

Pre-Chirp-Domain Index Modulation for Full-Diversity Affine Frequency Division Multiplexing towards 6G

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Abstract—As a superior multicarrier technique utilizing chirp signals for high-mobility communications, affine frequency division multiplexing (AFDM) is envisioned to be a promising candidate for sixth-generation (6G) wireless networks. AFDM is based on the discrete affine Fourier transform (DAFT) with two adjustable parameters of the chirp signals, termed the pre-chirp and post-chirp parameters, respectively. Whilst the post-chirp parameter complies with stringent constraints to combat the time-frequency doubly selective channel fading, we show that the pre-chirp counterpart can be flexibly manipulated for an additional degree of freedom. Therefore, this paper proposes a novel AFDM scheme with the pre-chirp index modulation (PIM) philosophy (AFDM-PIM), which can implicitly convey extra information bits through dynamic pre-chirp parameter assignment, thus enhancing both spectral and energy efficiency. Specifically, we first demonstrate that the subcarrier orthogonality is still maintained by applying distinct pre-chirp parameters to various subcarriers in the AFDM modulation process. Inspired by this property, we allow each AFDM subcarrier to carry a unique pre-chirp signal according to the incoming bits. By such an arrangement, extra bits can be embedded into the index patterns of pre-chirp parameter assignment without additional energy consumption. We derive asymptotically tight upper bounds on the average bit error probability (BEP) of the proposed schemes with the maximum-likelihood detection, and validate that the proposed AFDM-PIM can achieve full diversity under doubly dispersive channels. Based on the derived result, we further propose an optimal pre-chirp alphabet design to enhance the

bit error rate (BER) performance via intelligent optimization algorithms. Simulation results demonstrate that the proposed AFDM-PIM outperforms the classical benchmarks.

Index Terms—Index modulation (IM), affine frequency division multiplexing (AFDM), discrete affine Fourier transform (DAFT), doubly dispersive channel.

I. INTRODUCTION

THE beyond fifth-generation (B5G) and sixth-generation (6G) wireless networks are envisioned to deliver ultra-reliable, high data rate, and low-latency communications for high-speed mobile scenarios, including low-earth-orbit (LEO) satellite, high-mobility railway, unmanned aerial vehicles (UAV) and vehicle-to-vehicle (V2V) communications [2–8]. These scenarios inevitably suffer from severe Doppler shifts, which can cause time-frequency doubly selective fading (i.e., doubly dispersive channel) by involving the multi-path effects[9]. This makes the existing modulation formats, like the main-stream orthogonal frequency division multiplexing (OFDM) in 4G/5G standards, no longer suitable for next-generation networks [10]. Consequently, it is crucial to develop new waveforms for next-generation communication networks to adapt to the doubly selective channel.

To date, several novel modulation techniques have been designed to combat the time-frequency doubly selective fading, such as orthogonal time-frequency space (OTFS) [11–13] and orthogonal chirp-division multiplexing (OCDM) [14–16]. OTFS modulates information in the delay-Doppler (DD) domain using the inverse symplectic finite Fourier transform (ISFFT), which enables the transmission symbols to be multiplexed across the entire time-frequency domain [17–19]. OCDM utilizes a series of orthogonal chirp signals whose frequency varies with time to modulate information, which achieves better performance than the OFDM technique under doubly dispersive channels. However, the two-dimensional representation of the DD channel in OTFS incurs significant pilot overhead, and the diversity gain that OCDM can obtain depends on specific channel profiles.

Against this background, affine frequency division multiplexing (AFDM) has been proposed based on the discrete affine Fourier transform (DAFT) [20], which can also combat the time-frequency doubly selective fading, and more importantly has less complexity to implement than OTFS since it requires only one-dimension transformation. DAFT is defined as a generalized discrete form of the discrete Fourier transform (DFT) with a chirp-like basis specified by dual adjustable parameters, termed pre-chirp and post-chirp parameters, respectively. In AFDM, data symbols are multiplexed

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onto chirp-like subcarriers through DAFT and inverse DAFT (IDAF), which can separate the doubly dispersive channel into a sparse, quasi-static channel with a comprehensive DD channel representation by appropriately setting the chirp parameters. Therefore, AFDM achieves similar performance to OTFS, and demonstrates superior performance over OFDM and OCDM under doubly selective channels [21], making AFDM a promising waveform for future mobile communication systems [22].

There has been preliminary literature on AFDM [23–28]. A low-complexity embedded pilot-aided diagonal reconstruction (EPA-DR) channel estimation scheme was proposed in [23], which calculates the AFDM effective channel matrix directly without estimating the three channel parameters, eliminating the inherently severe inter-Doppler interference. In [24], the authors investigated the AFDM-empowered sparse code multiple access (SCMA) systems to support massive connectivity in high-mobility environments. An AFDM-based integrated sensing and communications (ISAC) system was studied in [25], demonstrating that the AFDM-ISAC system can maintain excellent sensing performance even under significant Doppler shifts. In [26], the authors proposed a bistatic sensing-aided channel estimation (SACE) scheme, which illustrates the capacity of AFDM to attain optimal sensing resolution and communication performance under reasonable parameter selection. A DFT-based modulation and demodulation technique for AFDM was studied in [27], with the potential to facilitate the adoption of AFDM for future deployments. The authors in [28] showed that utilizing a single pilot symbol achieves nearly the same sensing performance as employing the complete AFDM frame and proposed a low-complexity self-interference cancellation scheme for monostatic radar. The existing literature mostly explored the channel estimation, multiple access and ISAC issues under the classical AFDM architecture, whilst studies regarding further optimization/enhancement of the AFDM waveform are still at their infancy.

One promising research direction is to incorporate the index modulation (IM) philosophy for spectral and energy efficiency improvement [29–32], which conveys energy-free bits through the activation patterns of transmit entities, e.g., subcarriers [33], time slots [34], pulse positions [35], antennas [36], constellation modes [37], etc. IM-aided systems provide unique advantages, including flexible system architectures, high spectral efficiency (SE), and low hardware complexity, by leveraging additional energy-free information bits compared to conventional modulation systems.

In [38, 39], Tao *et al.* presented an IM-assisted scheme, which conveys energy-free information bits through the activation patterns of the subsymbols in the DAFT-domain, verifying that index bits have stronger diversity protection than modulation bits. A multicarrier system using the activation patterns of AFDM chirp subcarriers as indices was developed in [40], which indicates the potential of IM-assisted AFDM technology in enhancing bit error rate (BER) and energy efficiency performance. However, existing research has concentrated on post-chirp parameters in AFDM, with little attention paid to the considerable flexibility and degrees of freedom (DoFs) that the pre-chirp parameter offers. Although

the methods of index modulation based on the value of the pre-chirp parameter were discussed in [41], it focused more on reducing the complexity of the demodulation algorithm at the receiver, and no complete design method for the value of different pre-chirp parameters was proposed in this study.

Distinctively, this paper proposes a novel AFDM scheme with the pre-chirp-domain index modulation (AFDM-PIM) to enhance both spectral and energy efficiencies. Furthermore, performance evaluation of the proposed AFDM-PIM structure, including pairwise error probability (PEP) analysis and diversity analysis, is performed, and the numerical selection of the pre-chirp parameters is analyzed and optimized. The main contributions of this work are highlighted as follows:

- We prove that the subcarrier orthogonality is maintained by applying distinct pre-chirp parameters to different subcarriers during the AFDM modulation process. Based on this property, each AFDM subcarrier is constructed with a unique pre-chirp signal corresponding to the incoming bits. This configuration allows for the embedding of additional bits into the index patterns of pre-chirp parameter assignment without additional energy consumption.
- We derive the input-output relationship of the proposed AFDM-PIM scheme in the DAFT domain, and the asymptotically tight upper bounds on the average bit error probability (BEP) with the maximum likelihood (ML) detection based on the PEP analysis. Furthermore, we validate that the proposed AFDM-PIM scheme can achieve full diversity order under doubly dispersive channels.
- We propose an optimal pre-chirp alphabet design to enhance the BER performance via particle swarm optimization (PSO) algorithm. It is verified via extensive simulations that the optimized pre-chirp parameter alphabet results in a much better BER performance than the heuristic selection of pre-chirp parameter values. Our results also demonstrate that the proposed AFDM-PIM scheme is superior to classical AFDM and IM-aided OFDM algorithms in terms of BER performance.

The rest of the paper is organized as follows. In Section II, the AFDM system model is introduced. Section III details the proposed AFDM-PIM scheme. The performance analysis of AFDM-PIM under doubly dispersive channels is presented in Section IV, which includes the PEP and diversity analysis. The pre-chirp parameter optimization is provided in Section V. The simulation results and discussions are offered in Section VI, and Section VII draws the conclusions. The notations adopted in this paper are listed in Table I.

II. AFDM SYSTEM MODEL

The general system model of AFDM is presented in Fig. 1. For clarity, we provide a concise review of the fundamental concept of AFDM [21]. The transmitted bit stream is initially mapped onto a symbol vector, denoted as $\mathbf{x}_A = [x_A[0], x_A[1], \dots, x_A[N-1]]^T \in \mathbb{C}^{N \times 1}$, comprising N M -ary amplitude/phase modulations (APM) symbols in the DAFT domain. The resulting signals are then converted to time-

TABLE I
MATHEMATICAL NOTATIONS

Notation	Description
$\lfloor \cdot \rfloor$	Integer floor operator.
$s \sim \mathcal{CN}(0, \sigma^2)$	Random variable s follows a complex Gaussian distribution with zero mean and variance σ^2 .
x^*	Conjugate of the complex number x .
$a \mid b$	b is divisible by a without leaving a remainder.
$\max(a, b)$	Maximum value of a and b .
$\min(a, b)$	Minimum value of a and b .
$\binom{a}{b}$	Number of ways to choose b elements from a set of a elements.
$(\cdot)_N$	Modulo N operation.
$\mathbf{X}^T, \mathbf{X}^H$	Transpose and Hermitian operations of $\mathbf{X}$, respectively.
$\ \mathbf{X}\ _F$	Frobenius norm of $\mathbf{X}$.
$\mathbf{I}_N$	$N \times N$ identity matrix.
$Q(\cdot)$	Tail distribution function of the standard Gaussian distribution.
$E(\cdot)$	Expectation operator.
$\mathbb{I}$	Set of irrational numbers.
$\Re(x)$	Real part of the complex number x .
$j = \sqrt{-1}$	Imaginary unit (imaginary axis).
$\prod$	Product operator, representing the multiplication of a sequence of terms.
$\text{Tr}[\cdot]$	Trace operator.

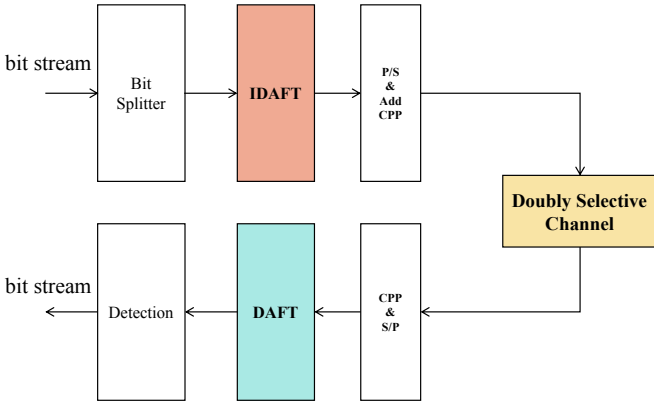


Fig. 1. The block diagram of AFDM system.

domain representations with an N -point IDAFD, formulated as

$$s_A[n] = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x_A[m] e^{j2\pi(c_1 n^2 + c_2 m^2 + \frac{nm}{N})}, \quad (1)$$

where $s_A[n]$ is the time domain signal, and $m, n \in \{0, 1, \dots, N-1\}$, while c_1 and c_2 are the post-chirp and pre-chirp parameters of the DAFD, respectively.

Similarly to OFDM, AFDM also necessitates the insertion of prefix to address the multi-path problem. By leveraging the inherent periodicity characteristic of DAFD, a chirp-periodic prefix (CPP) is inserted to serve the function analogous to the cyclic prefix (CP) in OFDM, which is given by

$$s_A[n] = s_A[N+n] e^{-j2\pi c_1(N^2 + 2Nn)}, \quad n = -L_{cp}, \dots, -1, \quad (2)$$

where L_{cp} is the length of the CPP.

Under high-mobility scenarios, the transmitted signals may experience time-frequency doubly dispersive channel

attributed to the severe Doppler shift and multi-path effects, which can be modeled as

$$h(\tau, \nu) = \sum_{p=1}^P h_p \delta(\tau - \tau_p) \delta(\nu - \nu_p), \quad (3)$$

where P is the number of the paths, ν_p and τ_p are the Doppler shift and delay of the p -th path, respectively, while $h_p \sim \mathcal{CN}(0, 1/P)$ is the p -th path's channel coefficient. The normalized delay and Doppler shift are given by $d_p = \tau_p \Delta f$ and $\alpha_p = N T \nu_p$, respectively, where Δf is the AFDM subcarrier spacing and T is the sampling interval with $T \Delta f = 1$. Furthermore, $\alpha_p \in [-\alpha_{\max}, \alpha_{\max}]$ and $d_p \in [0, d_{\max}]$, where $\alpha_{\max}$ and $d_{\max}$ denote the maximum Doppler shift and maximum delay, respectively [38]. For simplicity and without loss of generality, we mainly consider integer values of α_p in this paper.

At the receiver, by discarding the CPP, the received time domain symbols can be expressed as

$$r_A[n] = \sum_{p=1}^P h_p s_A[n - d_p] e^{-j2\pi \nu_p n} + w[n], \quad (4)$$

where $w[n] \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN). After the N -point DAFD, the received AFDM signals in the DAFD domain can be expressed as

$$y_A[\tilde{m}] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r_A[n] e^{-j2\pi(c_1 n^2 + c_2 \tilde{m}^2 + n\tilde{m}/N)} \quad \tilde{m} = 0, 1, \dots, N-1, \quad (5)$$

where $\tilde{m}$ are the indices in the DAFD domain. In matrix form, the received AFDM signals can be further written as

$$\mathbf{y}_A = \sum_{p=1}^P h_p \mathbf{A} \mathbf{\Gamma}_{\text{CPP}_p} \mathbf{\Delta}_{\nu_p} \mathbf{\Pi}^{d_p} \mathbf{A}^H \mathbf{x}_A + \mathbf{w}, \quad (6)$$

where the diagonal matrix $\mathbf{\Delta}_{\nu_p} = \text{diag}(e^{-j2\pi \nu_p \cdot 0}, e^{-j2\pi \nu_p \cdot 1}, \dots, e^{-j2\pi \nu_p \cdot (N-1)})$ represents the Doppler effect, $\mathbf{A}$ is the DAFD matrix, and $\mathbf{w}$ is the noise vector, while $\mathbf{\Pi}$ represents the forward cyclic-shift matrix given by

$$\mathbf{\Pi} = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}_{N \times N}, \quad (7)$$

and $\mathbf{\Gamma}_{\text{CPP}_p} = \text{diag}(\omega_{p,0}, \omega_{p,1}, \dots, \omega_{p,N})$ is the $N \times N$ diagonal matrix for CPP with

$$\omega_{p,n} = \begin{cases} e^{-j2\pi c_1(N^2 - 2N(d_p - n))}, & n < d_p, \\ 1, & n \geq d_p. \end{cases} \quad (8)$$

Upon receiving the signal $\mathbf{y}_A$, the ML detector can be employed for signal detection.

III. PROPOSED AFDM-PIM SCHEME

We first analyze the orthogonality of AFDM subcarriers and then derive the proposed AFDM-PIM framework. The input-output relation and parameter settings of the AFDM-PIM are also presented.

A. Orthogonality Analysis of AFDM Subcarriers

Following the modulation process of AFDM, (1) can also be expressed as

$$s_A[n] = \sum_{m=0}^{N-1} x_A[m] \phi_n(m), \quad n = 0, 1, \dots, N-1, \quad (9)$$

where $\phi_n(m)$ denotes the m -th chirp-like subcarrier given by

$$\phi_n(m) = \frac{1}{\sqrt{N}} e^{j2\pi(c_1 n^2 + c_2 m^2 + \frac{m n}{N})}. \quad (10)$$

The following Theorem demonstrates the flexibility of the pre-chirp parameter c_2 assignment for different subcarriers.

Theorem 1. Applying distinct c_2 to different subcarriers in the AFDM modulation process will still preserve their orthogonality.

Proof. The inner product between two subcarriers of AFDM, which utilize the same post-chirp parameter c_1 but distinct values of pre-chirp parameter c_2 , designated as $\phi_n^{c_1, c_2, 1}(m)$ and $\phi_n^{c_1, c_2, 2}(m)$, respectively, is given by

$$\begin{aligned} & \sum_{n=0}^{N-1} \phi_n^{c_1, c_2, 1}(m_1) (\phi_n^{c_1, c_2, 2}(m_2))^* \\ &= \frac{1}{N} e^{-j2\pi(c_{2,1}m_1^2 - c_{2,2}m_2^2)} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(m_1 - m_2)n} \\ &= \frac{1}{N} e^{-j2\pi(c_{2,1}m_1^2 - c_{2,2}m_2^2)} \frac{1 - e^{-j2\pi N(\frac{m_1 - m_2}{N})}}{1 - e^{-j2\pi(\frac{m_1 - m_2}{N})}} \\ &= \begin{cases} 1, & m_1 = m_2, \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (11)$$

It is evident that the orthogonality among AFDM subcarriers is maintained when different values of c_2 are employed. $\square$

TABLE II
MAPPING RULE BETWEEN THE INDEX BITS AND THE PCPs IN THE CASE OF $N_c = 4$ AND $\lambda = 4$.

Index bits	PCPs for Each Group			
	subcarrier 1	subcarrier 2	subcarrier 3	subcarrier 4
0000	$c_2^{(1)}$	$c_2^{(2)}$	$c_2^{(3)}$	$c_2^{(4)}$
0001	$c_2^{(1)}$	$c_2^{(2)}$	$c_2^{(3)}$	$c_2^{(4)}$
0010	$c_2^{(1)}$	$c_2^{(3)}$	$c_2^{(2)}$	$c_2^{(4)}$
0011	$c_2^{(1)}$	$c_2^{(3)}$	$c_2^{(4)}$	$c_2^{(2)}$
0100	$c_2^{(1)}$	$c_2^{(4)}$	$c_2^{(2)}$	$c_2^{(3)}$
0101	$c_2^{(1)}$	$c_2^{(4)}$	$c_2^{(3)}$	$c_2^{(2)}$
0110	$c_2^{(2)}$	$c_2^{(1)}$	$c_2^{(3)}$	$c_2^{(4)}$
0111	$c_2^{(2)}$	$c_2^{(1)}$	$c_2^{(4)}$	$c_2^{(3)}$
1000	$c_2^{(2)}$	$c_2^{(3)}$	$c_2^{(1)}$	$c_2^{(4)}$
1001	$c_2^{(2)}$	$c_2^{(3)}$	$c_2^{(4)}$	$c_2^{(1)}$
1010	$c_2^{(2)}$	$c_2^{(4)}$	$c_2^{(1)}$	$c_2^{(3)}$
1011	$c_2^{(2)}$	$c_2^{(4)}$	$c_2^{(3)}$	$c_2^{(1)}$
1100	$c_2^{(3)}$	$c_2^{(1)}$	$c_2^{(2)}$	$c_2^{(4)}$
1101	$c_2^{(3)}$	$c_2^{(1)}$	$c_2^{(4)}$	$c_2^{(2)}$
1110	$c_2^{(3)}$	$c_2^{(2)}$	$c_2^{(1)}$	$c_2^{(4)}$
1111	$c_2^{(3)}$	$c_2^{(2)}$	$c_2^{(4)}$	$c_2^{(1)}$

Inspired by Theorem 1, we design the AFDM-PIM scheme, as shown in Fig. 2, which utilizes the flexibility of c_2 assignment to convey additional information bits.

B. Transmitter

Consider the same AFDM symbol vector comprising N M -ary constellation symbols in the DAFT domain as in Section II. At the transmitter, unlike classical AFDM, the N AFDM subcarriers are divided into G groups, with each group comprising $N_c = N/G$ chirp subcarriers. The total B information bits are split into G parallel streams of $b = B/G$ bits for each subcarrier group. Each b -bit stream is further segmented into b_1 symbol bits and b_2 index bits, i.e., $b = b_1 + b_2$. Within the g -th group ($1 \leq g \leq G$), the

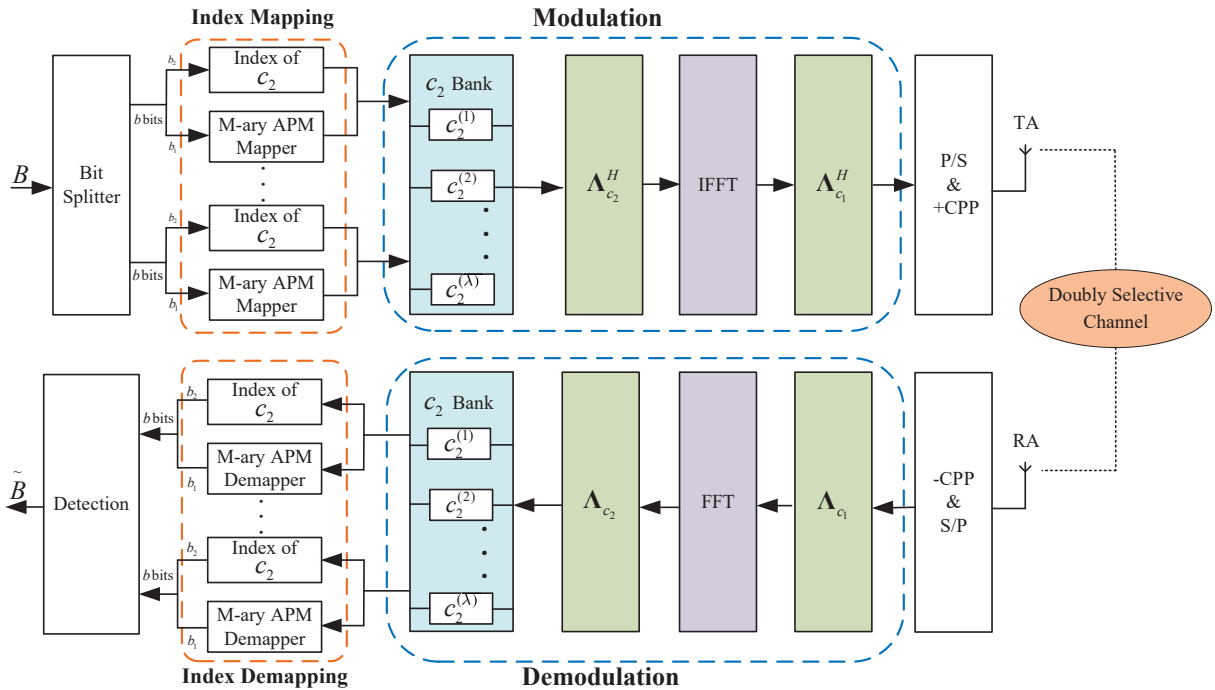


Fig. 2. Transceiver structure of the proposed AFDM-PIM scheme.

$b_1 = N_c \log_2(M)$ symbol bits are conveyed by N_c M -ary symbols, denoted as $\mathbf{x}^g = [x^g[0], x^g[1], \dots, x^g[N_c - 1]]^T \in \mathbb{C}^{N_c \times 1}$. On the other hand, each subcarrier is assigned with a unique c_2 value from a finite alphabet of λ legitimate c_2 realizations (corresponding to the ' c_2 Bank' in Fig. 2), i.e., $\mathcal{P}_c = \{c_2^{(1)}, c_2^{(2)}, \dots, c_2^{(\lambda)}\}$. Specifically, the pre-chirping pattern (PCP) of the c_2 values in the g -th group, denoted by $\mathbf{P}_{c_2}^g = [c_{2,N_c(g-1)}, c_{2,N_c(g-1)+1}, \dots, c_{2,N_cg-1}]^T \in \mathbb{C}^{N_c \times 1}$ is determined by the b_2 index bits according to the pre-defined relationship between the index-bit stream and the permutations of N_c elements from $\mathcal{P}_c$, where $c_{2,m} \in \mathcal{P}_c$ represents the pre-chirp parameter of the m -th subcarrier. $\mathbf{P}_{c_2} = [\mathbf{P}_{c_2}^1, \mathbf{P}_{c_2}^2, \dots, \mathbf{P}_{c_2}^G]$ represents the PCP for all the groups (PCPG), and all the possible PCPGs are combined into a set, termed $\mathcal{S}_p$. Table II exemplifies the mapping rule in the case of $N_c = 4$ and $\lambda = 4$. Every b_2 index bits correspond to one row in Table II. Hence, aside from the APM bits, additional b_2 information bits can be implicitly conveyed by the indices of $\mathcal{P}_{c_2}$, and

$$b_2 = \begin{cases} \lfloor \log_2(\mathcal{C}_{\lambda, N_c} N_c!) \rfloor, & \lambda \geq N_c, \\ \lfloor \log_2(\lambda!) \rfloor \frac{N_c}{\lambda}, & \lambda < N_c \text{ \& } \lambda \mid N_c, \\ \lfloor \log_2(\mathcal{C}_{\lambda, N_c} \lambda! \lambda^{(N_c - \lambda)}) \rfloor, & \text{otherwise,} \end{cases} \quad (12)$$

where $\mathcal{C}_{\lambda, N_c}$ is defined as

$$\mathcal{C}_{\lambda, N_c} = \begin{pmatrix} \max(\lambda, N_c) \\ \min(\lambda, N_c) \end{pmatrix}. \quad (13)$$

For ease of design and subsequent optimization, we typically select the case of $\lambda = N_c$, which implies $b_2 = \lfloor \log_2(N_c!) \rfloor$.

After the mapping for all the groups, the time-domain transmitted signals can be generated through N -point IDAFT operation, expressed as

$$s[n] = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x[m] e^{j2\pi(c_1 n^2 + c_{2,m} m^2 + nm/N)}. \quad (14)$$

The matrix form of (14) can be formulated as

$$\mathbf{s} = \mathbf{A}^H \mathbf{x} = \mathbf{\Lambda}_{c_1}^H \mathbf{F}^H \mathbf{\Lambda}_{c_2}^H \mathbf{x}, \quad (15)$$

where $\mathbf{x} = [(\mathbf{x}^1)^T, (\mathbf{x}^2)^T, \dots, (\mathbf{x}^G)^T]^T$, and $\mathbf{\Lambda}_{c_2}$ and $\mathbf{\Lambda}_{c_1}$ represent the pre-chirp and post-chirp diagonal matrices, respectively, expressed as

$$\mathbf{\Lambda}_{c_2} = \text{diag}(e^{-j2\pi c_{2,m} m^2}, m = 0, 1, \dots, N-1), \quad (16)$$

$$\mathbf{\Lambda}_{c_1} = \text{diag}(e^{-j2\pi c_1 n^2}, n = 0, 1, \dots, N-1), \quad (17)$$

while $\mathbf{F}$ denotes the DFT matrix with elements $\mathbf{F}(m, n) = e^{-j2\pi mn/N} / \sqrt{N}$, $m, n = 0, 1, \dots, N-1$.

Like classical AFDM, our proposed AFDM-PIM also requires the CPP to mitigate the effects of multi-path propagation effectively. Without loss of generality, the length of the CPP is assumed to be greater than the maximum channel delay spread.

C. Receiver

At the receiver, considering the doubly dispersive channel in (3), the received time-domain signals after removing the CPP can be written as

$$r[n] = \sum_{p=1}^P h_p s[n - d_p] e^{-j2\pi \nu_p n} + w_r[n], \quad (18)$$

where $w_r[n] \sim \mathcal{CN}(0, N_0)$ is the AWGN. The matrix form of (18) is given by

$$\mathbf{r} = \mathbf{H} \mathbf{s} + \mathbf{w} = \sum_{p=1}^P h_p \mathbf{\Gamma}_{\text{CPP}_p} \mathbf{\Delta}_{\nu_p} \mathbf{\Pi}^{d_p} \mathbf{s} + \mathbf{w}_r, \quad (19)$$

where $\mathbf{w}_r = [w_r[0], w_r[1], \dots, w_r[N-1]]^T$ is the time-domain noise vector. As given in Section II, $\mathbf{\Delta}_{\nu_p}$ represents the Doppler effect, $\mathbf{\Pi}$ is the forward cyclic-shift matrix with $\mathbf{\Pi}^{d_p}$ modeling the delay extension, and $\mathbf{\Gamma}_{\text{CPP}_p}$ is the effective CPP matrix.

By applying the DAFT operation, the received DAFT-domain symbols are obtained as

$$y[\bar{m}] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r[n] e^{-j2\pi(c_1 n^2 + c_{2,\bar{m}} \bar{m}^2 + n\bar{m}/N)}, \quad (20)$$

which can also be grouped into a vector as

$$\begin{aligned} \mathbf{y} &= \mathbf{A} \mathbf{r} = \sum_{p=1}^P h_p \mathbf{A} \mathbf{\Gamma}_{\text{CPP}_p} \mathbf{\Delta}_{\nu_p} \mathbf{\Pi}^{d_p} \mathbf{A}^H \mathbf{x} + \mathbf{A} \mathbf{w}_r \\ &= \mathbf{H}_{\text{eff}} \mathbf{x} + \mathbf{w}, \end{aligned} \quad (21)$$

where $\mathbf{H}_{\text{eff}}$ is the effective channel matrix in the DAFT-domain and $\mathbf{w} = \mathbf{A} \mathbf{w}_r$.

The time-frequency doubly dispersive channel can be estimated through pilot-aided channel estimation algorithms [23, 42, 43]. This paper will not provide further elaboration on this subject for brevity. Given the estimated $\hat{\mathbf{H}}_{\text{eff}}$ for the effective channel matrix $\mathbf{H}_{\text{eff}}$, the ML data detection is formulated as the following optimization

$$(\hat{\mathbf{x}}, \hat{P}_{c_2}) = \arg \min_{\mathbf{x}, P_{c_2}} \left\| \mathbf{y} - \hat{\mathbf{H}}_{\text{eff}} \mathbf{x} \right\|^2. \quad (22)$$

D. Input-Output Relation and Parameter Settings

Substituting (14) and (18) into (20), the input-output relation of AFDM-PIM can be obtained as

$$y[\bar{m}] = \frac{1}{N} \sum_{p=1}^P \sum_{m=0}^{N-1} h_p \xi_{(p, \bar{m}, m)} \eta_{(p, \bar{m}, m)} x[m] + w[\bar{m}], \quad (23)$$

where

$$\xi_{(p, \bar{m}, m)} = e^{j \frac{2\pi}{N} (N c_{2,m} m^2 - N c_{2,\bar{m}} \bar{m}^2 - m d_p + N c_1 d_p^2)}, \quad (24)$$

$$\begin{aligned} \eta_{(p, \bar{m}, m)} &= \sum_{n=0}^{N-1} e^{-j \frac{2\pi}{N} ((\bar{m} - m + \alpha_p + 2N c_1 d_p) n)} \\ &= \frac{e^{-j2\pi(\bar{m} - m + \alpha_p + 2N c_1 d_p)} - 1}{e^{-j \frac{2\pi}{N} (\bar{m} - m + \alpha_p + 2N c_1 d_p)} - 1}. \end{aligned} \quad (25)$$

In matrix representation, (23) can be rewritten as

$$\mathbf{y} = \sum_{p=1}^P h_p \mathbf{H}_p \mathbf{x} + \mathbf{w}, \quad (26)$$

where the elements of $\mathbf{H}_p$ are given by

$$H_p[\bar{m}, m] = \frac{1}{N} \xi_{(p, \bar{m}, m)} \eta_{(p, \bar{m}, m)} = \begin{cases} \xi_{(p, \bar{m}, m)}, & m = (\bar{m} + \text{loc}_p)_N, \\ 0, & \text{otherwise,} \end{cases} \quad (27)$$

where $\text{loc}_p = (\alpha_p + 2Nc_1d_p)_N$. Since the range of loc_p is $[-\alpha_{\max} + 2Nc_1d_p, \alpha_{\max} + 2Nc_1d_p]$, we define $\text{loc}_p \in \mathbb{K}_p$, with $\mathbb{K}_p = \{-\alpha_{\max} + 2Nc_1d_p, \dots, \alpha_{\max} + 2Nc_1d_p\}$.

It can be seen from (27) that for the two chirp parameters of AFDM-PIM, only the post-chirp parameter exerts an influence on the positions of non-zero entries in the matrix $\mathbf{H}_p$ determined by loc_p , which is independent of the pre-chirp parameter. Therefore, like classical AFDM, full diversity order can be obtained by the proposed AFDM-PIM under doubly dispersive channels by adjusting c_1 to avoid possible overlap between non-zero elements of $\mathbf{H}_i$ and $\mathbf{H}_j$ ($i \neq j$). Specifically, this requires that the intersection between the corresponding ranges for loc_i and loc_j is empty, i.e.,

$$\mathbb{K}_i \cap \mathbb{K}_j = \emptyset. \quad (28)$$

Without loss of generality, assume that $d_i \leq d_j$. Then the constraint (28) can be transformed into

$$c_1 > \frac{2\alpha_{\max}}{2N(d_j - d_i)}. \quad (29)$$

Since the minimum value of $(d_j - d_i)$ with $i \neq j$ equals one, it can be concluded that c_1 can be set as:

$$c_1 = \frac{2\alpha_{\max} + 1}{2N}. \quad (30)$$

Following the configuration (30) and $(d_{\max} + 1)(2\alpha_{\max} + 1) \leq N$, the channel paths with different delays or Doppler shifts can be distinguished within the DAFT domain, as illustrated in Fig. 3, which shows an example of the effective channel matrix of an AFDM-PIM system under a three-path channel. The rigorous diversity analysis will be provided in Section IV.

IV. PERFORMANCE ANALYSIS

We first derive the average BEP (ABEP) upper bounds for our AFDM-PIM scheme with the ML detection. Then we analyze the diversity order achieved by the system.

A. Error Performance Analysis

To facilitate the analysis, the received DAFT-domain signal (26) can be rewritten as

$$\mathbf{y} = \Phi(\mathbf{x})\mathbf{h} + \mathbf{w}, \quad (31)$$

where $\Phi(\mathbf{x}) = [\mathbf{H}_1\mathbf{x}, \mathbf{H}_2\mathbf{x}, \dots, \mathbf{H}_P\mathbf{x}] \in \mathbb{C}^{N \times P}$ and $\mathbf{h} = [h_1, h_2, \dots, h_P]^T \in \mathbb{C}^{P \times 1}$. The conditional PEP (CPEP) between $\Phi(\mathbf{x})$ and its estimate $\hat{\Phi}(\hat{\mathbf{x}})$ can be calculated as (32), shown at the bottom of this page, where $\chi = \mathbf{w}^H(\hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x}))\mathbf{h} + \mathbf{h}^H(\hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x}))^H\mathbf{w}$. Since χ follows Gaussian distribution with variance $2N_0\|(\hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x}))\mathbf{h}\|^2$, the CPEP can be expressed as

$$\Pr([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}]|\mathbf{h}) = Q\left(\sqrt{\frac{\delta}{2N_0}}\right), \quad (33)$$

where $\delta = \|(\hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x}))\mathbf{h}\|^2 = \mathbf{h}^H\Psi\mathbf{h}$ with $\Psi = (\hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x}))^H(\hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x}))$. According to [44], Q -function can be approximated as

$$Q(x) \approx \frac{1}{12}e^{-x^2/2} + \frac{1}{4}e^{-2x^2/3}. \quad (34)$$

Hence the CPEP can be approximated as

$$\Pr([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}]|\mathbf{h}) \approx \frac{1}{12}e^{-\varsigma_1\delta} + \frac{1}{4}e^{-\varsigma_2\delta}, \quad (35)$$

with $\varsigma_1 = \frac{1}{4N_0}$ and $\varsigma_2 = \frac{1}{3N_0}$. Consequently, the unconditional PEP (UPEP) can be calculated as

$$\begin{aligned} \Pr([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}]) &= \mathbb{E}\left(\Pr([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}]|\mathbf{h})\right) \\ &\approx \int_0^{+\infty} \left(\frac{1}{12}e^{-\varsigma_1\delta} + \frac{1}{4}e^{-\varsigma_2\delta}\right) p_\delta(\delta) d\delta, \end{aligned} \quad (36)$$

where $p_\delta(\delta)$ denotes the probability density function (PDF) of δ . Noting $p_\delta(\delta) = 0$ for $\delta < 0$ and utilizing the definition of the moment-generating function (MGF) $M_\eta(s) = \mathbb{E}(e^{s\eta}) = \int_{-\infty}^{+\infty} e^{s\eta} p_\eta(\eta) d\eta$, we can calculate the UPEP (36) as

$$\Pr([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}]) \approx \frac{1}{12}M_\delta(-\varsigma_1) + \frac{1}{4}M_\delta(-\varsigma_2). \quad (37)$$

Theorem 2 ([45]). *For an $N \times N$ Hermitian matrix $\mathbf{Q}$ and an $N \times 1$ zero-mean complex-valued random vector $\mathbf{v}$ with covariance matrix $\mathbf{L}$, the characteristic function of the quadratic form $f = \mathbf{v}^H\mathbf{Q}\mathbf{v}$ can be expressed as*

$$\varphi_f(t) = |\mathbf{I} - j \cdot t\mathbf{L}\mathbf{Q}|^{-1} = \prod_{\iota=1}^{\kappa} \frac{1}{1 - j \cdot t\lambda_\iota(\mathbf{L}\mathbf{Q})}, \quad (38)$$

where $\lambda_\iota(\mathbf{L}\mathbf{Q})$ is the ι -th non-zero eigenvalue of matrix $\mathbf{L}\mathbf{Q}$, and κ denotes the number of the non-zero eigenvalues, i.e.,

$$\begin{aligned} \Pr([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}]|\mathbf{h}) &= \Pr(\|\mathbf{y} - \hat{\Phi}(\hat{\mathbf{x}})\mathbf{h}\|^2 < \|\mathbf{y} - \Phi(\mathbf{x})\mathbf{h}\|^2) \\ &= \Pr\left(\text{Tr}\left[(\mathbf{y} - \hat{\Phi}(\hat{\mathbf{x}})\mathbf{h})^H(\mathbf{y} - \hat{\Phi}(\hat{\mathbf{x}})\mathbf{h}) - (\mathbf{y} - \Phi(\mathbf{x})\mathbf{h})^H(\mathbf{y} - \Phi(\mathbf{x})\mathbf{h})\right] < 0\right) \\ &= \Pr\left(\text{Tr}\left[\mathbf{h}^H\left(\Phi(\mathbf{x}) - \hat{\Phi}(\hat{\mathbf{x}})^H\right)\left(\Phi(\mathbf{x}) - \hat{\Phi}(\hat{\mathbf{x}})^H\right)\mathbf{h}\right] - C < 0\right) = \Pr\left(\chi > \|(\hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x}))\mathbf{h}\|^2\right), \end{aligned} \quad (32)$$

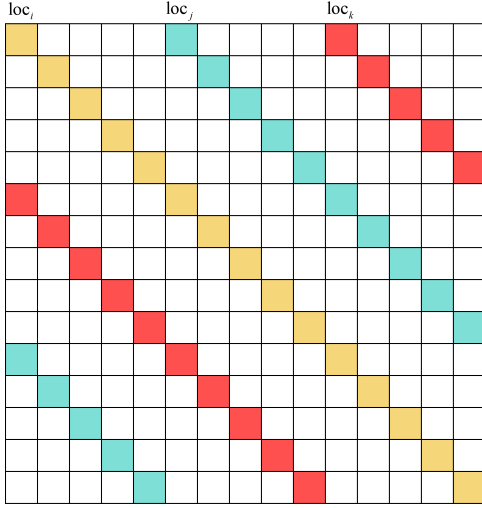


Fig. 3. Example of the effective channel matrix of a three-path channel.

the rank of $\mathbf{LQ}$. Using the relationship $M_f(s) = \varphi_f(-j \cdot s)$, the MGF of f is given by

$$M_f(s) = \prod_{\iota=1}^{\kappa} \frac{1}{1 - s\lambda_{\iota}(\mathbf{LQ})}. \quad (39)$$

Since $\mathbf{h}$ is a zero mean complex vector with the covariance matrix $\frac{1}{P}\mathbf{I}_P$ and Ψ is a Hermitian matrix, according to Theorem 2, the UPEP (37) can be further expressed as

$$\Pr([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}]) \approx \frac{1}{12} \prod_{\iota=1}^{\kappa} \frac{1}{1 + \frac{\lambda_{\iota}(\Psi)}{4PN_0}} + \frac{1}{4} \prod_{\iota=1}^{\kappa} \frac{1}{1 + \frac{\lambda_{\iota}(\Psi)}{3PN_0}}, \quad (40)$$

where we assume that the rank of Ψ is κ .

Moreover, based on the UPEP (40), the ABEP upper bound for the proposed AFDM-PIM scheme can be calculated by

$$\Pr_{\text{ABEP}} \leq \frac{1}{b2^b} \sum_{\mathbf{x}} \sum_{\hat{\mathbf{x}}} \sum_{\Phi} \sum_{\hat{\Phi}} \Pr([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}]) \times \tau([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}]), \quad (41)$$

where $\tau([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}])$ represents the number of error bits caused by the corresponding pairwise error event.

B. Diversity Analysis

At the high signal-to-noise ratio (SNR) region, the approximation of (40) can be simplified as

$$\begin{aligned} \Pr([\mathbf{x}, \Phi] \rightarrow [\hat{\mathbf{x}}, \hat{\Phi}]) &\approx \frac{1}{12} \prod_{\iota=1}^{\kappa} \frac{4PN_0}{\lambda_{\iota}(\Psi)} + \frac{1}{4} \prod_{\iota=1}^{\kappa} \frac{3PN_0}{\lambda_{\iota}(\Psi)} \\ &\approx \left(\prod_{\iota=1}^{\kappa} \lambda_{\iota}(\Psi) \right)^{-1} \left(\frac{(4P)^{\kappa}}{12} + \frac{(3P)^{\kappa}}{4} \right) \text{SNR}^{-\kappa}, \end{aligned} \quad (42)$$

where SNR is defined as $1/N_0$. As a result, the diversity order μ of the proposed AFDM-PIM scheme equals to the minimum value of κ , i.e.,

$$\mu = \min \text{rank}(\Psi). \quad (43)$$

Since the eigenvalues of Hermitian matrix Ψ can be calculated as the square of the singular values of $(\hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x}))$,

$\text{rank}(\Psi) = \text{rank}(\hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x}))$ and the diversity order μ can also be expressed as

$$\mu = \min \text{rank}(\hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x})). \quad (44)$$

Defining $\Phi(\bar{\delta}) = \hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x})$, the full diversity order analysis for the AFDM-PIM can be transformed into the full rank analysis of $\Phi(\bar{\delta})$ as stated in Theorem 3 below.

Theorem 3. *The proposed AFDM-PIM scheme is capable of achieving the full diversity order if the following two conditions are met.*

Condition 1: *The number of the paths satisfies*

$$P \leq (d_{\max} + 1)(2\alpha_{\max} + 1) \leq N. \quad (45)$$

Condition 2: *The pre-chirp parameters in $\mathcal{P}_c$ take the irrational numbers.*

Proof. See Appendix A. $\square$

C. Spectral Efficiency Analysis

This subsection examines the SE of the proposed AFDM-PIM scheme. Classic AFDM and IM-assisted AFDM (referred to as AFDM-IM) are utilized as benchmarks to highlight the superiority of the proposed approach.

The SEs of the classic AFDM [21] and AFDM-IM [40] are given by

$$S_{\text{AFDM}} = \log_2(M) \quad (46)$$

and

$$S_{\text{AFDM-IM}} = \frac{\lfloor \log_2 \left(\frac{N_c}{a} \right) \rfloor + a \log_2(M)}{N_c}, \quad (47)$$

where N_c and a represent the numbers of subcarriers in each group and activated subcarriers at each transmission, respectively, for the AFDM-IM scheme.

Based on the description in Subsection III-B, the SE of the proposed AFDM-PIM can be calculated as

$$S_{\text{AFDM-PIM}} = \frac{\lfloor \log_2(N_c!) \rfloor}{N_c} + \log_2(M). \quad (48)$$

As can be seen from the above analysis, the proposed AFDM-PIM demonstrates a higher SE than AFDM, due to the presence of index modulation. Furthermore, in contrast to AFDM-IM, which employs inactive subcarriers at each transmission, AFDM-PIM exhibits a higher SE, attributed to the capacity of all subcarriers to carry modulated data. In summary, compared with classic methods, the proposed AFDM-PIM scheme offers an enhancement in terms of SE, without incurring any additional hardware cost.

V. PARAMETER OPTIMIZATION

In this section, we establish an optimization problem for the optimal c_2 alphabet design, where the PSO algorithm is employed to enhance the BER performance.

A. Problem Formulation

For tractable analysis, the pre-chirp alphabet design is investigated based on the optimal BER detector [46], i.e., the ML detector based on the signal model (31), expressed as

$$(\hat{\mathbf{x}}, \hat{P}_{c_2}) = \arg \min_{\mathbf{x}, P_{c_2}} \|\mathbf{y} - \Phi(\mathbf{x})\mathbf{h}\|^2. \quad (49)$$

To achieve the optimal c_2 alphabet, the minimum Euclidean distance (MED) between different realizations of $\Phi(\mathbf{x})$ should be maximized. The Euclidean distance between two realizations of $\Phi(\mathbf{x})$ is formulated as

$$O_{k,j}(\mathcal{P}_c) = \sum_{\mathbf{x}', \mathbf{x}} \sum_{r=1}^{\mathcal{R}} \left\| \Phi_k^r(\mathbf{x}') - \Phi_j^r(\mathbf{x}) \right\|_{\mathbb{F}}^2, \quad (50)$$

where j and k represent the indices of $\mathcal{S}_p$, and the corresponding PCPGs are expressed as $P_{c_2}^{(j)} = [c_{2,1}, c_{2,2}, \dots, c_{2,N}]$ and $P_{c_2}^{(k)} = [c'_{2,1}, c'_{2,2}, \dots, c'_{2,N}]$, respectively, while r is the index of delay and Doppler selection. Specifically, each r corresponds to a specific combination of P paths with different delays and Doppler shifts under doubly dispersive channels with a maximum delay $d_{\max}$ and a normalized Doppler shift $\alpha_{\max}$, and the maximum value of r is $\mathcal{R} = \binom{P_{\max}}{P}$, where $P_{\max} = (d_{\max} + 1)(2\alpha_{\max} + 1)$.

According to Theorem 3 and the periodicity brought by 2π in (16), c_2 is an irrational number with a principal value range of $[0, 1]$. To enhance the BER performance of the proposed AFDM-PIM, we formulate the following problem to maximize the MED with optimal c_2 alphabet design:

$$\max_{\mathcal{P}_c} \min_{k,j \in [1, 2^{b_2}]} O_{k,j}(\mathcal{P}_c), \quad (51a)$$

$$\text{s.t.} \quad j \neq k, \quad (51b)$$

$$c_2 \in [0, 1], \quad (51c)$$

$$c_2 \in \mathbb{I}. \quad (51d)$$

To derive the objective function, consider the elements of $\mathbf{x}'$ and $\mathbf{x}$, x'_{loc_p+n} and x_{loc_p+n} , which can be expressed as $x'_{\text{loc}_p+n} = r_{p,n} e^{j\vartheta'_{p,n}}$ and $x_{\text{loc}_p+n} = r'_{p,n} e^{j\vartheta_{p,n}}$. Define

$$\begin{cases} \theta_n = 2\pi \left(c_{2,\text{loc}_p+n} (\text{loc}_p+n)^2 - c_{2,n} n^2 \right), \\ \theta'_n = 2\pi \left(c'_{2,\text{loc}_p+n} (\text{loc}_p+n)^2 - c'_{2,n} n^2 \right). \end{cases} \quad (52)$$

and

$$F_{p,n} = r_{p,n}^2 + r_{p,n}'^2 - 2r_{p,n}r_{p,n}' \cos(\theta'_n - \theta_n + \psi_{p,n}), \quad (53)$$

where $\psi_{p,n} = \vartheta'_{p,n} - \vartheta_{p,n}$. Then the objective function $O_{k,j}(\mathcal{P}_c)$ can be expressed as

$$O_{k,j}(\mathcal{P}_c) = \sum_{\mathbf{x}', \mathbf{x}} \sum_{r=1}^{\mathcal{R}} \sum_{p=1}^P \sum_{n=0}^{N-1} F_{p,n}. \quad (54)$$

The derivation of (54) is given in Appendix B.

B. Problem Transformation

1) *Transformation of $\mathbf{x}'$ and $\mathbf{x}$* : Consider the constant modulus constellation using phase shift keying (PSK) as an example. Since x'_{loc_p+n} and x_{loc_p+n} take values from the M -PSK constellation, $r_{p,n} = r'_{p,n} = 1$, and $\psi_{p,n}$ is given by

$$\psi_{p,n} = \frac{2\pi k}{M}, k = -(M-1), \dots, -1, 0, 1, \dots, M-1, \quad (55)$$

where each possible value is assigned with equal probability, i.e., $\psi_{p,n}$ follows a discrete uniform distribution. For the case where $\mathbf{x}' \neq \mathbf{x}$, $O_{k,j}(\mathcal{P}_c)$ can be expressed as

$$\begin{aligned} O_{k,j}^{(\mathbf{x}' \neq \mathbf{x})}(\mathcal{P}_c) &= \sum_{\mathbf{x}' \neq \mathbf{x}} \sum_{r=1}^{\mathcal{R}} \sum_{p=1}^P \sum_{n=0}^{N-1} (-\cos(\psi_{p,n} + \theta'_n - \theta_n)) \\ &= \sum_{r=1}^{\mathcal{R}} \sum_{p=1}^P \sum_{n=0}^{N-1} \left(\sum_{k=-M}^{M-1} \cos\left(\frac{2\pi k}{M} + \theta'_n - \theta_n\right) \right) = 0. \end{aligned} \quad (56)$$

Therefore, for the objective function $O_{k,j}(\mathcal{P}_c)$, it is sufficient to consider the case of $\mathbf{x}' = \mathbf{x}$. In this case, $\psi_{p,n} = 0$ for all $n = 0, 1, \dots, N-1$, and $O_{k,j}(\mathcal{P}_c)$ can be expressed as

$$O_{k,j}(\mathcal{P}_c) = \sum_{r=1}^{\mathcal{R}} \sum_{p=1}^P \sum_{n=0}^{N-1} (1 - \cos(\theta'_n - \theta_n)). \quad (57)$$

For non-constant modulus constellations, take quadrature amplitude modulation (QAM) as an example. Then x'_{loc_p+n} and x_{loc_p+n} take values from the M -QAM constellation and the constellation diagram is symmetric with respect to the origin. Hence, for each pair of x'_{loc_p+n} and x_{loc_p+n} with phase difference $\psi_{p,n} = \gamma$, there exist a pair of corresponding constellation points with phase difference $\psi_{\hat{p},\hat{n}} = \gamma + \pi$. Therefore, for the case of $\mathbf{x}' \neq \mathbf{x}$, we can obtain

$$\begin{aligned} &\sum_{\mathbf{x}' \neq \mathbf{x}} \sum_{r=1}^{\mathcal{R}} \sum_{p=1}^P \sum_{n=0}^{N-1} (r_{p,n} r'_{p,n} \cos(\theta'_n - \theta_n + \psi_{p,n})) \\ &= \sum_{r=1}^{\mathcal{R}} \sum_{p=1}^P \sum_{n=0}^{N-1} \left(\sum_{\psi_{p,n}} \cos(\gamma + \theta'_n - \theta_n) + \cos(\gamma + \pi + \theta'_n - \theta_n) \right) \\ &= 0. \end{aligned} \quad (58)$$

Consequently, it is sufficient to focus on the case of $\mathbf{x}' = \mathbf{x}$ for the objective function $O_{k,j}(\mathcal{P}_c)$. Under this condition, $r_{p,n} = r'_{p,n}$ and $\psi_{p,n} = 0$, and $O_{k,j}(\mathcal{P}_c)$ can be expressed as

$$O_{k,j}(\mathcal{P}_c) = \sum_{\mathbf{x}', \mathbf{x}} \sum_{r=1}^{\mathcal{R}} \sum_{p=1}^P \sum_{n=0}^{N-1} 2r_{p,n}^2 (1 - \cos(\theta'_n - \theta_n)), \quad (59)$$

where signal amplitudes $r_{p,n}$ are unrelated to optimization.

In summary, for both constant modulus and non-constant modulus constellations, the optimization problem (51) can be transformed into

$$\max_{\mathcal{P}_c} \min_{k,j \in [1, 2^{b_2}]} \sum_{r=1}^{\mathcal{R}} \sum_{p=1}^P \sum_{n=0}^{N-1} (1 - \cos(\theta'_n - \theta_n)), \quad (60a)$$

$$\text{s.t.} \quad (51b), (51c), (51d). \quad (60b)$$

2) *Transformation of $P_{c_2}^{(j)}$ and $P_{c_2}^{(k)}$* : In the problem (60), each pair of indices of PCPGs, i.e., each pair of j, k , corresponds to a specific set of $P_{c_2}^{(j)}$ and $P_{c_2}^{(k)}$. By substituting (52) into the objective function of (60), $O_{k,j}(P_c)$ becomes

$$O_{k,j}(P_c) = \sum_{r=1}^{\mathcal{R}} \sum_{p=1}^P \sum_{n=0}^{N-1} (1 - O'), \quad (61)$$

where

$$\begin{cases} O' = \cos(2\pi (\Delta c_{2,\text{loc}_p+n}(\text{loc}_p+n)^2 - \Delta c_{2,n}n^2)), \\ \Delta c_{2,\text{loc}_p+n} = c'_{2,\text{loc}_p+n} - c_{2,\text{loc}_p+n}, \\ \Delta c_{2,n} = c'_{2,n} - c_{2,n}. \end{cases} \quad (62)$$

It can be observed that in the case of $j \neq k$, i.e., $P_{c_2}^{(j)} \neq P_{c_2}^{(k)}$, as the discrepancy between $P_{c_2}^{(j)}$ and $P_{c_2}^{(k)}$ diminishes, the number of zero values in O' increases. Therefore, for the problem (60), it is sufficient to consider the case where $\|P_{c_2}^{(j)} - P_{c_2}^{(k)}\|_0 = 2$. Hence, the optimization problem can be further expressed as

$$\max_{P_c} \min_{k,j} \sum_{r=1}^{\mathcal{R}} \sum_{p=1}^P \sum_{n=0}^{N-1} (1 - \cos(\theta'_n - \theta_n)), \quad (63a)$$

$$\text{s.t. } j \neq k, \quad (63b)$$

$$\|P_{c_2}^{(j)} - P_{c_2}^{(k)}\|_0 = 2, \quad (63c)$$

$$c_2 \in (0, 1), \quad (63d)$$

$$c_2 \in \mathbb{I}. \quad (63e)$$

C. Problem Solver

The optimization (63) is a non-convex problem, and it is challenging to obtain a global optimal solution. To this end, the PSO-based algorithm is invoked to obtain a suboptimal solution, attributed to its rapid convergence and exemplary global searching capabilities.

First, a population of N_p particles with velocities and positions are initialized. The velocities of the particles are represented by $\mathcal{V}^{(0)} = \{v_1^{(0)}, v_2^{(0)}, \dots, v_{N_p}^{(0)}\}$, which is indicative of the extent of change occurring during the iterative process. The positions of the particles are denoted by $\mathcal{P}^{(0)} = \{p_1^{(0)}, p_2^{(0)}, \dots, p_{N_p}^{(0)}\}$, where each position represents a potential solution for the pre-chirp alphabet, i.e.,

$$p_{n_p}^{(0)} = \mathcal{P}_{c,n_p}^{(0)} = \{c_{2,n_p}^{(0,1)}, c_{2,n_p}^{(0,2)}, \dots, c_{2,n_p}^{(0,\lambda)}\}, \quad (64)$$

in which n_p denotes the index of particle. As an initial solution, the first particle $p_1^{(0)}$ is initialized as a heuristic pre-chirp alphabet, where the elements in $\mathcal{P}_c$ are evenly distributed within the interval $[0, 1]$, and the remaining particles are randomly initialized.

Subsequently, the fitness value of each particle is evaluated per the specified utility function. In light of the constraints imposed by (63b) and (63c), a brick wall penalty factor is introduced, and the utility function in the i_{ter} -th iteration is defined as

$$\mathcal{F}_P(p_{n_p}^{(i_{ter})}) = \begin{cases} \epsilon, & \text{if } p_{n_p}^{(i_{ter})} \text{ is feasible,} \\ -1, & \text{otherwise,} \end{cases} \quad (65)$$

Algorithm 1 PSO-Based Algorithm for c_2 Alphabet Design

Input: $N_p, \varpi, \varrho_{global}, \varrho_{local}, v_{\max}, I_{\max}, \lambda$ and all other parameters required to evaluate fitness function;

Output: $\mathcal{P}_c$;

```

1: Set  $i_{ter}=0$  and initialize  $N_p$  particles with positions  $\mathcal{P}^{(0)}$ 
   and zero velocities  $\mathcal{V}^{(0)}$ ;
2: Calculate fitness values of all particles by (65),  $\mathcal{F}_P(p_{n_p}^{(0)})$ ,
    $n_p = 1, 2, \dots, N_p$ ;
3: Initialize local optimal position of each particle
    $p_{n_p,local} = p_{n_p}^{(0)}$ , calculate global optimal position
    $p_{global} = \arg \max_{1 \leq n_p \leq N_p} (\mathcal{F}_P(p_{n_p}^{(0)}))$ ;
4: while  $i_{ter} \leq I_{\max}$  do
5:   for  $n_p = 1$  to  $N_p$  do
6:     Update velocity  $v_{n_p}^{(i_{ter})}$  according to (66);
7:     for  $i = 1$  to  $\lambda$  do
8:       if  $v_{n_p}^{(i_{ter})}[i] > v_{\max}$  then
9:          $v_{n_p}^{(i_{ter})}[i] \leftarrow v_{\max}$ ;
10:      else if  $v_{n_p}^{(i_{ter})}[i] < -v_{\max}$  then
11:         $v_{n_p}^{(i_{ter})}[i] \leftarrow -v_{\max}$ 
12:      end if
13:    end for
14:    Update position  $p_{n_p}^{(i_{ter})}$  based on (67);
15:    Calculate fitness value  $\mathcal{F}_P(p_{n_p}^{(i_{ter})})$  using (65);
16:    if  $\mathcal{F}_P(p_{n_p}^{(i_{ter})}) > \mathcal{F}_P(p_{n_p,local})$  then
17:       $p_{n_p,local} \leftarrow p_{n_p}^{(i_{ter})}$ ;
18:    end if
19:    if  $\mathcal{F}_P(p_{n_p}^{(i_{ter})}) > \mathcal{F}_P(p_{global})$  then
20:       $p_{global} \leftarrow p_{n_p}^{(i_{ter})}$ ;
21:    end if
22:  end for
23:   $i_{ter} \leftarrow i_{ter} + 1$ ;
24: end while
25: Obtain global optimal position  $\mathcal{P}_c \leftarrow p_{global}$ ;
26: return  $\mathcal{P}_c$ .
```

where $\epsilon = \min_{k,j} O_{k,j}(P_c)$ and $O_{k,j}(P_c)$ is calculated by (61). Then the particle with the greatest fitness value is considered to be the initial global optimal position p_{global} , and the local optimal position $p_{n_p,local}$ of each particle is initialized as $p_{n_p}^{(0)}$.

Each particle conveys its local optimal position to other particles during the iteration. The velocity and position of each particle are updated according to

$$v_{n_p}^{(i_{ter})} = \varpi v_{n_p}^{(i_{ter}-1)} + r_1 \varrho_{local} (p_{n_p,local} - p_{n_p}^{(i_{ter}-1)}) + r_2 \varrho_{global} (p_{global} - p_{n_p}^{(i_{ter}-1)}), \quad (66)$$

$$p_{n_p}^{(i_{ter})} = p_{n_p}^{(i_{ter}-1)} + v_{n_p}^{(i_{ter})}, \quad (67)$$

respectively, where ϖ is the inertia weight, ϱ_{global} and ϱ_{local} denote the global and local updating coefficients, respectively, while r_1 and r_2 are two random values drawn within the interval $[0, 1]$. The particle velocity is constrained to the range between $-v_{\max}$ and $v_{\max}$. Afterward, the fitness values of all the particles are evaluated, and the global and local optimal positions, p_{global} and $\{p_{n_p,local}\}_{n_p=1}^{N_p}$, are updated. The above

TABLE III
SIMULATION PARAMETERS

Parameter	Description	Value
c	Speed of light in free space	3×10^8 m/s
f_c	Carrier frequency	8 GHz
f_s	Subcarrier spacing	1.5 kHz
v_e	Maximum speed of mobile station	202.5/405 km/h
$v_{\max}$	Maximum particle velocity	0.05
$\mathcal{P}_c$	Alphabet of c_2 realizations for $\lambda = 2/3/4$	$\{0.20, 0.60\}$ $\{0.29, 0.62, 0.93\}$ $\{0.01, 0.20, 0.41, 0.80\}$
ϖ	Inertia weight	0.5
N_p	Number of particles	200
$I_{\max}$	Maximum iteration value	300
ϱ_{global}	Global updating coefficients	2
ϱ_{local}	Local updating coefficients	2

update process is repeated until the maximum number $I_{\max}$ of iterations is reached. The particle with the highest fitness value corresponds to the optimization result $\mathcal{P}_c$. The procedure of the proposed PSO-based algorithm is outlined in Algorithm 1.

VI. SIMULATION RESULTS AND ANALYSIS

In this section, we carry out simulations to evaluate the performance of the proposed AFDM-PIM schemes, and conduct the BER performance comparison between the proposed AFDM-PIM scheme and other existing counterparts. The accuracy of the PEP-based theoretical derivation is also investigated in comparison with the Monte Carlo simulation results. In simulations, the carrier frequency is set to $f_c = 8$ GHz, and the subcarrier spacing in the DAF domain is set to $f_s = 1.5$ kHz. Two distinct simulation scenarios are conducted, namely, when the full diversity condition (45) is satisfied and when it is not satisfied. The maximum normalized Doppler shifts for these two scenarios are set to $\alpha_{\max} = 1$ and $\alpha_{\max} = 2$, corresponding to the high-speed scenarios with the maximum speed of mobile station $v_e = 202.5$ km/h and the ultra high-speed scenarios with the maximum speed of mobile station $v_e = 405$ km/h, respectively.¹ The Doppler shift of each path is generated according to Jakes Doppler spectrum approach as $\alpha_p = \alpha_{\max} \cos(\theta_{p,d})$, where $\theta_{p,d} \in [-\pi, \pi]$ (for integer Doppler cases, the Doppler shift is $\lfloor \alpha_{\max} \cos(\theta_{p,d}) \rfloor$). We set the pertinent parameters of Algorithm 1 as $v_{\max} = 0.05$, $\varpi = 0.5$, $N_p = 200$, $I_{\max} = 300$, $\varrho_{\text{global}} = 2$ and $\varrho_{\text{local}} = 2$. The adopted simulation parameters are listed in Table III. Unless otherwise specified, the ML detector is employed for both the proposed AFDM-PIM and the classical benchmarks.

A. Effectiveness of Algorithm 1

We first validate the effectiveness of the proposed c_2 alphabet design with Algorithm 1. Fig. 4 compares the BER performance achieved by the c_2 alphabet designed by Algorithm 1 and that attained by the initial solution, namely, the first initial particle $\mathbf{p}_1^{(0)}$ for Algorithm 1, given two numbers of paths $P = 2$ and 3. Binary PSK (BPSK) is employed as the modulation scheme for the data bits, and the parameter settings are $(N, G, \lambda, d_{\max}, \alpha_{\max}) = (6, 2, 3, 1, 1)$. Since the full diversity order condition is satisfied, the BER performance

¹The relationship between speed and normalized Doppler shift is shown in Appendix C.

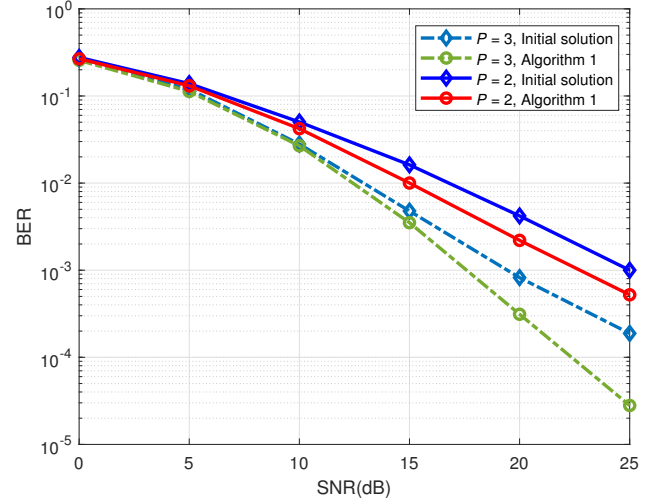


Fig. 4. BER Performance comparison between the c_2 alphabet designed by Algorithm 1 and the initial c_2 solution, given two different numbers of paths.

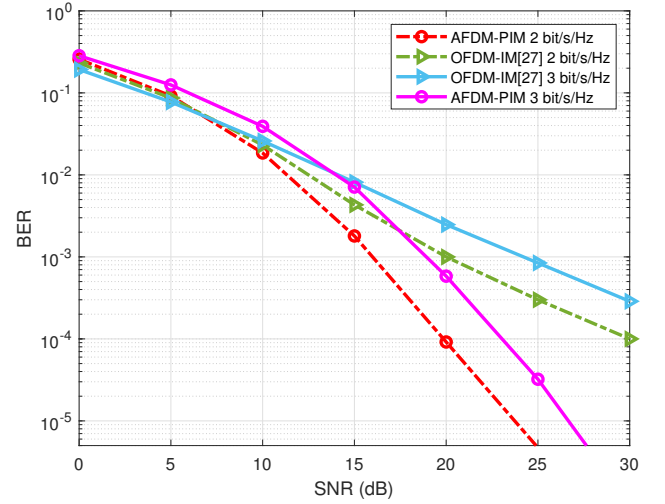


Fig. 5. Performance comparison between the proposed AFDM-PIM scheme and the OFDM-IM scheme given two different spectral efficiencies.

improves with the increase in the number of paths. Fig. 4 indicates that the c_2 alphabet designed by Algorithm 1 exhibits superior performance by about 3 dB in SNR than the initial solution at the BER level of 10^{-3} , given $P = 3$. The results of Fig. 4 demonstrate the effectiveness of Algorithm 1.

B. Performance Comparison with Existing Benchmarks

Fig. 5 compares the BER performance achieved by the proposed AFDM-PIM and the OFDM-IM [33] under a doubly dispersive channel with $P = 4$ and given two SEs of 2 and 3 bit/s/Hz. For the proposed AFDM-PIM, $(N, G, \lambda, d_{\max}, \alpha_{\max})$ are set to $(8, 2, 4, 0, 1)$, and BPSK and quadrature PSK (QPSK) are employed to reach the SEs of 2 and 3 bit/s/Hz, respectively. For the OFDM-IM scheme [33], each group comprises n subcarriers, with a subcarriers activated at each transmission. To reach the SEs of 2 and 3 bit/s/Hz, $(n, a) = (4, 2)$ and $(n, a) = (8, 7)$ are employed, respectively, and 8-PSK is utilized. It can be seen from Fig. 5 that at the BER level of 10^{-3} , our AFDM-PIM attain the SNR gains of about 5 dB and 4 dB over the OFDM-IM for

the cases of $SE = 3$ bits/s/Hz and 2 bits/s/Hz, respectively. This enhanced performance is attributable to the intrinsic advantage of AFDM-PIM, which can achieve full diversity order on doubly dispersive channels.

Fig. 6 compares the BER performance of our AFDM-PIM with that of the classical AFDM scheme [21] under the same doubly dispersive channel with $P = 4$. The proposed AFDM-PIM employs BPSK and QPSK to achieve the SEs of 2 and 3 bit/s/Hz, respectively. The other parameters are as $(N, G, \lambda, d_{\max}, \alpha_{\max}) = (8, 2, 4, 2, 2)$. To reach the same SE levels in the classical AFDM [21], the number of subcarriers is set to 8, and QPSK and 8-PSK are utilized correspondingly. The both schemes do not satisfy the full diversity order condition. Specifically, the proposed AFDM-PIM does not satisfy Condition 1 in Theorem 3, while the AFDM does not satisfy Theorem 1 of [21]. It can be seen from Fig. 6 that the AFDM-PIM demonstrates an about 2 dB gain in the SNR compared to the AFDM scheme at the BER level of 10^{-3} . This is because the AFDM-PIM allows lower order constellations to reach the same SE as the AFDM. The results of Fig. 6 indicates that the AFDM-PIM offers a viable alternative for communication under doubly dispersive channels.

In Fig. 7, we evaluate the BER performance of the proposed AFDM-PIM and the AFDM-IM [40] under a doubly dispersive channel with the number of paths $P = 3$ and given two SEs of 2 and 3 bit/s/Hz. For the proposed AFDM-PIM, $(N, G, \lambda, d_{\max}, \alpha_{\max})$ are set to $(8, 2, 4, 1, 2)$, and BPSK and QPSK are employed to reach the SEs of 2 and 3 bit/s/Hz, respectively. For the AFDM-IM scheme [40], each group comprises n subcarriers, with a subcarriers activated at each transmission. To reach the same SE levels for the AFDM-IM, $(n, a) = (4, 2)$ and $(n, a) = (8, 7)$ are employed, respectively, and 8-PSK is utilized. Note that the both schemes do not meet the full diversity order condition. It can be seen from Fig. 7 that our AFDM-PIM scheme yields better BER performance than the AFDM-IM scheme, with an SNR gain of more than 2 dB at the BER level of 10^{-3} . The enhanced performance is attributable to the capability of our AFDM-PIM to employ lower order constellations than the AFDM-PIM, while reach-

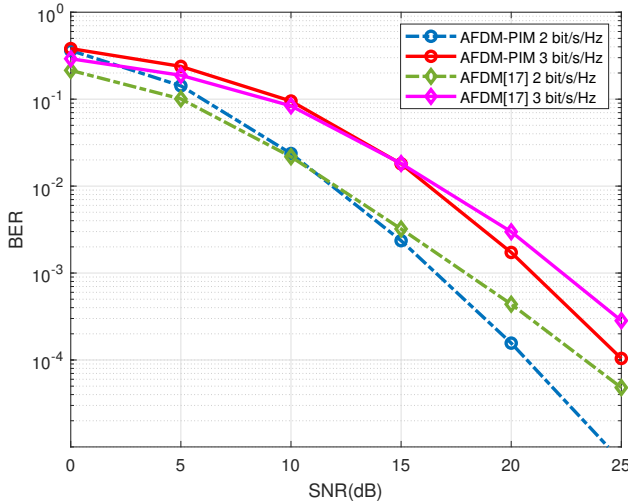


Fig. 6. Performance comparison between the proposed AFDM-PIM scheme and the classical AFDM scheme given two different spectral efficiencies.

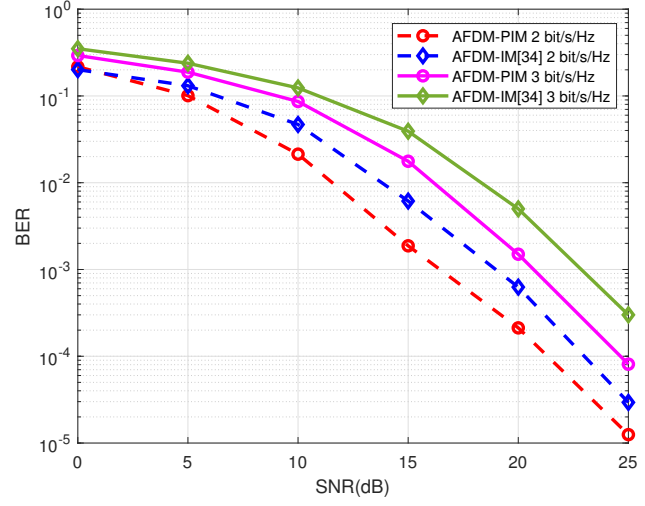


Fig. 7. Performance comparison between the proposed AFDM-PIM scheme and the AFDM-IM scheme given two different spectral efficiencies.

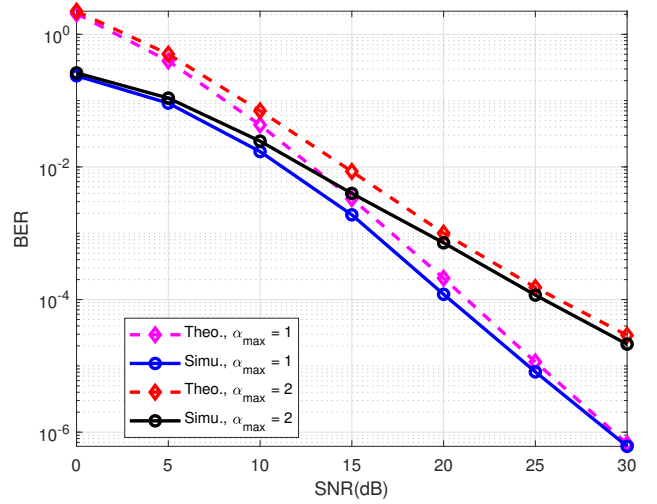


Fig. 8. Comparison of the theoretical ABEP upper bound and the simulated BER of the proposed AFDM-PIM given two different Doppler shifts.

ing the same SE as the AFDM-IM.

C. Accuracy of PEP-based Theoretical Analysis

Fig. 8 presents a comparison of the theoretical ABEP upper bound and the simulated BER of the proposed AFDM-PIM scheme over a doubly dispersive channel with the number of paths $P = 3$. The spectral efficiency of 1.5 bit/s/Hz is considered, $(N, G, \lambda, d_{\max})$ are set to $(4, 2, 2, 0)$ and BPSK is employed as the modulation scheme. Besides, the maximum normalized Doppler shifts are set to $\alpha_{\max} = 1$ and $\alpha_{\max} = 2$, corresponding to the high-speed and ultra high-speed scenarios, respectively. It can be seen from Fig. 8 that in the both cases, the theoretical ABEP results deviate from the simulated results in the low-SNR region. This is because the theoretical ABEP upper bound is subject to several approximations, which inevitably becomes inaccurate when the noise is dominant. On the other hand, the theoretical results closely match the simulated BER curves at sufficiently high SNRs, which demonstrates the validity of the PEP analysis for the AFDM-PIM presented in Subsection IV-A.

$$\begin{bmatrix} \hat{H}_a[0, \text{loc}_a] \hat{x}_{\text{loc}_a} - H_a[0, \text{loc}_a] x_{\text{loc}_a} \\ \hat{H}_a[1, (\text{loc}_a + 1)_N] \hat{x}_{(\text{loc}_a + 1)_N} - H_a[1, (\text{loc}_a + 1)_N] x_{(\text{loc}_a + 1)_N} \\ \vdots \\ \hat{H}_a[N - 1, (\text{loc}_a + N - 1)_N] \hat{x}_{(\text{loc}_a + N - 1)_N} - H_a[N - 1, (\text{loc}_a + N - 1)_N] x_{(\text{loc}_a + N - 1)_N} \end{bmatrix}, \quad (70)$$

$$\begin{bmatrix} \hat{H}_b[0, \text{loc}_b] \hat{x}_{\text{loc}_b} - H_b[0, \text{loc}_b] x_{\text{loc}_b} \\ \hat{H}_b[1, (\text{loc}_b + 1)_N] \hat{x}_{(\text{loc}_b + 1)_N} - H_b[1, (\text{loc}_b + 1)_N] x_{(\text{loc}_b + 1)_N} \\ \vdots \\ \hat{H}_b[N - 1, (\text{loc}_b + N - 1)_N] \hat{x}_{(\text{loc}_b + N - 1)_N} - H_b[N - 1, (\text{loc}_b + N - 1)_N] x_{(\text{loc}_b + N - 1)_N} \end{bmatrix}, \quad (71)$$

VII. CONCLUSIONS

In this paper, we have proposed the novel AFDM-PIM scheme to combat time-frequency doubly selective channel fading. In our proposed scheme, the pre-chirp parameters on subcarriers are no longer fixed but are selected from a predefined alphabet. This enables additional bits to be transmitted through indexing the specific pre-chirp parameter values on subcarriers, thereby enhancing both spectrum and energy efficiency. The PEP-based theoretical BER upper bound for the proposed AFDM-PIM scheme with the ML detection has been derived and verified through simulations, and the full diversity order conditions of the proposed AFDM-PIM under doubly dispersive channels have been derived. Furthermore, we have presented a PSO-based optimization algorithm to design the pre-chirp parameter alphabet. Both analytical and simulation results have demonstrated that the proposed AFDM-PIM exhibits enhanced spectral efficiency and superior error performance in comparison to classical multi-carrier modulation schemes.

APPENDIX

A. Proof of Theorem 3

Proof. First, we show that Condition 1 is necessary for the AFDM-PIM to achieve the full diversity order. Assume that Condition 1 is not met, i.e.,

$$P > (d_{\max} + 1)(2\alpha_{\max} + 1). \quad (68)$$

Under this assumption, according to the definition of $\text{loc}_p = (\alpha_p + 2Nc_1d_p)_N$, the following situation must be true

$$\exists a, b \in [1, \dots, P], a \neq b, \text{ such that } \text{loc}_a = \text{loc}_b. \quad (69)$$

The corresponding two columns in the matrix $\Phi(\delta)$ can be expressed as (70) and (71) at the top of this page, where $\hat{H}_{(\cdot)}[\cdot] \hat{x}_{(\cdot)}$ and $H_{(\cdot)}[\cdot] x_{(\cdot)}$ represent the corresponding elements in $\Phi(\hat{\mathbf{x}})$ and $\Phi(\mathbf{x})$, respectively.

Based on (27), the positions of the non-zero entries in the matrices $\mathbf{H}_a, \hat{\mathbf{H}}_a, \mathbf{H}_b$ and $\hat{\mathbf{H}}_b$ are consistent. In the instances, such as when both $\Phi(\hat{\mathbf{x}})$ and $\Phi(\mathbf{x})$ contain only a single

non-zero element at the same position, (70) and (71) are linearly correlated. Therefore, $\Phi(\delta)$ cannot be full rank, i.e., the assumption (68) is false.

Besides, when $\Phi(\delta)$ achieves the full diversity order, the channel paths with different delay values or distinct Doppler frequency shifts are distinguished within the DAFT domain, as illustrated in Fig. 3, i.e., $(d_{\max} + 1)(2\alpha_{\max} + 1) \leq N$. Therefore, Condition 1 is a prerequisite for achieving the full diversity order.

Next we prove that Condition 2 can ensure that $\Phi(\delta)$ has the full rank. $\Phi(\delta)$ can be expressed as $\Phi(\delta) = \hat{\Phi}(\hat{\mathbf{x}}) - \Phi(\mathbf{x}) = [\gamma_1, \dots, \gamma_P]$, where $\gamma_p \in \mathbb{C}^{N \times 1}$ with the entries $\gamma_p[n] = H_p[n, (\text{loc}_p + n)_N] \hat{x}_{(\text{loc}_p + n)_N} - H_p[n, (\text{loc}_p + n)_N] x_{(\text{loc}_p + n)_N}$, $p = 1, 2, \dots, P$, $n = 0, 1, \dots, N - 1$.²

Given constants $\ell_{n,m}$, $\hat{H}_p[n, \text{loc}_p + n]$ can be expressed as $\hat{H}_p[n, \text{loc}_p + n] = \ell_{n,m} H_p[n, \text{loc}_p + n]$. According to (27), the entries of γ_p are given by

$$\begin{aligned} \gamma_p[n] &= H_p[n, \text{loc}_p + n] \rho_{n,m} \\ &= e^{j2\pi[c_{2,\text{loc}_p+n}(\text{loc}_p + n)^2 - c_{2,n}n^2]} \\ &\quad \times e^{j\frac{2\pi}{N}[-(\text{loc}_p + n) + c_1d_p^2]} \rho_{n,m}, \end{aligned} \quad (72)$$

where $\rho_{n,m} = (x_{\text{loc}_p+n} - \ell_{m,n} \hat{x}_{\text{loc}_p+n})$. Now assume that a set of numbers $\{\beta_p\}_{p=1}^P$ that are not all zero satisfy

$$\sum_{p=1}^P \beta_p \gamma_p = \mathbf{0}. \quad (73)$$

Without loss of generality, β_b is assumed to be the non-zero number, $b \in \{1, 2, \dots, P\}$. The relationship for any index n can be obtained as

$$\sum_{p=1}^P \beta_p \gamma_p[n] = \sum_{p=1}^P \frac{\beta_p}{\beta_b} \gamma_p[n] = 0, \quad \forall n. \quad (74)$$

²For the convenience of typesetting, we use $\text{loc}_p + n$ to represent $(\text{loc}_p + n)_N$ throughout the following text.

$$\begin{aligned} \|(\Phi_k^r(\mathbf{x}') - \Phi_j^r(\mathbf{x}))\|_F^2 &= \sum_{p=1}^P \sum_{n=1}^N \left| H'_p[n, \text{loc}_p + n] x'_{\text{loc}_p+n} - H_p[n, \text{loc}_p + n] x_{\text{loc}_p+n} \right|^2 \\ &= \sum_{p=1}^P \sum_{n=1}^N \left| e^{j\frac{2\pi}{N}[Nc'_{2,\text{loc}_p+n}(\text{loc}_p + n)^2 - Nc'_{2,n}n^2]} x'_{\text{loc}_p+n} - e^{j\frac{2\pi}{N}[Nc_{2,\text{loc}_p+n}(\text{loc}_p + n)^2 - Nc_{2,n}n^2]} x_{\text{loc}_p+n} \right|^2, \end{aligned} \quad (80)$$

From (74), it is not difficult to obtain

$$\begin{aligned}\rho_{n,b} &= - \sum_{p=1, p \neq b}^P \frac{\beta_p}{\beta_b} \frac{\gamma_p[n]}{H_p[n, \text{loc}_p + n]} \\ &= -e^{-j2\pi c_{2,\text{loc}_b} \text{loc}_b^2} e^{j\frac{2\pi}{N} (Nc_1(-d_b^2 + \text{loc}_b d_b))} \\ &\quad \times \sum_{p=1, p \neq b}^P e^{j2\pi (c_{2,\text{loc}_p+n} (\text{loc}_p+n)^2 + c_1 d_p^2 - \frac{\text{loc}_p d_p}{N})} \frac{\beta_p}{\beta_b} \rho_{n,m}. \quad (75)\end{aligned}$$

Since $\rho_{0,b}$ is a signal error term unrelated to $c_{2,n}$, the phase of $\frac{\beta_p}{\beta_b} (m \neq b)$ must contain $2\pi (c_{2,\text{loc}_b} \text{loc}_b^2 - c_{2,\text{loc}_p+n} (\text{loc}_p+n)^2)$ to eliminate the influence of irrational numbers. On the other hand, if (75) holds, there exists another non-zero β_a ($a \neq b$). Similar to the derivation process of (75), the phase of $\frac{\beta_p}{\beta_a} (m \neq a)$ must contain $2\pi (c_{2,\text{loc}_a} \text{loc}_a^2 - c_{2,\text{loc}_p+n} (\text{loc}_p+n)^2)$. Hence, the relationship between $\frac{\beta_a}{\beta_b}$ and $\frac{\beta_b}{\beta_a}$ can be obtained as $\frac{\beta_a}{\beta_b} \cdot \frac{\beta_b}{\beta_a} = 1$, i.e.,

$$\begin{aligned}e^{j2\pi c_{2,\text{loc}_b} \text{loc}_b^2} e^{-j2\pi c_{2,\text{loc}_a+n} (\text{loc}_a+n)^2} \\ \times e^{j2\pi c_{2,\text{loc}_a} \text{loc}_a^2} e^{-j2\pi c_{2,\text{loc}_a+n} (\text{loc}_a+n)^2} \vartheta = 1, \quad \forall n, \quad (76)\end{aligned}$$

where ϑ is a complex number whose phase does not contain $c_{2,n}$. Given that all the $c_{2,n}$ are irrational numbers, the phase on the left-hand side of (76) cannot be an integer multiplying 2π . This implies that the imaginary part is not zero, and (76) does not hold. Consequently, the assumption of (73) is invalid, which means that the column vectors of the matrix $\Phi(\delta)$ are linearly independent, i.e., the rank of $\Phi(\delta)$ is P . Combining this result with the definition of diversity order (44), it is concluded that the AFDM-PIM can achieve the full diversity order. $\square$

B. Derivation of $O_{k,j}(\mathcal{P}_c)$ (54)

Substituting $\Phi_k^r(\mathbf{x}) = [\mathbf{H}_1 \mathbf{x}, \dots, \mathbf{H}_P \mathbf{x}]$ and $\Phi_k^r(\mathbf{x}') = [\mathbf{H}'_1 \mathbf{x}', \dots, \mathbf{H}'_P \mathbf{x}']$ into (50) leads to

$$O_{k,j}(\mathcal{P}_c) = \sum_{\mathbf{x}', \mathbf{x}} \sum_{r=1}^R \sum_{p=1}^P \|\mathbf{H}'_p \mathbf{x}' - \mathbf{H}_p \mathbf{x}\|_2. \quad (77)$$

According to (27), the elements of $\mathbf{H}_p \mathbf{x}$ can be expressed as

$$\begin{aligned}\mathbf{H}_p \mathbf{x}[n] &= H_p[n, \text{loc}_p + n] x_{\text{loc}_p + n} \\ &= e^{j2\pi (c_{2,\text{loc}_p+n} (\text{loc}_p+n)^2 - c_{2,n} n^2 d_p)} \\ &\quad \times e^{j\frac{2\pi}{N} (-(\text{loc}_p+n) + c_1 d_p^2)} x_{\text{loc}_p + n}. \quad (78)\end{aligned}$$

Moreover, the elements of $\mathbf{H}'_p \mathbf{x}' - \mathbf{H}_p \mathbf{x}$ can be expressed as

$$\begin{aligned}(\mathbf{H}'_p \mathbf{x}' - \mathbf{H}_p \mathbf{x})[n] &= H'_p[n, \text{loc}_p + n] x'_{\text{loc}_p + n} \\ &\quad - H_p[n, \text{loc}_p + n] x_{\text{loc}_p + n} \\ &= \left(e^{j\frac{2\pi}{N} (Nc'_{2,\text{loc}_p+n} (\text{loc}_p+n)^2 - Nc'_{2,n} n^2)} x'_{\text{loc}_p + n} \right. \\ &\quad \left. - e^{j\frac{2\pi}{N} (Nc_{2,\text{loc}_p+n} (\text{loc}_p+n)^2 - Nc_{2,n} n^2)} x_{\text{loc}_p + n} \right) \\ &\quad \times e^{j\frac{2\pi}{N} (-(\text{loc}_p+n) d_p + Nc_1 d_p^2)}. \quad (79)\end{aligned}$$

Therefore, the norm of $\|(\Phi_k^r(\mathbf{x}') - \Phi_k^r(\mathbf{x}))\|_F^2$ in (50) can be calculated as (80), shown at the bottom of last page, where

x'_{loc_p+n} and x_{loc_p+n} are the elements of $\mathbf{x}'$ and $\mathbf{x}$, respectively. We define $x_{\text{loc}_p+n} = r_{p,n} e^{j\vartheta_{p,n}}$ and $x'_{\text{loc}_p+n} = r'_{p,n} e^{j\vartheta'_{p,n}}$.

Utilizing the formula

$$|a - b|^2 = |a|^2 - 2\Re(ab^*) + |b|^2, \quad (81)$$

where a and b are any complex numbers, (80) can be expressed as

$$\begin{aligned}\|(\Phi_k^r(\mathbf{x}') - \Phi_k^r(\mathbf{x}))\|_F^2 &= \sum_{p=1}^P \sum_{n=1}^N \left| e^{j\vartheta'_{p,n}} x'_{\text{loc}_p+n} - e^{j\vartheta_{p,n}} x_{\text{loc}_p+n} \right|^2 \\ &= \sum_{p=1}^P \sum_{n=1}^N \left(|x_{\text{loc}_p+n}|^2 + |x'_{\text{loc}_p+n}|^2 \right. \\ &\quad \left. - 2\Re \left(x'_{\text{loc}_p+n} (x_{\text{loc}_p+n})^* e^{j(\vartheta'_{p,n} - \vartheta_{p,n})} \right) \right) \\ &= \sum_{p=1}^P \sum_{n=1}^N (r_{p,n}^2 + r'_{p,n}{}^2 \\ &\quad - 2r_{p,n} r'_{p,n} \Re \left(e^{j(\vartheta'_{p,n} - \vartheta_{p,n} + \vartheta'_{p,n} - \vartheta_{p,n})} \right)), \quad (82)\end{aligned}$$

where $\vartheta'_{p,n}$ and $\vartheta_{p,n}$ are given by (52).

According to Euler's formula, we further simplify (82) as

$$\|(\Phi_k^r(\mathbf{x}') - \Phi_k^r(\mathbf{x}))\|_F^2 = \sum_{p=1}^P \sum_{n=1}^N F_{p,n}, \quad (83)$$

where $F_{p,n}$ is given by (53). Substituting (83) into (50) leads to $O_{k,j}(\mathcal{P}_c)$ of (54).

C. Relationship between Speed and Normalized Doppler Shift

Let the maximum speed of the mobile station be v_e , the carrier frequency be f_c , and the speed of light be denoted by c . Then the maximum Doppler frequency $\nu_{\max}$ is given by

$$\nu_{\max} = \frac{v_e f_c}{c}. \quad (84)$$

As established in Section II, the maximum normalized Doppler shift is obtained through the process of normalization as

$$\alpha_{\max} = N T \nu_{\max} = \frac{\nu_{\max}}{f_s} = \frac{v_e f_c}{c f_s}. \quad (85)$$

where f_s represents the subcarrier spacing.

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