Supplementary Information for "Grasping New Material Densities".

S1. Relationships between log density ratio, centre of mass, object mass and torque.

The object's CoM depends on the geometric centres of the two components (GC_{Steel}, GC_{PVC}), the density of the two materials (ρ_{Steel}, ρ_{PVC}) and their volumes (Vol_{Steel}, Vol_{PVC}):

$$CoM = \frac{GC_{Steel} \times Vol_{Steel} \times \rho_{Steel} + GC_{PVC} \times Vol_{PVC} \times \rho_{PVC}}{Vol_{Steel} \times \rho_{Steel} + Vol_{PVC} \times \rho_{PVC}}$$

More simply, the CoM can be determined from the ratio of the two densities, i.e. let $\rho_{Steel} = k \times \rho_{PVC}$

$$CoM = \frac{GC_{Steel} \times Vol_{Steel} \times k \times \rho_{PVC} + GC_{PVC} \times Vol_{PVC} \times \rho_{PVC}}{Vol_{Steel} \times k \times \rho_{PVC} + Vol_{PVC} \times \rho_{PVC}}$$

$$= \frac{GC_{Steel} \times Vol_{Steel} \times k + GC_{PVC} \times Vol_{PVC}}{Vol_{Steel} \times k + Vol_{PVC}}$$

Similarly, only the ***ratio*** of the two volumes (or the ratio of the two lengths, given that length ∝ volume for cylinders) is required to determine the CoM. Let $Length_{Steel} = l \times Length_{PVC}$, then

$$CoM = \frac{GC_{Steel} \times Length_{PVC} \times l \times k + GC_{PVC} \times Length_{PVC}}{Length_{PVC} \times l \times k + Length_{PVC}}$$

$$CoM = \frac{GC_{Steel} \times l \times k + GC_{PVC}}{l \times k + 1}$$

As the LDR increases or decreases, the object's CoM asymptotes to the GC of one of the two components:

$$As\ LDR \to \infty,\ \ k \to \infty,\ \ CoM \to \frac{GC_{Steel} \times l \times k}{l \times k} = GC_{Steel}$$

$$As\ LDR \to -\infty,\ \ k \to 0,\ \ CoM \to \frac{GC_{PVC}}{1} = GC_{PVC}$$

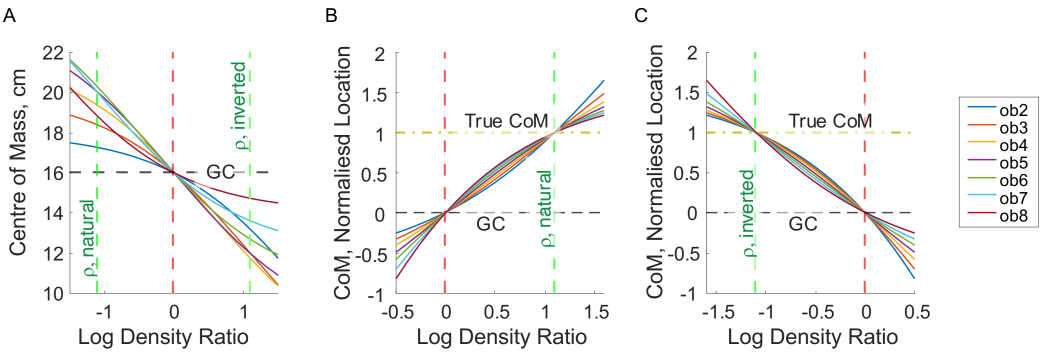

Figure S1. *Log density ratio and centre of mass.* ***(A)*** *Relationship between the log density ratio and each object's centre of mass, in cm.* ***(B)*** *Natural Density stimuli: the relationship between log density ratio and the CoM, in units of normalised location.* ***(C)*** *Inverted Density stimulus set. The normalised location metric allows us to combine grasping data from different objects within a common scale.*

S2. Estimation of material densities (or their ratio) from the forces experienced on lifting the object.

The torque, τ, experienced on lifting a stimulus object depends on the grasp position ($Grasp$), the geometric centres of the two components, their volume, and density:

$$\tau = (GC_{Steel} - Grasp) \times Vol_{Steel} \times \rho_{Steel} + (GC_{PVC} - Grasp) \times Vol_{PVC} \times \rho_{PVC}$$

The total object mass depends on the volume and density of the two parts:

$$Mass = Vol_{Steel} \times \rho_{Steel} + Vol_{PVC} \times \rho_{PVC}$$

Rearranging these equations gives:

$$\rho_{PVC} = \left(Mass - \frac{\tau}{(GC_{Steel} - Grasp)}\right) / Vol_{PVC} \times \left(1 - \frac{(GC_{PVC} - Grasp)}{(GC_{Steel} - Grasp)}\right)$$

$$\rho_{Steel} = (Mass - \rho_{PVC} \times Vol_{PVC})/Vol_{Steel}$$

Note that if $Length_{Steel}$ and $Length_{PVC}$ are substituted for Vol_{Steel} and Vol_{PVC}, the ***ratio*** of the two densities will remain correct.

S3: Model details

Fitted parameters: means and (std) for the preferred model (Model 1). Subsequent columns show the parameters of alternative models (p1-p4) and how these compare to the preferred model.

Model	1						1_lin	2	2_lin	3	3_lin	4	4_lin
		Exp1	Exp 2A	Exp 2B	Exp 3A	Exp 3B							
LDR_{start}	0						0	0	0	0	0	p1	p1
LDR_{end}	p1	0.88 (0.44)	0.81 (0.56)	-0.8 (0.55)	-0.93 (0.65)	0.90 (0.40)	p1	p1	p1	p1	p1	p2	p2
r_A	p2	1.17 (1.31)	0.44 (0.34)	1.46 (1.15)	0.36 (0.44)	1.50 (1.04)	p2	p2	p2	p2	p2	p3	p3
σ_N	p3	1.58 (0.39)	1.70 (0.80)	1.78 (0.63)	1.60 (0.47)	1.68 (0.55)	p3	p3	p3	0	0	p4	p4
σ_{LDR}	0						0	p4	p4	p3	p3	0	0
Log likelihood		133.9 (11.2)	135.1 (15.3)	136.9 (15.6)	133.6 (14.2)	136.1 (14.2)							
Learning space	log						linear	log	linear	log	linear	log	linear
N subs preferred							6, 9, 5, 13, 7*	0, 1, 0	0, 0, 0	5, 6, 2	5, 4, 1	3, 5, 7, 1, 5	5, 2, 4, 5, 5
vs. model							1	1	1_lin	1	1_lin	1	1_lin

***Table S1**. Model parameters and comparisons. Model 1 is the preferred model, presented in the manuscript. The fitted parameters (those maximising the log likelihood of the data) are presented for each experiment part. Alternative models are presented in terms of the free parameters and the learning space, i.e., whether learning of density ratio followed exponential trajectory in log space, i.e. log(ρ_{steel} /ρ_{PVC}) or linear space (shaded columns). For each alternative modle, N subs preferred gives the number of subjects (of 20) for which the alternative model was preferred. For models of equal complexity (e.g. 1 vs 1_lin), this is simply a comparison of log likelihoods. For models of different complexity, the comparison was made via F ratio tests. N subs preferred is given either for (i) Expts. 1, 2A, 2B, 3A and 3B or (ii) only for Expts. 1, 2A and 3A. *Note that there was little difference in log likelihood between model 1 and model 1_lin: less than 1% for all subjects.*

S4. Post-hoc comparisons for learning rate, following ANOVA, as shown in Fig. 6A

Group 1	Group 2	Mean Difference	Upper CI	Lower CI	p-value
Expt 1	Expt 2a	0.6363	-0.2484	1.5211	0.274
Expt 1	Expt 2b	-0.4577	-1.3425	0.427	0.6045
Expt 1	Expt 3a	1.1439	0.2591	2.0287	0.0046
Expt 1	Expt 3b	-0.5307	-1.4154	0.3541	0.4585
Expt 2a	Expt 2b	-1.0941	-1.9788	-0.2093	0.0076
Expt 2a	Expt 3a	0.5076	-0.3772	1.3923	0.5041
Expt 2a	Expt 3b	-1.167	-2.0518	-0.2823	0.0036
Expt 2b	Expt 3a	1.6016	0.7169	2.4864	0
Expt 2b	Expt 3b	-0.073	-0.9577	0.8118	0.9994
Expt 3a	Expt 3b	-1.6746	-2.5593	-0.7898	0

S5. ANOVA details for orientation effect, as shown in Fig. 8

Separate 2 factor repeated measures ANOVAs (object x orientation) per experiment show significant effects of both object and orientation on grasping position within each experiment.

Expt. 1: Effects of object: $F_{6,266}$=13.0, p<0.001 and orientation: $F_{1,266}$=7.2, p<0.01
Expt. 2A: Effects of object: $F_{6,266}$=5.5, p<0.001 and orientation: $F_{1,266}$=7.6, p<0.01
Expt. 2B: Effects of object: $F_{6,266}$=6.0, p<0.001 and orientation: $F_{1,266}$=5.71, p<0.05
Expt. 3A: Effects of object: $F_{6,266}$=3.7, p<0.01 and orientation: $F_{1,266}$=19.4, p<0.001
Expt. 3B: Effects of object: $F_{6,266}$=8.8, p<0.001 and orientation: $F_{1,266}$=8.3, p<0.01