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Technical Brief

Applying Multiobjective Cost and Weight Optimization to the Initial Design of Turbine Disks

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Aerospace design optimization typically explores the effects of structural performance and aerodynamics on the geometry of a component. This paper presents a methodology to incorporate manufacturing cost and fatigue life models within an integrated system to simultaneously trade off the conflicting objectives of minimum weight and manufacturing cost while satisfying constraints placed by structural performance and fatigue. A case study involving the design of a high pressure turbine disk from an aircraft engine is presented. Manufacturing cost and fatigue life models are developed in DECISIONPRO™, a generic modeling tool, whereas finite element analysis is carried out in the Rolls-Royce PLC proprietary solver SC03. A multiobjective optimization approach based on the nondominated sorting genetic algorithm (NSGA) is used to evaluate the Pareto front for minimum cost and volume designs. A sequential workflow of the different models embedded within a scripting environment developed in MATLAB™ is used for automating the entire process.

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1 Concurrent Design and Manufacturing Cost

Although manufacturing cost is a vital factor in product development, many organizations have a separate costing department with few cost estimation tools integrated in the design process, making it extremely difficult for designers to estimate the cost effects of changes made by them [1]. In aerospace component design, multidisciplinary design optimization (MDO) is widely used to shorten and improve the design process [2]. Though MDO is a relatively mature field, optimization with cost as the objective function has been rarely used in the past due to the following reasons:

- lack of detailed information for determining accurately the cost of manufacture at the design stage
- difficulty in modeling manufacturing cost in terms of geometry parameters and design variables
- minimizing mass or weight considered analogous to minimizing cost [3].

Weight used as a surrogate for cost in the absence of a detailed cost model can be potentially misleading as subtractive material removal increases processing cost and scrap and not all low volume designs are low in manufacturing cost. Parametric cost models, also known as cost estimating relationships (CERs), allow optimizers to operate with a highly abstract design definition and are known to be accurate with well defined classes of components. However, this approach is primarily based on statistical assumptions between component properties and cost [4]. These assumptions typically require data that are in limited supply in a low volume aerospace manufacturing context [1,5].

Here, we develop a workflow of computer aided design (CAD), finite element (FE) analysis, cost estimation, and fatigue life prediction models within a scripting environment to carry out the shape optimization of a high pressure turbine (HPT) disk from a civil aircraft engine. Cost estimation is carried out using a detailed manufacturing process based model, which is more accurate compared to previously used methods [1,6]. The objective in this work is twofold, first, to demonstrate an MDO process driven by cost alongside structural performance in early design; second, to show that there is a definite difference between minimizing weight and minimizing cost in this case and to investigate the tradeoffs existing between low weight and low cost designs.

2 Design Problem

2.1 High Pressure Turbine Disk.

A schematic two dimensional axisymmetric section of a HPT disk is shown in Fig. 1. The rationale behind the basic shape of the disk is as follows; mass needs to be placed as near as possible to the axis of rotation for a low stress, weight efficient design to survive failure/burst or release of the blades at high rotational speeds. The disk also has to be wider at the bore to achieve the requisite strength needed to accommodate a large diameter shaft within the bore. Turbine disks are relatively heavy and function at high rotary speeds, and failure would produce uncontained high energy debris and catastrophic consequences. Therefore, manufacturers have to fulfill various regulatory requirements to control their integrity and life [7]. In a preliminary design, one of the requirements involves designing to withstand 120% of the highest shaft speed without burst. This is typically converted to four limiting stress values at a given tem-

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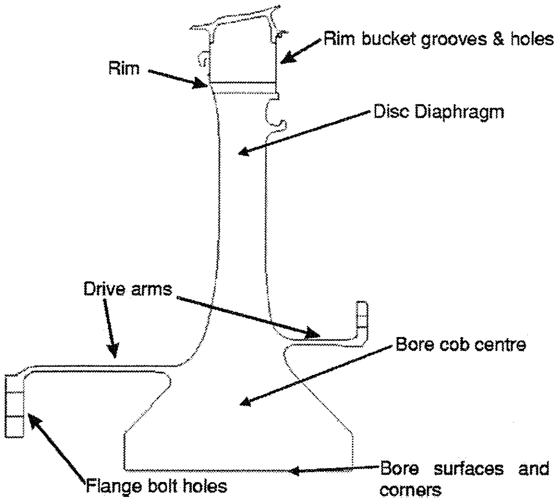


Fig. 1 An axisymmetric section of the HPT disk

perature, speed, and rim load [7]. In this study, the problem formulated is to optimize the initial shape by minimizing the volume of the disk profile (for low weight) while satisfying the four stress constraints to ensure overspeed integrity.

Due to restrictions imposed by suppliers and of ultrasonic inspection techniques, the final disk profile is machined from a rectilinear initial forging. Minimizing only volume/weight may lead to excessive machining to obtain the final shape from the initial forging profile, leading to increased material, machining, and scrap costs. Therefore, we need to utilize a cost model that is sufficiently detailed to estimate accurately the increase in cost due to material removal. Moreover, a large percentage of the cost is due to surface condition, finish, and tolerance requirements. The cost model used in this study possesses the requisite detail to allow modifications made to surface finish, and tolerance requirements to be reflected in cost of manufacture. However, these specifications cannot be modified on an ad hoc basis as they have a huge influence on fatigue life, which acts as a fifth constraint in the design process and must allow for the effects of varying surface finish and residual compressive stresses induced by shot peening.

The above design problem may be reformulated as the following multiobjective optimization (MO) problem.

Minimize

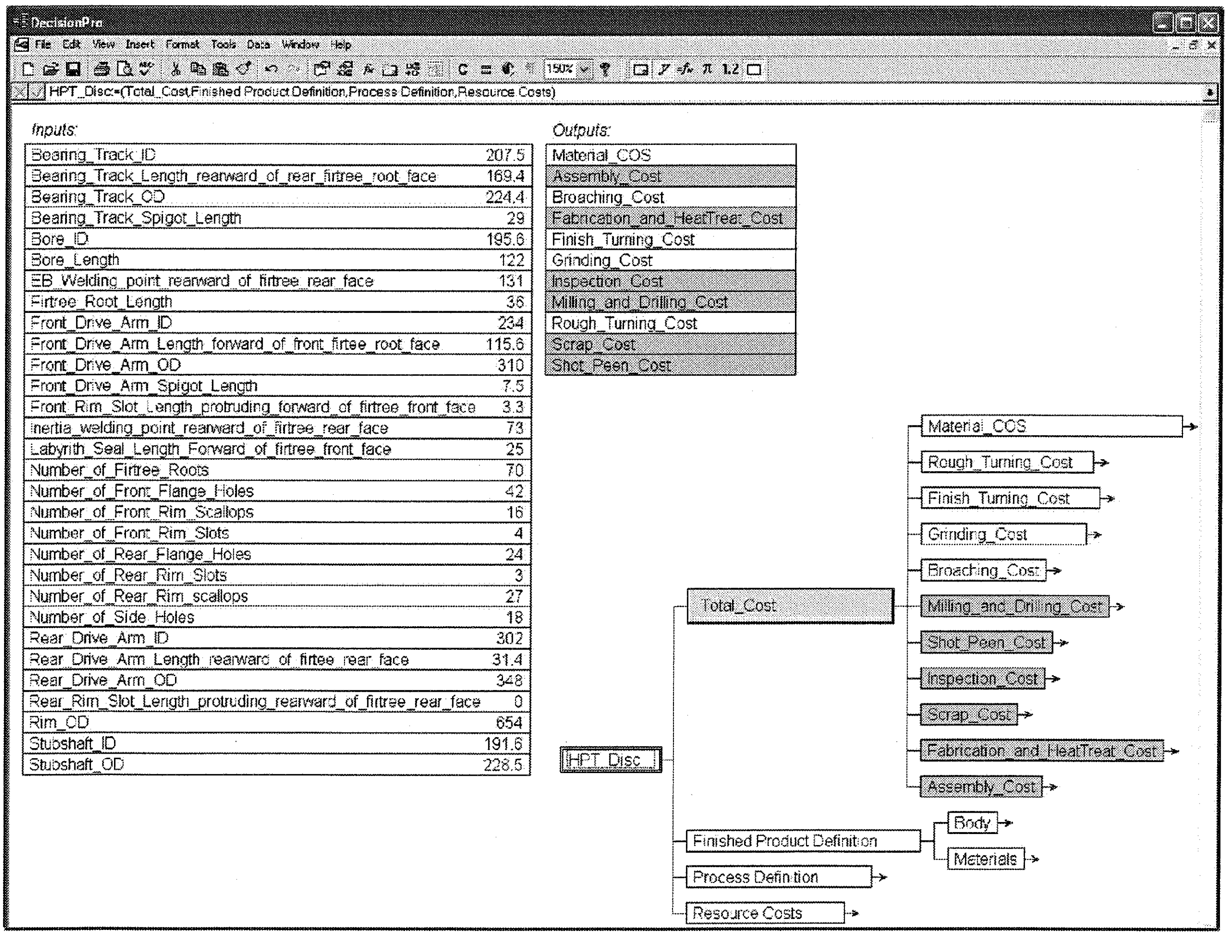
$$f(V,C_M) \tag{1}$$

subject to

$$\sigma_i \leqslant \sigma_{i_{\max}} \tag{2}$$

$$N \geqslant N_{\min} \tag{3}$$

with



HPT_Disc=(Total_Cost,Finished Product Definition,Process Definition,Resource Costs)

Inputs:

Bearing_Track_ID	207.5
Bearing_Track_Length_rearward_of_rear_firtree_root_face	169.4
Bearing_Track_OD	224.4
Bearing_Track_Spigot_Length	29
Bore_ID	195.6
Bore_Length	122
EB_Welding_point_rearward_of_firtree_rear_face	131
Firtree_Root_Length	36
Front_Drive_Arm_ID	234
Front_Drive_Arm_Length_forward_of_front_firtee_root_face	115.6
Front_Drive_Arm_OD	310
Front_Drive_Arm_Spigot_Length	7.5
Front_Rim_Slot_Length_protruding_forward_of_firtree_front_face	3.3
inertia_welding_point_rearward_of_firtree_rear_face	73
Labyrith_Seal_Length_Forward_of_firtree_front_face	25
Number_of_Firtree_Roots	70
Number_of_Front_Flange_Holes	42
Number_of_Front_Rim_Scallops	16
Number_of_Front_Rim_Slots	4
Number_of_Rear_Flange_Holes	24
Number_of_Rear_Rim_Slots	3
Number_of_Rear_Rim_scallops	27
Number_of_Side_Holes	18
Rear_Drive_Arm_ID	302
Rear_Drive_Arm_Length_rearward_of_firtee_rear_face	31.4
Rear_Drive_Arm_OD	348
Rear_Rim_Slot_Length_protruding_rearward_of_firtree_rear_face	0
Rim_OD	654
Stubshaft_ID	191.6
Stubshaft_OD	228.5

Outputs:

- Material_COS
- Assembly_Cost
- Broaching_Cost
- Fabrication_and_HeatTreat_Cost
- Finish_Turning_Cost
- Grinding_Cost
- Inspection_Cost
- Milling_and_Drilling_Cost
- Rough_Turning_Cost
- Scrap_Cost
- Shot_Peen_Cost

Fig. 2 HPT disk cost model in DECISIONPRO

Table 1 List of geometry variables used in optimization

No.	Design variables	Description
1	Base–height	Height of the bore cob
2	Bore–length	Axial length of the cob
3	FDA–OD	Front drive arm outer diameter
4	RDA–ID	Rear drive arm inner diameter
5	FSDI	Front diaphragm inclination
6	RSDI	Rear diaphragm inclination
7	RDA diap width	Width of the diaphragm base at the blend with rear drive arm
8	FDA–Diap–Width	Width of the diaphragm base at the blend with front drive arm

$$X_{j_{\min}} \leqslant X_j \leqslant X_{j_{\max}} \tag{4}$$

where $i=1,\ldots,4$ and $j=1,\ldots,m$. In this problem, $f(V,C_M)$ represents the two objectives: volume (V), which is a substitute for weight, and manufacturing cost (C_M). The four limiting stress constraints are represented by σ_i and N is the fatigue life prediction in the number of cycles for a candidate design. X_j is the value of the jth design variable and Eq. (4) represents the bound constraints on m design variables.

2.2 Multiobjective Optimization. Multiobjective design problems lead to a solution set of designs known as the Pareto set or front [8]. Here, the objectives are low volume tensioned against manufacturing cost incurred for material removal. Approaches for generating Pareto fronts for MO problems may be classified into various types. The simplest though computationally expensive approach is the weighted sum (WS) method, which uses a weighting parameter to assign relative importance to each of the objectives and combines all of them into a single function. Single objective optimization techniques are then utilized to minimize this aggregate function, leading to a single point on the front. The weightings are then modified, and repeated searches reveal further points on the Pareto front. In recent years, several approaches have been proposed to solve multiobjective problems using evolutionary algorithms (MOEAs) [9]. Since many MOEAs work with a population of solutions in each iteration, it is easy to apply them to find multiple solutions at each stage, while iteratively moving toward the true Pareto-optimal region. One of the best known MOEA methods is the nondominated sorting genetic algorithm (NSGA-II) method introduced by Deb et al. [10]. In that approach, a GA is used to carry out the search, while the goal function used to drive the process is based on the relative ranking and spacing of the designs in the current solution set rather than on the combined weighted performance of the objectives, thus reducing the need for subjective weight assignment [11]. An improved version of the NSGA-II called OPTIONS-NSGA2 [12], which combines the GA available within the OPTIONS[1] design exploration system with ranking and crowded comparison functions, is employed in this study to seek the Pareto front.

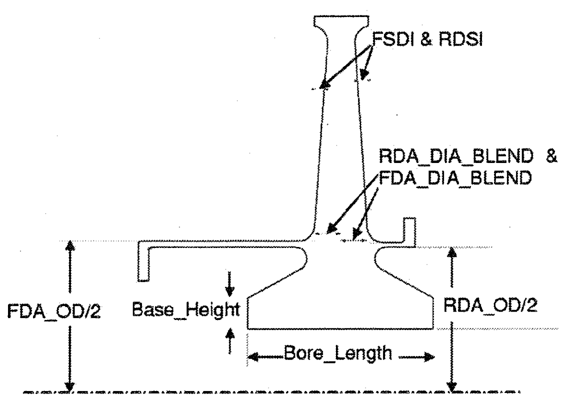


Fig. 3 2D axisymmetric section of the disk showing design variables used in optimization

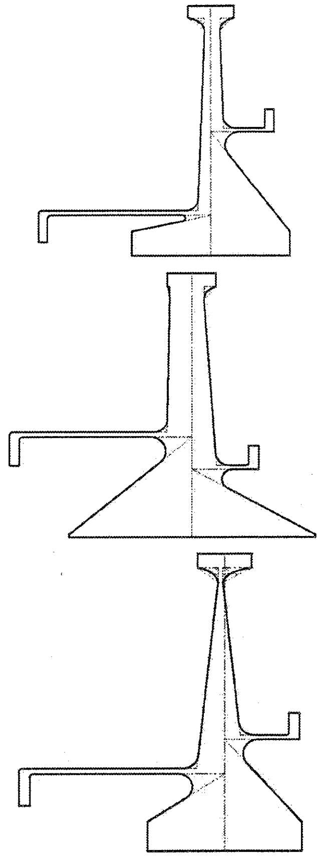


Fig. 4 Examples of parametrically modified geometries

Table 2 Limiting stress values

Stresses	MPa
Allowable area weighted mean hoop stress (AWMHS)	762
Maximum diaphragm radial stress	1200
Maximum rim hoop stress	700
Maximum bore hoop stress	1007
Overall temperature	603 °C

3 Profile Optimization of the High Pressure Turbine Disk

Deployment of optimization methods is possible only after the creation of a design automation system that allows alternative designs to be created and analyzed against the formulated problem. This requires a direct link between the various software and analysis codes, with the system inputs and outputs controlled automatically by the optimizer. The five elements essential to the MDO process described here are (1) a manufacturing process based cost model, (2) a parametrized solid model of the HPT disk, (3) a suitable FE model for carrying out a structural analysis of the candidate disk profile, (4) a fatigue life prediction model that allows for the effects of changing surface finish and shot peening,

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[1] http://www.soton.ac.uk/~ajk/options/welcome.html

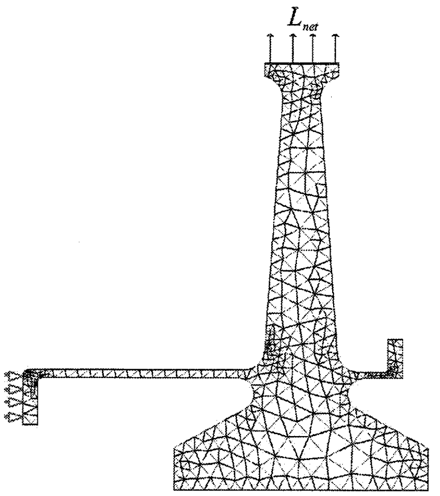

Fig. 5 Typical FE mesh in SC03, showing adaptive refinement

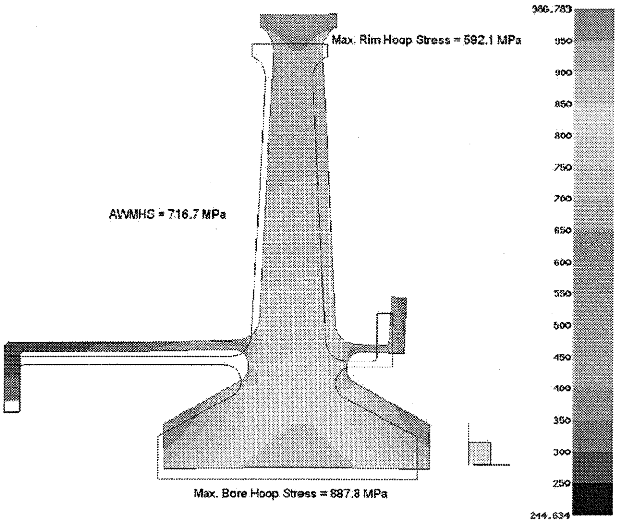

Fig. 6 Hoop stress contours for the initial design in SC03

and (5) a script for automating the data flow and invoking various models. The following sections elaborate on the above five elements.

3.1 Cost Model Development. Cost estimation is carried out on an individual process/operation basis by using dimensions and solid model properties from the CAD model [1,6,13]. This allows for a detailed estimate of incremental change in cost following a geometry modification. The cost calculations for every process are embedded in an object-oriented environment and treated as black-box models reused by the designer interactively or in batch by the invoking script. A component cost model is formed by instantiating various manufacturing process cost objects needed to produce a particular component. An integral part of this cost model is DECISIONPRO™,[2] a generic modeling software which enables object-oriented encapsulation of cost data without writing a bespoke code [1,13]. The first page listing all the inputs and outputs for the HPT disk cost model is shown in Fig. 2. The list of inputs contains all the design parameters that influence cost. The designer may choose to modify only a few of them during optimization and leave the rest with default values carried forward in

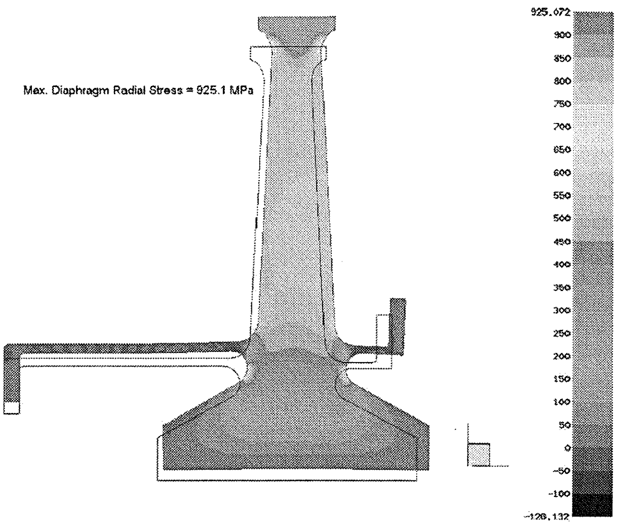

Fig. 7 Radial stress contours for the initial design in SC03

each design. By way of example, consider the "Rough–Turning–Cost" element in the figure. This element uses the relationship

$$\text{Rough–Turning–Cost} = \text{volume\_removed}$$
$$\times \text{ machine–cost–rate/removal–rate}$$

where the volume removed is calculated directly from the CAD model and the machine cost and removal rates come from the machine shop data embedded in the DECISIONPRO model. All the other items in the model are built up in a similar hierarchical manner, using appropriate machining rates and costing figures.

The total manufacturing cost is expressed as

$$C_M = (R_o + R_m)\left(\frac{T_{\text{su}}}{Q} + T_{\text{ot}} + T_{\text{no}}\right) + C_{\text{ma}} + C_o + C_s + C_c \qquad (5)$$

where C_M is the total manufacturing cost, C_{ma} is the cost of material/fabrication cost, C_o is the overhead cost, C_s is the scrap/rework cost, and C_c is the cost of consumables, such as coolant, tool wear and change, and fixture cost. R_o and R_m are the unit cost rates of the operator and machine, respectively, Q is the batch quantity, T_{su} is the setup time for tooling and machines over the entire batch, T_{ot} is the operation time required to carry out the various processes to achieve the final shape, and T_{no} is the non-operation time involved in loading, unloading, and operator allowances. T_{ot} is the total process time for all operations in the manufacturing sequence and is expressed as

$$T_{\text{ot}} = \sum_{i=1}^{n} T_{f_i}$$

where T_{f_i} = manufacturing time required per process i.

The manufacturing time required per process T_f depends on the following: (a) the material removal rate (MRR) for roughing operations and (b) the surface generation rate for finishing operations. The ratio of the volume of material removed to the MRR gives the time required for roughing, and similarly the ratio of the finished surface area to the surface generation rate provides the

Table 3 Material constants used in life prediction

Ultimate tensile strength (σ_{TS}) in MPa	1420
Young's modulus (E) in MPa	208,500
Slope of the S–N curve (b) in MPa	–0.06
Stress life curve intercept (σ_f') in MPa	1573

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[2] http://www.vanguardsw.com/

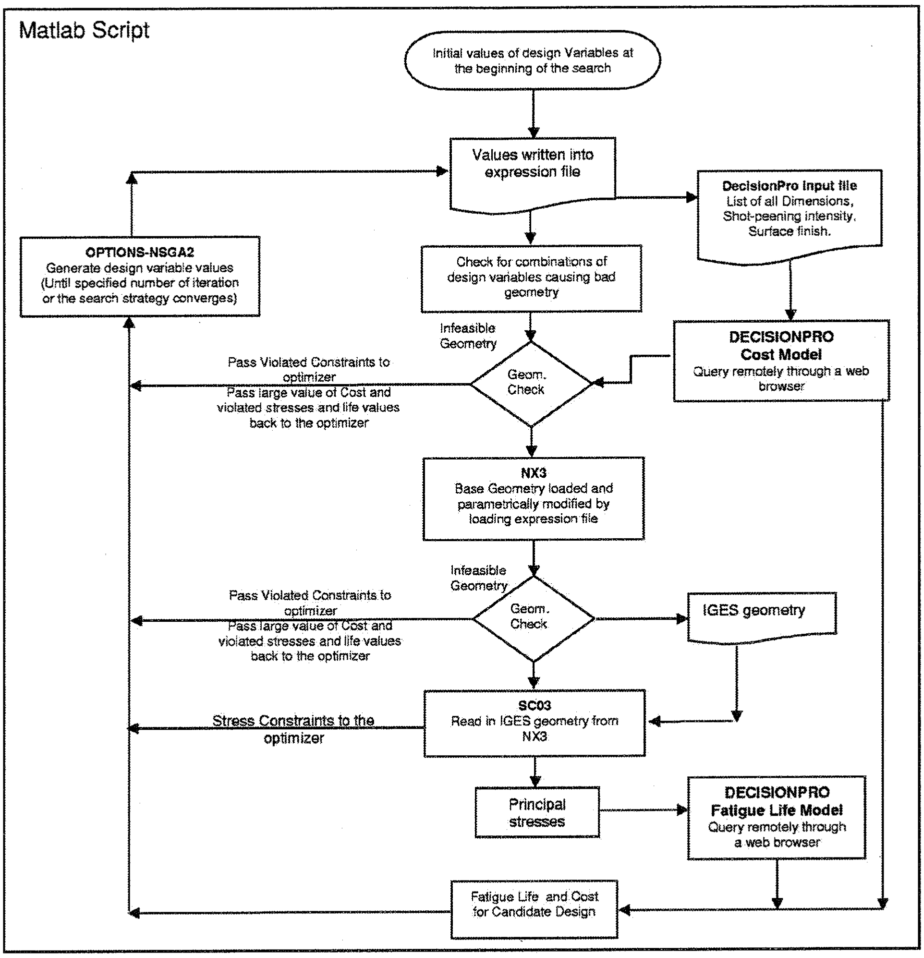

Fig. 8 The design process sequence

process time for finishing operations. The MRR, surface generation rate, and other parameters, such as scrap rate, overhead, operator, and machine per hour rates, are adapted from values obtained from proprietary machining databases.

3.2 Computer Aided Design Based Geometry Parametrization. The basic purpose of geometry parametrization is to provide powerful, quick, and automated manipulation of product geometry and definitions in response to analysis [14]. Most aerospace organizations use a CAD system over other means of geometry parametrization as it is conducive for modeling complex geometries and a common model can be developed, parametrized, and used as a geometry repository for the entire product life cycle across the organization.

In this study, we use Unigraphics NX3®[3] to parametrize and externally control the geometry through an optimizer. The geometry was parametrized, taking into account the rationale behind the basic shape of the disk outlined in Sec. 2 while maintaining a reasonable number of design variables within the problem. We do not modify the rim and bore diameter of the disk as these parameters are fixed by an aerothermal analysis of the gas path through the engine. Checks are also programmatically included to recognize and discard infeasible geometries, which are directly proportional to increasing design detail [15]. In total, eight geometry parameters are derived for controlling the two dimensional cross section of the disk, as listed in Table 1 and shown in Fig. 3.

The other dimensions of the disk are evolved from these eight basic variables using geometric relationships in the sketch. Constant dimensional constraints are also used but not included in the optimization process, for example, constant rim to bore height. Finally, conditional statements are used to ensure that conflicting values are not passed to the sketch. Figure 4 shows three extreme examples of resulting shapes as the position of the drive arms and the inclination of the diaphragm are parametrically modified. When a candidate design is generated and converted to the IGES format, it is transferred by the script running the workflow to the FE solver for a structural analysis.

3.3 Structural Analysis. An axisymmetric two dimensional FE analysis is carried out on the disk profile to test candidate geometries for overspeed integrity. Engine regulatory rules state that "the disk should withstand 120% of engine red line speed (RLS) without burst or 105% of calculated terminal speed (TS) reached after shaft failure." This is achieved by designing for four limiting stress conditions at RLS and an overall area weighted temperature. The four stresses and temperature used in the analysis are given in Table 2.

The boundary conditions for evaluating the above stresses are applied as follows: A total load representing the blade mass is applied to the rim along with axial displacement constrained on the front drive arm face. The total load for the blade, disk finger

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[3] http://www.ugs.com/products/nx/

lock plate, and damper is 266 kN per blade at 1228 rad/s.[4] A net load (L_{net}) for a two dimensional asymmetric approximation is calculated by incorporating corrections for speed, number of blades, and rim outer diameter of the disk. A structural analysis is performed here in a proprietary Rolls-Royce FE solver known as SC03. The disk is remeshed at each design iteration with a minimum of 750 triangular elements, followed by an adaptive refinement in areas of high stress to allow for geometry changes. Figures 5, 6, and 7 show, respectively, the mesh, boundary conditions, and hoop and radial stresses induced in one of the initial geometries. The induced stress values are saved in a result file and sent back to the optimizer as constraint values. Principal stresses from the FE model are also passed on through the workflow script to the fatigue life model.

3.4 Fatigue Life Prediction. Disk lifing methodologies fundamentally affect safe component life, material usage, and maintenance procedures; therefore, optimizing the disk profile is valid only after fatigue effects on the design are considered. At an early stage in design where feasible concepts are investigated, a total-life model, though conservative in nature, provides a reasonable estimate of fatigue life [16]. The basis of the method used here is the material's *S-N* curve, which is a plot of the stress amplitude against the number of cycles to failure at that amplitude. Most *S-N* curves can be redrawn on a log-log scale to fit a linear relationship known in the literature as Basquin's equation given below:

$$\frac{(\sigma_{\max}-\sigma_{\min})}{2} = \frac{\Delta\sigma}{2} = \sigma_a = \sigma'_f(2N_f)^b$$

where σ'_f is the fatigue strength coefficient and b is the fatigue strength exponent, both being material specific constants. $\Delta\sigma/2 = \sigma_a$ is the constant fully reversible stress amplitude, $\sigma_{\max}$ and $\sigma_{\min}$ are the maximum and minimum induced stresses of the fatigue loading cycle, and N_f is the number of cycles to failure [16]. The modified Goodman relation [16] is used to incorporate the mean stress effects as fatigue loading is not fully reversible in disk design. In this study, the life model uses the principal stresses induced in the component from the disk FE model to calculate equivalent stress amplitudes using the von Mises criterion. $\sigma_{\max}$ are the principal stresses in three directions and $\sigma_{\min}$ is equal to zero. The compressive residual stress (σ_{cr}) induced by shot peening and the surface finish modifying factor (K_{sf}) are used to accommodate changing peening intensity and surface finish. DECISIONPRO is used for modeling the life prediction calculations and is queried in the same manner as the cost model. The material constants used for this study are tabulated in Table 3. It is recognized that actual disk life depend on numerous details of a highly localized nature. Such details cannot be known until a finalized design has been achieved, and the intention in this study is to demonstrate a methodology for including various design models within a common optimization strategy.

3.5 Process Sequence. The sequential automation of the various models used in this study is achieved through a scripting environment developed in MATLAB™. The optimizer invokes this script for each candidate design to compare the results obtained against the objective and constraint functions. Figure 8 shows the framework and a detailed flowchart for this process. Table 4 shows the list of design variables, objectives, and constraints for the present problem.

Table 4 List of optimization parameters

Design variables	1	Base–height	Height of the bore cob
	2	Bore–length	Axial length of the cob
	3	FDA–OD	Front drive arm outer diameter
	4	RDA–ID	Rear drive arm inner diameter
	5	FSDI	Front diaphragm inclination
	6	RSDI	Rear diaphragm inclination
	7	RDA–diap–width	Width of the diaphragm base at the blend with rear drive arm
	8	FDA–diap–width	Width of the diaphragm base at the blend with front drive arm
	9	Surface finish (μm) $1<s<2$ (1.1, 1.2, ..., 2.0)	
	10	Shot-peening intensity (MPa) $\sigma_{\text{cr}}=(0,100,200,300)$	Compressive residual stress imparted by shot peening
Objectives	Cost Volume		
Constraints	AWMHS Maximum diaphragm radial stress Maximum rim hoop stress Maximum bore hoop stress Fatigue life		

The continuous geometry variables are used for geometry generation in NX3. The geometry variables are also used in the cost model to estimate the size of the initial forging and manufacturing cost of the disk to its final size. The surface finish is used to set both the finished area generation speed in the disk turning operation of the cost model and the surface finish correction factor (K_{sf}) in the fatigue life prediction model. The shot-peening intensity is used to calculate the net stress amplitude for fatigue loading and to set the cost value for the shot-peening operation.

4 Optimization Results and Discussion

For the optimization problem formulated in Sec. 2, the Pareto front resulting from optimization, along with the entire search space, is shown in Fig. 9. These designs are further divided into feasible or nonfeasible depending on whether they violate any of the given constraints. A clear tradeoff can be seen between manufacturing cost and volume. The design variable, objective, and constraint values are tabulated in Table 5 for three designs from the Pareto front marked in Fig. 9. In this plot, the cost and volume

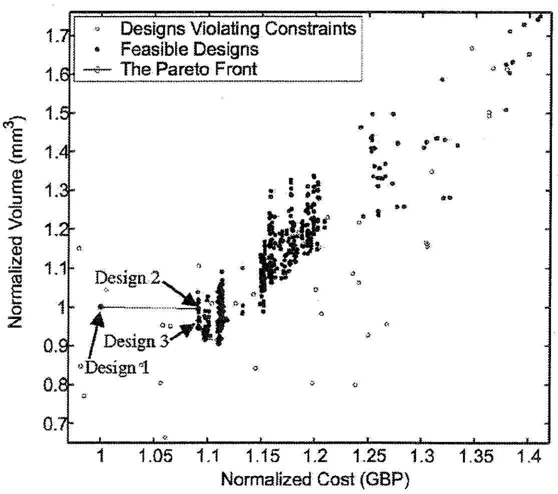

Fig. 9 The Pareto front

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[4] Loads and speeds are representative of the values used in the design of the disk for modern turbofan engines.

Table 5 A comparison of five candidate designs on the Pareto front

Design variables	Lower Bound	Upper Bound	Designs on the Pareto front (see Fig. 9) 1	2	3
Base–height (mm)	2	27	19.9	2.9	6.8
Bore–length (mm)	50	200	122.3	126.2	126.2
FDA–OD (mm)	205	500	309	353.3	352.7
RDA–ID (mm)	205	500	301	352.6	352.6
FSDI (deg)	1	6	18	17.0	17.2
RSDI (deg)	1	6	18	14.7	14.7
RDA–diap–Width (mm)	8	20	3	2.5	3.4
FDA–diap–Width (mm)	8	20	3	2.2	4.7
Surface finish (μm)	1	2	1.6	1.7	1.6
Shot-peening intensity (MPa)	0	300	100	100.0	100.0
Objective function					
Total cost (GBP)			1	1.091	1.091
Volume (mm^3)			1	0.995	0.962
Constraints	Limit				
Allowable AWMHS	762 MPa		717	740	721
Maximum bore hoop stress	1007 MPa		888	925	880
Maximum diaphragm radial stress	1200 MPa		872	993	1062
Maximum rim hoop stress	700 MPa		592	666	678
Fatigue life	20,000 cycles		28,926	34,391	39,457

values are normalized using Design 1 as the base design. The Pareto front is achieved after evaluating 50 generations of the GA with a population size of 50 designs per generation, requiring a total computing time (in serial mode) of 66 h.

A manufacturing cost difference of approximately 11% is seen between the two extreme points on the front. This proves the hypothesis stated at the beginning of this paper that using a process based cost model provides substantially different results as compared to a single objective weight/volume minimization problem. It can be seen from Table 5 that Designs 1 and 2 have similar surface condition requirements. However, the total manufacturing cost of Design 2 is much higher than that for Design 1. This is solely due to the added machining cost incurred in generating the profile with a smaller base and a high position of the drive arms along the disk diaphragm. On the other hand, a comparison of Designs 2 and 3 shows a large difference in volume but a very small difference in cost. Volume is added primarily in the diaphragm area in Design 2 due to the small inclination (2.2 deg and 2.4 deg). However, subtractive machining has relatively smaller influence here on cost as compared to the bore region. This is another indication of reduction in volume not being reflected in reduced cost at all. Figure 10 shows the disk profiles generated in Designs 1, 2, and 3.

5 Conclusions

The methodology described in this paper demonstrates the application of a multiobjective search algorithm to search for a low cost, better performing design by modeling complex relationships between geometry, induced stresses, manufacturing cost, and fatigue life. Disparate analysis models typically used in an aerospace concept design are integrated within a single system to optimize the geometry of a component. To achieve this, a substantial effort has been invested in developing highly flexible two dimensional parametrization schemes for manipulating geometry and for automating the process workflow. We introduce a manufacturing process based cost model to provide incremental cost estimates as a direct function of modifying the CAD geometry. The results show significant differences between volume/mass minimization and cost minimization, and a definite tradeoff exists between them.

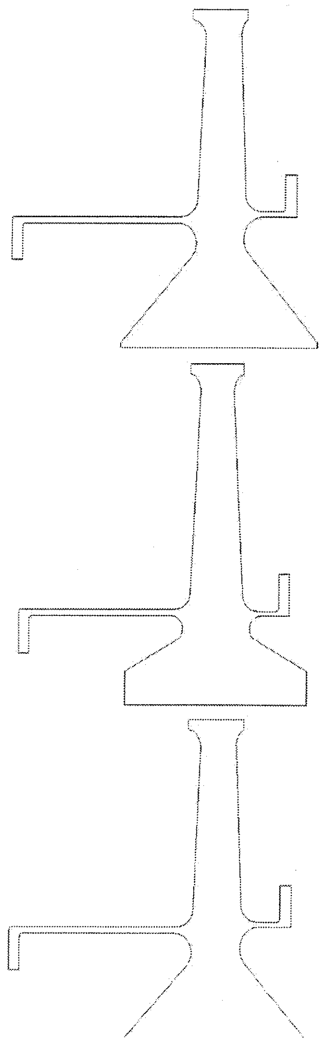

Fig. 10 Designs 1, 2, and 3

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