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Measuring and Pricing Macroeconomic Uncertainty: a Machine Learning Econometric Approach

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by

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Abstract

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This thesis measures and prices macroeconomic (or aggregate) uncertainty with non-parametric (AI/ML) methods, benchmarking against the current parametric standard in the literature. Long-short term memory deep neural networks (LSTMs) are the current method of preference to measure time varying phenomena such as macroeconomic uncertainty in chapter 2. Before examining whether a non-parametric measure of macroeconomic uncertainty is priced in the cross section of US stock returns in chapter 4, chapter 3 inquires into the common empirical finding of a negative uncertainty premium. To do so, chapter 3 exploits monthly data for the US AMEX, Nasdaq and NYSE stocks between 1993 and 2022, to build a dynamic hedging strategy across calm and turbulent sub-periods, examining the corresponding uncertainty premia. All along, parametric and non-parametric macroeconomic uncertainty measures are compared between (e.g. deploying VARs) and in terms of their pricing effects (e.g. with suitable tests for nested and non-nested specifications). The results indicate that the non-parametric measure of uncertainty has superior explanatory and predictive power for stock returns compared to the traditional parametric measures.

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I declare that this thesis and the work presented in it is my own and has been generated by me as the result of my own original research.

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Chapter 1

Introduction

What exactly is uncertainty? It represents situations where decision-makers are unable to forecast potential outcomes or understand their associated probability distributions (Cascaldi-Garcia et al. (2023)). This contrasts with risk, which involves unknown outcomes but known probability distributions. Volatility, often linked with risk, is actually a statistical measure of variations in observed outcomes. The critical distinction between these concepts became evident during the market's response to the COVID-19 pandemic: a fundamental shock escalated into widespread panic, effectively transforming risk into uncertainty. When market participants realize that their previous assumptions about risk no longer hold true and uncertainty prevails, their fear of unforeseen losses can wreak havoc on financial markets. From a broader perspective, uncertainty profoundly influences consumer and business decisions, affects how monetary and fiscal policies are executed and perceived, alters expected returns on assets and alters risk assessments, and impacts international trade and globalization. As noted by various studies, uncertainty plays a significant role in economic decision-making and policy effectiveness. Bernanke (1983) emphasizes that under uncertainty, irreversible investment decisions are profoundly affected by fluctuating information, creating cyclical investment patterns. Auerbach and Gorodnichenko (2012) highlight the varying effectiveness of fiscal multipliers in different economic regimes, underscoring the importance of accounting for predictable fiscal components during recessions. Additionally, Handley and Limao (2015) demonstrate how trade policy uncertainty affects firms' export investments, using Portugal's EC accession as a case study. Given its prevalence and significance, understanding macroeconomic uncertainty has become a renewed focus for economists and practitioners, aiming to improve economic forecasting and policy intervention strategies (Bloom (2009)).

1.1 Background

The literature on measuring uncertainty is still in its developing stages. Existing research has largely depended on volatility and dispersion measures as proxies for uncertainty. In his influential study, Bloom (2009) discovered a strong countercyclical relationship between real activity and uncertainty, using stock market volatility as a proxy. His VAR estimates indicate that uncertainty impacts output and employment within six months following an innovation in these measures, initially depressing real activity and then increasing it, leading to an overshoot of its long-run level—consistent with models that highlight uncertainty as a driving force of macroeconomic fluctuations. Furthermore, Bloom et al. (2018) highlighted a correlation between real activity and uncertainty, using proxies such as dispersion in firm-level earnings, industry-level earnings, total factor productivity, and forecasters' predictions. A recurring theme in these studies is that these proxies for uncertainty are strongly countercyclical. While these analyses provide valuable insights, it is important to note that measures of dispersion and stock market volatility may not be closely aligned with true economic uncertainty. In fact, one widely-used proxy, the VIX, predominantly reflects financial market volatility driven by time-varying risk aversion rather than true economic uncertainty Bekaert et al. (2013).

Contrasting with theories that portray uncertainty as a catalyst for economic downturns, another perspective suggests that increased uncertainty is actually a consequence of such downturns. The notion that economic downturns foster risky behavior is supported by researchers like Bachmann et al. (2011) and Fostel and Geanakoplos (2012). According to Bachmann et al. (2011), recessions are characterized by heightened uncertainty and volatility at both macro and micro levels. Commonly, this pattern is attributed to uncertainty shocks adversely affecting economic activity. However, they propose an alternative hypothesis: negative shocks may induce risky behavior, which in turn elevates observed volatility. Focusing on consumer price changes, they introduce a model where firms, faced with imperfect information about demand, learn about market conditions through sales volume. This model suggests that economic downturns are opportune times for price experimentation, as the cost of pricing errors is lower, making market exit more imminent. Empirical evidence from Consumer Price Index (CPI) microdata supports their prediction that high price volatility increases the likelihood of market exit.

Similarly, Van Nieuwerburgh and Veldkamp (2006), Fajgelbaum et al. (2017), and Ilut and Saijo (2021) suggest that reduced access to reliable information during periods of distress can weaken the ability to accurately forecast future outcomes. Van Nieuwerburgh and Veldkamp (2006) argues that the end of economic booms leads to sharp downturns due to reduced precision in forecasting, as low production during recovery results in noisy information. This impairs learning, slows recovery, and creates asymmetry between booms and busts. Fajgelbaum et al. (2017) presents a model where high uncertainty about economic fundamentals during low activity periods slows information flow and discourages investment, creating ”uncertainty traps” that extend recessions. Ilut and Saijo (2021) suggest that firms face uncertainty about profitability, and information gathered through production influences economic activity. This creates feedback loops, affecting forecasting precision and generating countercyclical economic patterns. Together, these studies suggest that during economic distress, the scarcity of reliable information significantly weakens the ability to forecast future outcomes accurately, contributing to prolonged economic instability. The uncertainty about future conditions can lead to the adoption of new and potentially unfamiliar economic policies, as proposed by Pástor and Veronesi (2013), which may themselves contribute to increased uncertainty. These perspectives highlight a complex feedback loop, where economic downturns lead to heightened uncertainty, potentially exacerbating economic instability.

Another line of research focuses on cross-sectional dispersion in the subjective expectations of analysts or firms as a measure of uncertainty. In such context, Bachmann et al. (2013) adopted this approach with a survey of German firms, suggesting that uncertainty is more a result of recessions rather than a cause, in opposition to theoretical models like those proposed by Bloom (2009) and Bloom et al. (2018). D'Amico and Orphanides (2008) examined analyst uncertainty and disagreement measures from the Survey of Professional Forecasters in an earlier study. Although analysts' forecasts are intriguing, several limitations exist when using them to measure uncertainty. Firstly, subjective expectations are available for only a limited number of series. Of the 132 monthly macroeconomic series considered in this dissertation, fewer than a fifth have corresponding expectations series. Secondly, it is uncertain whether survey responses accurately reflect the economy's conditional expectations. The surveyed forecasters often have known systematic biases or omit relevant information, and financial incentives may further bias their forecasts. Thirdly, disagreement in survey forecasts may more accurately represent differences in opinion rather than actual uncertainty. As Diether et al. (2002) provide evidence, stocks with higher dispersion in analysts' earnings forecasts often yield lower future returns, especially in small or previously underperforming stocks. This suggests that dispersion in analysts' forecasts may serve as a proxy for opinion differences about a stock. Similarly, Mankiw et al. (2003) analyzed 50 years of inflation expectations data and highlighted significant disagreement among both consumers and professionals, indicating variation over time with inflation changes. This insight implies that a thorough model of economic dynamics must incorporate these business-cycle moments, as most macroeconomic models do not naturally generate disagreement. The sticky-information model, in particular, aligns with many of these observations.

Furthermore, Lahiri and Sheng (2010) demonstrate that even unbiased forecasts can generate divergence between point forecast disagreement and average forecast error uncertainty unless the variance of accumulated aggregate shocks over the forecast horizon is zero. Using the Survey of Professional Forecasters, they show that variance in these shocks can create a substantial gap between uncertainty and disagreement during notable economic changes or when the forecast horizon is not extremely short. Addressing these challenges, Bachmann et al. (2013) suggested additional uncertainty proxies, such as an ex-post measure of forecast error variance derived from survey expectations. In a similar vein, Scotti (2016) applied this approach to series with real-time data availability.

While those studies focus on variation in outcomes around subjective survey expectations for a few variables, Jurado et al. (2015) analysis centers on uncertainty around objective statistical forecasts for hundreds of economic series. In their influential work, Jurado et al. (2015) significantly enhance the methodology for assessing macroeconomic uncertainty by employing cross-sectional aggregate measures of conditional volatility. This innovative metric centers on statistical forecasts derived from a broad spectrum of economic indicators. The framework's strength lies in its capacity to evaluate uncertainty across the entire economy, aggregating conditional variance measures from a comprehensive set of economic variables. This approach provides a more robust depiction of uncertainty compared to traditional benchmarks, which often fail to capture prolonged unemployment levels, such as those experienced during the 2007-2009 recession, as highlighted by Schaal (2012).

According to the methodology used, recent contributions to the measurement of uncertainty can be classified into four different categories. Beyond econometric-based measures, as introduced by Jurado et al. (2015), there are alternative methods for quantifying uncertainty, including text-based, survey-based, and market-based measures, each offering unique perspectives depending on the type of uncertainty under examination.

Text-based measures analyze the impact of policy changes, events, or news on market uncertainty and risk. A notable example is the Economic Policy Uncertainty Index proposed by Baker et al. (2016), calculated based on the proportion of news articles discussing uncertainties in economic policy. Similarly, monetary policy uncertainty is measured by the percentage of articles focused on U.S. monetary policy actions and their implications, as detailed by Husted et al. (2020). Trade policy uncertainty is evaluated by the share of articles on trade policy uncertainties, according to Caldara et al. (2020). The World Uncertainty Index measures the occurrence of "uncertainty" in reports from 143 countries, utilizing data from the Economist Intelligence Unit, as described by Ahir et al. (2022).

Survey-based measures target the confidence levels among consumers or businesses and examine how market expectations influence economic activities. A significant example is the Survey of Business Uncertainty, which involves a panel survey revealing firms' one-year-ahead uncertainties regarding their sales and employment (Altig et al. (2022)). Consumer-perceived uncertainty is captured through reports about uncertainties concerning car purchases over the next 12 months, as studied by Leduc and Liu (2016) using data from the University of Michigan Survey of Consumers. Macroeconomic uncertainty is assessed from squared deviations of surprises in macroeconomic data releases, analyzed by Scotti (2016) using Bloomberg survey expectations. Professional forecasters' uncertainty is derived from the distribution tails of errors in forecasting U.S. GDP growth, utilizing data from the Survey of Professional Forecasters by Rossi and Sekhposyan (2015).

Market-based measures explore financial market behaviors and dynamics to evaluate uncertainty. Common examples include realized volatility, which is the sum of squared intra-daily returns of the S&P 500, as explained by Andersen et al. (2006). The Volatility Index (VIX) serves as an inferred gauge of market-expected volatility, derived from S&P 500 index options provided by the Chicago Board Options Exchange (CBOE). The variance risk premium, which reflects the difference between option-implied and expected realized variances of S&P 500 returns, is discussed by Bollerslev et al. (2009). Lastly, market-based monetary policy uncertainty is assessed by examining the 90% confidence interval of the market-implied distribution for the effective Fed Funds rate, as described by Swanson (2006).

In comparison to these methods, the key advantage of the econometric-based measurement lies in its ability to incorporate a diverse array of economic indicators for comprehensive analysis. It relies on observed data rather than subjective questionnaires or news sources, helping to mitigate biases and the limitations associated with narrower information sources. Consequently, this research focuses on advancing the econometric-based approach to measuring uncertainty, aiming to leverage its broad applicability and robust analytical foundation.

Hence, focusing on econometric-based measures, Jurado et al. (2015) offer an effective approach to capturing macroeconomic and financial uncertainty through a robust econometric framework. This method utilizes extensive cross-sections of economic and financial variables to estimate conditional volatilities of time series, which are aggregated to form indicators of macroeconomic or financial uncertainty. Formally, Jurado et al. (2015) define h-period ahead uncertainty for a single variable as the conditional volatility of the variable's unforecastable component. This component is the difference between its future value and its expectation based on current information at time t. Aggregate macroeconomic uncertainty, thus, is averaged across all macro variables. The resulting comprehensive aggregate uncertainty measure is broader than traditional proxies and is more effective at identifying and predicting periods of economic distress than limited-variable measures.

1.2 Measuring Uncertainty

While Jurado et al. (2015) set a high standard in measuring macroeconomic uncertainty, there remains room for further enhancement. Their use of traditional linear statistical models, such as PCA and FAVAR, presents notable limitations. These models often struggle to capture the nonlinear relationships among macroeconomic variables, resulting in an incomplete measure of the unpredictable components. Implementing a machine learning model, specifically a Long Short-Term Memory (LSTM) neural network, allows for a more effective analysis of these nonlinear interactions.

With advancements in statistical and computer sciences, a broad array of machine learning algorithms has emerged, including neural networks (NNs). These algorithms are particularly well-suited for managing high-dimensional but persistent datasets, given sufficient computational power. Though their non-parametric nature and reliance on nonlinear equations can present challenges in interpretation and manipulation, LSTMs have proven effective in handling time series data, especially in the context of volatility.

In this dissertation, we adopt the LSTM model as a methodological advancement. Our analysis indicates that uncertainty measured using LSTM tends to register lower levels than traditional linear statistical models during periods of low uncertainty. Furthermore, the predictive accuracy of our LSTM-based measure in forecasting key macroeconomic variables within a VAR framework is comparable to, and in some cases surpasses, the measure developed by Jurado et al. (2015).

The primary objective of Chapter 2 in this dissertation is to enhance the capture of predictable variation in macroeconomic and financial time series. To achieve this, we propose using LSTM neural networks to measure macroeconomic uncertainty. Similar to the approach in Jurado et al. (2015), our LSTM NNs aim to remove predictable variation stemming from nonlinearities and interactions between macroeconomic and financial variables, thereby improving the measurement of macroeconomic uncertainty. Unlike the parametric framework based on principal component analysis (PCA) and factor-augmented vector autoregressions (FAVAR) used by Jurado et al. (2015), our approach leverages non-parametric methods for recursively forecasting macroeconomic and financial time series. Our method involves two LSTM NNs and a recursive procedure that generates estimates of macroeconomic uncertainty, which more accurately reflect the underlying uncertainty in each economic variable. By combining these refined estimates of idiosyncratic uncertainty from modeling the conditional variance of a wide range of economic variables into a single uncertainty measure, we propose an improved measure of macroeconomic uncertainty. As an added benefit, we demonstrate that this alternative measure, derived from applying machine learning tools, allows for more effective identification of economic crises and relatively stable periods.

Additionally, our proposed measure of macroeconomic uncertainty exhibits promising forecasting properties for various macroeconomic variables, often outperforming the uncertainty measure developed by Jurado et al. (2015). These comparisons are statistically formalized using Diebold-Mariano tests, which evaluate the differences in forecasting capabilities between our nonlinear framework and the traditional linear framework. Moreover, Granger-causality tests are employed to explore the capacity of macroeconomic uncertainty to predict out-of-sample macroeconomic variables within a VAR setting.

To further demonstrate the advantages of the LSTM model in measuring uncertainty, we propose two alternative frameworks as robustness checks in Chapter 2 by partially integrating LSTM components into the original linear framework. In the first framework, LSTM autoencoders are utilized to extract factors from macroeconomic and financial time series, replacing the PCA component of the linear framework while retaining the remaining structure. These new factors extracted by the LSTM autoencoders are then used in the same FAVAR model, with macroeconomic uncertainty estimated using the same dataset. Results indicate that incorporating the LSTM autoencoder significantly enhances the performance of the original linear framework compared to the PCA model. The estimates of macroeconomic uncertainty show similarities to those produced by our primary framework with two LSTM models (the LSTMs framework), yet a noticeable disparity remains. This suggests that although the LSTM model outperforms PCA in factor extraction, the continued use of the original FAVAR model limits the efficiencies observed in the full LSTM framework. This first robustness check highlights the superior predictive capabilities of the LSTM model over the FAVAR model in forecasting macroeconomic series, a conclusion further supported by the results from the second robustness check.

In the second robustness check, we use the LSTMs framework as a reference for direct comparison with the FAVAR model. Specifically, we implement a FAVAR model incorporating the factors extracted by the LSTM autoencoder from the first robustness check to forecast macroeconomic series while retaining the LSTM model for forecasting financial series in the recursive system of the LSTMs framework. Compared to the complete LSTM framework, which is our main methodological contribution, this second robustness check strategically incorporates the FAVAR model while preserving the LSTM components as much as possible. The findings from this exercise indicate that, despite involving only one FAVAR model, the estimates of macroeconomic uncertainty are notably inferior and lag behind those achieved with the complete LSTM framework. This outcome further emphasizes the LSTM model's superior performance relative to the FAVAR model, reinforcing the LSTM's effectiveness and robustness in handling and forecasting macroeconomic uncertainties.

1.3 Pricing Uncertainty

In Chapter 2, we introduce a novel approach to measuring macroeconomic uncertainty using an LSTM model. Then, in Chapters 3 and 4, we primarily concentrate on evaluating uncertainty through the lens of cross-sectional stock returns. This includes assessing macroeconomic and financial uncertainty, as measured by Jurado et al. (2015), over an extended time period and analyzing different market conditions, specifically turbulent versus calm periods. Additionally, we evaluate the macroeconomic uncertainty derived from our model in Chapter 2, comparing it to the measures provided by Jurado et al. (2015).

In a foundational work, Merton (1973) demonstrates that investors are motivated to hedge against unpredictable future changes in consumption and investment opportunities. In this framework, state variables that correlate with these changes are reflected in capital market prices, with the covariance between a stock's return and the state variable being linearly related to its expected return. According to this approach, common risk factors for pricing the cross-section of risky assets are state variables derived from macroeconomic indicators. This is because fluctuations in these indicators can significantly influence expected returns due to evolving economic conditions and, consequently, the investment opportunities available in risky assets. The expected return on risky assets varies with their sensitivity to these state variables.

Recent studies, such as those by Gomes et al. (2003), Bloom (2009), Allen et al. (2012), Drechsler (2013), Jurado et al. (2015), and Ludvigson et al. (2021), provide both theoretical and empirical evidence that variations in economic uncertainty serve as a significant state variable with the capacity to predict future consumption and investment decisions, along with other macroeconomic indicators. Other notable contributions, including those by Ang et al. (2006), Ang et al. (2009), and Bali et al. (2017), examine the impact of macroeconomic uncertainty on pricing individual stocks and equity portfolios. These researchers employ different measures: Ang et al. (2006) consider changes in the VIX index as a measure of aggregate volatility, while Bali et al. (2017) use the macroeconomic uncertainty index proposed by Jurado et al. (2015), defined as the conditional volatility of the forecast error from a factor-augmented vector autoregressive (FAVAR) model applied to a broad set of economic and financial indicators. Both groups of authors identify a significantly negative risk premium associated with the uncertainty pricing factor.

The uncertainty premium aligns with the intertemporal capital asset pricing model (ICAPM) proposed by Merton (1973), Campbell (1993), and Campbell (1996). Ang et al. (2006) discover that stocks with high sensitivities to innovations in aggregate volatility tend to have lower average returns compared to those with low exposure, even when controlling for traditional pricing factors such as size, book-to-market, momentum, and liquidity. Similarly, Bali et al. (2017) contend that rising economic uncertainty diminishes future investment and consumption opportunities. In response to this unfavorable shift, investors are inclined to hold stocks whose returns increase during periods of economic uncertainty. This preference for intertemporal hedging suggests that investors are willing to pay higher prices and accept lower returns for stocks with a higher covariance with economic uncertainty, or a higher 'uncertainty beta'. Both Ang et al. (2006) and Bali et al. (2017) also provide evidence of time variation in the (negative) uncertainty premium, noting that it is significantly higher during recessions and periods of heightened aggregate uncertainty compared to expansionary and relatively calm periods.

Considering the time-varying property of risk premium related to uncertainty, we anticipate that the risk premium on macroeconomic uncertainty—interpreted as innovations in the conditional volatility of macroeconomic and financial shocks—not only varies over time but also fluctuates around zero to satisfy investors' dynamic hedging needs during periods of elevated uncertainty. While this prediction may appear to conflict with the findings of Ang et al. (2006) and Bali et al. (2017), who report a consistently negative uncertainty premium across different uncertainty regimes, our focus is on a different measure of uncertainty. Ang et al. (2006) use innovations in the VIX as a proxy for aggregate uncertainty, whereas Bali et al. (2017) employ the direct measure of uncertainty developed by Jurado et al. (2015), distinct from the measure of its innovations that we use. Because the VIX index is widely regarded as a fear index rather than a true measure of uncertainty, and the uncertainty measure itself tends to show greater time dependence than the innovations in uncertainty.

To support our hypothesis regarding the predictive role and dynamics of the macroeconomic uncertainty risk premium, we employ three distinct empirical strategies. First, we construct long-minus-short investment strategies based on the cross-sectional rankings of stock returns from five non-overlapping periods of calm and turbulent times between 1998 and 2022. These rankings are based on firms' exposures to macroeconomic uncertainty, allowing us to evaluate the profitability of these strategies across different uncertainty regimes. The portfolios are designed to be long on stocks in the highest quintile of uncertainty beta exposure and short on those in the lowest quintile, effectively 'buying high and selling low exposures.' Second, we estimate the dynamics of the uncertainty premium during these five sample periods using Fama-MacBeth (Fama and MacBeth, 1973) cross-sectional regressions. Third, we construct a mimicking (hedging) portfolio using a methodology similar to Engle et al. (2020) to estimate, on a monthly basis, the dynamics of the uncertainty premium. This approach involves constructing an adjusted portfolio with base assets that closely mirror the maximum level of variation in the uncertainty measure each month. Such process is independent of the selection criteria for turbulent or calm evaluation subsamples, which are defined by ad-hoc methods, the exogenous CFNAI index methods, and the endogenous methods related to thresholds of uncertainty changes.

To test our main hypothesis, we analyze data on the cross-section of stock prices from the CRSP database, encompassing all available stocks from NYSE, NASDAQ, and AMEX. Our findings indicate that the beta exposure of stock returns to macroeconomic uncertainty increases monotonically across different uncertainty regimes, even after controlling for the Fama and French (2015) five-factor model (hereafter referred to as FF5). Consistent with existing literature, these beta loadings are negative for stocks in the lower quintiles and positive for those in the higher quintiles when stocks are sorted by their uncertainty exposure. However, contrary to previous studies, we observe that the risk premium on macroeconomic uncertainty is negative during periods of declining conditional volatility (calm periods) and positive during periods of rising conditional volatility (turbulent periods). This finding is further supported by the performance dynamics of the 5-1 portfolio strategy, which involves buying stocks in the highest quintile and selling those in the lowest quintile. This strategy tends to result in negative average returns during calm periods and positive average returns during turbulent periods. This outcome is largely attributed to our choice of uncertainty proxy, which is based on the innovations to the conditional volatility measure developed by Jurado et al. (2015). Contrary to the main findings of Ang et al. (2006) and Bali et al. (2017), which suggest that the risk premium of uncertainty is consistently negative with larger magnitude during turbulent times, our findings with innovations in uncertainty provide a more plausible explanation from a cross-period hedging perspective. Specifically, investors effectively pay for insurance during calm periods and receive compensation during turbulent times.

These results are further validated through estimates of the uncertainty risk premium obtained from Fama-MacBeth cross-sectional regressions applied to an augmented FF5 model that incorporates our macroeconomic uncertainty proxy. We identify a negative risk premium for this variable during calm periods (2003-2006 and 2012-2016) and a positive risk premium during turbulent periods (1998-2002, 2007-2011, and 2017-2022). Additional confirmation comes from applying a mimicking portfolio approach similar to that of Engle et al. (2020). In this approach, the returns on our mimicking portfolio are determined by projecting the macroeconomic uncertainty state variable onto a set of base assets ranked by their exposure to uncertainty, while controlling for the FF5 factors.

Uncertainty has garnered increasing attention due to its prevalence and persistence, leading to various advancements in its measurement, as reviewed by Cascaldi-Garcia et al. (2023). In Chapter 3, we primarily focus on the econometric-based uncertainty measures introduced by Jurado et al. (2015) for macroeconomic uncertainty and Ludvigson et al. (2021) for financial uncertainty. Instead of using the conditional volatility indices these authors developed, we consider the innovations to these processes as proxies for economic uncertainty. This approach aligns with strategies used by (i) Ang et al. (2006), who analyze first differences of the VIX index, and (ii) Engle et al. (2020), who examine innovations to an autoregressive process of order one fitted to an index measuring climate change news. In our work, we derive innovations from fitting a stochastic volatility model to the errors of a FAVAR-type process.

As additional robustness checks, we apply the novel asset pricing model developed by Giglio and Xiu (2021) to demonstrate that our uncertainty premium estimates are resilient to potential omitted variable bias. In this model, asset pricing factors are derived from principal components as a preliminary step before conducting the two-pass regression by Fama and MacBeth (1973). We also examine our selection of calm and turbulent episodes, discussing alternative methods for segmenting the sample period. Furthermore, the robustness section evaluates the impact of microcaps on the role of economic uncertainty in pricing the cross-section of stock returns. Following the approach of Hou et al. (2015), we trim the bottom 20% of NYSE stocks based on market value.

The significance of macroeconomic uncertainty in cross-sectional asset pricing is further confirmed through recent statistical tests developed by Barillas and Shanken (2017, 2018), and Barillas et al. (2020), which compare competing nested and non-nested asset pricing models. We find robust statistical evidence in favor of augmenting the FF5 model with econometric-based measures of macroeconomic and financial uncertainty, using differences in squared Sharpe ratios and Bayesian procedures. The model comparison exercise provides evidence from statistical tests for non-nested models, indicating that the applicability of each uncertainty measure for explaining the cross-section of stock returns varies with the uncertainty regime. For instance, during the 2007-2011 financial crisis, the uncertainty measure derived from innovations in financial uncertainty outperforms the macroeconomic measure. Conversely, during the 2017-2022 period dominated by the COVID-19 pandemic, the macroeconomic uncertainty measure proves to be more effective.

1.4 Comparing LSTM-based Uncertainty with Linear Measures

In Chapter 2, we introduce a novel LSTM-based measurement of macroeconomic uncertainty. This approach more effectively captures predictable variations in macroeconomic and financial data, reduces forecast errors, and enhances the overall predictive accuracy of uncertainty measures. In Chapter 3, we examine and price the linear measures of uncertainty proposed by Jurado et al. (2015) within the cross-section of stock returns, demonstrating that uncertainty serves as a valid pricing factor with risk premiums that vary over time. Consequently, in Chapter 4, we apply a similar framework to price our LSTM-based macroeconomic uncertainty. This chapter focuses on comparing the LSTM-based measure with linear measures, providing evidence that incorporating machine learning models and such nonlinear factors can improve the predictive power of linear asset pricing models while maintaining their interpretability.

Since the introduction of the three-factor model by Fama and French (1993), the multi-factor pricing framework has remained a central paradigm in asset pricing research. However, with the ongoing increase in the complexity of financial markets, traditional models have gradually shown their limitations. As noted in Fama and French (2015), the linear assumption may miss nonlinear risk exposures. In addition, the construction of factors faces numerous limitations and has been the subject of extensive research. As Kozak et al. (2020) point out, traditional factors exhibit high collinearity and significant information overlap. Many factors are highly correlated in terms of economic logic or statistical characteristics (e.g., value factor and profitability factor). While an increase in the number of factors might lead to an inflated in-sample explanatory power, it often results in a decrease in out-of-sample predictive ability. This collinearity makes it difficult to distinguish the economic significance of factor premiums (e.g., whether returns stem from risk compensation or data mining).

Furthermore, McLean and Pontiff (2016) highlighted the issues of traditional factors being prone to data mining pollution and the slowing pace of new factor discoveries. They collected 97 stock return predictive factors published in academic journals, covering the period from 1963 to 2014, and compared the performance of these factors before publication, after publication (but before public dissemination), and after public dissemination. They found that average factor returns decreased by about 32% after publication (out-of-sample), with the decline being statistically significant. For example, the momentum factor's return dropped by nearly 50% post-publication. Many factors performed well in-sample (before publication) but failed out-of-sample (after publication), suggesting they may be the result of data mining. Some financial ratio factors (such as accruals) were significant in early studies but could not be replicated later. Over time, the economic significance of new factor discoveries (ratio of mean returns to volatility) has gradually diminished. For example, factors discovered after 2000 have returns approximately 40% lower on average than those discovered in the 1980s. Easily discoverable simple factors (e.g., market size, value) have been extensively mined, leaving the remaining potential factors requiring more complex data or methodologies, with significantly increased discovery costs.

To address these issues, an increasing number of studies involve PCA models, which also improve on the traditional model's reliance on manual feature selection and its associated information omissions. Giglio and Xiu (2021) discussed that certain risk factors (e.g., liquidity, consumption) should earn risk premiums according to asset pricing theory. However, in practice, models often fail to fully capture all risk factors, resulting in omitted variable bias in standard estimation methods. They proposed a new three-step method that uses PCA to extract the factor space of test asset returns, followed by additional regression to obtain observed factor risk premiums. This method was applied to a large dataset containing 647 portfolios, including stocks, bonds, and currencies. The results showed significant differences in the risk premium conclusions compared to those obtained using standard methods in the existing literature.

Similarly, Kelly et al. (2019) introduced a new method for modeling the cross-section of stock returns, namely Instrumented Principal Component Analysis (IPCA). This approach allows for observable characteristics to serve as instrumental variables for unobservable dynamic loadings and enables the identification of latent risk factors that are related to expected returns. Their study found that four IPCA factors could explain the cross-section of average returns more accurately than existing factor models, with most characteristics in the literature being statistically significant in the IPCA specification. Not only does this model perform well in explaining systematic risk, but it also effectively describes the differences in average returns between stocks.

These studies on PCA and latent factors provide strong evidence of information omission by traditional model factors, and further research on IPCA and dynamic loadings highlights issues within the multi-factor pricing framework related to the static factor assumption that fixes factor loadings as constant parameters, hindering the capture of the dynamics of asset risk in response to economic cycle fluctuations or market structural changes. Although these latent factor methods effectively improve the predictive ability of asset risk premiums, it is noteworthy that these latent factors lack practical economic significance and are difficult to interpret, leading to the "black-box" issue. Investors expect a clear explanation of investment logic to deal with ever-changing market conditions. Therefore, we focus on a method that can both leverage the machine learning model's ability to handle complex data and retain a certain level of interpretability in asset pricing.

Based on our measurement and pricing of uncertainty, LSTM-based uncertainty is an excellent entry point. First, the LSTM model is highly capable of predicting financial data, which can be effectively used to construct factors thanks to its superior predictive capability. Fischer and Krauss (2018) systematically studied the performance of LSTM in financial time-series forecasting and demonstrated its superiority over traditional linear models. Linear models (such as ARIMA, linear regression), while widely used, face two major challenges in financial series prediction: inadequate capture of nonlinear relationships—markets are affected by multiple factors such as investor behavior and macroeconomic shocks, often displaying nonlinear and nonstationary characteristics; difficulty modeling long-term dependencies—traditional models struggle to capture the association between distant time points effectively (e.g., the long-term impact of events on markets). As an improved variant of RNN, LSTM can effectively model long-term dependencies due to its gating mechanism (input gate, forget gate, output gate) and memory cell design. They used high-frequency or daily time-series data from multiple financial markets (such as stock prices, index returns, exchange rates), covering different market cycles, and compared the LSTM model with linear models: ARIMA, linear regression, LASSO; traditional machine learning models: support vector machines (SVM), random forest; simple neural networks: conventional RNN, feedforward neural network (FFNN). The results showed that in most financial time series, the LSTM had lower prediction errors, higher directional prediction accuracy, and greater stability during high volatility periods (e.g., financial crisis) than traditional models, as it could capture the nonlinear panic transmission mechanism. Their results also suggest that combining LSTM with fundamental data (like financial report texts) can extract nonlinear factors, enhancing the explanatory power of multi-factor models.

Besides that, LSTM-based uncertainty comes with interpretability. In our Chapter 2, we employe LSTM to improve predictions of macroeconomic and financial time-series, subsequently refining the measurement of uncertainty. We use the LSTM model to construct a nonlinear factor describing macroeconomic uncertainty while verifying that uncertainty is an effective pricing factor with time-varying risk premium through testing and pricing linear uncertainty measures in Chapter 3. Compared to directly extracting factors from data using LSTM or PCA models, our LSTM-based uncertainty is interpretable, because of the definition of the uncertainty concept. We use the LSTM model to remove the predictable part of the time series as much as possible, defining the conditional volatility of the unpredictable part as uncertainty. Therefore, although LSTM-based predictions of sequence data are hard to interpret, the factor we built on the unpredictability aligns with the economic significance of linear uncertainty measures by Jurado et al. (2015).

In Chapter 4, we incorporate the LSTM-based uncertainty measure from Chapter 2 as a risk factor into the asset pricing model. We then compare it to asset pricing models that use linear measures of uncertainty, as discussed in Chapter 3. Hence, our method combines machine learning's data handling capabilities with interpretability in asset pricing by applying nonlinear methods to macroeconomic and financial data, measure uncertainty, and introduce this as a pricing factor in a linear model, retaining the model's ability to explain risk premiums while infusing it with machine learning insights.

Why use uncertainty as a baseline? First, as discussed in Chapter 2, uncertainty captures global economic conditions effectively, such as those influenced by events like the coronavirus pandemic and the Russo-Ukrainian conflict, which have greatly impacted both macroeconomics and stock markets. Jurado et al. (2015) introduced an econometric approach using the Factor-Augmented Vector Autoregressive (FAVAR) method to capture subtle economic uncertainties. Building on this, we replace traditional PCA and FAVAR with LSTM models, suited for high-dimensional data with nonlinear changes during economic shifts. These advances enhance uncertainty measurement, improving prediction capabilities in our research domain. Second, as shown in Chapter 3, we demonstrate that linear measures of uncertainty based on stock returns can serve as valid pricing factors with a fluctuating risk premium over time. Hence, in Chapter 4, we incorporate the LSTM-based uncertainty measure into the multifactor asset pricing model, examining its risk premium and then comparing the LSTM-based uncertainty against linear measures of uncertainty based on the same asset pricing model. Such comparison allows us to evaluate the explanatory ability and predictive power of nonlinear versus linear factors, both reflecting similar informational perspectives. Moreover, it enables us to determine whether machine learning models can augment traditional linear factor models by developing more effective factors. Through this analysis, we aim to assess the advantages of incorporating machine learning into conventional asset pricing frameworks.

The main exercises in Chapter 4 are divided into two main parts: analyzing the risk premiums associated with LSTM-based uncertainty and empirically comparing LSTM-based uncertainty measures with traditional linear measures.

To validate our hypothesis regarding the predictive capability and dynamics of our LSTM-based macroeconomic uncertainty risk premium, we employ three distinct empirical strategies, with the same three empirical exercise conducted in Chapter 3 on the linear measures. First, we construct a mimicking (or hedging) portfolio by ranking the cross-section of stock returns which are based on firms' exposure to LSTM-based macroeconomic uncertainty. This involves creating a mimicking portfolio that projects economic uncertainty measures directly against a selection of base asset returns and such mimicking portfolio is further employed in the empirical comparison part. Second, we explore the dynamics of the monthly uncertainty premium through Fama-MacBeth cross-sectional regressions (Fama and MacBeth, 1973). This approach allows us to assess how the uncertainty premium changes over time and across various market conditions. Lastly, we employ the novel asset pricing model proposed by Giglio and Xiu (2021) to demonstrate the robustness of the uncertainty premium estimates. This step ensures that our estimates remain reliable and unaffected by potential omitted variable biases, providing a comprehensive analysis of the role of uncertainty in asset pricing.

Our primary hypothesis is evaluated using cross-sectional stock price data sourced from CRSP, which includes all available stocks listed on the NYSE, NASDAQ, and AMEX. Our analysis shows that the beta exposure of stock returns to macroeconomic uncertainty—estimated by the LSTM model—displays a consistent increase across different uncertainty regimes, even after adjusting for the Fama and French (2015) five-factor model (FF5). Consistent with existing literature and our empirical findings using a linear uncertainty measure detailed in Chapter 3, these beta loadings are negative for stocks in the lower quintiles and positive for those in the higher quintiles based on their exposure to uncertainty. We observe the same departure from traditional literature: the risk premium on macroeconomic uncertainty derived from LSTM is negative during periods of decreasing conditional volatility (calm periods) and positive during periods of increasing conditional volatility (turbulent periods). This result stems from the choice of our uncertainty proxy based on innovations to the LSTM-based macroeconomic uncertainty, as the same pattern emerges when using innovations in linear uncertainty measures. Our findings are reinforced by the estimates of the uncertainty risk premium derived from Fama-MacBeth cross-sectional regressions applied to an FF5 model, which is enhanced with the LSTM-based uncertainty proxy. Additionally, we employ the 3-Stage latent factor regression methodology from Giglio and Xiu (2021), which also provides similar results.

The secondary aim of the Chapter 4 is to conduct a thorough comparison of our LSTM-based macroeconomic uncertainty measure with the linear measures proposed by Jurado et al. (2015), within the framework of asset pricing theory. We perform this comparison through two primary approaches, from the risk premium perspective and the explanatory power on cross-sectional stock returns. We assess the returns of stocks with different loadings on both types of uncertainty and evaluate their associated risk premiums during calm and turbulent periods. To identify these periods, we follow the method suggested by Bali et al. (2017), using the median of the uncertainty measure. By taking the medians of both the LSTM-based and linear macroeconomic measures, we analyze the differences in stock returns and risk premiums when uncertainty shocks are captured by each measure. This approach allows us to see how each measure performs under varying economic conditions. Following that, we use the asset pricing model comparison technique developed by Barillas and Shanken (2017, 2018); Barillas et al. (2020) to evaluate the explanatory power of the uncertainty measures. By including these measures as an additional risk factor in the FF5 model, we compare the predictive ability of these expanded 6-factor models against the baseline FF5 model and against each other, considering both nested and non-nested settings. Through these analyses, we aim to determine which uncertainty measure provides a more robust and reliable explanation of stock returns and risk premiums in turbulent and calm periods, enhancing our understanding of the role of uncertainty in asset pricing.

The results of our comparisons indicate that stocks with higher loadings on macroeconomic uncertainty estimated by the LSTM model tend to deliver higher returns during turbulent periods and lower returns during calm periods. The disparity in returns between portfolios with the highest and lowest uncertainty loadings is more pronounced when using the LSTM-based measure as a proxy for macroeconomic uncertainty, compared to the linear measure. Accordingly, the LSTM-based measure results in the same positive risk premiums during turbulent months and negative premiums during calm months but larger in magnitude. This means that investors can choose to pay higher (insurance) premia during calm periods to secure greater compensation (positive uncertainty premia) in turbulent periods when using the LSTM-based measure to hedge against macroeconomic uncertainty. Furthermore, the LSTM-based uncertainty measure significantly enhances the explanatory power of the FF5 model compared to the linear measure, with this improvement being particularly evident during the 2007-2008 financial crisis. This enhancement is attributed to the machine learning models' ability to handle large-dimensional data and capture nonlinear changes more effectively than linear models. This capability allows the LSTM-based measure to better reflect the complexities and dynamics of macroeconomic uncertainty, thereby providing a more robust framework for analyzing asset prices.

The remainder of this dissertation is organized as follows: Chapter 2 introduces a novel nonlinear measurement of macroeconomic uncertainty using LSTM models. Chapter 3 evaluates and prices linear uncertainty measures in the context of cross-sectional stock returns. Chapter 4 deals with pricing our novel LSTM-based uncertainty measure and then focuses on comparing LSTM-based measures with traditional linear measures.

Chapter 2

Measuring Macroeconomic Uncertainty

We deploy non-parametric Long Short-Term Memory (LSTM) models to measure time-varying macroeconomic and financial uncertainty, benchmarking them against Jurado et al. (2015)'s parametric factor augmented vector autoregressive (FAVAR) framework. Diebold-Mariano and Granger-causality tests in vector autoregressive specifications of the macroeconomy confirm that LSTMs outperform parametric methods, better capturing predictable variation, reducing forecast errors, and enhancing the predictive capabilities of uncertainty measures. The improvements stem from both LSTMs' superiority over principal component analysis (PCA) in factor extraction and their ability to flexibly handle non-linearities and complex interactions in data-rich environments.

2.1 Introduction

As introduced in Chapter 1, Jurado et al. (2015) formally define h-period ahead uncertainty for a single variable as the conditional volatility of the unforecastable component of its future value, i.e. the difference between the future value of the variable and its expectation based on the information available at time t. Their econometric framework consists of two main components: a PCA+FAVAR forecasting part and a time-varying stochastic volatility modeling part. In the forecasting component, they employ large cross-sections of macroeconomic and financial variables as predictors. PCA model is employed to extract factors from these variables, which are then used to construct a FAVAR model for forecasting macroeconomic series. For the modeling of conditional volatility, a stochastic volatility model is applied to estimate the time-varying conditional volatility of the forecast errors from the FAVAR model. This estimated volatility serves as the measure of uncertainty for individual macroeconomic series. Aggregate macroeconomic uncertainty is calculated as the average of these uncertainty measures across all macro variables.

Our main objective in Chapter 2 is to better capture predictable variation in macroeconomic and financial time series with machine learning models. To do this, our proposed approach implies to recursively forecast macroeconomic and financial time series deploying non-parametric methods such as the LSTM neural networks, instead of the parametric forecasting by PCA+FAVAR. Comparing with PCA+FAVAR, our LSTM NNs are able to further remove predictable variation due to nonlinearities and interactions between macroeconomic and financial variables, thereby isolating a cleaner unpredictable component used to measure idiosyncratic uncertainty. By integrating these refined estimates of idiosyncratic uncertainty—which are derived from modeling the conditional variance of a comprehensive set of economic variables—into a unified uncertainty measure, we propose an improved metric for macroeconomic uncertainty. From empirical analysis, we show that this alternative measure of uncertainty obtained from applying machine learning tools, allows for more effective identification of economic crises and relatively calm periods.

Moreover, our proposed measure of macroeconomic uncertainty also possesses appealing forecasting properties for a variety of macroeconomic variables, outperforming in many instances the uncertainty measure developed in Jurado et al. (2015). The comparisons are formalized statistically through the application of Diebold-Mariano tests, which assess the difference in forecasting capabilities between our nonlinear framework and the traditional linear framework, and Granger-causality tests to explore the ability of macroeconomic uncertainty to predict out-of-sample macroeconomic variables within a VAR setting.

To further substantiate the advantages of the LSTM model in uncertainty measurement, we propose two alternative frameworks as robustness checks given by partially integrating LSTM components into the original linear framework. In the first robust framework, LSTM autoencoders are employed to extract factors from macroeconomic and financial time series, effectively replacing only the PCA model in the linear framework while retaining the rest of its structure. The new factors extracted by the LSTM autoencoders are employed in the same FAVAR model, and macroeconomic uncertainty is estimated using the same dataset. The results indicate that incorporating the LSTM autoencoder significantly enhances the performance of the original linear framework with the PCA model. The estimates of macroeconomic uncertainty reveal some characteristics akin to those produced by our main framework with two LSTM models (LSTMs framework), yet there remains a noticeable disparity when compared to our primary framework. This finding implies that although the LSTM model outperforms PCA, the retention of the original FAVAR model in this robustness check framework hinders it from fully realizing the efficiencies observed in the LSTMs framework. This first robustness check highlights the superior predictive capabilities of the LSTM model over the FAVAR model in forecasting macroeconomic series, which is corroborated further by the results of the second robustness check.

The second robustness check is performed using the LSTMs framework as a reference. Specifically, to facilitate a direct comparison between LSTM and FAVAR, we use a FAVAR model which incorporates the factors extracted by the LSTM autoencoder from the first robustness check to forecast the macroeconomic series. Meanwhile, the LSTM model used for forecasting the financial series in the recursive system of the LSTMs framework is retained. Therefore, compared to the full LSTM framework - that we propose as our main methodological contribution - this second robustness check strategically employs the FAVAR model while maximally preserving the LSTM components. The findings from this second robustness exercise indicate that, even though it involves only one FAVAR model, the estimates of macroeconomic uncertainty are notably inferior and lag behind those achieved using the complete LSTM framework. This outcome further highlights the superior performance of the LSTM model relative to the FAVAR model, reinforcing the LSTM's effectiveness and robustness in handling and forecasting macroeconomic uncertainties.

The remainder of Chapter 2 is organized as follows: First, Section 2.2 presents a concise literature review that covers previous empirical and theoretical research on uncertainty. Second, the methodology employed in this research, including the framework of the LSTM model and its application in the measurement process, is detailed in Section 2.3. Third, we conduct an empirical analysis and compare the results of the estimated macroeconomic uncertainty with those of Jurado et al. (2015) in Section 2.4. Fourth, the robustness checks are elaborated upon and discussed in Section 2.5. Finally, Section 2.6 concludes with a summary of the major findings.

2.2 Literature Review

In this chapter, we explore and analyze existing research into uncertainty. One prevalent theory posits that uncertainty may be a driving factor behind reduced economic growth. Studies such as those by Sim et al. (2010) and Arellano et al. (2010) demonstrate how uncertainty can impact financing constraints. Additionally, research by Basu and Bundick (2017), Leduc and Liu (2016), and Fernández-Villaverde et al. (2011) suggests that precautionary saving behaviors can be influenced by uncertainty.

Bloom (2009) identifies a pronounced countercyclical relationship between real activity and stock market volatility, which is often considered a representative indicator of uncertainty. Using a VAR specification for the dynamics of several economic variables, these authors observe that uncertainty affects output and employment in a manner where an increase in volatility initially reduces real economic activity, but subsequently leads to an increase, potentially causing an overshoot in the long term. Further support for the countercyclical relationship between uncertainty proxies and real activity is provided by Bloom et al. (2018), where dispersion is used as a measure of uncertainty. Bloom's findings indicate that dispersion and stock market volatility are strongly countercyclical, though they do not directly link to actual uncertainty. Bekaert et al. (2013) discusses the use of the VIX index, one of the most popular proxies for uncertainty, suggesting that it is driven more by time-varying risk aversion rather than uncertainty itself. This adds a layer of complexity to understanding and measuring uncertainty's true impact on economic variables.

In contrast to theories that view uncertainty as a catalyst for economic downturns, another school of thought suggests that higher uncertainty is actually a consequence of such downturns. A notable study by Bachmann et al. (2013) finds that uncertainty often results from recessions rather than being a precipitating factor, which stands in contrast to the findings of Bloom (2009) and Bloom et al. (2018). The perspective that economic downturns encourage risky behavior is supported by researchers like Bachmann et al. (2011) and Fostel and Geanakoplos (2012). Similarly, Van Nieuwerburgh and Veldkamp (2006), Fajgelbaum et al. (2017), and Ilut and Saijo (2021) suggest that diminished access to reliable information during distress periods can degrade the ability to forecast future outcomes accurately. The uncertainty over future conditions can prompt the adoption of new and potentially unfamiliar economic policies, as Pástor and Veronesi (2013) proposes, which may themselves contribute to increased uncertainty. These perspectives highlight a complex feedback loop where economic downturns lead to increased uncertainty, which in turn may exacerbate economic instability.

Jurado et al. (2015) have significantly advanced the methodology for measuring macroeconomic uncertainty by using cross-sectional aggregate measures of conditional volatility. This new metric focuses on statistical forecasts obtained from a broad array of economic indicators. The strength of their framework lies in its ability to assess uncertainty across the entire economy by aggregating individual conditional variance measures obtained from a large set of economic variables. This approach offers a more robust depiction compared to traditional benchmarks which historically fail to capture prolonged levels of unemployment, particularly those observed during the 2007-2009 recession, as highlighted by Schaal (2012).

In addition to econometric-based measures of uncertainty as introduced in Jurado et al. (2015), there are alternative approaches to quantifying uncertainty, including text-based, survey-based, and market-based measures. Each offers unique insights and methodologies depending on the type of uncertainty being examined. Text-based measures involve analyzing how policy changes, events, or news impact market uncertainty and risk. A prominent example is the economic policy uncertainty index proposed by Baker et al. (2016), which is calculated based on the share of news articles discussing uncertainty regarding various aspects of economic policy. Survey-based measures focus on examining confidence levels among consumers or businesses and how market expectations influence economic activities. One significant example of this approach is the survey of business uncertainty, which entails a panel survey revealing the one-year-ahead uncertainties that firms have about their own sales and employment (Altig et al. (2022)). Market-based measures explore financial market behaviors and dynamics to assess uncertainty. These commonly include metrics like realized volatility and the Volatility Index (VIX), which are widely used to gauge market sentiment and expectations.

When compared to these methods, the advantage of econometric-based measurement lies in its ability to incorporate a wide range of economic indicators for a comprehensive analysis. It relies on observed data rather than subjective questionnaire or news, which helps to avoid biases and the limitations of narrower information sources. Consequently, this research focuses on enhancing the econometric-based approach to measuring uncertainty, aiming to leverage its broad applicative value and robust analytical foundation.

Despite setting a formidable standard in the measurement of uncertainty, we argue that the approach by Jurado et al. (2015) invites further enhancement. Their reliance on PCA and FAVAR models—traditional linear statistical models—introduces notable limitations. Notably, these models struggle to capture the nonlinear relationships among macroeconomic variables, resulting in an incomplete measure of the unpredictable components. By incorporating a machine learning model, specifically the LSTM model, it becomes possible to discern and analyze these nonlinear interactions more effectively.

This research chooses the LSTM model as a methodological advancement, given its proven capabilities with time series data, especially within the context of volatility. Throughout our analysis, we find that uncertainty measured using LSTM exhibits lower levels than those measured by linear statistical models during periods of low uncertainty. Moreover, the predictive accuracy of our measure of uncertainty to forecast key macroeconomic variables in a VAR setting is comparable and even improves over Jurado et al. (2015)'s measure in some cases.

2.3 Methodology

In this study, our main benchmark is the framework proposed by Jurado et al. (2015), hereafter referred to as PCA+FAVAR. A brief introduction to this framework is provided below to furnish basic knowledge before we delve into our LSTM-enhanced framework.

2.3.1 Linear PCA+FAVAR Model

Let $U_{j,t}(h)$ denote the h-period ahead uncertainty obtained from a variable $x_{j,t}$. Uncertainty for this variable is characterized by the conditional volatility of the time series $x_{j,t}$. More formally, let

$$U_{j,t}(h) = \sqrt{E\left(\left(x_{j,t+h} - E\left(x_{j,t+h} \mid I_t\right)\right)^2 \mid I_t\right)},$$

where $\mathrm{E}\left(x_{j,t+h} \mid I_t\right)$ denotes the expectation of the random variable $x_{j,t+h}$ evaluated at period t+h conditional on the set of available information at time t. This set I_t contains present and past information on the variable $x_{j,t}$ but also on the remaining variables used to forecast the dynamics of the variable of interest.

Let $\boldsymbol{X}_t = (x_{1,t}, x_{2,t}, ..., x_{N,t})$ denote all the predictors available, where N is the number of macroeconomic and financial series used for the measurement of uncertainty. The sequential procedure introduced by Jurado et al. (2015) is to extract the factors from $\boldsymbol{X}_t$ via PCA. Let

$$x_{j,t} = \lambda^F \boldsymbol{F}_t + e^x_{j,t},$$

where $\boldsymbol{F}_t$ are the latent common factors, λ^F are the latent factor loadings and $e^x_{j,t}$ are the idiosyncratic errors for the variable, $x_{j,t}$. To capture potential nonlinearities, these authors introduce two additional predictors, denoted by $\boldsymbol{W}_t$, given by the squares of the first component of $\boldsymbol{F}_t$ and the first factor extracted from the squared values of $\boldsymbol{X}_t$ via another PCA model as introduced above.

To calculate the uncertainty in the series $x_{j,t}$, its forecast for period $h \geq 1$ is estimated with a factor-augmented forecasting model.

$$x_{j,t+1} = \Phi^x_j(L)x_{j,t} + \gamma^F_j(L)\boldsymbol{F}_t + \gamma^W_j(L)\boldsymbol{W}_t + v_{j,t+1}.$$

In this formula, L represents the lag operator indicating that the dependent variable as well as the observable and unobservable regressors are dynamic and include lags in the predictive regression model. Furthermore, Jurado et al. (2015) also allow for the presence of time-varying volatility in each of these variables. This feature introduces time-varying uncertainty into the series $x_{j,t}$. When these factors display autoregressive dynamics, the forecasting model enriched with these factors is concisely described as a FAVAR model. Let $\boldsymbol{Z}_t = \left(\boldsymbol{F}'_t, \boldsymbol{W}'_t\right)'$, which aggregates both latent factors and additional predictors and define $\mathcal{Z}_t = \left(\boldsymbol{Z}_t, \boldsymbol{Z}_{t-1}, \ldots, \boldsymbol{Z}_{t-q+1}\right)'$ and $X_{j,t} = \left(x_{j,t}, x_{j,t-1}, \ldots, x_{j,t-q+1}\right)'$. A suitable vector autoregressive specification for modeling the joint dynamics of all the variables in the system is the following;

$$\begin{pmatrix} \mathcal{Z}_t \\ X_{j,t} \end{pmatrix} = \begin{pmatrix} \Phi^{\mathcal{Z}} & 0 \\ \lambda'_j & \Phi^X_j \end{pmatrix} \begin{pmatrix} \mathcal{Z}_{t-1} \\ X_{j,t-1} \end{pmatrix} + \begin{pmatrix} V^{\mathcal{Z}}_t \\ V^X_{j,t} \end{pmatrix}.$$

$$\mathcal{X}_{j,t} = \Phi_j^{\mathcal{X}} \mathcal{X}_{j,t-1} + \mathcal{V}_{j,t}^{\mathcal{X}}$$

The h-step-ahead forecast of macroeconomic series will be the conditional mean under quadratic loss.

$$E_t\left(\mathcal{X}_{j,t+h}\right) = \left(\Phi_j^{\mathcal{X}}\right)^h \mathcal{X}_{j,t}$$

The forecast error variance at time t will be:

$$\Omega_{j,t}^{\mathcal{X}}(h) = E_t\left[\left(\mathcal{X}_{j,t+h} - E_t\mathcal{X}_{j,t+h}\right)\left(\mathcal{X}_{j,t+h} - E_t\mathcal{X}_{j,t+h}\right)'\right].$$

The one-step-ahead prediction errors of $\mathcal{Z}_t$ and $X_{j,t}$ are permitted to exhibit time-varying volatility. Consequently, the stochastic volatility models are constructed for the forecast errors, both $V_t^{\mathcal{Z}}$ and $V_{j,t}^X$. For any $V_{t+1} \in \{V_{t+1}^{\mathcal{Z}}, V_{j,t+1}^X\}$:

$$V_{t+1} = \sigma_{t+1}\varepsilon_{t+1}, \varepsilon_{t+1} \overset{iid}{\sim} N(0,1)$$

$$\log \sigma_{t+1}^2 = \alpha + \beta \log \sigma_t^2 + \tau\eta_{t+1}, \eta_{t+1} \overset{iid}{\sim} N(0,1),$$

In stochastic volatility models, a shock to the second moment is allowed to be independent of the first moment, which implies that:

$$E_t\left(\sigma_{t+h}^2\right) = \exp\left[\alpha \sum_{s=0}^{h-1} \beta^s + \frac{\tau^2}{2} \sum_{s=0}^{h-1} \beta^{2s} + \beta^h \log \sigma_t^2\right],$$

since $\varepsilon_t \overset{iid}{\sim} (0,1)$, it is fact that $E_t\left(V_{t+h}^2\right) = E_t\left(\sigma_{t+h}^2\right)$, then the forecast error variance for $h > 1$ can be computed according to the decomposition of forecast error variance by Jurado et al. (2015). The h-step-ahead forecast error variance for $X_{j,t+h}$ can be expressed as:

$$\Omega_{j,t}^X(h) = \Phi_j^X \Omega_{j,t}^X(h-1)\Phi_j^{X\prime} + \Omega_{j,t}^{\mathcal{Z}}(h-1) + E_t\left(V_{jt+h}^X V_{jt+h}^{X}{}'\right) + 2\Phi_j^X \Omega_{j,t}^{X\mathcal{Z}}(h-1) \quad (2.1)$$

and the h-period ahead uncertainty at time t is defined as the square root of the forecast error variance such that

$$U_{j,t}^X(h) = \sqrt{1_j' \Omega_{j,t}^X(h) 1_j},$$

with 1_j being a $N \times 1$ selection vector where only the j-th element is 1 and all other elements are 0. In the first formula, $\Omega_{j,t}^X(h)$ denotes the forecast error variance for $X_{j,t}$, $\Omega_{j,t}^{\mathcal{Z}}(h)$ represents the forecast error variance for $\mathcal{Z}_t$ and $\Omega_{j,t}^{X\mathcal{Z}}$ is the covariance between $V_{t+1}^{\mathcal{Z}}$ and $V_{j,t+1}^X$. The uncertainty in each macroeconomic series is estimated via the formula above and aggregated to obtain Jurado et al. (2015) estimates of macroeconomic uncertainty:

$$\sum_{j=1}^N \omega_j U_{j,t}^X(h) \quad (2.2)$$

where ω_j are suitable weights allocated to each idiosyncratic uncertainty measure. A straightforward weighting approach is to assign an equal weight to each idiosyncratic uncertainty measure, such that $\omega_j = 1/N$, for $j = 1, \ldots, N$. However, if individual uncertainties exhibit a factor structure, the weights can be determined using the eigenvector associated with the largest eigenvalue of the $N \times N$ covariance matrix derived from the matrix of individual uncertainties.

2.3.2 LSTM Model

In this subsection, we outline and discuss in detail the primary structure of the LSTM model and our main framework for measuring uncertainty with the LSTM model. Initially, we introduce the essential concepts and background of the LSTM model, starting with a generic example of a neural network model. Subsequently, we delve into the architecture of the cells within the LSTM neural network model. Building on this foundation, we explore how LSTM models are employed in the estimation of macroeconomic uncertainty.

Machine learning, a rapidly advancing branch of artificial intelligence, is evolving continuously. Chen et al. (2024) proposes a model that integrates feedforward networks, LSTM networks for recognizing economic state processes, and generative adversarial networks, highlighting that machine learning can analyze time series data with a breadth and precision that traditional econometric methods struggle to achieve. With advancements in statistics, machine learning techniques have expanded beyond the confines of traditional statistical models, enhancing their capability to capture patterns in data and facilitate more precise predictions. The definition of machine learning is broad but can be generally summarized into several key processes: selecting an appropriate model based on the data, continuously optimizing this model, and ultimately deploying the model to predict new datasets. Machine learning finds extensive applications across various fields, including image recognition and classification, speech recognition, natural language processing, and algorithmic recommendations. In the context of time series analysis, which holds significant importance in the area of economics and finance, machine learning-based methods have shown promising improvements in performance, as suggested by Siami-Namini et al. (2018). These advancements emphasize the potential of machine learning to enhance analytical precision and effectiveness in these critical areas.

Neural networks represent a potent category of models within the realm of machine learning techniques. Their structure is inspired by the human brain and mimics the manner in which biological neurons transmit signals to one another. Typically, neural networks comprise several layers: an input layer, multiple hidden layers, and an output layer, all of which consist of numerous cells. This Chapter illustrates the concept using a neural network model with a simplified structure as an example, providing a clear and brief explanation of its components and functions.

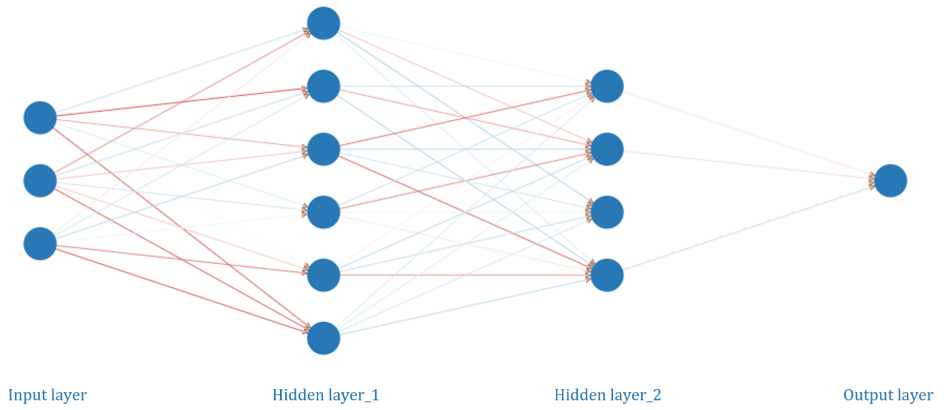


FIGURE 2.1: Neural network model

Figure 3.1 displays a neural network model featuring two hidden layers, where each blue node or cell represents the fundamental units of the model. Each node corresponds to an input value in the input layer or to a processed value through an activation function in the hidden and output layers. The basic workflow within a cell includes receiving inputs, applying weights, accumulating these weighted inputs, and then processing them through an activation function. Subsequently, the final output is relayed to a cell in the subsequent layer or to other nodes within the same layer. The connections between various nodes are depicted by lines, which illustrate the direction of information flow, typically from left to right. Each line is assigned a weight that modulates the strength of the signal transmitted. In the figure, the varying shades of the lines denote different weight values, visually indicating the degree of influence each connection holds within the network.

To perform computations using a neural network, the input data begins at the input layer, passes through each layer via a weighted sum and activation function, and ultimately arrives at the output layer. Here, the model's predictions are compared with the true values, and the prediction error is used for computing a loss function. The gradients of the loss function with respect to each weight are computed, and these gradients are employed to update the weights, thereby minimizing the prediction error across the entire network. This computational sequence involves forward propagation (where data moves through the network and outputs are generated), loss calculation (assessing prediction accuracy), backpropagation (propagating the error back through the network for learning), and weight updates. These steps are repeated across multiple epochs until the model's performance on the training set stabilizes or meets a predefined stopping criterion. Common types of neural networks include Multi-Layer Perceptrons (MLP), Convolutional Neural Networks (CNN), and Recurrent Neural Networks (RNN). The focus of this research, however, will be on the

LSTM (Long Short-Term Memory) neural network (NN), a specialized type of RNN that is particularly effective in handling long-term dependencies in data.

The LSTM NN serves as the primary methodology in this research to replace traditional linear statistical models, such as PCA and FAVAR, in the measurement of uncertainty. The LSTM neural network is a specialized type of recurrent neural network, developed by Hochreiter and Schmidhuber (1997) and further enhanced by Gers et al. (2000), who introduced the concept of forgetting gates. This innovation helps to effectively solve the issue of vanishing gradients that often plagues general recurrent neural networks. The LSTM model is particularly adept at handling long-term dependencies within a sequence and predicting sequence data, making it an invaluable tool for analyzing macroeconomic data series. Similar to conventional neural network models, the LSTM architecture is composed of specific cells. The main structure of cells within an LSTM model is depicted in Figure 2.2:

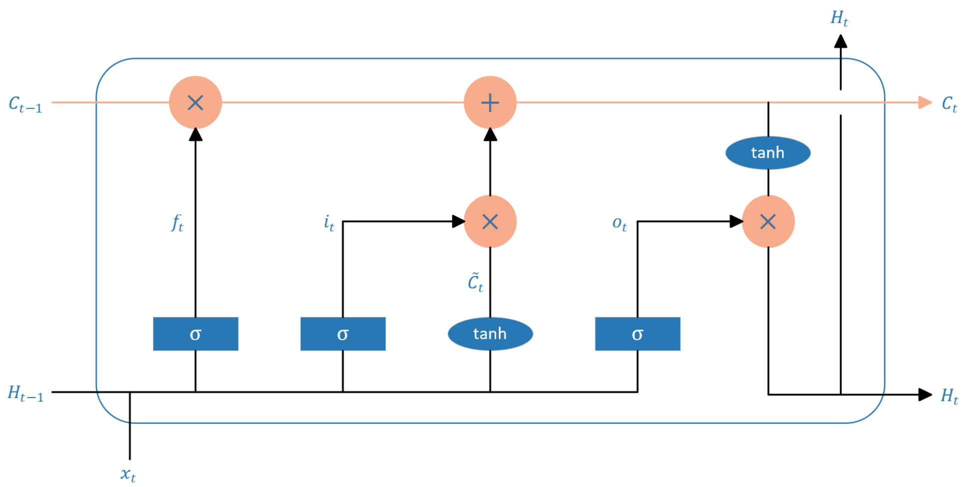


FIGURE 2.2: The structure of LSTM cell, reproduced from Olah (2015)

In the LSTM cell, x_t denotes the current input, H_{t-1} and C_{t-1} represent the output and cell state (or memory) from the previous LSTM cell, respectively. Simultaneously, h_t and C_t are the output and cell state of the current cell, which will subsequently be passed to the next cell. The operations within the cell are sequentially introduced from left to right. The initial step within the cell involves determining which information to discard from the previous cell state. This decision is governed by a sigmoid function, denoted as σ, also known as the 'forget gate'. The forget gate considers both the output from the previous cell and the current input to decide which information is no longer necessary and can be omitted as the process moves forward. Thus,

$$f_t = \sigma\left(W_f\left[H_{t-1}, x_t\right] + b_f\right)$$

$$\sigma(X) = \frac{1}{1 + e^{-X}}$$

where W_f represents the weight matrix and b_f is the bias term associated with the forget gate. The sigmoid function is capable of mapping variables to the interval $(0, 1)$. Through the computation facilitated by the sigmoid layer, we are able to identify which aspects of the previous cell state are relevant and should therefore be retained, allowing this information to flow into the current cell state.

The subsequent step involves determining what new information will be incorporated into the cell state of the current cell. This process is divided into two parts. The first part is handled by the 'input gate', which employs another sigmoid layer to decide which values should be updated, represented by i_t. The second part involves the hyperbolic tangent function, commonly referred to as the tanh layer, to generate new candidate values, denoted by $\widetilde{C}_t$.

$$i_t = \sigma\left(W_i\left[H_{t-1}, x_t\right] + b_i\right)$$

$$\widetilde{C}_t = tanh\left(W_C\left[H_{t-1}, x_t\right] + b_C\right)$$

$$tanh\left(X\right) = \frac{\mathrm{e}^X - \mathrm{e}^{-X}}{\mathrm{e}^X + \mathrm{e}^{-X}}$$

In which, all the W and b represent corresponding weight matrices and bias terms, respectively. The tanh function maps variables to the interval $(-1, 1)$. Both i_t and $\widetilde{C}_t$ will be combined in order to create an update to the cell state of current cell in the following step as shown in the formula below:

$$C_t = f_t * C_{t-1} + i_t * \widetilde{C}_t.$$

In this expression, the first term represents the portion of information from the previous cell state to be forgotten, as determined at the beginning. The second term represents the new candidate values generated in the current cell, scaled by the amount of new information to be updated.

Ultimately, the determination of the output for the current cell is dependent on the cell state. The LSTM model is constructed such that the initial output is determined by a sigmoid layer; this output is then modulated by the cell state, using a tanh layer to refine it.

$$o_t = \sigma\left(W_o\left[H_{t-1}, x_t\right] + b_o\right)$$

$$H_t = o_t * tanh\left(C_t\right)$$

where W_o represents the weight matrix and b_o donates the bias term associated with the initial layer. The cell state undergoes processing through the tanh layer, which yields values between -1 and 1 serving as the judgment condition. The initial output is multiplied by this judgment condition, ensuring that only crucial parts are expressed by the current cell. By defining and taking advantage of such cells, LSTM models can be constructed, enabling more complex operations. The LSTM models used in this study will be illustrated upon in subsequent sections.

2.3.3 Measuring Macroeconomic Uncertainty using LSTM

This section explores the framework for measuring uncertainty using LSTM models. Our framework is adapted from Jurado et al. (2015) PCA+FAVAR setting introduced above. Our model incorporates two LSTM models to forecast macroeconomic and financial series through recursive procedure. Let $\boldsymbol{X}$ represent the data series — the predictors used to conduct our analysis. This series consists of two parts: the first part is the macroeconomic data series, denoted by $\boldsymbol{X}^m$, and the second part is the financial data series, denoted by $\boldsymbol{X}^f$. Together, these series contain all available information for estimating uncertainty at time t.

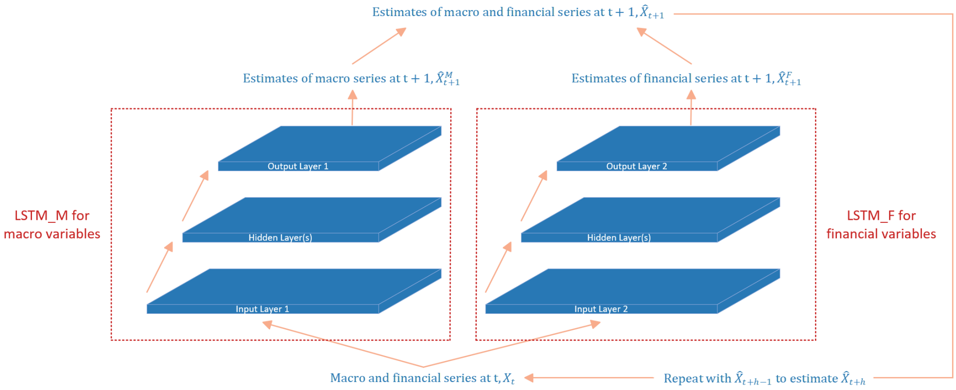


FIGURE 2.3: Forecasting with LSTM models

Figure 2.3 above illustrates the process of forecasting macroeconomic and financial series h steps ahead using two LSTM models with recursive procedure. At time t, the macroeconomic and financial series serve as inputs, and the two LSTM models respectively output the estimates for these series at time $t+1$. Subsequently, these estimates are combined and re-inputted into the LSTM models to generate estimates for time $t+2$. Through this recursive process, the forecasts h steps ahead are derived, following the constraints defined by the uncertainty model strictly, which only considers the information available at time t. The methodology of our framework for measuring macroeconomic uncertainty with LSTM models will be delineated step by step, starting from the forecasting of macroeconomic and financial series.

Let $\boldsymbol{X}_t^m$ and $\boldsymbol{X}_t^f$ denote the vector of macroeconomic and financial variables, respectively. The predictive LSTM model can be expressed as

$$\hat{\boldsymbol{X}}^m_{t+1} = f^M\left(\boldsymbol{X}^m_t, \boldsymbol{X}^f_t\right) + \boldsymbol{v}^m_{t+1}.$$

In the above formula, $f^M(\cdot,\cdot)$ represents the LSTM model that processes the vector of time series, which predicts all the macroeconomic variables for the subsequent time step. $\boldsymbol{v}^m_{t+1} = (v^m_{1,t+1}, v^m_{2,t+1}, ..., v^m_{n,t+1})'$ denotes the one-step-ahead forecast error vector of $\boldsymbol{X}^m_{t+1}$. The time-varying volatility is permitted within $v^m_{i,t+1}$, and for any $i \neq j$, we have $\mathrm{Cov}(v^m_{i,t+1}, v^m_{j,t+1}) = 0$, indicating that the forecast errors are uncorrelated. Similarly, another LSTM model is employed for predicting the estimates of financial variables at the next time step:

$$\hat{\boldsymbol{X}}^f_{t+1} = f^F\left(\boldsymbol{X}^m_t, \boldsymbol{X}^f_t\right) + \boldsymbol{v}^f_{t+1}$$

Here, $f^F(\cdot,\cdot)$ represents the LSTM model designated for forecasting financial variables, and $\boldsymbol{v}^f_{t+1} = (v^f_{1,t+1}, v^f_{2,t+1}, ..., v^f_{n,t+1})$ represents the one-step-ahead forecast error vector for the corresponding financial variables, where each component $v^f_{j,t+1}$ is uncorrelated.

In this study, as we focus primarily on estimating macroeconomic uncertainty, our attention is chiefly on the macroeconomic data series. However, the inclusion of financial variables in the information set improves considerably the forecasting performance of the macroeconomic variables. The recursive process for forecasting the macroeconomic variables h periods ahead is as follows:

$$\begin{aligned}\hat{\boldsymbol{X}}^m_{t+2} &= f^M\left(\hat{\boldsymbol{X}}^m_{t+1}, \hat{\boldsymbol{X}}^f_{t+1}\right) + \boldsymbol{v}^m_{t+2}\\ &= f^M\left(f^M\left(\boldsymbol{X}^m_t, \boldsymbol{X}^f_t\right), f^F\left(\boldsymbol{X}^m_t, \boldsymbol{X}^f_t\right)\right) + \boldsymbol{v}^m_{t+2}\end{aligned}$$

$$\begin{aligned}\hat{\boldsymbol{X}}^m_{t+3} &= f^M\left(\hat{\boldsymbol{X}}^m_{t+2}, \hat{\boldsymbol{X}}^f_{t+2}\right) + \boldsymbol{w}^m_{t+3}\\ &= f^M\left(f^M\left(f^M\left(\boldsymbol{X}^m_t, \boldsymbol{X}^f_t\right), f^F\left(\boldsymbol{X}^m_t, \boldsymbol{X}^f_t\right)\right)\right.\\ &\quad \left., f^F\left(f^M\left(\boldsymbol{X}^m_t, \boldsymbol{X}^f_t\right), f^F\left(\boldsymbol{X}^m_t, \boldsymbol{X}^f_t\right)\right)\right) + \boldsymbol{v}^m_{t+3}\end{aligned}$$

$$\cdots$$

$$\hat{\boldsymbol{X}}^m_{t+h} = f^M\left(\hat{\boldsymbol{X}}^m_{t+h-1}, \hat{\boldsymbol{X}}^f_{t+h-1}\right) + \boldsymbol{v}^m_{t+h}$$

The estimated value of the macroeconomic variables at the $t+2$ time step is forecasted using the estimated values of both macroeconomic and financial variables at the $t+1$ time step. This recursive procedure allows the estimation of the h-step-ahead macroeconomic variable. It is important to note that the LSTM models are only trained once for predicting the estimates at $t+1$. The same models are then used recursively to make further predictions without any modifications or retraining. This approach is necessary to follow the original constraints within the PCA+FAVAR framework, which considers only the available information at time t to generate the forecasts.

After forecasting the macroeconomic variables from time step $t+1$ to $t+h$, the next task involves modeling the time-varying volatility of the forecast error $v^m_{j,t}$, and the forecast error variance, $\Omega_{j,t}^{m}$ for every macroeconomic variable, $x_{j,t}^{m}$. Given that the forecast error exhibits time-varying stochastic volatility and uncorrelated across predictors according to the assumption from Jurado et al. (2015), the log volatility is expected to follow an autoregressive structure. When $h = 1$:

$$v_{j,t+1}^{m} = \sigma_{j,t+1}^{m} \varepsilon_{j,t+1}^{m}, \varepsilon_{j,t+1}^{m} \overset{iid}{\sim} N(0,1)$$

$$\log \left(\sigma_{j,t+1}^{m}\right)^{2} = \alpha_{j}^{m} + \beta^{m} \log \left(\sigma_{j,t}^{m}\right)^{2} + \tau_{j}^{m} \eta_{j,t+1}^{m}, \eta_{j,t+1}^{m} \overset{iid}{\sim} N(0,1),$$

The shock to the second moment is allowed to be independent of the first moment in the stochastic volatility model, which implies that

$$E_{t} \left(\sigma_{t+h}^{m}\right)^{2} = \exp \left[\alpha^{m} \sum_{s=0}^{h-1} \left(\beta^{m}\right)^{s} + \frac{\left(\tau^{m}\right)^{2}}{2} \sum_{s=0}^{h-1} \left(\beta^{m}\right)^{2s} + \left(\beta^{m}\right)^{h} \log \left(\sigma_{t}^{m}\right)^{2}\right]$$

where $\varepsilon_{i,t+h}^{m} \overset{iid}{\sim} (0,1)$ such that $E_{t} \left(v_{t+h}^{m}\right)^{2} = E_{t} \left(\sigma_{t+h}^{m}\right)^{2}$. For $h > 1$, the forecast error variance for $x_{i,t}^{m}$, denoted as $\Omega_{i,t}^{m}(h)$, is computed by the recursion:

$$\Omega_{i,t}^{m}(h) = \Phi_{i} \left(\Omega_{i,t}^{m}(h-1)\right) \Phi_{i}^{\prime} + E_{t}(v_{i,t+h}^{m}, v_{i,t+h}^{m\prime}). \tag{2.3}$$

where Φ_{i} denotes the corresponding coefficients in $f^{M}(\cdot,\cdot)$ when forecasting the macro variables at next time step. Comparing the decomposition expression (2.1) of the PCA+FAVAR approach and expression (2.3) of our framework, it is worth noticing that the components related to factors in forecast error variance are omitted reasonably, as the non-parametric LSTM models are deployed, which leads to lower estimates of macro uncertainty as shown in the following sections.

Under our framework, macroeconomic uncertainty is obtained as in expression (2.2) but aggregating the above idiosyncratic conditional volatility measures $U_{i,t}^{m}(h) = \sqrt{1_{i}^{\prime} \Omega_{i,t}^{m}(h) 1_{i}}$ obtained from expression (2.3). Compared with the PCA+FAVAR approach, a notable enhancement in our framework is the adoption of two LSTM models for forecasting macroeconomic and financial variables up to h time steps ahead through recursive procedure, as opposed to using PCA and FAVAR models. This shift capitalizes on the strengths of LSTM models, particularly their superior capacity for handling large datasets and their adeptness in managing nonlinearity, distinguishing them from linear models.

2.4 Empirical Implementation

This section introduces the dataset employed in this study, the structure of the LSTM models used for empirical implementation, and the estimates of macroeconomic uncertainty for three forecast horizons: $h = 1, 3$, and 12 steps ahead, respectively.

Additionally, a forecast comparison is made, for each forecast horizon, with the estimates of macroeconomic uncertainty presented in Jurado et al. (2015), which serves as benchmark model. The comparisons are formalized statistically through the application of Diebold-Mariano tests, which assess the difference in forecast ability between our nonlinear framework and the traditional linear framework, and Granger-causality tests to explore the ability of macroeconomic uncertainty to predict out-of-sample macroeconomic variables within a VAR setting.

2.4.1 Data and Measurement of Macroeconomic Uncertainty

In this research, we consider a dataset consisting of 281 data series to estimate macroeconomic uncertainty, maintaining consistency with Jurado et al. (2015) by using the same dataset. This approach helps eliminate any discrepancies that could arise from using different datasets. The dataset is divided into two parts: the first contains 133 monthly macroeconomic series as introduced by Ludvigson and Ng (2016), encompassing a comprehensive range of macroeconomic indicators such as real income and output, manufacturing, trade sales, hours, real retail, inventories, inventory-sales ratios, orders, labor costs, compensation, housing starts, price indexes, bond and stock market indexes, foreign exchange rates, and others. The second part comprises 148 monthly financial time series from the study by Ludvigson and Ng (2007), featuring variables like the dividend-price ratio, earnings-price ratio, defaults, term spreads, growth rates of aggregated dividends and prices, yields on corporate bonds, Treasury yields, yield spreads, and cross-sections of industry, book-market, size, and equity returns. These series extend from January 1960 to May 2022.

In the measurement of PCA+FAVAR, the factors were extracted from the whole dataset which contained all the available information in the time series. Jurado et al. (2015) subsequently use these factors along with the 133 macroeconomic series to implement the FAVAR model and estimate uncertainty within each individual series. Hence, in our LSTM-based framework of measuring uncertainty, we employ the all the macroeconomic and financial series as the input of our LSTM models and forecast macroeconomic and financial series at next time step with two LSTM model respectively, as introduced by Figure 2.3.

Our framework to estimate macroeconomic uncertainty starts with a first LSTM model, which processes macroeconomic series. This is illustrated in Figure 2.4 below.

This LSTM model comprises three layers, setting tanh activation functions in the cells, as defined in Section 2.3.2. The initial layer serves as the input layer, accommodating an input size of $(6, 281)$. The dataset includes a total of 749 time steps, and has been separated into smaller series, each with a time spread of 6 steps. These smaller sequences each consist of 6 time steps and 281 series of macroeconomic and financial

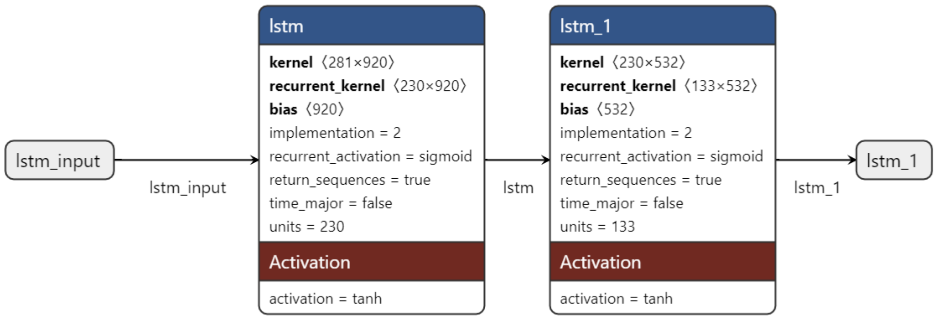


FIGURE 2.4: Structure of LSTM forecasting macroeconomic series

variables at every step. The second and third layers function as forecasters. The output dimension of the second layer is $(6, 230)$, while the third layer outputs $(6, 133)$, which constitutes the final output we require. The input dataset comprises macroeconomic and financial data series from time step 1 to 748, with the labels being the macroeconomic data series from time step 2 to 749. Thus, we use all available data at time step t to predict the value of macroeconomic variables at time step $t + 1$.

A second LSTM model forecasts the financial data series. Although our primary focus is on macroeconomic uncertainty, forecasting the macroeconomic series several periods ahead necessitates including the financial series as an input into the LSTM model due to our recursive structure. The structure of the second LSTM model is illustrated in Figure 2.5 below.

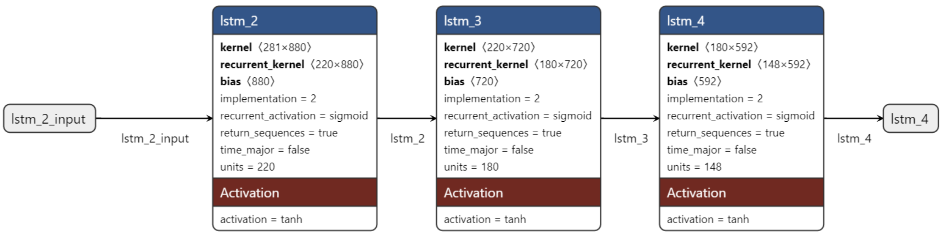


FIGURE 2.5: Structure of LSTM forecasting financial series

The second LSTM model consists of four layers [1], with tanh activation functions in the cells. As illustrated in Figure 2.5, the first layer is the input layer, featuring an input size of $(6, 281)$, identical to the first LSTM model. This input dimension reflects that the dataset, encompassing both macroeconomic and financial series, is separated into smaller sequences each containing 6 time steps and 281 series. The subsequent layers, from the second to the fourth, serve forecasting and output purposes. The ultimate output dimension is $(6, 148)$, representing the forecasted financial data series for the $t + 1$ time step. The input dataset mirrors that of the first LSTM model, spanning from

[1]Compared to X_t^m, there are certain multicollinearity problems in X_t^f, To address these, we devise a more complex LSTM model after pre-processing the data. The $f^F()$ incorporates an extra hidden layer, refined hyperparameter tuning, and L1 regularization, which tends to generate sparse weight matrices and helps with feature selection.

time step 1 to 748, and uses the financial data series from time step 2 to 749 as labels. Overall, the second LSTM model uses all available data at time step t to forecast the financial series at the next time step.

Prior to initiating the training, and following the methodology employed by Jurado et al. (2015), the entire dataset is standardized using the Z-score given by $Z_i = \frac{(X_i - \mu_i)}{\sigma_i}$, where X_i is the orginal series, μ_i and σ_i are mean and standard deviation. The dataset is divided into two segments: 80% for the training set and 20% for the test set. The training phase involves 300 epochs, employed model optimization algorithm (Adaptive Moment Estimation optimizer) to minimize the mean square error loss. Additionally, dimensional reduction and weight matrix optimization methods such as L1 and L2 normalization, are implemented in this LSTM model to curb overfitting during training and improve the generalization ability. Post-training, the LSTM models are employed to predict the estimated values of the macroeconomic and financial series at time $t + 1$. The outputs from the LSTM models comprise numerous small sequences, each sized $(6, 133)$ or $(6, 148)$, which can be reorganized by time step to reconstruct the entire series at the $t + 1$ time step. Employing these two LSTM models enables predictions of the macroeconomic and financial series at time $t + 1$. Subsequently, the forecasts of the macroeconomic and financial series at the $t + 1$ time step are inputted back into the LSTM models to generate predictions for both series at time $t + 2$. This recursive process facilitates the generation of h-step-ahead forecasts for the macroeconomic series and helps measure the associated forecast errors, essential for quantifying uncertainty in the macroeconomic series

Panel A, B and C of Figure 2.6 display the estimates of macroeconomic uncertainty represented by blue lines for $h = 1, 3$, and 12, compared against the macroeconomic uncertainty measured by PCA+FAVAR, depicted in red dash-dot lines.

The horizontal dashed lines in the figures represent 1.64 standard deviations from the mean of each series, delineating periods of relatively high uncertainty[2]. Four significant peaks emerge from the time series estimates. These periods of heightened uncertainty align with major crises known for their extensive and profound impacts: the energy crisis around 1974, the recession stemming from monetary policy around 1981, the financial crisis around 2008, and the onset of the coronavirus pandemic starting in 2020. Our LSTM-based framework effectively identifies all these crises, corroborating historical records. As the forecast horizon h extends, the average level of uncertainty generally increases. However, the variability or fluctuation of uncertainty decreases, indicating that the forecast gradually converges towards the unconditional mean as the forecast horizon approaches infinity, aligning with the measurements obtained via PCA+FAVAR.

[2]Since macroeconomic uncertainty is calculated using the weighted uncertainty of individual series, confidence intervals are not disclosed. Instead, the 1.64 standard deviations are derived from the unconditional sample standard deviation. This methodology follows the same approach as Jurado et al. (2015) to identify periods of high uncertainty.

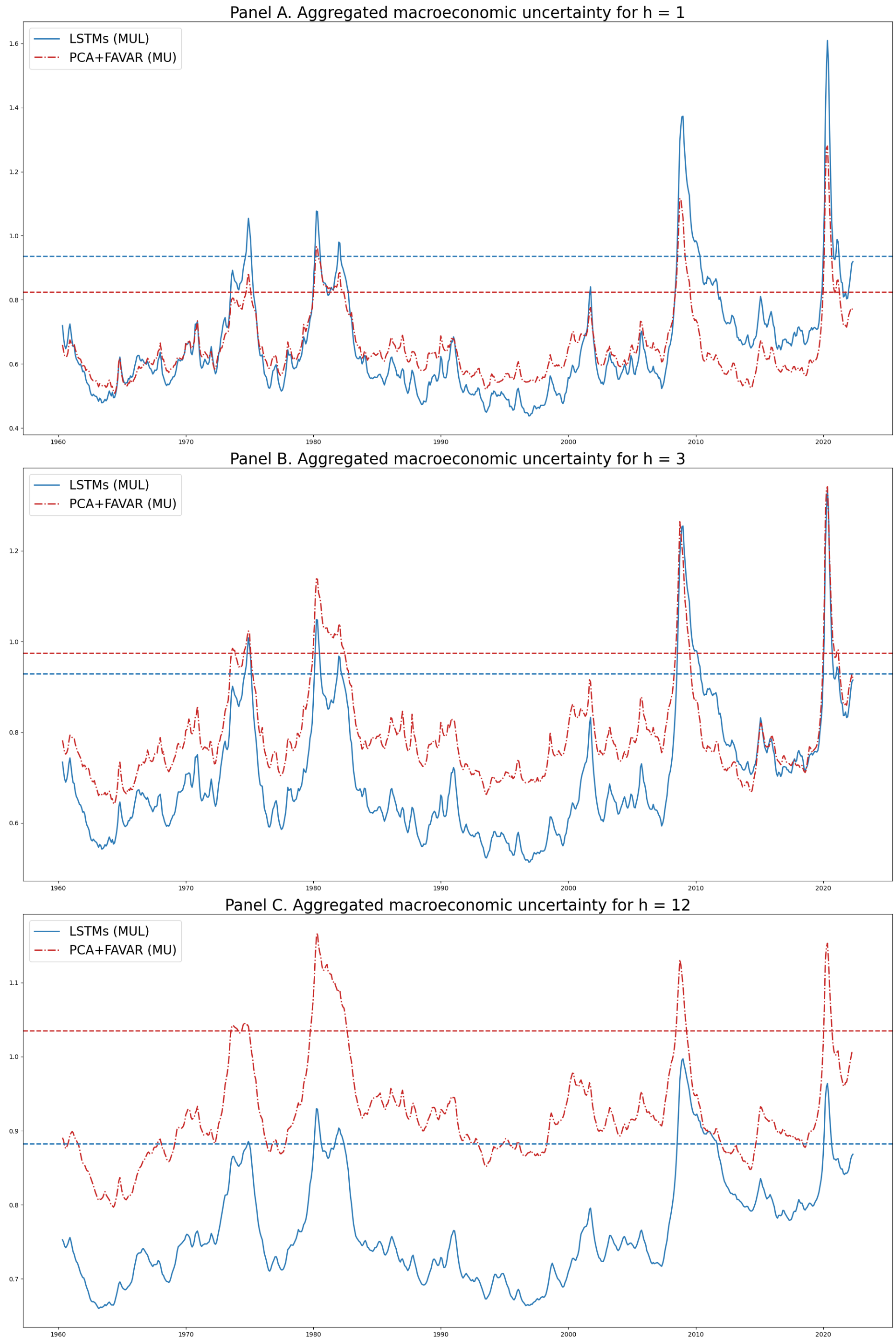


FIGURE 2.6: Aggregated macroeconomic uncertainty

Figure 2.6 is divided into Panels A, B, and C, each illustrating aggregated macroeconomic uncertainty derived from an LSTM-based framework (shown by blue lines) for horizons $h = 1, 3$, and 12. These are compared to macroeconomic uncertainty measured using PCA+FAVAR (depicted with red dash-dot lines). The horizontal lines on each panel indicate 1.64 standard deviations from the mean of the series, marking periods of relatively high uncertainty.

In comparison to the estimates by PCA+FAVAR, those derived from our LSTM framework show stronger nonlinearity and more pronounced fluctuations, particularly for $h = 1$ and 3. During periods of high uncertainty, our framework adeptly detects and identifies surges in uncertainty. Our estimates either mirror or exceed those derived from PCA+FAVAR in terms of magnitude. In contrast, during periods of low uncertainty, our framework efficiently lowers the estimates of macroeconomic uncertainty. The overall tendency of our nonlinear macroeconomic uncertainty estimates is to remain significantly lower than those from PCA+FAVAR, while still retaining the same or enhanced ability to explain and predict macroeconomic variables, which will be discussed below. Before this, we present a forecast evaluation exercise to compare the macro uncertainty estimates obtained from the two models. To do this, we consider the Diebold-Mariano (DM) test.

2.4.2 Forecast Evaluation Exercise

A refined version of the DM test of equal predictive ability allows us to determine the presence of superior predictive ability between pairs of forecasts. Thus, the null hypothesis of the DM test can be written as $H_0 : g(\varepsilon_{1t}) \geq g(\varepsilon_{2t})$ whereas the alternative hypothesis corresponds to $H_A : g(\varepsilon_{1t}) < g(\varepsilon_{2t})$, and $g(\cdot)$ denotes some suitable loss function such as the mean square prediction error. In this setting, rejection of the null hypothesis is interpreted as evidence of superior predictive ability of the model characterized by the error term ε_{1t}.

The DM test is implemented as follows. Let $d_t = g(e_{1t}) - g(e_{2t})$ denote the loss differential between the two predictive loss functions evaluated with the residuals e_{1t} and e_{2t} of the two forecast models, and let $\bar{d}$ be its sample mean such that $\bar{d} = \frac{1}{T}\sum_{t=1}^{T}[g(e_{1t}) - g(e_{2t})]$. To account for the presence of serial correlation in the residuals, we compute the Newey-West estimator of the long run variance of d_t according to Newey and West (1987), given by

$$\hat{s}_{d,T} = \hat{\sigma}_{d,0} + 2\sum_{l=1}^{q}\widetilde{\omega}_l\hat{\sigma}_{d,l}$$

with $\hat{\sigma}_{d,l} = \frac{1}{T}\sum_{t=l+1}^{T} d_t d_{t-l}$ for $l = 0, 1, \ldots$ and $\widetilde{\omega}_l = 1 - \frac{l}{q+1}$ a suitable kernel function; q denotes the truncation lag. Consequently, the DM statistic is defined as

$$DM = \sqrt{T}\frac{\bar{d}}{\sqrt{\hat{s}_{d,T}}}.$$

The critical values of the test are obtained under the least favourable case given by equal predictive ability ($g(\varepsilon_{1t}) = g(\varepsilon_{2t})$). In this case, the DM test statistic converges to a $N(0,1)$ distribution and rejection of the null hypothesis is assessed by the left tail of this distribution. Thus, Model 1 exhibits superior predictive performance compared to Model 2 if the p-value obtained from the left tail is smaller than the significance level α. More specifically, let Model 1 be the LSTM approach and Model 2 the PCA+FAVAR method. The outcomes of these tests for forecast horizons $h = 1, 3$, and 12 are presented in Table 2.1 below.

TABLE 2.1: DM test for macroeconomic uncertainty (LSTMs vs PCA+FAVAR)

h = 1		h = 3		h = 12	
DM	p-value	DM	p-value	DM	p-value
5.068	0.999	-10.003	0.000	-5.538	0.000

Table 2.1 shows that for h $= 1$ there are no statistically significant differences in predictive ability across macroeconomic uncertainty estimates. A plausible explanation is that although our framework generally reduces the estimates of macroeconomic uncertainty during calm periods compared to the benchmark model, the LSTM framework predicts increased uncertainty during the four prominent peaks that correspond to the major crisis periods. The latter effect compensates for the reduced uncertainty in the remaining periods such that the two methods report similar average loss functions computed over the evaluation period. For forecast horizons h $= 3, 12$, the macroeconomic uncertainty estimates by the LSTMs framework are significantly lower than those obtained by the PCA+FAVAR across the evaluation period. While the trend of macroeconomic uncertainty remains generally consistent with that of PCA+FAVAR, the reduction primarily manifests for periods of lower uncertainty at h $= 3$. For h $= 12$, our estimates of macroeconomic uncertainty remain much lower across all stages. This difference suggests that LSTM models exhibit superior predictive ability compared to PCA+FAVAR, a factor that becomes increasingly relevant at longer forecast horizons.

The following empirical exercise aims to determine the forecast ability of our measure of macroeconomic uncertainty. Similar to the approach by Jurado et al. (2015), we incorporate key macroeconomic variables and macroeconomic uncertainty using monthly data into the same VAR(11) model characterized by 10 macro variables and the measure of uncertainty studied by these authors. The macroeconomic variables are the log of real IP, log of employment, log of real consumption, log of PCE deflator, log of real new orders, log of real wages, hours, federal funds rate, log of S&P 500 Index, the growth rate of M2. To evaluate the predictive effectiveness of the LSTM derived uncertainty measure and its comparative strength against the PCA+FAVAR derived uncertainty measure, we conduct a predictive performance assessment. This involves employing Granger causality tests to verify the statistical significance of the uncertainty measures and, more critically, applying the DM test to rigorously compare their predictive power.

The null hypothesis for the Granger-causality test asserts that there is no predictive ability in the VAR specification from the uncertainty measures to the macroeconomic variables, while the alternative hypothesis suggests that the macroeconomic uncertainty measures possess predictive power over these variables. Table 2.2 displays the p−values from the Granger-causality tests for the VAR(11) model, which is augmented with various macroeconomic uncertainty measures. The first three columns report the p−values utilizing the uncertainty measures by Jurado et al. (2015), constructed from different forecast horizons ($U1J$, $U3J$ and $U12J$). The remaining three columns present the results for analogous models that incorporate macroeconomic uncertainty measures developed by the LSTMs framework ($U1L$, $U3L$ and $U12L$).

TABLE 2.2: VAR(11) Granger-causality p-values (LSTMs)

	$U1J$	$U3J$	$U12J$	$U1L$	$U3L$	$U12L$
log(real IP)	0.000	0.000	0.000	0.000	0.000	0.000
log(employment)	0.000	0.000	0.000	0.000	0.000	0.000
log(real consumption)	0.000	0.000	0.000	0.000	0.000	0.000
log(PCE deflator)	0.000	0.000	0.000	0.000	0.000	0.000
log(real new orders)	0.019	0.000	0.001	0.000	0.000	0.000
log(real wage)	0.081	0.217	0.075	0.147	0.206	0.260
hours	0.000	0.000	0.000	0.000	0.000	0.001
federal funds rate	0.000	0.000	0.001	0.000	0.000	0.002
log(S&P 500 Index)	0.002	0.000	0.001	0.002	0.002	0.002
growth rate of M2	0.000	0.000	0.000	0.000	0.000	0.008

The results from Table 2.2 reveal that most macroeconomic variables demonstrate significant Granger-causality with both types of macroeconomic uncertainty estimates. The only exception is log wages, for this variable none of the uncertainty measures shows predictive ability. This empirical finding may be the result of the sticky character of wages that do not rapidly react to changes in the macroeconomic outlook.

Building upon the strong results obtained from the Granger-causality tests, the next empirical exercise compares the forecasts of the ten macroeconomic variables obtained from the VAR(11) model, augmented to incorporate the different macroeconomic uncertainty estimates. The comparison is carried out using the DM tests under the competing specifications (LSTMs and PCA+FAVAR). The results of these tests are detailed in Table 2.3 below.

The results presented in Table 2.3 are based on the two-sided version of the DM test. To compare the predictive ability of the models under the alternative hypothesis we consider the sign of the DM statistic reported in the table. A statistically significant negative DM statistic implies that the VAR(11) model incorporating macroeconomic uncertainty from the LSTM framework possesses better predictive performance, whereas a statistically significant positive DM statistic indicates superior predictive ability of the VAR(11) model including the macroeconomic uncertainty measure obtained from the PCA+FAVAR. In 24 out of 30 instances, the VAR(11) model that includes macroeconomic uncertainty estimates from the LSTM framework

TABLE 2.3: DM test for VAR(11) models (LSTMs vs PCA+FAVAR)

	h = 1		h = 3		h = 12	
	DM	p-value	DM	p-value	DM	p-value
log (real IP)	6.429	0.000	2.074	0.038	0.298	0.766
log (employment)	−4.172	0.000	−1.232	0.218	−0.545	0.586
log (real consumption)	−6.083	0.000	−0.471	0.638	1.665	0.096
log (PCE deflator)	−1.320	0.187	−2.201	0.028	−0.526	0.599
log (real new orders)	−6.206	0.000	−2.598	0.010	−1.508	0.132
log (real wage)	−6.997	0.000	−0.027	0.979	2.164	0.031
hours	0.282	0.778	1.050	0.294	−0.394	0.694
federal funds rate	−2.893	0.004	0.459	0.646	0.943	0.346
log(S&P 500 Index)	6.596	0.000	3.100	0.002	0.597	0.551
growth rate of M2	3.682	0.000	1.040	0.299	0.691	0.490

demonstrate superior or comparable forecast ability compared to those using estimates from the PCA+FAVAR. The results from these tests provide strong empirical evidence in favour of the LSTM-induced uncertainty measure for forecasting purposes compared to Jurado et al. (2015) uncertainty measure.

Considering the significant advantages of the LSTM model over linear models in measuring uncertainty, an additional empirical exercise is to employ the LSTM model, instead of the VAR(11) model, for predicting the above macroeconomic variables. Unlike the VAR(11) model, the LSTM can adeptly capture and learn nonlinear relationships as well as manage long-term dependencies in lengthy data sequences. While the VAR(11) model tries to accommodate long-term dependencies by the lag length, this approach typically leads to a marked rise in model complexity and often falls short of achieving the efficacy of LSTM techniques. Consequently, to conduct a more thorough comparison of the predictive ability of the macroeconomic uncertainty measures on macroeconomic variables, further analysis using the LSTM model is necessary. In the following analysis, we use the macroeconomic variables and macroeconomic uncertainty estimated from our LSTMs framework alongside those from Jurado et al. (2015), to compile two distinct datasets—each containing eleven variables—and train two separate LSTM models to predict the ten macroeconomic variables. By employing the DM test to assess the predictive performance of these LSTM models, we aim not only to determine the uncertainty measure with superior predictive ability but also to illustrate their practical effectiveness within a nonlinear predictive modeling context. The objective of this exercise is to gauge the predictive ability of the two uncertainty measures in a nonlinear setting given by the LSTM that replaces the linear VAR specification.

The findings from these analyses are collated and displayed in Table 2.4 below, which helps further substantiate the predictive efficiency and robustness of the applied models.

Table 2.4 implements the DM test that compares the predictive ability of the two competing uncertainty measures in a nonlinear LSTM setting. There are only 5

TABLE 2.4: DM test for LSTM-VAR(11) models (LSTMs vs PCA+FAVAR)

	h = 1		h = 3		h = 12	
	DM	p-value	DM	p-value	DM	p-value
log (real IP)	−3.9496	0.0001	−14.7853	0.000	−2.2647	0.0238
log (employment)	−5.1876	0.000	−14.9482	0.000	−0.9009	0.3679
log (real consumption)	−8.0636	0.000	−12.1360	0.000	−0.7853	0.4326
log (PCE deflator)	−7.1100	0.000	−15.9269	0.000	−2.6523	0.0082
log (real new orders)	−11.4145	0.000	9.2127	0.000	−3.7954	0.0002
log (real wage)	−5.6710	0.000	−13.9576	0.000	−1.1965	0.2319
hours	7.8513	0.000	10.0987	0.000	1.1657	0.2441
federal funds rate	21.0856	0.000	5.4253	0.000	−0.5471	0.5845
log(S&P 500 Index)	−3.6717	0.0003	−8.1042	0.000	2.8128	0.0050
growth rate of M2	−23.6882	0.000	−12.3132	0.000	−2.2770	0.0231

scenarios in which the null hypothesis of equal predictive ability could not be rejected. In contrast, in 19 instances, the double LSTM model (LSTM to obtain the macroeconomic uncertainty and LSTM to forecast the macroeconomic variables) outperformed the LSTM model that uses the macroeconomic uncertainty measures obtained from PCA+FAVAR. In the remaining 6 cases the LSTM model with PCA+FAVAR-derived uncertainty exhibited superior predictive ability compared to the double LSTM approach.

2.5 Robustness

To assess the robustness of our framework with LSTM models and further explore why the LSTM model demonstrates significant advantages over linear models, we propose two alternative frameworks. One uses an LSTM autoencoder to substitute the PCA model in the measurement approach of Jurado et al. (2015), and the other employs a FAVAR model in place of an LSTM model within our main framework. Both alternative frameworks and their respective results are discussed in this section[3]

2.5.1 LSTM vs PCA: Robustness Check 1

In this section, the LSTM model replaces the PCA model in the PCA+FAVAR specification proposed by Jurado et al. (2015). The objective of this robustness exercise is to further elucidate the contribution of the LSTM model to measuring macroeconomic uncertainty. In this case, we explore the potential of this nonlinear method to extract factors. The main advantage compared to standard PCA is the ability to capture nonlinearities in the unobserved components.

[3]In addition to the alternative frameworks discussed in this section, another potential robustness check could involve mimicking the structure of PCA and FAVAR models using two LSTM models, which would differ from our non-parametric main framework. However, such a robustness check is not considered in this section. The reasoning is that mimicking the structure of the FAVAR model with LSTM models is unnecessary, given the impressive capability of LSTM models in handling large-dimensional time series data.

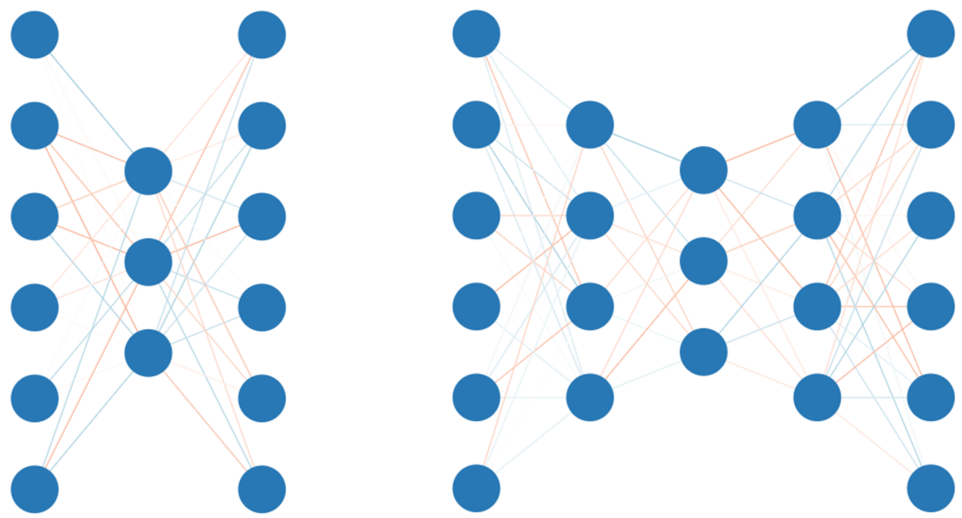


FIGURE 2.7: Neural network representation of PCA and LSTM autoencoder

Figure 2.7 illustrates the structure of the PCA model as an autoencoder alongside the structure of the LSTM autoencoder. For the PCA model, the left part of the figure depicts the process of factor extraction, with the input and output layers representing the original and estimated datasets, respectively, and the middle layer denoting the factors extracted from the dataset. Compared to the PCA model, the LSTM model used to extract factors is more complex, incorporating additional hidden layers. The original dataset is encoded by the LSTM model to extract factors, and then decoded back to the estimated dataset at the output layer. By continuously optimizing the structure and training of the model to minimize the loss function, an effective LSTM autoencoder can be developed, and the output from the middle layer will be used as the factors.

Let $\boldsymbol{X}$ denote the entire dataset used in this research. Specifically, $\boldsymbol{X}^m$ represents the macroeconomic dataset and $\boldsymbol{X}^f$ represents the financial dataset. We employ the LSTM model to extract a vector of factors, $\boldsymbol{F}_t$, from the information set $\{\boldsymbol{X}^m, \boldsymbol{X}^f\}$. More formally,

$$\boldsymbol{F}_t = f^E\left(\boldsymbol{X}_t^m, \boldsymbol{X}_t^f\right)$$

$$x_{i,t} = \lambda^F \boldsymbol{F}_t + e_{i,t}^x$$

The first formula delineates the encoder component of our LSTM model, which extracts factors from the observables. The subsequent formula describes the decoder component, which employs a linear model to reconstruct the original dataset from these factors. In this context, λ^F represents the factor loadings and $e_{i,t}^x$ denotes the idiosyncratic error terms associated to each variable. An important hyperparameter of the LSTM model is the number of cells in the code layer, that determines the number of factors extracted. Following factor extraction, a factor-augmented forecasting model similar to Jurado et al. (2015) FAVAR model can be devised. Let $x_{j,t+1}^m$ represent one of the macroeconomic series that we aim to forecast. Then,

$$x^m_{j,t+1} = \Phi^x_j(L)x^m_{j,t} + \gamma^F_j(L)\boldsymbol{F}_t + \gamma^W_j(L)\boldsymbol{W}_t + v^m_{j,t+1}$$

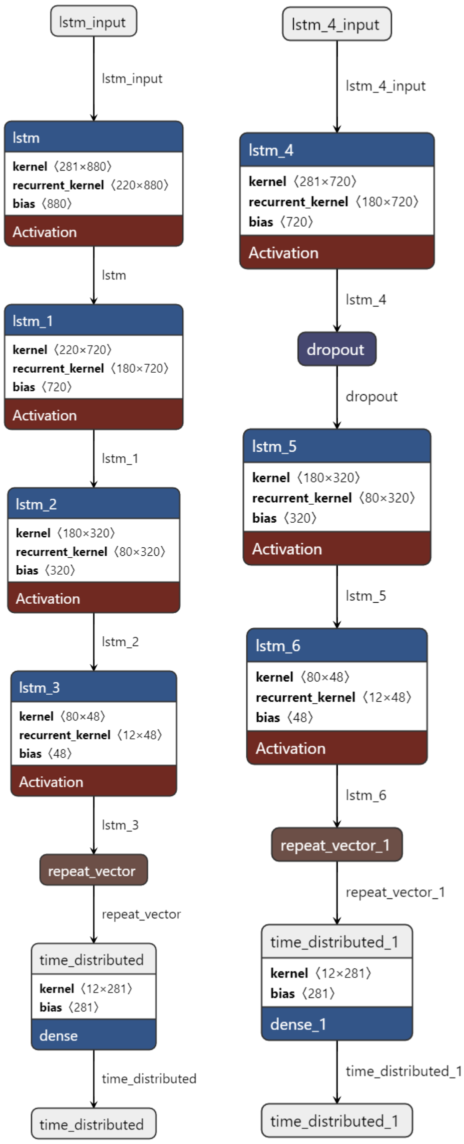
In this formula, L denotes the lag operator; $\boldsymbol{F}_t$ refers to the factors, while $\boldsymbol{W}_t$ represents an additional predictor that includes the square of the first factor in $\boldsymbol{F}_t$, as well as the first factor derived from the squared values of $\boldsymbol{X}_t$. This model mimicks the specification of the FAVAR model proposed in Jurado et al. (2015), but employ the nonlinear factors extracted by the LSTM autoencoder instead of the linear factors obtained from PCA.

There are two LSTM models designated to replace the PCA models in factor extraction. Before detailing the structure of the LSTM models, it is necessary to determine the number of factors. In the research by Jurado et al. (2015), the number of factors was chosen based on the methodology established by Bai and Ng (2002), which set the convergence rate for factor estimates and provided several criteria for selecting the number of factors in datasets with large cross sections and time dimensions. According to information criteria selection rules, Jurado et al. (2015) selected 14 factors. Of these, 12 factors were initially extracted using the PCA model; the 13^{th} factor was the square of the first among these 12 factors, and the 14^{th} factor was the first factor extracted from the squared values of the entire dataset by the PCA model.

For comparison, we decide to extract the same number of factors but employing the LSTM model instead. In a second stage, we expand the number of factors to 18 using the panel criteria in Bai and Ng (2002)[4]. The structure of the LSTM model for extracting 12 factors is depicted in the left part of Figure 2.8 below.

The LSTM model employed comprises seven layers, with all activation functions being tanh, except for those in the last layer. From the second to the fifth layers, the LSTM model performs the task of extracting factors. The input size is $(10, 281)$, which includes all macroeconomic and financial time series. The total time steps are 749, and the data series are separated into small sequences with a time step span of 10 prior to input. In these four layers, the dimensionality of the data series is reduced to a vector of 12, which can be regarded as our 12 factors, thus completing the dimensionality reduction process. The sixth layer involves simply repeating the vector 10 times to form a matrix, which is used to restore the factors. The last layer facilitates the restoration process, wherein the factor matrix is transformed back to the size of $(10, 281)$ through a linear function. The input dataset is the entire data series, and the labels mirror the input dataset. This setup is designed so that in the LSTM model, the dataset is compressed into factors and then restored, with the loss being computed from the variance generated during this process.

[4] As a further robustness check, we also consider a model with 18 or 20 factors, which is the upper bound considered by Jurado et al. (2015). The results from this additional empirical exercise are presented in Appendix **??**.



(A) (B)

FIGURE 2.8: Structure of LSTM autoencoders

The other LSTM model used in the Robustness 1 framework is similar, but processes the dataset differently as factors are extracted from the squared values of the data series. The structure of this LSTM model is delineated in the right part of Figure 2.8. This model also consists of seven layers and employs tanh activation functions, except for in the output layer. From layers one to five, the input data series is compressed into a vector of 12. The second layer includes a dropout feature to aid in preventing overfitting. The succeeding layers of the LSTM then restore the factors back into the data series.

Prior to training, the dataset is standardized using their Z-scores so as to prevent disproportionately large metric values overshadowing other important features. The training dataset comprises 70% of the entire dataset, and the test set comprises 30%.

The training procedure involves 500 epochs, conducting the model optimization algorithm (ADAM optimizer). The loss is calculated via MSE, and dimensional reduction and weight matrix optimization methods (L1 and L2 normalization) are implemented to avoid overfitting. Post-training, we input the entire dataset and extract the vectors from the fourth layer. Upon completion of the training for the LSTM models, the 14 factors required are obtained, which are used to construct the FAVAR model for forecasting macroeconomic series. This setup enables us to model stochastic volatility and estimate macroeconomic uncertainty as described in the Robustness 1 framework. The estimates of macroeconomic uncertainty derived from the Robustness 1 framework are presented in Figures 2.9 below, comparing them with estimates from the full LSTM framework and PCA+FAVAR.

The estimates of macroeconomic uncertainty by the Robustness 1 framework represented in yellow lines are largely similar to those by PCA+FAVAR, yet they display more pronounced nonlinearities and stronger fluctuations, as evidenced by the increased estimates during four crises and reduced estimates during calm periods. It is reasonable to find such similarities between the two sets of estimates since the FAVAR model is retained in both frameworks, exerting a stronger influence on the prediction of macroeconomic sequences than the PCA model. Nonetheless, the factors extracted by the LSTM autoencoder still contribute positively to the measurement of uncertainty. This enhancement observed in the Robustness 1 framework, relative to the PCA+FAVAR estimates, can be attributed solely to the involvement of the LSTM autoencoder. To substantiate our inference, DM tests are conducted on the uncertainty forecasts obtained from both the Robustness 1 and PCA+FAVAR frameworks.

TABLE 2.5: DM test for macroeconomic uncertainty (Robustness 1 vs PCA+FAVAR)

h = 1		h = 3		h = 12	
DM	p-value	DM	p-value	DM	p-value
9.079	0.999	−8.777	0.000	−5.011	0.000

The results presented in Table 2.5 demonstrate that the Robustness 1 framework significantly reduces the estimates of macroeconomic uncertainty for forecast horizons $h = 3$ and 12. The same predictive effectiveness checks discussed in Section 2.4.2 are conducted now for this exercise. Thus, Table 2.6 presents the p-values from the Granger-causality tests for the VAR(11) model augmented with various measures of macroeconomic uncertainty. The first three columns of Table 2.6 display p-values using the uncertainty measures derived by Jurado et al. (2015) obtained under different forecast horizons. The latter three columns consider counterpart models incorporating macroeconomic uncertainty measured by the LSTM autoencoder with 14 factors.

The empirical findings in Table 2.6 demonstrate that replacing the PCA model with the LSTM autoencoder improves the predictive ability of the corresponding

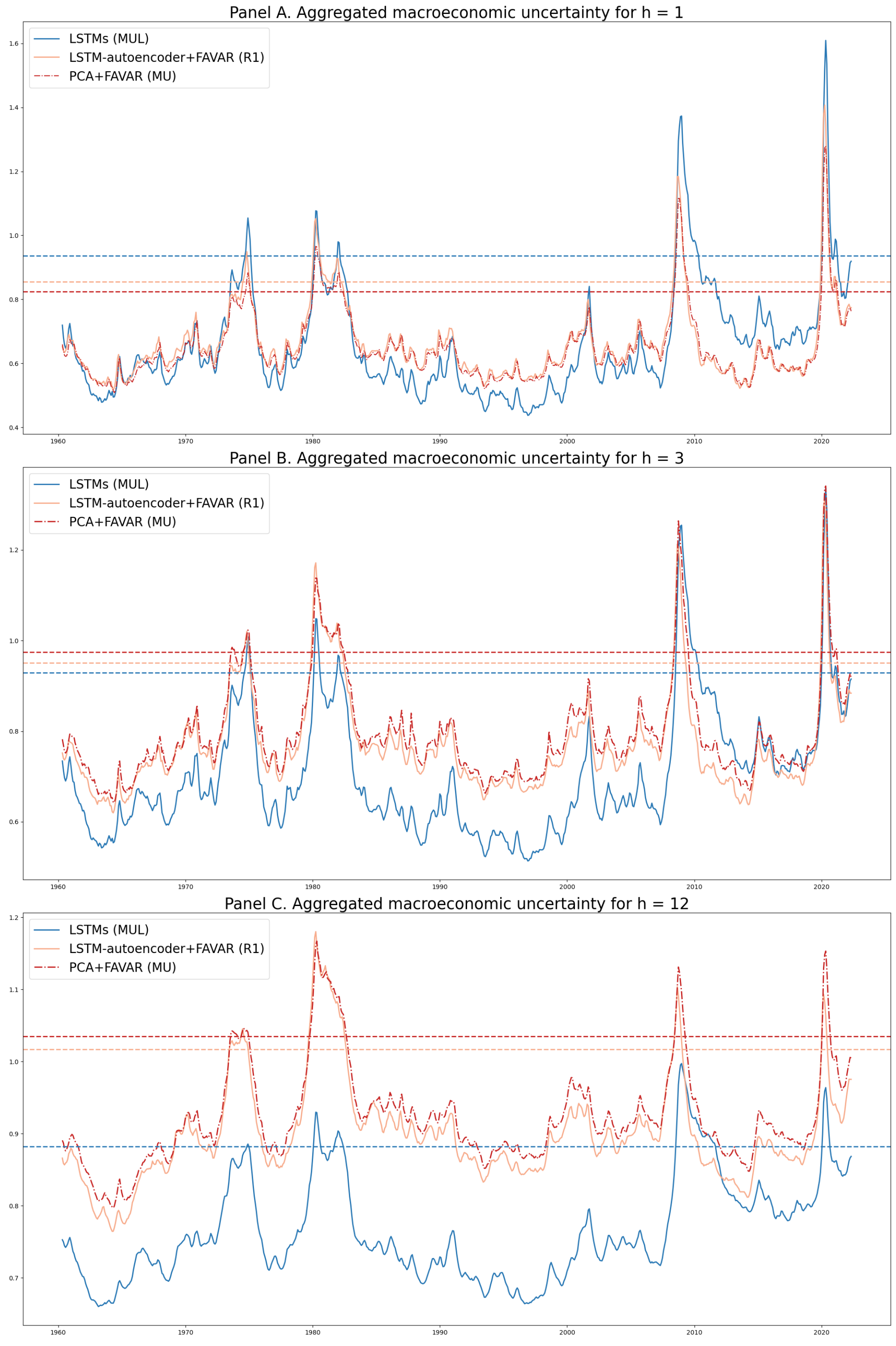


FIGURE 2.9: Aggregated macroeconomic uncertainty by LSTM autoencoder

Figure 2.9 is divided into Panels A, B, and C, each illustrating aggregated macroeconomic uncertainty derived from the Robustness 1 LSTM autoencoder + FAVAR framework (shown by yellow lines) for horizons $h = 1, 3$, and 12. These are compared to macroeconomic uncertainty measured using LSTMs and PCA+FAVAR (depicted with blue straight lines and red dash-dot lines). The horizontal lines on each panel indicate 1.64 standard deviations from the mean of the series, marking periods of relatively high uncertainty.

TABLE 2.6: VAR(11) Granger-causality p-value (Robustness 1)

	$U1J$	$U3J$	$U12J$	$U1R1F14$	$U3R1F14$	$U12R1F14$
log(real IP)	0.000	0.000	0.000	0.000	0.000	0.000
log(employment)	0.000	0.000	0.000	0.000	0.000	0.000
log(real consumption)	0.000	0.000	0.000	0.000	0.000	0.000
log(PCE deflator)	0.000	0.000	0.000	0.000	0.000	0.000
log(real new orders)	0.019	0.000	0.001	0.000	0.000	0.007
log (real wage)	0.081	0.217	0.075	0.020	0.018	0.056
hours	0.000	0.000	0.000	0.000	0.000	0.003
federal funds rate	0.000	0.000	0.001	0.001	0.001	0.000
log(S&P 500 Index)	0.002	0.000	0.001	0.000	0.000	0.000
growth rate of M2	0.000	0.000	0.000	0.000	0.000	0.003

uncertainty measures in several instances. Specifically, for certain variables such as wages, the macroeconomic uncertainty estimates from Robustness 1 for $h = 1$ and $h = 3$ exhibit significant explanatory power, unlike the estimates by PCA+FAVAR.

The next exercise compares the predictive ability of the uncertainty measure by Jurado et al. (2015) against the uncertainty measure from applying the LSTM autoencoder with 14 factors. These results are documented in Tables 2.7.

TABLE 2.7: DM test for VAR(11) model (Robustness 1 vs PCA+FAVAR)

	h = 1		h = 3		h = 12	
	DM	p-value	DM	p-value	DM	p-value
log(real IP)	5.988	0.000	2.079	0.038	−1.759	0.079
log(employment)	5.362	0.000	1.923	0.055	−1.767	0.078
log(real consumption)	−5.191	0.000	−2.647	0.008	−5.168	0.000
log(PCE deflator)	1.703	0.089	0.300	0.765	−3.192	0.002
log(real new orders)	11.633	0.000	5.357	0.000	2.149	0.032
log(real wage)	−11.633	0.000	−5.894	0.000	−5.809	0.000
hours	10.379	0.000	4.477	0.000	1.726	0.085
federal funds rate	9.609	0.000	3.968	0.000	−0.324	0.746
log(S&P 500 Index)	5.352	0.000	2.369	0.018	−0.505	0.614
growth rate of M2	−11.084	0.000	−4.745	0.000	−1.056	0.291

The VAR(11) model that incorporates uncertainty estimates from the Robustness 1 framework exhibit superior predictive power for most variables and forecast horizons, illustrating the superiority of the LSTM technology to extract the factors and, hence, to obtain estimates of macroeconomic uncertainty that exhibit better forecast ability compared to the standard PCA+FAVAR setting. The only exceptions are at $h = 12$ for the federal funds rate, the S&P 500 financial index, and the growth rate of money supply.

To assess the robustness of the number of factors in the Robustness 1 framework on the estimates of macroeconomic uncertainty, we re-estimated the model using the same framework but extracted 18 and 20 factors instead. The 18 factors were selected based on the panel criteria outlined in Bai and Ng (2002), while the 20 factors represent the upper bound considered by Jurado et al. (2015). The structures of the LSTM autoencoder for extracting 18 and 20 factors are similar to that for extracting 14 factors;

the primary distinction lies in the vector size within the LSTM model. Panel A, B and C in Figures 2.10 display the corresponding estimates of macroeconomic uncertainty.

The comparison of the macroeconomic uncertainty estimates from the Robustness 1 framework using 14, 18, and 20 factors reveals very minor differences which suggest that the improvement from increasing the number of factors is negligible. Tables 2.8 details the corresponding p-values from the Granger-causality tests for the VAR(11) model augmented with various measures of macroeconomic uncertainty in Robustness Check 1. The first three columns of Table 2.8 display p-values using the uncertainty measures when considering 18 factors in Robustness 1 framework from different forecast horizons. The latter three columns consider counterpart models incorporating macroeconomic uncertainty measured by LSTM autoencoder setting with 20 factors.

TABLE 2.8: VAR(11) Granger-causality p-value for (Robustness 1 with 18 & 20 factors)

	$U1R1F18$	$U3R1F18$	$U12R1F18$	$U1R1F20$	$U3R1F20$	$U12R1F20$
log(real IP)	0.000	0.000	0.000	0.000	0.000	0.000
log(employment)	0.000	0.000	0.000	0.000	0.000	0.000
log(real consumption)	0.000	0.000	0.000	0.000	0.000	0.000
log(PCE deflator)	0.000	0.000	0.000	0.000	0.000	0.000
log(real new orders)	0.000	0.000	0.016	0.000	0.000	0.005
log (real wage)	0.022	0.027	0.122	0.016	0.015	0.061
hours	0.000	0.000	0.005	0.000	0.000	0.001
federal funds rate	0.002	0.001	0.000	0.000	0.000	0.000
log(S&P 500 Index)	0.000	0.000	0.000	0.000	0.000	0.000
growth rate of M2	0.000	0.000	0.010	0.000	0.000	0.002

Comparing these results with those for the 14 factors case detailed in Table 2.6, it becomes evident that increasing the number of factors in the Robustness 1 framework does not enhance the explanatory power of the uncertainty measures.

The comparative results of the forecasting performance between VAR(11) models augmented with macroeconomic uncertainty measures by Jurado et al. (2015) and those using the LSTM autoencoder with 18 and 20 factors are documented in Tables 2.9, and 2.10, respectively.

TABLE 2.9: DM test for VAR(11) models (Robustness 1 with 18 factors vs PCA+FAVAR)

	h = 1		h = 3		h = 12	
	DM	p-value	DM	p-value	DM	p-value
log (real IP)	5.939	0.000	2.381	0.018	0.166	0.868
log (employment)	5.293	0.000	2.199	0.028	0.122	0.903
log (real consumption)	−4.967	0.000	−2.015	0.044	−2.453	0.014
log (PCE deflator)	1.790	0.074	0.701	0.484	−0.756	0.450
log (real new orders)	11.588	0.000	5.206	0.000	2.401	0.017
log (real wage)	−11.367	0.000	−5.398	0.000	−5.055	0.000
log (hours)	10.178	0.000	4.290	0.000	2.026	0.043
federal funds rate	9.698	0.000	4.328	0.000	1.520	0.129
log(S&P 500 Index)	5.577	0.000	2.620	0.009	0.757	0.449
growth rate of M2	−11.105	0.000	−4.865	0.000	−1.879	0.061

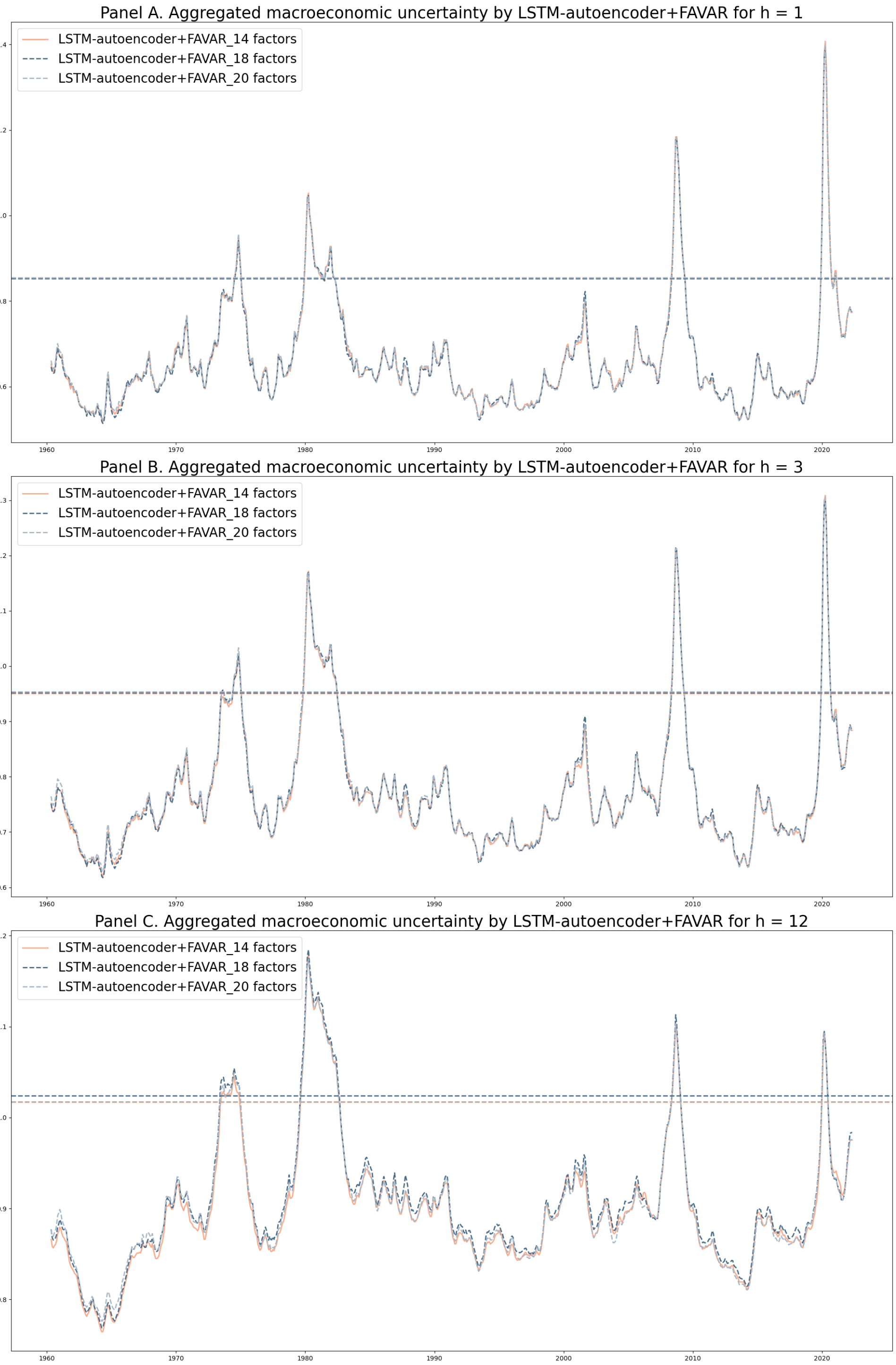


FIGURE 2.10: Aggregated macroeconomic uncertainty by LSTM autoencoder with 14, 18 and 20 Factors

Figure 2.10 is divided into Panels A, B, and C, each illustrating aggregated macroeconomic uncertainty derived from the Robustness 1 LSTM autoencoder + FAVAR framework with 14, 18 or 20 factors for horizons $h = 1, 3$, and 12. The horizontal lines on each panel indicate 1.64 standard deviations from the mean of the series, marking periods of relatively high uncertainty.

TABLE 2.10: DM test for VAR(11) models (Robustness 1 with 20 factors vs PCA+FAVAR)

	h = 1		h = 3		h = 12	
	DM	p-value	DM	p-value	DM	p-value
log (real IP)	5.267	0.000	1.722	0.085	−0.735	0.463
log (employment)	4.844	0.000	1.615	0.107	−0.750	0.454
log (real consumption)	−5.793	0.000	−3.193	0.002	−3.944	0.000
log (PCE deflator)	1.166	0.244	−0.083	0.934	−1.915	0.056
log (real new orders)	11.861	0.000	5.319	0.000	2.243	0.025
log (real wage)	−12.717	0.000	−6.897	0.000	−5.598	0.000
log (hours)	10.130	0.000	4.293	0.000	1.868	0.062
federal funds rate	9.186	0.000	3.745	0.000	0.616	0.538
log(S&P 500 Index)	5.335	0.000	2.207	0.028	0.157	0.876
growth rate of M2	−11.036	0.000	−4.621	0.000	−1.472	0.141

The predictive capability of the uncertainty measures remains equivalent to the case with 14 factors, showing no significant improvement. Therefore, we suggest that the 14 factors selected based on the information criteria selection rule exhibit robustness.

The following empirical exercise aims to compare the forecast ability of our proposed model given by the full LSTM framework against the model discussed in this section that mimics Jurado et al. (2015) approach but uses an LSTM model to extract the common unobserved factors. The results of the comparison using the DM test are presented in Table 2.11.

TABLE 2.11: DM test for macroeconomic uncertainty (LSTMs vs Robustness 1)

h = 1		h = 3		h = 12	
DM	p-value	DM	p-value	DM	p-value
1.779	0.962	−7.557	0.000	−5.3188	0.000

The results in Table 2.11 support the hypothesis that the macroeconomic uncertainty obtained from the LSTMs framework is significantly lower than under the mixed framework that comprises the LSTM autoencoder and the FAVAR model, particularly for $h = 3$ and 12. This observation aligns with Panel B and C in Figures 2.9. For $h = 1$, although the difference is not statistically significant, the LSTM-based measure presents more extreme nonlinear changes than the measure by Robustness 1 framework.

Table 2.12 presents the results of the Granger-casuality tests, in which the first three columns report the p-values using macroeconomic uncertainty obtained from the LSTMs framework and the latter three columns consider models incorporating macroeconomic uncertainty constructed using the mixed Robustness 1 framework.

The predictive power of both macroeconomic uncertainty measures is strong and consistent across most scenarios, except for the variable wages, as previously highlighted. The macroeconomic uncertainty estimated by Robustness 1 displays statistically significant predictive power for $h = 1$ and $h = 3$, which is further

TABLE 2.12: VAR(11) Granger-causality p−values (LSTMs & Robustness 1)

	$U1L$	$U3L$	$U12L$	$U1R1F14$	$U3R1F14$	$U12R1F14$
log(real IP)	0.000	0.000	0.000	0.000	0.000	0.000
log(employment)	0.000	0.000	0.000	0.000	0.000	0.000
log(real consumption)	0.000	0.000	0.000	0.000	0.000	0.000
log(PCE deflator)	0.000	0.000	0.000	0.000	0.000	0.000
log(real new orders)	0.000	0.000	0.000	0.000	0.000	0.007
log(real wage)	0.147	0.206	0.260	0.020	0.018	0.056
hours	0.000	0.000	0.001	0.000	0.000	0.003
federal funds rate	0.000	0.000	0.002	0.001	0.001	0.000
log(S&P 500 Index)	0.002	0.002	0.002	0.000	0.000	0.000
growth rate of M2	0.000	0.000	0.008	0.000	0.000	0.003

confirmed when increasing the number of factors within Robustness 1 framework, as detailed in Appendix **??**.

TABLE 2.13: DM test for VAR(11) models (LSTMs vs Robustness 1)

	h = 1		h = 3		h = 12	
	DM	p-value	DM	p-value	DM	p-value
log (real IP)	-5.871	0.000	-2.064	0.039	1.714	0.087
log (employment)	-5.526	0.000	-1.966	0.050	1.702	0.089
log (real consumption)	5.000	0.000	2.704	0.007	5.144	0.000
log (PCE deflator)	-1.786	0.074	-0.355	0.723	3.127	0.002
log (real new orders)	-10.131	0.000	-4.422	0.000	-1.714	0.087
log (real wage)	-11.226	0.000	-5.290	0.000	-2.177	0.030
hours	11.565	0.000	6.166	0.000	5.755	0.000
federal funds rate	-10.131	0.000	-4.422	0.000	-1.714	0.087
log(S&P 500 Index)	-5.085	0.000	-2.293	0.022	0.537	0.591
growth rate of M2	9.834	0.000	4.449	0.000	1.061	0.289

The last exercise in this section, reported in Table 2.13, shows the results of the DM tests for the comparison of the predictive ability of the relevant uncertainty measures for forecasting the macro variables in a VAR(11) model. The results provide strong support to the view that the macroeconomic uncertainty obtained from the LSTMs framework is superior or comparable to the mixed approach in 22 out of 30 instances. Hence, the results of the Robustness 1 framework can be considered to be half way between the full LSTM framework and the PCA+FAVAR approach.

2.5.2 LSTM vs FAVAR: Robustness Check 2

To further investigate the superiority of non-parametric LSTM model comparing with FAVAR model, we propose another robustness check based on both the LSTMs framework and Robustness 1 framework, which aims to facilitate a deeper comparison. In this robustness check, the LSTM model $f^m(\cdot,\cdot)$, used for forecasting macroeconomic variables at the next time step in the recursive process within the LSTM framework, is replaced by an LSTM autoencoder and a factor-augmented forecasting model, as specified in the Robustness 1 framework. Hence, the robustness check 2 employs a parametric forecasting model enhanced by nonlinear factors, so as to prove the superiority of non-parametric LSTM model when comparing the uncertainties obtained from LSTMs framework and Robustness 2 approach. Figure 2.11 below illustrates the process in the second robustness exercise.

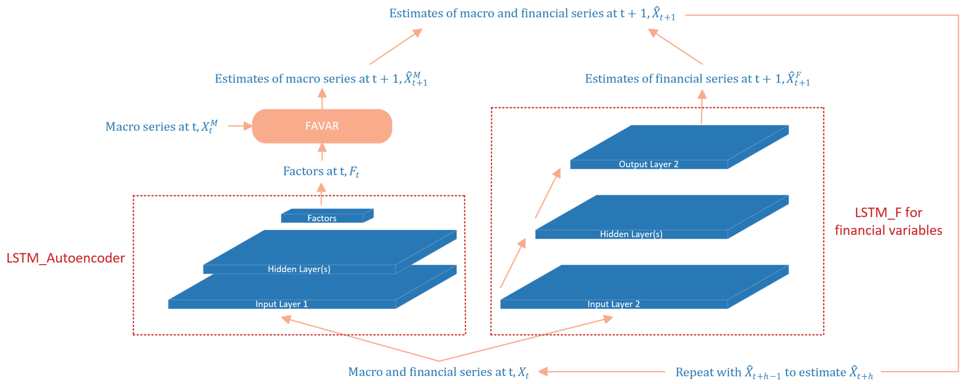


FIGURE 2.11: Robustness 2 framework

In the second robustness exercise, the one-step-ahead forecasts of macroeconomic variables leverage the LSTM autoencoder and the FAVAR system from the Robustness 1 approach, while the one-step-ahead forecasts for financial variables use the LSTM model from our main framework. For multi-step forecasts, an iterative process is employed: factors are extracted up to the (h-1)th step, then used to forecast both macroeconomic and financial variables at step h, which then facilitate factor extraction for the (h+1)th step predictions.

The procedure is as follows. Let $\boldsymbol{X} = (\boldsymbol{X}^m, \boldsymbol{X}^f)$ represent the complete dataset, processed via the LSTM autoencoder:

$$\boldsymbol{F}_t^* = f^E\left(\boldsymbol{X}_t^m, \boldsymbol{X}_t^{f^*}\right)$$

$$x_{i,t} = \lambda^F \boldsymbol{F}_t^* + e_{i,t}^x.$$

In this context, the term $\boldsymbol{X}_t^{f^*}$ refers to the various financial series from Robustness Check 1 used in factor extraction and $\boldsymbol{F}_t^*$ denotes the corresponding factors extracted. For one-step ahead forecasts, we employ the financial dataset at time t as input to an LSTM autoencoder. For $h > 1$, however, the future values $\boldsymbol{X}_{t+h}^{f*}$ are estimated using the LSTM model outlined in Section 2.3.3, which will be further discussed in subsequent sections.

Following factor extraction, the factor-augmented forecasting model can be devised with $\boldsymbol{F}_t^*$ and $\boldsymbol{W}_t^*$, which are the additional predictors constructed using the X_t^{f*} different from Robustness Check 1.

Let $x^m_{j,t+1}$ represent one of the macroeconomic series that we aim to forecast. Then,

$$x^m_{j,t+1} = \Phi^x_j(L)x^m_{j,t} + \gamma^F_j(L)\boldsymbol{F}^*_t + \gamma^W_j(L)\boldsymbol{W}^*_t + v^m_{j,t+1}$$

where L denotes the lag operator. For conveniently describing the recursive process within the Robustness 2 framework, the next-step forecast of macroeconomic variables is succinctly represented as:

$$\hat{\boldsymbol{X}}^m_{t+h} = f^{FA}\left(\hat{\boldsymbol{X}}^m_{t+h-1}, \boldsymbol{F}^*_{t+h-1}, \boldsymbol{W}^*_{t+h-1}\right) + \boldsymbol{v}^m_{t+h}$$

Here, $f^{FA}(\cdot,\cdot)$ denotes the factor-augmented forecasting model mimicking the specification of the FAVAR model for predicting macroeconomic variables. The term $\boldsymbol{v}^m_{t+h}$ denotes the forecast error, which is allowed to exhibit time-varying volatility.

Regarding the future values $\boldsymbol{X}^{f*}_{t+h}$ for $h > 1$, the LSTM model, $f^F(\cdot,\cdot)$, outlined in Section 2.3.3 is employed:

$$\hat{\boldsymbol{X}}^{f*}_{t+h} = f^F(\hat{\boldsymbol{X}}^m_{t+h-1}, \hat{\boldsymbol{X}}^{f*}_{t+h-1}) + \boldsymbol{v}^f_{t+h}$$

In which the $\hat{\boldsymbol{X}}^m_{t+h-1}$ is the estimated macroeconomic variables from $f^{FA}(\cdot,\cdot)$, hence the estimates of financial variables is different from our main framework using LSTM model to forecast macroeconomic variables and denoted by $\hat{\boldsymbol{X}}^{f*}_{t+h}$.

In this exercise, the estimates of macroeconomic uncertainty obtained under the latter approach are compared against the original estimates from our proposed framework employing the DM test. Panel A, B and C in Figure 2.12 illustrate the macroeconomic uncertainty as estimated by Robustness 2 for $h = 1, 3$, and 12, in comparison to macroeconomic uncertainty estimated by our main framework consisting of two LSTM models.

For $h = 3$ and 12, the results of the DM test in Table 2.14 provide overwhelming evidence of the superiority of the LSTM macroeconomic uncertainty measure compared to the uncertainty measure obtained under the approach developed in Robustness check 2.

TABLE 2.14: DM test for macroeconomic uncertainty by LSTMs against LSTM&FAVAR

h = 1		h = 3		h = 12	
DM	p-value	DM	p-value	DM	p-value
2.133	0.984	−11.022	0.000	−5.651	0.000

To evaluate the difference in predictive power between the macroeconomic uncertainty obtained by the LSTMs framework and Robustness 2 when forecasting ten important macroeconomic variables, we constructed identical VAR(11) and LSTM models as previously mentioned, containing macroeconomic uncertainty by

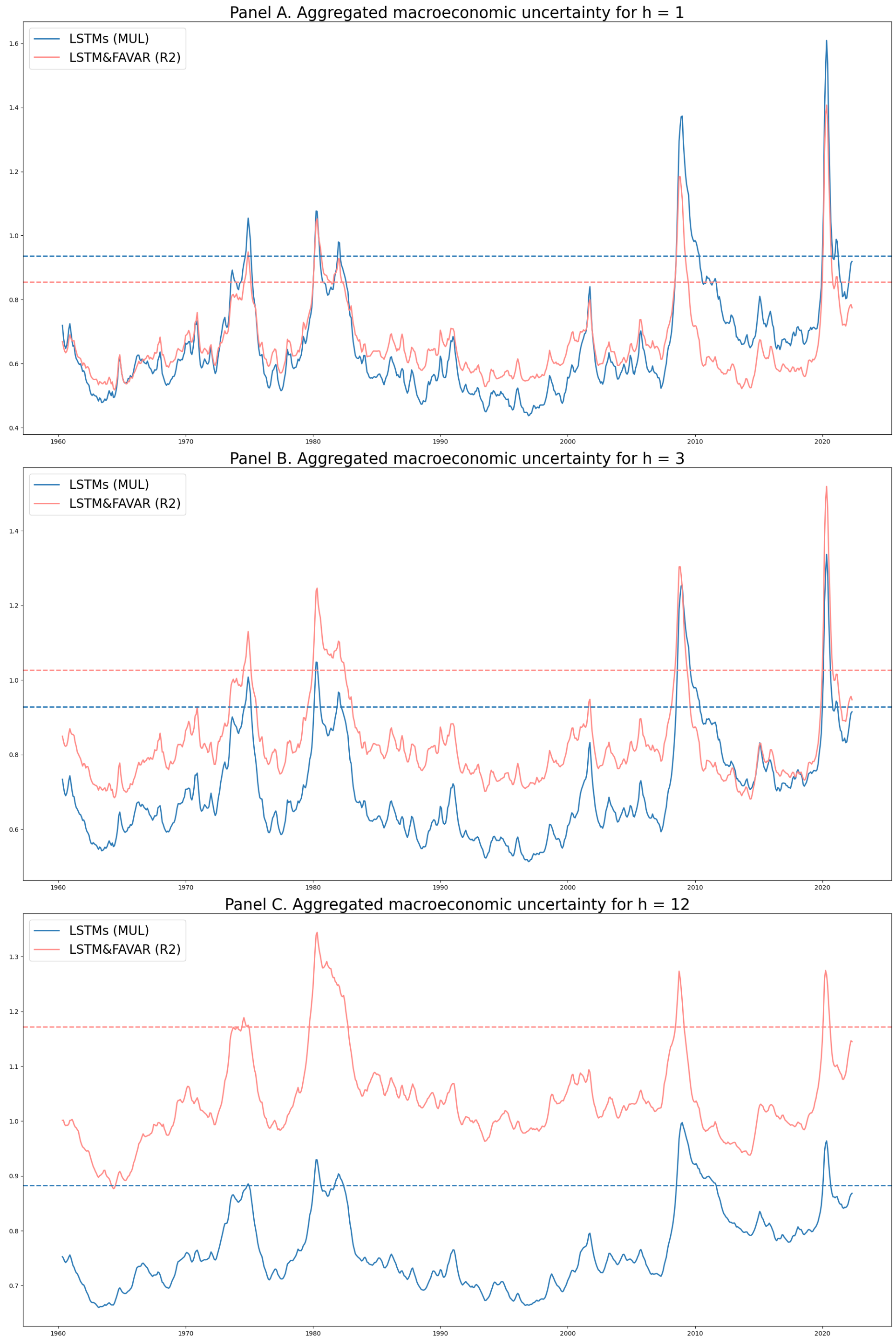


FIGURE 2.12: Aggregated macroeconomic uncertainty by LSTM&FAVAR

Figure 2.12 is divided into Panels A, B, and C, each illustrating aggregated macroeconomic uncertainty derived from the Robustness 2 LSTM&FAVAR framework (shown by yellow lines) for horizons $h = 1, 3,$ and 12. These are compared to macroeconomic uncertainty measured using LSTMs (depicted with blue straight lines). The horizontal lines on each panel indicate 1.64 standard deviations from the mean of the series, marking periods of relatively high uncertainty.

Robustness 2. DM tests were conducted to assess the differences, with the results displayed in the tables below.

TABLE 2.15: DM test for VAR(11) models (LSTMs vs Robustness 2)

	h = 1		h = 3		h = 12	
	DM	p-value	DM	p-value	DM	p-value
log (real IP)	-12.427	0.000	-5.264	0.000	-0.678	0.498
log (employment)	-0.814	0.416	-0.305	0.760	-1.525	0.128
log (real consumption)	-4.615	0.000	-6.599	0.000	-0.839	0.402
log (PCE deflator)	-6.303	0.000	-3.272	0.001	-1.284	0.200
log (real new orders)	-7.799	0.000	-3.872	0.000	-1.322	0.186
log (real wage)	5.630	0.000	5.400	0.000	2.013	0.045
hours	-0.948	0.344	-0.473	0.636	-0.186	0.852
federal funds rate	0.598	0.550	1.851	0.065	-1.419	0.156
log(S&P 500 Index)	6.818	0.000	2.533	0.012	-0.937	0.349
growth rate of M2	-2.756	0.006	-1.542	0.124	-1.391	0.165

TABLE 2.16: DM test for LSTM-VAR(11) models (LSTMs vs Robustness 2)

	h = 1		h = 3		h = 12	
	DM	p-value	DM	p-value	DM	p-value
log (real IP)	-18.664	0.000	-6.849	0.000	-6.202	0.000
log (employment)	-17.821	0.000	-6.931	0.000	-5.618	0.000
log (real consumption)	-18.833	0.000	-6.526	0.000	-4.860	0.000
log (PCE deflator)	-19.608	0.000	-6.571	0.000	-5.550	0.000
log (real new orders)	-12.769	0.000	-0.017	0.986	-3.596	0.000
log (real wage)	-14.793	0.000	-6.828	0.000	-4.429	0.000
hours	6.236	0.000	4.736	0.000	0.271	0.787
federal funds rate	8.168	0.000	5.637	0.000	-3.557	0.000
log(S&P 500 Index)	-13.924	0.000	-7.955	0.000	-6.102	0.000
growth rate of M2	1.807	0.071	6.728	0.000	4.218	0.000

From Table 2.15, it is evident that the macroeconomic uncertainty obtained from the LSTMs framework outperforms the procedure introduced under the Robustness check 2. In 25 instances, the VAR(11) model incorporating macroeconomic uncertainty from the LSTMs framework demonstrates the same or better predictive capability than the competitor. The results in Table 2.16 further validate that macroeconomic uncertainty by the LSTMs framework is superior, noting 27 situations where the LSTM model incorporating macroeconomic uncertainty from the LSTMs framework shows the same or enhanced predictive ability.

Utilizing the FAVAR model to forecast the macroeconomic dataset, we observe a significant increase in the estimates of macroeconomic uncertainty at $h = 3$ and 12, while the predictive power for forecasting ten macroeconomic variables is diminished. As a linear model, the FAVAR model may not capture the non-linear relationships within the macroeconomic dataset as effectively as the LSTM model.

2.6 Conclusion

In Chapter 2, we propose a nonlinear framework for measuring macroeconomic uncertainty using neural network models. Specifically, two LSTM models and a recursive procedure are employed to forecast macroeconomic and financial time series, respectively. The time-varying volatility of the forecast error is modeled to estimate the uncertainty underlying each series and is aggregated to construct an econometric measure of macroeconomic uncertainty. Moreover, to further study and illustrate the advantages of LSTM models over linear time series models and ensure robustness, two additional nonlinear frameworks are proposed.

The first robustness check involves comparing the LSTM and PCA models, using the LSTM autoencoder to replace the PCA in the linear measurement of macroeconomic uncertainty. Factors extracted by the LSTM autoencoder are used to conduct the same FAVAR model as the linear measurement to estimate macroeconomic uncertainty and facilitate a horizontal comparison.

The second robustness check involves comparing the LSTM and FAVAR models. In this setup, the FAVAR model, configured with factors from the LSTM autoencoder in the first robustness check, is used to forecast the macroeconomic series, and the LSTM model from our main framework is used to forecast the financial series. The estimates of macroeconomic uncertainty from our main framework, the two robustness checks, and the econometric-based linear framework are compared and analyzed horizontally.

Based on the results of the estimates of macroeconomic uncertainty and the applications of Granger-causality and Diebold-Mariano tests, the estimates of macroeconomic uncertainty obtained under our framework are significantly lower overall, highlighting the presence of nonlinear dynamics and interactions between the variables. The proposed models are more capable of efficiently identifying economic crises and calm periods, while maintaining or even enhancing the explanatory and predictive abilities of uncertainty measurement compared to those measured by linear models for macroeconomic variables.

The core contributions to the existing body of knowledge from this study are threefold: First, considering the estimates of macroeconomic uncertainty, the LSTM model is proposed as a forecasting model for large-dimensional datasets. As macroeconomic uncertainty is primarily measured by forecast error variance, our LSTM models fit the macroeconomic and financial datasets better than the FAVAR model, as indicated by the smaller forecast errors and our estimates of macroeconomic uncertainty. Second, our research applies the LSTM model in the measurement of macroeconomic uncertainty and demonstrates that using LSTM models can reduce the estimated value of macroeconomic uncertainty, and their explanatory and predictive power are on par with those measured by linear frameworks. Third, compared to linear models, the LSTM model proves to be more effective in extracting factors and forecasting time series.

Chapter 3

Hedging Economic Uncertainty from Cross-section of Stock Returns

We examine the pricing of economic uncertainty in the cross-section of stock returns. Uncertainty is proxied by innovations to the macroeconomic and financial measures in Jurado et al. (2015) and Ludvigson et al. (2021). In contrast to existing literature, a negative uncertainty risk premium is found in calm periods, turning positive in turbulent ones. These findings are rationalized by means of a hedging portfolio that delivers positive returns in turbulent periods to compensate for the cost of insuring the portfolio in calm periods. We also provide statistical evidence of the uncertainty factors' added value and their relative predictive power across uncertainty regimes.

3.1 Introduction

In Chapter 3, we focus on examining and pricing the econometric-based uncertainty by Jurado et al. (2015) with cross sections of stock returns. Currently, there are several representative studies about pricing uncertainty. Ang et al. (2006) use the innovations in VIX to proxy for uncertainty (Classified as market-based measure of uncertainty by Cascaldi-Garcia et al. (2023)) and price it with daily frequency data, while the Bali et al. (2017) examine the macroeconomic uncertainty by Jurado et al. (2015) and price it on a monthly basis. Both of these research find a negative cross-sectional relationship that stocks with high sensitivities to uncertainty have low average returns (compared to stocks with low exposure), hence the uncertainty has a negative risk premium. The key mechanism behind such negative risk premiums of uncertainty is that an increase in economic uncertainty reduces future investment and consumption opportunities. To hedge against such an unfavorable shift, investors prefer to hold stocks with returns that increase in times of economic uncertainty. Such intertemporal hedging demand implies that investors are willing to hold stocks with higher covariance with economic uncertainty, paying higher prices and accepting lower returns for stocks with higher 'uncertainty beta'. Ang et al. (2006) and Bali et al. (2017) also provide evidence of time variation in the (negative) uncertainty premium, finding it to be larger in magnitude during recessions and periods of high aggregate uncertainty than in expansionary and relatively calm times.

We partially agree with the perspective that the risk premium associated with uncertainty is both time-varying and tends to be negative during periods of calm due to the demand for hedging. However, during turbulent times, we believe investors should receive compensation for holding stocks that have higher exposure to uncertainty rather than incurring higher costs compared to calmer periods. Since people have already paid for insurance to hedge against potential increases in uncertainty during calm times, this insurance should provide payouts when uncertainty rises, rather than requiring additional payments. Consequently, the risk premium associated with uncertainty transitions from negative to positive as conditions shift from calm to turbulent. Our hypothesis contrasts with the findings of Ang et al. (2006) and Bali et al. (2017), and several potential explanations exist for this discrepancy. Firstly, the VIX index is often viewed more as a fear index than a pure uncertainty index and may inherently include components of jump risk. This structural bias could lead to skewed results when analyzing the risk premiums related to uncertainty. Secondly, our measurement research in Chapter 2 identifies an inherent time dependence within the uncertainty index, which could introduce bias in estimations. These limitations in current research have driven our investigation into the risk premium associated with uncertainty.

Our key point is supported by the similar research of Engle et al. (2020), hedging against climate changes. These authors explore the dynamic characteristics of climate to construct a mimicking portfolio that hedges against the realizations of climate risk using publicly traded assets. The proposed dynamic hedging approach is similar to Black and Scholes (1973) and Merton (1973), but instead of buying a security that directly pays off in the event of a future climate disaster, the hedging strategy builds portfolios whose short-term returns hedge news about climate change over the holding period. By hedging period by period the innovations in news about long-run climate change, an investor can ultimately cover her long-run exposure to climate risk. The mimicking portfolio proposed by these authors is based on exploring which stocks rise in value and which stocks fall in value when negative news about climate change materializes. Then, investors aim to construct investment portfolios that overweight stocks that perform well on the arrival of such negative news to profit in climate risk episodes. The results in Engle et al. (2020) suggest that the risk premium on innovations to climate risk is positive in the presence of negative shocks to climate risk and negative in calm periods characterized by the absence of climate events.

Following a similar rationale, we expect that the risk premium on macroeconomic uncertainty - interpreted as innovations to the conditional volatility of macroeconomic and financial shocks - not only is time varying but also fluctuates around zero to meet investors' dynamic hedging demands in periods of high uncertainty. To support our hypothesis on the predictive role and dynamics of the macroeconomic uncertainty risk premium, we follow three different empirical strategies. We first compute long-minus-short investment strategies formed on rankings of the cross-section of stock returns monthly between 1998 and 2022. The rankings are based on firms' exposures to macroeconomic uncertainty, assessing the profitability of these strategies across uncertainty regimes. These portfolios are long on the fifth quintile of stocks ranked on the uncertainty beta exposure and short on stocks in the first quintile. Second, we estimate the dynamics of the monthly uncertainty premium estimated from Fama-MacBeth (Fama and MacBeth, 1973) cross-sectional regressions. Third, we construct a mimicking (hedging) portfolio following a similar procedure to Engle et al. (2020) and estimate, on a monthly basis, the presence of dynamics in the uncertainty premium.

Our main hypothesis is tested with data on the cross-section of stock prices obtained from CRSP and including all available stocks from NYSE, NASDAQ, and AMEX. We find that the beta exposure of stock returns to macroeconomic uncertainty, after controlling for the FF5 factors by Fama and French (2015), is monotonically increasing across uncertainty regimes. In line with existing literature, these beta loadings are negative for stocks in the lower quintiles and positive for stocks in the higher quintiles of the distribution of stocks sorted on their uncertainty exposure. However, in contrast to this literature, we find that the corresponding risk premium on macroeconomic uncertainty is negative in periods of decreasing conditional volatility (calm periods) and positive in periods of increasing conditional volatility (turbulent periods). This result is mainly due to our choice of uncertainty proxy that is given by the innovations to the conditional volatility measure developed in Jurado et al. (2015). Intuitively, the expected excess return on risky assets is obtained as the product of the uncertainty beta loading and the macroeconomic uncertainty measure. Because our proxy of macroeconomic uncertainty is a sequence of innovations, it takes positive and negative values over time, by construction. Therefore, the excess return for stocks in the lower quintiles is expected to be positive when the sequence of innovations takes negative values (calm episodes) and negative when the sequence of innovations proxying macroeconomic uncertainty takes positive values (turbulent episodes). For stocks in the higher quintiles, the rationale is the opposite: the excess return is expected to be positive when the innovations are positive and negative when the innovations are negative. Hence, 5-1 investment strategies yield negative average returns in calm periods and positive average returns in turbulent periods.

These results are confirmed from estimates of the uncertainty risk premium obtained from Fama-MacBeth cross-sectional regressions applied to the FF5 model that is augmented with the macroeconomic uncertainty proxy. We find a negative risk premium on the latter variable during calm periods (2003-2006 and 2012-2016) and a positive risk premium during turbulent periods (1998-2002, 2007-2011 and 2017-2022). A further confirmation of these empirical findings is obtained from applying a mimicking portfolio approach similar to Engle et al. (2020). The returns on our mimicking portfolio are obtained from projecting the macroeconomic uncertainty state variable on a set of base assets ranked on exposure to uncertainty, controlling for the FF5 factors. The dynamics of the returns on this hedging portfolio confirm the findings discussed above and show a cyclical behavior around zero with negative values in calm periods and positive values during turbulent periods.

As additional robustness checks, we apply the novel asset pricing model developed by Giglio and Xiu (2021) to show the insensitivity of the uncertainty premium estimates to potential omitted variable bias. We also analyze our choice of calm and turbulent episodes and discuss alternative methods to split the sample period with corresponding results, such as the CFNAI index. The robustness section also assesses the effect of microcaps on the role of economic uncertainty for pricing the cross-section of stock returns. As in Hou et al. (2015), we trim the bottom 20% of NYSE stocks sorted on market value and find that removing microcaps from the cross-section of returns quantitatively but not qualitatively alters our findings on the fluctuations of the uncertainty premium estimates around zero.

The importance of macroeconomic uncertainty for cross-sectional asset pricing is confirmed by recent statistical tests developed in Barillas and Shanken (2017, 2018) and Barillas et al. (2020), comparing nested and non-nested asset pricing models respectively. We find (overwhelmingly) robust statistical evidence in support of augmenting the FF5 model with econometric-based measures of macroeconomic and financial uncertainty, obtained from differences of squared Sharpe ratios and Bayesian procedures. The model comparison exercise also provides evidence from statistical tests for non-nested models, suggesting that the suitability of each uncertainty measure for explaining the cross-section of stock returns depends on the uncertainty regime. For example, during the 2007-2011 crisis, we find that the measure of uncertainty built from innovations to financial uncertainty is superior to the macroeconomic uncertainty measure. In contrast, during the 2017-2022 period, characterized by the COVID-19 pandemic, the opposite is true.

The rest of this Chapter is organized as follows. Section 3.2 examines how macroeconomic uncertainty is priced in the cross-section of stock returns monthly spanning the period 1998 to 2022 and characterized by the occurrence of calm and turbulent episodes. Section 3.3 presents several robustness checks to assess the sensitivity of the results to an alternative definition of economic uncertainty based on financial innovations, a different methodology to estimate the risk premium, a reduced cross-section of returns without microcaps, and discusses the choice of the evaluation periods. Section 3.4 introduces a model comparison exercise to statistically assess the role of the macroeconomic and financial uncertainty mimicking portfolios as additional risk factors in Fama-French empirical asset pricing models. The exercise enables comparison in terms of the predictive ability of macroeconomic and financial uncertainty measures in nested and non-nested settings. Finally, Section 3.5 concludes.

3.2 Pricing Economic Uncertainty in the Cross-section

3.2.1 Theoretical Background

This section discusses the methodology to price aggregate uncertainty in the cross-section of risky assets along with data and variable definitions. The main approaches found in the literature to price the cross-section of stock returns are Fama-Macbeth (Fama and MacBeth, 1973) cross-sectional regressions and the construction of a mimicking portfolio that maximizes the correlation between the pricing anomaly (*e.g.* climate risk, macro uncertainty) and a set of base asset returns. More recently, Giglio and Xiu (2021) have proposed a three-stage procedure to price the cross-section of stock returns. This method extends the Fama-Macbeth approach by considering a preliminary step to estimate a set of unobservable pricing factors using principal components. This methodology combines the principal component analysis with two-pass cross-sectional regressions to provide consistent estimates of the risk premium for any observed factor. In this section, we focus on the first two methods to compute the risk premium on economic uncertainty and implement Giglio and Xiu (2021)'s approach as a robustness check in Section 3.3.

We denote by r_t^i the excess returns over the risk-free rate on stock i at time t. These returns are assumed to follow a linear multifactor model, in which asset returns are driven by innovations in the pricing factors $\{f_{kt}, u_{kt}\}$ and an idiosyncratic term ε_t^i:

$$r_t^i = a^i + \sum_{k=1}^{K} \beta_k^i \left(f_{kt} - E[f_{kt}]\right) + \beta_u^i \left(u_t - E[u_t]\right) + \varepsilon_t^i, \tag{3.1}$$

where a^i denotes the risk premium on risky asset i and β_k^i are asset i's risk exposures to $k = 1, \ldots, K$ risk factors. Similarly, β_u^i denotes the risk exposure to the uncertainty factor, or asset i's sensitivity to uncertainty risk; $f_{kt} - E[f_{kt}]$ denote the factor innovations and $u_t - E[u_t]$ represents the innovations in the factor capturing economic uncertainty risk. In this basic setup, the risk exposures are constant over time. This is relaxed in the empirical application in which the pricing methods are updated on a monthly basis using rolling regressions. The risk premium is given by

$$a^i = \sum_{k=1}^{K} \beta_k^i \lambda_k + \beta_u^i \lambda_u, \tag{3.2}$$

where λ_k is the price of risk of each of the k factors, λ_u is the price of macro uncertainty risk, and β_k^i and β_u^i are now interpreted as the quantity of risk associated to the pricing factors.

3.2.2 Data and Variable Definitions

To investigate how economic uncertainty is priced in the cross-section of stock returns we consider a sample that includes all common stocks traded on the NYSE, AMEX, and NASDAQ exchanges from 1993 through 2022. We eliminate stocks with a price per share less than \$5 or more than \$1,000. The monthly return and volume data are from the CRSP. We adjust stock returns for delisting to avoid survivorship bias. In contrast to Ang et al. (2006), we only use monthly returns to match the frequency of the uncertainty measures reported in Jurado et al. (2015).

The sample data is divided into five evaluation periods of similar length with the aim of covering different uncertainty regimes. The first period covers the dotcom stock market bubble that peaked on Friday, March 10, 2000. This period of sustained increase in stock prices coincided with the widespread adoption of the World Wide Web and the Internet, resulting in the rapid growth of valuations in new dot-com startups. The second interval spans the years 2003 to 2006 and covers a period of low and relatively stable inflation and low volatility of financial markets. This period contained the longest economic expansion since World War II. Unfortunately, the financial crisis of 2007-08 broke the calm of the Great Moderation and came to be known as the Great Recession. The latter is the third episode considered in our analysis that spans from 2007 through December 2011. This period also captures short episodes of financial distress such as the European sovereign debt crisis that took place in the European Union from 2009 until the mid to late 2010s. This crisis was characterized by several Eurozone member states (Greece, Portugal, Ireland and Cyprus) that were unable to repay or refinance their government debt or to bail out over-indebted banks. The fourth period is defined by the interval 2012-2016 and covers a period of booming stock markets and relative calm in financial markets. The last episode that spans from 2017 to 2022 includes a global economic recession given by the outbreak of the COVID-19 pandemic from February 2020. This was followed by a sustained period of economic slowdown that saw stagnation of economic growth and consumer activity. The robustness of the results to this specific choice of evaluation periods is discussed below.

Following Bali et al. (2017), for each stock and month in the different evaluation periods, we estimate the uncertainty beta from monthly rolling regressions of excess stock returns on the different measures of economic uncertainty over a 60−month fixed window while controlling for the FF5 factors. Thus, the first regression to obtain the pre-formation beta loadings runs from January 1993 to December 1997. Monthly data on the risk-free rate, proxied by the one-month US Treasury bill rate, and the FF5 pricing factors are obtained from Kenneth French's data library. These factors are given by the excess market return (MKT), the size factor (SMB) that is given by a portfolio return constructed from a small-minus-large investment strategy, the value factor (HML) that is given by a long-minus-short portfolio that exploits differences in profitability based on stocks' book-to-market ratio, the profitability factor (RMW) given by a robust-minus-weak portfolio return, and the investment factor (CMA) that is constructed as a conservative-minus-aggressive portfolio return.

Economic uncertainty is proxied by the innovations to the measures developed in Jurado et al. (2015) and Ludvigson et al. (2021). These authors differentiate between macroeconomic and financial uncertainty. To estimate macroeconomic uncertainty Jurado et al. (2015) develop a factor-augmented predictive regression model from a rich set of macroeconomic and financial time series that include, among others, real output, income, employment, consumer spending, price indexes, bond and stock market indexes, and foreign exchange measures. These authors interpret economic uncertainty as the conditional volatility of the unpredictable component of the future value of each series, and then aggregate individual conditional volatilities into a macro uncertainty index for horizons of one-, three- and 12-months. Financial uncertainty is constructed in a similar way but using only information from a large set of financial variables as detailed in Ludvigson et al. (2021)[1].

Our measures of economic uncertainty follow Ang et al. (2006) and Engle et al. (2020) and focus on innovations to a state variable with ability to predict the cross-section of returns, see Bali et al. (2017). Ang et al. (2006) use the first differences of VIX to remove the strong persistence in the volatility index and Engle et al. (2020) remove the presence of serial correlation in their measure of climate risk news by considering the innovations from an AR(1) process. In a similar vein, we fit the following stochastic volatility process to model the time series of conditional volatilities obtained in Jurado et al. (2015):

$$\log\left(\sigma_{t+1}^{j}\right)^{2} = \gamma_0^j + \gamma_1^j \log\left(\sigma_t^j\right)^2 + \eta_{t+1}^j, \eta_{t+1}^j \overset{iid}{\sim} N(0, \tau^j), \tag{3.3}$$

with σ_{t+1}^j the conditional volatility of the forecast error term of a FAVAR process applied to time series of macroeconomic ($j = m$) or financial variables ($j = f$). The innovations η_{t+1}^j to the conditional volatility process are our main measure of

[1]The macroeconomic and financial uncertainty data can be accessed at https://www.sydneyludvigson.com/macro-and-financial-uncertainty-indexes.

macroeconomic uncertainty (η^m_{t+1}) and financial uncertainty (η^f_{t+1}); τ^j is the standard deviation of the innovations and γ_0 and γ_1 are the coefficients driving the presence of serial dependence in the conditional volatility process.

The above approach to obtain our economic uncertainty measures is not intended to fully remove the serial dependence in the shocks. In fact, exhibiting time series persistence is a necessary condition for time series effects because investors will only dynamically adjust their risk premium in response to shocks that are informative about future levels of economic uncertainty. This is particularly relevant for the periods under study, as we identify calm uncertainty regimes as episodes exhibiting decreasing uncertainty and hence characterized by persistent negative shocks in model (3.3). Similarly, turbulent periods are characterized by sharp increases in uncertainty that will be reflected in model (3.3) by runs of positive shocks.

The dynamics of our benchmark uncertainty measures η^m_{t+1} and η^f_{t+1}, reported in Figure 3.1 with blue and red lines, respectively, provide clear indication of time series persistence over the different evaluation periods. This is given by the presence of short-term trends that are negative during calm periods and positive for turbulent periods. Interestingly, although both time series exhibit positive comovement there is certain decoupling that suggests that the uncertainty measures react differently to economic shocks and may be considered as complementary uncertainty measures. For example, the fluctuations in the financial uncertainty measure during the 2007-2008 crisis are much stronger than for the macroeconomic uncertainty proxy. In contrast, the latter measure reacts earlier to the outbreak of the COVID-19 pandemic compared to the financial uncertainty proxy that reaches a peak soon afterwards.

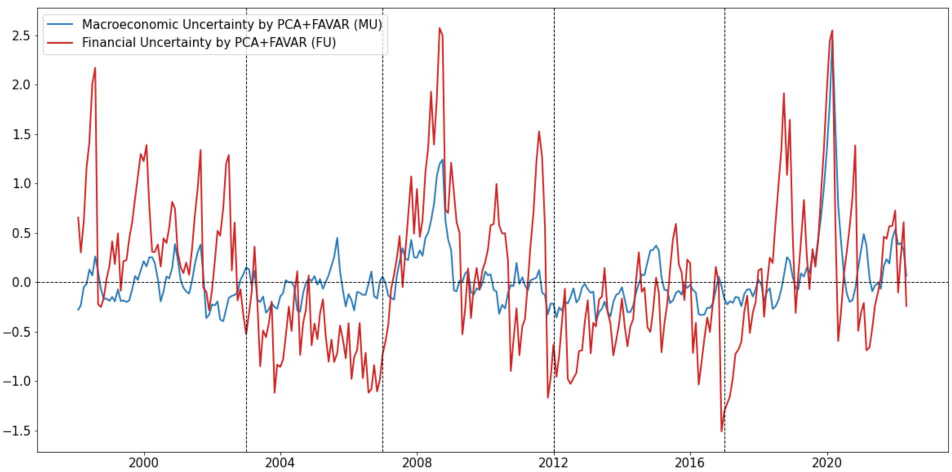


FIGURE 3.1: One-month-ahead innovations in macroeconomic and financial uncertainty

These sequences are obtained from model (3.3) using the conditional volatility measures developed in Jurado et al. (2015) and Ludvigson et al. (2021) over the period 1998 to 2022. Blue line for the macroeconomic uncertainty measure (η^m_{t+1}) and red line for the financial uncertainty measure (η^f_{t+1}).

3.2.3 Empirical Analysis

Our goal is to test whether the cross-section of average returns depends on their exposure to economic uncertainty and whether these returns vary with the uncertainty regime. To do this, we first consider our measure of macroeconomic uncertainty, denoted hereafter as MU, and follow a procedure similar to Ang et al. (2006) to estimate the pre-formation regression between the excess returns on the cross-section of stocks and the FF5 factor augmented with our uncertainty measure. The empirical model that we examine is

$$r_t^i = \beta_0^i + \beta_{mkt}^i MKT_t + \beta_{smb}^i SMB_t + \beta_{hml}^i HML_t + \beta_{rmw}^i RMW_t + \beta_{cma}^i CMA_t + \beta_{mu}^i MU_t + \epsilon_t^i, \tag{3.4}$$

with $i = 1, \ldots, n$, where n is the number of assets in the cross-section. The quantity of interest in this regression is the sensitivity of the excess returns to variations in the uncertainty measure over time. This is captured by the uncertainty loading β_{mu}. The uncertainty index developed in Jurado et al. (2015) is constructed with monthly data. This implies that our uncertainty measure also has this frequency forcing, in turn, the time series regression equation (3.4) to use monthly observations. A suitable methodology using our dataset is to consider monthly rolling regressions over a 60−month fixed window starting from January 1993. A similar approach is followed in Bali et al. (2017) for the analysis of economic uncertainty in a similar setting.

3.2.3.1 Ranking the Cross-section into Quintile Portfolios

We sort firms from the cross-section of returns on the uncertainty loadings β_{mu} obtained from the above time series regression applied to all stocks. The objective of this empirical exercise is to construct a set of base assets that are sufficiently disperse in exposure to economic uncertainty. Pástor and Stambaugh (2003), Ang et al. (2006) and Bali et al. (2017) follow similar approaches as pre-formation regressions to sort stocks from the cross-section. Firms in the first quintile have the lowest beta loadings on uncertainty whereas firms in the highest quintile have the highest coefficients. For each quintile, we construct value-weighted portfolios and form post-ranking portfolio returns for the next month.

Table 3.1 reports several summary statistics for each quintile portfolio sorted by past β_{mu} obtained from the previous 60−month regression window. Panels A to E report the statistics for each of the five evaluation periods considered in our empirical analysis. The first two columns report the average return and standard deviation of the value-weighted portfolio return for each quintile. The average returns reported in the first column are increasing over the quintile portfolios for the high uncertainty periods 1998-2002, 2007-2011 and 2017-2022, and negative for the low uncertainty periods. Similarly, the average return of the 5-1 portfolio strategy is positive for the high uncertainty periods and negative for the low uncertainty periods. These strategies are statistically significant at 5% for 1998-2002 and 2017-2022. The third column reports the time series average of the pre-formation uncertainty loadings β_{mu} obtained from running regression (3.4) using as test assets the quintile portfolios evaluated over all $60-$month rolling windows starting from 1993. The only difference with the previous regression is that we consider the formed quintile portfolios as test assets instead of the cross-section of returns. By construction, the factor loadings on macroeconomic uncertainty are monotonically increasing with the magnitude of the coefficients varying across evaluation periods. The magnitude of the coefficients is larger for the first period and remains in the interval $(-6\%, 6\%)$ for the remaining sample periods. An interesting pattern emerges across evaluation periods. The pre-formation beta loadings are negative for the lower quintiles and turn positive for the higher quintiles. Similar results are obtained in Ang et al. (2006) for innovations to VIX over a different evaluation period and Bali et al. (2017) for the raw uncertainty measure of Jurado et al. (2015). The presence of negative and positive beta loadings amplifies the magnitude of the return on the long-minus-short portfolio because it takes full advantage of the spread in average returns across quintiles.

3.2.3.2 Mimicking Portfolio Approach

The monotonicity in pre-formation beta loadings and the corresponding differences in profitability between top and bottom quintile portfolios suggest that macroeconomic uncertainty may have power to predict the risk premium in the cross-section of stock returns. However, these results need to be confirmed within the framework of an unconditional factor model. To do this, we construct an ex-post factor that mimics macroeconomic uncertainty by projecting monthly measures of our uncertainty proxy MU on a set of base assets, see Breeden et al. (1989). The portfolio weights are obtained as parameter estimates $\widehat{w}$ of an OLS regression between the MU factor and the vector of quintile portfolio returns X_t:

$$MU_t = c + w' X_t + v_t, \tag{3.5}$$

with v_t the error term. The mimicking portfolio return is obtained as the linear projection of the uncertainty measure on the base assets $RMU_t = \widehat{w}' X_t$. Returns are constructed as excess returns so the coefficients w are interpreted as weights in a zero-cost portfolio. The multifactor asset pricing model is obtained by replacing MU_t by RMU_t in expression (3.4) and presented below for completeness:

$$r_t^i = \beta_0^i + \beta_{mkt}^i MKT_t + \beta_{smb}^i SMB_t + \beta_{hml}^i HML_t + \beta_{rmw}^i RMW_t + \beta_{cma}^i CMA_t + \beta_{rmu}^i RMU_t + \varepsilon_t^i. \tag{3.6}$$

The pre-formation factor loadings β_{rmu} are obtained by running this regression over $60-$month rolling windows covering each evaluation period. The reported coefficients in the next-to-last column of Table 3.1 are time series averages of the factor beta loading estimates obtained from each rolling regression over the evaluation period using the quintile portfolios as test assets. The results are similar to the pre-formation beta loadings reported in Column 3 but the magnitude of the coefficients is significantly larger for the extreme quintiles. To examine ex-post factor exposure to macroeconomic uncertainty risk consistent with an unconditional factor model approach, we need to calculate post-ranking uncertainty betas over the full evaluation periods and not only using $60-$month rolling regressions. The last column of Table 3.1 shows similar patterns of post-formation factor loadings obtained from the time series regression (3.6) that uses the five quintile portfolio returns as test assets.

3.2.3.3 Pricing Macroeconomic Uncertainty

Table 3.1 suggests that macroeconomic uncertainty is a priced factor that varies over time. The next step to formally assess this is to estimate the price of economic uncertainty using Fama and MacBeth (1973) cross-sectional regressions:

$$r^i = c + \beta^i_{mkt}\lambda_{MKT} + \beta^i_{smb}\lambda_{SMB} + \beta^i_{hml}\lambda_{HML} + \beta^i_{rmw}\lambda_{RMW} + \beta^i_{cma}\lambda_{CMA} + \beta^i_{rmu}\lambda_{RMU} + \epsilon^i, \tag{3.7}$$

where the λs represent unconditional prices of risk of the various factors. Engle et al. (2020), in their appendix, also discuss a similar approach to price the occurrence of shocks to climate news. However, in contrast to these authors, our framework considers a tradable factor RMU_t instead of the state variable MU_t. This implies that the regressor in (3.7) is β_{rmu} obtained from the first stage regression instead of β_{mu}. Both approaches lead to similar interpretations given that RMU_t is the portfolio return that is maximally correlated to MU_t, however, the use of a portfolio return in the cross-sectional regression allows us to obtain further insights. For example, in the latter case, the uncertainty risk premium λ_{RMU} can be interpreted as the average return of the hedging portfolio RMU_t computed over each evaluation period. This interpretation is not valid if the cross-sectional pricing regression includes the raw uncertainty proxy MU_t instead.

Table 3.2 presents the unconditional cross-correlations between the pricing factors. Panel A reports the unconditional correlations computed over the period 1998-2022 and Panel B computes the correlations between RMU and the FF5 factors for each evaluation period. The unconditional correlation of the mimicking portfolio return with MKT and SMB is negative. This result also holds across subsamples and increases in magnitude in periods of financial distress. Episodes with positive returns on these factor portfolios are corresponded by negative returns on the uncertainty mimicking portfolio. These empirical findings provide further evidence on the role of

TABLE 3.1: Portfolios sorted by exposure to macro uncertainty

Rank	Mean	Std. Dev.	Pre-Formation β_{mu}	Pre-Formation β_{rmu}	Post-Formation β_{rmu}
Panel A. 1998-2002					
1	0.3337	4.9109	-10.3837	-16.8527	-14.6253
2	0.3750	4.5993	-2.2925	-2.1257	-3.0908
3	-0.1356	5.0838	-2.4098	-2.8173	-3.3723
4	-0.0141	5.9841	2.6627	2.1994	2.4438
5	1.8503	8.0180	7.0383	14.7105	16.3221
5-1	1.5166				
	[2.7496]				
Panel B. 2003-2006					
1	1.3870	3.1648	-6.5613	-12.2143	-15.8916
2	1.3664	2.8415	-3.6210	-3.8340	-3.5744
3	1.4391	3.1147	-2.6745	-4.8721	-4.9730
4	1.3269	3.0220	3.1451	4.1355	4.3580
5	1.0552	3.2488	6.8887	19.3259	13.5811
5-1	-0.3319				
	[-0.8472]				
Panel C. 2007-2012					
1	-0.3883	8.2087	-6.9287	-17.0256	-18.3317
2	0.3589	6.0683	-1.9212	-3.6412	-4.3951
3	0.2742	4.8341	-0.5972	-2.4954	-0.8518
4	0.5264	5.3070	2.5653	4.2123	3.8429
5	0.9505	5.4122	5.3868	12.3000	10.5344
5-1	1.3388				
	[1.0177]				
Panel D. 2012-2016					
1	1.3882	3.8460	-4.3625	-18.9237	-17.4776
2	1.2220	3.3612	-1.3361	-5.0397	-5.8922
3	1.1097	2.8610	0.1800	-0.2079	-0.2918
4	1.4713	3.0390	2.2973	3.5609	5.1329
5	0.7783	3.4051	4.1600	9.6917	11.4019
5-1	-0.6099				
	[-1.1497]				
Panel E. 2017-2022					
1	0.0723	5.2673	-5.0615	-16.6689	-16.2592
2	0.8873	4.1908	-1.9841	-4.9459	-2.9668
3	1.3013	4.4052	-0.2995	-0.9516	-0.9262
4	1.5447	5.2038	1.4776	4.6199	5.0425
5	2.4313	5.5845	3.7299	12.8208	13.6963
5-1	2.3589				
	[3.0676]				
Panel F. 1998-2022					
1	0.5179	5.4475	-6.6417	-16.5110	-16.1931
2	0.8210	4.3955	-2.1705	-3.9348	-4.2885
3	0.7783	4.2172	-1.0862	-2.1438	-2.3121
4	0.9642	4.7504	2.3872	3.7416	3.6926
5	1.4418	5.4950	5.3578	13.5284	13.6432
5-1	0.9239				
	[1.6836]				

Note: We form value-weighted quintile portfolios every month by regressing excess individual stock returns on MU, controlling for FF5 factors as in equation (3.4), using monthly data over $60-$month rolling regressions. Stocks are sorted into quintiles based on coefficient β_{mu} from lowest (quintile 1) to highest (quintile 5). The statistics in columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. The row 5-1 refers to difference in monthly returns between portfolio 5 and 1. The pre-formation betas refer to value-weighted β_{mu} or β_{rmu} within each quintile portfolio and are obtained from rolling regressions with quintile portfolios as test assets. The last column reports ex-post β_{rmu} factor loadings over each evaluation period, where RMU is factor mimicking portfolio obtained from (3.5). We compute ex-post betas by running FF5 model augmented with RMU factor. Robust t-statistics, adjusted according to Newey and West (1987), are reported in square brackets. The sample period is divided into five evaluation periods.

the macroeconomic uncertainty mimicking portfolio as a hedging instrument in periods of heightened uncertainty in which standard pricing factors such as MKT and SMB, typically providing a positive return, may yield negative returns. The unconditional correlation with the other pricing factors is slightly positive. As shown in Panel B, the sign of these correlations changes over time across sample periods but, in general, the magnitude is small suggesting that the HML, RMW and CMA factors are independent of the uncertainty pricing factor.

TABLE 3.2: Factor correlations

Panel A. Cross-correlations between factors 1998-2022						
	MKT	SMB	HML	RMW	CMA	RMU
MKT	1	0.2666	-0.0905	-0.3904	-0.3335	-0.4855
SMB	0.2666	1	0.0174	-0.4683	0.0114	-0.1736
HML	-0.0905	0.0174	1	0.3989	0.6153	0.0264
RMW	-0.3904	-0.4683	0.3989	1	0.2692	0.1647
CMA	-0.3335	0.0114	0.6153	0.2692	1	0.1077
RMU	-0.4855	-0.1736	0.0264	0.1647	0.1077	1
Panel B. Cross-correlations with RMU						
Period	MKT	SMB	HML	RMW	CMA	RMU
1998-2002	-0.3207	-0.0328	0.3719	0.2168	0.4139	1
2003-2006	-0.3504	-0.3061	0.1982	0.1689	-0.0166	1
2007-2012	-0.7090	-0.3846	-0.1084	0.3208	-0.0093	1
2012-2016	-0.4838	-0.1678	-0.0891	0.2179	-0.0496	1
2017-2022	-0.4356	-0.1133	-0.0973	-0.0985	-0.0298	1

Note: The table reports correlations of the RMU factor with the FF5 risk factors. The variable RMU represents the monthly return on the mimicking portfolio obtained from regression (3.5). The factors MKT, SMB, HML, RMW and CMA are the FF5 factors. Panel A reports the unconditional correlations computed over the period 1998-2022 and Panel B computes the correlations between RMU and the FF5 factors for each evaluation period.

To estimate the factor premiums λs, we first construct a set of test assets with returns r_t^i, whose factor loadings on macroeconomic uncertainty risk are sufficiently disperse so that the cross-sectional regressions are informative. Following Ang et al. (2006), at the end of each month, the cross-section of stocks is sorted first into five quintiles based on β_{mkt} and then within each quintile into β_{mu} quintiles to yield 25 portfolios that are used as test assets in the cross-sectional asset pricing model. Both loadings β_{mkt} and β_{mu} are obtained from the regression equation (3.4) applied to monthly data over the past 60 months. The Fama-MacBeth procedure is estimated in two stages. In a first stage, the betas in (3.7) are obtained from the time series regression (3.4) using the full sample. In the second stage, the risk premia are estimated from the cross-sectional regression (3.7) using monthly data.

Table 3.3 reports the price of risk associated to the six factor model given by the Fama and French (2015) pricing model augmented with the uncertainty mimicking portfolio RMU. The results reveal important heterogeneity in the risk premium associated to each factor over the evaluation period. The risk premium on the market portfolio is positive in all periods except in the calm 2012-2016 period. During this period, test assets with more exposure to this factor require a lower expected return compared to test assets that exhibit a lower beta exposure to the market portfolio. The magnitude of the risk premium on this factor is larger during the high uncertainty periods 2007-2011 and 2017-2022 compared to the remaining periods under evaluation. Similarly, we find strong evidence of a positive risk premium on the size factor across evaluation periods. In contrast, the risk premium on the book-to-market risk factor is negative in the high uncertainty periods 1998-2002 and 2007-2011 and positive in the remaining periods. The sign and magnitude of the risk premium on RMW and CMA also show time variation across evaluation periods.

TABLE 3.3: Fama-Macbeth factor risk premiums using RMU

	1998-2002	2003-2006	2007-2011	2012-2016	2017-2022	1998-2022
const	0.4284	1.2453	0.7662	1.4218	0.9975	0.9610
	[1.2827]	[7.0930]	[3.3202]	[5.3482]	[3.8517]	[6.9207]
MKT	0.1841	0.1991	0.3294	-0.3526	0.3181	0.1355
	[0.2538]	[0.3407]	[0.3808]	[-1.1338]	[0.5089]	[0.4528]
SMB	3.2248	0.7834	0.1346	0.2504	1.1118	1.1142
	[3.5997]	[0.8735]	[0.1972]	[0.3880]	[1.4237]	[2.6617]
HML	-0.3648	0.9288	-1.3769	0.7848	0.7484	0.1201
	[-0.2305]	[1.2563]	[-2.2744]	[1.8957]	[1.7255]	[0.2859]
RMW	0.0712	-1.4712	-0.8105	0.0644	-0.6105	-0.5143
	[0.0373]	[-1.4219]	[-1.2182]	[0.1453]	[-1.4834]	[-1.0687]
CMA	-0.0268	0.9600	-0.4218	-0.1662	0.2905	0.0951
	[-0.0196]	[1.9884]	[-0.8511]	[-0.4035]	[0.6310]	[0.2812]
RMU	0.0350	-0.0048	0.0644	-0.0094	0.0813	0.0355
	[1.3482]	[-0.2766]	[0.8903]	[-0.3589]	[2.3275]	[1.7602]

Note: The table reports the Fama and MacBeth (1973) factor premiums on 25 portfolios sorted first on β_{mkt} and then on β_{mu} for the FF5 model augmented with the RMU risk factor obtained as a mimicking portfolio return from regression (3.5) using macroeconomic uncertainty MU as proxy. Robust t-statistics that account for the first-stage estimation in the factor loadings, adjusted according to Newey and West (1987), are reported in square brackets. Each column reports the estimates of the factors risk premium for different evaluation.

Interestingly, the estimated risk premium obtained for the uncertainty risk factor RMU is positive in high uncertainty regimes and negative in the low uncertainty regime taking place between 2003 and 2006, suggesting that test assets with more exposure to economic uncertainty have lower expected returns (higher prices) in calm periods than in turbulent periods. The variation in the risk premium on this risk factor over time suggests that a dynamic hedging strategy can be implemented by investing in the mimicking portfolio RMU. Investors require a positive expected excess return on this portfolio in high uncertainty periods to compensate for the negative expected returns realized by the portfolio in low uncertainty periods. Investors pay a premium in low uncertainty regimes to dynamically hedge against high uncertainty periods.

TABLE 3.4: Ex-post factor loadings on *RMU* for 2007-2011

	Pre-ranking on β_{mu}				
Pre-ranking on β_{mkt}	Low 1	2	3	4	High 5
Low 1	-10.7503	-1.8150	-1.2948	7.2287	16.4946
	[-4.0165]	[-1.8494]	[-0.8803]	[3.5392]	[8.4647]
2	-14.3055	0.8148	-2.3684	5.2664	13.7770
	[-4.6116]	[0.2848]	[-2.4754]	[3.0884]	[5.1361]
3	-15.5728	-4.1505	-0.6087	2.8103	11.3135
	[-7.5760]	[-2.4403]	[-0.1790]	[1.9911]	[3.5606]
4	-20.4733	-4.6844	-0.1342	4.2198	7.9285
	[-4.3363]	[-2.3698]	[-0.0763]	[2.2083]	[2.7688]
High 5	-20.7484	-9.9361	-6.0013	-0.4732	8.2238
	[-9.1371]	[-3.5750]	[-1.4010]	[-0.1222]	[2.5477]

Note: The table reports ex-post factor loadings on β_{rmu} obtained from the first-stage Fama and MacBeth (1973) regression using the time series specification (3.7) applied to 25 portfolios sorted first on β_{mkt} and then on β_{mu} using the risk factor *MU* as proxy for economic uncertainty. Robust t-statistics, adjusted according to Newey and West (1987), are reported in square brackets. The sample period is from January 2007 to December 2011.

Table 3.4 reports the factor loadings β_{rmu} for each of the 25 base assets used in the first-pass time series regression from Fama and MacBeth (1973). Due to space constraints, we only report the loadings computed over the period January 2007 to December 2011, that reflects an episode of high turbulence in financial markets, but qualitatively similar results are obtained for the remaining sample periods. There is strong monotonicity in the uncertainty factor loadings for each quintile of stock returns sorted on market beta. For example, for the bottom β_{mkt} quintile, the parameter estimates range from -10.75 to 16.49 and for the corresponding top market quintile, the first-stage factor loading parameter estimates range between -20.75 and 8.22. There is wide dispersion in the uncertainty factor loadings across market quintile portfolios. Similar monotonicity results are obtained for the first-stage beta estimates for each quintile portfolio ranked on β_{mu}, in this case the beta loadings decrease across quintile portfolios. Interestingly, we find a clear monotonic pattern for most of the 25 values of the first-stage factor loadings that starts from -20.75 for the pair (High 5, Low 1) portfolio and ends with the value 16.49 for the pair (Low 1, High 5) portfolio, with the only exception of some values in the middle column. The results on this table confirm that portfolios formed on market return and macroeconomic uncertainty beta loadings exhibit exposure to macroeconomic uncertainty risk that is captured by the mimicking portfolio *RMU*.

The risk premium λ_{RMU} in the cross-sectional regression (3.7) can be interpreted as the discount rate applied to the uncertainty beta factor loadings. This coefficient can be also estimated from the time series average of the mimicking portfolio return RMU[2] over the evaluation period. Using this strategy, a noisy measure of the monthly risk premium is the excess return of the mimicking portfolio. Figure 3.2 presents the dynamics of the mimicking portfolio returns over the period 1998 to 2022. The results illustrate graphically the fluctuations in the uncertainty risk premium over time and the dynamics of the mimicking portfolio returns. Green lines describe the dynamics of the returns over turbulent periods and red lines over calm episodes. The blue line is obtained by applying a locally estimated scatterplot smoothing (LOESS) method that is obtained by fitting a weighted polynomial regression to the dynamics of the mimicking portfolio returns. This nonparametric regression method allows the fit to adapt to local variations while still preserving the overall trend. The dynamics of the smooth curve shed important insights on the evolution of the uncertainty risk premium over time. First, calm periods are associated to episodes of negative returns of the mimicking portfolio and turbulent periods to episodes of positive returns. Second, our division of the sample into five non-overlapping evaluation periods is ad-hoc and does not fully reflect the underlying differences in stock markets between calm and turbulent periods. For instance, the period 2007-2011 should be further classified into a tubulent and calm period, as revealed by the evolution of the uncertainty risk premium within that interval. The period 2007-2009 shows a steep increase in the return of the mimicking portfolio that can be interpreted as a positive risk premium on economic uncertainty whereas the episode 2010-2011 shows negative returns of the mimicking portfolio that reflect a negative risk premium on uncertainty.

Panel A of Table 3.5 reports the least squares parameter estimates of the time series regression (3.5) using the ex-post quintile portfolios computed over the full sample period (1998-2022). Therefore, these estimates are invariant to the choice of breakpoints defining each evaluation period. After suitable standardization of the portfolio weights - obtained by dividing by the largest estimate (0.038) - we see that the composition of the mimicking portfolio is $(-1, -0.1, -0.12, 0.15, 0.65)$ that yields similar portfolio weights compared to the 5-1 portfolio given by $(-1, 0, 0, 0, 1)$. The mimicking portfolio can be further refined by noting that the middle parameter estimates are not statistically significant. In this case the mimicking portfolio can be constructed using only the bottom and top quintiles. For comparison purposes, the dynamics of the 5-1 portfolio constructed on macroeconomic uncertainty are reported in the bottom panel of Figure 3.2. The dynamics of both portfolios are very similar with just minor differences in magnitude across investment strategies.

Panel B of Table 3.5 reports average market capitalization of the quintile portfolios over the evaluation periods. For each month, the aggregate market capitalization of stocks in a given quintile is calculated. This value is divided by the total market

[2] A similar strategy can be used to test the suitability of time series asset pricing equations using the Fama-MacBeth two-pass regression approach.

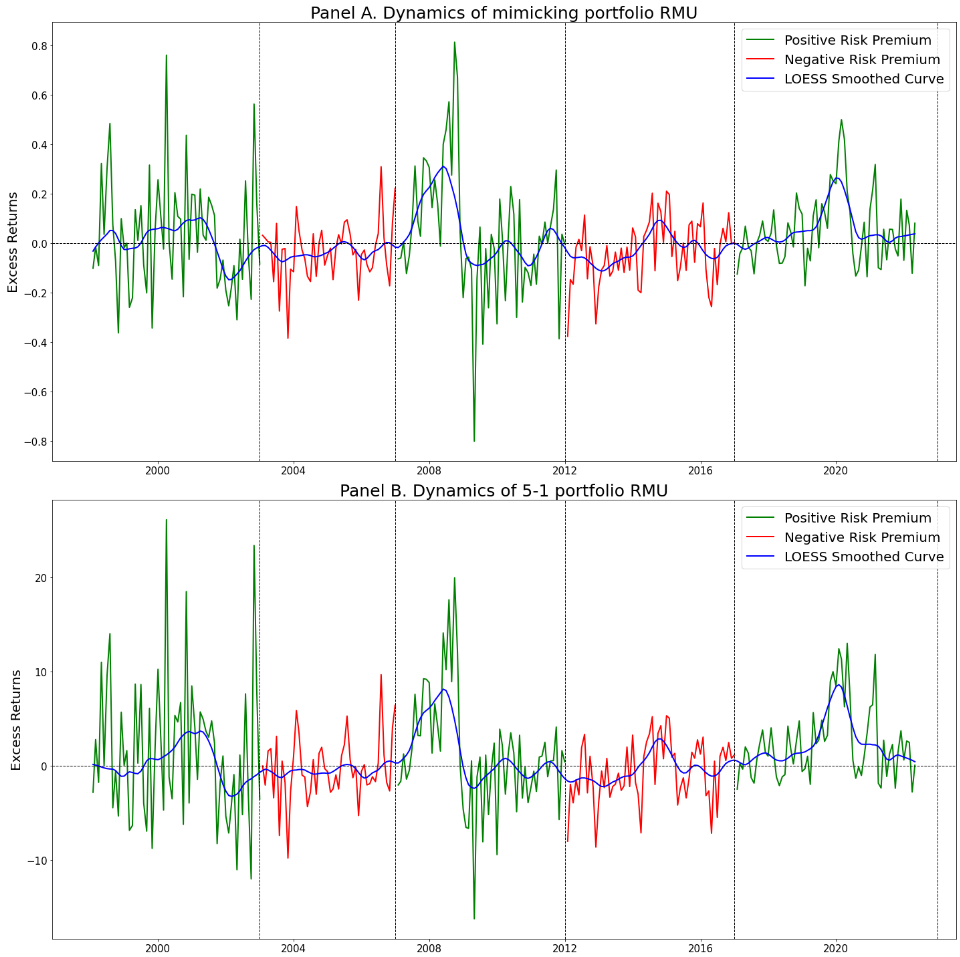


FIGURE 3.2: Dynamics of mimicking portfolio return RMU

Top panel reports the dynamics of the mimicking portfolio return RMU obtained from the regression equation (3.5). Bottom panel reports the dynamics of 5-1 portfolio constructed from sorting the cross-section of stock returns into five quintiles ranked on β_{mu}. The sample period is 1998 to 2022.

capitalization of the cross-section of stocks in that month. Subsequently, for each subsample, the monthly average ratio for all months within each quintile is computed and reported in the above table. The market capitalization of each quintile portfolio is stable across evaluation periods with the bottom and top quintiles reporting smaller market capitalization than the middle quintile portfolios. This result may suggest that stocks with small market capitalization have a prominent role on the construction of the mimicking portfolio and, hence, on the dynamics of the risk premium. This hypothesis is studied in more detail below and rejected using a subset of the cross-section of stock returns.

TABLE 3.5: Properties of mimicking portfolio, *RMU*

Panel A. Mimicking portfolio weights					
	X_1	X_2	X_3	X_4	X_5
w	-0.0380	-0.0041	-0.0045	0.0059	0.0245
	[-7.2602]	[-0.5734]	[-0.6095]	[0.8412]	[4.9952]
Panel B. Distribution of market capitalization					
	X_1	X_2	X_3	X_4	X_5
1998–2002	0.1450	0.1738	0.2243	0.2673	0.1895
2003–2006	0.1607	0.1969	0.2050	0.2770	0.1605
2007–2011	0.1402	0.2003	0.2648	0.2404	0.1543
2012–2016	0.1192	0.2087	0.2729	0.2564	0.1429
2017–2022	0.1568	0.2573	0.2567	0.2016	0.1276
All Sample	0.1438	0.2085	0.2465	0.2467	0.1544

Note: Panel A reports the OLS parameter estimates of the regression equation (3.5). The dependent variable is the macroeconomic uncertainty measure *MU* and the sample period 1998 to 2022. Robust t-statistics, adjusted according to Newey and West (1987), are reported in square brackets. Panel B reports the monthly average ratio of market capitalization of each quintile portfolio. For each month, the aggregate market capitalization of stocks in each quintile is calculated and divided by the total market capitalization of the cross-section of stocks in that month.

3.3 Robustness Exercises

This section conducts several robustness checks. First, we specify an alternative proxy of economic uncertainty given by the innovations to the financial uncertainty index proposed in Ludvigson et al. (2021). Second, we implement Giglio and Xiu (2021)'s three-stage procedure to correct for biases arising from the potential misspecification of the FF5 asset pricing model used as benchmark. The third robustness check consists of removing microcaps from the cross-section of stock returns to assess the extent to which the results are driven by small stocks. The last exercise provides a detailed analysis of the time-varying property of uncertainty's risk premium, with turbulent and calm months defined by different criteria formally.

3.3.1 Sensitivity of the Cross-section to Financial Uncertainty Proxy

We first investigate the robustness of our results to the choice of economic uncertainty measure. The following exercise replaces the innovations to the macroeconomic uncertainty measure developed in Jurado et al. (2015) by the innovations to the financial uncertainty measure proposed by Ludvigson et al. (2021). This last measure is constructed in a similar way to the macroeconomic uncertainty index, but is based on a large set of financial variables only. To obtain the sequence of innovations defining our proxy of uncertainty, we use the same empirical strategy discussed above given by fitting a stochastic volatility model of order one to the financial uncertainty index, and retaining the zero-mean innovations to this process.

Table 3.6 reports the descriptive statistics of the quintile portfolios ranked on the financial uncertainty beta loadings and the regression coefficients obtained from the pre-formation rolling regressions using the quintile portfolios as test assets. The main difference with Table 3.1 is that the dispersion in parameter estimates of the beta loadings is smaller than for the macroeconomic uncertainty measure. Except for the first period in Panel A, we observe monotonicity in the average return of the quintile portfolios. Portfolios with higher exposure to financial uncertainty have lower average returns in Panels B and D, and higher returns in Panels C and E. Similarly, the 5-1 investment portfolios are negative for Panels B and D, and positive and statistically significant at 1% for Panels C and E. The differences in profitability across long-minus-short investment strategies are greater than for the macroeconomic uncertainty measure, which suggest that the financial uncertainty pricing factor may have more predictive power about the cross-section of stock returns than the macroeconomic uncertainty factor.

To obtain an investment portfolio mimicking the dynamics of the financial uncertainty index, FU, we follow the same procedure introduced in (3.5) and regress the sequence of financial innovations against the set of quintile portfolios constructed from the pre-formation beta loadings using $60-$month rolling regressions. We denote the corresponding mimicking portfolio as RFU. Table 3.7 presents the cross-correlations between the uncertainty measures MU and FU and the corresponding mimicking portfolio returns RMU and RFU. The correlation between the uncertainty measures and their corresponding mimicking portfolio returns are between 0.5 and 0.6. The correlation between the macroeconomic and financial uncertainty measures takes similar values. These results suggest that although the different uncertainty measures capture similar changes in the information set there are also sizeable differences across the uncertainty proxies that point out the importance of considering each measure of uncertainty as a separate pricing factor. This is formalized in the following Fama-MacBeth cross-sectional regression.

Table 3.8 reports the coefficients of the cross-sectional regression (3.7) using the return on the financial mimicking portfolio RFU as additional pricing factor. The set of test assets is given by 25 investment portfolios ranked according to the beta loadings of the market and the exposure to the financial uncertainty loadings over each evaluation period. The risk premium on the financial uncertainty factor is positive in turbulent periods and negative in calm periods. The magnitude of the risk premium is also greater than for the asset pricing model that considers the macroeconomic uncertainty proxy. The coefficients of the remaining factors' risk premium is also different with respect to Table 3.3. This is because the test assets are obtained from a different ranking of the cross-section of returns that is based on the financial uncertainty beta loadings.

As an additional robustness check, we report in Figure 3.3 the dynamics of the mimicking portfolio returns based on the financial uncertainty measure and the corresponding 5-1 investment portfolio. The dynamics of both portfolios are similar to those of the macroeconomic uncertainty mimicking portfolios. The risk premium on uncertainty is positive and large over the first years of the 2007-2011 financial crisis episode and also for 2017-2022. Interestingly, the magnitude of the portfolio returns is larger for the financial crisis period compared to the mimicking portfolio constructed from macroeconomic uncertainty. This result suggests that financial uncertainty may be more informative than macroeconomic uncertainty in periods of financial turbulence whereas macroeconomic uncertainty may be more relevant in periods of wide economic distress such as during the COVID-19 pandemic[3].

3.3.2 The Role of Omitted Factors

Omitted variable bias arises in standard risk premia estimators whenever the model used in the estimation does not fully account for all priced sources of risk in the economy. To correct for possible biases derived from incomplete choices of the space of pricing factors, Giglio and Xiu (2021) propose a three-pass method to estimate the risk premium of an observable factor, which is valid even when not all factors in the model are specified or observed. This procedure is implemented in this section as an alternative to the FF5 model. These pricing factors are replaced by a set of unobservable factors that are recovered by principal components applied to the cross-section of asset returns. This robustness exercise contrasts with existing methods in the literature that conduct robustness checks to assess the sensitivity of the estimated risk premia to alternative definitions of the pricing factors or by including additional variables such as momentum, liquidity, investment, and profitability factors of Fama and French (1993), Carhart (1997), Pástor and Stambaugh (2003), and Hou et al. (2015), among others. In this sense, Giglio and Xiu (2021) is more powerful than these methods in correcting for omitted variable biases in estimating the risk premium of uncertainty.

Table 3.9 reports the time series average of the uncertainty risk premium estimates obtained from Giglio and Xiu (2021)'s procedure over each evaluation period. The availability of different number of assets in the cross-section for each month of the sample period does not allow us to conduct PCA once and extract a single set of factors that is valid over the full sample period 1998 to 2022. Instead, we implement the same rolling window scheme followed in our main framework (or in Bali et al.

[3] Section 3.4 offers further statistical evidence on the relative predictive ability of MU and FU over these two turbulent evaluation periods.

TABLE 3.6: Portfolios sorted by exposure to financial uncertainty

Rank	Mean	Std. Dev.	Pre-Formation β_{fu}	Pre-Formation β_{rfu}	Post-Formation β_{rfu}
Panel A. 1998-2002					
1	0.7124	6.7302	-3.6025	-7.7299	-8.1980
2	-0.5064	5.9259	-1.0694	-4.9531	-4.0490
3	0.1971	5.0866	-1.5110	-1.9920	-2.7907
4	-0.0167	5.0615	-0.2035	-0.3434	-0.3154
5	1.2162	6.7693	1.3748	2.8525	2.3161
5-1	0.5037				
	[0.3327]				
Panel B. 2003-2006					
1	1.7289	2.3957	-1.8520	-7.2068	-3.6290
2	1.5053	3.7817	-1.1583	-4.3510	-6.8892
3	1.2663	2.9696	-1.0826	-2.1162	0.3032
4	1.0799	2.7829	-0.3939	0.6860	1.1076
5	0.7915	3.1688	1.6247	2.8474	4.8480
5-1	-0.9374				
	[-4.6960]				
Panel C. 2007-2012					
1	-1.1701	7.4377	-1.7951	-5.0654	-4.8387
2	0.2474	6.0331	-1.1986	-4.4248	-4.1211
3	0.2659	5.3742	-0.1828	-0.8567	-1.5341
4	0.6897	5.1138	1.0625	1.6105	1.4179
5	1.5197	6.2462	2.1372	4.7877	4.9773
5-1	2.6898				
	[2.3838]				
Panel D. 2012-2016					
1	1.7556	3.8055	-1.8051	-5.3005	-5.2168
2	1.3264	3.3014	-0.9193	-3.2081	-2.0035
3	1.1862	3.0216	-0.1467	-1.0368	-1.4034
4	1.0161	2.8869	0.7306	0.9696	1.1727
5	0.8666	3.1419	2.3266	5.6750	6.3717
5-1	-0.8890				
	[-1.4509]				
Panel E. 2017-2022					
1	0.6521	5.3549	-1.8718	-5.5120	-6.0099
2	1.0396	4.2858	-0.7720	-2.1174	-2.6281
3	1.0562	4.3385	-0.2376	-0.7026	-0.2229
4	1.4675	4.6741	0.7051	2.1922	2.1184
5	1.9212	6.6313	1.8538	6.0797	5.6146
5-1	1.2692				
	[2.7209]				
Panel F. 1998-2022					
1	0.6938	5.6211	-2.1947	-6.1111	-6.2563
2	0.6947	4.8530	-1.0145	-3.7655	-3.4288
3	0.7785	4.3205	-0.6082	-1.3003	-1.4892
4	0.8462	4.2924	0.4164	1.0528	0.9125
5	1.2914	5.5247	1.8731	4.5366	4.5449
5-1	0.5976				
	[1.0068]				

Note: We form value-weighted quintile portfolios every month by regressing excess individual stock returns on FU, controlling for the FF5 factors as in equation (3.4), using monthly data over 60−month rolling regressions. Stocks are sorted into quintiles based on the coefficient β_{fu} from lowest (quintile 1) to highest (quintile 5). The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. The row 5-1 refers to the difference in monthly returns between Portfolios 5 and 1. The pre-formation betas refer to the value-weighted β_{fu} or β_{rfu} within each quintile portfolio and are obtained from rolling regressions with the quintile portfolios as test assets. The last column reports ex-post β_{rfu} factor loadings over each evaluation period, where RFU is the factor mimicking portfolio obtained from (3.5). We compute the ex-post betas by running the FF5 model augmented with the RFU factor. Robust t-statistics, adjusted according to Newey and West (1987), are reported in square brackets. The sample period is divided into five evaluation periods.

TABLE 3.7: Correlations between macroeconomic and financial measures

	MU	FU	RMU	RFU
MU	1	0.5369	0.5926	0.2998
FU	0.5369	1	0.3811	0.6255
RMU	0.5926	0.3811	1	0.4735
RFU	0.2998	0.6255	0.4735	1

Note: The table reports pairwise correlations among the uncertainty measures MU and FU and the corresponding mimicking portfolio returns RMU and RFU obtained from regression equation (3.5). The sample period is 1998 to 2022.

TABLE 3.8: Fama-Macbeth factor risk premiums using RFU

	1998-2002	2003-2006	2007-2011	2012-2016	2017-2022	1998-2022
const	0.0937	1.0513	0.7555	1.1889	0.9027	0.7895
	[0.1484]	[5.3063]	[1.6771]	[4.6385]	[2.7889]	[3.9086]
MKT	0.2772	0.0702	-0.1076	0.1416	0.5479	0.1956
	[0.2874]	[0.1715]	[-0.1201]	[0.3712]	[0.7818]	[0.5854]
SMB	1.2687	-0.1049	1.0071	-0.0958	0.3674	0.5112
	[0.9593]	[-0.1507]	[2.7735]	[-0.2709]	[0.5132]	[1.3833]
HML	-0.6769	1.2632	-1.0555	0.0482	0.2559	-0.0823
	[-0.4587]	[2.1388]	[-1.2376]	[0.0963]	[0.4984]	[-0.1992]
RMW	0.1607	-0.8594	1.0776	-0.1372	-0.4332	-0.0100
	[0.1142]	[-1.9237]	[2.1104]	[-0.2358]	[-1.2938]	[-0.0266]
CMA	-0.9013	0.6497	0.0626	-0.9930	0.1639	-0.2337
	[-0.8874]	[1.2848]	[0.0158]	[-1.6078]	[0.3336]	[-0.7081]
RFU	0.0975	-0.1372	0.2352	-0.1046	0.1231	0.0513
	[0.7791]	[-5.3629]	[1.3428]	[-1.5447]	[1.6996]	[0.8909]

Note: The table reports the Fama and MacBeth (1973) factor premiums on 25 portfolios sorted first on β_{mkt} and then on β_{fu} for the FF5 model augmented with the RFU risk factor obtained as a mimicking portfolio return from regression (3.5) using financial uncertainty FU as proxy. Robust t-statistics that account for the first-stage estimation in the factor loadings, adjusted according to Newey and West (1987), are reported in square brackets. Each column reports the estimates of the factors risk premium for a different evaluation period.

(2017)) to conduct the three-stage estimation exercise and price uncertainty dynamically. In the first stage, the unobserved factors are dynamically extracted from 60−month rolling windows together with the corresponding factor loadings. These loadings are used in a second stage to compute the risk premium associated to each of the unobserved pricing factors for the same rolling window, denoted as λ_{ug}. In the third stage, we run a time series regression between the mimicking portfolio returns RMU_t (or RFU_t) and the estimated factors over the same rolling sample to obtain the loadings β_{gx}. The monthly uncertainty risk premium λ_{gx} is obtained by multiplying the estimated risk premium from the unobserved estimated factors with the loadings from the latter time series regression such that $\lambda_{gx} = \lambda'_{ug}\beta_{gx}$. This exercise is repeated over the full sample period. For each rolling window, the number of factors is set to explain 99.9% of the variance of the cross-section of returns. The average number of

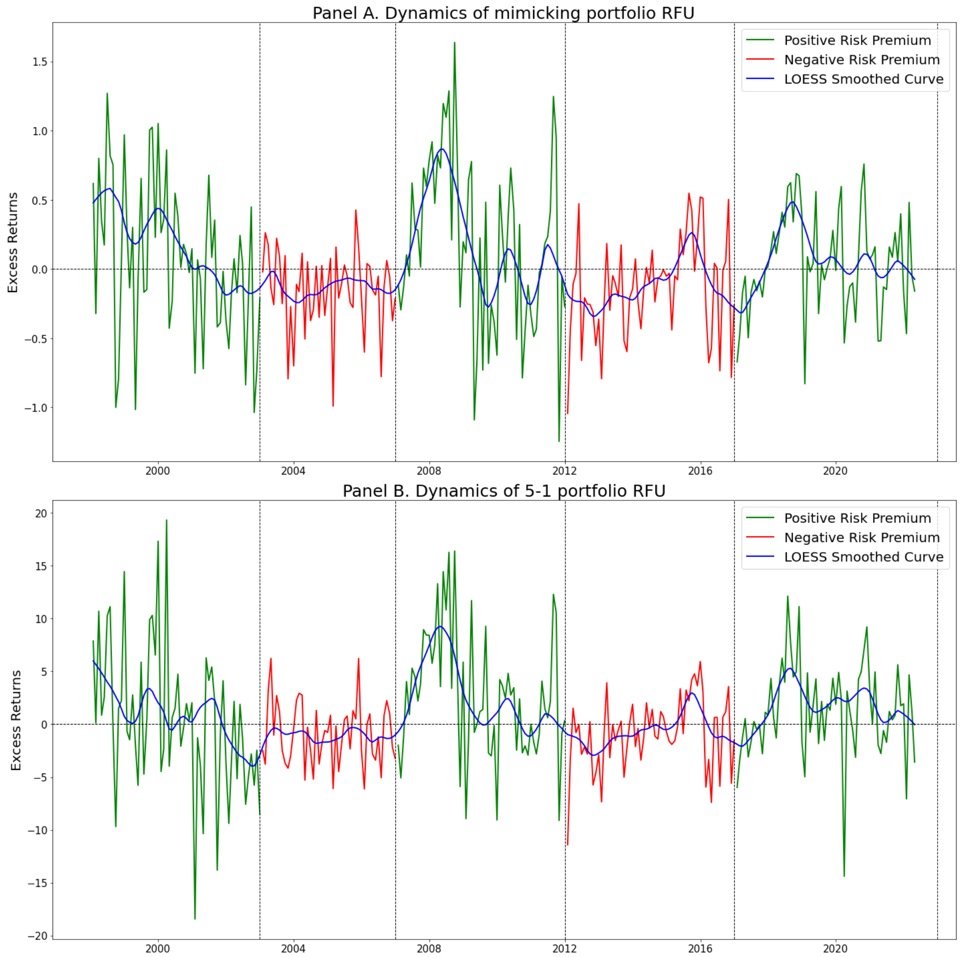


FIGURE 3.3: Dynamics of mimicking portfolio return RFU

Top panel reports the dynamics of the mimicking portfolio return RFU obtained from the regression equation (3.5). Bottom panel reports the dynamics of 5-1 portfolio constructed from sorting the cross-section of stock returns into five quintiles ranked on β_{fu}. The sample period is 1998 to 2022.

factors across rolling windows is greater than 30. The objective of considering such a large number of factors is to guarantee that the estimates of the uncertainty risk premium are free from potential biases due to the omission of relevant pricing factors. The results in Table 3.9 confirm the evidence from previous exercises that suggests that the uncertainty risk premium is negative in calm periods and positive in turbulent periods.

But even if we restrict the number of factors to a range between 7 and 9 (cumulating 70% to 80% of the variance) and repeat the estimation of the uncertainty risk premium using Giglio and Xiu (2021)'s estimation procedure, we obtain very similar dynamics for the uncertainty risk premium to those observed in the top panels of Figures 3.2 and 3.3. For ease of comparison, the two panels in Figure 3.4 display four curves given by (i) the macro (financial) uncertainty mimicking factor RMU (RFU); (ii) the corresponding LOESS smoothed curve; (iii) the risk premium estimates obtained from

TABLE 3.9: Macroeconomic and financial uncertainty risk premia by 3-Stage approach

	1998-2002	2003-2006	2007-2011	2012-2016	2017-2022	1998-2022
RMU	0.0325	-0.0025	0.0013	-0.0165	0.0094	0.0052
	[14.5883]	[-1.0131]	[0.4896]	[-5.4013]	[3.4783]	[2.8853]
RFU	0.1135	-0.0152	0.0092	-0.0060	0.0092	0.0235
	[17.4746]	[-1.5995]	[1.4266]	[-0.6483]	[2.1440]	[4.6489]

Note: We employ a fixed 60-month rolling window analysis consistent with our main framework and conduct the same three-stage approach as in Giglio and Xiu (2021) to estimate the monthly risk premiums of macroeconomic and financial uncertainty. The number of latent factors in each rolling window is defined dynamically by the cumulative variance explained by these latent factors in PCA. This table reports the average risk premiums of the five evaluation periods, and the Newey and West (1987) adjusted t-statistic is reported in square brackets.

the three-stage procedure considering seven unobserved factors, and (iv) its corresponding LOESS smoothed curve.[4] Comparing the risk premium estimates across asset pricing models in both panels reveals similar dynamics. Interestingly, for the top panel, the FF5 model augmented with the macroeconomic uncertainty factor yields higher returns and lower volatility than the model that estimates the factors dynamically[5]. The latent factor asset pricing model also reveals two separate uncertainty episodes during the 2007-2011 evaluation period, and reacts less markedly during the COVID-19 pandemic uncertainty window.

The findings obtained from the bottom panel share some similarities with the analysis of the macroeconomic uncertainty risk premium but there are also some distinct features that are worth mentioning. The risk premium from the latent factor model using financial uncertainty is highly volatile compared to the mimicking portfolio return *RFU* and is positive over the period 2003-2006 and mainly negative during 2007-2011. For the remaining periods, both estimates of the uncertainty risk premium reflect similar patterns. There are also significant differences between the risk premium estimates of the latent factor model augmented with macroeconomic uncertainty (top panel) and the latent factor model with financial uncertainty (bottom panel) that reveal that the beta exposures (β_{gx}) of the *RMU* and *RFU* pricing factors to the latent seven factor model are very different. These observations suggest that financial uncertainty risk premium estimates obtained from the latent seven factor model are not as reliable as the risk premium estimates of macroeconomic uncertainty.

As a final remark on this section, we note that although the factor model approach proposed by Giglio and Xiu (2021) alleviates potential biases in the risk premium estimates due to the omission of relevant pricing factors, the static risk premium estimates reported in Table 3.9 may change with the choice of the evaluation periods,

[4]The dynamics of the uncertainty risk premia for the asset pricing models that consider 8 and 9 latent factors are very similar and not reported for space considerations.

[5]Yet, Giglio and Xiu (2021) show that it is possible to improve the performance of the latent factor models by optimally choosing the number of factors, instead of fixing them 'ad hoc'.

as apparent from the dynamics in Figure 3.4. Different strategies to deal with this problem are discussed in the next section.

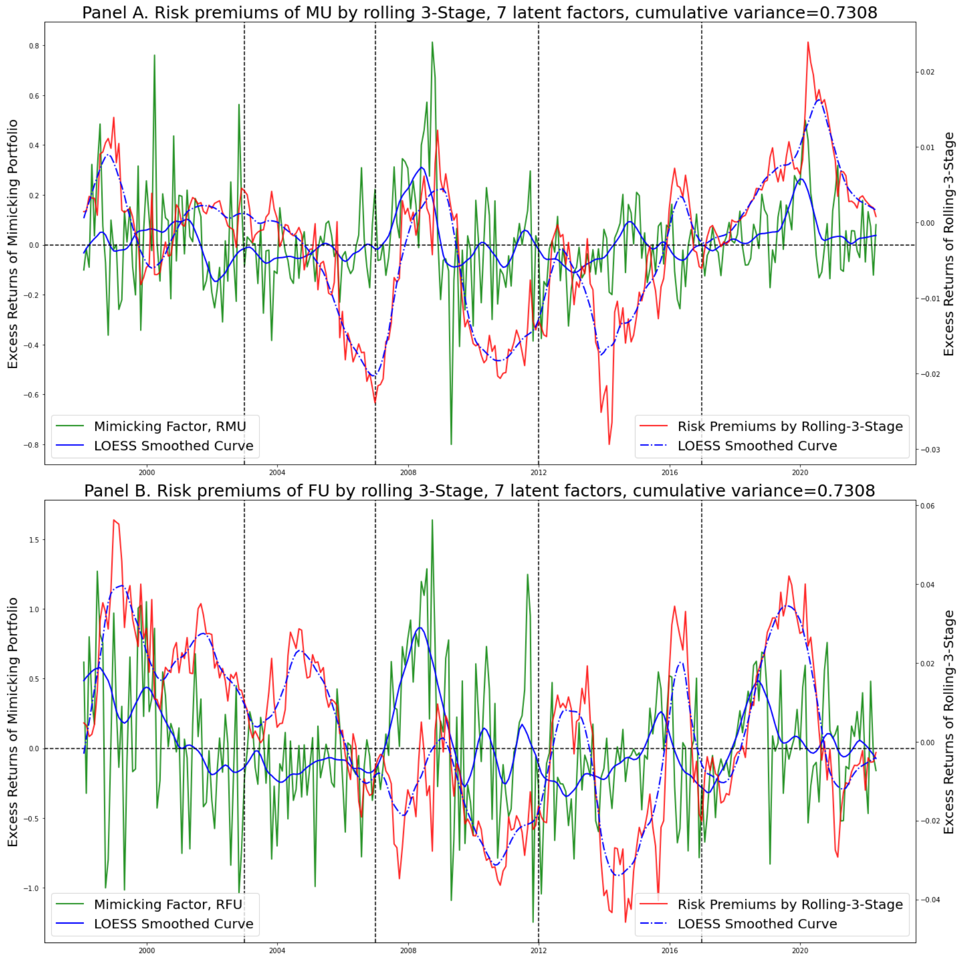


FIGURE 3.4: Dynamics of risk premiums estimated by different approaches

In this figure, top panel reports the dynamics of monthly risk premiums on macroeconomic uncertainty and bottom panel reports the monthly risk premiums on financial uncertainty. Green lines for the mimicking portfolio returns obtained from macro and financial uncertainty, respectively. Red lines for the risk premium estimates obtained from the latent factor model obtained from Giglio and Xiu (2021) three-stage procedure. The LOESS smoothed curves are also reported for each approach. Estimates from the latter approach are obtained from 60−month rolling windows and fixing the number of latent factors to 7 throughout the whole sample period 1998 to 2022.

3.3.3 The Effect of Microcaps

Microcaps are widely acknowledged to have the highest equal weighted returns and the largest cross-sectional dispersions in returns and pricing anomalies. Panel B of Table 3.5 also reports evidence that suggests that small firms may have a prominent role on the construction of the economic uncertainty factor.

Therefore, to assess empirically if the above results are driven by these stylized facts on microcap firms our second robustness exercise consists of assessing the replicability of the above results for a restricted cross-section of stock returns given by removing microcap firms. Following standard practice in the literature, see Hou et al. (2015), we consider a subset of the cross-section of all stocks listed in the NYSE that is obtained by removing stocks below the $20th$ percentile in terms of market capitalization. Table 3.10 reproduces the empirical exercise in Tables 3.1 and 3.6 using only the restricted cross-section of stock returns. The results do not present significant differences with respect to the analysis of the full cross-section of returns. There is monotonicity in the average returns of the quintile portfolios ranked on the pre-formation uncertainty beta loadings. Panels A, C and E report positive average returns for the 5-1 investment strategy, and Panels B and D report negative average returns. The magnitude of the average returns is smaller than in the previous analysis suggesting that the inclusion of microcap stocks magnifies the differences between top and bottom portfolios. We also observe monotonicity in the pre-formation and post-formation factor loadings and the dispersion of the coefficients is similar to the results obtained for the full cross-section.

The robustness exercise on the dynamics of the risk premium to the selection of stocks in the cross-section is completed in Table 3.11 by reporting the parameter estimates of the cross-sectional Fama-MacBeth regression after removing microcap stocks. The sign of the coefficients associated to the factor loadings β_{rmu} is consistent with previous results and shows a negative risk premium for calm periods and a positive risk premium for turbulent periods. Importantly, removing microcap stocks affects the magnitude of the risk premium that is smaller in this exercise than for the full cross-section of stock returns. This result is consistent with the average profitability of the 5-1 strategies computed in Table 3.10.

3.3.4 Choice of Evaluation Periods

In all our main research, we construct the mimicking portfolio of uncertainty and estimate the risk premium of uncertainty on a monthly basis. We then group these monthly excess returns into five turbulent and calm evaluation periods to facilitate discussion of the time-varying nature of the risk premium. Each evaluation period is determined ad hoc, and we emphasize that this grouping is independent of the estimation of both the mimicking portfolio and the risk premium. In other words, the reported values are simply the averages of the monthly estimated excess returns in each evaluation period, and these values are not affected by how the monthly estimates are grouped. In this section, we formally define turbulent and calm months using two approaches, including an exogenous method based on the Chicago FED National Activity Index (CFNAI), and an alternative method that relies on a threshold

TABLE 3.10: Portfolios sorted by exposure to macro uncertainty without microcaps

Rank	Mean	Std. Dev.	Pre-Formation β_{mu}	Pre-Formation β_{rmu}	Post-Formation β_{rmu}
Panel A. 1998-2002					
1	0.2885	4.9012	-9.5192	-16.3936	-15.3156
2	0.6780	4.5845	-2.3261	-2.9238	-3.0741
3	-0.2885	4.9216	-1.0598	-1.3123	-3.4146
4	0.1879	6.3974	0.9253	-0.6755	-0.1789
5	1.5136	7.5169	7.3534	14.8549	14.879
5-1	1.2251				
	[1.5164]				
Panel B. 2003-2006					
1	1.2873	3.1615	-6.2988	-12.8732	-14.5321
2	1.4290	2.9486	-3.7453	-4.7624	-5.7695
3	1.4637	2.9975	-0.8938	-1.4268	2.5913
4	1.2530	2.9046	1.0881	1.1964	-0.5345
5	1.1307	3.2470	6.6514	17.4271	14.4566
5-1	-0.1566				
	[-0.4886]				
Panel C. 2007-2012					
1	-0.4461	8.1402	-6.5079	-16.8483	-17.9029
2	0.3166	6.0682	-2.1923	-4.3538	-5.2923
3	0.3439	5.0057	0.0580	1.0333	0.9558
4	0.5149	5.2792	1.8075	2.0697	2.2895
5	1.0283	5.2306	5.1675	12.5086	11.0088
5-1	1.4744				
	[1.1362]				
Panel D. 2012-2016					
1	1.4979	3.8287	-4.4456	-18.1326	-16.9263
2	1.2289	3.3053	-0.9387	-5.1928	-5.6681
3	1.0827	2.8651	0.1856	0.4706	0.0373
4	1.4146	2.9730	2.0214	1.4543	4.1225
5	0.9674	3.4761	3.8933	10.6857	12.4689
5-1	-0.5304				
	[-0.7947]				
Panel E. 2017-2022					
1	0.1671	5.4052	-5.1163	-16.4194	-15.8980
2	0.9065	4.0575	-1.6094	-4.5646	-2.9620
3	1.3107	4.4841	-0.4601	-0.6111	-0.8004
4	1.4393	5.0894	1.4794	3.8395	2.7013
5	2.4622	5.6175	3.5297	13.4066	14.3214
5-1	2.2951				
	[3.0247]				
Panel F. 1998-2022					
1	0.5236	5.4536	-6.3635	-16.2713	-16.2726
2	0.8905	4.3657	-2.0897	-4.3457	-4.5852
3	0.7617	4.2299	-0.4036	-0.3292	-1.2773
4	0.9565	4.7939	1.4800	1.6235	2.0447
5	1.4466	5.3212	5.2398	13.6215	13.3988
5-1	0.9230				
	[1.6908]				

Note: We form value-weighted quintile portfolios every month by regressing excess stock returns on MU, controlling for the FF5 factors as in equation (3.4), using monthly data over $60-$month rolling regressions. The sample considers a subset of the cross-section of all stocks listed in the NYSE that is obtained by removing stocks below the $20th$ percentile in terms of market capitalization. Stocks are sorted into quintiles based on the coefficient β_{mu} from lowest (quintile 1) to highest (quintile 5). The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. The row 5-1 refers to the difference in monthly returns between portfolio 5 and portfolio 1. The pre-formation betas refer to the value-weighted β_{mu} or β_{rmu} within each quintile portfolio and are obtained from rolling regressions with the quintile portfolios as test assets. The last column reports ex-post β_{rmu} factor loadings over each evaluation period, where RMU is the factor mimicking portfolio obtained from (3.5). We compute the ex-post betas by running the FF5 model augmented with the RMU factor. Robust t-statistics, adjusted according to Newey and West (1987), are reported in square brackets. The sample period is divided into five evaluation periods.

TABLE 3.11: Fama-Macbeth factor risk premiums without microcaps

	1998-2002	2003-2006	2007-2011	2012-2016	2017-2022	1998-2022
const	0.9038	0.9063	0.6728	1.2875	1.0010	0.9569
	[1.9654]	[3.3605]	[2.3688]	[5.9678]	[3.8712]	[6.2385]
MKT	-0.7389	-0.1346	0.2919	-0.0594	0.3307	-0.0537
	[-0.9363]	[-0.2999]	[0.4070]	[-0.1642]	[0.5911]	[-0.1896]
SMB	3.4497	-0.2543	-0.4055	0.1063	0.4095	0.6953
	[2.0918]	[-0.2785]	[-0.7216]	[0.2254]	[0.4287]	[1.2622]
HML	2.0798	1.0580	-1.1491	0.5922	0.2146	0.5339
	[0.9858]	[1.8525]	[-2.3867]	[1.1470]	[0.3492]	[1.0105]
RMW	-0.3909	0.0438	-0.2611	-0.5415	0.3112	-0.1698
	[-0.4601]	[0.0584]	[-0.3286]	[-1.0282]	[0.8324]	[-0.5432]
CMA	1.1326	1.6166	-0.7249	0.0883	-0.2993	0.3021
	[0.9314]	[2.4169]	[-1.5700]	[0.2064]	[-0.5782]	[0.8436]
RMU	0.0381	-0.0266	0.0675	-0.0144	0.0807	0.0321
	[1.1693]	[-1.5687]	[0.9548]	[-0.5351]	[2.4621]	[1.5576]

Note: The table reports the Fama and MacBeth (1973) factor premiums computed from the cross-section of stocks listed in the NYSE after stocks in the bottom 20% of the distribution in terms of market capitalization. The dependent variable in the cross-sectional regression is given by 25 portfolios sorted first on β_{mkt} and then on β_{mu} for the FF5 model augmented with the RMU risk factor obtained as a mimicking portfolio return from regression (3.5) using MU as proxy for economic uncertainty. Robust t-statistics that account for the first-stage estimation in the factor loadings are reported in square brackets. Each column reports the estimates of the factors risk premium for a different evaluation period.

value for the uncertainty index itself, MU or FU respectively. By applying these statistical criteria, we identify turbulent and calm months more precisely and then compare the risk premium of uncertainty during turbulent versus calm periods in detail.

There are 292 months in the entire sample, and the turbulent or calm months are identified using either the CFNAI index or a threshold of the uncertainty index. Then, for these selected turbulent and calm months—which are typically noncontinuous—we employ a cross-section regression to estimate the risk premiums instead of using the Fama–MacBeth regression in our main analysis. Specifically, rather than running the cross-sectional regression in each month as in the second stage of Fama–MacBeth and then averaging the monthly estimated risk premiums, we conduct a single cross-sectional regression for turbulent or calm months using the average excess returns of our test assets, which consist of 25 portfolios sorted by β_{MKT} and β_{MU}.

Regarding the exogenous definition based on the CFNAI index, we use both the monthly index's three-month moving average (CFNAI-MA3) and the diffusion in CFNAI. According to the Federal Reserve Bank of Chicago, the thresholds are set at -0.7 for CFNAI-MA3 and -0.35 for diffusion in CFNAI. Thus, months with CFNAI-MA3 below -0.7 or diffusion in CFNAI below -0.35 are defined as turbulent, while all others are classified as calm. By these criteria, CFNAI-MA3 yields 27 turbulent months and 271 calm months, and the diffusion in CFNAI identifies 38 turbulent months and 254 calm months.

Table 3.12 presents the estimated risk premiums for macroeconomic and financial uncertainty from cross-section regressions in turbulent and calm months, as defined by the CFNAI-MA3 and diffusion in CFNAI, respectively. For macroeconomic uncertainty, Panels A and B show that the risk premium is positive and significant in turbulent months, consistent with our main findings, whereas it hovers around zero and is not significant in calm months. For financial uncertainty in Panel C and D, the risk premium is positive in turbulent months and negative in calm months, both significant and again in line with our main findings. The magnitudes of these risk premiums differ somewhat from the Fama–MacBeth estimates, and the risk premium of RMU is close to zero in calm months. The difference stems from the selection criteria of the CFNAI index, whereby only extremely turbulent months are classified as such, while months with relatively mild uncertainty shocks are deemed calm. Nonetheless, the risk premiums in turbulent months remain larger in magnitude than in calm months, reinforcing the evidence for the time-varying property of uncertainty's risk premium under the exogenous definition.

For the alternative definition based on the threshold value of the uncertainty index, we follow the suggestion in Jurado et al. (2015), setting the threshold as the mean of the uncertainty index plus 1.65 times its standard deviation. Months with an uncertainty index above this threshold are defined as turbulent, and the remaining months are correspondingly defined as calm. Under this definition by threshold of uncertainty index, there are 18 turbulent months and 274 calm months identified for MU, while 14 turbulent months and 278 calm months are identified for FU.

Table 3.13 presents the estimated risk premiums for RMU and RFU using cross-sectional regressions in turbulent versus calm months, as determined by the threshold of uncertainty index. Relative to the exogenous approach, the signs of the risk premiums in turbulent and calm months remain consistent, though the magnitudes differ partly because fewer turbulent months are identified by the threshold. Nonetheless, the results again show an increase in risk premiums when moving from calm to turbulent months, supporting our main findings.

3.4 Comparing Asset Pricing Models

We obtain two main conclusions from the previous exercise. Economic uncertainty is a priced factor in the cross-section of stock returns and, more importantly, the associated risk premium is negative in calm periods and positive in turbulent periods. The pricing factor can be proxied by a hedging portfolio with weights obtained from projecting the uncertainty measures on a set of test assets. The magnitude and

TABLE 3.12: Risk premium by cross-sectional regression in turbulent vs calm, defined by CFNAI

Panel A. *RMU* turbulent vs calm, defined by CFNAI-MA3 (-0.7)							
Turbulent (21 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RMU*
Risk premium	0.9977	0.2032	1.9728	-0.7073	-0.5914	0.4831	0.1691
	[2.3880]	[0.3030]	[1.4770]	[-0.5350]	[-0.4120]	[0.3630]	[7.9030]
Calm (271 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RMU*
Risk premium	1.0002	0.6624	0.0460	0.1397	0.4098	0.2460	0.0023
	[30.8960]	[12.7500]	[0.4440]	[1.3630]	[3.6870]	[2.3820]	[1.4080]
Panel B. *RMU* turbulent vs calm, defined by Diffusion in CFNAI (-0.35)							
Turbulent (38 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RMU*
Risk premium	1.3367	1.1332	1.0188	-0.6223	-0.6897	-0.2632	0.1085
	[4.8050]	[2.6310]	[1.0220]	[-0.6310]	[-0.9050]	[-0.3710]	[7.8520]
Calm (254 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RMU*
Risk premium	0.9496	0.5540	0.0598	0.1836	0.4915	0.3417	0.0002
	[22.8190]	[8.5960]	[0.4010]	[1.2440]	[4.3100]	[3.2170]	[0.1170]
Panel C. *RFU* turbulent vs calm, defined by CFNAI-MA3 (-0.7)							
Turbulent (21 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RFU*
Risk premium	0.6448	0.7497	2.6789	-2.6133	0.7447	-1.8693	0.2672
	[1.7330]	[1.2770]	[3.0740]	[-1.7840]	[0.5370]	[1.7450]	[4.9730]
Calm (271 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RFU*
Risk premium	1.0275	0.6200	-0.0087	0.2874	0.3063	0.4282	-0.0119
	[35.6370]	[13.6340]	[-0.1290]	[2.5310]	[2.8500]	[5.1580]	[-2.8470]
Panel D. *RFU* turbulent vs calm, defined by Diffusion in CFNAI (-0.35)							
Turbulent (38 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RFU*
Risk premium	1.2604	0.7655	1.6458	-0.3311	0.6786	0.4062	0.1538
	[4.0210]	[1.5300]	[1.7920]	[-0.3110]	[0.6900]	[0.5740]	[3.3810]
Calm (254 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RFU*
Risk premium	0.9610	0.6090	-0.0340	0.1401	0.2868	0.2416	-0.0136
	[20.4910]	[8.1360]	[-0.2480]	[0.8790]	[1.9510]	[2.2830]	[-1.9930]

Note: The table reports the factor premiums estimated by cross-sectional regressions in turbulent and calm months for *RMU* and *RFU*, corresponding to Panels A–D. Turbulent months are defined by the statistical criteria of CFNAI-MA3 < -0.7 or diffusion in CFNAI < -0.35, and all other months are designated as calm. The dependent variable in each cross-sectional regression is the average excess return of 25 portfolios, which are sorted first on β_{mkt} and then on β_{mu}. The regressions follow the FF5 model augmented with the *RMU* risk factor, obtained as a mimicking portfolio return from regression (3.5) using *MU* as a proxy for economic uncertainty (and β_{fu} for the estimations involving *RFU*). Robust t-statistics, adjusted according to Newey and West (1987), are reported in square brackets.

statistical significance of the risk premium varies across uncertainty regimes but is robust to the characterization of the economic uncertainty measure.

To confirm these results and provide further statistical support to the above findings, we carry out a model comparison exercise. The objective of this exercise is twofold. First, we assess the added predictive ability of the economic uncertainty risk factors against the FF5 model that acts as benchmark. This is done in a nested setting using the recent methodologies discussed in Barillas and Shanken (2017) based on the difference of squared Sharpe ratios and Barillas and Shanken (2018) through the comparison of posterior probabilities of each model candidate. Second, we also compute the posterior probabilities of a battery of model candidates that include different combinations of the FF5 model augmented with our macro and financial

TABLE 3.13: Risk premium by cross-sectional regression in turbulent vs calm, defined by uncertainty (threshold = mean + 1.65*std)

Panel A. *RMU* turbulent vs calm, defined by threshold of *MU*							
Turbulent (18 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RMU*
Risk premium	0.3626	-0.0932	2.6688	-1.1245	-0.2908	2.1981	0.1054
	[1.1140]	[-0.2050]	[2.1590]	[-1.0420]	[-0.2800]	[1.9610]	[7.3790]
Calm (274 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RMU*
Risk premium	1.0419	0.6768	0.0214	0.1578	0.3791	0.1359	0.0083
	[48.7440]	[22.6290]	[0.2640]	[2.2260]	[5.5470]	[1.8450]	[8.8940]
Panel B. *RFU* turbulent vs calm, defined by threshold of *FU*							
Turbulent (14 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RFU*
Risk premium	-0.4527	-1.6759	2.3749	-2.3599	0.3949	-0.5005	0.2033
	[-0.9660]	[-2.1220]	[2.8980]	[-1.3970]	[0.3170]	[-0.3220]	[3.4550]
Calm (278 months)	const	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*RFU*
Risk premium	1.0732	0.7454	0.0743	0.2016	0.3349	0.3015	-0.0016
	[45.4530]	[18.7410]	[1.800]	[2.3690]	[5.3390]	[3.8500]	[-0.5430]

Note: The table presents the factor premiums estimated by cross-sectional regressions in turbulent and calm months for *RMU* and *RFU*. Turbulent months are defined using the threshold criterion such that *MU* or *FU* falls below its designated threshold, with the remaining months classified as calm. The dependent variable in each cross-sectional regression is the average excess return on 25 portfolios sorted first by β_{mkt} and then by β_{mu}. The model specification follows the FF5 framework, augmented by the *RMU* risk factor, which is obtained as the mimicking portfolio return from regression (3.5) using *MU* as a proxy for economic uncertainty (and by β_{fu} for estimations involving *RFU*). Robust t-statistics, adjusted according to Newey and West (1987), are reported in square brackets.

uncertainty measures, see Barillas et al. (2020) for the implementation of this procedure.

Barillas and Shanken (2017) show that it is sufficient to regress the pricing factor candidate into the FF5 model to determine its additional value to predict the cross-section of stock returns. The factor increases the predictive ability of the benchmark asset pricing model if it is not spanned by the FF5 model. This is statistically assessed by testing the significance of the coefficient alpha (abnormal excess returns not explained by the asset pricing model) in the following time series regression:

$$RMU_t = \alpha + \beta_{mkt} MKT_t + \beta_{smb} SMB_t + \beta_{hml} HML_t + \beta_{rmw} RMW_t + \beta_{cma} CMA_t + \epsilon_t. \tag{3.8}$$

Table 3.14 reports the estimates of this regression over the five evaluation periods. If the null hypothesis given by $\alpha = 0$ is statistically rejected then the proposed additional factor increases the predictive ability of the benchmark model. In contrast, if the null hypothesis is not rejected then the FF5 model is capable of explaining the returns of the mimicking portfolio and, hence, the additional pricing factor is spanned by a linear combination of the existing risk factors. This table also reports the estimates of the factor regression that considers the financial uncertainty mimicking portfolio, *RFU*, as the pricing factor proxying economic uncertainty. The results provide overwhelming evidence on the added value of the uncertainty pricing factors. The coefficient alpha is statistically significant for most periods and both asset pricing models. The evidence is stronger for the macroeconomic uncertainty risk factor than for the financial uncertainty factor. The latter uncertainty factor does not seem to add value to the FF5 model in 1998-2002 and 2012-2016 and marginally does at 5% significance level during the period 2017-2022. Interestingly, the sign of the alpha coefficient is also informative about the sign of the uncertainty risk premium. This is so because the intercept of the time series regression equation can be interpreted as the mean return of the mimicking portfolio once we control for the effect of the remaining pricing factors.

TABLE 3.14: Regression analysis of *RMU* & *RFU* across evaluation periods

	alpha	*MKT*	*SMB*	*HML*	*RMW*	*CMA*
			Panels A. 1998-2002			
RMU	0.0394	0.0054	-0.0069	0.0032	0.0003	-0.0069
	[2.5327]	[0.6944]	[-0.5971]	[0.3334]	[0.0318]	[-0.3775]
RFU	0.1183	-0.0257	-0.0070	0.0187	-0.0146	-0.0600
	[0.9323]	[-1.4011]	[-0.3435]	[0.7869]	[-1.2613]	[-2.9736]
			Panels B. 2003-2006			
RMU	-0.0199	-0.0152	-0.0093	0.0157	-0.0104	-0.0025
	[-2.4846]	[-3.6720]	[-1.5673]	[2.8167]	[-2.4119]	[-0.2666]
RFU	-0.0615	-0.0630	0.0174	-0.0575	-0.0512	-0.0215
	[-4.4971]	[-5.6298]	[2.1489]	[-3.8001]	[-4.2302]	[-1.6906]
			Panels C. 2007-2011			
RMU	0.0648	-0.0393	-0.0102	0.0273	-0.0091	-0.0230
	[2.1821]	[-9.3380]	[-1.3878]	[3.1437]	[-1.3843]	[-2.4022]
RFU	0.2587	-0.0958	0.0029	0.0880	-0.0342	-0.1158
	[4.0623]	[-10.6268]	[0.1964]	[10.2993]	[-0.9786]	[-4.2357]
			Panels D. 2012-2016			
RMU	-0.0103	-0.0193	0.0025	0.0024	0.0128	-0.0170
	[-0.5664]	[-6.8975]	[0.4944]	[0.2493]	[2.7566]	[-1.5490]
RFU	-0.0400	-0.0723	-0.0299	-0.0195	-0.0155	-0.0203
	[-0.9245]	[-8.6359]	[-1.7926]	[-1.8134]	[-0.7320]	[-1.5223]
			Panels E. 2017-2022			
RMU	0.0763	-0.0158	0.0037	0.0004	0.0004	-0.0134
	[2.6819]	[-7.3302]	[0.4970]	[0.1134]	[0.0905]	[-3.5749]
RFU	0.0968	-0.0331	0.0034	0.0061	-0.0056	-0.0357
	[1.9080]	[-6.9733]	[1.9260]	[0.4242]	[-0.3928]	[-1.3594]

Note: The table reports the factor loadings from the time series factor regressions (4.8) that consider *RMU* and *RFU*, respectively, as dependent variables and the FF5 model as regressors. Both economic and financial uncertainty factors are obtained from the mimicking portfolio approach in regression equation (3.5). Panels A to E report the estimates of the factor loadings over different evaluation periods. Robust t-statistics, adjusted according to Newey and West (1987), are reported in square brackets.

The above comparison can be formalized statistically. Applying the results in Gibbons et al. (1989), we know that the standardized squared alpha coefficient can be expressed as the difference in squared Sharpe ratios between the FF5 model augmented with the economic uncertainty risk factor and the FF5 benchmark model. In this case suitable Wald type tests are sufficient to determine the statistical significance of the uncertainty factor in a nested setting.

The Barillas and Shanken (2017), Barillas and Shanken (2018), Barillas et al. (2020) propose a series of statistical tests based on the maximum achievable squared Sharpe ratio and the Bayes inference to evaluate various risk factors in asset pricing models, which includes comparisons of both nested and nonnested settings. Specifically, the comparison of nested models in this study focuses on analyzing the FF5 model against the 6-factor model, which incorporates all factors from the FF5 model plus a mimicking factor for innovations in uncertainty. The comparison of nonnested models explores the competition among all 6-factor models that include mimicking factors for different types of uncertainty or volatility. We will discuss these statistical tests related to our empirical conduction in our research, starting with the Sharpe ratio test.

For the Sharpe ratio tests in nested setting, suppose an given asset pricing model:

$$R = \alpha + \beta f + \varepsilon$$

The α coefficient, the intercept term in the regression of the test assets' excess returns on the factors, is recognized as the deviation of the test asset's returns from those predicted by the asset pricing model. Therefore, the α coefficient reflects the performance of asset pricing models or factors and measures mispricing. A non-zero α indicates a deviation, suggesting that the asset pricing model is not perfect and could be enhanced to achieve a higher Sharpe Ratio.

For two asset pricing models, M_1 and M_2:

$$M_1 : R = \alpha_1 + \beta_1 f_1 + \varepsilon_1$$

$$M_2 : R = \alpha_2 + \beta_2 f_2 + \varepsilon_2$$

Ideally, α_1 and α_2 should be zero, and the factors, f_1 and f_2, should attain the maximum achievable Sharpe ratio, indicating no mispricing in both asset pricing models. In an optimal setting, the factors within these models would cover the tangency portfolio for the entire investment universe, thereby maximizing the achievable Sharpe ratio. However, suppose both M_1 and M_2 are potentially imperfect and exhibit varying performances. The extent of model mispricing can be evaluated through a quadratic expression involving the alphas, which represents the potential for enhancing the squared Sharpe ratio. M_1 is considered to outperform M_2 if the improvement in the Sharpe ratio from addressing mispricing by M_1 is less than the improvement derived from mispricing by M_2, as described.

$$Sh^2\left(f_1, f_2, R\right) - Sh^2\left(f_1\right) < Sh^2\left(f_2, f_1, R\right) - Sh^2\left(f_2\right)$$

$Sh^2\left(\cdot\right)$ signifies the maximum squared Sharpe ratio achievable from portfolios composed of the given returns. The formula positions M_1 on the left side and M_2 on the right side, illustrating the Sharpe ratio improvement achieved by incorporating factors from one model into the factor set of the other (specifically, adding f_2 into M_1 and f_1 into M_2). Additionally, the formula considers adding the same test assets to both factor sets. Since these added test assets are identical for both sides of the equation, they can be omitted, resulting in the following adjustment to the formula:

$$Sh^2(f_1, f_2) - Sh^2(f_1) < Sh^2(f_2, f_1) - Sh^2(f_2)$$

Therefore, the critical mispricing for comparing models relates to the misalignment of factors in one model with those in another. The returns of the test assets, denoted as R, are inconsequential in this evaluation, as the ranking of models and the discrepancies in metrics persist irrespective of varying test asset selections. This remains valid whether or not the test assets are accurately priced by either model. Both inequalities can be employed to compare asset pricing models based on model mispricing. Clearly, these are equivalent to the direct comparison of model Sharpe ratios:

$$Sh^2(f_1) > Sh^2(f_2)$$

In this context, M_1 is considered the superior model, assuming that a higher Sharpe ratio is achievable by investing in f_1 compared to f_2. Currently, the comparison of nested asset pricing models based on Sharpe ratios is accomplished. The model that demonstrates a higher Sharpe ratio or requires a smaller improvement in the maximum achievable Sharpe ratio can be regarded as the better model.

Regarding the Sharpe ratio test in nonnested setting, for the two nonnested models A and B, which include factors f_{At} and f_{Bt} respectively,

$$M_A : R_t = \alpha_{At} + \beta_A f_{At} + \varepsilon_{At}$$

$$M_B : R_t = \alpha_{Bt} + \beta_B f_{Bt} + \varepsilon_{Bt}$$

it is assumed that all time series, including the factor returns and test assets returns, are jointly stationary and ergodic with finite fourth moments. The squared maximum Sharpe ratios achievable from the factors are given by:

$$\theta_A^2 = \mu_A' V_A^{-1} \mu_A$$

$$\theta_B^2 = \mu_B' V_B^{-1} \mu_B$$

Where, the μs represent the nonzero means of the two sets of factors, and the Vs are the invertible covariance matrices. The corresponding sample quantities will be:

$$\hat{\theta_A^2} = \hat{\mu_A'} \hat{V_A^{-1}} \hat{\mu_A}$$

$$\hat{\theta_B^2} = \hat{\mu_B'} \hat{V_B^{-1}} \hat{\mu_B}$$

Where, $\hat{V}$ is the maximum likelihood estimator of V, the population covariance matrix. For the two nonnested models, the asymptotic distribution of the difference in their sample squared Sharpe ratios is given by:

$$\sqrt{T}\left(\left[\hat{\theta_A^2} - \hat{\theta_B^2}\right] - \left[\theta_A^2 - \theta_B^2\right]\right) \quad \overset{A}{\sim} \quad \mathcal{N}(0, \mathbb{E}\left[d_t^2\right])$$

$$d_t = 2\left(u_{At} - u_{Bt}\right) - \left(u_{At}^2 - u_{Bt}^2\right) + \left(\theta_A^2 - \theta_B^2\right)$$

in which $u_{At} = \mu_A' V_A^{-1}\left(f_{At} - \mu_A\right)$ and $u_{Bt} = \mu_B' V_B^{-1}\left(f_{Bt} - \mu_B\right)$.

Based on the asymptotic distribution of the difference between two nonnested models, a direct test of $\theta_A^2 = \theta_B^2$ can be conducted to evaluate the models. If the difference in squared Sharpe ratios is significant, a superior model supported by a higher Sharpe ratio will be substantiated. The foundation for testing nonnested models lies in the asymptotic variance. However, when two models include overlapping factors, differentiating between scenarios where the null hypothesis might hold becomes crucial from both economic and statistical perspectives.

One scenario is that the common factors span the (true) maximum Sharpe ratio portfolio based on factors from both models. In this setting, the squared Sharpe ratio of each model equals that of the common-factors model, making the other factors redundant. To assess this spanning condition, an alpha-based test can be utilized, where the factors excluded from each model collectively serve as the left-hand-side returns. If the spanning condition is rejected, it suggests that some or all of the additional factors contribute to an increased squared Sharpe ratio, and the equality between the models may or may not be maintained.

In situations where the spanning is not supported, as indicated by $\mathbb{E}\left[d_t^2\right] > 0$, a direct test of $\theta_A^2 = \theta_B^2$ utilizing the established framework is feasible. Alternatively, if there is a prior belief that exact spanning is implausible, ruling it out ahead of time, the direct test becomes the preferable approach. Empirical analyses consistently reject the spanning condition via the alpha-based test in all cases considered. Thus, our focus typically shifts to the direct test in practical applications.

In addition to the Sharpe ratio test, the Bayes test developed by Barillas and Shanken (2018) offers another effective method. This test is based on the joint alpha restriction for a set of test assets within a Bayesian framework. The prior beliefs about the extent of mispricing in the asset pricing models are economically motivated. The posterior probability that α is restricted to be zero is shown to be a function of the Gibbons Ross Shanken (GRS) F-statistic. The Barillas and Shanken test extends from nested models to nonnested models, providing a robust approach to model comparison and validation.

In the nested setting of such Bayes test, similarly to the Sharpe ratio test, the core concept of the Bayes test is that the comparison of asset pricing models containing tradable factors focuses primarily on determining whether one asset pricing model can adequately price the factors in the other models. This approach evaluates the efficacy of models based on their ability to explain the variability and returns of factors considered by competing frameworks.

For a given multifactor model with N test assets and K factors:

$$r_t = \alpha + \beta f_t + \varepsilon_t$$

Where r_t represents the excess returns of the test asset, f_t is the vector of factors, and ε_t represents the residuals, which follow a normal distribution. Both r_t and ε_t are vectors of length N, while α is also a vector of N. The factor f_t is a vector of length K, and β is a matrix of dimensions $N \times K$. Under the null hypothesis of zero-alpha, the expected returns are linearly related to the βs as follows:

$$E\left(r_t\right) = \beta E\left(f_t\right)$$

where $E\left(f_t\right)$ represents the factors' risk premium, a vector of length K. The GRS test of the null hypothesis, based on the F-statistic with degrees of freedom N and $T - N - K$, will equal the Wald statistic times $(T - N - K)/(NT)$.

$$W = T \frac{\hat{\alpha}' \hat{\sum}^{-1} \hat{\alpha}}{1 + Sh\left(F\right)^2}$$

$$\hat{\alpha}' \hat{\sum}^{-1} \hat{\alpha} = \left(Sh\left(F, R\right)^2 - Sh\left(F\right)^2\right)$$

Where, $Sh(F)^2$ denotes the maximum squared sample Sharpe ratio achievable over portfolios composed of factors; $\hat{\Sigma}$ is the covariance matrix estimated by maximum likelihood estimates. $Sh(F, R)^2$ represents the squared Sharpe ratio calculated over both the factors and the returns of test assets. This formula implies that under the null hypothesis, where $\alpha = 0$, the tangency portfolio derived from both the factors and the returns of the test assets aligns with the tangency portfolio that is based solely on the factors.

Under the alternative hypothesis, where $\alpha \neq 0$, the F-statistic follows a noncentral F distribution and includes a noncentrality parameter λ:

$$\lambda\left(1 + Sh\left(F\right)^2\right))/T = \alpha' \sum^{-1} \alpha = sh\left(F, R\right)^2 - sh\left(F\right)^2$$

The $sh(F, R)^2$ and $sh(F)^2$ represent the population Sharpe ratios. Under the null hypothesis where $\lambda = 0$, the tangency portfolio corresponding to the factor and asset returns is equivalent to the one based solely on the factors.

Then, the Bayesian procedure can be established, initiating with diffuse priors for β and Σ. The diffuse prior for β and Σ refers to Jeffreys (1998), is shown as follows.

$$P\left(\beta, \sum\right) \propto \left|\sum\right|^{-(N+1)/2}$$

Under the null hypothesis, the prior for α concentrates at zero. For the alternative hypothesis, we assume a multivariate normal informative prior for α, conditional on β and $\sum$.

$$P\left(\alpha|\beta, \sum\right) = MVN\left(0, k\sum\right)$$

The parameter k reflects the belief regarding the potential magnitude of deviations from the expected return relation. For a test asset, this formula indicates that k represents the prior expectation of the squared alpha divided by the residual variance, which is the expected increase in the maximum squared Sharpe ratio by adding the asset to the given factor. Consequently, when setting the maximum value of the Sharpe ratio, denoted as Sh_{max}, the square root of the maximum expected squared Sharpe ratio under the alternative hypothesis, the required k will be:

$$k = \left(Sh_{max}^2 - Sh\left(f\right)^2\right)/N$$

The next step involves estimating the Bayes factor, which quantifies the relative support between the null hypothesis $H_0 : \alpha = 0$ and the alternative hypothesis $H_1 : \alpha \neq 0$.

$$BF = ML\left(H_0\right)/ML\left(H_1\right)$$

For the specified multifactor model and given priors for β and Σ, the restricted marginal likelihood $ML\left(H_0\right)$, where α is constrained to zero, is equivalent to:

$$\left|F'F\right|^{-N/2}\left|S_R\right|^{-(T-K)/2}$$

The unrestricted marginal likelihood $ML\left(H_1\right)$, where α is unconstrained to zero, is equivalent to:

$$\left|F'F\right|^{-N/2}\left|S\right|^{-(T-K)/2}Q$$

Here, S_R and S represent the cross-product matrices of the OLS residuals under the respective hypotheses. The scalar Q is given by:

$$Q = \left(1 + \frac{a}{(a+k)}\left(W/T\right)\right)^{\frac{T-K}{2}}\left(1 + \frac{a}{k}\right)^{-N/2}$$

$$a = \left(1 + Sh\left(F\right)^2\right)/T$$

Where, W, as previously mentioned, is the GRS F-statistic multiplied by $(NT)/(T-N-K)$. The Bayes factor for the null hypothesis against the alternative hypothesis is estimated as follows:

$$BF = \frac{ML\left(H_0\right)}{ML\left(H_1\right)} = \frac{1}{Q}\left(\frac{|S|}{|S_R|}\right)^{(T-K)/2}$$

Under the prior value k_0 and let the Q_{k_0} equal to the value of Q, the Bayes factor for k_0 against k will be:

$$BF_{k_0,k} = Q_{k_0}/Q$$

The Bayes factor evaluates the preference for testing a factor-pricing model against a more general alternative, namely, the extent to which the model's zero-alpha restriction aligns with the empirical estimates. To compare measures of uncertainty based on asset pricing models, a further Bayes test called the relative test is necessary. This test allows us to compare one factor-pricing model against other such models. The relative test for comparing measures of uncertainty based on the FF5 model serves as an example to elucidate the framework of the relative test.

$$M : r = \alpha_1 + \beta_1[MKT, SMB, HML, RMW, CMA] + \varepsilon_1$$

$$M_a : r = \alpha_2 + \beta_2[MKT, SMB, HML, RMW, CMA, RMU] + \varepsilon_2$$

$$Relative : RMU = \alpha^* + \beta^*[MKT, SMB, HML, RMW, CMA] + \varepsilon^*$$

Where, M represents the FF5 model and M_a refers to the six-factor model after incorporating the mimicking factor of macroeconomic uncertainty, RMU, as the sixth factor into the FF5 model (Or mimicking factor of financial uncertainty, RFU). The relative test primarily relies on the *Relative* model, in which the additional factors RMU are used as test assets and estimated by the factors in the FF5 model. If $\alpha^* = 0$, the tangency portfolio (and associated Sharpe ratio) based on all the factors in M_a can be achieved through investment in the factors from M, which has fewer factors. If $\alpha^* \neq 0$, a higher (squared) Sharpe ratio can be obtained by exploiting all the factor investment opportunities, which means all the factors in FF5 model and the additional factor, RMU.

Hence, following the same concept as used in the Sharpe ratio test and the Barillas and Shanken test, the relative test is conducted to evaluate α^* in the *Relative* model. The null hypothesis in the relative test is $\alpha^* = 0$, which suggests that M is the superior model, since the FF5 model can achieve the same performance or Sharpe ratio but using fewer factors than the six-factor model. The alternative hypothesis in the relative test is $\alpha^* \neq 0$, indicating that the six-factor model, M_a, will outperform the FF5 model, since a higher Sharpe ratio can be achieved. This increase in performance reflects the potential improvement of the FF5 model when RMU or RFU is added as the sixth factor.

Through the Bayes factor and the posterior probability for the null and alternative hypotheses in the relative test, the Bayes test indicates the preference for a hypothesis or the more suitable asset pricing model. Suppose the Bayes factor favors the null hypothesis with a zero α^*. In this scenario, the factors uninvolved in the six-factor model are priced perfectly by the factors in the original FF5 model, suggesting that the FF5 model is superior as it provides the same pricing ability using fewer factors. Conversely, if the Bayes factor prefers the alternative hypothesis, it indicates that the multifactor model including all the factors (the six-factor model) performs better. This preference arises because the factors uninvolved in the original FF5 model cannot be perfectly priced, leading to more significant mispricing than in the original model.

For the nonnested setting of Bayes test, based on the framework discussed in the section on nested models, it is possible to conduct the Bayes test of a factor model against a more general alternative as well as between two nested factor models, which are the foundation of the Barillas and Shanken test for comparing nonnested models. The comparison between nonnested models can be achieved by examining a collection of asset pricing models, both nested and nonnested, and decomposing the marginal likelihood for each model. For example, when comparing the factors in FF3 models, there is a collection of four models which includes the CAPM model, FF3 model and two nonnest models {MKT HML} and {MKT SMB}.

$$r = \alpha_1 + \beta_1 MKT + \varepsilon_1$$

$$r = \alpha_2 + \beta_2 [MKT, HML] + \varepsilon_2$$

$$r = \alpha_3 + \beta_3 [MKT, SMB] + \varepsilon_3$$

$$r = \alpha_4 + \beta_4 [MKT, HML, SMB] + \varepsilon_4$$

Given the marginal likelihood, ML_j, for each model j in the collection with the corresponding prior probability $P\left(M_j\right)$, the posterior probability under Bayes' rule is given by:

$$P\left(M_j \mid D\right) = \{ML_j \times P\left(M_j\right)\} / \{\sum_i ML_i \times P\left(M_i\right)\}$$

Where D represents the entire sample of factors and test assets. This formula requires that the posterior probability of every model be conditional on the same data D, hence the restrictions of each model apply not only to the omitted factors denoted as f^* but also to the test assets represented by r, when computing the ML. Conversely, the ML for the encompassed factors f relies on their unrestricted joint density. Hence, the aspect of multivariate regression becomes crucial.

$$f = \alpha + \beta MKT + \varepsilon$$

Suppose that the multivariate regressions of f on MKT, f^* on (MKT, f) and r on (MKT, f, f^*) satisfy the distributional conditions. The ML for a model M containing factor f is given by:

$$ML = ML_U\left(f \mid MKT\right) \times ML_R\left(f^* \mid MKT, f\right) \times ML_R\left(r \mid MKT, f, f^*\right)$$

Where the restricted and unrestricted regression ML can be estimated according to the framework in the last section, respectively. For $ML_U\left(f \mid MKT\right)$, the f serves as the left-hand-side returns and MKT as the right-hand-side returns in the calculation. Similarly, $ML_R\left(f^* \mid MKT, f\right)$ is computed by configuring f^* as the left-hand-side returns and (MKT, f) as the right-hand-side returns. The posterior model probabilities are calculated by substituting the corresponding ML values for each model. By using uniform prior model probabilities, we ensure that no model receives preferential treatment, an approach that suits this research context well. This method places greater emphasis on the influence of the data on our beliefs concerning the models. From the posterior probabilities, it becomes possible to rank all the asset pricing models in the collection and make comparisons.

It is vital to highlight a key distinction between the framework of nonnested model comparison and other, more typical asset-pricing test methods, including variations of the classical GRS test and Bayesian tests for nested model comparisons discussed in the last section. These conventional tests focus mainly on whether alpha equals zero when regressing excess returns on the model factors.

In contrast, the comparison of nonnested models introduces an additional criterion—that all of the model's factors must be truly essential for pricing. This leads to the unrestricted component of a model's ML, assuming that every included factor improves the attainable Sharpe ratio. The comprehensive joint measure of model likelihood is therefore calculated as the product of both restricted and unrestricted components. Thus, our task involves not only identifying which set of factors generates the highest Sharpe ratio but also evaluating whether a model achieves this efficiently and economically, considering our prior beliefs about alphas.

Panel A of Table 3.15 reports the test statistic and corresponding p-value for both proxies of uncertainty across the five evaluation periods. The p-values of the asymptotic tests based on the difference of squared Sharpe ratios in the row *Sharpe Diff vs FF5* confirm the insights obtained from the time series regressions and show statistically the value of including economic uncertainty as an additional pricing factor. This is observed for both measures of economic uncertainty and across all sample periods. The above results also suggest that appropriate proxies for uncertainty may depend on the economic outlook. Thus, macroeconomic uncertainty may be more suitable to explain the cross-section of returns during periods of economic distress whereas financial uncertainty may be more relevant in periods of financial turmoil. To add further support to these claims, we compare both FF5-augmented models in a non-nested setting. Barillas et al. (2020) show that, under

TABLE 3.15: Model comparison based on Sharpe ratio and Bayes tests (RMU & RFU)

	Panel A: Six-factor models vs FF5									
	1998-2002		2003-2006		2007-2012		2012-2016		2017-2022	
	FF5+RMU	FF5+RFU	FF5+RMU	FF5+RFU	FF5+RMU	FF5+RFU	FF5+RMU	FF5+RFU	FF5+RMU	FF5+RFU
Squared Sharpe	0.1900	0.2054	0.4848	0.5143	0.4357	0.6838	0.1951	0.2139	0.5429	0.261
Sharpe Diff vs FF5	0.0338	0.0491	0.0353	0.0648	0.1426	0.3907	0.0092	0.0281	0.3752	0.0934
	[0.0014]	[0.0002]	[0.0027]	[0.0005]	[0.0013]	[0.0000]	[0.0046]	[0.0014]	[0.0062]	[0.0002]
Sharpe Diff vs FF6	-0.0153		-0.0295		-0.2481		-0.0189		0.2890	
	[0.0002]		[0.0004]		[0.0001]		[0.0014]		[0.0001]	
Bayes Factor	0.9025	0.7945	1.0212	0.8014	0.3070	0.0382	1.1365	0.9525	0.0612	0.5081
Post. prob. vs FF5	0.5256	0.5572	0.4947	0.5551	0.7651	0.9632	0.4681	0.5122	0.9423	0.6631

	Panel B: Tests based on non-nested posterior probabilities				
	1998-2002	2003-2006	2007-2012	2012-2016	2017-2022
FF5	0.2181	0.2276	0.0200	0.2642	0.0285
FF5+RMU	0.2416	0.2229	0.0651	0.2325	0.4652
FF5+RFU	0.2744	0.2840	0.5236	0.2774	0.0560
FF5+$RMU + RFU$	0.2659	0.2655	0.3916	0.2260	0.4503

Note: Panel A reports the statistics and p-values of different tests for model comparison across the five evaluation periods. The first row reports the squared Sharpe ratios obtained from the FF5+RMU and FF5+RFU asset pricing models. The second row reports the difference of squared Sharpe ratios between the former models and the FF5 benchmark. The p-values in this case are obtained from a nested Wald test using Gibbons et al. (1989) procedure. The fourth row reports the difference of squared Sharpe ratios between FF5+RMU and FF5+RFU. The p-values in this case are obtained from a Normal test using Barillas et al. (2020) non-nested procedure. Rows 6 and 7 report the Bayes Factor statistics and posterior probabilities of models FF5+RMU or FF5+RFU, depending on the column, against the FF5 model. Robust p-values are reported in square brackets. Panel B reports the posterior probabilities of different model candidates in a non-nested setting. Posterior probabilities are computed using the Bayesian approach and assumptions in Barillas and Shanken (2018). FF5+$RMU + RFU$ denotes a seven factor asset pricing model obtained by augmenting FF5 model with the macro and financial uncertainty factors constructed as mimicking portfolios.

the null hypothesis given by equality of the squared maximum Sharpe ratios attainable from the two sets of non-nested factors, the difference of squared Sharpe ratios follows a zero-mean Normal distribution. The row *Sharpe Diff vs FF6* in Panel A of Table 3.15 reports the difference of squared Sharpe ratios and corresponding p-values for the FF5+RMU model against FF5+RFU. The results provide strong evidence in favor of the latter model except for the period 2017-2022. This result shows that, during the outbreak of the COVID-19 pandemic, a wider measure of economic uncertainty such as our macroeconomic uncertainty proxy carried more informational content on the cross-section of stock returns than the financial uncertainty proxy.

Panel A also reports the Bayes Factor (BF) computed as the ratio of maximum likelihood functions under the null and alternative hypotheses and the value of the posterior probability (pp) of the augmented six-factor models against the FF5 benchmark. The posterior probability of the alternative FF5 model is $1 - pp$. To compute these posterior probabilities, we follow the Bayesian procedure in Barillas and Shanken (2018) and fit the FF5 model to the set of 25 test assets discussed above. A popular diffuse prior for β and Σ, see also Jeffreys (1998), is $P(\beta, \Sigma) \propto |\Sigma|^{-(n+1)/2}$, where n is the number of test assets. The parameter β denotes the vector of factor loadings in the FF5 specification and Σ is the covariance matrix of the vector of residuals obtained from fitting the FF5 asset pricing model to the set of test assets. Under the null hypothesis $\alpha = 0$ (restricted model), the prior for alpha is concentrated to zero. For the alternative hypothesis (unrestricted model), the informative prior for α conditional on β and Σ is assumed to be a multivariate normal distribution $MVN(0, k\Sigma)$. The parameter k represents the prior expectation of the squared alpha divided by the residual variance and reflects individuals' beliefs about the potential magnitude of deviations from the expected return relationship. Suitable choices for k are discussed in Barillas and Shanken (2018). The results in Table 3.15 provide support for the augmented asset pricing model that includes economic uncertainty as an additional pricing factor. However, there are differences in predictive ability across uncertainty proxies and evaluation periods. Thus, the support for the economic uncertainty measure obtained from the posterior probability is usually stronger for the financial uncertainty measure than for the macroeconomic uncertainty measure except for 2017-2022.

The following exercise aims to determine the robustness of this result to a battery of model candidates in a non-nested framework. The alternative models that we consider in Panel B of Table 3.15 are (i) FF5 model, (ii) FF5+RMU, (iii) FF5+RFU, (iv) FF5+$RMU + RFU$. The latter model is given by the FF5 factor model augmented with both macroeconomic and financial uncertainty factors. The posterior probabilities between non-nested models are computed using the methodology in Barillas and Shanken (2018). For each evaluation period, the sum of the probabilities assigned to each model is equal to one. The results vary across periods but the overall conclusion is in line with previous findings: the FF5 model augmented with the financial uncertainty pricing factor (mimicking portfolio return) receives the highest posterior probability in most periods with the exception of 2017-2022. In the latter period, the asset pricing model with the macroeconomic uncertainty proxy clearly outperforms the FF5+RFU model. The differences in the distribution of posterior probabilities between 2007-2012 and 2017-2022 are also worth discussing. During the first turbulent period the financial uncertainty factor adds explanatory power to the FF5 model to predict the cross-section of returns. This is a clear indication that the source of distress in financial markets was due to shocks in the financial sector. In contrast, in 2017-2022, uncertainty in the financial sector is not as relevant for explaining the cross-section of stock returns. In this period, economic uncertainty was due to shocks to the wider economy triggered by the outbreak of the COVID-19 pandemic that spread across the entire economy. In this case, most posterior probability is assigned to the FF5 model augmented with the macroeconomic uncertainty pricing factor.

3.5 Conclusion

In Chapter 3, we find that stocks with high past exposure to aggregate economic uncertainty have lower expected returns than stocks with low exposures in calm periods, and higher expected returns in turbulent periods. This result is interpreted as evidence of an insurance premium paid by investors in low uncertainty regimes to hedge against higher uncertainty periods. This evidence is further exploited to construct pricing risk factors based on variables that proxy macroeconomic and financial uncertainty. Our indicators of economic uncertainty are given by innovations to well known econometric-based measures of aggregate conditional volatility introduced by Jurado et al. (2015) and Ludvigson et al. (2021).

Our empirical findings provide overwhelming support to the existence of a time-varying risk premium on economic and financial uncertainty. The magnitude and sign of this premium on the cross-section of risky assets vary with the uncertainty regime and choice of uncertainty proxy. The magnitude is, in general, stronger for financial uncertainty than for macroeconomic uncertainty and the sign is negative for calm periods and positive for turbulent periods. This evidence is robust to the methodology used for estimating the risk premium and, to a large extent, the presence of unobserved factors, although our proposed approach is the Fama and French (2015) five factor model augmented with the economic uncertainty risk factors. Our empirical results on the relationship between the uncertainty risk premium and the uncertainty regime appear robust to the choice of evaluation period, as revealed by the dynamics of the macroeconomic and financial mimicking portfolio returns.

The predictive ability of the economic uncertainty risk premium is also supported statistically. Thus, we find overwhelming statistical evidence in support of augmenting the FF5 model with our measures of macroeconomic and financial uncertainty. The suitability of each uncertainty measure for explaining the cross-section of stock returns depends on the uncertainty regime and the type of shock producing the turbulent episode. Thus, our financial uncertainty measure is found to explain better the cross-section of stock returns than the corresponding macroeconomic uncertainty proxy during the 2007-2011 financial crisis period whereas the latter uncertainty measure is superior during the 2017-2022 period characterized by the COVID-19 pandemic and a sustained period of stagflation.

Chapter 4

Pricing LSTM-measured Macroeconomic Uncertainty

We examine the pricing of economic uncertainty in the cross-section of stock returns, when uncertainty is proxied by innovations to the non-parametric counterparts of the macroeconomic and financial volatility measures of Jurado et al. (2015) and Ludvigson et al. (2021). Long short-term memory neural networks (LSTMs) extract additional forecastable variation from uncertainty indices' conditional expectations, relative to Jurado et al. (2015) and Ludvigson et al. (2021) parametric counterparts (PCA extracted factors informing a FAVAR econometric specification). Yet, we show that both deliver negative uncertainty risk premia in calm periods and positive ones in turbulent times, stemming from a hedging portfolio that funds the cost of insuring the portfolio in calm periods with the positive excess returns accrued during uncertain ones. Comparing both, we provide (nested and non-nested) statistical evidence in support of the relatively bigger risk premia of non-parametric uncertainty factors (when exposed to similar uncertainty shocks) as well as of their better predictive performance, particularly during financial crisis.

4.1 Introduction

As the complexity of financial markets continues to escalate, the limitations of multi-factor pricing frameworks are gradually becoming apparent. These issues include the static factor assumption, where factor loadings are assumed to be constant and unable to capture the dynamic impact of economic cycles or market structure changes on asset risk. Additionally, traditional factor construction presents challenges such as high multicollinearity, the rapid obsolescence of newly discovered factors, and the difficulty of extracting remaining potential factors, leading to information omission. To address these issues, an increasing number of studies focus on applying machine learning models in asset pricing. While these methods can enhance the ability to predict risk premiums, they also face insurmountable black-box limitations, as discussed in 4.2.1.

In contrast to these studies, we propose in Chapter 2 a method for constructing nonlinear factors using the LSTM model. We use the LSTM model to forecast macroeconomic and financial data and further measure uncertainty. By leveraging the LSTM model's superior predictive ability for time-series data, we aim to eliminate the predictable components within the forecasting as much as possible. LSTM-based uncertainty, calculated from the conditional volatility of the unpredictable components, retains the same economic significance as linear uncertainty measures and is an interpretable factor constructed through nonlinear methods in asset pricing models. Incorporating nonlinear factors into multi-factor pricing models will result in significant improvements.

According to our analysis in Chapter 2, LSTM-based uncertainty offers better explanatory and predictive capabilities for macroeconomic variables. Uncertainty has already been validated in Chapter 3 as a pricing factor with a time-varying risk premium. Therefore, we believe that LSTM-based uncertainty, compared to linear uncertainty measures, will exhibit a similar time-varying risk premium and offer better explanatory and predictive capabilities for risk premiums. By comparing LSTM-based and linear uncertainty measures, we will also verify the feasibility of using LSTM to construct factors.

To validate our hypothesis regarding the predictive function and dynamics of our LSTM-based macroeconomic uncertainty risk premium, we employ three distinct empirical strategies prior to comparing the LSTM-derived uncertainty measure with the linear measure. Initially, we create a mimicking (or hedging) portfolio by ranking the cross-section of stock returns. These rankings are determined by firms' exposure to LSTM-based macroeconomic uncertainty, using a methodology akin to that of Engle et al. (2020), and are estimated on a monthly basis. This involves creating a mimicking portfolio as direct projections of economic uncertainty measures against a selection of base asset returns. Secondly, we examine the dynamics of the uncertainty premium over these five periods through Fama-MacBeth (Fama and MacBeth, 1973) cross-sectional regressions. Lastly, we employ the novel asset pricing model proposed by Giglio and Xiu (2021) to demonstrate the robustness of the uncertainty premium estimates, ensuring they remain unaffected by potential omitted variable biases.

Our primary hypothesis is evaluated using cross-sectional stock price data obtained from CRSP, which includes all available stocks listed on the NYSE, NASDAQ, and AMEX. Our analysis reveals that the beta exposure of stock returns to macroeconomic uncertainty estimated by the LSTM model, exhibits a monotonic increase across different uncertainty regimes, even after adjusting for the Fama and French (2015) FF5 model. Consistent with existing literature and our empirical findings using a linear uncertainty measure detailed in Chapter 3, these beta loadings are negative for stocks in the lower quintiles and positive for those in the higher quintiles, based on their exposure to uncertainty. However, we observe the same result as Chapter 3 and divergence from the traditional literature: the risk premium on macroeconomic uncertainty by LSTM is negative during periods of decreasing conditional volatility (calm periods) and positive during periods of increasing conditional volatility (turbulent periods).

These findings are corroborated by estimates of the uncertainty risk premium derived from Fama-MacBeth cross-sectional regressions applied to an FF5 model augmented with the LSTM-based uncertainty proxy, as well as through the 3-Stage latent factor regression methodology from Giglio and Xiu (2021). We observe a negative risk premium for the uncertainty proxy during calm periods (2003-2006 and 2012-2016) and a positive risk premium during turbulent periods (1998-2002, 2007-2011, and 2017-2022).

The other main objective of this research is to offer a thorough comparison between our LSTM-based macroeconomic uncertainty measure and the linear measures proposed by Jurado et al. (2015), within the context of asset pricing theory. This comparison is conducted in two primary ways. First, we evaluate the risk premium perspective by comparing the returns of stocks with varying loadings on both types of uncertainty and their associated risk premiums during calm and turbulent periods. Specifically, we identify calm and turbulent months formally based on the CFNAI index as Section 3.3.4 and the median of the uncertainty measure, as suggested by Bali et al. (2017), then compare differences in stock returns and risk premiums when uncertainty shocks are captured by both measures. Secondly, we assess the explanatory power of the uncertainty measures on cross-sectional stock returns using the asset pricing model comparison technique proposed by Barillas and Shanken (2017, 2018); Barillas et al. (2020). Including these uncertainty measures as an additional risk factor in the FF5 model, we compare the predictive ability of these 6-factor models against the FF5 model and against each other in both nested and non-nested settings.

The results of these comparisons reveal that stocks with higher loadings on macroeconomic uncertainty estimated by the LSTM model exhibit higher returns during turbulent periods and lower returns during calm periods. The return disparity between portfolios with the highest and lowest uncertainty loadings is more pronounced when using the LSTM-based measure as a proxy for macroeconomic uncertainty, compared to the linear measure. Consequently, the LSTM-based measure yields positive risk premiums during turbulent months and negative premiums during calm months, with larger absolute values. Investors can choose to pay higher insurance during calm periods to secure greater compensation in turbulent periods when using the LSTM-based measure to hedge against macroeconomic uncertainty. Additionally, during the 2007-2011 financial crisis and the calm evaluation periods of 2003-2006 and 2012-2016, the LSTM-based uncertainty measure enhances the explanatory power of the FF5 model effectively compared to the linear measure, as evidenced by the significantly larger Sharpe ratio and higher posterior probability in Bayesian model comparison tests. This improvement stems solely from the application of machine learning models to measure uncertainty, which excel in handling large-dimensional data and nonlinear changes compared to linear models.

The structure of this Chapter is as follows: Section 4.2 reviews the literature on using machine learning models for asset pricing and outlines the theoretical basis for assessing and pricing macroeconomic uncertainty. Section 4.3 analyzes how LSTM-based macroeconomic uncertainty is priced across the cross-section of stock returns over five non-overlapping subsamples between 1998 and 2022, characterized by calm and turbulent episodes. Section 4.4 presents the model comparison exercise to statistically evaluate the risk premiums associated with LSTM and linear frameworks, comparing the explanatory power of the LSTM-based measure against linear measures, which also include financial uncertainty. Finally, Section 4.5 offers conclusions.

4.2 Theoretical Background

4.2.1 Literature Review

This section offers a broader review of the applications of machine learning models in the field of asset pricing. It categorizes these methods into feature-based machine learning approaches and end-to-end deep learning approaches.

In the category of feature-based methods, most approaches focus on dimensionality reduction for high-dimensional data. For example, the 3-Stage latent factor regression used in this context is a dimensionality reduction framework that employs Principal Component Analysis (PCA). PCA is a statistical technique used to simplify complex data sets by transforming them into a series of linearly uncorrelated variables called principal components. This helps address the challenges posed by the interaction of multiple variables in traditional financial data analysis and prediction. PCA assists in resolving multicollinearity issues in multivariate regression equations for financial time series data, making it an effective method for handling large multivariable datasets.

Lettau and Pelger (2020) extends PCA by combining it with arbitrage pricing to explain the non-arbitrage factors in data, offering a model for expected excess returns on high-dimensional financial panel data. It considers pricing errors in expected returns, and by adding a non-arbitrage penalty term to PCA, it addresses the low signal-to-noise ratio in financial data while acquiring information related to kernel pricing. Ait-Sahalia and Xiu (2017) proposes a method to estimate common factors in high-dimensional data using high-frequency data, even as sampling frequency and covariance matrix dimensions increase. This approach represents the covariance matrix of stock portfolios as a low-rank common structure with a sparse residual matrix. The factors revealed by PCA explain a larger proportion of asset returns and provide better out-of-sample estimation than those from observable portfolio factors, such as market portfolios, Fama-French portfolios, or ETF portfolios. These studies illustrate PCA's strength in capturing the most meaningful sub-signals amidst random signals by including them in uncorrelated random variables with gradually decreasing variance, proving effective for high-frequency, high-dimensional financial data. PCA can decompose matrices of large panel datasets to extract latent factors without the need for pre-specification of observable common factors, reducing expertise requirements for researchers.

Similar to PCA, Singular Value Decomposition (SVD) is another effective method for dimensionality reduction widely used in machine learning. It is applied in areas such as feature decomposition, compression, noise reduction, recommendation systems, and natural language processing. Wang (2017) proposes a tunable SVD algorithm that extracts financial factors from stock return models. By adjusting algorithm parameters, it reduces sensitivity to data matrix errors, enhancing the model's robustness and effectiveness.

Independent Component Analysis (ICA), a more recent mathematical tool, is an extension of PCA. While PCA focuses on second-order statistics of financial data, ICA is better at identifying intrinsic features in higher-order, non-Gaussian distribution data. It effectively uncovers relationships among random variables, observed data, and hidden variables. For stock market data, identifying the most independent signals can minimize risks and achieve optimal portfolio allocation. Back and Weigend (1997) applies ICA to multivariable financial time series in stock investment portfolios. ICA's main idea is to map observed multivariate time series linearly into an independent component space. This captures features that cause significant changes in stock prices and those with high frequency but little contribution to overall stock levels. Noise size depends on amplitude rather than frequency, providing a new perspective for understanding mechanisms impacting stock market data.

To handle the high dimensionality of financial market data, a common approach is to first reduce high-dimensional data into a lower dimension. This reduced data is then classified or predicted, and subsequently transformed back into high-dimensional data to address the issues of dimensionality and overfitting. Autoencoders and Support Vector Machines (SVM) are examples of such methods, using nonlinear approaches for dimensionality reduction, unlike PCA.

In Gu et al. (2021), an autoencoder is used to compress returns into a low-dimensional set of factors. This introduces a conditional autoencoder model for individual stock returns, allowing asset characteristic covariates to have a nonlinear effect on factor exposure. Nonlinear features are mapped through covariate neural networks and expressed in beta form, creating an autoencoder with economic insights. By embedding neural networks, the traditional autoencoder is enhanced under beta constraints. Suimon et al. (2020) discusses a flexible curve model to describe yield rates, providing insights into government bond market pricing. In Huang (2012), a stock selection method using Support Vector Regression (SVR) and Genetic Algorithm (GA) is proposed. This approach ranks stocks based on returns using SVM, selecting the top-ranking stocks to form a portfolio. The Genetic Algorithm is used for feature selection and global optimization within the parameter space to find the optimal parameter solution, thus identifying the best subset of input variables.

Besides handling data dimensionality, Bayesian classification is a probability-based approach. Fulop and Yu (2017) implements a Bayesian learning method, assuming that during normal periods, asset prices divided by dividends follow a dynamic regression process around a long-term mean. In bubble periods, asset prices fluctuate wildly, and Bayesian learning is used for real-time joint estimation of the model's latent states and parameters. An empirical analysis on the S&P 500 shows this method effectively detects market bubbles.

Machine learning classification theories are also widely applied in asset pricing. Nti et al. (2019) examines how macroeconomic variable fluctuations affect liquidity in Ghana's stock market, proposing a model that uses these variables to predict the stock market with a hybrid RF and RNN machine learning model. This approach addresses multicollinearity issues of macro factors in market predictions. Krauss et al. (2017) evaluates the effectiveness of deep neural networks, gradient boosting trees, and random forests for statistical arbitrage on the S&P 500 dataset. The empirical results indicate that random forests outperform gradient boosting trees and deep neural networks.

Compared to feature-based machine learning methods, end-to-end approaches can extract features directly from large amounts of raw data, representing data from various sources in matrix form. By merging these vectors as a unified input, they enhance data diversity. Deep learning excels in extracting complex features and fitting nonlinear functions, with some methods eliminating intricate internal logic design. The evolution of financial big data has made end-to-end methods well-suited for big data and high-frequency financial data analysis.

Gudelek et al. (2017) proposes using a Convolutional Neural Network (CNN) to predict ETF price changes. It generates image snapshots within a limited time window each day, using common trend indicators, momentum indicators, and fundamental analysis metrics as input features. The use of multiple ETFs expands the dataset size and reduces information variance. To explore correlations across different markets, Hoseinzade and Haratizadeh (2018) presents a specialized CNN framework applicable to various data sources, such as the S&P 500, NASDAQ, Dow Jones NYSE, Dow Jones DJI, and Russell Index. These are combined into a three-dimensional tensor table, allowing each prediction model to use all information within the tensor to forecast a specific market's future trends. Kim and Kim (2019) employs different representations of the same data, generating stock time series and stock chart images (including candlestick charts, high and low price line charts, and volume bar charts). By integrating LSTM and CNN models, it predicts stock prices and creates model variations to adapt to changing data. These studies reveal that while CNN's predictive performance is average, it effectively integrates diverse data sources from multiple perspectives, stabilizing the model and enhancing algorithm robustness.

Reinforcement learning is a subfield of machine learning focused on making decisions based on environmental inputs to maximize expected returns. Cao et al. (2021) examines using reinforcement learning to derive optimal hedging strategies for derivatives when transaction costs are involved. Cao and Zhai (2022) develops a deep neural network-enhanced recurrent model to estimate both short-term and long-term trading impacts, evaluating the effect of prices on stock returns.

Advances in computing have expanded data sources. Multi-source textual data often feature high redundancy, low informational density, large volumes, and high frequency, making them useful for monitoring market sentiment, public opinion analysis, and investor viewpoints. Ding et al. (2015) represents documents as dense vectors, utilizes an event embedding neural tensor network for training, and applies a deep convolutional neural network to semantically combine input event sequences to predict stock price changes. As large language models rapidly evolve, they not only handle text but also have some capacity for images and structured data. Increasingly, research aims to leverage multimodal data to dynamically capture complex relationships between key events and market responses, enhancing the accuracy and interpretability of return predictions.

It's important to note that machine learning methods often face a trade-off between accuracy and model complexity. Simpler models like linear regression are highly interpretable but cannot handle complex data relationships. In contrast, deep neural networks can manage these complexities but may become "black boxes," relying on metrics like accuracy rather than traditional asset pricing evaluation standards. Investors, however, prefer clear explanations of investment logic to navigate changing market conditions. Our research aims to compute pricing factors using machine learning models as introduced in Chapter 2, while maintaining asset pricing on a linear factor model basis. This approach retains some explanatory power for understanding risk premiums.

4.2.2 Pricing LSTM-based Uncertainty

This section outlines the methodology used to price aggregate uncertainty in the cross-section of risky assets, alongside data and variable definitions. Two main approaches commonly identified in the literature for pricing the cross-section of stock returns are Fama-MacBeth (Fama and MacBeth, 1973) cross-sectional regressions and the creation of a mimicking portfolio that optimizes the correlation between the pricing anomaly (e.g., climate risk, macroeconomic uncertainty) and a set of base asset returns. More recently, Giglio and Xiu (2021) introduced a three-stage method to price the cross-section of stock returns. This approach builds upon the Fama-MacBeth method by incorporating an initial stage that estimates a set of unobservable pricing factors using principal component analysis. This methodology merges principal component analysis with two-pass cross-sectional regressions to yield consistent estimates of the risk premium for any observed factor. In this research, we apply all these methods to compute the risk premium on macroeconomic uncertainty as determined by the LSTM model in Section 4.3.

We denote r_t^i as the excess return over the risk-free rate for stock i at time t. These returns are modeled under a linear multifactor framework, where asset returns are influenced by innovations in the pricing factors $\{f_{kt}, u_{kt}\}$ and an idiosyncratic component ε_t^i:

$$r_t^i = a^i + \sum_{k=1}^{K} \beta_k^i \left(f_{kt} - E[f_{kt}]\right) + \beta_u^i \left(u_t - E[u_t]\right) + \varepsilon_t^i, \tag{4.1}$$

where a^i denotes the risk premium for the risky asset i and β_k^i represents asset i's risk exposure to the $k = 1, \ldots, K$ risk factors. Similarly, β_u^i denotes the asset's sensitivity to uncertainty risk, capturing its exposure to the uncertainty factor; $f_{kt} - E[f_{kt}]$ represents the factor innovations, while $u_t - E[u_t]$ reflects innovations in the uncertainty risk factor. In this basic setup, risk exposures are considered constant over time. However, this assumption is relaxed in the empirical application by updating the pricing methods on a monthly basis using rolling regressions. The risk premium can be expressed as:

$$a^i = \sum_{k=1}^{K} \beta_k^i \lambda_k + \beta_u^i \lambda_u, \tag{4.2}$$

where λ_k is the price of risk for each of the k factors, and λ_u is the price of macroeconomic uncertainty risk. Here, β_k^i and β_u^i are viewed as the quantity of risk associated with the respective pricing factors.

To examine how LSTM-based macroeconomic uncertainty is priced across the cross-section of stock returns, we consider a comprehensive sample of common stocks traded on the NYSE, AMEX, and NASDAQ exchanges from 1993 to 2022. We exclude stocks with a share price less than \$5 or exceeding \$1,000. The dataset comprises monthly return and volume data sourced from CRSP, with stock returns adjusted for delisting to mitigate survivorship bias. Differing from Ang et al. (2006), our analysis exclusively uses monthly returns to align with the frequency of our uncertainty measures.

The sample data is segmented into five evaluation periods of roughly equal length, designed to encompass various uncertainty regimes. The first period captures the dotcom stock market bubble, peaking on Friday, March 10, 2000. This era experienced a substantial rise in stock prices, driven by the widespread adoption of the World Wide Web and the Internet, which fueled the rapid valuation growth of new dot-com startups. The second period, from 2003 to 2006, is marked by low and stable inflation and minimal financial market volatility, coinciding with the long economic expansion since World War II. Unfortunately, the Great Moderation's tranquility was disrupted by the 2007-08 financial crisis, referred to as the Great Recession, forming the third period in our analysis from 2007 to December 2011. This era also includes short-lived financial turmoil, such as the European sovereign debt crisis from 2009 into the mid-to-late 2010s, characterized by several Eurozone states (Greece, Portugal, Ireland, and Cyprus) facing challenges in repaying or refinancing government debt or bailing out over-indebted banks. The fourth period, spanning 2012-2016, is defined by booming stock markets and relative financial market stability. The final period, from 2017 to 2022, encompasses a global economic recession caused by the COVID-19 pandemic outbreak in February 2020, followed by prolonged economic slowdown, manifesting as stagnation in economic growth and consumer activity. A more thorough investigation into the definition of calm and turbulent evaluation periods and the corresponding results are presented in Section 4.4.1.

In line with Bali et al. (2017), we estimate the uncertainty beta for each stock and each month within the different evaluation periods by conducting monthly rolling regressions of excess stock returns on the LSTM-based measure of macroeconomic uncertainty, while controlling for the FF5 factors, over a fixed 60-month window. Accordingly, the first regression to determine the pre-formation beta loadings spans from January 1993 to December 1997. The monthly risk-free rate, represented by the one-month US Treasury bill rate, along with the FF5 pricing factors, is sourced from Kenneth French's data library. These factors include the excess market return (MKT), the size factor (SMB), defined by a portfolio return derived from a small-minus-large investment strategy, the value factor (HML), representing a high-minus-low portfolio that leverages differences in profitability based on stocks' book-to-market ratios, the profitability factor (RMW), defined by a robust-minus-weak portfolio return, and the investment factor (CMA), constructed as a conservative-minus-aggressive portfolio return.

Our estimation of macroeconomic uncertainty using an LSTM nonlinear framework builds upon the econometric-based measures from Jurado et al. (2015) and Ludvigson et al. (2021), who distinguish between macroeconomic and financial uncertainty. Jurado et al. (2015) developed a factor-augmented predictive regression model utilizing a comprehensive set of macroeconomic and financial time series, which includes variables such as real output, income, employment, consumer spending, price indexes, bond and stock market indexes, and exchange rates. Economic uncertainty is interpreted as the conditional volatility of the unpredictable component of the future value of each series. These individual conditional volatilities are then aggregated into a macroeconomic uncertainty index for horizons of one, three, and twelve months. Financial uncertainty is similarly derived, relying on a broad set of financial variables, as detailed in Ludvigson et al. (2021). In our previous research presented in Chapter 2, we proposed a nonlinear framework employing two LSTM models with a recurrence structure, in contrast to the linear factor-augmented predictive regression, to forecast these macroeconomic and financial time series. We adhere to the same definition of uncertainty, thus modeling the conditional volatility of the unpredictable forecast errors from the LSTM models, as in Jurado et al. (2015).

To gauge economic uncertainty, we adopt approaches similar to those in Ang et al. (2006) and Engle et al. (2020), which emphasize innovations to a predictive state variable for the cross-section of returns, as explained in Bali et al. (2017). Specifically, Ang et al. (2006) mitigate the persistence in the volatility index by using the first differences of the VIX, while Engle et al. (2020) address serial correlation in climate risk news by applying an AR(1) process to capture innovations. Similarly, we employ a stochastic volatility model to characterize the time series of conditional volatilities, specified as follows:

$$\log{(\sigma^m_{t+1})^2} = \gamma^m_0 + \gamma^m_1 \log{(\sigma^m_t)^2} + \eta^m_{t+1}, \eta^m_{t+1} \overset{iid}{\sim} N(0, \tau^m), \tag{4.3}$$

where σ^m_{t+1} represents the conditional volatility of the forecast error term from our LSTM forecasting model applied to macroeconomic time series. The innovations η^m_{t+1} are used as a primary measure of macroeconomic uncertainty; τ^j denotes the standard deviation of these innovations, and γ^m_0 and γ^m_1 are the parameters capturing serial dependence in the conditional volatility process.

Our methodology for deriving uncertainty measures is not designed to completely eliminate the serial dependence in shocks. Instead, a certain degree of time series persistence is essential for capturing time series effects since investors are likely to adjust their risk premiums dynamically only in reaction to shocks that provide information about future economic uncertainty levels. This is particularly significant for the periods analyzed, as we define calm uncertainty regimes as those occasions marked by declining uncertainty, which are depicted by persistent negative shocks in model (4.3). Conversely, turbulent periods are distinguished by sudden surges in uncertainty, represented by sequences of positive shocks in model (4.3).

The dynamics of our LSTM-based macroeconomic uncertainty measure, denoted as η^m_{t+1}, alongside the linear measure from Jurado et al. (2015)—illustrated in Figure 4.1 with blue and red lines, respectively—demonstrates conspicuous time series persistence across different evaluation periods. This persistence is marked by short-term trends that are negative during calm periods and positive during turbulent periods. Notably, although both time series serve as proxies for macroeconomic uncertainty under similar definitions and exhibit high correlation, they diverge in the magnitude of identified uncertainty shocks. The LSTM-based measure indicates a more pronounced shock during the 2007-2008 financial crisis and a milder shock during the COVID-19 pandemic compared to the linear measure by Jurado et al. (2015). In less volatile periods, such as those between these significant shocks, our LSTM-based measure captures greater disturbances. Our prior empirical analysis in Chapter 2 provides evidence that the LSTM-based measure possesses comparable or superior explanatory and predictive capabilities concerning key macroeconomic variables than the linear measure by Jurado et al. (2015). We will first conduct an empirical analysis of our LSTM-based macroeconomic uncertainty, examining its pricing implications in the cross-section of returns, followed by a thorough comparison between the LSTM-based and linear measures within the context of asset pricing theory.

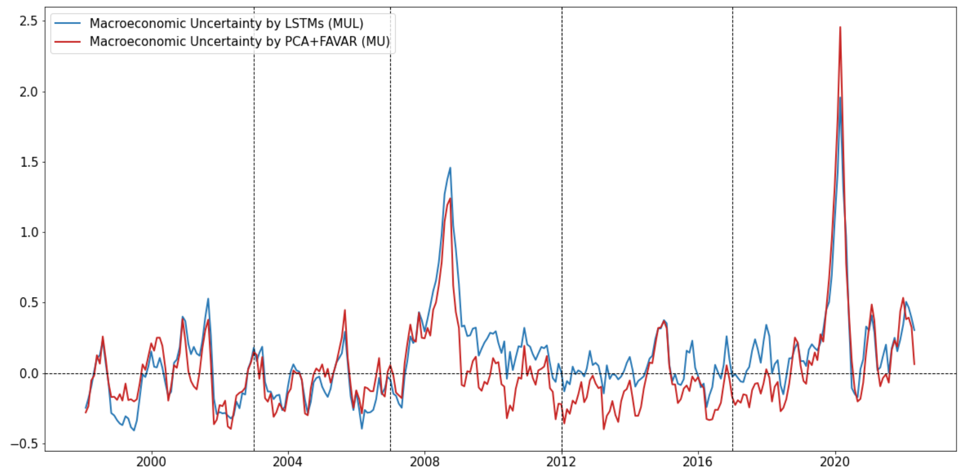


FIGURE 4.1: One-month-ahead innovations in LSTM-based macroeconomic uncertainty

The sequences displayed are derived from model (4.3), using the conditional volatility measures generated by our LSTM-based nonlinear framework and the linear framework from Jurado et al. (2015), spanning the period from 1998 to 2022. The blue line represents the LSTM-based macroeconomic uncertainty measure, while the red line corresponds to the linear macroeconomic uncertainty measure.

4.3 Empirical Analysis

To investigate the role of macroeconomic uncertainty measured by LSTM as a factor in asset pricing models, and to assess whether the risk premiums associated with this measure vary according to the uncertainty regime, we conduct three main analyses in this section: the mimicking portfolio approach, the Fama-Macbeth regression, and the 3-Stage latent factor regression. Initially, we develop an empirical model, considering the LSTM-based macroeconomic uncertainty measure, henceforth denoted as MUL. We adopt a procedure similar to Ang et al. (2006), estimating the pre-formation regression between excess returns on cross-sectional stocks and the Fama-French five-factor model (FF5), augmented with our MUL measure, represented by the innovations in uncertainty. Given the constraints of the monthly frequency in econometric-based uncertainty measurement, we employ monthly rolling regressions over a fixed 60-month window, aligning with the approach from Bali et al. (2017), rather than daily analysis as in Ang et al. (2006). The primary empirical model we examine is the following:

$$r_t^i = \beta_0^i + \beta_{mkt}^i MKT_t + \beta_{smb}^i SMB_t + \beta_{hml}^i HML_t + \beta_{rmw}^i RMW_t + \beta_{cma}^i CMA_t + \beta_{mul}^i MUL_t + \epsilon_t^i, \tag{4.4}$$

where $i = 1, \ldots, n$, and n indicates the number of assets in the cross-section. The focus of this regression is on the sensitivity of excess returns to temporal changes in the uncertainty measure, captured by the uncertainty loading β_{mul}.

In our empirical model (4.4), we employ the FF5 model as the baseline, opting for it over the CAPM model for estimating pre-loadings and instead of the FF3 model used for ex-post loadings by Ang et al. (2006). In contrast, Bali et al. (2017) considers a 7-factor model that includes the market factor MKT, size factor SMB, high-minus-low factor HML, winner-minus-losers factor UMD, liquidity factor LIQ, investment factor $R_{I\backslash A}$, and profitability factor R_{ROE}. While our empirical model accounts for fewer factors, the subsequent 3-Stage asset pricing exercise ensures robustness, effectively addressing all the omitted factors.

4.3.1 Mimicking Portfolio Approach

We categorize firms from the cross-section of returns based on the uncertainty loadings β_{mul}, derived from the monthly rolling time series regression performed on all stocks. The objective of this empirical analysis is to establish a set of base assets with diverse exposure to uncertainty. This methodology is akin to the approaches used by Pástor and Stambaugh (2003), Ang et al. (2006), and Bali et al. (2017), who conduct pre-formation regressions to sort stocks from the cross-section; however, we incorporate our macroeconomic uncertainty measure, enhanced by the LSTM model.

Firms are sorted into five quintiles, with those in the first quintile exhibiting the lowest loadings on uncertainty β_{mul}, and those in the fifth quintile showing the highest loadings. For each quintile, we construct value-weighted portfolios and determine the post-ranking portfolio returns.

Table 4.1 presents summary statistics for each quintile portfolio, sorted by their loadings β_{mul}, as estimated from the preceding 60-month regression window. Panels A through E detail statistics for each of the five evaluation periods defined in our empirical analysis, categorized as either calm or turbulent. The first two columns show the average return and standard deviation for each value-weighted quintile portfolio. Consistent with findings in Chapter 3, average returns increase across quintile portfolios during high uncertainty periods (1998-2002, 2007-2011, and 2017-2022), while they decrease during low uncertainty periods. Similarly, the average returns for the 5-1 portfolio strategy are positive in these high uncertainty periods and negative in low uncertainty periods. These strategies, or the differences between quintile portfolios with the highest and lowest uncertainty loadings, are statistically significant at the 5% level for 2003-2006, 2012-2016, and 2017-2022, and at the 10% level for 1998-2002. The third column provides the time series average of pre-formation uncertainty loadings, β_{mul}, derived from regression (4.4) using the quintile portfolios as test assets, evaluated over 60-month rolling windows starting in 1993. Unlike the cross-section of returns used in previous regressions, quintile portfolios are considered as test assets here. By design, the macroeconomic uncertainty loadings of quintile portfolios increase monotonically, with coefficients varying across evaluation periods. Interestingly, pre-formation beta loadings are negative for the lower quintiles and positive for the higher ones. Similar patterns are reported in Ang et al. (2006) for VIX innovations during different periods, and in Bali et al. (2017) for the raw uncertainty measure of Jurado et al. (2015), as well as in our own empirical findings in Chapter 3. The existence of both negative and positive beta loadings enhances the return on the high-minus-low (uncertainty exposure) portfolio by fully exploiting the spread in average returns across quintiles.

Building on the empirical findings in Chapter 3, which analyzed econometric-based macroeconomic and financial uncertainty, as well as similar analyses for LSTM-based macroeconomic uncertainty shown in Table 4.1, the observed increase in pre-formation beta loadings and the differences in average returns between the top and bottom quintile portfolios imply that the LSTM-based macroeconomic uncertainty potentially has predictive power over risk premiums in cross-sectional stock returns. This potential can be further validated within the framework of an unconditional factor model. We generate an ex-post factor that mirrors the LSTM-based macroeconomic uncertainty by projecting our monthly proxy, MUL, onto a set of base assets. The portfolio weights are obtained as parameter estimates $\hat{\omega}$ from an OLS

TABLE 4.1: Portfolios sorted by exposure to macro uncertainty by LSTM

Rank	Mean	Std. Dev.	Pre-Formation β_{mul}	Pre-Formation β_{rmul}	Post-Formation β_{rmul}
Panel A. 1998-2002					
1	0.2715	5.9779	-9.0026	-18.7070	-15.8581
2	-0.1568	4.8795	-4.6729	-9.3631	-6.8632
3	0.6923	5.4476	-0.1042	-1.3259	-1.8885
4	0.4980	6.2081	1.6360	-0.1418	2.5533
5	1.1718	7.9125	5.8550	14.8512	15.3839
5-1	0.9003				
	[1.7180]				
Panel B. 2003-2006					
1	1.5582	3.2783	-6.0724	-13.9672	-15.5681
2	1.2666	3.1704	-4.3475	-5.4735	-10.3296
3	1.4983	2.6772	0.1941	-0.6732	-1.9466
4	1.4092	2.9852	4.2546	4.3710	8.2784
5	0.8089	3.0123	6.2025	18.0958	11.4056
5-1	-0.7493				
	[-4.2208]				
Panel C. 2007-2011					
1	-0.7199	7.9415	-5.2021	-17.3321	-19.7108
2	0.0254	5.1986	-1.9688	-5.9046	-3.8806
3	0.2274	5.3769	0.4239	-1.4387	0.1743
4	0.8764	5.3243	1.9884	4.2284	2.9051
5	1.3943	5.8010	4.5028	14.0697	13.6552
5-1	2.1142				
	[1.6142]				
Panel D. 2012-2016					
1	1.9618	3.8761	-4.1819	-17.7098	-16.1222
2	1.2968	2.9423	-0.8108	-5.0777	-6.1412
3	1.1210	2.9912	0.5236	-1.1647	-2.2465
4	1.2532	3.1592	2.2360	3.4154	5.2221
5	0.8141	3.7552	6.5251	14.5613	14.7713
5-1	-1.1477				
	[-2.4815]				
Panel E. 2017-2022					
1	0.1648	5.0498	-6.0437	-16.3839	-16.1368
2	0.8086	4.2478	-3.1527	-6.0578	-5.9857
3	1.2992	4.6383	-0.9382	-2.1585	-0.9427
4	1.5721	4.8930	1.5254	4.5042	4.2788
5	2.4368	5.6774	5.6647	14.7809	15.3630
5-1	2.2719				
	[4.3020]				
Panel F. 1998-2022					
1	0.6032	5.5939	-6.1009	-16.9313	-16.1636
2	0.6249	4.2379	-2.9370	-6.4080	-6.1099
3	0.9504	4.4398	-0.0004	-1.3912	-1.0589
4	1.1161	4.7289	2.2379	3.2472	3.6104
5	1.3616	5.5851	5.7303	15.1490	15.4250
5-1	0.7584				
	[1.3583]				

Note: Every month, we construct value-weighted quintile portfolios by regressing individual stock excess returns on MUL, while controlling for the FF5 factors as specified in equation (4.4). We employ monthly data over 60-month rolling regression periods. Stocks are categorized into quintiles based on the coefficient β_{mul}, ranging from the lowest (quintile 1) to the highest (quintile 5). In the columns labeled Mean and Std. Dev., statistics are presented in monthly percentage terms reflecting total, not excess, simple returns. The row labeled 5-1 indicates the difference in monthly returns between portfolio 5 and portfolio 1. Pre-formation betas represent the value-weighted β_{mul} or β_{rmul} for each quintile portfolio, obtained from rolling regressions using quintile portfolios as test assets. The final column presents the ex-post β_{rmul} factor loadings for each evaluation period, where $RMUL$ corresponds to the factor mimicking portfolio derived from (4.5). We calculate the ex-post betas by augmenting the FF5 model with the $RMUL$ factor. Robust t-statistics, adjusted according to Newey and West (1987), are reported in square brackets. Our analysis divides the sample period into five distinct evaluation periods.

regression between the MUL factor and the vector of quintile portfolio returns X_t:

$$MUL_t = c + \omega' X_t + v_t, \tag{4.5}$$

where v_t represents the error term. The mimicking portfolio returns or mimicking factor is derived by linearly projecting the LSTM-based uncertainty measure onto the base assets, resulting in $RMUL_t = \widehat{\omega}' X_t$. These returns are constructed as excess returns, enabling the coefficients, ω, to serve as weights in a zero-cost portfolio. The multifactor asset pricing model is then obtained by substituting MUL_t with the mimicking factor $RMUL_t$ in equation 4.4, as shown below:

$$r_t^i = \beta_0^i + \beta_{mkt}^i MKT_t + \beta_{smb}^i SMB_t + \beta_{hml}^i HML_t + \beta_{rmw}^i RMW_t + \beta_{cma}^i CMA_t + \beta_{rmul}^i RMUL_t + \varepsilon_t^i. \tag{4.6}$$

The pre-formation factor loadings β_{rmul} are computed by conducting this regression over the same 60-month rolling windows. The corresponding coefficients, reported in the next-to-last column of Table 4.1, represent the time series average of these pre-formation factor loadings, estimated from each rolling regression using the quintile portfolios as test assets. The pre-formation factor loadings β_{rmul} reveal the same monotonic behavior seen in β_{mul} between the top and bottom quintile portfolios, with β_{rmul} exhibiting significantly greater magnitude.

To evaluate the ex-post factor loadings related to LSTM-based macroeconomic uncertainty, consistent with an unconditional factor model approach, we estimate the post-ranking uncertainty betas over the full evaluation subsamples rather than only using 60-month rolling regressions. The last column of Table 4.1 reveals the post-formation β_{rmul} estimated from the time series regression 4.6 using the five quintile portfolios as test assets, which display similar patterns.

Through the mimicking portfolio approach and corresponding empirical models, we initially demonstrate that the LSTM-based macroeconomic uncertainty has the capacity to predict risk premiums in cross-sectional stock returns. This leads to the construction of the mimicking portfolio or factor for the LSTM-based macroeconomic uncertainty, providing an investable portfolio that adheres to asset pricing theory principles as compared to the index itself. Consequently, the mimicking factor, denoted as $RMUL$, can be further utilized in Fama-Macbeth regressions or latent factor regressions to investigate its time-varying risk premiums, or in comparisons with other econometric-based uncertainty measures.

4.3.2 Fama-Macbeth Regression

Following the observed time-varying cross-sectional relationship between loadings on LSTM-based macroeconomic uncertainty and average stock returns—where stocks with higher uncertainty loadings exhibit lower average returns during calm periods and higher returns during turbulent periods—the LSTM-based macroeconomic uncertainty emerges as a priced factor that varies over time. To further explore these time-varying risk premiums, we employ the Fama and MacBeth (1973) cross-sectional regression:

$$r^i = c + \beta^i_{mkt}\lambda_{MKT} + \beta^i_{smb}\lambda_{SMB} + \beta^i_{hml}\lambda_{HML} + \beta^i_{rmw}\lambda_{RMW} + \beta^i_{cma}\lambda_{CMA} + \beta^i_{rmul}\lambda_{RMUL} + \epsilon^i, \tag{4.7}$$

where λs denote the unconditional risk premiums of the factors. Unlike in existing literature, such as Bali et al. (2017) and Engle et al. (2020), where the state variable (in our case, MUL) is incorporated in the Fama-Macbeth regressions, we opt for the tradable mimicking factor $RMUL$. This approach means that the regressor in (4.7) is β_{rmul} from the first stage regression, rather than β_{mul}. Since $RMUL$ is designed to have maximal correlation with MU, both approaches are expected to yield similar results. However, employing the mimicking factor in the Fama-Macbeth regression provides additional insights. Notably, the uncertainty risk premium λ_{RMUL} can be interpreted as the average excess return of the hedging portfolio $RMUL$, computed over each evaluation subsample. Such an interpretation would not be valid if the cross-sectional pricing regression included the non-tradable raw uncertainty index MUL instead, because it violates the multifactor model assumption that all factors must be investable portfolios.

Table 4.2 presents the unconditional cross-correlations between pricing factors, with Panel A displaying the correlations across the entire sample and Panel B detailing the correlations between $RMUL$ and the FF5 factors within each evaluation subsample. The unconditional correlation between the mimicking factor $RMUL$ and both MKT and SMB is consistently negative throughout subsamples, with the magnitude intensifying during turbulent periods. An exception is noted in the $RMUL$ and SMB correlation for the 2017-2022 period, where these factors appear independent of one another. Conversely, the unconditional correlation of $RMUL$ with other pricing factors, such as HML, RMW, and CMA, is slightly positive, though the sign of these correlations varies across subsamples over time and maintains a small magnitude, as demonstrated in Panel B. This suggests an independence of HML, RMW, and CMA from $RMUL$.

To estimate the factor premiums λs, we begin by constructing a set of test assets with returns r^i_t, ensuring that their factor loadings on macroeconomic uncertainty risk are sufficiently varied to yield informative cross-sectional regressions. Following the methodology of Ang et al. (2006), we sort the cross-section of stocks at the end of each month into five quintiles based on β_{mkt}. Within each quintile, stocks are further sorted into quintiles based on β_{mul}, resulting in 25 portfolios that serve as test assets in the cross-sectional asset pricing model. These loadings, β_{mkt} and β_{mul}, are derived from regression equation (4.4), applied to monthly data over the preceding 60 months. The

TABLE 4.2: Correlations of $RMUL$ with FF5 factors

Period	MKT	SMB	HML	RMW	CMA
1998-2002	-0.0609	-0.2302	0.2086	0.1817	0.0808
2003-2006	-0.4795	-0.3339	0.1339	0.2027	-0.0379
2007-2011	-0.6742	-0.2961	0.0226	0.2663	-0.0087
2012-2016	-0.4413	-0.2130	-0.0934	0.1294	-0.1261
2017-2022	-0.3866	0.0231	0.0129	-0.1727	0.0375
The whole sample	-0.3702	-0.2029	0.0597	0.1584	0.0321

Note: The table presents the correlations between the $RMUL$ factor and the FF5 risk factors. Here, $RMUL$ denotes the monthly return on the mimicking portfolio derived from regression (4.5). The factors MKT, SMB, HML, RMW, and CMA represent the established FF5 factors. The last row shows the unconditional correlations calculated over the entire 1998-2022 period, while the first to fourth rows detail the correlations between $RMUL$ and each of the FF5 factors across the distinct evaluation periods.

Fama-MacBeth procedure is performed in two stages. During the first stage, the betas in (4.7) are obtained from the time series regression (4.4) using the full sample. In the second stage, the risk premia are estimated via the cross-sectional regression (4.7) using monthly data.

Table 4.3 presents the risk premiums associated with the six-factor model based on the Fama and French (2015) pricing model, augmented by the LSTM-based uncertainty mimicking portfolio $RMUL$. The results highlight significant heterogeneity in the risk premium for each factor throughout the evaluation periods. The risk premium on the market portfolio is positive across all periods, except during the calm 2012-2016 period, where test assets with greater exposure to this factor demand a lower expected return compared to those with lower beta exposure. The magnitude of the market factor's risk premium is notably larger during the high uncertainty periods compared to other periods. Likewise, there is substantial evidence of a positive risk premium on the size factor across the evaluation periods. Conversely, the risk premium for the book-to-market factor is negative in the high uncertainty periods of 1998-2002 and 2007-2011 but positive in other periods. The sign and magnitude of the risk premiums on RMW and CMA also vary over time across different evaluation periods.

Table 4.3 reveals that the estimated risk premium for the LSTM-based macroeconomic uncertainty, represented by $RMUL$, is positive during turbulent subsamples and negative during calm subsamples. This suggests that test assets with higher exposure to economic uncertainty tend to have lower expected returns and consequently higher prices in calm periods as opposed to turbulent periods. The variation in the risk premium over time for LSTM-based macroeconomic uncertainty implies the potential for a dynamic hedging strategy by investing in the mimicking portfolio $RMUL$. In turbulent periods, investors require a positive expected excess return from this portfolio to offset the negative expected returns experienced in calm periods, effectively suggesting that investors are willing to pay a premium in calm periods to

TABLE 4.3: Fama-Macbeth factor risk premiums using $RMUL$

	1998-2002	2003-2006	2007-2011	2012-2016	2017-2022	1998-2022
const	0.3970	1.0697	0.6519	1.5305	1.0172	0.9288
	[1.1786]	[4.5677]	[1.8492]	[4.3989]	[3.8327]	[5.6371]
MKT	1.2403	-0.2333	0.3717	-0.1476	0.4232	0.3553
	[1.5800]	[-0.7276]	[0.4198]	[-0.3438]	[0.4687]	[1.0110]
SMB	1.4663	-0.2560	1.0585	2.0994	0.5579	1.0303
	[2.1054]	[-0.4693]	[1.4495]	[3.4436]	[0.5900]	[2.7529]
HML	-0.2531	1.8799	-0.5991	0.7724	-0.3759	0.2102
	[-0.1427]	[2.4974]	[-0.9310]	[2.4893]	[-0.4144]	[0.4393]
RMW	-3.3238	0.0818	-0.9944	-0.8547	-0.1277	-1.0775
	[-2.3143]	[0.0885]	[-1.6478]	[-1.1187]	[-0.1546]	[-2.1492]
CMA	-1.3964	0.1935	-0.6298	0.3226	0.2165	-0.2708
	[-2.5459]	[0.2992]	[-0.9669]	[0.8955]	[0.3675]	[-0.9499]
$RMUL$	0.0564	-0.0230	0.0874	-0.0341	0.0769	0.0356
	[1.1459]	[-1.2200]	[1.3681]	[-2.3187]	[3.4411]	[1.7230]

Note: The table presents the Fama and MacBeth (1973) factor premiums for 25 portfolios sorted initially by β_{mkt} and subsequently by β_{mul}. This is within the framework of the FF5 model, which is enhanced by the $RMUL$ risk factor. The $RMUL$ factor is derived as a mimicking portfolio return from regression (4.5), using LSTM-based macroeconomic uncertainty MUL as a proxy. Robust t-statistics, adjusted according to Newey and West (1987) to account for first-stage estimation in the factor loadings, are reported in square brackets. Each column provides the estimates of the factor risk premiums for different evaluation periods.

hedge against potential turbulence. This finding aligns with the cross-sectional relationship between stock returns and loadings on macroeconomic uncertainty by LSTM, as illustrated in Table 4.2, and is corroborated by similar results reported in Chapter 3, using econometric-based uncertainty measures by Jurado et al. (2015).

TABLE 4.4: Ex-post factor loadings on $RMUL$ for 2007-2011

	Pre-ranking on β_{mul}				
Pre-ranking on β_{mkt}	Low 1	2	3	4	High 5
Low 1	-6.4810	-4.2984	0.5583	7.6363	17.1941
	[-2.7967]	[-1.5591]	[0.2231]	[2.6296]	[6.4268]
2	-15.5090	-2.6191	0.1100	4.0129	15.3480
	[-4.8404]	[-0.9454]	[0.0740]	[1.0517]	[5.4505]
3	-16.5052	-2.3447	1.0935	2.8665	11.5981
	[-5.9711]	[-1.2280]	[0.6069]	[1.3351]	[5.0111]
4	-27.4854	-8.5137	-2.1616	5.0009	10.1644
	[-3.7795]	[-3.0120]	[-0.6048]	[2.4705]	[4.6830]
High 5	-22.3386	-7.4986	1.1574	4.7209	16.5842
	[-6.6430]	[-3.9969]	[0.3596]	[1.2432]	[3.2654]

Note: The table displays the ex-post factor loadings β_{rmul}, derived from the first-stage Fama and MacBeth (1973) regression using the time series specification (4.7). These loadings are applied to 25 portfolios, which are sorted initially by β_{mkt} and subsequently by β_{mul}, utilizing the risk factor MUL as a proxy for economic uncertainty. Robust t-statistics, adjusted according to Newey and West (1987), are provided in square brackets to ensure reliability of the estimates. The analysis covers the sample period from January 2007 to December 2011.

Table 4.4 presents the factor loadings β_{rmul} for each of the 25 base assets used in the first-pass time regression modeled after Fama and MacBeth (1973), during the 2007-2011 subsample. This subsample containing the financial crisis is employed as an illustrative example for the linear RMU in Table 3.4 of Section 3.2.3.3, we choose the same subsample here because the $RMUL$ significantly outperforms the others during the 2007-2011 by the further comparison between LSTM-based and linear measures in Section 4.4.2, warranting further consideration.

A pronounced monotonicity in the uncertainty factor loadings is observed for each quintile of stock returns sorted by market beta. For the quintile portfolios with the lowest loadings on the MKT factor in the top row, the estimated factor loadings β_{rmul} range from -6.48 to 17.19. In contrast, for the quintile portfolios with the highest loadings on the MKT factor in the bottom row, the β_{rmul} values range from -22.34 to 16.58. The presence of monotonicity persists across portfolios controlled by varying MKT factor loadings, indicating a broad dispersion in uncertainty factor loadings across market quintile portfolios. Regarding the five portfolios along the diagonal, from the "High 5 Low 1" to the "Low 1 High 5," the first-stage factor loadings β_{rmul} progress from -22.34 to 17.19, demonstrating a clear increasing pattern. These findings verify that the 25 portfolios, formed on the basis of market returns and LSTM-based macroeconomic uncertainty, exhibit substantial exposure to macroeconomic uncertainty risk, as encapsulated by the mimicking factor $RMUL$.

Comparing with the ex-post factor loadings on RMU in Table 3.4, the ex-post loadings on $RMUL$ shows stronger monotonicity when controlling for the same level of pre-ranking β_{mkt}, that in each row of Table 4.4, the ex-post loadings on $RMUL$ are monotonically increasing along with the increasing pre-ranking β_{mul}. The ex-post loadings on $RMUL$ are larger in magnitude than the ex-post loadings on RMU at the same location in most cases. These results might reflect that the 25 portfolios are more dispersed here, due to the higher uncertainty identified by MUL than MU during 2007-2011, as shown in Figure 4.1.

The risk premium λ_{RMUL} can also be estimated as the time series average of the mimicking portfolio return $RMUL$[1] over the evaluation period. Figure 4.2 illustrates the dynamics of the mimicking portfolio returns from 1998 to 2022. The graphical representation highlights the fluctuations in the uncertainty risk premium over time as well as the dynamics of the mimicking portfolio returns. Green lines depict the returns during turbulent periods, while red lines represent calm episodes. A blue line is drawn using a locally estimated scatterplot smoothing (LOESS) method, fitting a weighted polynomial regression to the mimicking portfolio returns. This nonparametric method adapts to local variations while maintaining the overall trend, providing key insights into the evolution of the uncertainty risk premium. The

[1] A similar methodology can test the effectiveness of time series asset pricing equations using the Fama-MacBeth two-pass regression approach.

estimated risk premium by Fama-Macbeth regression is on monthly basis, then we take the average for 5 evaluation periods for the convenience to discuss the difference between calm and turbulent time. A more detailed comparison is provided in Section 4.4.1. In general, the uncertainty risk premium exhibits time-varying properties, with calm periods typically showing negative returns for the mimicking portfolio and turbulent periods associated with positive returns.

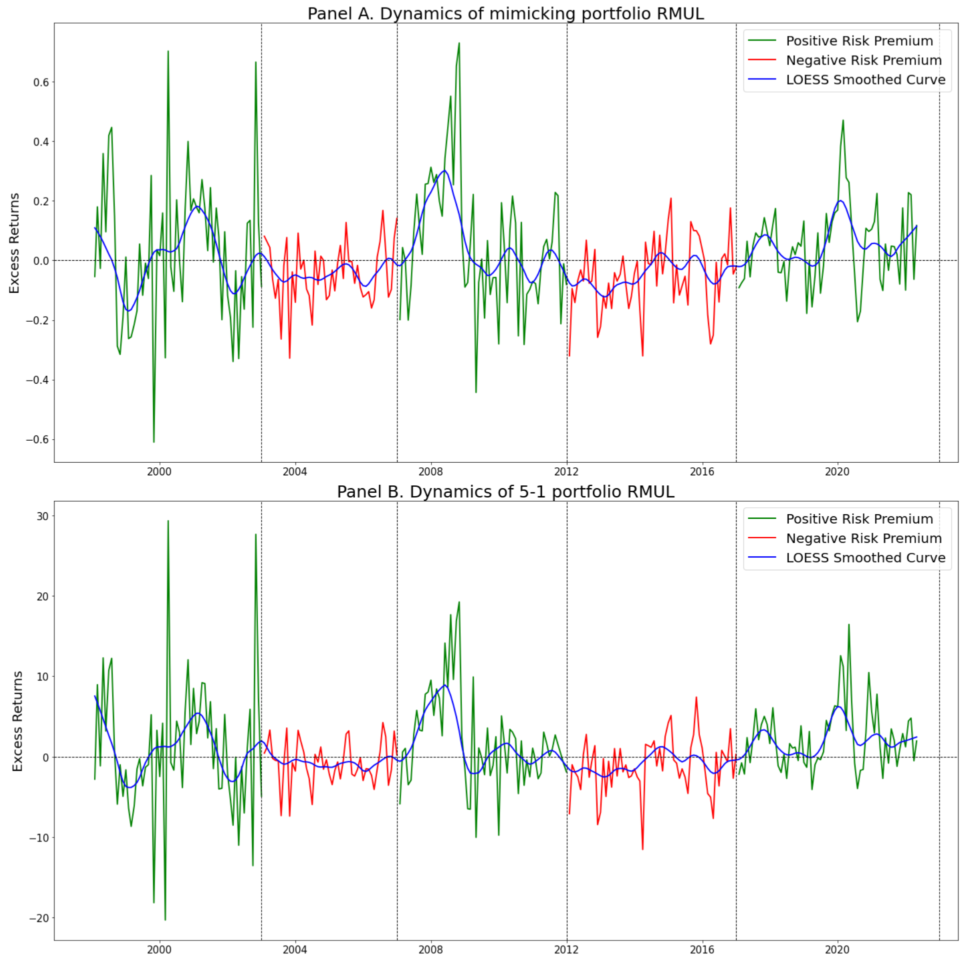


FIGURE 4.2: Dynamics of mimicking portfolio return $RMUL$

The top panel illustrates the dynamics of the mimicking portfolio return $RMUL$, which is derived from the regression equation (4.5). Meanwhile, the bottom panel shows the dynamics of the 5-1 portfolio, created by sorting the cross-section of stock returns into five quintiles based on their β_{mul} rankings. This analysis covers the sample period from 1998 to 2022.

Panel A of Table 4.5 presents the least squares parameter estimates from the time series regression (4.5), using the ex-post quintile portfolios over the entire sample period (1998-2022). Consequently, these estimates remain consistent, irrespective of the breakpoints used to define each evaluation period. When the portfolio weights are duly standardized—by dividing them by the largest estimate (0.028)—the resulting composition of the mimicking portfolio is $(-1, -0.68, -0.15, 0.28, 0.93)$. This yields comparable portfolio weights to those of the 5-1 portfolio, which is constructed as

$(-1, 0, 0, 0, 1)$. In this scenario, the mimicking portfolio can essentially be created using only the bottom and top quintiles. For the sake of comparison, the dynamics of the 5-1 portfolio based on macroeconomic uncertainty are depicted in the bottom panel of Figure 4.2. The dynamics of both portfolios exhibit remarkable similarity, with only minor differences in magnitude observed across different investment strategies.

TABLE 4.5: Properties of mimicking portfolio, $RMUL$

Panel A. Mimicking portfolio weights					
	X_1	X_2	X_3	X_4	X_5
w	-0.0279	-0.0191	-0.0041	0.0078	0.0259
	[-6.2665]	[-2.6624]	[-0.5890]	[1.2785]	[5.9814]
Panel B. Distribution of market capitalization					
	X_1	X_2	X_3	X_4	X_5
1998–2002	0.1187	0.1830	0.2601	0.2636	0.1746
2003–2006	0.1512	0.1606	0.2684	0.2630	0.1568
2007–2011	0.1425	0.2047	0.2687	0.2499	0.1342
2012–2016	0.1038	0.2097	0.2763	0.2756	0.1345
2017–2022	0.1498	0.2484	0.2543	0.2250	0.1224
All Sample	0.1327	0.2036	0.2653	0.2547	0.1437

Note: Panel A presents the OLS parameter estimates for the regression equation (4.5), where the dependent variable is the macroeconomic uncertainty measure MUL. The analysis spans the sample period from 1998 to 2022, with robust t-statistics provided in square brackets for reference. Meanwhile, Panel B reports the monthly average ratio of market capitalization for each quintile portfolio. For each month, the total market capitalization of stocks within each quintile is calculated and then divided by the entire market capitalization of all stocks in the cross-section for that month, reflecting the relative size of each quintile portfolio within the market.

Panel B of Table 4.5 presents the average market capitalization of the quintile portfolios over the evaluation periods. For each month, the total market capitalization of stocks within a given quintile is calculated and then normalized by dividing it by the total market capitalization of all stocks in that month's cross-section. The monthly average ratio for all months within each quintile is subsequently computed for each subsample and reported in the table. Consistently across evaluation periods, the bottom and top quintile portfolios exhibit smaller market capitalizations compared to the middle quintile portfolios.

4.3.3 3-Stage Latent Factor Regression

Omitted variable bias occurs in standard risk premium estimators when the estimation model fails to incorporate all priced risk sources in the economy. To address biases stemming from an incomplete selection of pricing factors, Giglio and Xiu (2021) propose a three-pass method for estimating the risk premium of an observable factor. This method remains valid even if not all model factors are specified or observed. In this section, we implement this procedure as an alternative to the FF5 model. Instead of the standard pricing factors, we employ a set of unobservable factors obtained via principal component analysis applied to the cross-section of asset returns. The 3-Stage latent factor regression represents a departure from traditional methods that rely on robustness checks, which typically evaluate the sensitivity of estimated risk premiums to various definitions of pricing factors or by incorporating additional variables like momentum, liquidity, investment, and profitability factors as suggested by Fama and French (1993), Carhart (1997), Pástor and Stambaugh (2003), and Hou et al. (2015), among others. Giglio and Xiu (2021)'s method surpasses these techniques in accurately addressing omitted variable biases during the estimation of the risk premium of uncertainty.

The main framework of Giglio and Xiu (2021) is based on a three-stage procedure, firstly extracting the principal components of by returns by conducting the PCA model, where the principal components or so called the latent factors is a full set of factors which could explain the returns without any omitted factors bias; secondly, a cross-sectional regression of average returns onto the estimated latent factors loadings is conducted to estimate the risk premiums of the estimated latent factors; thirdly, the risk premiums of observable factors is estimated by the product of corresponding loadings on the latent factors and the estimated risk premiums of latent factors. The main framework of Giglio and Xiu (2021) assumes the constant loadings and risk premia for the given sample, but allows for time-varying risk premia and risk exposures by conducting certain conditional models. When pricing the time-varying risk premiums of uncertainty in our case, it brings the cost of greater statistical complexity to employ the appropriate conditioning information when modeling. Hence, we follow the same rolling window analysis as our main framework to conduct the 3-Stage exercise and price uncertainty dynamically. Moreover, the noncontinuous stock returns make it impossible to conduct PCA for the whole period and take in account for all available stocks at the same. If we extract latent factors from the continuous stocks only, such latent factors will have limited predictive power to the noncontinuous stocks and there will always be omitted factors existing. Hence, we employ the rolling window analysis and repeat the 3-Stage exercise for each monthly window, which helps avoid the statistical complexity of using conditional models to estimate time-varying risk premiums while also allowing us to include as many noncontinuous stocks as possible. Specifically, the same 60-month rolling window is used which is consistent with our main framework and Bali et al. (2017), then we conduct the same 3-Stage exercise for each monthly window and use the estimated risk premiums of uncertainty as the estimates at the end of each monthly window. Hence, for a given monthly window, the loadings and risk premia is assumed to be consistent as Giglio and Xiu (2021), and the rolling window brings the dynamic to the monthly estimated risk premiums. In particular, our rolling-3-Stage framework is as follows:

For the first PCA stage, suppose $\overline{R}$ is the $n \times T$ matrix of demeaned excess returns, where $T = [t_i, t_{i+1}, ..., t_{i+59}]$ representing the i_{th} rolling windows with 60-month's time spread, and extract the latent factors, $\hat{L}$, $p \times T$ matrix by conducting the PCA of the matrix $n^{-1}T^{-1}\overline{R}^{\intercal}\overline{R}$

$$\hat{L} = T^{1/2}\left(\xi_1 : \xi_2 : ... : \xi_p\right)^{\intercal} \quad \text{and} \quad \hat{\beta}_l = T^{-1}\overline{R}\hat{L}^{\intercal}$$

where $\xi_1, \xi_2, ..., \xi_p$ are the normalized eigenvectors corresponding to the largest $\hat{p}$ eigenvalues. In Giglio and Xiu (2021), the number of latent factors, p, is determined by a consistent estimator $\hat{p}$. For our 3-Stage exercise, we control p either by the cumulative variance explained of these latent factors in PCA within each rolling window or by fixing p across all rolling windows using an ad-hoc approach. And hence the excess returns can be expressed as

$$\begin{aligned} R &= \beta_l \lambda_l^{\intercal} + \beta_l L + U \\ \overline{R} &= \beta_l \overline{L} + \overline{U} \end{aligned}$$

For the second cross-sectional regression stage, the risk premiums of latent factors, λ_l, is estimated by a cross-sectional regression of average returns onto the estimated factor loadings, $\hat{\beta}_l$:

$$\hat{\lambda}_l = \left(\hat{\beta}_l^{\intercal}\hat{\beta}_l\right)^{-1}\hat{\beta}_l^{\intercal}\overline{r}$$

In the third time series regression stage, run a time series regression of mimicking factors of uncertainty, $RMUL$, onto the latent factors to estimate the loadings of uncertainty on latent factors, γ_l.

$$\overline{RMUL} = \gamma_l \overline{L} + \overline{Z}$$

So, the estimator of loadings and fitted value of the mimicking factors of uncertainty can be expressed as:

$$\hat{\gamma}_l = \overline{RMUL}\hat{L}^{\intercal}\left(\hat{L}\hat{L}^{\intercal}\right)^{-1} \quad \text{and} \quad RM\hat{U}L = \hat{\gamma}_l \hat{L}$$

The risk premiums of uncertainty are estimated by the product of their loadings on latent factors and the risk premiums of latent factors:

$$\hat{\lambda}_{RMUL,t_{i+59}} = \hat{\gamma}_l \hat{\lambda}_l$$

which has a more compact form estimator as

$$\hat{\lambda}_{RMUL,t_{i+59}} = \overline{RMUL}\hat{L}^{\intercal}\left(\hat{L}\hat{L}^{\intercal}\right)^{-1}\left(\hat{\beta}_l^{\intercal}\hat{\beta}_l\right)^{-1}\hat{\beta}_l^{\intercal}\overline{r}$$

Table 4.6 provides the time series average of the uncertainty risk premium estimates derived from the Giglio and Xiu (2021) approach for each evaluation period. For each rolling window, we set the number of factors to explain 99.9% of the variance in the cross-section of returns. The average number of factors across rolling windows exceeds 30, ensuring that the uncertainty risk premium estimates remain unaffected by the omission of relevant pricing factors. The results in Table 4.6 corroborate previous findings, indicating that the uncertainty risk premium is negative during calm periods and positive during turbulent periods.

TABLE 4.6: LSTM-based uncertainty risk premia by 3-Stage approach

	1998-2002	2003-2006	2007-2011	2012-2016	2017-2022	1998-2022
RMUL	0.0314	-0.0093	0.0004	-0.0074	0.0119	0.0061
	[14.9506]	[-3.9900]	[0.1624]	[-2.7871]	[5.9639]	[3.7515]

Note: We use a fixed 60-month rolling window analysis consistent with our primary framework and employ the three-stage approach outlined in Giglio and Xiu (2021) to estimate the monthly risk premiums associated with macroeconomic and financial uncertainty. The number of latent factors in each rolling window is dynamically determined by the cumulative variance explained by these factors in the PCA. This table reports the average risk premiums across the five evaluation periods, with the Newey and West (1987) adjusted t-statistics provided in square brackets for robustness check.

Even when we constrain the number of factors to a range between 7 and 9 (capturing 70% to 80% of the variance) and re-estimate the uncertainty risk premium using Giglio and Xiu (2021)'s procedure, we observe similar dynamics for the uncertainty risk premium to those shown in the top panels of Figures 4.2. For ease of comparison, Figure 4.3 features three panels displaying four curves: (i) the LSTM-based macroeconomic uncertainty mimicking factor $RMUL$; (ii) the corresponding LOESS-smoothed curve; (iii) the risk premium estimates derived from the three-stage procedure with 7, 8 and 10 unobserved factors; and (iv) theirs LOESS-smoothed counterpart. Comparing risk premium estimates across models in these panels, we find consistent dynamics. Notably, the FF5 model augmented with the macroeconomic uncertainty factor results in higher returns and lower volatility compared to the dynamically estimated factors model. Moreover, the latent factor asset pricing model identifies two distinct uncertainty episodes within the 2007-2011 evaluation period and exhibits a subtler reaction during the COVID-19 pandemic uncertainty window.

As a concluding observation for this section, it is important to note that while the factor model approach proposed by Giglio and Xiu (2021) mitigates potential biases in risk premium estimates arising from the omission of relevant pricing factors, the static risk premium estimates reported in Table 4.6 may vary with different evaluation period selections, as evidenced by the dynamics illustrated in Figure 4.3.

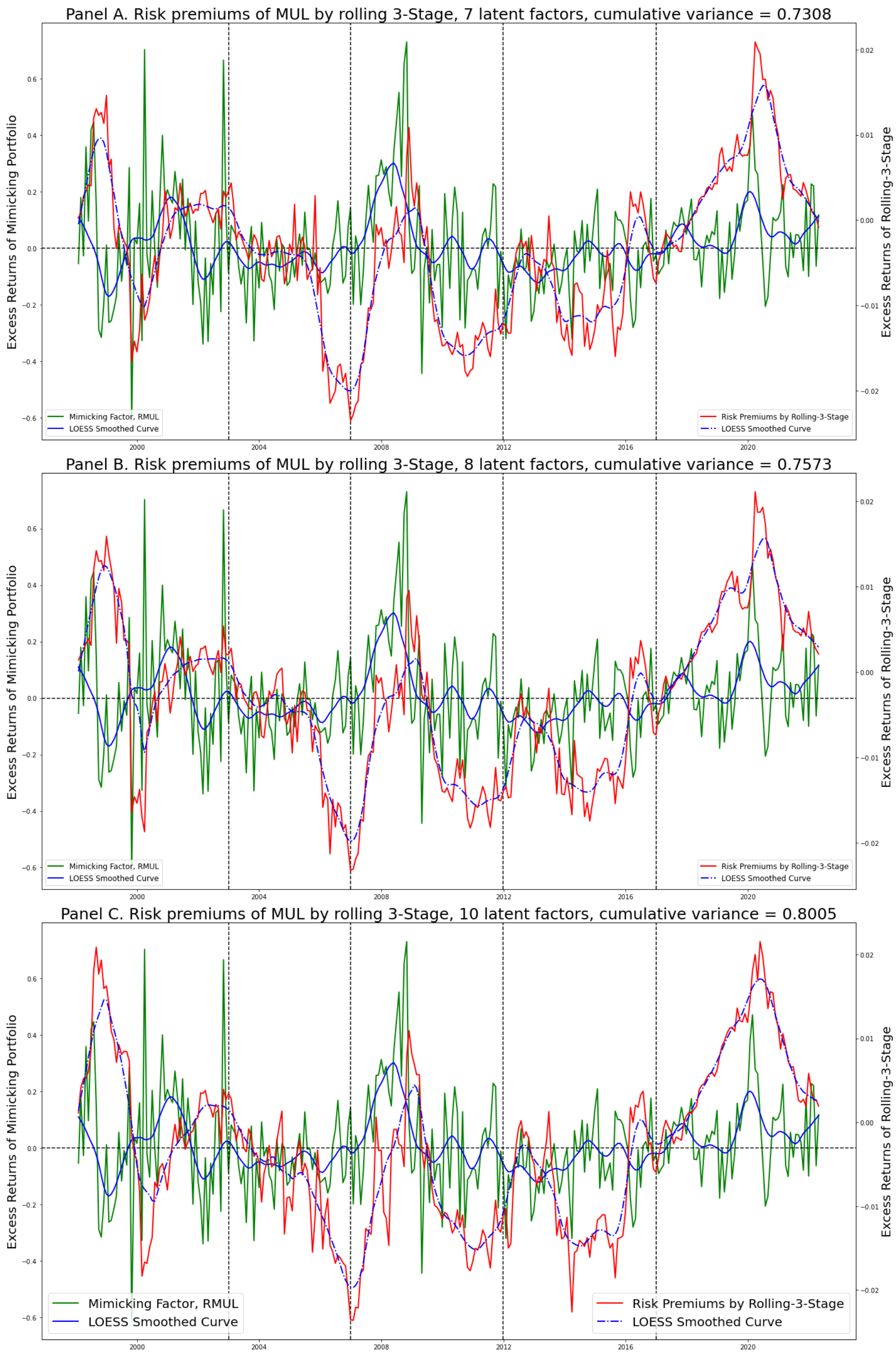


FIGURE 4.3: Dynamics of risk premiums estimated by different approaches

This figure comprises three panels that depict the dynamics of monthly risk premiums on macroeconomic uncertainty using different numbers of latent factors. The green lines represent the mimicking portfolio returns derived from regression (4.5), while the red lines indicate the risk premium estimates obtained through the latent factor model using the three-stage procedure proposed by Giglio and Xiu (2021). Additionally, LOESS smoothed curves are shown for each approach. Estimates from the latter method are calculated using 60-month rolling windows, with the number of latent factors fixed at 7, 8, and 10 across the entire sample period from 1998 to 2022.

4.4 Comparison of Uncertainty by LSTM and Linear Framework

Building on our previous empirical analysis, we use the cross-section of stock returns to assess and price the LSTM-based macroeconomic uncertainty, arriving at two principal conclusions consistent with the findings in Chapter 3. First, the LSTM-based macroeconomic uncertainty measure is a priced factor in the cross-section of stock returns. Second, the associated risk premium is time-varying, being estimated as negative during calm periods and positive during turbulent periods. This pricing factor can be represented by a hedging portfolio, with weights derived from projecting the uncertainty measures onto a set of test assets. While the magnitude and statistical significance of the risk premium vary across different uncertainty regimes, robustness remains with respect to the characterization of the economic uncertainty measure.

To validate these findings and conduct a key empirical comparison between the macroeconomic uncertainty measured by the LSTM-based framework and the linear framework, we evaluate the time-varying risk premiums over calm and turbulent periods, alongside a model comparison exercise in this section. Table 4.7 presents the cross-correlations between the macroeconomic uncertainty measures MUL and MU, as well as their corresponding mimicking portfolio returns $RMUL$ and RMU. The correlations between the uncertainty indices and their corresponding mimicking portfolio returns are relatively high, which aligns with expectations since we employ the same econometric definition and framework as Jurado et al. (2015), albeit employing a time-series machine learning model to measure macroeconomic uncertainty. Despite both measures capturing similar shifts in the macroeconomic information set, slight differences induced by the use of the nonlinear LSTM model persist, which is a focal point of interest for further investigation in this research.

TABLE 4.7: Correlations between LSTM-based and linear measures

	MUL	MU	$RMUL$	RMU
MUL	1	0.8910	0.5729	0.5621
MU	0.8910	1	0.5371	0.5926
$RMUL$	0.5729	0.5371	1	0.7608
RMU	0.5621	0.5926	0.7608	1

Note: The table reports pairwise correlations among the uncertainty measures MUL and MU and the corresponding mimicking portfolio returns $RMUL$ and RMU obtained from regression equation (4.5). The sample period is 1998 to 2022.

4.4.1 Differences in Risk Premiums

In this section, we compare the macroeconomic uncertainty estimated by the LSTM framework and a linear framework by examining the difference in their respective mimicking portfolio returns and risk premiums during the same calm and turbulent months. Such examination involves identifying calm and turbulent months using two methods: an exogenous approach based on the CFNAI index and an endogenous approach relying on the median of uncertainty measures.

As introduced in Section 3.3.4, the exogenous criteria for defining calm and turbulent months employs both the three-month moving average of the monthly index CFNAI-MA3, and the diffusion index of CFNAI. We apply the same criteria here to compare the risk premiums of $RMUL$ and RMU during these defined turbulent and calm periods.

Table 4.8 presents the estimated risk premiums for $RMUL$ derived from cross-sectional regressions during turbulent and calm months. In both turbulent and calm months, the risk premiums for $RMUL$ are significantly positive and negative, respectively, aligning with our main findings and reinforcing the evidence for the time-varying nature of uncertainty's risk premium. The differences in magnitude compared to the Fama–MacBeth estimates arise from the CFNAI index's classification criteria, which only designate extreme turbulence, while less severe uncertainty shocks are considered calm. When comparing these to the risk premiums of RMU from Table 3.12, the $RMUL$ risk premiums are notably larger in magnitude and even negative and significant during calm months, unlike those for RMU. Thus, the LSTM-based measure proves to be a more effective proxy for macroeconomic uncertainty than the linear measure, allowing investors to hedge more effectively against increases in uncertainty.

Regarding the endogenous definition of calm and turbulent months, following Bali et al. (2017), we define calm and turbulent months using the median of uncertainty measures (or innovations in uncertainty). To compare the mimicking portfolio returns and risk premiums of both macroeconomic uncertainty measures under equivalent levels of uncertainty shocks, we first identify calm and turbulent months for each measure based on their medians and then focus on the overlapping months where both measures indicate calm or turbulent periods simultaneously. This approach results in 140 months designated as turbulent and 91 months as calm for both uncertainty measures. It is important to note that while defining calm and turbulent periods based on the median of uncertainty measures may lack the robustness of more formal statistical criteria or macroeconomic indices like those from NBER, our aim is to observe differences in the asset loadings and corresponding average returns when considering macroeconomic uncertainty derived from different frameworks.

TABLE 4.8: Risk premium of $RMUL$ in turbulent vs calm periods, defined by CFNAI

Panel A. turbulent vs calm, defined by CFNAI-MA3 (-0.7)							
Turbulent (21 months)	const	MKT	SMB	HML	RMW	CMA	$RMUL$
Risk premium	0.5763	-0.2389	4.2373	-0.9474	1.3371	1.5678	0.2127
	[1.4580]	[-0.2940]	[2.7960]	[-0.6830]	[0.7510]	[1.2320]	[7.7220]
Calm (271 months)	const	MKT	SMB	HML	RMW	CMA	$RMUL$
Risk premium	1.0328	0.6966	-0.1295	0.1583	0.2604	0.1619	-0.0044
	[33.7270]	[11.0520]	[-1.1020]	[1.4720]	[1.8860]	[1.6420]	[-2.0510]
Panel B. turbulent vs calm, defined by Diffusion in CFNAI (-0.35)							
Turbulent (38 months)	const	MKT	SMB	HML	RMW	CMA	$RMUL$
Risk premium	1.0599	1.3561	1.9998	-1.6173	0.4345	-0.0073	0.1619
	[3.1350]	[2.1360]	[2.0560]	[-1.5360]	[0.4340]	[-0.0070]	[9.5550]
Calm (254 months)	const	MKT	SMB	HML	RMW	CMA	$RMUL$
Risk premium	0.9910	0.5206	-0.0870	0.3325	0.3233	0.3034	-0.0113
	[19.5970]	[5.4820]	[-0.5980]	[2.1100]	[2.1600]	[1.8880]	[-4.4590]

Note: The table reports the factor premiums estimated by cross-sectional regressions in turbulent and calm months for $RMUL$, corresponding to Panels A and B. Turbulent months are defined by the statistical criteria of CFNAI-MA3 < -0.7 or diffusion in CFNAI < -0.35, and all other months are designated as calm. The dependent variable in each cross-sectional regression is the average excess return of 25 portfolios, which are sorted first on β_{mkt} and then on β_{mul}. The regressions follow the FF5 model augmented with the $RMUL$ risk factor, obtained as a mimicking portfolio return from regression (4.5) using MUL as a proxy for economic uncertainty. Robust t-statistics, adjusted according to Newey and West (1987), are reported in square brackets.

We select the five quintile portfolios constructed in Section 4.3 for overlapping months. Table 4.9 presents the summary statistics for these quintile portfolios during the same turbulent and calm months for macroeconomic uncertainty measured by LSTM in Panel A and by the linear framework in Panel B. Compared to the results in Table 4.1, which defined subsamples in an ad-hoc manner and included more noisy data within each evaluation period, these results display stronger monotonicity. In turbulent months, the average returns of stocks within the five quintile portfolios increase monotonically, while they decrease monotonically in calm months as the loadings on macroeconomic uncertainty rise, as indicated by the pre-formation βs of both uncertainty indices and their corresponding mimicking factors. Consequently, the average returns of the 5-1 portfolios are of greater magnitude due to the exclusion of noisy moments. Additionally, when comparing the results for macroeconomic uncertainty as measured by LSTM and the linear framework, the average returns of the 5-1 portfolio associated with LSTM-based macroeconomic uncertainty consistently exhibit greater magnitudes. This means that investors receive higher compensation during turbulent periods and incur greater costs during calm periods when employing the LSTM-based uncertainty measure for hedging against the same source of macroeconomic uncertainty. The LSTM-based hedge portfolio proves more effective, benefiting from the superior capability of machine learning models to manage large-dimensional datasets, as discussed in Chapter 2.

Table 4.9 indicates that stocks with higher loadings on macroeconomic uncertainty tend to have higher average returns during turbulent months and lower returns

TABLE 4.9: Portfolios sorted by exposure to macroeconomic uncertainty in turbulent and calm periods

Panel A. Macroeconomic uncertainty from innovations in LSTM-based measure								
	Turbulent Periods				Calm Periods			
Rank	Mean	Std. Dev.	Pre-Formation β_{mul}	Pre-Formation β_{rmul}	Mean	Std. Dev.	Pre-Formation β_{mul}	Pre-Formation β_{rmul}
1	-0.6076	6.2180	-6.1677	-16.8223	2.4146	4.5519	-6.3613	-16.9576
2	-0.0184	4.5022	-2.9254	-6.4683	1.6942	3.7712	-3.1181	-6.5669
3	0.6804	4.5689	-0.1961	-1.7482	1.2982	4.3803	0.1794	-0.9839
4	1.1825	5.3240	2.1603	3.6560	0.9737	4.0354	2.3873	2.6883
5	2.4003	6.1288	5.5018	14.8837	0.5583	4.9308	5.8962	15.4515
5-1	3.0079				-1.8563			
Panel B. Macroeconomic uncertainty from innovations in Jurado et al. (2015) FAVAR measure								
	Turbulent Periods				Calm Periods			
Rank	Mean	Std. Dev.	Pre-Formation β_{mu}	Pre-Formation β_{rmu}	Mean	Std. Dev.	Pre-Formation β_{mu}	Pre-Formation β_{rmu}
1	-0.5895	6.1664	-6.6633	-16.6363	2.2281	4.3033	-6.8333	-16.3157
2	0.3371	4.7081	-2.0795	-3.7548	1.7237	3.8197	-2.2854	-4.2151
3	0.4172	4.4614	-1.0413	-2.0068	1.2969	3.8889	-1.4014	-2.2974
4	0.9831	5.2293	2.2788	3.9671	0.8080	4.3524	2.5022	3.3955
5	2.3019	6.2883	5.1557	13.2985	0.9047	4.6641	5.5746	13.8967
5-1	2.8914				-1.3235			

Note: Turbulent and calm periods are defined using the median of the uncertainty measure, following Bali et al. (2017), and the overlapping months for both uncertainty measures are selected as the subsamples of interest. Five quintile portfolios are constructed through rolling monthly regressions as described in Section 4.3. The statistics under the columns labeled Mean and Std. Dev are expressed in monthly percentage terms, calculated for the overlapping turbulent and calm months identified for both uncertainty measures. The pre-formation betas are estimated via rolling monthly regressions and reported as the average values corresponding to the turbulent and calm months.

during calm periods. When using the LSTM-based measure instead of the linear measure, the average returns of the 5-1 portfolios maintain the same signs but are larger in absolute value. This suggests that the LSTM-based uncertainty measure is a more effective factor for hedging against macroeconomic uncertainty. To further evaluate this, Table 4.10 reports the risk premiums of both uncertainty measures, as estimated by the Fama-Macbeth regression, during the defined overlapping turbulent and calm months.

Firstly, consistent with our previous findings, the estimated risk premiums for macroeconomic uncertainty measures are positive during turbulent periods and negative during calm periods. This pattern suggests that assets with greater exposure to macroeconomic uncertainty have lower expected returns or higher prices during calm periods compared to turbulent periods, thus confirming the dynamic hedging rationale associated with macroeconomic uncertainty.

Secondly, akin to the average returns of the 5-1 portfolios, and compared to the innovations in Jurado et al. (2015) linear macroeconomic uncertainty measure, the risk premiums of LSTM-based measure are of similar signs but larger in absolute value in both turbulent and calm months. Investors may choose to pay higher (insurance) premia during calm periods to secure greater compensation (positive uncertainty premia) in turbulent periods from investing in the mimicking portfolio $RMUL$.

TABLE 4.10: Risk premiums by Fama-Macbeth in turbulent and calm periods

	Panel A. *RMUL*			
	Turbulent Periods		Calm Periods	
	Mean	Std. Dev.	Mean	Std. Dev.
const	1.0757	3.5184	0.5378	2.9517
MKT	0.4577	8.0696	0.8162	5.9231
SMB	1.9685	8.2595	0.6203	6.6299
HML	0.0240	10.3125	0.7162	8.5224
RMW	-1.8363	10.0376	-0.0955	7.4413
CMA	-0.5040	6.1171	-0.1742	5.8860
RMUL	0.1232	0.2093	-0.0839	0.1569
	Panel B. *RMU*			
	Turbulent Periods		Calm Periods	
	Mean	Std. Dev.	Mean	Std. Dev.
const	0.9559	3.1224	0.7382	2.5752
MKT	0.1688	7.5188	0.6962	4.9592
SMB	2.0185	7.4825	1.7405	6.3356
HML	0.5487	7.9974	0.0503	7.3651
RMW	-1.0099	7.8276	0.1108	6.8114
CMA	-0.0649	7.2549	0.2558	5.4665
RMU	0.1203	0.2322	-0.0752	0.1493

Note: The table provides the Mean and Std. Dev of the monthly risk premiums for macroeconomic uncertainty measures during overlapping turbulent and calm periods, as estimated by the Fama and MacBeth (1973) regression. The risk factors *RMUL* and *RMU* represent the mimicking factors for macroeconomic uncertainty, derived from the LSTM-based measure *MUL* and the linear framework measure *MU*, respectively. These factors are priced individually using different sets of 25 portfolios, each sorted initially by β_{mkt} and subsequently by β_{mul} or β_{mu}, which serve as test assets.

4.4.2 Differences based on Asset Pricing Models

In this section, we perform a model comparison exercise with a dual objective. First, we evaluate the enhanced predictive capability of the LSTM-based macroeconomic uncertainty against the FF5 model, which serves as a benchmark. This assessment as introduced in Section 3.4 is conducted within a nested framework using recent methodologies suggested by Barillas and Shanken (2017), which focus on the difference in squared Sharpe ratios, and Barillas and Shanken (2018), which compare posterior probabilities of each model candidate. Second, we calculate the posterior probabilities for a range of model candidates that incorporate various combinations of the FF5 model, augmented with macroeconomic uncertainty measures obtained from LSTM, as well as econometric-based macroeconomic and financial measures from a linear framework. This procedure follows the implementation guidelines outlined by Barillas et al. (2020).

Barillas and Shanken (2017) demonstrate that to determine the additional value of a pricing factor candidate in predicting the cross-section of stock returns, it is sufficient to regress this factor onto the FF5 model. The factor enhances the benchmark asset pricing model's predictive ability if it is not explained by the FF5 model. This is statistically evaluated by testing the significance of the alpha coefficient (abnormal excess returns not accounted for by the asset pricing model) in the following time series regression:

$$RMUL_t = \alpha + \beta_{mkt} MKT_t + \beta_{smb} SMB_t + \beta_{hml} HML_t + \beta_{rmw} RMW_t + \beta_{cma} CMA_t + \epsilon_t. \tag{4.8}$$

Table 4.11 presents the estimates of this regression across the five evaluation periods. A statistical rejection of the null hypothesis $\alpha = 0$ indicates that the proposed additional factor enhances the benchmark model's predictive ability. Conversely, if the null hypothesis is not rejected, it implies that the FF5 model adequately explains the returns of the mimicking portfolio, suggesting that the additional pricing factor is spanned by a linear combination of the existing risk factors. The results strongly support the added value of the LSTM-based macroeconomic uncertainty, with alpha being statistically significant in most periods. Interestingly, the alpha coefficient's sign also provides insights into the sign of the uncertainty risk premium, as the intercept of the time series regression can be interpreted as the mean return of the mimicking portfolio once the effects of other pricing factors are controlled for.

The comparison mentioned above can be statistically formalized. As per the findings in Gibbons et al. (1989), the standardized squared alpha coefficient can be expressed as the difference in squared Sharpe ratios between the FF5 model augmented with the economic uncertainty risk factor and the standard FF5 benchmark model. In this context, suitable Wald-type tests are sufficient to assess the statistical significance of the uncertainty factor within a nested model framework. Panel A of Table 4.12 presents the test statistic and corresponding p-value across the five evaluation periods. The p-values from the asymptotic tests, based on the difference in squared Sharpe ratios shown in the row *Sharpe Diff vs FF5*, confirm the insights gained from the time series regressions. They statistically demonstrate the value of incorporating macroeconomic uncertainty measured by LSTM as an additional pricing factor, observed consistently across all evaluation periods.

The results above indicate that the macroeconomic uncertainty measured by the LSTM framework serves as an appropriate proxy during the subsamples of 2003-2006, 2007-2011, and 2012-2016, in comparison to the linear framework's uncertainty measure. For the periods 1998-2002 and 2017-2022, while the LSTM-based macroeconomic uncertainty slightly lags behind the linear framework by a difference of only 0.01 in the squared Sharpe ratio, the gap is relatively minor. For the uncertainty measures derived from the linear framework, the optimal proxies for uncertainty depend on the economic context, as shown in Chapter 3. The linear

TABLE 4.11: Regression analysis of $RMUL$ across evaluation periods

	alpha	MKT	SMB	HML	RMW	CMA
Panels A. 1998-2002						
$RMUL$	0.0334	-0.0009	-0.0102	0.0162	-0.0063	-0.0080
	[1.7474]	[-0.1267]	[-0.8170]	[2.6918]	[-0.4989]	[-0.5235]
Panels B. 2003-2006						
$RMUL$	-0.0226	-0.0209	-0.0052	0.0079	-0.0101	0.0008
	[-2.2549]	[-4.3414]	[-1.1930]	[1.9260]	[-2.8135]	[0.1813]
Panels C. 2007-2011						
$RMUL$	0.0925	-0.0367	-0.0029	0.0350	-0.0117	-0.0336
	[3.5327]	[-10.6598]	[-0.4528]	[6.0584]	[-2.0011]	[-4.3744]
Panels D. 2012-2016						
$RMUL$	-0.0323	-0.0166	-0.0038	0.0051	0.0032	-0.0218
	[-2.4230]	[-10.3346]	[-0.9104]	[0.5339]	[0.5626]	[-2.1997]
Panels E. 2017-2022						
$RMUL$	0.0700	-0.0132	0.0064	0.0029	-0.0044	-0.0098
	[4.0803]	[-6.7917]	[0.9432]	[0.6713]	[-1.2393]	[-2.8703]

Note: The table presents the factor loadings from the time series factor regressions (4.8), with $RMUL$ as the dependent variable and the FF5 model factors serving as regressors. The LSTM-based uncertainty factors are derived using the mimicking portfolio approach specified in regression equation (3.5). Panels A to E provide the estimates of these factor loadings across different evaluation periods. Robust t-values, adjusted according to Newey and West (1987), are included in square brackets to ensure the reliability of the estimates.

framework's macroeconomic uncertainty is more effective in explaining the cross-section of returns during economic distress, whereas financial uncertainty is more pertinent during financial crises. However, the LSTM-based macroeconomic uncertainty demonstrates enhanced predictive capacity for cross-sectional returns in both economic distress and financial turmoil periods. This improvement is attributed to the advanced machine learning model used in the measurement, as confirmed by further non-nested comparisons.

To substantiate these claims further, we compare all FF5-augmented models in a non-nested setting. Barillas et al. (2020) demonstrate that under the null hypothesis—assuming equality of the squared maximum Sharpe ratios derivable from two sets of non-nested factors—the difference in squared Sharpe ratios follows a zero-mean Normal distribution. The row *Sharpe Diff vs FF6* in Panel A of Table 4.12 presents the difference in squared Sharpe ratios and corresponding p-values for the FF5+$RMUL$ model compared against FF5+RMU and FF5+RFU. The findings provide compelling evidence in favor of the FF5+$RMUL$ model for the period 2007-2011. This indicates that during the financial crisis, although the financial uncertainty proxy contained more informational content regarding the cross-section of stock returns than the macroeconomic uncertainty[2], the $RMUL$ still outperformed the RFU significantly, owing to the LSTM employed in the measurement.

Panel A also illustrates the Bayes Factor (BF), calculated as the ratio of maximum likelihood functions under the null and alternative hypotheses, and the value of the posterior probability (pp) of the augmented six-factor models compared to the FF5 benchmark. The posterior probability of the alternate FF5 model is given by $1 - pp$. To determine these posterior probabilities, we adhere to the Bayesian procedure outlined in Barillas and Shanken (2018). A widely used diffuse prior for β and Σ, as discussed in Jeffreys (1998), is $P(\beta, \sum) \propto |\sum|^{-(n+1)/2}$, where n denotes the number of test assets. The parameter β signifies the vector of factor loadings in the FF5 specification, and Σ is the covariance matrix of residuals derived from implementing the FF5 asset pricing model to the set of test assets. Under the null hypothesis $\alpha = 0$ (restricted model), the prior for alpha is concentrated at zero. For the alternative hypothesis (unrestricted model), the informative prior for α, conditioned on β and Σ, is presumed to follow a multivariate normal distribution $MVN(0, k\Sigma)$. Here, the parameter k reflects the prior expectation of the squared alpha divided by the residual variance, capturing beliefs about potential deviations from the expected return relationship. Suitable values for k are discussed in Barillas and Shanken (2018). The findings in Table 4.12 lend support to the augmented asset pricing model that incorporates macroeconomic uncertainty measured by LSTM as an additional pricing factor.

The following exercise aims to evaluate the robustness of the results across multiple model candidates in a non-nested framework. Panel B of Table 4.12 considers alternative models: (i) FF5 model, (ii) FF5+$RMUL$, (iii) FF5+RMU, (iv) FF5+RFU, (v) FF5+$RMUL + RFU$, and (vi) FF5+$RMU + RFU$. These models augment the FF5 factor model with macroeconomic uncertainty factors from LSTM and/or the linear framework and financial uncertainty factors. The posterior probabilities among non-nested models are calculated using the method in Barillas and Shanken (2018). For each period, the probabilities assigned to all models sum to one. The results vary across periods, but overall, the conclusions align with previous findings:

Firstly, during calm evaluation periods, the FF5 model augmented with the $RMUL$ pricing factor receives the highest posterior probability, with RFU also adding explanatory power to predict the cross-section of returns, while RMU does not. Given the ad-hoc definition of these periods, calm subsamples may still include some noise. Hence, comparing the posterior probabilities of FF5+$RMUL$ and FF5+$RMUL + RFU$ is insightful. The noisy moments possibly relate more to financial uncertainty in 2003-2006, as including RFU into the FF5+$RMUL$ model improves its explanatory power. However, for 2012-2016, adding RFU to the FF5+$RMUL$ model decreases its

[2]The FF5 model augmented with financial uncertainty from the linear framework had a significantly larger squared Sharpe ratio than FF5 augmented by macroeconomic uncertainty from the same framework.

explanatory power, as indicated by a lower posterior probability. A similar pattern is observed for RMU when comparing posterior probabilities of FF5+RMU and FF5+$RMU + RFU$.

Secondly, for the period 2007-2011, reflecting significant uncertainty, $RMUL$, RMU, and RFU all enhance the FF5 model's explanatory power. The linear framework's macroeconomic and financial uncertainty shows the latter stems mainly from the financial crisis, aligning with real-world events. However, the LSTM-based framework significantly improves macroeconomic uncertainty, offering stronger explanatory power than the linear framework's financial uncertainty. Notably, this enhancement is solely due to the application of machine learning models, as the FF5+$RMUL$ model can still be refined by incorporating RFU.

Thirdly, during the turbulent periods 1998-2002 and 2017-2022, financial crises and macroeconomic distress are pivotal. Financial uncertainty excels in explaining returns during the first subsample, while macroeconomic measures have superior explanatory power in the latter. Despite RMU generally outperforming, the differences in posterior probabilities between $RMUL$ and RMU are relatively small and comparable.

4.5 Conclusion

We observe that stocks with high past exposure to macroeconomic uncertainty measured by LSTM tend to have lower expected returns than those with low exposures during calm periods, and conversely, higher expected returns during turbulent periods. This finding suggests that investors pay an insurance premium in low uncertainty regimes to hedge against heightened uncertainty in turbulent periods. We leverage this evidence to build pricing factors using Fama-Macbeth regression and a 3-Stage latent factor regression. Our analysis of LSTM-based macroeconomic uncertainty yields consistent results with econometric-based measures of aggregate conditional volatility, such as those introduced by Jurado et al. (2015) and Ludvigson et al. (2021), as discussed in Chapter 3.

Our empirical findings robustly support the existence of a time-varying risk premium on macroeconomic uncertainty captured by LSTM. The magnitude and sign of this premium on the cross-section of risky assets shift with the uncertainty regime and the choice of uncertainty proxy, whether LSTM-based or linear. Generally, the LSTM-based macroeconomic uncertainty shows a stronger magnitude, with a negative sign during calm periods and positive during turbulent periods. This conclusion holds true across different methodologies for estimating the risk premium and, to a large extent, even with the presence of unobserved factors, albeit with the preferred approach being the Fama and French (2015) five-factor model enhanced with economic uncertainty risk factors. The relationship between the uncertainty risk

TABLE 4.12: Model comparison based on Sharpe ratio and Bayes tests (RMUL)

	Panel A: Six-factor models vs FF5				
	1998-2002	2003-2006	2007-2011	2012-2016	2017-2022
	FF5+$RMUL$	FF5+$RMUL$	FF5+$RMUL$	FF5+$RMUL$	FF5+$RMUL$
Squared Sharpe	0.1768	0.5180	0.7467	0.2888	0.5288
FF5+$RMUL$ vs FF5	0.0205	0.0684	0.4536	0.1029	0.3612
	[0.0053]	[0.0051]	[0.0001]	[0.0080]	[0.0079]
FF5+$RMUL$ vs FF5+RMU	-0.0133	0.0332	0.3110	0.0937	-0.0140
	[0.0195]	[0.0025]	[0.0057]	[0.0025]	[0.0037]
FF5+$RMUL$ vs FF5+RFU	-0.0286	0.0037	0.0629	0.0748	0.2679
	[0.0003]	[0.0004]	[0.0000]	[0.0013]	[0.0001]
Bayes Factor	0.9962	0.7781	0.0243	0.4935	0.0669
Post. prob. vs FF5	0.5009	0.5624	0.9763	0.6696	0.9373

	Panel B: Non-nested comparison of all models				
	1998-2002	2003-2006	2007-2011	2012-2016	2017-2022
FF5	0.1561	0.1412	0.0074	0.1340	0.0152
FF5+$RMUL$	0.1567	0.1814	0.3056	0.2716	0.2276
FF5+RMU	0.1656	0.1382	0.0242	0.1179	0.2490
FF5+RFU	0.1763	0.1761	0.1945	0.1407	0.0300
FF5+$RMUL + RFU$	0.1703	0.1984	0.3230	0.2210	0.2372
FF5+$RMU + RFU$	0.1715	0.1647	0.1454	0.1147	0.2410

	Panel C: Non-nested comparison of six-factor models				
	1998-2002	2003-2006	2007-2011	2012-2016	2017-2022
FF5+$RMUL$	0.3143	0.3659	0.5829	0.5122	0.4494
FF5+RMU	0.3322	0.2788	0.0461	0.2224	0.4914
FF5+RFU	0.3536	0.3553	0.3710	0.2654	0.0592

	Panel D: Non-nested comparison of seven-factor models				
	1998-2002	2003-2006	2007-2011	2012-2016	2017-2022
FF5+$RMUL + RFU$	0.4933	0.5464	0.6896	0.6584	0.4960
FF5+$RMU + RFU$	0.5067	0.4536	0.3104	0.3416	0.5040

Note: Panel A reports the statistics and p-values of different tests for model comparison across the five evaluation periods. The first row reports the squared Sharpe ratios obtained from the FF5+$RMUL$ asset pricing model. The second row reports the difference of squared Sharpe ratios between the former models and the FF5 benchmark. The p-values in this case are obtained from a nested Wald test using Gibbons et al. (1989) procedure. The fourth and sixth row reports the difference of squared Sharpe ratios between FF5+$RMUL$ and FF5+RMU or FF5+$RMUL$ and FF5+RFU. The p-values in these case are obtained from a Normal test using Barillas et al. (2020) non-nested procedure. Rows 8 and 9 report the Bayes Factor statistics and posterior probabilities of models FF5+$RMUL$ against the FF5 model. Robust p-values are reported in square brackets. Panel B, C&D report the posterior probabilities of different model candidates in non-nested setting. Posterior probabilities are computed using the Bayesian approach and assumptions in Barillas and Shanken (2018). FF5+$RMUL + RFU$ and FF5+$RMU + RFU$ denote the seven factor asset pricing models obtained by augmenting FF5 model with the macroeconomic uncertainty by LSTM or linear framework respectively, and financial uncertainty.

premium and the uncertainty regime appears consistent across evaluation periods, as illustrated by the dynamics of the mimicking portfolio returns.

In terms of comparing our enhanced LSTM-based framework to the linear framework for measuring uncertainty, the predictive ability of the LSTM-based risk premium is statistically supported. There is significant statistical evidence in favor of augmenting the FF5 model with our LSTM-based macroeconomic uncertainty measure.

Chapter 5

Conclusions

5.1 Main Findings

In this thesis, we present an extensive study of uncertainty, focusing specifically on the econometric-based measurement of uncertainty. In addition, we examine its impact on the predictive performance of macroeconomic variables and the returns on financial assets.

In Chapter 2, we address macroeconomic uncertainty measurement by Jurado et al. (2015) and propose enhancements using the LSTM model. Such nonlinear machine learning model is designed to capture complex changes in large-dimensional time-series data. For financial uncertainty measurement, Ludvigson et al. (2021) employ a similar linear framework as Jurado et al. (2015) using a FAVAR model for forecasting financial series instead. Although our nonlinear framework is not applied to financial variables, we offer a baseline example and guidance, providing a foundation for improving the linear forecasting framework, which is entirely feasible for forecasting financial variables and measuring financial uncertainty in a similar manner.

Our approach enhances econometric-based uncertainty measurement by using a nonlinear framework with machine learning models to forecast high-dimensional time series and eliminate predictable components. We employ two LSTM models and a recursive procedure to predict macroeconomic and financial variables. This contrasts with the linear approach using PCA and FAVAR models as in Jurado et al. (2015). We model the time-varying volatility of forecast errors using the same stochastic volatility model to estimate individual macroeconomic series uncertainty, aggregating it into the econometric measure of macroeconomic uncertainty. While our measurement aligns with the definition and assumptions of Jurado et al. (2015), the key difference is our nonlinear forecasting procedure.

Our LSTM-enhanced frameworks exhibit heightened sensitivity to nonlinear changes in macroeconomic uncertainty during both calm and turbulent periods, offering more accurate and dynamic estimates compared to traditional linear models. Specifically, these models produce significantly lower uncertainty estimates during calm periods and forecasts extending 3 to 12 steps ahead. This improved sensitivity allows our models to better capture intricate nonlinear dynamics, outperforming PCA and FAVAR models, which tend to overestimate uncertainty in calm times and underestimate it during turbulence due to their linear constraints.

Our findings highlight the advantages of nonlinear frameworks incorporating LSTM models over linear models, particularly when dealing with large-dimensional data and nonlinear interactions. By focusing on conditional forecast error variance, LSTM models coupled with a recursive procedure demonstrate superior performance, evidenced by smaller forecast errors and reduced macroeconomic uncertainty estimates. Notably, our framework not only maintains but often surpasses the explanatory and predictive power for macroeconomic variables offered by traditional linear approaches, emphasizing the enhanced capability of LSTM models in handling macroeconomic and financial complexities for a more accurate measure of uncertainty.

To validate and showcase the advantages of nonlinear machine learning models over traditional linear models, we propose alternative nonlinear frameworks replacing linear components like PCA or FAVAR with LSTM models. These new frameworks yield uncertainty estimates that sit statistically between our nonlinear approach and Jurado et al. (2015)'s linear model, providing robust evidence for the superiority of nonlinear approaches.

Specifically, in our first robustness check, we replace the linear PCA model with an LSTM autoencoder in the framework for measuring uncertainty. This LSTM autoencoder extracts nonlinear factors which are used in the same linear FAVAR model to forecast macroeconomic variables. The following procedure of estimating uncertainty are the same except for forecasting parts. This setup allows us to directly compare the effectiveness of factor extraction between the LSTM autoencoder and PCA, with all other components unchanged. Results indicate that LSTM outperforms PCA in extracting factors, yielding better uncertainty measures. However, despite improvements, there remains a significant gap when compared to our main framework, suggesting that LSTM also surpasses the FAVAR model in time series forecasting, a conclusion supported by our second robustness check.

In the second check, we substitute one LSTM model in our recursive forecasting framework with a FAVAR model enhanced by nonlinear factors from the LSTM autoencoder. The remaining LSTM model continues to forecast financial variables. This configuration allows a direct comparison between LSTM and FAVAR models for forecasting large-dimensional data. Confirming findings from the first check, the

LSTM proves more effective than FAVAR, as evidenced by more accurate uncertainty measurements from our main framework. Thus, the LSTM model demonstrates superior performance over linear models in both factor extraction and time series forecasting.

In Chapter 3, we transition from measuring uncertainty to evaluating and pricing linear macroeconomic uncertainty by Jurado et al. (2015). This sets the groundwork for the comparison between LSTM-based and linear measures in Chapter 4.

Building on the research by Ang et al. (2006) and Bali et al. (2017) on pricing uncertainty, we identify key limitations. Ang et al. (2006) use VIX innovations as a proxy for uncertainty with daily stock returns, but the VIX is criticized for inefficacy and is often labeled as a fear index. Conversely, Bali et al. (2017) use an econometric-based uncertainty measure from Jurado et al. (2015), but their approach may suffer from autocorrelation due to modeling conditional forecast error variance with time-varying stochastic volatility. Hence, our indicators of economic uncertainty are derived from innovations to well-established econometric-based measures of aggregate conditional volatility, as introduced by Jurado et al. (2015) and Ludvigson et al. (2021).

We find that stocks with higher past exposure to aggregate economic uncertainty tend to yield lower expected returns than those with minimal exposure during calm periods, but higher expected returns during turbulent periods. This finding is interpreted as evidence of an insurance premium that investors pay in low uncertainty regimes to hedge against periods of heightened uncertainty. We further leverage this evidence to develop pricing risk factors using variables that serve as proxies for macroeconomic and financial uncertainty.

Our empirical findings strongly support the existence of a time-varying risk premium associated with economic and financial uncertainty. The magnitude and direction of this premium across a range of risky assets vary according to the uncertainty regime and the choice of uncertainty proxy. Generally, the premium is more pronounced for financial uncertainty than for macroeconomic uncertainty, with a negative sign during calm periods and a positive sign during turbulent periods. This evidence holds true across different methodologies used for estimating the risk premium and, to a large extent, considers the presence of unobserved factors. Our preferred approach is the Fama and French (2015) five-factor model, enhanced with economic uncertainty risk factors. Our empirical results regarding the relationship between the uncertainty risk premium and the uncertainty regime remain robust regardless of the evaluation period, as demonstrated by the dynamics of the macroeconomic and financial mimicking portfolio returns.

The predictive ability of the economic uncertainty risk premium is also supported statistically by the model comparison tests from Barillas and Shanken (2017), Barillas and Shanken (2018), and Barillas et al. (2020). Thus, we find overwhelming statistical evidence in support of augmenting the FF5 model with our measures of macroeconomic and financial uncertainty. The suitability of each uncertainty measure for explaining the cross-section of stock returns depends on the uncertainty regime and the type of shock producing the turbulent episode. Thus, our financial uncertainty measure is found to explain better the cross-section of stock returns than the corresponding macroeconomic uncertainty proxy during the 2007-2011 financial crisis period whereas the latter uncertainty measure is superior during the 2017-2022 period characterized by the COVID-19 pandemic and a sustained period of stagflation.

In Chapter 4, we examine and price our nonlinear estimates of macroeconomic uncertainty, derived from LSTM models, following the same path for the robustness to compare with the linear measures as examined in Chapter 3. The innovations in LSTM-based uncertainty are employ as the proxy and incorporated as a sixth factor within the FF5 model, we apply the same rolling window regression methodology using monthly data to analyze the cross-sectional relationship between stock returns and their loadings on LSTM-based uncertainty. This analysis reveals a consistent time-varying relationship: stocks with higher exposure to LSTM-based uncertainty tend to have lower expected returns during calm periods and higher expected returns during turbulent periods. This finding aligns with the results for linear macroeconomic and financial measures discussed in Chapter 3.

After constructing the five quintile portfolios and the mimicking portfolio for LSTM-based uncertainty, the risk premiums estimated through the Fama-MacBeth regression and the latent factor pricing model further confirm the existence of time-varying risk premiums on LSTM-based uncertainty. These premiums exhibit negative values during calm periods and positive values during turbulent periods. While there are differences in magnitude when compared to estimates derived from linear measures, the consistent change in sign across calm and turbulent periods underscores a robust and persistent relationship. This indicates that the observed time-varying cross-sectional relationship and risk premiums are inherent and stable properties of these econometric-based uncertainty measures, especially when they adhere to the same definitions and assumptions.

In addition to pricing LSTM-based uncertainty, a key component of our analysis involved comparing the LSTM-based uncertainty with linear measures. To do this, we augmented the FF5 model with mimicking factors of different uncertainty indices and conducted comparisons using both nested and non-nested tests. Our primary findings reveal that LSTM-based uncertainty significantly enhances the FF5 model, consistently outperforming estimates from linear frameworks. This holds true across both calm and turbulent periods. Unlike the results in Chapter 3, where the linear measure of macroeconomic uncertainty was more effective during the 2017-2022 period marked by the COVID-19 pandemic and stagflation, and financial uncertainty excelled during the 2007-2011 financial crisis, the LSTM-based macroeconomic uncertainty measure demonstrates a clear advantage across these subsamples. This outcome underscores the superior ability of LSTM models to capture nonlinear variations in large-dimensional data, which translates into impressive predictive power for stock returns and significantly enhances the performance of asset pricing models. Compared to other nonlinear asset pricing methods that use machine learning models, our approach effectively integrates nonlinearity by introducing a nonlinear factor into existing factor models, while maintaining a level of explainability that is crucial within asset pricing theory.

In conclusion, this thesis presents comprehensive research aimed at enhancing the econometric-based measurement of uncertainty by leveraging nonlinear machine learning models. These models offer improved handling of large-dimensional data and capture nonlinear variations more effectively, addressing the limitations inherent in traditional linear models. Furthermore, the study provides detailed evidence of the time-varying risk premiums associated with uncertainty, thus addressing gaps in existing research. Finally, the thesis introduces a novel nonlinear factor, demonstrating its potential to significantly enhance asset pricing models.

5.2 Further Research

In reflecting on the limitations of this research, several key aspects warrant consideration for future exploration. Firstly, while our analysis has primarily concentrated on econometric-based measures of uncertainty, it's crucial to acknowledge the potential benefits of incorporating a broader spectrum of uncertainty measures. Econometric-based methods have been valuable in providing structured quantitative insights. However, they may not fully capture the spectrum of uncertainty present in complex, real-world environments. Future studies could enrich the analysis by including text-based measures of uncertainty, which leverage the vast amount of unstructured data available in news articles, financial reports, and social media. These sources can provide nuanced insights into market sentiment and perceived risk that traditional econometric indicators might miss. Additionally, market-based measures, which directly reflect investor sentiment through financial instruments like options and futures, could offer a more immediate and possibly more accurate gauge of market uncertainty. Secondly, while we have employed the LSTM model, known for its strengths in capturing time-series data dependencies due to its recurrent structure, there is a promising avenue for exploring other advanced machine learning models. Alternative models such as Recurrent Neural Networks (RNNs) and Transformer models offer unique advantages. RNNs, while similar to LSTMs, come with variations that might uncover different aspects of data patterns. Meanwhile, Transformer models have revolutionized the approach to sequence data with their attention mechanisms, allowing them to handle dependencies across vast datasets effectively. Their widespread adoption in large language models underscores their robustness and adaptability, making them worthy of exploration for financial data analysis as well. These models could potentially provide deeper insights into the nonlinear patterns characteristic of economic and financial datasets. Thirdly, our empirical analysis has been limited to data available up through 2022. This restriction is primarily due to data availability at the time of research. Yet, the landscape of global macroeconomic events continues to evolve, with significant occurrences such as the Ukraine War presenting new variables that could impact both uncertainty and asset pricing. The implications of such events are profound, influencing global economic stability and investor behavior. A more detailed examination of these post-2022 developments is necessary, as they could alter risk assessments and financial models significantly. Understanding how these events impact market sentiment and investor confidence could provide critical insights for future economic modeling and risk management strategies. To sum up, expanding the scope of uncertainty measures, exploring advanced machine learning methodologies, and updating empirical analyses to include recent macroeconomic events are all promising directions for future research. Each of these steps would contribute to a more comprehensive understanding of uncertainty's role in financial markets, potentially leading to improved forecasting models and more robust asset pricing strategies. As the financial landscape continues to change, these considerations will help ensure that research remains relevant and responsive to new challenges and opportunities.

For further research, there are two promising directions that offer substantial potential for advancing our understanding of asset pricing and financial market dynamics. First, enhancing the latent factor asset pricing framework proposed by Giglio et al. (2023) through the integration of machine learning models presents an exciting opportunity. Traditionally, the PCA (Principal Component Analysis) model has been employed to extract latent factors; however, recent advancements in machine learning suggest that models like the LSTM-autoencoder could provide significant improvements. As demonstrated in Chapter 2, our comparisons have shown that the LSTM-autoencoder surpasses the PCA model in extracting factors from large-dimensional datasets. This superiority is largely attributed to its enhanced capability for capturing nonlinear changes, which are often prevalent in complex financial data. By adopting a similar machine learning-based approach within the latent factor framework, researchers can potentially achieve a more nuanced and comprehensive analysis of the omitted factors that influence asset pricing. This advancement could lead to improved predictive models and a deeper understanding of the underlying dynamics that drive market behavior.

Second, exploring the idiosyncratic volatility of stock returns presents another intriguing avenue for research. The study by Ang et al. (2006) emphasizes that idiosyncratic volatility is linked to negative risk premiums when accounting for the VIX index, which is commonly used as a measure of market uncertainty. However, the current research framework still has gaps concerning the incorporation of various uncertainty measures. Our findings indicate that econometric-based uncertainty measures can often provide more nuanced and relevant insights than the VIX index, especially during periods of heightened market turbulence. Therefore, substituting the VIX with econometric-based uncertainty measures, alongside other types of uncertainty, to perform an analysis of idiosyncratic volatility could be highly beneficial. By controlling for these alternative uncertainty measures instead of relying solely on the VIX index, we may arrive at more accurate and reasonable outcomes. This approach could offer a deeper understanding of how idiosyncratic volatility interacts with different forms of uncertainty, ultimately leading to more robust asset pricing models. Such research could illuminate the intricate dynamics of market risk and investor behavior, allowing for more sophisticated strategies in risk management and investment decisions.

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tocchapterReferences

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