+\}, \tag{1}$$

where

$$\{\mathbf{a}^\pm\} = \begin{Bmatrix} \mathbf{a}_1^\pm \\ \mathbf{a}_2^\pm \end{Bmatrix}, \tag{2}$$

and $\boldsymbol{s}_{jj} = \boldsymbol{r}_{jj}$ and $\boldsymbol{s}_{jk} = \boldsymbol{t}_{jk}$ for $j \neq k$. Here, $\boldsymbol{s}$ is the scattering matrix, $\boldsymbol{r}$ and $\boldsymbol{t}$ denote the reflection and transmission coefficients, respectively. Superscript + and − denote the wave direction. For the structure shown in figure 1, it is assumed that the interface degree of freedoms of the joint are compatible with those of the WFE models. The mathematical model of the joint between interface nodes is obtained from

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a FEM model after dynamic condensation under no excitation of the non-interface nodes ($\boldsymbol{F_n} = \mathbf{0}$) as follows:

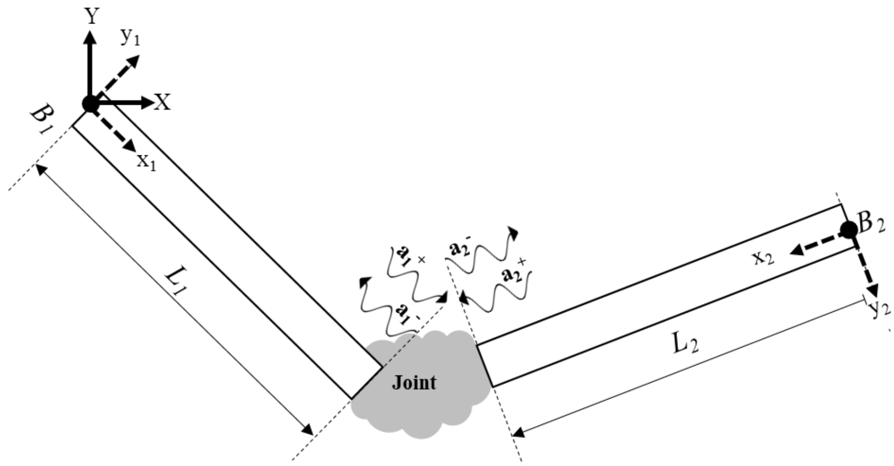

Figure 1. Waves propagating in two finite jointed waveguides and waves on the joint.

$$\mathbf{D}_{ii}\mathbf{Q}_i = \mathbf{F}_i \ . \tag{3}$$

Here, $\mathbf{Q}$ and $\mathbf{F}$ are the degree of freedoms and internal nodal forces in global coordinates. Subscript i represents the interface nodes. The continuity and equilibrium conditions of the joint is expressed as:

$$\mathbf{Q}_i = \mathbf{Rq} \ , \tag{4}$$

$$\mathbf{F}_i - \mathbf{Rf} = \mathbf{0} \ , \tag{5}$$

respectively. Here, $\boldsymbol{R}$ is the global rotation matrix which is formed by diagonally concatenating the rotation matrices ($\mathbf{R}_j$) of the individual waveguides, $\mathbf{q}$ and $\mathbf{f}$ are nodal displacement vector and nodal force vector. Using the relationship between the wave amplitudes, the displacement and force field, equations (4) and (5), one may obtain the scattering matrix as follows:

$$\mathbf{s} = -\left[\mathbf{R\Phi}_f^- - \mathbf{D}_{ii}\mathbf{R\Phi}_q^-\right]^{-1}\left[\mathbf{R\Phi}_f^+ - \mathbf{D}_{ii}\mathbf{R\Phi}_q^+\right], \tag{6}$$

where, the global wavemode matrices, $\mathbf{\Phi}_q^{\pm}$, $\mathbf{\Phi}_f^{\pm}$, $\mathbf{\Psi}_q^{\pm}$ and $\mathbf{\Psi}_f^{\pm}$ are obtained in a similar way as the global rotation matrix. Note that, subscript q and f denote the displacement and force field, respectively.

One may calculate the efficiencies by using the scattering matrix elements. Considering a waveguide carrying an in-plane flexural and longitudinal waves, one may determine the reflection efficiency (Ω) and transmission efficiency (T) for beams excited by flexural waves as follows:

$$\Omega_{BB} = \left|r\right|^2, \tag{7}$$

$$\mathrm{T}_{BB} = \kappa\gamma\left|t\right|^2, \tag{8}$$

$$\Omega_{BL} = \frac{c_{L1}}{2c_{B1}}\left|r_{BL}\right|^2, \tag{9}$$

$$\mathrm{T}_{BL} = \frac{1}{2\beta_2}\left|t_{BL}\right|^2. \tag{10}$$

Here, r and t denote the reflection and transmission coefficients of flexural waves, subscript BL shows the property of mode conversion from bending wave to longitudinal wave, c_{Li} and c_{Bi} are the bending and longitudinal wave speed for structure i, $\beta_2 = m_1' c_{B1}/m_2' c_{L2}$, $\kappa = k_2/k_1$ and $\gamma = E_2 I_2 \kappa^2 / E_1 I_1$ where m_i', E_i and I_i are the mass per unit length, Young's modulus and area moment of inertia of the structure i.

3. Polynomial Chaos Expansion (PCE)

According to the Polynomial Chaos Expansion (PCE), any uncertain variable (X) can be represented by a truncated polynomial with constant deterministic coefficients (x_j) as follows [19,20]:

$$X(\omega) \approx \sum_{j=0}^{N} x_j(\omega) P_j(\xi) \tag{11}$$

Here, ξ_j are uncertain parameters and P_j is an orthogonal polynomial so that, $\langle P_i \cdot P_j \rangle = \delta_{ij} \langle P_j^2 \rangle$ where $\langle \cdot \rangle$ represents the mean value, δ_{ij} is Kronecker delta and ω is the frequency in rad/s. The series summation is truncated at a finite number (N) which is the order of the polynomial for a single uncertain parameter.

The unknown deterministic coefficients in equation (11) might be determined by the collocation point method. The method calculates the coefficients by using a set of collocation points of the uncertain parameter ($\boldsymbol{\xi} = \{\xi^{(1)}, \xi^{(2)}, \cdots, \xi^{(N_{CP})}\}^T$) and the corresponding uncertain variable realizations ($\mathbf{X}(\omega) = \{X^{(1)}(\omega), X^{(2)}(\omega), \cdots, X^{(N_{CP})}(\omega)\}^T$). Note that, the number of collocation points (N_{CP}) should be equal or greater than number of the finite terms (N+1). One may refer to Ref. [21] for further details of the collocation point method. In this paper, ξ will be the uncertain parameter to describe uncertain physical property (e.g. joint thickness, etc.) whilst X is the resulting uncertain parameter (transmission or reflection efficiency) and the evaluation is performed frequency by frequency.

4. Analytical and numerical results

In this study, scattering properties of an uncertain right-angled joint that couples two beams are analysed by combined hybrid WFE and PCE, namely WFE-PCE. The beams are considered to be identical to each other with the following properties: $E = 70$ GPa, $\rho = 2600$ kg/m^3, $L = 1$ m, $h = 1$ mm, $b = 10$ mm and $\eta = 0.005$ where b, h and η are width, thickness and loss factor, respectively. The joint is initially assumed to have the same mechanical and physical properties of the beams for the deterministic parameters with a joint length of 5 cm. The analyses are carried out between 1-10000 Hz. For the structure considered, the wavelength of flexural waves coincides with the length of joint at approximately 3700 Hz whereas the longitudinal wavelength is always greater than the joint length in the frequency range considered. Before going through the uncertain joint problem, the hybrid WFE method is verified with the analytical computations for the scattering properties. One may refer to Ref. [22] for the analytical formulations. The length of the finite element utilized in WFE is 0.005 m whereas the joint is modelled by 10 elements. The analyses are verified as shown in figure 2.

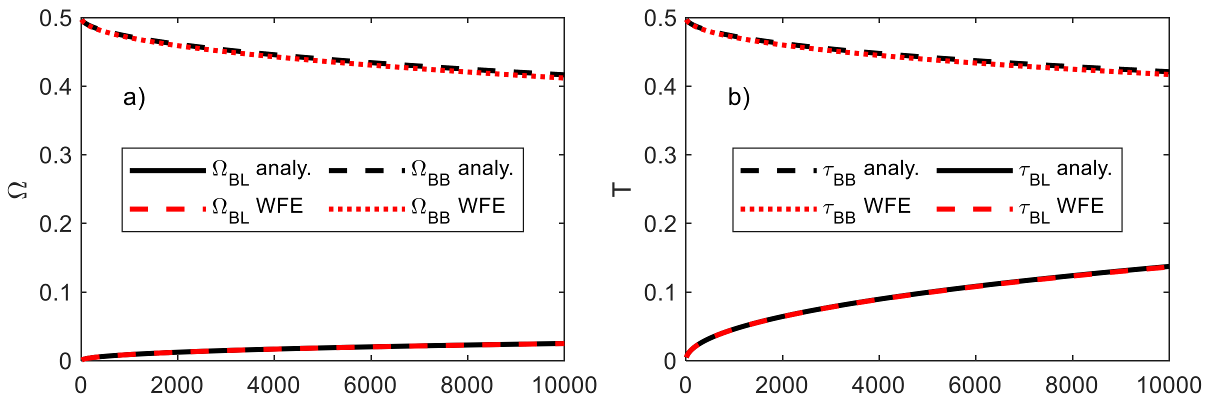

Figure 2. Reflection and transmission efficiencies for a deterministic right-angled joint.

As may be inferred from figure 2, the scattering properties are consistent with each methodology considered. Some negligible deviations for the bending to bending wave reflection and transmission between the analytical and numerical results are observed as the frequency increases. This is due to fact that the hybrid WFE evaluates the results for a damped joint, whereas it is not the case for the analytical calculation. It must be mentioned that the transmission from bending to longitudinal wave power increases with increasing frequency as expected.

After verification of the results, the uncertainty analysis is carried out by WFE-PCE. The joint is assumed to have uniformly distributed thickness between 0.95 mm and 1.05 mm, i.e., U(0.95,1.05) mm and loss factor 0.00475 and 0.00525, i.e., U(0.00475,0.00525), where the upper and lower bounds of the uncertain parameters lay between 5% of their mean. Typically, one would expect manufacturing tolerances to be no greater than this range, but applying this range should be a more significant assessment of the possible fitting and the greater range and variation of the scattering properties. It must be noted that, these uncertain parameters are assumed to be uncorrelated.

The parameters of the PCE are selected as *N*=2 for each uncertain parameter and ξ_η and ξ_h are both assumed to be represented by Legendre polynomials uniformly distributed between -1 and 1. The reason for selecting such a lower order polynomial is that there is no significant increase/decrease in the response parameters (i.e., no resonance behaviour in the frequency response functions) as inferred from figure 2). One may refer to Ref. [23] for more detailed information on the choice, consequences and general preference for selecting the polynomial order. The vector that contains the orthogonal polynomial terms is $P_i = \{1 \quad \xi_i \quad 1.5\xi_i^2 - 0.5\}$ where *i* is h for uncertain thickness and η for uncertain loss factor. In regard to these parameters, the resulting transmission or reflection efficiency of interest at a frequency is expressed as follows:

$$X(\omega) \approx \sum_{j=0}^{2}\sum_{t=0}^{2} x_{jt}(\omega) P_{jt}(\xi_\eta, \xi_h), \tag{12}$$

where P_{jt} is the element of the vector P which is calculated by $P_h \otimes P_\eta$ and $\otimes$ is the Kronecker product.

The scattering efficiencies obtained by hybrid WFE are verified with a combination of the hybrid WFE and Monte Carlo Simulations (MCS). For MCS, 50 samples of each uncertain parameter are created therefore $50^2 = 2500$ simulations are performed for the verification. The mean and the ratio of standard deviation to the mean of the efficiencies for the uncertain joint considered are presented in figures 3-5, respectively. To observe the effect of the uncertain loss factor, the mean and the ratio of standard deviation to the mean of the sum of the efficiencies are presented in figures 6-7.

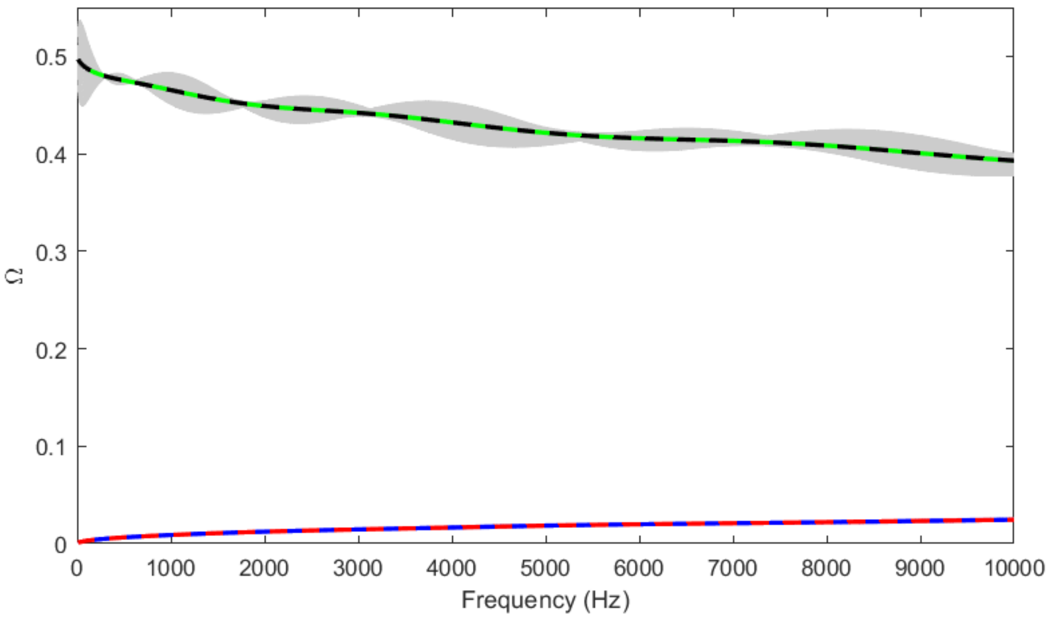

Figure 3. The mean of the uncertain reflection efficiency (grey: WFE-MCS samples, green: mean of Ω_{BB} obtained by WFE-MCS, black: mean of Ω_{BB} obtained by WFE-PCE, blue: mean of Ω_{BL} obtained by WFE-MCS and red: mean of Ω_{BL} obtained by WFE-PCE).

 doi:10.1088/1742-6596/2909/1/012031

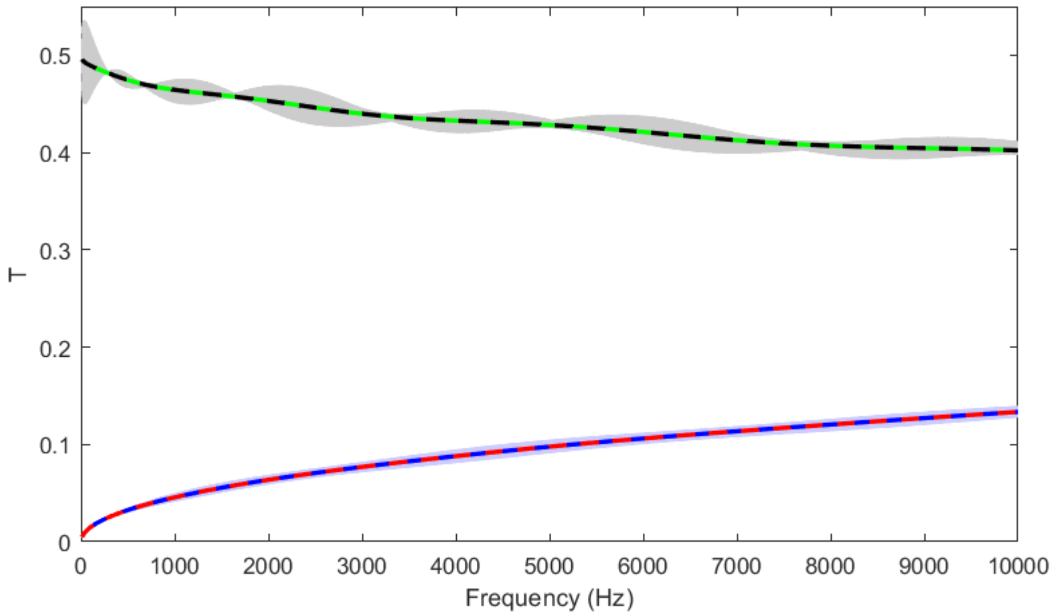

Figure 4. The mean of the uncertain transmission efficiency (grey: WFE-MCS samples, green: mean of T_{BB} obtained by WFE-MCS, black: mean of T_{BB} obtained by WFE-PCE, blue: mean of T_{BL} obtained by WFE-MCS and red: mean of T_{BL} obtained by WFE-PCE).

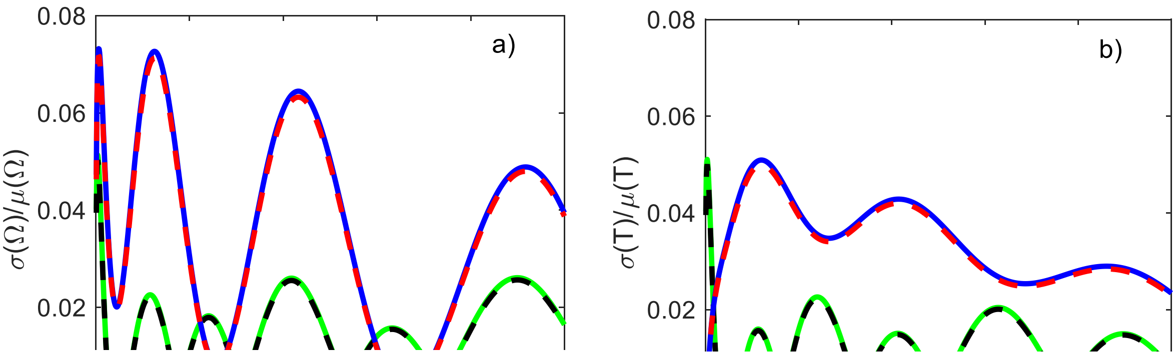

Figure 5. The ratio of standard deviation to the mean of the a) uncertain reflection and b) uncertain transmission efficiencies (green: results of bending to bending wave by WFE-MCS, black: results of bending to bending wave by WFE-PCE, blue: results of bending to longitudinal wave by WFE-MCS, red: results of bending to longitudinal wave by WFE-PCE).

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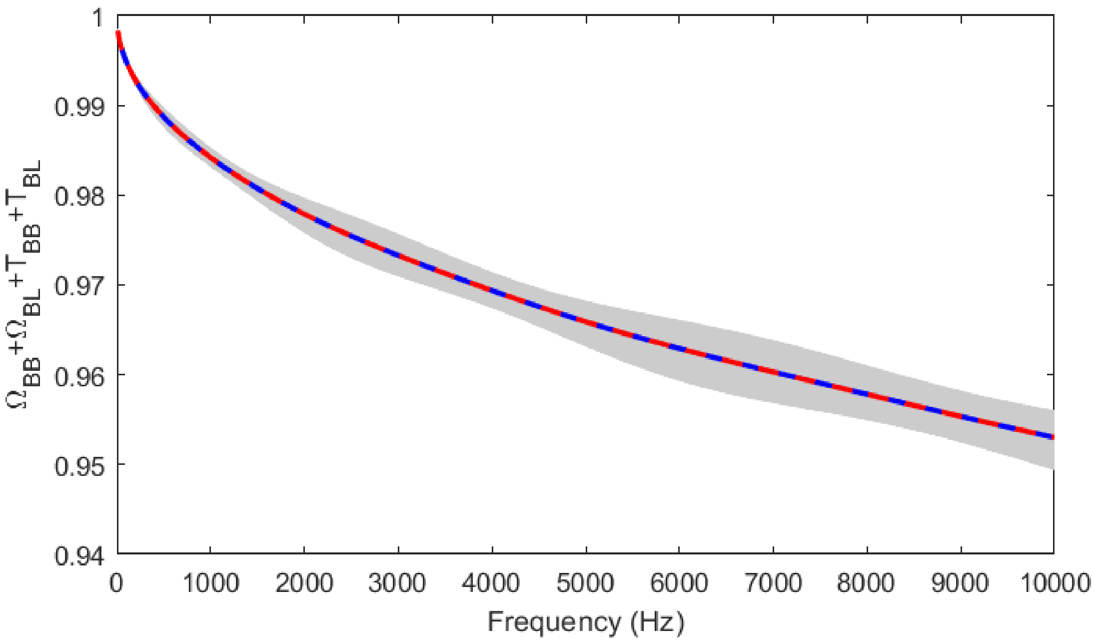

Figure 6. The mean of the total uncertain transmission and reflection efficiencies (grey: WFE-MCS samples, blue: mean obtained by WFE-MCS and red: mean obtained by WFE-PCE).

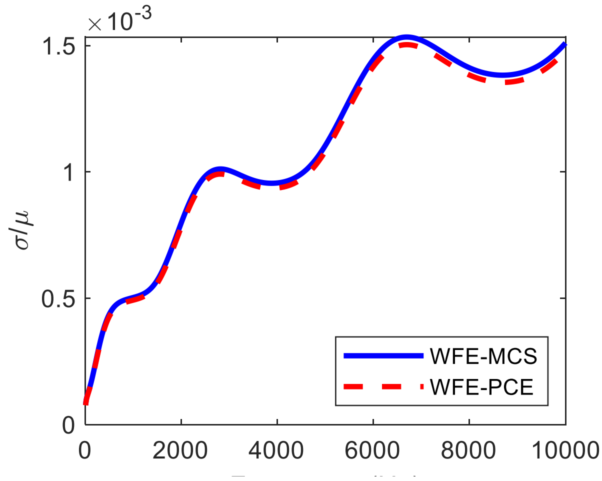

Figure 7. The ratio of standard deviation to the mean of the sum of the efficiencies.

It is clearly observed in figures 3-5 that, the results obtained by WFE-MCS and WFE-PCE are consistent with each other. The simulation time is 21 s for WFE-PCE whereas approximately 3400 s is required to obtain the results by WFE-MCS with the same computer. Therefore, one may state that WFE-PCE is an efficient and accurate alternative method for analysing the structures possessing joint uncertainty. Apart from the conclusions about the methodology proposed, one may infer the following results for the structure considered. The local uncertainty in the joint is more effective on the scattering properties between the bending to longitudinal wave transmission or reflection rather than efficiencies related to bending to bending waves. Also, the uncertainty in the resulting efficiencies is not always more effective with higher frequency. However, normalised standard deviations with respect to the mean does not have a steady behaviour as may be inferred from figures 3 and 4. The reason of this behaviour is having a step at the joint due to uncertainty of the joint. Th step at the joint affects the wave scattering properties and non-monotonic behaviour with respect to frequency is obtained. It should be noted that, this non-monotonic behaviour would not be observed if a shorter joint were analysed in the study. Selecting a long joint (the joint has a length of 5 cm in both perpendicular directions in the plane of the analysis) yields shifting the natural frequencies and corresponding modes of the joint to lower frequencies and hence they become observable in the frequency range considered. Also, for the joint considered with free-free boundary conditions, there are six nonzero elastic modes in the frequency range up to 10 kHz. Figures 6-7 show the effect of the uncertain loss factor of the joint. For a joint without damping, the summed total of the efficiencies must be equal to one. The results indicate that the uncertainty is gradually more effective on dissipating power with the increasing frequency as expected. However, there are also variations on the value of the normalised standard deviation.

5. Conclusions

In this study, the scattering properties of an uncertain right-angled joint connecting two isotropic beams was considered. The loss factor and the thickness of the joint are uncertain and assumed to be uniformly distributed and independent. The structure is analysed by hybrid Wave Finite Element (WFE) whereas the uncertainty is evaluated by two different methods for comparison, i.e., Monte Carlo Simulation (MCS) and Polynomial Chaos Expansion (PCE). The results show that the combined hybrid WFE and PCE (namely WFE-PCE) is an effective method compared to combined hybrid WFE and MCS (WFE-MCS) in terms of computational time and accuracy. On the other hand, it is observed that the local uncertainty on the joint is more effective on the scattering properties between the bending to longitudinal wave transmission or reflection rather than bending to bending wave scattering. It is also inferred that the normalised standard deviation of the efficiencies does not always increase with increasing frequency, whereas that for the total efficiency does increase. Therefore, the local uncertainty problems must be specifically analysed for the structures being considered and for the frequency range of interest.

Acknowledgements

This study is supported by "The Scientific and Technological Research Council of Turkey, TUBITAK" through the 2219-International Postdoctoral Research Fellowship Program for Turkish citizens.

References

1. Lyon RH, DeJong RG. Theory and application of statistical energy analysis. 2nd ed. Oxford: Butterworth-Heinemann; 1995. 277 p.
2. Mace BR, Duhamel D, Brennan MJ, Hinke L. Finite element prediction of wave motion in structural waveguides. J Acoust Soc Am [Internet]. 2005 May 28 [cited 2020 Jul 17];117(5):2835–43. Available from: http://asa.scitation.org/doi/10.1121/1.1887126
3. Duhamel D, Mace BR, Brennan MJ. Finite element analysis of the vibrations of waveguides and periodic structures. J Sound Vib. 2006 Jun 27;294(1–2):205–20.
4. Waki Y, Mace BR, Brennan MJ. Free and forced vibrations of a tyre using a wave/finite element approach. J Sound Vib. 2009 Jun 19;323(3–5):737–56.
5. Waki Y, Mace BR, Brennan MJ. Numerical issues concerning the wave and finite element method for free and forced vibrations of waveguides. J Sound Vib. 2009 Oct 23;327(1–2):92–108.
6. Renno JM, Mace BR. Calculation of reflection and transmission coefficients of joints using a hybrid finite element/wave and finite element approach. J Sound Vib. 2013 Apr 29;332(9):2149–64.
7. Renno JM, Mace BR. Vibration modelling of structural networks using a hybrid finite element/wave and finite element approach. Wave Motion. 2014 Jun 1;51(4):566–80.
8. Fan Y, Collet M, Ichchou M, Li L, Bareille O, Dimitrijevic Z. Energy flow prediction in built-up structures through a hybrid finite element/wave and finite element approach. Mech Syst Signal Process. 2016 Jan 1;66–67:137–58.
9. Masri EE, Ferguson NS, Waters T. Application of the wave finite element method to reinforced concrete structures with damage. In: Journal of Physics: Conference Series [Internet]. Institute of Physics Publishing; 2016 [cited 2021 Apr 16]. p. 012053. Available from: https://iopscience.iop.org/article/10.1088/1742-6596/744/1/012053

10. Mitrou G, Ferguson NS, Renno JM. Wave transmission through two-dimensional structures by the hybrid FE/WFE approach. Vol. 389, Journal of Sound and Vibration. Academic Press; 2017. p. 484–501.
11. Masri EE, Ferguson NS, Waters T. Wave propagation and scattering in reinforced concrete beams. J Acoust Soc Am [Internet]. 2019 Nov 12 [cited 2021 Mar 2];146(5):3283–94. Available from: http://asa.scitation.org/doi/10.1121/1.5131644
12. Aimakov N, Tanner G, Chronopoulos D. A wave finite element approach for modelling wave transmission through laminated plate junctions. Sci Rep [Internet]. 2022 Feb 3 [cited 2022 Feb 7];12(1):1–15. Available from: https://www.nature.com/articles/s41598-022-05685-y
13. Fabro AT, Ferguson NS, Mace BR. Wavenumber and natural frequency statistics of waveguides with spatially correlated material variability from finite element analysis. In: 18th International Congress on Sound and Vibration (14/07/11). 2011.
14. Fabro AT, Ferguson NS, Jain T, Halkyard R, Mace BR. Wave propagation in one-dimensional waveguides with slowly varying random spatially correlated variability. J Sound Vib. 2015 May 12;343:20–48.
15. Fabro AT, Ferguson NS, Mace BR. Wave propagation in slowly varying waveguides using a finite element approach. J Sound Vib. 2019 Mar 3;442:308–29.
16. Ben Souf MA, Bareille O, Ichchou M, Haddar M. The wave finite element method for uncertain systems with model uncertainty. Proc Inst Mech Eng C J Mech Eng Sci [Internet]. 2016 Apr 30 [cited 2021 Apr 16];230(6):974–85. Available from: http://journals.sagepub.com/doi/10.1177/0954406215617197
17. Henneberg J, Gomez Nieto JS, Sepahvand K, Gerlach A, Cebulla H, Marburg S. Periodically arranged acoustic metamaterial in industrial applications: The need for uncertainty quantification. Applied Acoustics. 2020 Jan 1;157:107026.
18. Kara M, Ferguson NS. A hybrid methodology for the frequency response function variability due to joint uncertainty. Mech Syst Signal Process. 2023;202.
19. Ghanem RG, Spanos PD. Stochastic finite elements: A spectral approach. New York: Dover Publications; 2003.
20. Sepahvand K, Marburg S, Hardtke HJ. Uncertainty quantification in stochastic systems using polynomial chaos expansion. Int J Appl Mech [Internet]. 2010 [cited 2017 May 16];02(02):305–53. Available from: http://www.worldscientific.com/doi/abs/10.1142/S1758825110000524
21. Choi SK, Grandhi R v., Canfield RA, Pettit CL. Polynomial chaos expansion with latin hypercube sampling for estimating response variability. AIAA Journal. 2004 May 2;42(6):1191–8. Available from: https://arc.aiaa.org/doi/abs/10.2514/1.2220
22. Cremer L, Heckl M, Petersson BAT. Structure-borne sound: Structural vibrations and sound radiation at audio frequencies. Structure-Borne Sound: Structural Vibrations and Sound Radiation at Audio Frequencies. 2005.
23. Kara M, Ferguson NS. A two-stage Polynomial Chaos Expansion application for bound estimation of uncertain FRFs. J Sound Vib. 2023;566.