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Optical superresolution through super-oscillations

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Abstract.

We demonstrate that a quasi-periodic array of nano-holes in a metal screen can focus light into subwavelength spots in the far field without contributions from evanescent fields. The subwavelength spots were observed with a conventional optical microscope and mapped to the far-field. We relate the formation of subwavelength light localizations in the far field to the phenomenon of superoscillations. This effect offers a new way to achieve subwavelength imaging, which differs from approaches based on the recovery of evanescent fields.

Keywords: superresolution, superoscillation, Quasicrystal, diffraction

1. Introduction: Superresolution and super-oscillation

Most ideas for achieving super-resolution in optics are based on the recovery of evanescent fields which are commonly believed to be the necessary components to form subwavelength field concentrations. For instance, the famous Veslago-Pendry idea for a far-field super-lens is based on the recovery of evanescent information by "amplifying" the fading evanescent components in a slab of a negative-index material [1]. Optical superlenses capable of imaging an object from the near-field on one side of the "lense" to the near field on its other side [2], and from the near field to the far field [3, 4] have been demonstrated. However, a recent remarkable theoretical discovery suggests that evanescent fields may *not* be needed to achieve subwavelength concentration of light in the far field: Berry and Popescu predicted that diffraction on a grating structure could create subwavelength localizations of light that propagate further into the far field than more familiar evanescent waves [5]. They relate this effect to the fact that band-limited functions are able to oscillate arbitrarily faster than the highest Fourier components they contain, a phenomenon called superoscillations [6, 7, 8].

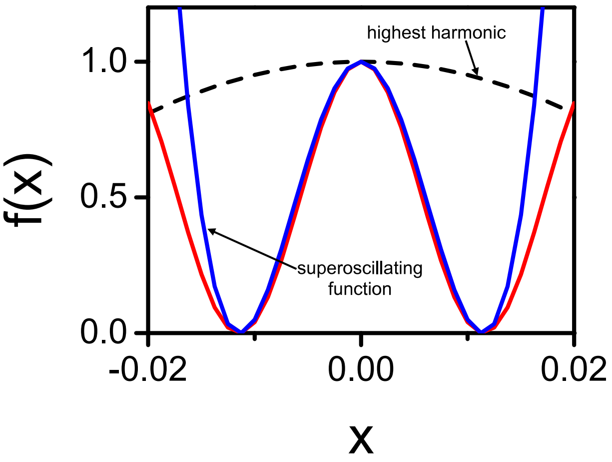


Figure 1. A function, super-oscillating at $x = 0$ (blue line). The dashed black line shows the highest Fourier harmonic of the Fourier spectrum of this function. The super-oscillating feature changes nearly 9 times faster than the highest harmonic.

A simple example of a superoscillating function is a limited series of harmonics such as

$$f(x) = \sum a_n \cos(2\pi n x). \tag{1}$$

By appropriately choosing the constants a_n of the series, one is able to create features which oscillate much faster than the highest frequency component. Consider, for example, $f(x)$ with $a_0 = 1$, $a_1 = 13295000$, $a_2 = -30802818$, $a_3 = 26581909$, $a_4 = -10836909$, $a_5 = 1762818$ and $a_n = 0$ for $n > 5$. It is plotted on Fig. 1, solid blue curve. In comparison, we also plot function $f_{max}(x) = cos(2\pi \times 5x)$, which is the highest frequency component of $f(x)$ (black dashed line). One can observe that at $x = 0$ function $f(x)$ has a feature that oscillates much faster than $f_{max}(x)$. Actually, near $x = 0$ function

$f(x)$ can be well approximated by a harmonic function $f_+(x)$=0.5($\cos(2\pi \times 43.6x)+1$) (solid red curve) oscillating nearly 9 times faster than the highest harmonic of $f(x)$.

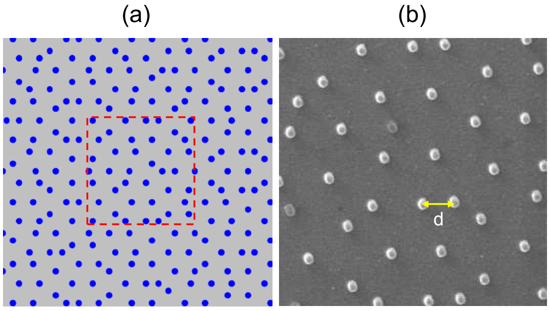


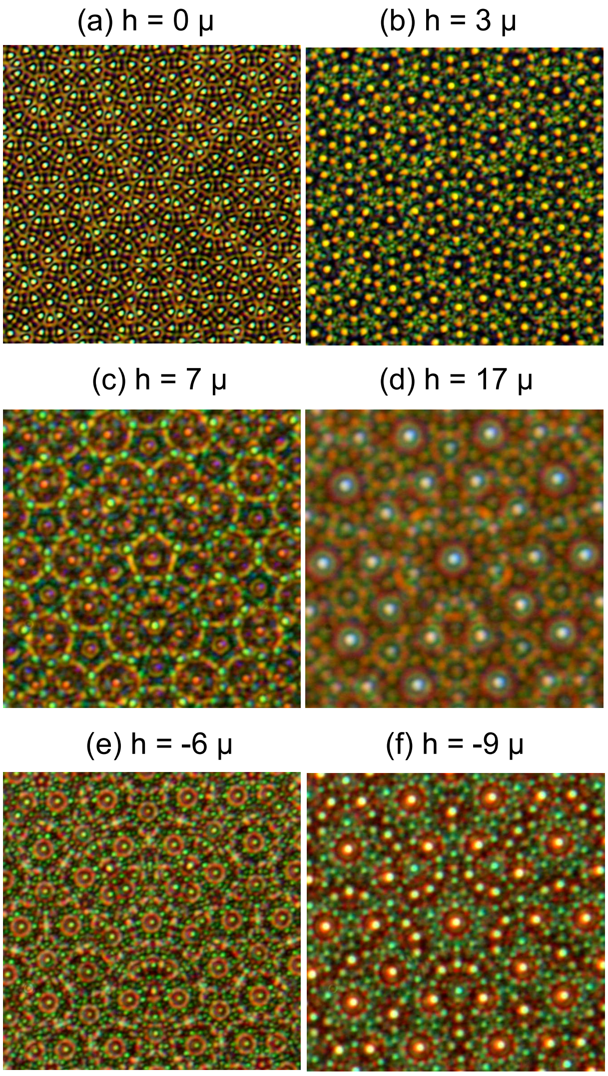
Figure 2. Quasi-crystal sample: (a) A fragment of a Penrose-like quasi-periodic pattern of holes; (b) SEM image of the fragment similar to quasi-periodic nanohle array marked in (a). The minimum distance between two neighbouring holes is $d = 1.2\mu m$.

2. Subwavelength spots imaged by a conventional optical microscope

Recently we saw the first evidence of superoscillation in optics: using a scanning subwavelength aperture we mapped subwavelength localizations generated by diffraction of monochromatic light on a quasi-periodic array of nanoholes on a metal screen [9], i.e. we essentially detected the sub-wavelength features in the *near-field* of a probe. In this paper we make a substantial step forward and show that the subwavelength features can be imaged by a *conventional lens* and projected into the *far-field* with high amplification. We relate the effect to the super-oscillation phenomenon. We therefore demonstrate a new way of achieving subwavelength concentration of light and its mapping into the far-field, which is different from the approaches based on the recovery of evanescent fields [1, 3]. We also show that a quasi-periodic array generates incredibly rich diffraction patterns when illuminated with white light or coherent sources, a phenomenon analogue to the classical Talbot effect observed on periodic structures [10]. Particularly we show the formation of patterns with sparsely distributed bright hot-spots receiving the majority of the light energy transmitted by the array, indicating that such an array may be used as a far-field sub-wavelength focusing device.

These phenomena were observed with a quasi-periodic array of nano-holes arranged in a famous Penrose-like pattern (see Fig. 2(a)) which was known in Eastern design for about 500 years [11]. The array had approximate 10-fold symmetry and contained about $14,000$ holes of 200 nm diameter with the minimum distance between two neighboring holes $d = 1.2$ μm(see Fig. 2(b)). The overall diameter of the array was ~ 0.2 mm. It was manufactured by electron beam lithography in a 100 nm aluminium film deposited on a silica substrate.

When the structure was illuminated with a white light source, it created colorful, quasi-periodic patterns at different heights from the array (see Fig. 3). The patterns were imaged using the CCD array of a digital camera through an optical system


(a) h = 0 μ

(b) h = 3 μ

(c) h = 7 μ

(d) h = 17 μ

(e) h = -6 μ

(f) h = -9 μ

Figure 3. Colorful "photonic carpets" observed with a conventional optical microscope focused at different heights h above the quasi-periodic nanohole array when it is illuminated with a thermal white light source. The image size is 35×35 μm^2.

comprising a high-numerical aperture objective lens (NA = 0.7, 50×) and another lens with a magnification of 2.5. The imaging system was focused at different heights h above the array. In what follows, negative values of h correspond to focal points below the structure. The patterns are intensely colored and even show "white" spots at certain heights (see Fig. 3) (d, f)).

Monochromatic "photonic carpets" can also be seen with laser illumination (in our experiments we used the spectrally filtered emission of a crystal-fiber super-continuum source). Fig. 4(a) shows the image obtained when the microscope (high numerical aperture lens NA = 0.95, 150×) is focused on the quasi-periodic hole array. As the focus is moved away from the plane of the sample the diffraction pattern evolves dramatically (Figs. 4(b-f)). An important feature, previously unobserved in regular patterns, is that the quasi-crystal array can produce large dark fields with sparsely distributed hot-spots

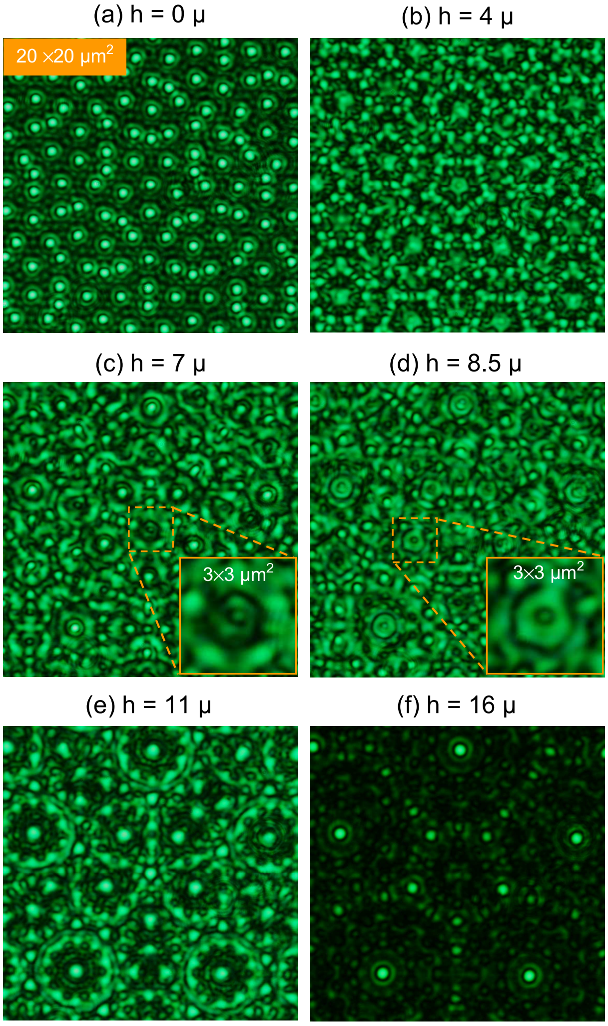


Figure 4. Monochromatic "photonic carpets" observed with a conventional optical microscope focused at different heights h above the quasi-periodic nano-hole array when it is illuminated with coherent light at a wavelength $\lambda = 500\ nm$. Areas $(3 \times 3 \mu m^2)$ containing subwavelength spots are zoomed in image (c) and (d), with the diameters of the central small spots of $\sim 420\ nm$ (c) and $\sim 340\ nm$ (d), respectively. Well-isolated hot-spots with diameter of $\sim 900\ nm$ are seen at distance $h = 16\ \mu m$. In all cases the image size is $20 \times 20\ \mu m^2$.

of energy concentration (Fig. 4(f), $h = 16\ \mu$). The density of the hot-spots is about 40 times smaller than that of the nano-holes in the array (compare Fig. 4(a) with 4(f)), so they actually harvest the majority of the energy from the light transmitted through a number of holes in the array.

Numerous sub-wavelength features can be seen in Fig. 4(b-e). It may look surprising that field structures smaller than the Rayleigh resolution limit were observed with a conventional optical microscope. Indeed, for a lense with NA=0.95 at $\lambda = 500nm$ the Rayleigh resolution limit for a feature with cylindrical symmetry is 642 nm ($1.22\ \lambda$ /NA). We therefore conclude that the quasi-crystal array of holes is a generator of super-oscillating field as recently suggested in [5]. Berry and Popescu theoretically predicted

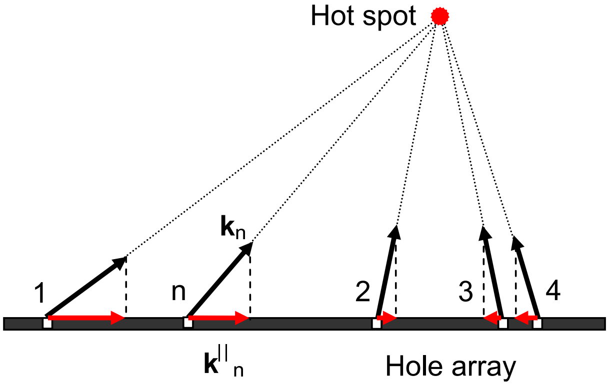


Figure 5. Formation of a super-oscillating feature by light diffracted on the hole array.

that a complex optical grating structure could create super-oscillating fields with sub-wavelength features propagating into the far-field without the need for evanescent components. Indeed, as sketched in Fig. 5, the field cross-section at the hot-spot along the direction parallel to the plane of the array may be presented as a superposition of partial waves emanated from individual holes of the quasi-crystal array:

$$E(x) = \sum a_n \cos(k_n^{||} x + \varphi_n). \tag{2}$$

Such a superposition resembles the structure of superoscillting function (1) : for a certain combination of partial amplitudes a_n, spatial frequencies $k_n^{||}$, and phases φ_n, superoscillation features are created, similar to that illustrated in Fig. 1.

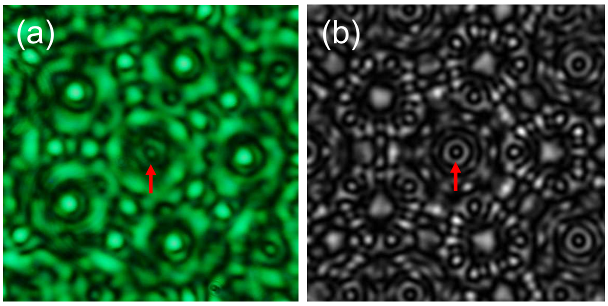


Figure 6. Details of the diffraction pattern at a height $h = 7\ \mu m$.(a) Fragment of the optical microscope image at a height $h = 7 \pm (0.1)\ \mu m$. The image size is $10 \times 10\ \mu m^2$. (b) The corresponding calculated pattern at $h = 7.1\ \mu m$.

3. Simulations

To confirm that the observed subwavelength spots were indeed formed only by propagating waves, we performed simulations of the diffraction pattern near the subwavelength spots at different distances z from the array and compared them with the microscope observations (see Fig. 6). The simulations are based on a slightly more

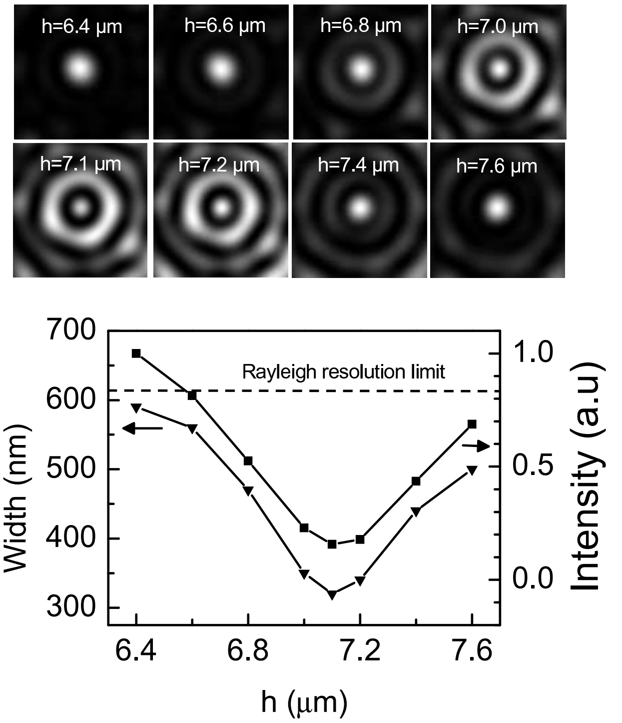


Figure 7. "Focusing" by the nano-hole array: Evolution of a subwavlength spot at different heights above a quasi-periodic nanohole array. Top: diffraction patterns at different heights to the array. Bottom: the width and relative intensity of the central spot. The dashed line indicates the position of Rayleigh resolution limit.

sophisticated version of formula (2). It is given by the diffraction theory employing angular spectrum representation of the diffracted fields, with only propagating waves involved:

$$E(x,y,z) = \int F(u,v) e^{-i(ux+vy)} e^{iz\sqrt{k^2-u^2-v^2}} dudv, \tag{3}$$

i.e. for $u^2 + v^2 \leq k^2$. Here k is the wavevector of light and $F(u,v)$ is the Fourier component of the electromagnetic field at the plane of the array, at $z = 0$. In our calculation only an area of the array containing about $\sim$ 530 holes was taken into account. Excellent agreement is found between the experimental results and the theoretical calculations. In particular, in the center of Fig. 6(a) there is an Airy-like ring structure wherein the diameter of the first dark ring is 420 nm. This is smaller than the Rayleigh resolution of 642nm of the lens used for imaging, which is calculated as $(1.22\lambda/NA)$ at $\lambda = 500$ nm. The calculated diffraction pattern (see Fig. 6(b)), shows even smaller central dark ring with a diameter of just 320 nm (the smaller value for the calculated ring diameter, as compared to the experimental value, is likely to be explained by the manufacturing tolerance of the holes' positions in the array).

Intriguingly, focusing by the nano-hole array resembles focusing by a conventional lens. Indeed, Fig. 7 shows that the structure of the focusing spot is similar to that of a lens with a limited aperture. In the bottom of Fig. 7 we plot both the intensity and the full width of the spots, which indicates that the smaller the spot, the weaker the intensity.

4. Conclusions

In conclusion, we have demonstrated that a quasi-periodic array of nano-holes can be used to focus light into small spots in free space. The subwavelength spots can be formed without evanescent waves, can be directly imaged by conventional optical microscope and mapped into the far-field with high amplification. This effect is linked to superoscillation phenomenon, which may provide a new way to achieve subwavelength imaging in the far field, different from the approaches based on the recovery of evanescent waves.

Acknowledgments

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